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# PETS Recirculation Theory and Data Analysis

Volker Ziemann  
Uppsala University  
Department of Physics and Astronomy

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R. Ruber, V. Ziemann, *An analytical model for PETS recirculation*, CTF3-Note-092  
V. Ziemann, *Data analysis for PETS recirculation*, CTF3-Note-094



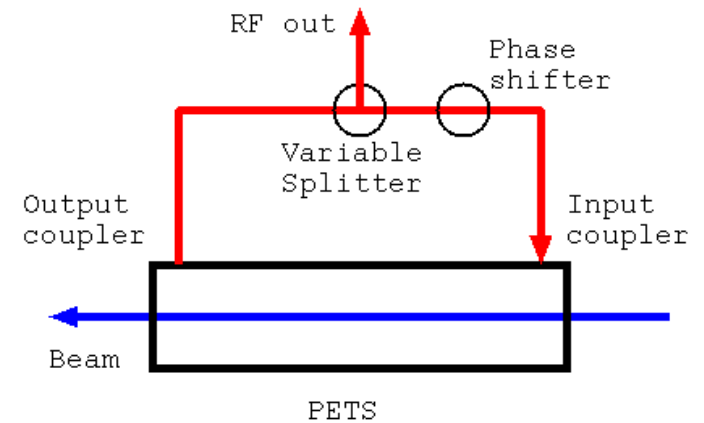
# Overview

- Simple Model
- Analytical solution with **constant** recirculation parameters (CRP) and constant bunch intensity
- Numerical solution with CRP with real data
- **Varying** recirculation parameters (VRP)
  - fit the gain and phase
  - recover coupling to beam and phase to detector
- Detector non-linearities and Breakdown
- Software

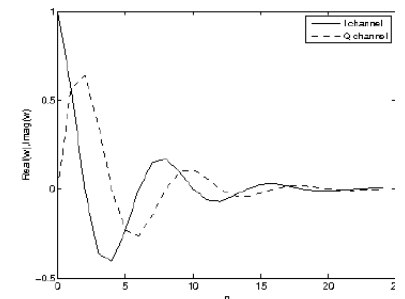


# Simple Recirculation Model

- When an electron bunch passes through PETS it generates a field burst
- that shows up in the PETS again after a round-trip time  $\tau$  and
  - is attenuated by factor  $g=e^{-\alpha}$
  - returns with a phase shift  $\phi$
- After one turn  $q=e^{i(\phi+i\alpha)}$
- 'Wake' or field in PETS of single bunch after  $n$  turns (Greens function, impulse response)



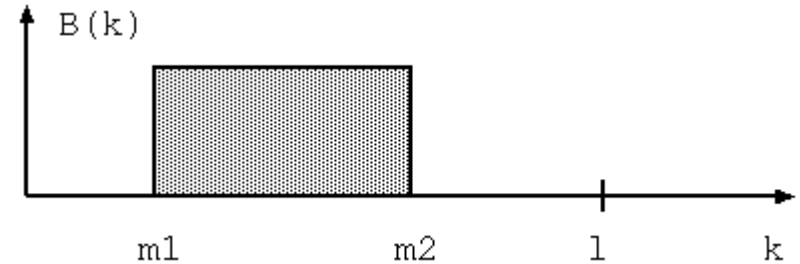
$$w(n) = g^n e^{in\phi} = e^{in(\phi+i\alpha)} = q^n$$





# Wake function from bunch train

- Assume box-like bunch distribution
- Field after time 'l' (in units of round-trip-time)



– convolution

$$f(l) = \sum_{k=m_1}^l w(l-k)B(k) = B \sum_{k=0}^{l-m_1} e^{ik(\phi+i\alpha)} = B \sum_{k=0}^{l-m_1} q^k$$

- just geometric sums
  - within bunch-train
  - after bunch-train

$$f(l) = B \frac{1 - q^{l-m_1+1}}{1 - q} \quad \text{for } m_1 \leq l \leq m_2$$

$$f(l) = B \sum_{k=m_1}^{m_2} w(l-k) = Bq^{l-m_2} \frac{1 - q^{m_2-m_1+1}}{1 - q} \quad \text{for } m_2 < l$$

- Simplify with

$$|\sin(\phi+i\alpha)|^2 = \sin^2 \phi + \sinh^2 \alpha$$

- Power =  $|f|^2$

$$p(l) = B^2 e^{-(l-m_1)\alpha} \frac{\sin^2((l-m_1+1)\phi/2) + \sinh^2((l-m_1+1)\alpha/2)}{\sin^2(\phi/2) + \sinh^2(\alpha/2)}$$

