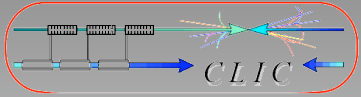


Application of a simple recirculation model to first 12 GHz PETS tests with beam

Assumptions and validity

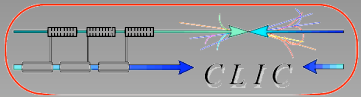
CLIC RF meeting

Erik Adli, University of Oslo and CERN



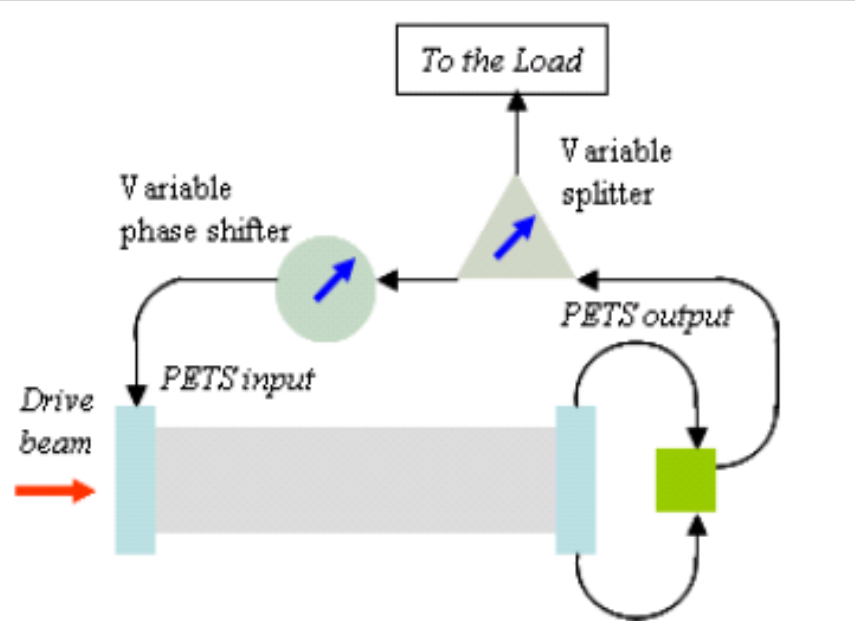
Outline

- **MODEL**
- **MEASUREMENT**
- **FIT**
- **REALITY CHECK**
- **APPLICATION TO NORMAL PULSES SERIES**
- **APPLICATION TO SHORTENED PULSES SERIES**
- **ATTEMPT TO PHASE-FIT SHORTENED PULSES**



Simple model of recirculation

In an attempt to the recirculated power and predict the power for a given current we assume the following simple field model :



$$E_{n+1} = \lambda E_n + E_0$$

↓

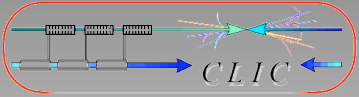
$$E_{n+1} - E_n = (\lambda - 1)E_n + E_0$$

↓

- r** : the ratio of the field being recirculated
- η** : estimated ohmic losses around the circulation
- φ** : the field phase change after one recirculation
- λ = r × η × exp(jφ)** : field reduction factor after one recirculation

$$\frac{E_{n+1} - E_n}{\Delta t} = \frac{(\lambda - 1)}{\Delta t} E_n + \frac{1}{\Delta t} E_0$$





Simple model: predictions

Approx as differential eq, with solution (for initial condition $E(0) = 0$)

$$E(t) = \frac{E_0}{1 - \lambda} (1 - \exp(-(1 - \lambda)t/\Delta t)) = A(1 - e^{-t/\tau})$$

