AdS$_4$ solutions of massive IIA from dyonic supergravity and their simple Chern-Simons duals

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Based on

- A. Guarino, D. Jafferis, OV, Phys. Rev. Lett. **115** (2015) 9 [arXiv:1504.08009]
- A. Guarino, OV, arXiv:1508.04432
- A. Guarino, OV, arXiv:1509.02526
- OV, arXiv:1509.07117

Three puzzles

Puzzle 1: AdS/CFT for the simplest CS?

- A natural way to obtain a CFT_3 is to start from Chern-Simons (CS) theory and add couplings. CS with a simple gauge group $\mathrm{SU}(N)$ and adjoint matter seemed like a good candidate to describe the M2-brane CFT_3.

  However: [Schwarz '04]

- Such CFT_3s cannot preserve maximal supersymmetry.
- The CS term seems to be related to the Romans mass.

Puzzle 1: AdS/CFT for the simplest CS?

- The M2-brane CFT_3 is now known to be described by an ($\mathcal{N} = 6$) CS-matter theory with non-simple gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$ at levels k and $-k$.

  [Aharony, Bergman, Jafferis, Maldacena '08]

- The question remained: do the simplest type of CS-matter theory enjoy AdS_4 duals? Prospects looked bleak. For most of these theories the spectrum has light higher spin operators and exponential growth. [Minwalla, Narayan, Sharma, Umesh, Yin '11]

- Some of these simplest CS-matter theories can still have conventional AdS_4 duals. But none has been found until now.

Puzzle 2: AdS_4 backgrounds of type IIA

- M-theory and type IIB have many known AdS_4 and AdS_5 backgrounds, respectively, in and beyond the Freund-Rubin class.

- This is related to the presence of $\hat{F}_{(4)}$ and $\hat{F}_{(5)}$ in the respective field contents. This is in turn related to the fact that the dual CFTs should be conformal phases of the M2 and D3 brane field theories.

- Massless and massive type IIA also have an $\hat{F}_{(4)}$. However, excluding the massless IIA solutions obtained from M-theory on S^1, essentially only one class of AdS_4 solutions, of massive IIA, is explicitly know analytically.

Puzzle 2: AdS_4 backgrounds of type IIA

- This is the class of direct products $\mathrm{AdS}_4 \times M_6$, where M_6 is nearly-Kähler, and where the IIA forms take values along the nearly-Kähler forms on M_6. [Behrndt, Cvetic '04] This class generalises to M_6 half-flat. [Lüst, Tsimpis '04]

- These manifolds can be thought to be six-dimensional counterparts of five-dimensional Sasaki-Einstein manifolds.

- However, while infinitely many Sasaki-Einstein five-manifolds are known, *e.g.* in the cohomogeneity-one class, [Gauntlett, Martelli, Sparks, Waldram '04] the only explicitly known nearly-Kähler manifolds are homogeneous. Similarly, only homogeneous examples are known in the half-flat case [Koerber, Lüst, Tsimpis '08]

- Generalisations with $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure can be studied [Lüst, Tsimpis '09] but there is no known analytical example in massive IIA (see however [Rota, Tomasiello '15])

Puzzle 3: dyonic $\mathcal{N} = 8$ supergravity

- $D = 4$ $\mathcal{N} = 8$ gauged supergravity often admits continuous or discrete symplectic deformations that respect $\mathcal{N} = 8$ supersymmetry and the gauge group

  [Dall'Agata, Inverso, Trigiante, '12]

- E.g. the covariant derivatives acquire a new coupling to the magnetic vectors proportional to a parameter c,

$$D = d - g\,(\mathcal{A}^{\Lambda} - c\,\tilde{\mathcal{A}}_{\Lambda})\ .$$

- At finite gauge coupling g, electric/magnetic duality is broken and the theory typically becomes sensitive to the symplectic frame specified by c. The physical couplings of the supergravity develop a c dependence.

Puzzle 3: dyonic $\mathcal{N} = 8$ supergravity

- Do these $\mathcal{N} = 8$ gaugings enjoy a string or M-theory origin? For dyonic gaugings with AdS vacua, do these have CFT_3 duals?

- For example, the purely electric $\mathcal{N} = 8$ SO(8) gauging arises from consistent truncation of $D = 11$ supergravity on S^7. [de Wit, Nicolai '87]

- All the solutions of the $D = 4$ theory give rise to solutions in $D = 11$. In particular, the $D = 4$ vacua uplift to $\text{AdS}_4 \times S^7$ M-theory backgrounds.

- Some of these have known CFT_3 duals. Eg, the central vacuum uplifts to Freund-Rubin, which is dual to ABJM.

Our new results

- Massive type IIA supergravity admits an $\mathcal{N} = 8$ consistent truncation on S^6.

- The resulting $D = 4$ theory has a dyonically-gauged $\mathrm{ISO}(7) = \mathrm{SO}(7) \ltimes \mathbb{R}^7$ gauge group.