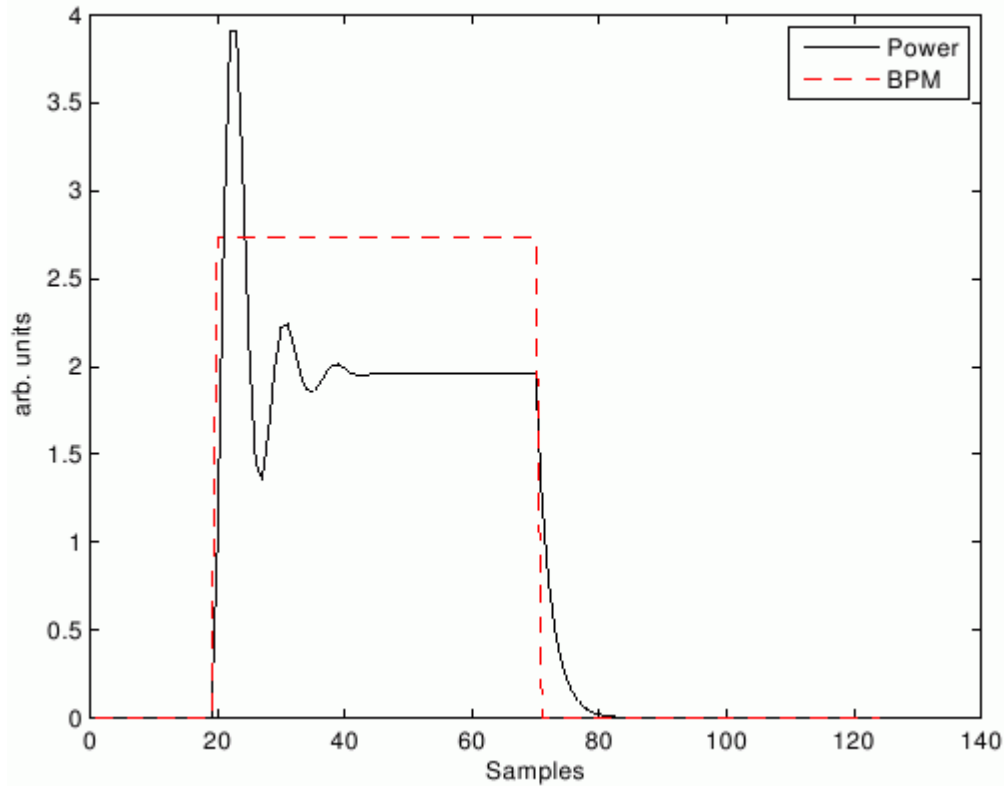
- in and behind

$$p(l) = p(m_2) e^{-2(l-m_2)\alpha} \quad \text{for } m_2 < l$$

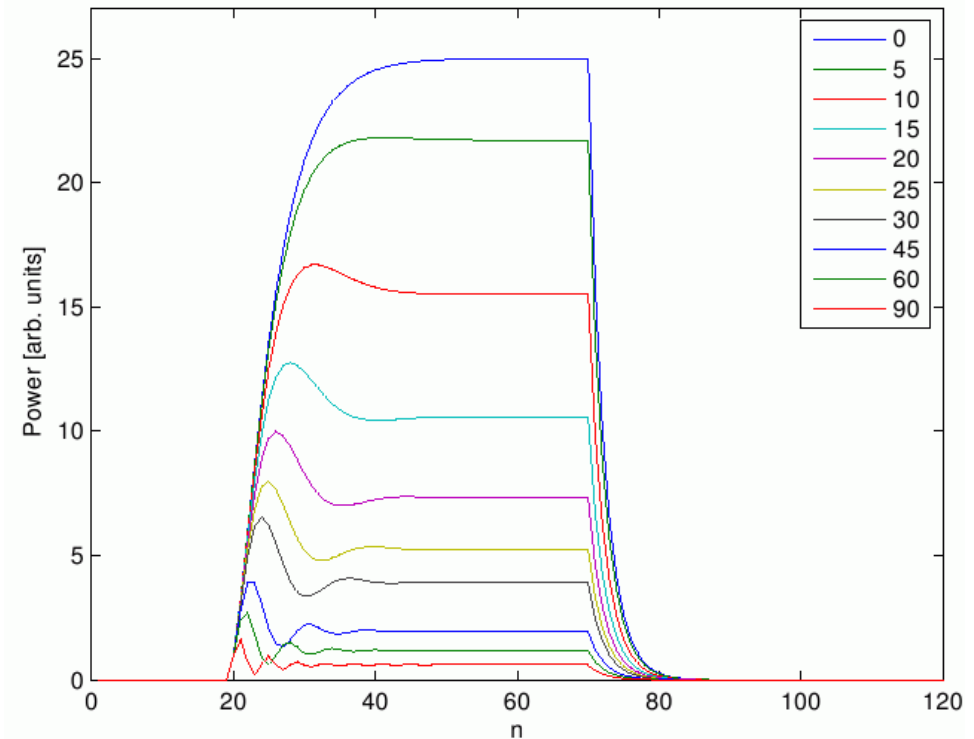


# Example

- Varying the phase with  $g=0.8$



- $g=0.8$ ,  $\phi=45$  degree





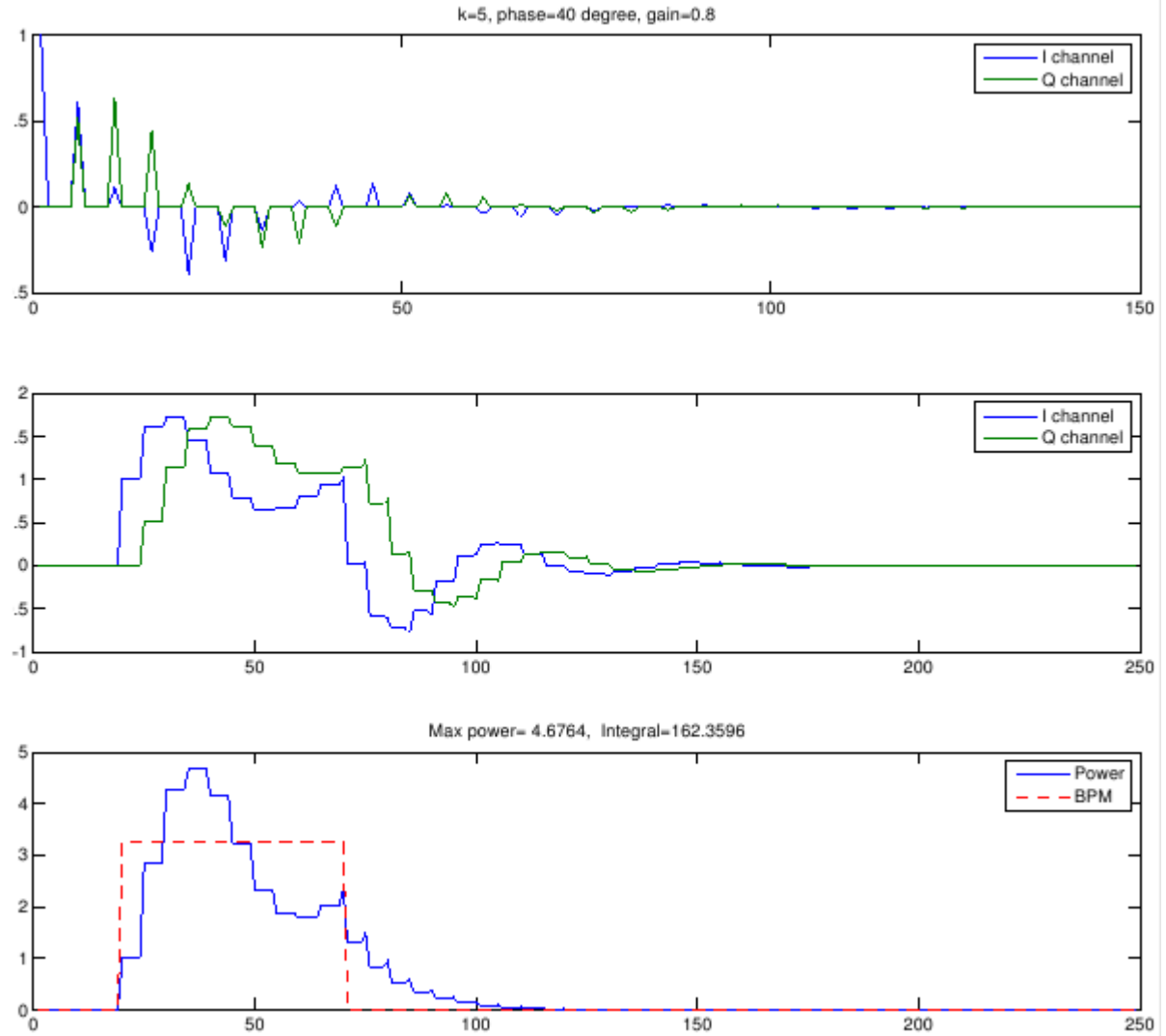
# Numerical model

- Wake with round trip time

$$w(n) = \begin{cases} e^{in(\phi+i\alpha)/k} & \text{if } n = 0, k, 2k, \dots \\ 0 & \text{else} \end{cases}$$