thus

$$E(t) = A(1 - e^{-t/\tau})$$

with

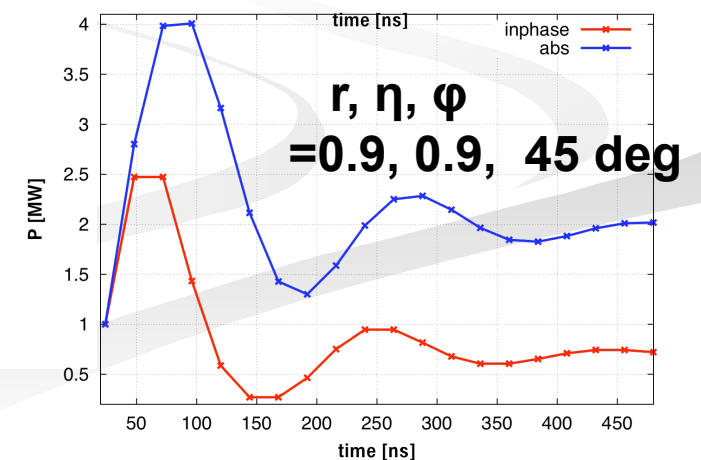
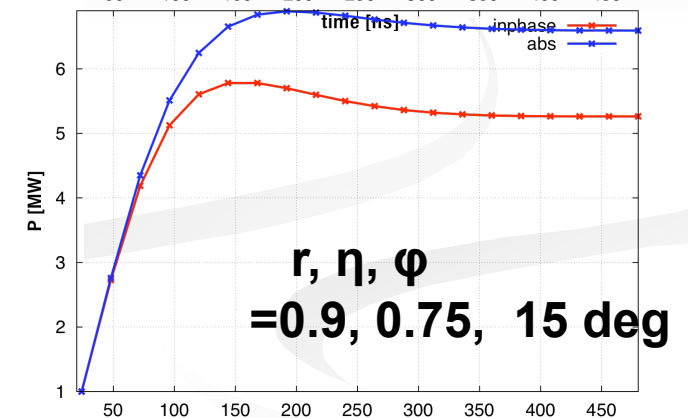
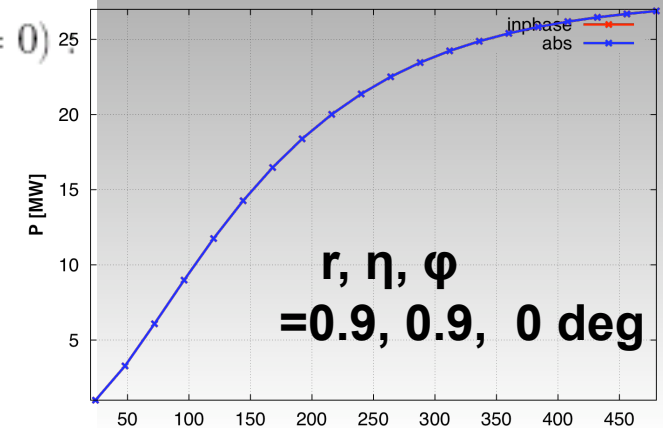
$$\lambda = r \times \eta_{ohm} \times e^{j\theta}$$

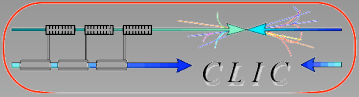
$$A = \frac{E_0}{1 - \lambda}$$

$$\tau = \frac{t_{recirc}}{(1 - \lambda)}$$

complex solution:
real(E) : works on beam
abs(E)² \propto measured P

$$P \propto E^2$$





Recirculation reconstruction with beam pus

$$E_{out,n} = c\bar{I}_n$$

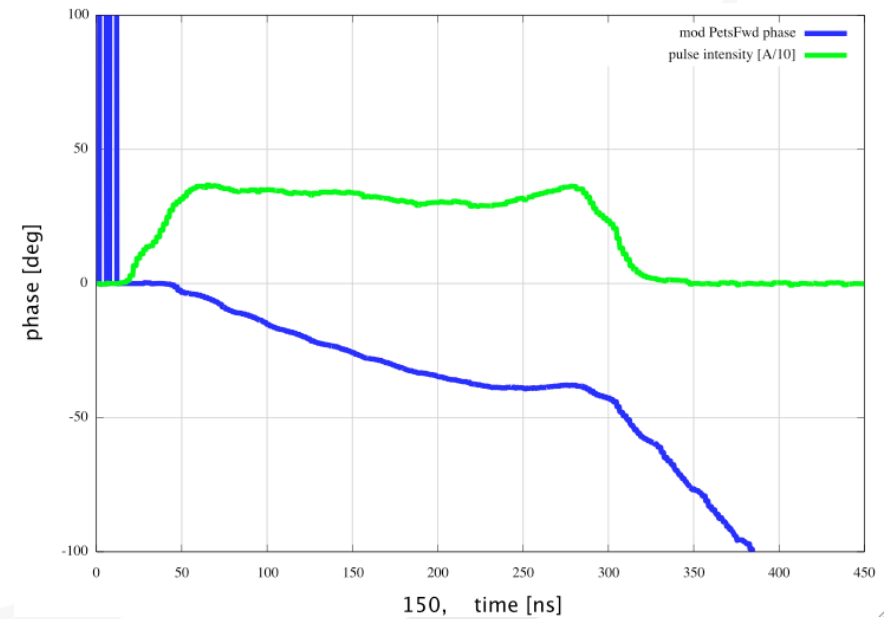
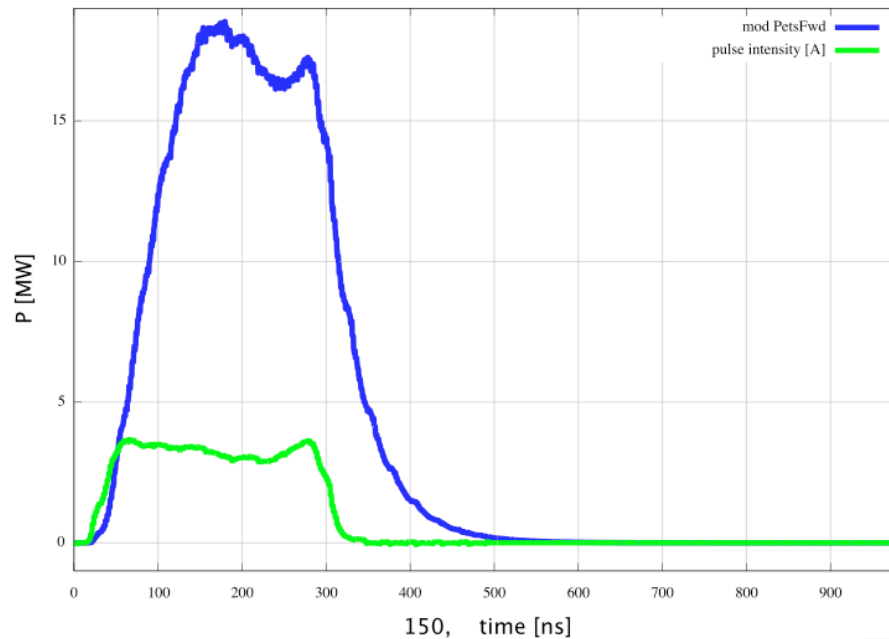
$$E_{out,n} = c\bar{I}_n + c\lambda\bar{I}_{n-a}$$