- All the solutions of the $D = 4$ theory give rise to solutions in $D = 10$. In particular, the $D = 4$ vacua uplift to $\mathrm{AdS}_4 \times S^6$ massive type IIA backgrounds.

  We found the first explicit $\mathcal{N} = 2$ such solution, a new $\mathcal{N} = 1$ solution and recover other solutions. All of these have the $\mathrm{SU}(3) \times \mathrm{SU}(3)$ structure of [Lüst, Tsimpis '09].

Our new results

- Massive type IIA on these $\mathrm{AdS}_4 \times S^6$ backgrounds is dual to the simple CS theories of type discussed above. We gave the first $\mathrm{AdS}_4/\mathrm{CFT}_3$ precision match.

- The $D = 4$ magnetic coupling $m \equiv gc$, the $D = 10$ Romans mass $\hat{F}_{(0)}$ and the CS level k are related by

$$m = \hat{F}_{(0)} = k/(2\pi\ell_s) \; ,$$

where $\ell_s = \sqrt{\alpha'}$ is the string length.

Outline

1. Three puzzles
2. Dyonic ISO(7) supergravity
3. Consistent truncation of massive type IIA on S^6
4. AdS_4 solutions of massive IIA
5. Further discussion

Dyonic ISO(7) supergravity

The $D = 4$ $\mathcal{N} = 8$ tensor hierarchy

The bosonic fields of $D = 4$ $\mathcal{N} = 8$ supergravity come in irreps of $\mathrm{E}_{7(7)}$. The p-forms, $p = 1, 2, 3, 4$, generate a 'tensor hierarchy'. [de Wit, Nicolai, Samtleben '08]

1	metric :	ds_4^2
56	coset representatives :	$\mathcal{V}_{\mathbb{M}}{}^{ij}$,
56	vectors :	$\mathcal{A}^{\mathbb{M}}$,
133	two-forms :	$\mathcal{B}_\alpha$,
912	three-forms :	$\mathcal{C}_\alpha{}^{\mathbb{M}}$,
$\mathbf{133 + 8645}$	four-forms	

The $D = 4$ $\mathcal{N} = 8$ duality hierarchy

The higher-rank forms carry dynamical degrees of freedom, albeit not independent ones. They can be expressed in terms of the lower-rank forms, scalars and metric via the 'duality hierarchy' [Bergshoeff, Hartong, Hohm, Huebscher, Ortin '09]

$$\begin{aligned}
\tilde{\mathcal{H}}_{(2)\Lambda} &= \mathcal{R}_{\Lambda\Sigma}\,\mathcal{H}^{\Sigma}_{(2)} - \mathcal{I}_{\Lambda\Sigma} * \mathcal{H}^{\Sigma}_{(2)} \; , \\
\mathcal{H}_{(3)\alpha} &= -\tfrac{1}{12}\,(t_\alpha)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{N}\mathbb{P}} * D\mathcal{M}^{\mathbb{M}\mathbb{N}} \; , \\
\mathcal{H}_{(4)\alpha}{}^{\mathbb{M}} &= -\tfrac{1}{84}\,(t_\alpha)_{\mathbb{P}}{}^{\mathbb{R}} X_{\mathbb{N}\mathbb{Q}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{M}\mathbb{N}} \Big(\mathcal{M}^{\mathbb{P}\mathbb{Q}} \mathcal{M}_{\mathbb{R}\mathbb{S}} + 7\,\delta^{\mathbb{P}}_{\mathbb{S}}\,\delta^{\mathbb{Q}}_{\mathbb{R}} \Big)\,\mathrm{vol}_4 \; .
\end{aligned}$$

The embedding tensor

In $\mathcal{N} = 8$ supergravity, all effects of the gauging are codified in the embedding tensor $\Theta_{\mathbb{M}}{}^{\alpha}$.

[de Wit, Samtleben, Trigiante '07]

This is subject to

- quadratic constraints, which ensure the consistency of the gauging, and
- linear constraints, which restrict it to the **912** of $E_{7(7)}$.

The dyonic ISO(7) embedding tensor

- To formulate the ISO(7) gauging, it is natural to branch out $E_{7(7)}$ into SL(7), since

$$\mathrm{ISO}(7) \equiv \mathrm{SO}(7) \ltimes \mathbb{R}^7 \subset \mathrm{SL}(7) \ltimes \mathbb{R}^7 \subset \mathrm{GL}(7) \ltimes \mathbb{R}^7 \subset \mathrm{SL}(8) \subset \mathrm{E}_{7(7)} \ .$$

- The embedding tensor of dyonic ISO(7) supergravity takes values in the $\mathbf{28} + \mathbf{1}$ of SL(7): [Dall'Agata, Inverso, '11]

$$\Theta_{[AB]}{}^{C}{}_{D} = 2\,\delta^{C}_{[A}\theta_{B]D} \quad , \quad \Theta^{[AB]C}{}_{D} = 2\,\delta^{[A}_{D}\xi^{B]C} \ .$$

where

$$\theta = \mathrm{diag}(\mathbb{I}_7, 0) \ , \quad \xi = \mathrm{diag}(0_7, 1) \ ,$$

