The Opposite Ends of Supersymmetry and their Implications for the LHC

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Outline of Lectures

1. Lecture 1
    - a) Minimal SUSY
    - b) Challenges of low-energy SUSY
    - c) The Higgs crisis in SUSY
    - d) Dismissing naturalness and finetuning
    - e) Retaining the good things of SUSY
2. Lecture 2
    - a) Brief philosophical interlude
    - b) Theory home for "1st extreme of SUSY"
    - c) Implications for cosmology and the LHC
    - d) Further inquiries on the Higgs mass crisis
3. Lecture 3
    - a) The "2nd extreme of SUSY"
    - b) Challenges of zero scalar mass boundary conditions
    - c) Dark Matter Considerations
    - d) g-2 connections
    - e) LHC Implications

Borrowing from John Steinbeck….

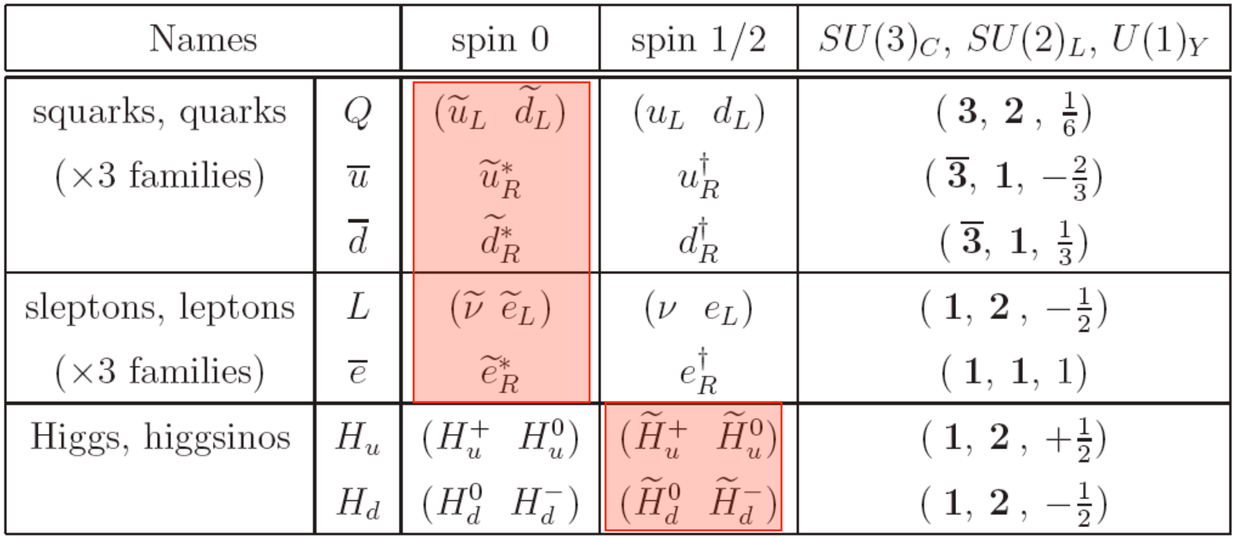
“There are some people who deeply and basically dislike theories and are hostile to speculations. These are usually unsure people who, whirling in uncertainties, try to steady themselves by grabbing and tightly holding on to facts…. To such a person a theory is a lie until it is proven and then it becomes a truth or a fact. But there’s no joy in it. Now -- to get to my theory.”