$$\lambda = A \exp(j\phi)$$

$$a = t_{circ} f_{BPM}$$

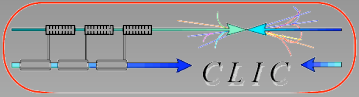
$$E_{out,n} = c \sum_{m=0}^M \lambda^m \bar{I}_{n-am}$$

**3 parameter model:
c, A, phi**



$$P_{,n} = \text{const} (E_{out,n})^2$$

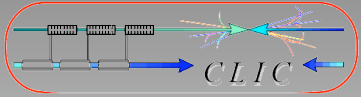
$$\theta_{,n} = \arctan(\text{Im}(E_{out,n}) / \text{Re}(E_{out,n}))$$



CODE

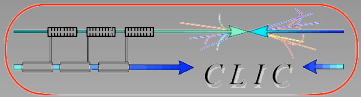
- The computer algorithm becomes very simple

```
for m=0:n_recirculations,  
    recirced_field = [zeros(1,round(n_recirc_step*m)) (lambda^m)*I_beam(1,:)];  
    E_mod_total = postpad(E_mod_total, length(recirced_field),0,2);  
    E_mod_total += recirced_field;  
end
```



RF MEASUREMENTS

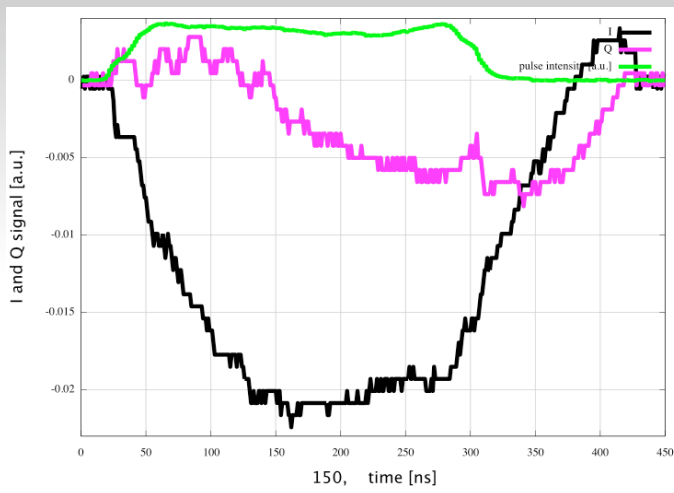
- Diode (calibrated at time of run)
 - less noise than IQ signal
- I and Q channel, field measurements with $\pi/2$ relative phase (calibration not known at time of run)
 - However: with IQ both RF power and phase can be reconstructed, so we use IQ for fit
 - Diode is used to compare absolute values of model and measurements



Power and phase from RF signals

Using I and Q channel :