- The ISO(7)-covariant derivatives that follow from this are

$$D = d - g\,\mathcal{A}^{IJ}\,t_{[I}{}^{K}\,\delta_{J]K} + \big(g\,\delta_{IJ}\,\mathcal{A}^{I} - m\,\tilde{\mathcal{A}}_{J}\big)\,t_{8}{}^{J} \ .$$

A restricted duality hierarchy

For the ISO(7) gauging, a restricted duality hierarchy can be identified, still $\mathcal{N} = 8$ but only SL(7)-covariant:

$$
\begin{array}{rcl}
\mathbf{1} & \text{metric :} & ds_4^2 \\
\mathbf{21}' + \mathbf{7}' + \mathbf{21} + \mathbf{7} & \text{coset representatives :} & \mathcal{V}^{IJ\,ij} \;,\; \mathcal{V}^{I8\,ij} \;,\; \tilde{\mathcal{V}}_{IJ}{}^{ij} \;,\; \tilde{\mathcal{V}}_{I8}{}^{ij} \;, \\
\mathbf{21}' + \mathbf{7}' + \mathbf{21} + \mathbf{7} & \text{vectors :} & \mathcal{A}^{IJ} \;,\quad \mathcal{A}^{I} \;,\quad \tilde{\mathcal{A}}_{IJ} \;,\quad \tilde{\mathcal{A}}_{I} \;, \\
\mathbf{48} + \mathbf{7}' & \text{two-forms :} & \mathcal{B}_I{}^J \;,\quad \mathcal{B}^I \;, \\
\mathbf{28}' & \text{three-forms :} & \mathcal{C}^{IJ} \;.
\end{array}
$$

A restricted duality hierarchy

This restricted duality hierarchy has closed supersymmetry transformations and field equations. For example, the Bianchi identities close into

$$D\mathcal{H}^{IJ}_{(2)} = 0 \ , \quad D\mathcal{H}^{I}_{(2)} = m\,\mathcal{H}^{I}_{(3)} \ , \quad D\tilde{\mathcal{H}}_{(2)IJ} = -2\,g\,\mathcal{H}_{(3)[I}{}^{K}\,\delta_{J]K} \ , \quad D\tilde{\mathcal{H}}_{(2)I} = g\,\delta_{IJ}\,\mathcal{H}^{J}_{(3)} \ ,$$

$$D\mathcal{H}_{(3)I}{}^{J} = \mathcal{H}^{JK}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)IK} + \mathcal{H}^{J}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)I} - 2g\,\delta_{IK}\,\mathcal{H}^{JK}_{(4)} - \tfrac{1}{7}\,\delta^{J}_{I}\,(\text{trace}) \ ,$$

$$D\mathcal{H}^{I}_{(3)} = -\mathcal{H}^{IJ}_{(2)} \wedge \tilde{\mathcal{H}}_{(2)J} \quad , \qquad D\mathcal{H}^{IJ}_{(4)} \equiv 0 \ .$$

A restricted duality hierarchy

The duality relations close into

$$\tilde{\mathcal{H}}_{(2)IJ} = -\tfrac{1}{2}\mathcal{I}_{[IJ][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[IJ][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[IJ][KL]}\,\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[IJ][K8]}\,\mathcal{H}^{K}_{(2)} \,,$$

$$\tilde{\mathcal{H}}_{(2)I} = -\tfrac{1}{2}\mathcal{I}_{[I8][KL]} * \mathcal{H}^{KL}_{(2)} - \mathcal{I}_{[I8][K8]} * \mathcal{H}^{K}_{(2)} + \tfrac{1}{2}\mathcal{R}_{[I8][KL]}\,\mathcal{H}^{KL}_{(2)} + \mathcal{R}_{[I8][K8]}\,\mathcal{H}^{K}_{(2)} \,,$$

$$\mathcal{H}_{(3)I}{}^{J} = -\tfrac{1}{12}(t_I{}^J)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} - \tfrac{1}{7}\,\delta^J_I\,(\text{trace}) \,,$$

$$\mathcal{H}_{(3)}{}^{I} = -\tfrac{1}{12}(t_8{}^I)_{\mathbb{M}}{}^{\mathbb{P}}\,\mathcal{M}_{\mathbb{NP}} * D\mathcal{M}^{\mathbb{MN}} \,,$$

$$\mathcal{H}^{IJ}_{(4)} = \tfrac{1}{84}X_{\mathbb{NQ}}{}^{\mathbb{S}}\big((t_K{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)K\,\mathbb{N}} + (t_8{}^{(I|})_{\mathbb{P}}{}^{\mathbb{R}}\mathcal{M}^{|J)8\,\mathbb{N}}\big)\big(\mathcal{M}^{\mathbb{PQ}}\mathcal{M}_{\mathbb{RS}} + 7\,\delta^{\mathbb{P}}_{\mathbb{S}}\,\delta^{\mathbb{Q}}_{\mathbb{R}}\big)\text{vol}_4 \,.$$

Supersymmetric critical points

The dyonic ISO(7) gauging displays a rich structure of critical points, both supersymmetric and non-supersymmetric, all of them AdS. In contrast, the purely electric gauging has no known vacua.