Some motivations to study Supersymmetry

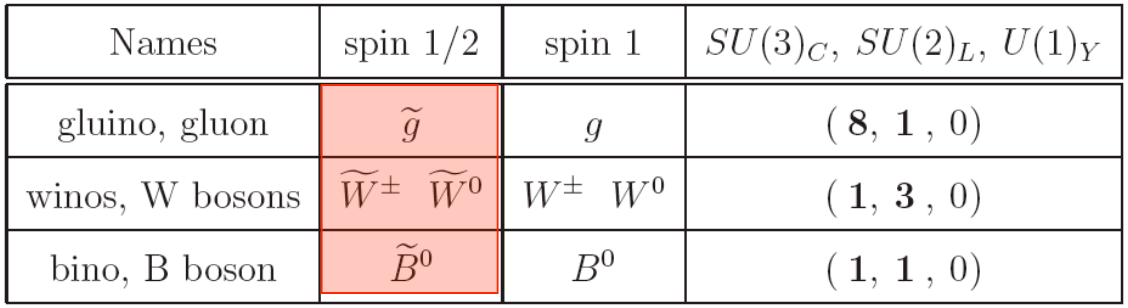
1. Gauge Coupling Unification
2. “Obvious” space-time symmetry extension to explore
3. String theory seems to like it
4. Source of dark matter (R-parity)
5. Radiative electroweak symmetry breaking
6. Can solve gauge hierarchy problem
7. Rich, calculable, self-consistent beyond-the-SM theory

Other reasons: QFT laboratory, etc.

The Particle Spectrum of Minimal Supersymmetry



Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\ \mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$



Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

Excellent source from which to learn the fundamentals.

Superpartners highlighted in red.

SUSY Primer: Martin, hep-ph/9709356v5 (Dec 08)

Mixed States: Charginos of the MSSM

Charginos in the $\chi_i^\pm = \left\{ \widetilde{W}^\pm, \widetilde{H}^\pm \right\}$ basis,

$$U^\dagger X V^{-1} = \mathrm{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}), \quad \text{where}$$

$$X = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{2} c_\beta m_W & \mu \end{pmatrix}$$

Mixed States: Neutralinos of the MSSM

Neutralinos in the $\chi_i^0 = \{\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0\}$ basis,

$$N^* Y N^{-1} = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}), \quad \text{where}$$

$$Y = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}$$

Mixed States: Sfermions

For sfermions in the $\tilde{f}_i = \{\tilde{f}_L, \tilde{f}_R\}$ basis,

$$m^2_{\tilde{f}_i} = \begin{pmatrix} m^2_{\tilde{f}_L} + m^2_f + \Delta_{\tilde{f}_L} & m_f(A_f - \mu\eta_f) \\ m_f(A_f - \mu\eta_f) & m^2_{\tilde{f}_R} + m^2_f + \Delta_{\tilde{f}_R} \end{pmatrix}$$

where

$$\eta_f = \begin{cases} 1/\tan\beta, & \text{for up type fermions} \\ \tan\beta, & \text{for down type fermions} \end{cases}$$

and

$$\Delta_f = (T_3^f - Q_{em}^f \sin^2\theta_W)\cos 2\beta m_Z^2.$$

The mixing of $\tilde{f}_L$ and $\tilde{f}_R$ is defined such that

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

Description of SUSY Breaking

SUSY breaking resides in <F> of chiral multiplet

$$X = x + \sqrt{2}\psi\theta + F\theta^2$$

This leads to gravitino mass: $m_{3/2}^2 \sim \frac{F^\dagger F}{M_{\rm Pl}^2}$

Gravitino is spin 3/2 particle. ψ is the absorbed ±1/2 spin component (goldstino).

Gaugino masses: $\int d^2\theta \frac{X}{M_{\rm Pl}} \mathcal{W}\mathcal{W} \sim m_{3/2}\lambda\lambda$

Scalar masses: $\int d^2\theta d^2\bar{\theta} \frac{X^\dagger X}{M_{\rm Pl}^2} \Phi_i^\dagger \Phi_i \rightarrow m_{3/2}^2 \phi_i^* \phi_i$

Everybody $\sim m_{3/2}$, and $m_{3/2} \sim m_W$ for naturalness.