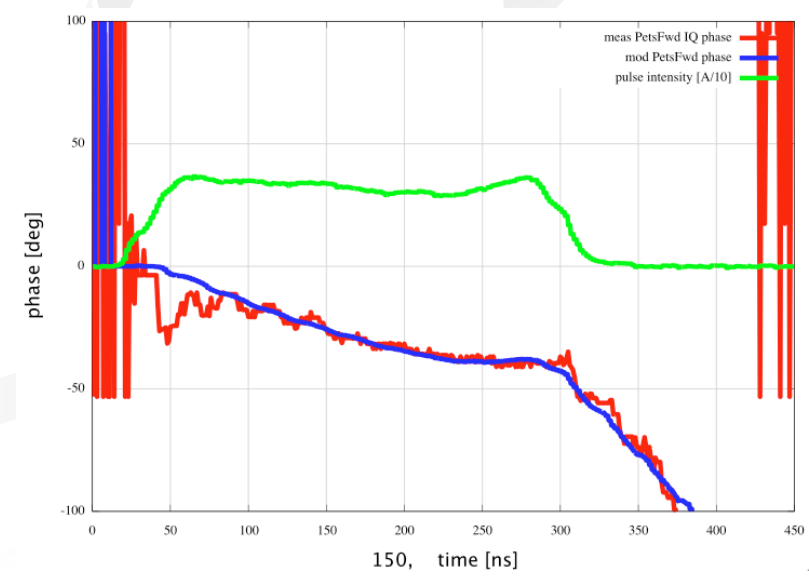
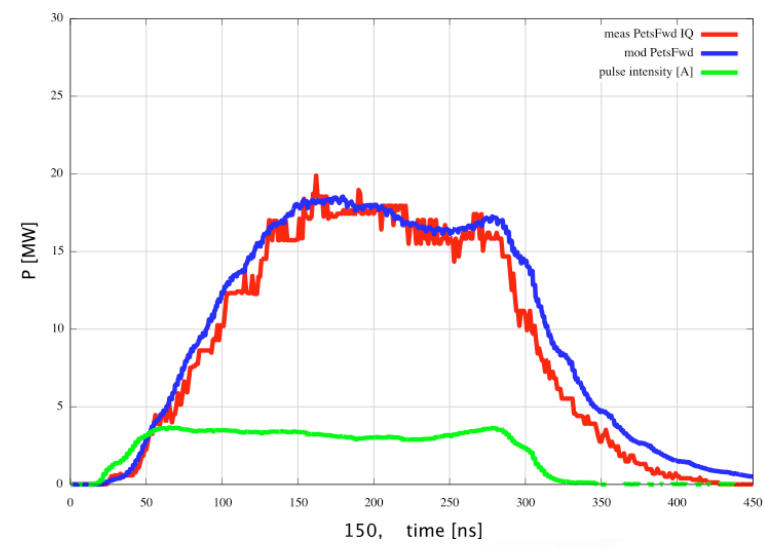
Needed timing adjustment (IQ to pulse) "**~10 ns scale**"
 - **same shift** for most bunches (some timing jitter)

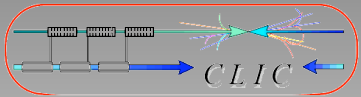


Needed phase adjustment
 "**~10 ps scale**"
 - **variable shift** for bunches required¹

$$P_{\text{meas}} = \text{const} (I^2 + Q^2)$$

$$\theta_{\text{meas}} = \arctan(I / Q)$$





ROBUST FIT?

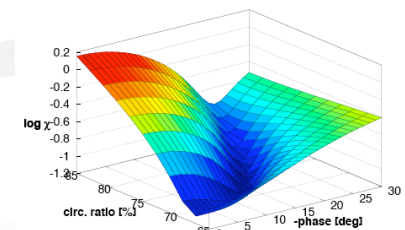
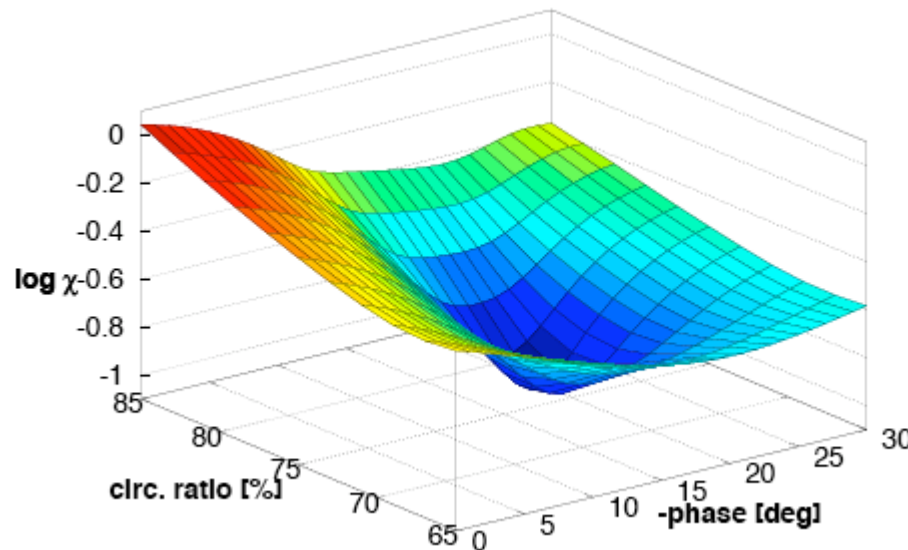
Performed: scan in 3D parameter space (c, A, phi)