SUSY	bos. sym.	ref.
$\mathcal{N}=3$	SO(4)	[Gallerati, Samtleben, Trigiante '14]
$\mathcal{N}=2$	$\mathrm{SU}(3)\times\mathrm{U}(1)$	[Guarino, Jafferis, OV '15]
$\mathcal{N}=1$	G_2	[Borghese, Guarino, Roest '12]
$\mathcal{N}=1$	SU(3)	[Guarino, OV '15]

Consistent truncation of massive type IIA on S^6

The strategy

In order to find the full embedding of dyonic ISO(7) supergravity in massive IIA, we follow two steps:

- We adapt the de Wit-Nicolai $D = 11$ approach to IIA. The IIA bosonic and fermionic field content and supersymmetry transformations are rewritten with $\mathrm{SO}(1,3) \times \mathrm{SL}(7)$ and $\mathrm{SO}(1,3) \times \mathrm{SU}(8)$ covariance.

- We develop a new technique: exploit the $D = 4$ restricted duality hierarchy.

Type IIA with only SO(1,3) manifest

Under

$$\mathrm{SO}(1,9) \to \mathrm{SO}(1,3) \times \mathrm{SO}(6) \; ,$$

the IIA fields split as

$$
\begin{aligned}
d\hat{s}^2_{10} &= \Delta^{-1}\, ds^2_4 + g_{mn}\big(dy^m + B^m\big)\big(dy^n + B^n\big) \; , \\
\hat{A}_{(3)} &= \tfrac{1}{6} A_{\mu\nu\rho}\, dx^\mu \wedge dx^\nu \wedge dx^\rho + \tfrac{1}{2} A_{\mu\nu m}\, dx^\mu \wedge dx^\nu \wedge \big(dy^m + B^m\big) \\
&\quad + \tfrac{1}{2} A_{\mu m n}\, dx^\mu \wedge \big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \\
&\quad + \tfrac{1}{6} A_{mnp}\big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \wedge \big(dy^p + B^p\big) \; , \\
\hat{B}_{(2)} &= \tfrac{1}{2} B_{\mu\nu}\, dx^\mu \wedge dx^\nu + B_{\mu m}\, dx^\mu \wedge \big(dy^m + B^m\big) + \tfrac{1}{2} B_{mn}\big(dy^m + B^m\big) \wedge \big(dy^n + B^n\big) \; , \\
\hat{A}_{(1)} &= A_\mu\, dx^\mu + A_m\big(dy^m + B^m\big) \; ,
\end{aligned}
$$

Type IIA with only SO(1,3) manifest

- SO(6) can by straightforwardly promoted to SL(6). Then we have the SO(1,3)-covariant field content in SL(6) representations:

$$
\begin{array}{rcl}
\mathbf{1} & \text{metric}: & ds_4^2 \ , \\
\mathbf{21}+\mathbf{6}+\mathbf{1}+\mathbf{20}+\mathbf{15} & \text{scalars}: & g_{mn} \ , \ A_m \ , \ \hat{\phi} \ , \ A_{mnp} \ , \ B_{mn} \ , \\
\mathbf{6}'+\mathbf{1}+\mathbf{15}+\mathbf{6} & \text{vectors}: & B_\mu{}^m \ , \ A_\mu \ , \ A_{\mu mn} \ , \ B_{\mu m} \ , \\
\mathbf{6}+\mathbf{1} & \text{two-forms}: & A_{\mu\nu m} \ , \ B_{\mu\nu} \ , \\
\mathbf{1} & \text{three-form}: & A_{\mu\nu\rho} \ .
\end{array}
$$

- These can be grouped up into SL(7) irreps, too.

Non-linear redefinitions

Further redefinitions are needed so that the p-forms comply with the $D = 4$ transformations dictated by the tensor hierarchy:

- Vectors:

$$C_\mu{}^{m8} \equiv B_\mu{}^m \,, \qquad C_\mu{}^{78} \equiv A_\mu \,, \qquad \tilde{C}_{\mu\, mn} \equiv A_{\mu mn} - A_\mu B_{mn} \,, \qquad \tilde{C}_{\mu\, m7} \equiv B_{\mu m} \,,$$

- Two-forms:

$$C_{\mu\nu\, m} \equiv -A_{\mu\nu m} + C_{[\mu}{}^{n8}\, \tilde{C}_{\nu]nm} + C_{[\mu}{}^{78}\, \tilde{C}_{\nu]m7} \,, \qquad C_{\mu\nu\, 7} \equiv -B_{\mu\nu} + C_{[\mu}{}^{m8}\, \tilde{C}_{\nu]m7} \,,$$