Challenges for Low-Energy SUSY

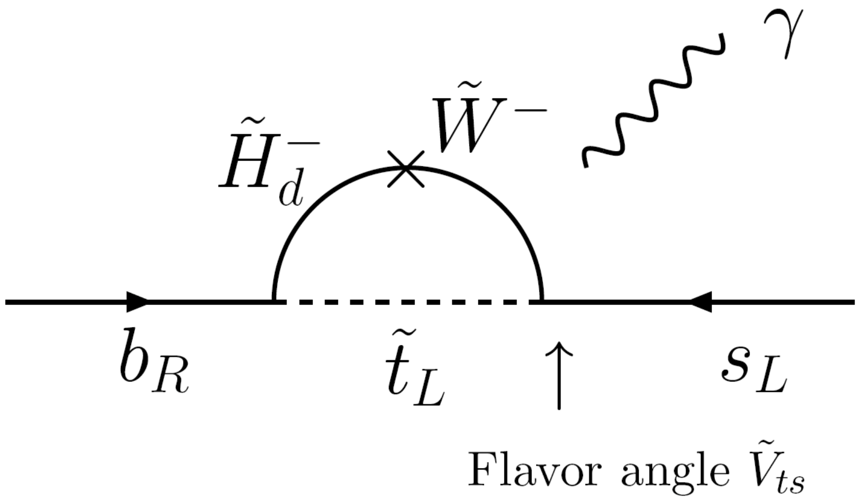
Throw a dart into Minimal SUSY parameter space,
And what do you get?

Observable predictions would be wildly
Incompatible with experiment.

Briefly review these challenges ….

Flavor Changing Neutral Currents

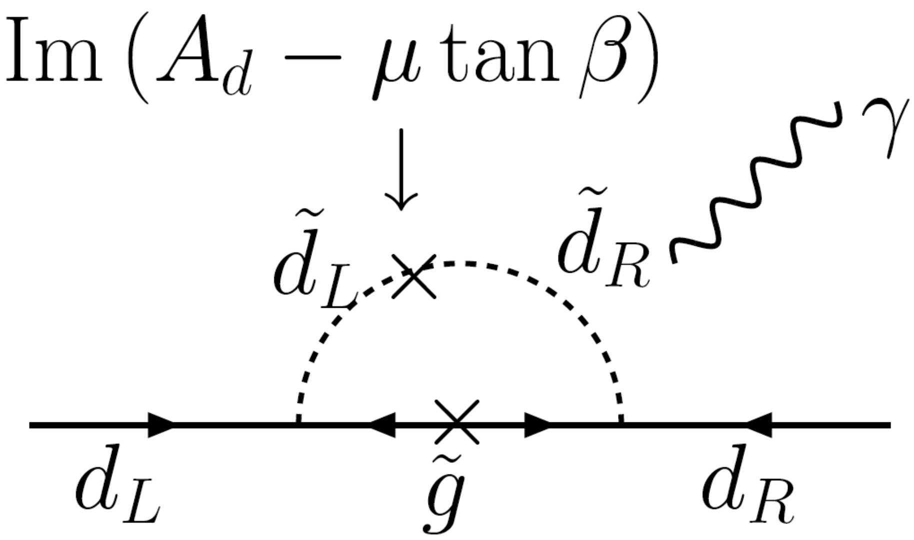
Random superpartner masses and mixing angles would generate FCNC far beyond what is measured:



However: heavy scalars would squash these FCNCs

CP Violation

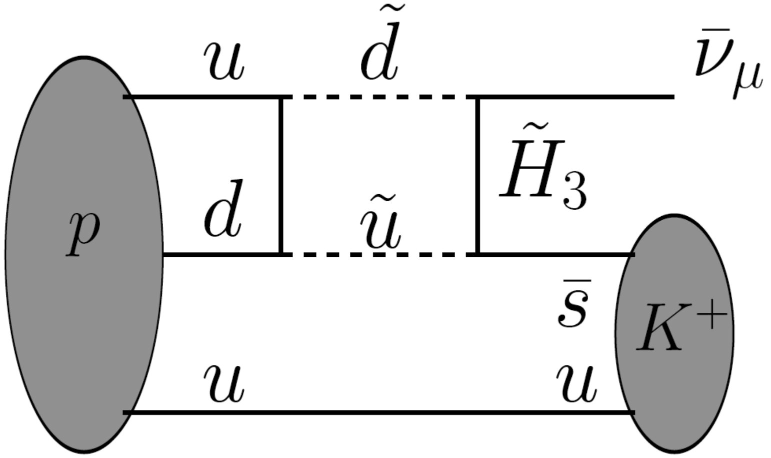
Supersymmetry has many new sources of CP violation:



Large unless CP angle small or scalar masses heavy.