- $c = \text{Real}(w)$
- $s = \text{Imag}(w)$
- Matlab code

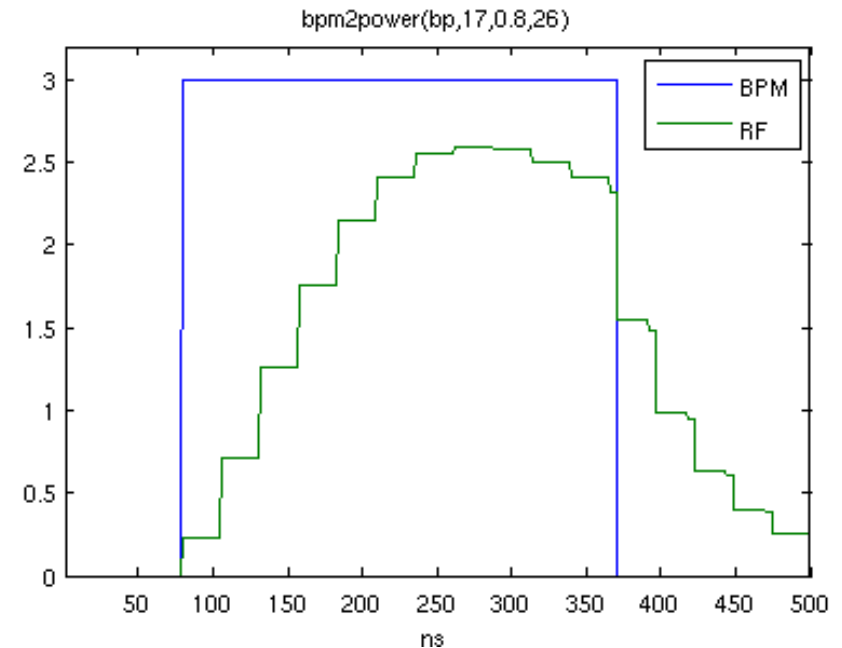
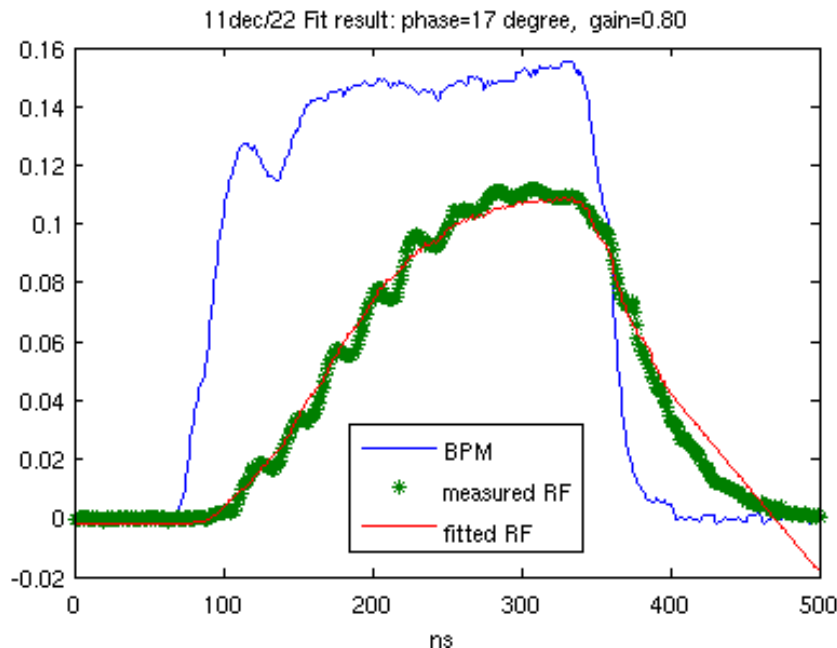
```
realf = conv(bpm,c);  
imagf = conv(bpm,s);  
power=realf.^2+imagf.^2;
```





# Steps

- Finite round-trip time causes steps in the recorded power
- Not properly reproduced



- But the simulation is OK for sharp BPM
- Limited bandwidth of the BPM smooths out step-like features.