- scan over 200 consecutive pulses in time window w/o pulse shortening

Minimized metric (both P and theta) :

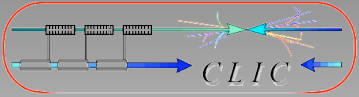
$$\chi^2(c, A, \phi) = \frac{1}{N_p} \sum_{N_p} \{ [P_{meas} - P_{mod}(c, A, \phi)]^2 / \text{pulselength} / \max(P_{meas}) \}$$

$$+ c_{PP} \frac{1}{N_p} \sum_{N_p} \{ [\theta_{meas} - \theta_{mod}(c, A, \phi)]^2 / \text{pulselength} \}$$

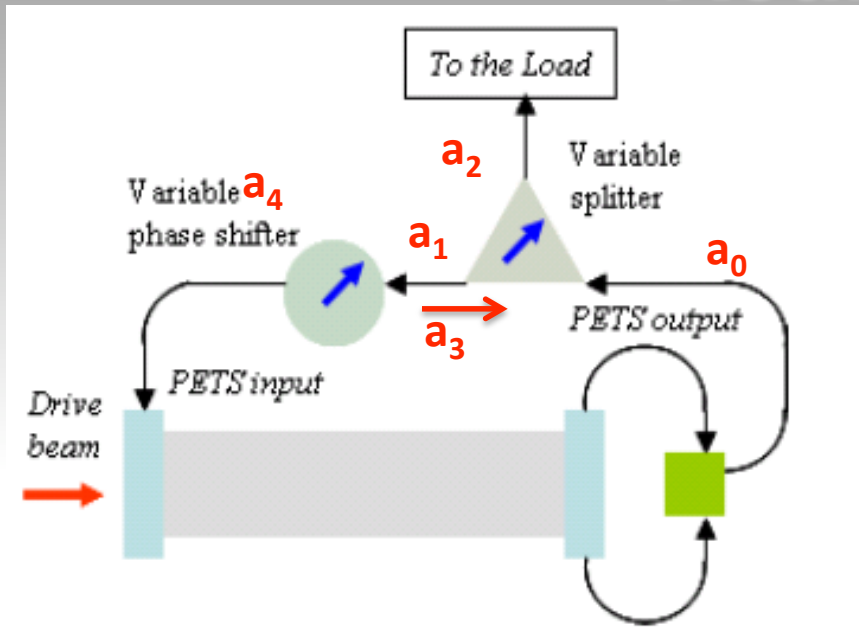


(power fit alone did not give very well defined minimum)

well defined minimum result: A=0.75, phi=18 deg -> indicates robust model



Reality check: splitter ratio



Measurement :

$$r \equiv \frac{a_4}{a_0}$$

where

$$a_0^2 = a_1^2 + a_2^2$$

and

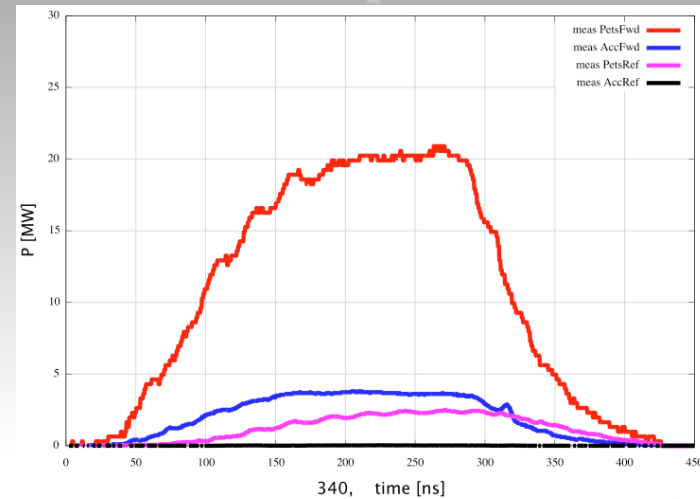
$$a_1^2 = a_3^2 + a_4^2$$

yielding

$$r^2 = \left(\frac{a_4}{a_0}\right)^2 = \frac{a_1^2 - a_3^2}{a_0^2} = \frac{a_0^2 - a_2^2 - a_3^2}{a_0^2} = 1 - \frac{A_{FWD}}{P_{FWD}} - \frac{P_{REF}}{P_{FWD}} = 1 - 0.19 - 0.11 = 0.70$$