Proton Decay

Perhaps less troublesome…. Proton decay can be problematic, even in R-parity conserving supersymmetry.



Dim-5 operator suppressed by heavy triplet or
Much heavier scalar mass superpartners

Model Building

Many clever solutions exist to overcome these challenges.

To me, the most challenging one is the flavor problem.

The two “opposite ends” of supersymmetry that I will discuss solve the flavor problem in different ways.

But first, a recent problem for supersymmetry has arisen: the prediction of the light Higgs boson mass.

Two Higgs Doublets of Supersymmetry

Supersymmetry requires two Higgs doublets. One to give mass to up-like quarks (H_u), and one to give mass to down quarks and leptons (H_d).

8 degrees of freedom. 3 are eaten by longitudinal components of the W and Z bosons, leaving 5 physical degrees of freedom: $H^{\pm}$, A, H, and h.

As supersymmetry gets heavier ($m_{3/2} >> MZ$), a full doublet gets heavier together (H±,A,H) while a solitary Higgs boson (h) stays light, and behaves just as the SM Higgs boson.

Coupling of the neutral scalar Higgses

ϕ		$g_{\phi\bar{t}t}$	$g_{\phi\bar{b}b}$	$g_{\phi VV}$
SM	H	1	1	1
MSSM	h^o	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H^o	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A^o	$1/\tan\beta$	$\tan\beta$	0

Haber et al. ‘01

Heavy Higgs

$$HVV: \quad \cos(\beta-\alpha) \rightarrow \boxed{0} + \mathcal{O}(m_Z^4/m_A^4)$$

$$H\bar{t}t: \quad \frac{\sin\alpha}{\sin\beta} \rightarrow \boxed{\frac{1}{\tan\beta}} + \mathcal{O}(m_Z^2/m_A^2)$$

$$H\bar{b}b: \quad \frac{\cos\alpha}{\cos\beta} \rightarrow \boxed{\tan\beta} + \mathcal{O}(m_Z^2/m_A^2)$$

Light Higgs

$$hVV: \quad \sin(\beta-\alpha) \rightarrow 1$$

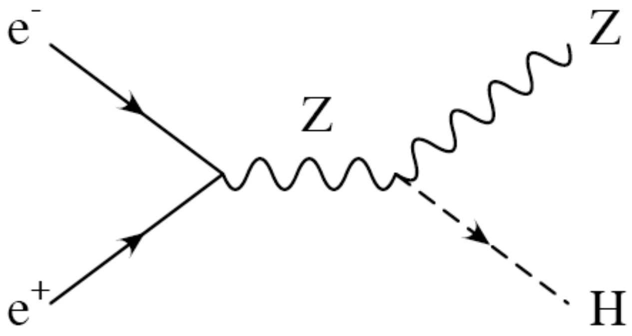
$$htt: \quad \frac{\cos\alpha}{\sin\beta} \rightarrow 1$$