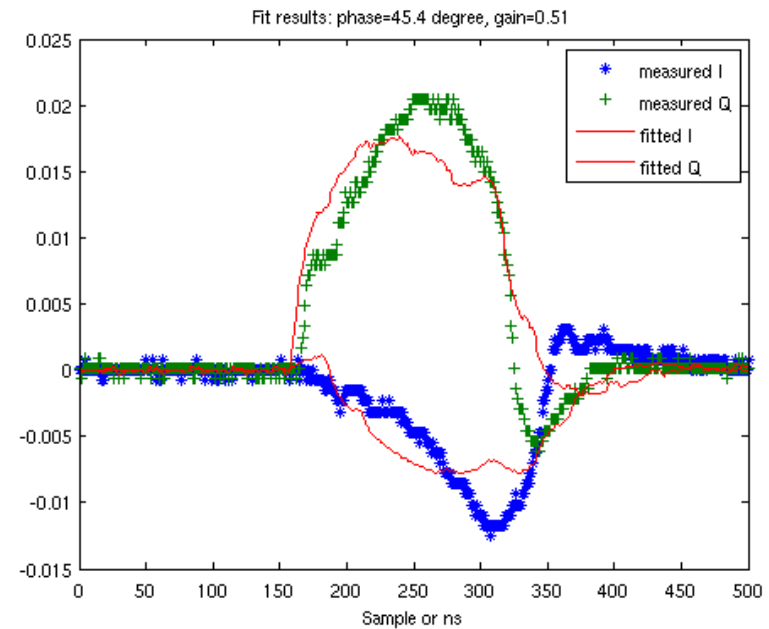
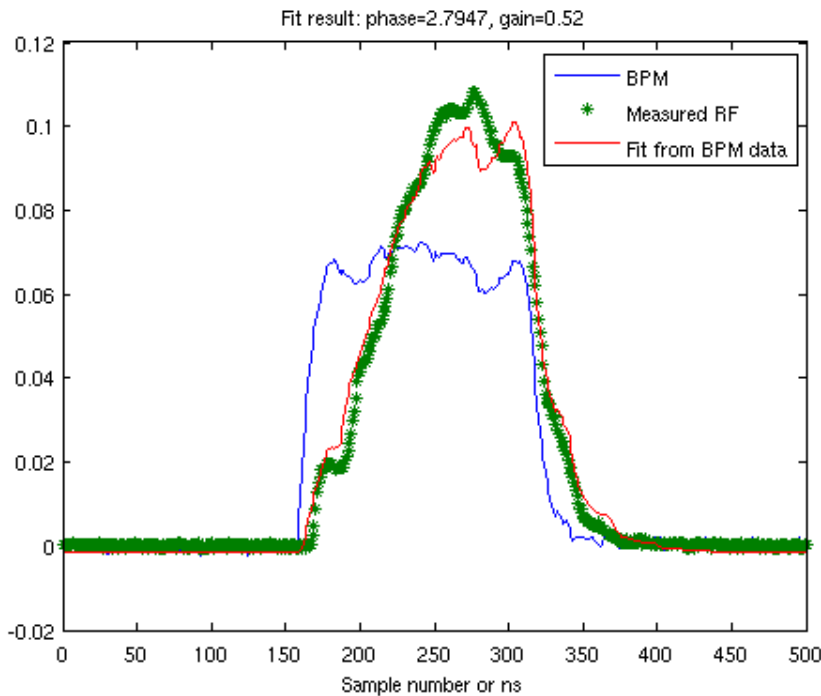


# Fit real data from Nov 28, 2008

- Fit Power profile from BPM data (26 ns rt)
  - offset, amplitude, phase, gain, delay

- Fit I-Q profile from BPM data
  - phase, gain, amplitude, extra phase

- Unsatisfactory IQ-fit







# Variable Recirculation Parameters

- What's wrong with the IQ fit?
- What information can be extracted from the IQ data? Some sort of phase? If so, what phase?
- How do the recirculation parameters change when a discharge/breakdown occurs?
- Idea: Make the recirculation parameter  $q=e^{i(\phi+i\alpha)}$  variable, i.e.  $q_m=e^{i(\phi_m+i\alpha_m)}$  for DAQ-sample  $m$ .
- Now use the following equivalent representation

$$E_m = a I_m + q_m E_{m-1} \quad (m-1 \text{ means one rt-time back})$$

where  $I_m$  = BPM signal,  $E_m$  = complex field in PETS



# Determining the $q_m$

- If we measure  $E_m$  and  $I_m$  we can calculate  $q_m$

$$q_m = \frac{E_m - aI_m}{E_{m-1}}$$

- But we measure  $E$  in the IQ detector, not  $E$  in the PETS with  $\bar{E} = e^{i\psi} E$

- Rewrite  $q_m$  in terms of the measurable  $\bar{E}$

$$q_m = \frac{e^{-i\psi} \bar{E}_m - aI_m}{e^{-i\psi} \bar{E}_{m-1}} = \frac{\bar{E}_m - ae^{i\psi} I_m}{\bar{E}_{m-1}}$$

- with unknown coupling  $a$  and phase  $\psi$



# Finding the coupling $a$ and $\psi$

- The generalized coupling  $c=ae^{i\psi}$  should be rather more slowly varying than the  $q_m$ .
- Minimize the variation  $\chi^2$  of the  $q_m$