- Three-form:

$$C_{\mu\nu\rho} \equiv A_{\mu\nu\rho} - C_{[\mu}{}^{m8}\, C_\nu{}^{n8}\, \tilde{C}_{\rho]mn} + C_{[\mu}{}^{m8}\, C_\nu{}^{78}\, \tilde{C}_{\rho]m7} + 3\, C_{[\mu}{}^{78} C_{\nu\rho]7} \,.$$

Similar redefinitions were first considered in type IIB. [Ciceri, de Wit, OV '14; Samtleben, Hohm '15]

KK ansatz and consistency of the truncation

The KK ansatz naturally relates the SL(6)-covariant IIA field content to the restricted tensor hierarchy for the ISO(7) gauging and quantities on S^6:

- Vectors:

$$C_\mu{}^{m8}(x,y) = \tfrac{1}{2}\, g\, K^m_{IJ}(y)\, \mathcal{A}_\mu{}^{IJ}(x) \quad , \quad C_\mu{}^{78}(x,y) = -\mu_I(y)\, \mathcal{A}_\mu{}^I(x) \ ,$$

$$\tilde{C}_{\mu\, mn}(x,y) = \tfrac{1}{4}\, K^{IJ}_{mn}(y)\, \tilde{\mathcal{A}}_{\mu\, IJ}(x) \quad , \quad \tilde{C}_{\mu\, m7}(x,y) = -g^{-1}\, (\partial_m \mu^I)(y)\, \tilde{\mathcal{A}}_{\mu\, I}(x) \ ,$$

- Two-forms:

$$C_{\mu\nu\, m}(x,y) = -g^{-1}\, (\mu_I \partial_m \mu^J)(y)\, \mathcal{B}_{\mu\nu\, J}{}^I(x) \quad , \quad C_{\mu\nu\, 7}(x,y) = \mu_I(y)\, \mathcal{B}_{\mu\nu}{}^I(x) \ .$$

- Three-form:

$$C_{\mu\nu\rho}(x,y) = (\mu_I \mu_J)(y)\, \mathcal{C}_{\mu\nu\rho}{}^{IJ}(x) \ ,$$

- Similarly with the metric, scalars and fermions.

When these ansatze are introduced into the SL(7)-covariant IIA susy transformations, the S^6 dependence drops out and the susy transformations of the restricted $D=4$ hierarchy arise.

The full non-linear embedding

$$d\hat{s}_{10}^2 = \Delta^{-1}\, ds_4^2 + g_{mn}\, Dy^m\, Dy^n \ ,$$

$$\begin{aligned}\hat{A}_{(3)} &= \mu_I \mu_J \left(\mathcal{C}^{IJ} + \mathcal{A}^I \wedge \mathcal{B}^J + \tfrac{1}{6} \mathcal{A}^{IK} \wedge \mathcal{A}^{JL} \wedge \tilde{\mathcal{A}}_{KL} + \tfrac{1}{6} \mathcal{A}^I \wedge \mathcal{A}^{JK} \wedge \tilde{\mathcal{A}}_K \right) \\ &\quad + g^{-1} \left(\mathcal{B}_J{}^I + \tfrac{1}{2} \mathcal{A}^{IK} \wedge \tilde{\mathcal{A}}_{KJ} + \tfrac{1}{2} \mathcal{A}^I \wedge \tilde{\mathcal{A}}_J \right) \wedge \mu_I D\mu^J + \tfrac{1}{2} g^{-2}\, \tilde{\mathcal{A}}_{IJ} \wedge D\mu^I \wedge D\mu^J \\ &\quad - \tfrac{1}{2}\, \mu_I\, B_{mn}\, \mathcal{A}^I \wedge Dy^m \wedge Dy^n + \tfrac{1}{6} A_{mnp}\, Dy^m \wedge Dy^n \wedge Dy^p \ ,\end{aligned}$$

$$\hat{B}_{(2)} = -\mu_I \left(\mathcal{B}^I + \tfrac{1}{2} \mathcal{A}^{IJ} \wedge \tilde{\mathcal{A}}_J \right) - g^{-1}\, \tilde{\mathcal{A}}_I \wedge D\mu^I + \tfrac{1}{2} B_{mn}\, Dy^m \wedge Dy^n \ ,$$

$$\hat{A}_{(1)} = -\mu_I\, \mathcal{A}^I + A_m\, Dy^m \ .$$

Here, the covariant derivatives are

$$Dy^m \equiv dy^m + \tfrac{1}{2}\, g\, K^m_{IJ}\, \mathcal{A}^{IJ} \qquad , \qquad D\mu^I \equiv d\mu^I - g\, \mathcal{A}^{IJ} \mu_J \ .$$