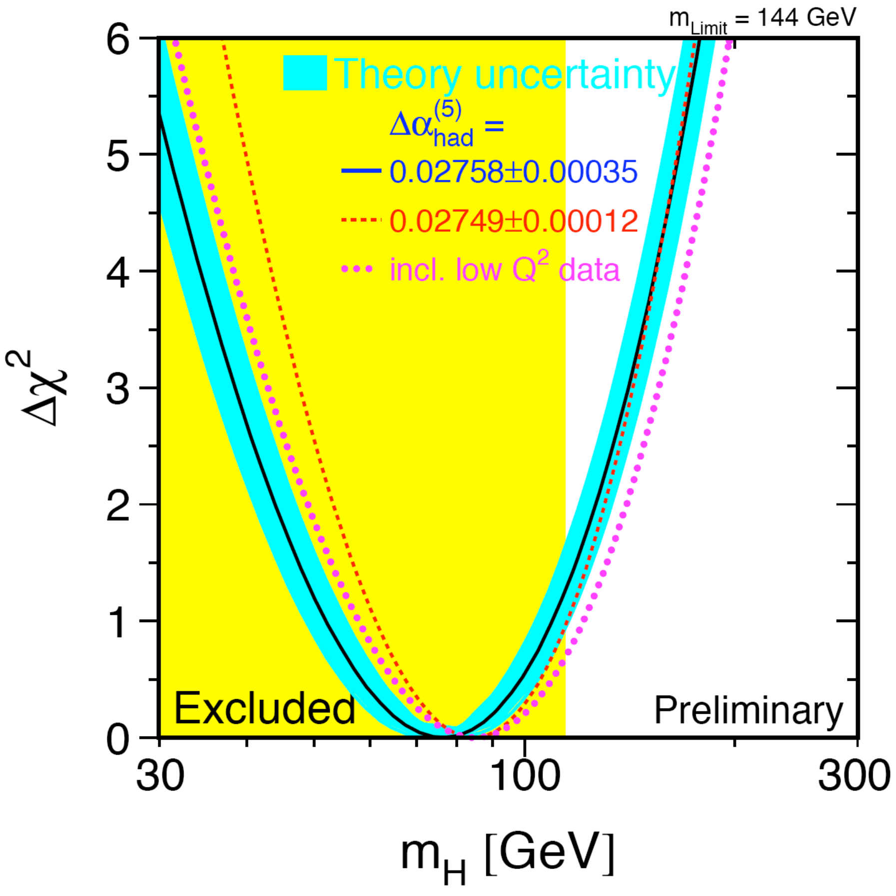
$$hbb: \quad \frac{-\sin\alpha}{\cos\beta} \rightarrow 1$$

Higgs mass limits

Higgs boson mass upper limit (95% CL) from precision Electroweak is less than 182 GeV.

Lower limit from lack of direct signal at LEP 2 is about 115 GeV.