$$\chi^2 = \frac{1}{M} \sum_{m=1}^M (q_m - \bar{q})^2$$

- with 
$$\bar{q} = \frac{1}{M} \sum_{m=1}^M q_m$$

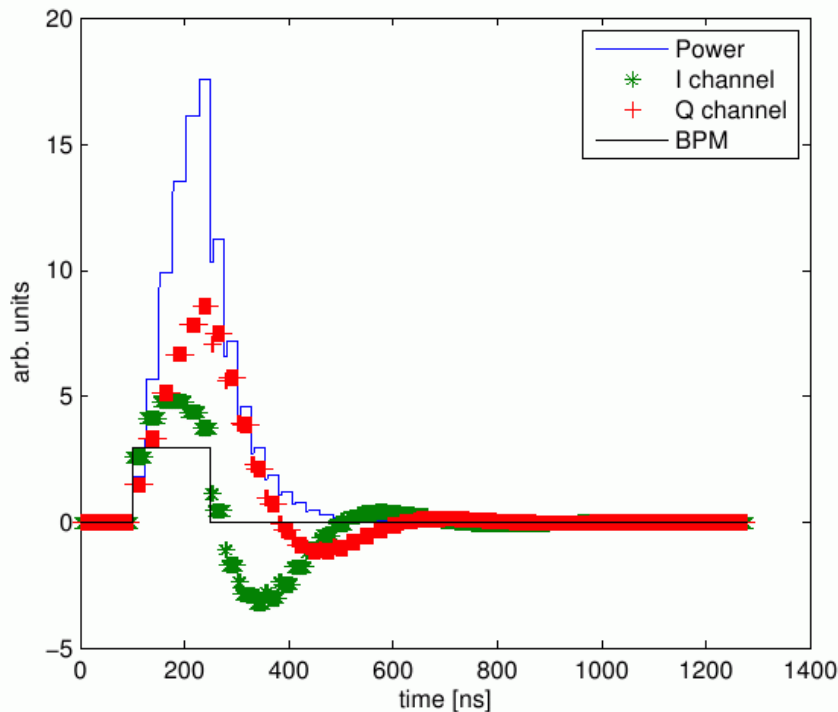
- with respect to  $a$  and  $\psi$



# Testing the Algorithm

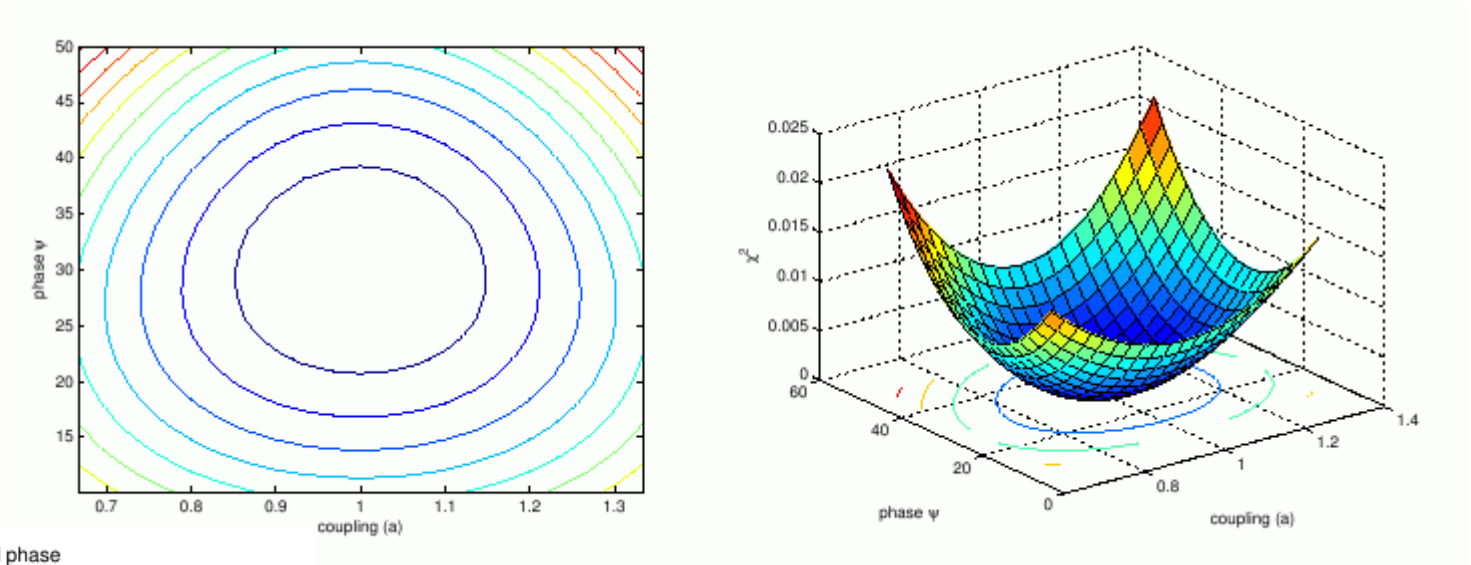
- Synthetic data
  - 26 ns,  $g=0.8$ ,  $\phi=20^\circ$
  - $a=1$ ,  $\psi=30^\circ$

```
Ebar=chI+ichQ;  
c=x(1)+i*x(2);  
cut=0.1*max(abs(Ebar));  
q=zeros(1,length(Ebar));  
for j=ktime+1:length(Ebar)  
    if abs(Ebar(j-ktime))>cut  
        q(j)=(Ebar(j)-c*II(j))/Ebar(j-ktime);  
    end  
end
```

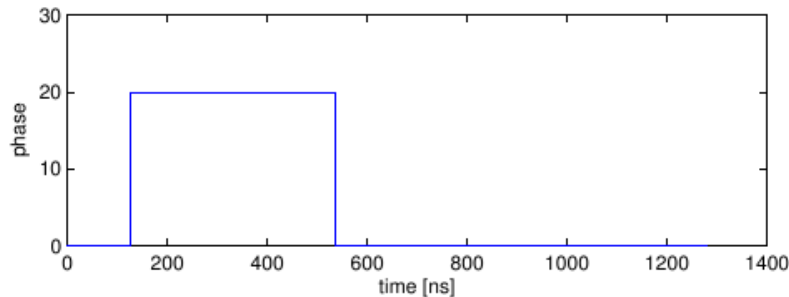
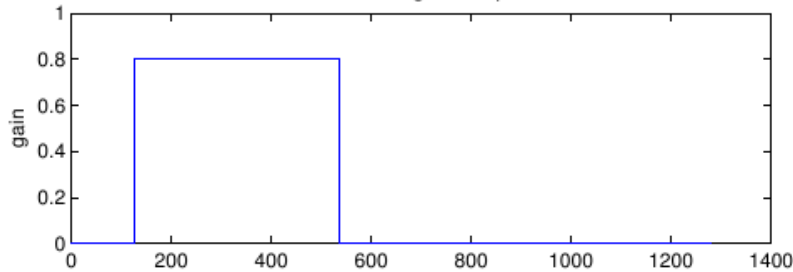