thus

$$r_{meas} = 0.84$$



- a_0 : FIELD amplitude of PETS output - P_{FWD}
- a_1 : FIELD amplitude of recirc. arm of splitter
- a_2 : FIELD amplitude of field at load - A_{FWD}
- a_3 : FIELD amplitude field reflected (believed: from splitter) - P_{REF}
- a_4 : FIELD amplitude recirculate field

Model fit :

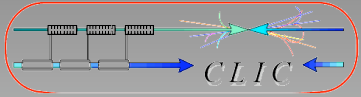
Our fit gave:

$$A = \eta_{ohm} \times r_{fit} = 0.75$$

and we estimate $\eta_{ohm} = 0.9$ giving

$$r_{fit} = 0.83$$

-> Seems consistent



Reality check: constant factor

Comparing the RF signal from the calibrated diode with the RF signal from model (using the BPMs, calibrated with test-current, but be confirmed end-to-end calibration) :

$$P_{meas} = 0.67 \times P_{mod}(FF=1)$$

sources :

$$P \approx (1/4) I^2 L_{pets}^2 FF^2 (R'/Q) \omega / v_g$$

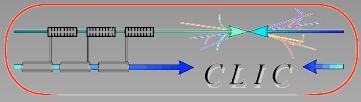
- calibration
- FF (Form Factor)
- detuning beam frequency wrt. PETS fundamental mode

If we assume ok calibrations and no detuning:

$$P_{meas} = P_{mod}(FF=0.82)$$

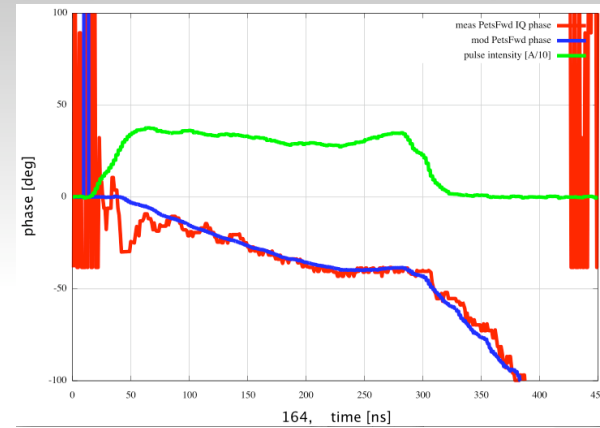
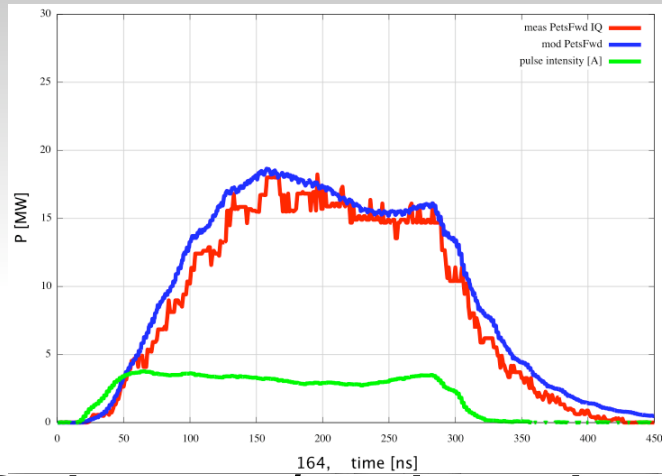
-corresponding to an rms bunch length of 2.5 mm

- > numbers seem reasonable
- > however we stress again: good rms bunch length measurement is imperative to compare measurement and model