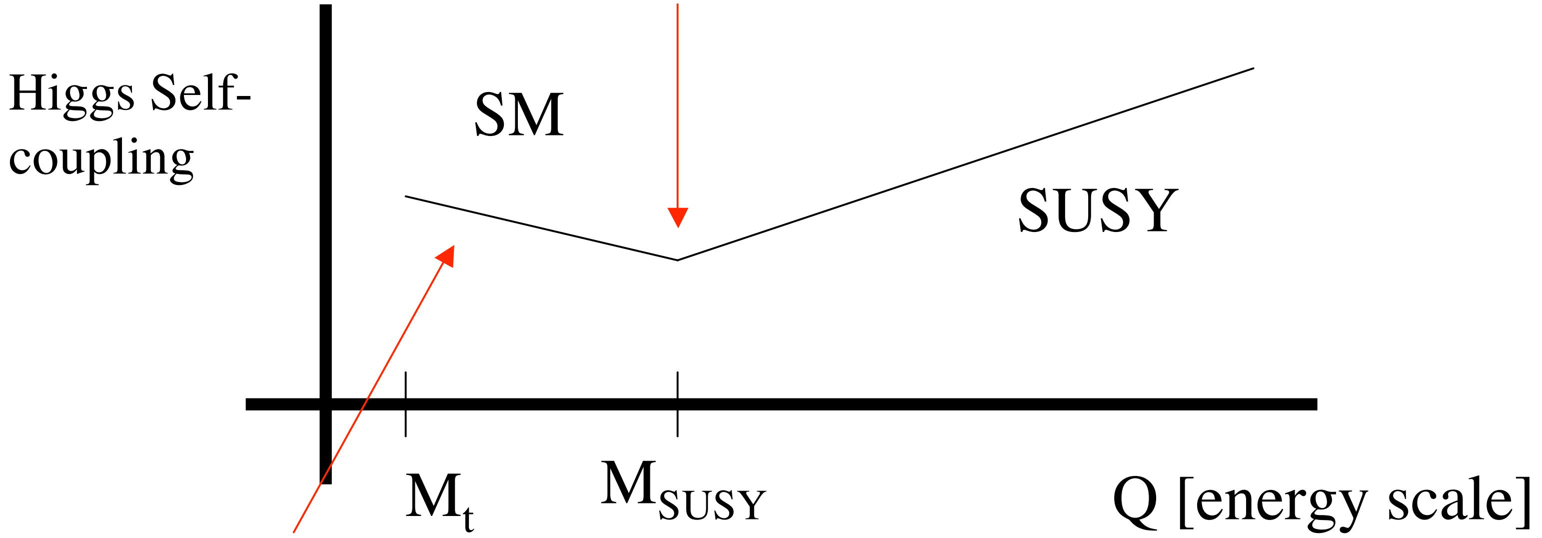


LEPEWWG, 0712.0929

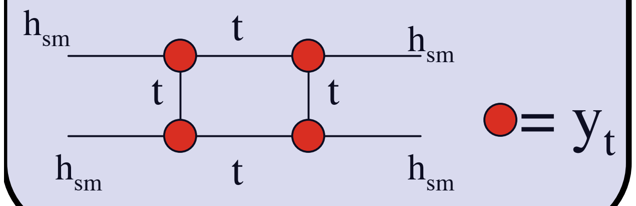
Experiment: 115 GeV $< m_h <$ 182 GeV

Understanding Lightest Higgs Mass Computation

$$\lambda(M_{SUSY}) = \frac{1}{8}(g^2+g'^2)\cos^2 2\beta$$



$$\frac{d\lambda}{d\log Q} = -\frac{3}{4\pi^2}y_t^4+\cdots$$



$$\begin{aligned} m_h^2 &= 2\lambda v^2 = 2\left(\lambda(M_{SUSY}) + \frac{3}{4\pi^2}y_t^4\log\frac{M_{SUSY}}{M_t}\right)v^2 \\ &= M_Z^2\cos^2 2\beta + \frac{3M_t^4}{\pi^2 v^2}\log\frac{M_{SUSY}}{M_t} \end{aligned}$$

Higgs boson mass

In minimal supersymmetry the lightest Higgs mass is computable:

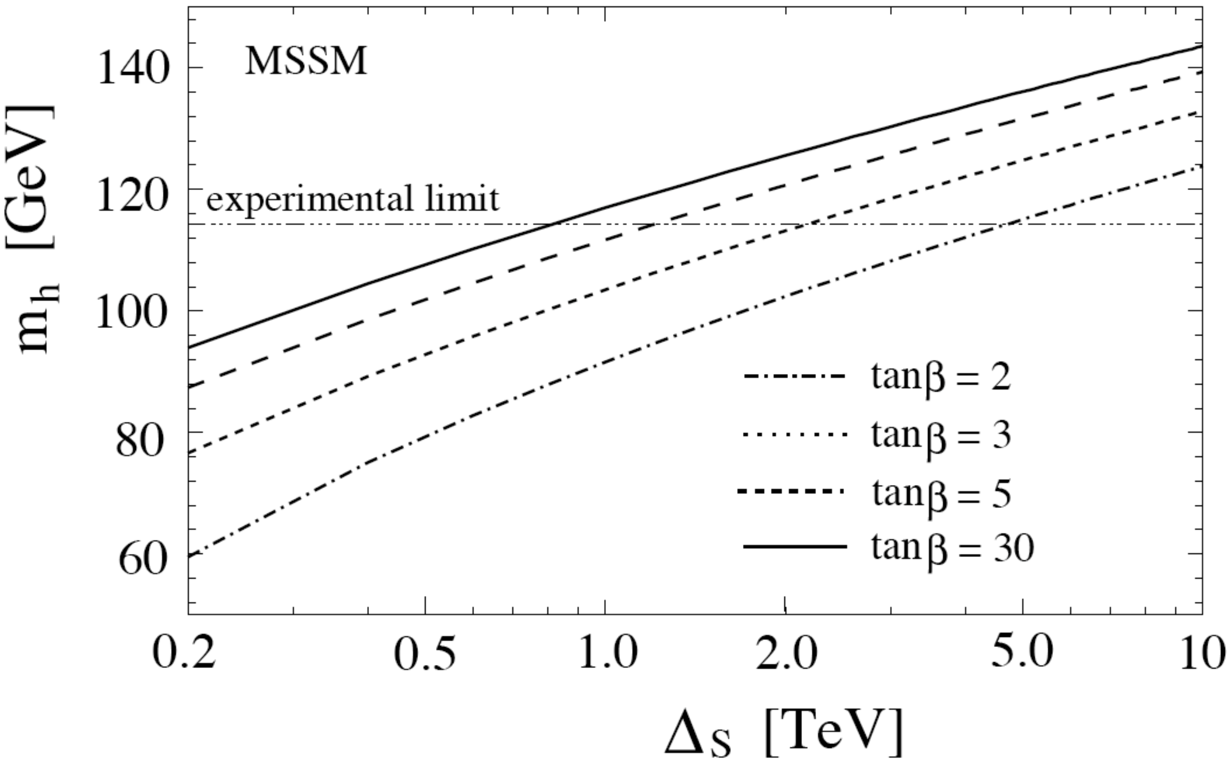
$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{\tilde{m}_t^2}{m_t^2} + \cdots$$