- Matlab code
  - $chI/chQ$ : measured data
  - $c=ae^{i\psi} = x_1 + ix_2 = \text{fit param}'$
  - ignore values with small  $\bar{E}$
- and then minimize the rms variation of the  $q$

# Test Results with clean data



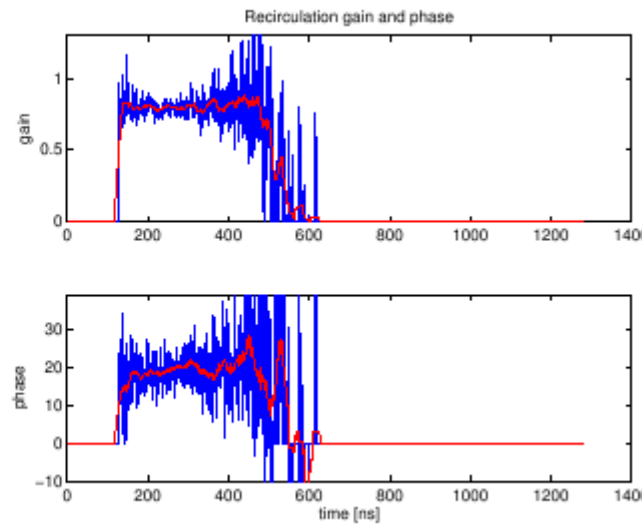
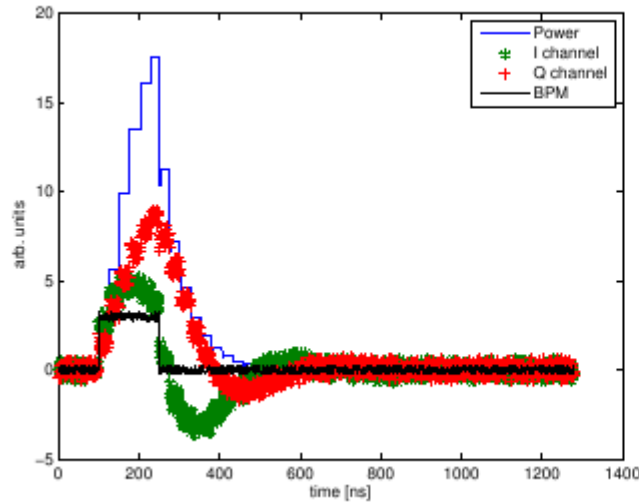
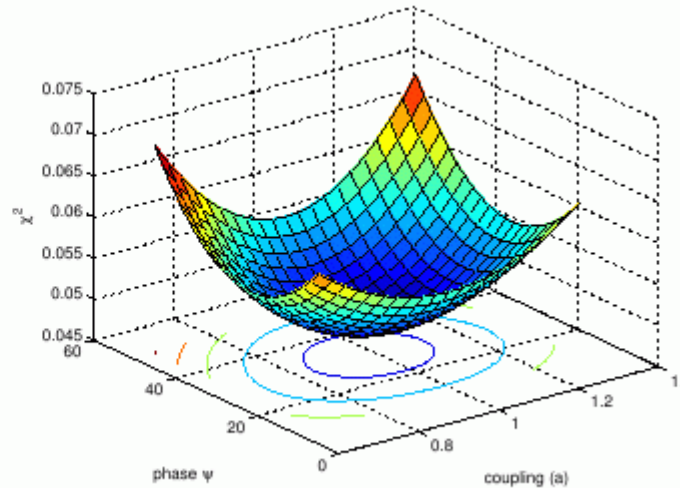
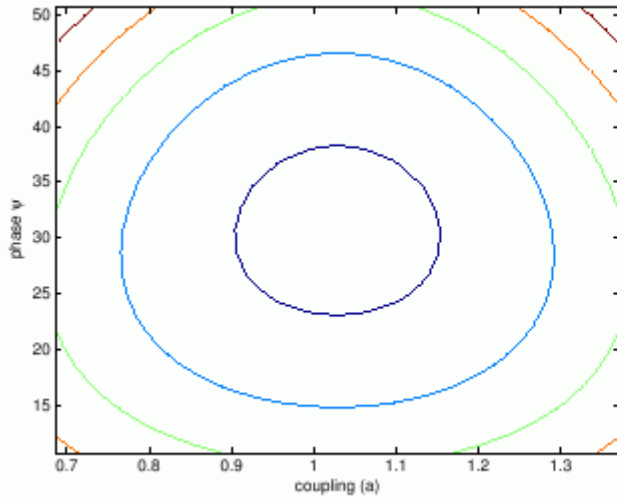
Recirculation gain and phase



- The  $X^2$  minimization yields coupling  $a$  and  $\psi$
- and the perfectly reconstructed  $g$  and  $\phi$  within the pulse