The full non-linear embedding

The internal (inverse) metric and forms are given in terms of SL(7)-covariant blocks of the $D = 4$ scalar matrix $\mathcal{M}_{\mathbb{M}\mathbb{N}}$ and S^6 quantities as

$$\begin{aligned} g^{mn} &= \tfrac{1}{4} g^2 \, \Delta \, K^m_{IJ} \, K^n_{KL} \, \mathcal{M}^{IJ\,KL} \, , \\ A_m &= \tfrac{1}{2} \, g \, \Delta \, g_{mn} \, K^n_{IJ} \, \mu_K \, \mathcal{M}^{IJ\,K8} \, , \\ B_{mn} &= -\tfrac{1}{2} \, \Delta \, g_{mp} \, K^p_{IJ} \, \partial_n \mu^K \, \mathcal{M}^{IJ}{}_{K8} \, , \\ A_{mnp} &= A_m B_{np} + \tfrac{1}{8} \, g \, \Delta \, g_{mq} \, K^q_{IJ} \, K^{KL}_{np} \mathcal{M}^{IJ}{}_{KL} . \end{aligned}$$

Field strengths

The embedding can be given in terms of independent $D = 4$ degrees of freedom.

- Compute the field strengths using their type IIA definitions:

$$\hat{F}_{(4)} = \mu_I \mu_J \, \mathcal{H}^{IJ}_{(4)} + g^{-1} \, \mathcal{H}_{(3)\,J}{}^{I} \wedge \mu_I D\mu^J + \tfrac{1}{2} \, g^{-2} \, \tilde{\mathcal{H}}_{(2)\,IJ} \wedge D\mu^I \wedge D\mu^J + \ldots \,,$$

$$\hat{H}_{(3)} = -\mu_I \, \mathcal{H}^I_{(3)} - g^{-1} \, \tilde{\mathcal{H}}_{(2)\,I} \wedge D\mu^I + \ldots \,,$$

$$\hat{F}_{(2)} = -\mu_I \mathcal{H}^I_{(2)} + g^{-1} \left(g \, \delta_{IJ} \, \mathcal{A}^J - m \, \tilde{A}_I \right) \wedge D\mu^I + \ldots \,,$$

Here, $\mathcal{H}^{IJ}_{(4)}$, etc., turn out to be the field strengths of the restricted $D = 4$ duality hierarchy.

- These field strengths are now regarded as short-hand for the corresponding $D = 4$ dualised expressions.

The Freund-Rubin term

- An elegant expression can be found for the Freund Rubin term using the duality condition for $\mathcal{H}^{IJ}_{(4)}$. It can be written as $\hat{F}_{(4)} = U\,\text{vol}_4$, where

$$U = -\frac{g}{84}\, X'_{\mathbb{MP}}{}^{\mathbb{R}} X_{\mathbb{NQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MN}} \Big(\mathcal{M}^{\mathbb{PQ}} \mathcal{M}_{\mathbb{RS}} + 7\, \delta^{\mathbb{P}}_{\mathbb{S}}\, \delta^{\mathbb{Q}}_{\mathbb{R}} \Big)$$

closely parallels the scalar potential of $D = 4$ $\mathcal{N} = 8$ gauged supergravity

$$V = \frac{g^2}{168}\, X_{\mathbb{MP}}{}^{\mathbb{R}} X_{\mathbb{NQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MN}} \Big(\mathcal{M}^{\mathbb{PQ}} \mathcal{M}_{\mathbb{RS}} + 7\, \delta^{\mathbb{P}}_{\mathbb{S}}\, \delta^{\mathbb{Q}}_{\mathbb{R}} \Big) \, .$$

- $X'_{\mathbb{MN}}{}^{\mathbb{P}} = \Theta'_{\mathbb{M}}{}^{\alpha}\, (t_\alpha)_{\mathbb{N}}{}^{\mathbb{P}}$ is defined in terms of an S^6-dependent $\Theta'_{\mathbb{M}}{}^{\alpha}$, with

$$\Theta'_{[AB]}{}^{C}{}_{D} = 2\, \delta^{C}_{[A}\, \theta'_{B]D} \qquad , \qquad \Theta'^{[AB]C}{}_{D} = 2\, \delta^{[A}_{D}\, \xi'^{B]C} \; ,$$

and

$$\theta'_{IJ} = \mu_I \mu_J \; , \quad \theta'_{I8} = 0 \; , \quad \theta'_{88} = 0 \; ; \qquad \xi'^{\,AB} = 0 \; .$$

AdS$_4$ solutions of massive IIA

Uplift of supersymmetric critical points

- By the consistency of the truncation, all solutions of the $D = 4$ supergravity give rise to solutions of massive type IIA. In particular, the (AdS) critical points uplift to $AdS_4 \times S^6$ solutions of massive IIA.

SUSY	bos. sym.	ref.
$\mathcal{N} = 3$	$SO(4)$	[Pang, Rong '15]
$\mathcal{N} = 2$	$SU(3) \times U(1)$	[Guarino, Jafferis, OV '15]
$\mathcal{N} = 1$	G_2	[Behrndt, Cvetic '04]
$\mathcal{N} = 1$	$SU(3)$	[OV '15]

- Will focus on susy solutions, but we also find new and previously known non-susy ones. In particular, we recover a non-susy solution first discussed by [Lüst, Marchesano, Martucci, Tsimpis '08] We find this solution to be perturbatively stable.