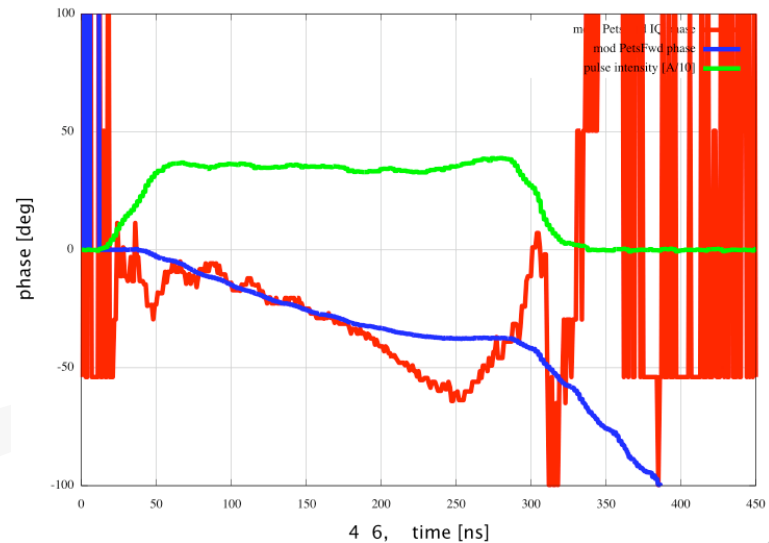
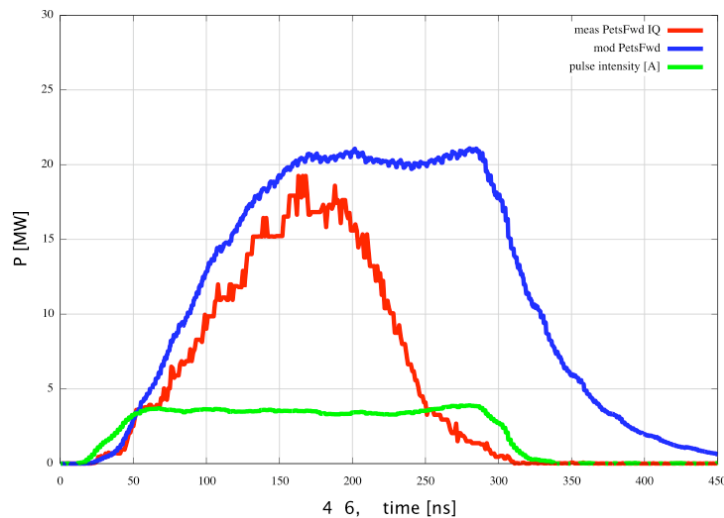


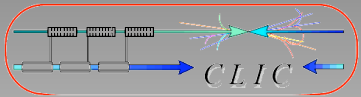
Demonstration of goodness of fit

- Pulses w/o pulse shortening (demo: many pulses)



- Pulses w/ pulse shortening (demo: many pulses)