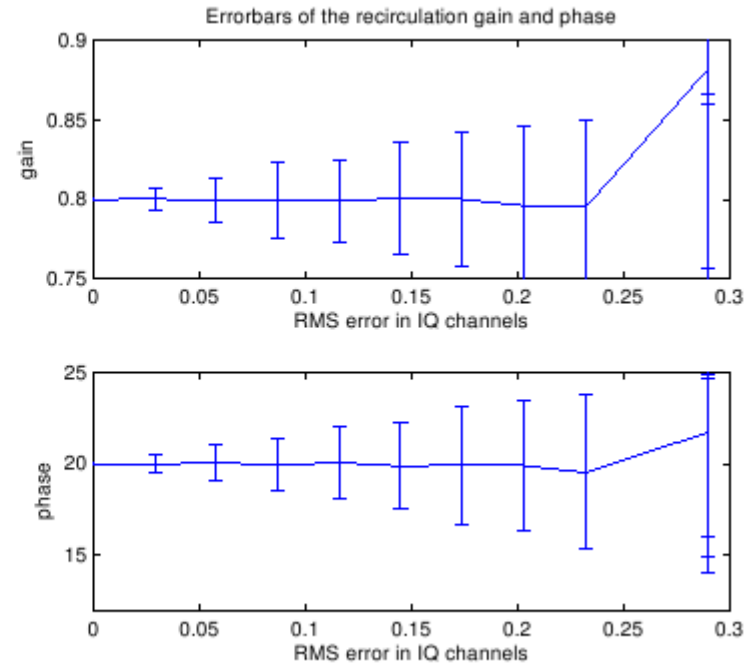
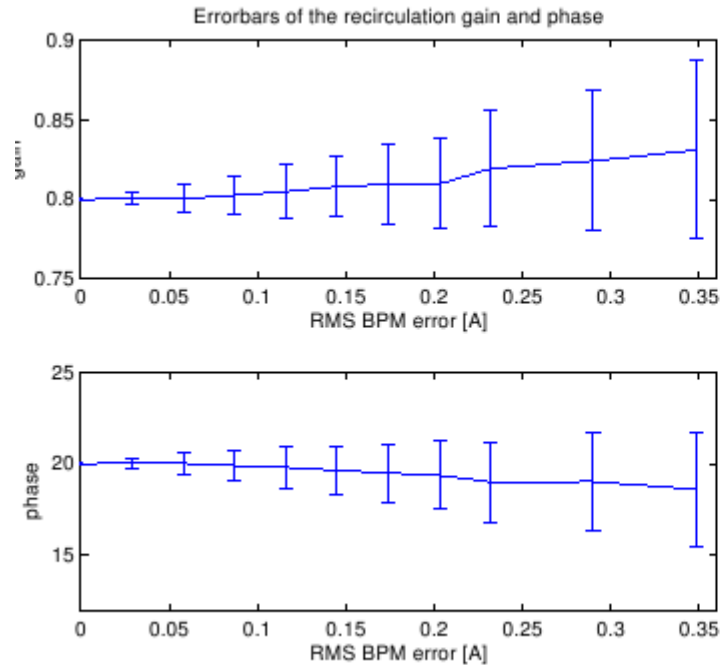
# Noisy data



- $BPM = \pm 0.25$
- $IQ = \pm 0.5$
- hard-edge
- $a = 1.04$  (1)
- $\psi = 30.7^\circ$  (30)
  
- phase and gain roughly correct
- very noisy after bpm pulse



# Noise scaling

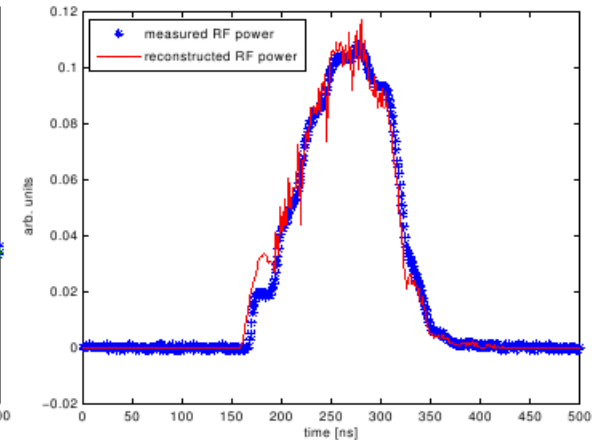
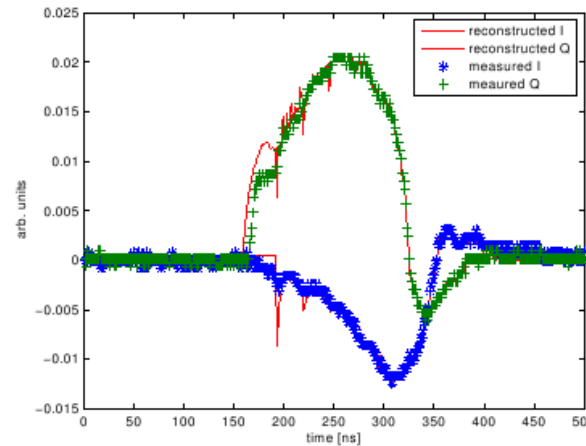
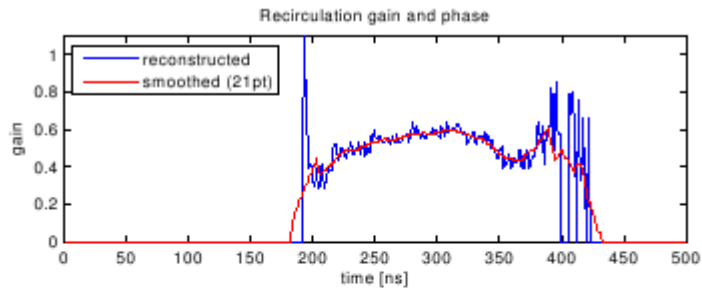
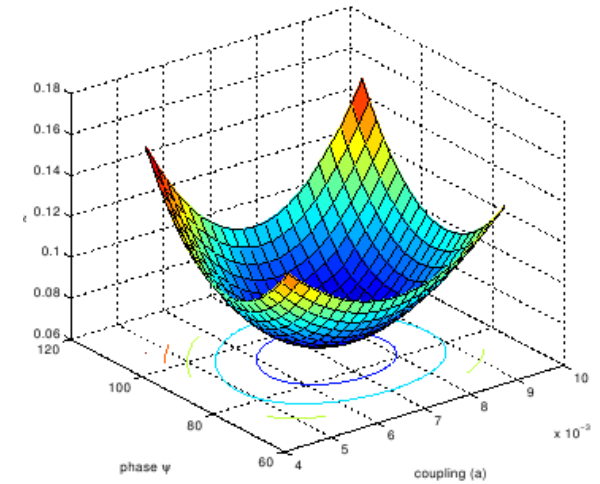
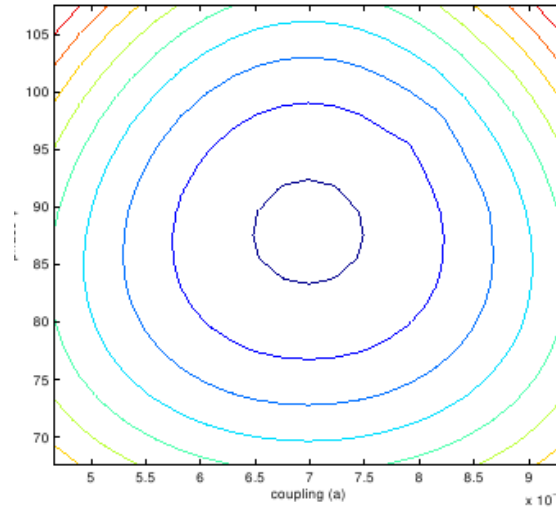
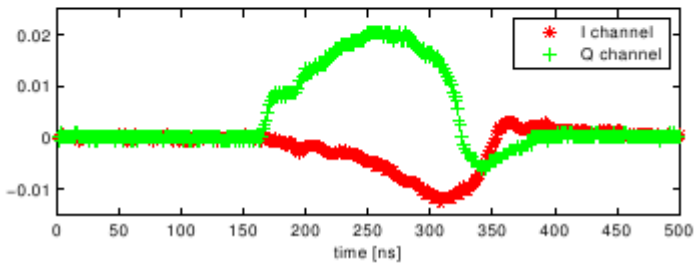
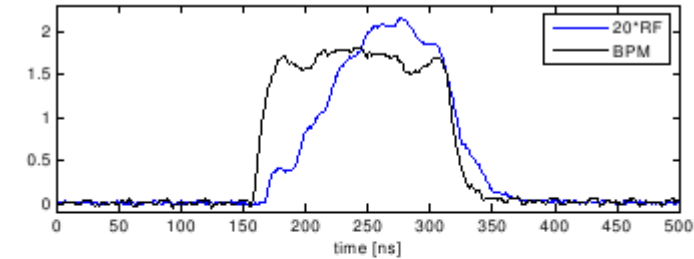