AdS_4 solutions with at least SU(3) symmetry

- The solutions typically have SU(3)×SU(3)-structure [Lüst, Tsimpis '09]
- Locally, S^6 is foliated with S^5 leaves. The S^5 is equipped with its usual Sasaki-Einstein structure $(\boldsymbol{J}, \boldsymbol{\Omega}, \boldsymbol{\eta})$. The solutions thus generalise to arbitrary Sasaki-Einstein.
- For the G_2–invariant solution, the SU(3)×SU(3)-structure reduces to and SU(3) structure. $(\boldsymbol{J}, \boldsymbol{\Omega}, \boldsymbol{\eta})$ conspire to produce the nearly-Kähler structure $(\mathcal{J}, \Omega)$ on S^6:

$$\mathcal{J} = \sin^2\alpha \cos\alpha\, \boldsymbol{J} + \sin^3\alpha\, \mathrm{Re}\, \boldsymbol{\Omega} + \sin\alpha\, d\alpha \wedge \boldsymbol{\eta} ,$$

$$\mathrm{Re}\,\Omega = -\sin^3\alpha\, \boldsymbol{J} \wedge d\alpha + \sin^2\alpha \cos\alpha\, \mathrm{Re}\,\boldsymbol{\Omega} \wedge d\alpha - \sin^3\alpha\, \mathrm{Im}\,\boldsymbol{\Omega} \wedge \boldsymbol{\eta} ,$$

$$\mathrm{Im}\,\Omega = -\sin^4\alpha\, \boldsymbol{J} \wedge \boldsymbol{\eta} + \sin^3\alpha \cos\alpha\, \mathrm{Re}\,\boldsymbol{\Omega} \wedge \boldsymbol{\eta} + \sin^2\alpha\, \mathrm{Im}\,\boldsymbol{\Omega} \wedge d\alpha .$$

AdS_4 solutions with at least SU(3) symmetry

- The $\mathcal{N} = 2$ solution is the first explicit, analytically known solution in this class.
- The solutions are typically cohomogeneity-one. All previously known $\mathcal{N} = 1$ solutions are have SU(3)-structure and are homogeneous. These include:
  - Nearly-Kähler [Behrndt, Cvetic '04]
  - Half-flat [Koerber, Lüst, Tsimpis '08; Tomasiello '07]
- This does not exclude the existence of inhomogeneous solutions in the SU(3)-structure class. Numerical evidence of nearly-Kähler metrics has been recently found [Foscolo, Haskins '15]
- Also recently, other $\mathcal{N} = 1$, cohomogeneity-one AdS_4 solutions with SU(3)×SU(3)-structure have been constructed [Rota, Tomasiello '15; Apruzzi, Fazzi, Passias, Tomasiello '15]

An $\mathcal{N} = 2$ AdS$_4$ solution of massive IIA

$$d\hat{s}^2_{10} = L^2 \big(3 + \cos 2\alpha\big)^{1/2} \big(5 + \cos 2\alpha\big)^{1/8} \Big[ds^2(\text{AdS}_4) + \frac{3}{2}\, d\alpha^2 + \frac{6 \sin^2\alpha}{3 + \cos 2\alpha}\, ds^2(\mathbb{CP}^2) + \frac{9 \sin^2\alpha}{5 + \cos 2\alpha}\, \boldsymbol{\eta}^2 \Big]$$

$$e^{\hat{\phi}} = e^{\phi_0} \frac{\big(5 + \cos 2\alpha\big)^{3/4}}{3 + \cos 2\alpha} \ ,$$

$$L^{-2} e^{-\frac{1}{2}\phi_0} \hat{H}_{(3)} = 24\sqrt{2}\, \frac{\sin^3\alpha}{\big(3 + \cos 2\alpha\big)^2}\, \boldsymbol{J} \wedge d\alpha \ ,$$

$$L^{-3} e^{\frac{1}{4}\phi_0} \hat{F}_{(4)} = 6\, \text{vol}(\text{AdS}_4) + 12\sqrt{3}\, \frac{7 + 3\cos 2\alpha}{\big(3 + \cos 2\alpha\big)^2}\, \sin^4\alpha\ \text{vol}(\mathbb{CP}^2)$$
$$+ 18\sqrt{3}\, \frac{\big(9 + \cos 2\alpha\big) \sin^3\alpha \cos\alpha}{\big(3 + \cos 2\alpha\big)\big(5 + \cos 2\alpha\big)}\, \boldsymbol{J} \wedge d\alpha \wedge \boldsymbol{\eta} \ ,$$

$$L^{-1} e^{\frac{3}{4}\phi_0} \hat{F}_{(2)} = -4\sqrt{6}\, \frac{\sin^2\alpha \cos\alpha}{\big(3 + \cos 2\alpha\big)\big(5 + \cos 2\alpha\big)}\, \boldsymbol{J} - 3\sqrt{6}\, \frac{\big(3 - \cos 2\alpha\big)}{\big(5 + \cos 2\alpha\big)^2}\, \sin\alpha\ d\alpha \wedge \boldsymbol{\eta} \ ,$$