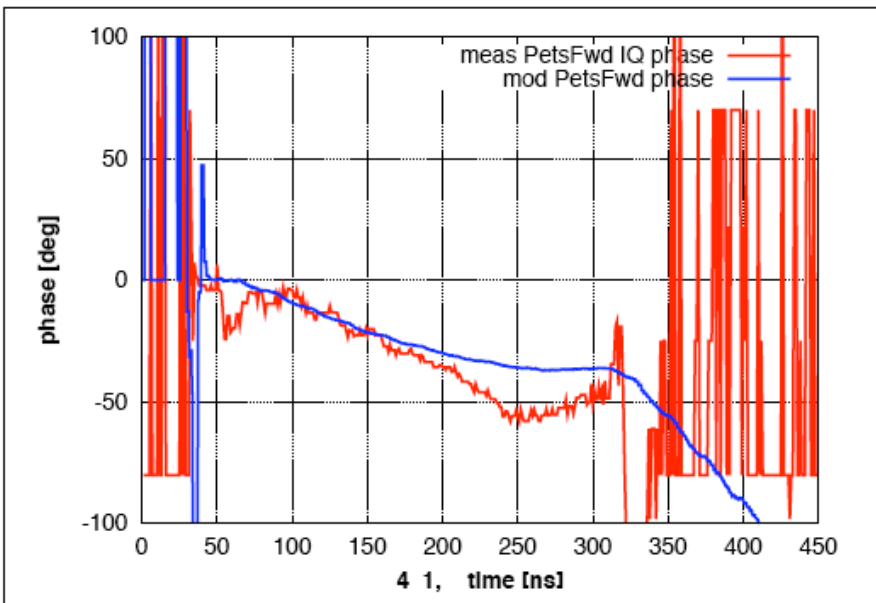




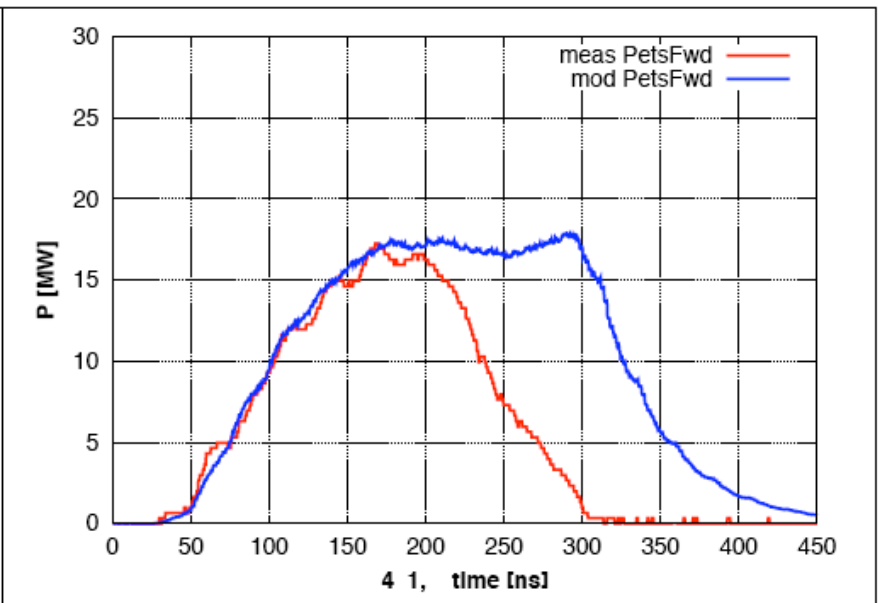
Phase behaviour of pulses w/ pulse shortening

- With fit parameters

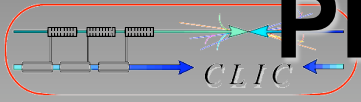
Constant recirculation phase :



PHASE: IQ versus MOD



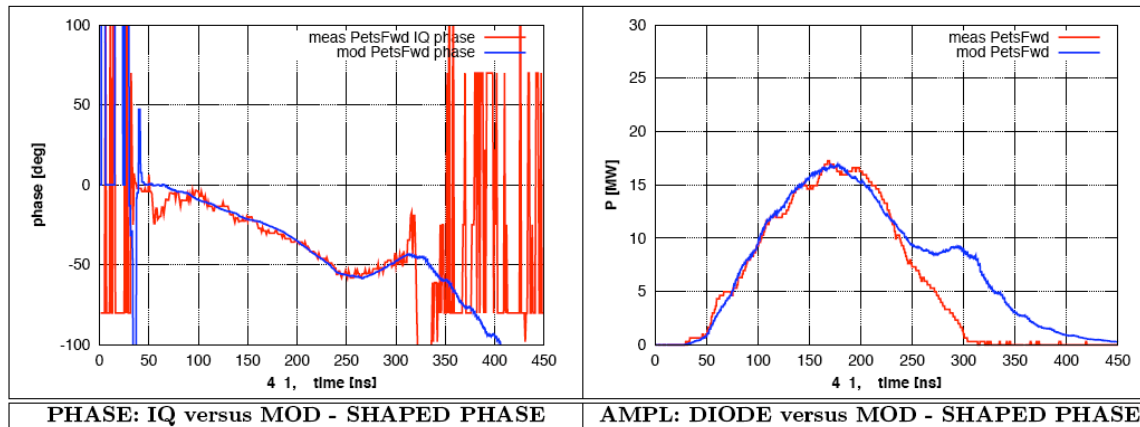
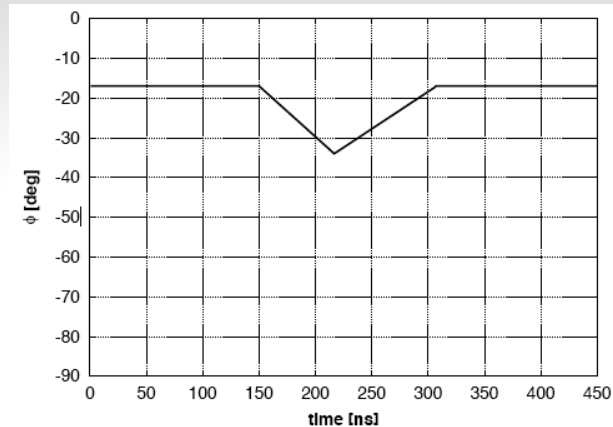
AMPL: DIODE versus MOD



Phase behaviour of pulses w/ pulse shortening

- Adjusting recirc phase to fit beam phase

Phase-dip in recirculation phase



If attenuation is also adjusted we can also fit the measured power much better
 -> however, physical models of break down is out of my current scope