Tree-level value is bounded by $m_Z = 91\,\mathrm{GeV}$. Current lower limit on Higgs boson mass is $114\,\mathrm{GeV}$. Thus, we need $\sim (70\,\mathrm{GeV})^2$ contribution from quantum correction.

Need $\tilde{m}_t \gtrsim 5\,\mathrm{TeV}(0.8\,\mathrm{TeV})$ for $\tan\beta = 2(30)$

Log-sensitivity keeps m_h below the Precision EW bound (~ 200 GeV)

Lightest Higgs Mass in the MSSM



Tobe, JW, '02

$$\begin{aligned} m_h^2 &= M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \frac{3m_t^2}{v^2} c_{\tilde{t}}^2 s_{\tilde{t}}^2 (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \ln \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} + \cdots \\ &\equiv M_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \ln \frac{\Delta_S^2}{m_t^2} \quad \text{where } \Delta_S^2 \gtrsim m_{\tilde{t}_1} m_{\tilde{t}_2} \end{aligned}$$

Naturalness

Naturalness is strained if M_{SUSY} becomes too large.

From the EW scalar potential of supersymmetry,
the minimization conditions yield

$$\frac{1}{2}m_Z^2 + \mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$$

This is of the generic form of one large number subtracting another and getting a small number:

$$\tilde{m}_1^2 - \tilde{m}_2^2 = m_Z^2$$

Example of extreme finetuning

Bush v. Gore Florida vote in 2000 U.S. Presidential election:

M_1^2 = Bush’s votes = 2,912,790
M_2^2 = Gore’s votes = 2,912,253

Normalizing $M_1^2 - M_2^2 = M_Z^2$ (multiply by 15.5) one gets the scale of ‘supersymmetry masses’ of this election to be

Sqrt[15.5* M_1^2] = 6.7 TeV [Well above Higgs mass needs.]

Obama-McCain a “250 GeV” election.
Sarkozy-Royal a “270 GeV” election.

First Extreme of Supersymmetry

Scalar superpartners (squarks and sleptons) are much, much heavier than fermionic superpartners (charginos, neutralinos and gluinos).

This goes under the names of Split Supersymmetry (Arkani-Hamed, Dimopolous, Giudice, Romanino) or PeV Scale supersymmetry (JW).

Let’s begin by building the rationale for this approach.

EW-Scale Naturalness

Appeals to naturalness are murky and controversial.
Incompatible views can be reasonable.

Agnostic approach: Delete all reference to naturalness and ask what is the “best” susy model consistent data.