Flux quantisation

- On our topologically S^6 solution, one can only impose quantisation conditions on $\hat{F}_{(0)}$ and $\hat{F}_{(6)}$:

$$k = 2\pi\ell_s\,\hat{F}_{(0)} \equiv 2\pi\ell_s\,m\ ,$$

$$N = -\frac{1}{(2\pi\ell_s)^5}\int_{S^6} e^{\frac{1}{2}\hat{\phi}}\ \hat{*}\hat{F}_{(4)} + \hat{B}_{(2)}\wedge d\hat{A}_{(3)} + \frac{1}{6}m\,\hat{B}_{(2)}\wedge\hat{B}_{(2)}\wedge\hat{B}_{(2)}$$

- The classical parameters g and m (or L, e^{ϕ_0}) become fixed in terms of the quantum numbers k, N.

The dual CFT_3

- These backgrounds should arise as the near horizon of D2-branes on a smooth space with RR and NS fluxes.
- The presence of the Romans mass adds ($\mathcal{N} = 2$) Chern-Simons terms at level k to maximal 3D super-Yang-Mills with gauge group $SU(N)$.
- Three adjoint chirals with $\mathcal{W} = \mathrm{Tr}(X[Y, Z])$, SU(3) flavour and U(1) R-symmetry.

AdS/CFT match of the free energy

We have independently computed the free energy of the above $\mathcal{N} = 2$ solution and in the candidate dual field theory.

- On the gravity side, the free energy is given by

$$F = \frac{16\pi^3}{(2\pi\ell_s)^8} \int_{S^6} e^{8A} \, \mathrm{vol}_6 = \frac{\pi}{5} \, 2^{1/3} \, 3^{1/6} \, N^{5/3} \, k^{1/3} \, ,$$

  where e^{2A} is the warp factor.

- On the field theory side, the free energy is computed by localisation to a solvable matrix model. The result matches the supergravity result! See also [Fluder, Sparks '15]

This provides the first $\mathrm{AdS}_4/\mathrm{CFT}_3$ test in the context of massive type IIA string theory.

Further discussion

Our new results in a wider context

- The Romans mass has long been known to induce magnetic gaugings in $D = 4$. [Polchinski, Strominger '95; Louis, Micu '02] Our result is a certain $\mathcal{N} = 8$ extension.

- Different arguments single out the usual $D = 11$ and IIB truncations on S^7, S^4 and S^5. [Nastase, Vaman '00; Cvetic, Lu, Pope '00; Lee, Strickland-Constable, Waldram '14; Samtleben, Hohm '14] These arguments seem to miss massive IIA on S^6.

- The latter truncation is arguably different, in that dyonic ISO(7) supergravity doesn't have an $\mathcal{N} = 8$ AdS vacuum that uplifts to a maximally supersymmetric $\text{AdS}_4 \times S^6$ massive IIA background.

  Yet, an $\mathcal{N} = 8$ truncation does exist at the level of the supergravities.

Our new results in a wider context

- Massless type IIA was conjectured in [Hull, Warner '88] to truncate consistently on S^6 to the purely electric ($m = 0$) ISO(7) gauging of [Hull, Warner '84]
  Our consistency proof is independent of the Romans mass. We thus prove that conjecture, too.

- We gave the *full* embedding of the $D = 4$ theory in $D = 10$ by exploiting the restricted tensor and duality hierarchies. In comparison, for $D = 11$ on S^7, only the non-linear embedding of the scalars is known –the embedding of the vectors is only known at the linear level.

Our new results in a wider context

- Massive IIA on $S^6 = G_2/SU(3)$ admits a different, $\mathcal{N}=2$ truncation when S^6 is equipped with its G_2-invariant nearly-Kähler structure. [Kashani-Poor '07] This is analogous to M-theory on Sasaki-Einstein. [Gauntlett,OV '07]
  - The $\mathcal{N}=8$ truncation retains modes at the bottom of the KK towers about the AdS vacua.
  - The $\mathcal{N}=2$ truncation retains modes up the KK towers about the AdS vacua.
  - Both truncations have a common sector: the $\mathcal{N}=1$, G_2–invariant sector of the $\mathcal{N}=8$ theory.

- A certain $\mathcal{N}=1$ subsector of dyonic ISO(7) supergravity is described by a superpotential

$$W = -g\,\left(2 + 6\,T^2\,U^2 + 6\,S\,T\,U^2\right) + 2\,m\,U^3\ ,$$

which contains non-geometric terms. This model, however, enjoys a perfectly geometric origin in massive IIA on S^6.

Thank you!