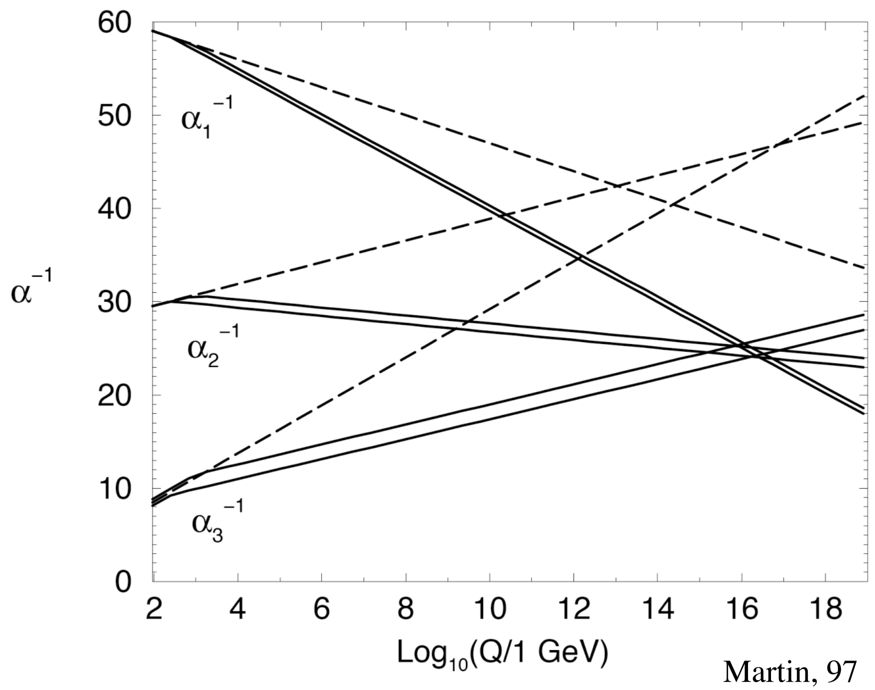
Arbitrary heavy SUSY?

After deleting naturalness from consideration, we should not conclude that SUSY is at some arbitrarily large scale, where it can’t cause harm.

We wish to retain good things about SUSY:

- Gauge Coupling unification
- Light Higgs boson mass prediction
- Cold Dark Matter

Gauge Coupling Unification



Martin, 97

"Proximity Factor" for gauge coupling unification is defined to be the factor A needed such that

Generic quantum correction

$$g_U = g_1(M_U) = g_2(M_U) = g_3(M_U) + A\frac{g_U^3}{16\pi^2}$$

In weak-scale MSSM $M_U \simeq 2 \times 10^{16}\,\text{GeV}$ and $A \simeq 1$.

Unification success sensitive to -inos, but not scalars [Giudice, Romanino; etc.]

Relic Abundance

Weinberg ‘83 : LSP is stable -- Problem? No -- Might be good

Goldberg ‘83 : LSP Majorana -- Good CDM Candidate

LSPs annihilate as universe expands until they can’t
find each other any more (freeze-out T ~ m/20)

$$\Omega h^2 = \frac{A}{\langle \sigma v \rangle} = \frac{A \tilde{m}^2}{\alpha}, \text{ where } \langle \sigma v \rangle = \frac{\alpha}{\tilde{m}^2}$$

CDM Limits and SUSY Mass

Experiment tells us

$$0.09 < \Omega_{CDM} h^2 < 0.13$$

Leads to upper bound constraint on lightest susy mass (neutralino), but others can be much heavier (squarks and sleptons).

$$\frac{A\tilde{m}^2}{\alpha} < 0.13 \quad \rightarrow \quad \tilde{m} < \sqrt{0.13\alpha/A} < \text{ few TeV}$$

Where we are at

Ignoring Naturalness

Eliminating bad things:

1. FCNC
2. Proton decay strains
3. CP Violation
4. Too light Higgs mass

Preserving good things:

- SUSY
- Light Higgs prediction
- Gauge Coupling Unification
- Dark Matter

Accomplished by large scalar susy masses, but light fermion susy masses (gauginos, higgsinos)

Good theory for this? Yes. The -ino masses charged under symmetries (R and PQ) whereas scalars are not. [Split SUSY literature.]

End of Lecture 1

Next time:

Brief philosophical insert

Pick up where we left off last time:

- Nice theory home for Split Supersymmetry
- Unique signatures for cosmology and colliders

Back to naturalness: Further inquiries on the Higgs mass crisis