- BPM RMS-errors on the order of 0.1 A/3A
- few percent

- IQ RMS-errors on the order of 0.1/10
- few percent



# Measurements (28.11)



well-fitted I and Q

fitted I/Q and measured power

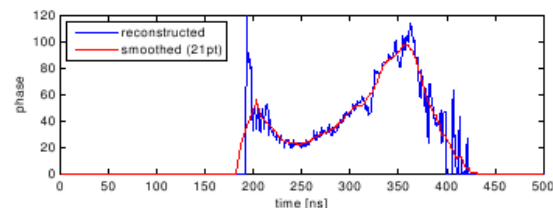
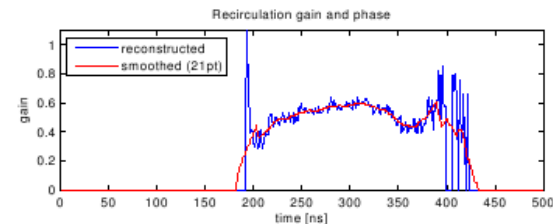




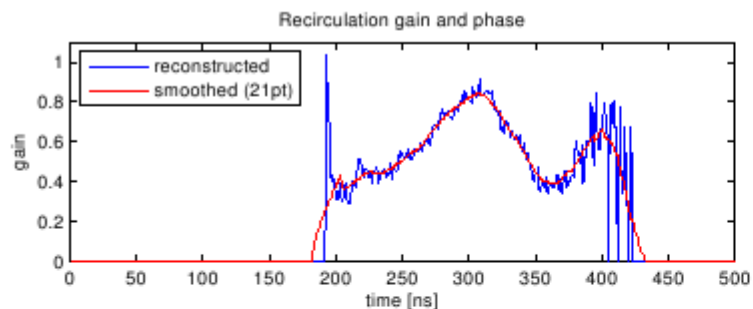
# Non-linearity of the IQ detector

- Introduce curvature in IQ
- Trade flat gain vs flat phase

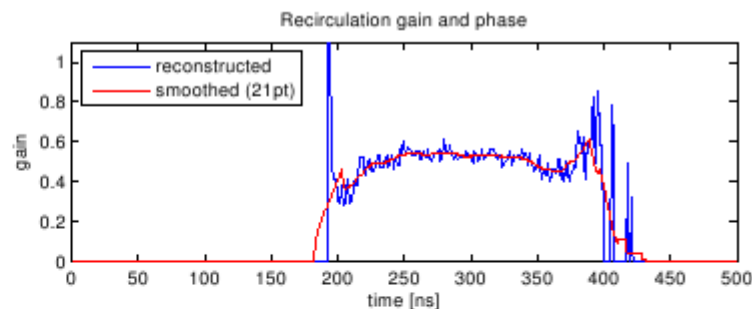
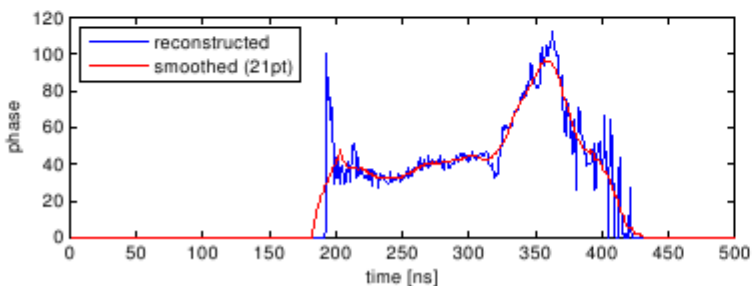
```
fudge=8; % parameter for non-linearity
chI=chI-fudge*(chI+0.03).^2+fudge*0.03^2;
chQ=chQ-fudge*(chQ+0.03).^2+fudge*0.03^2;
```



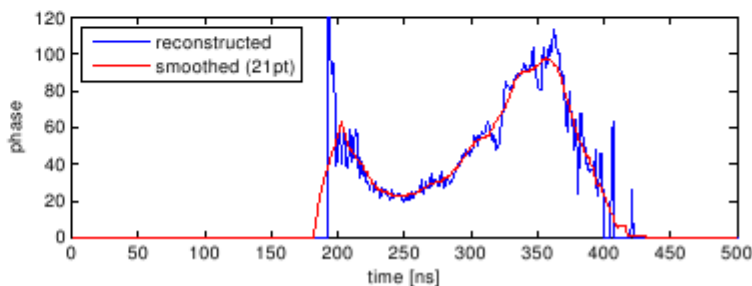
0



+8

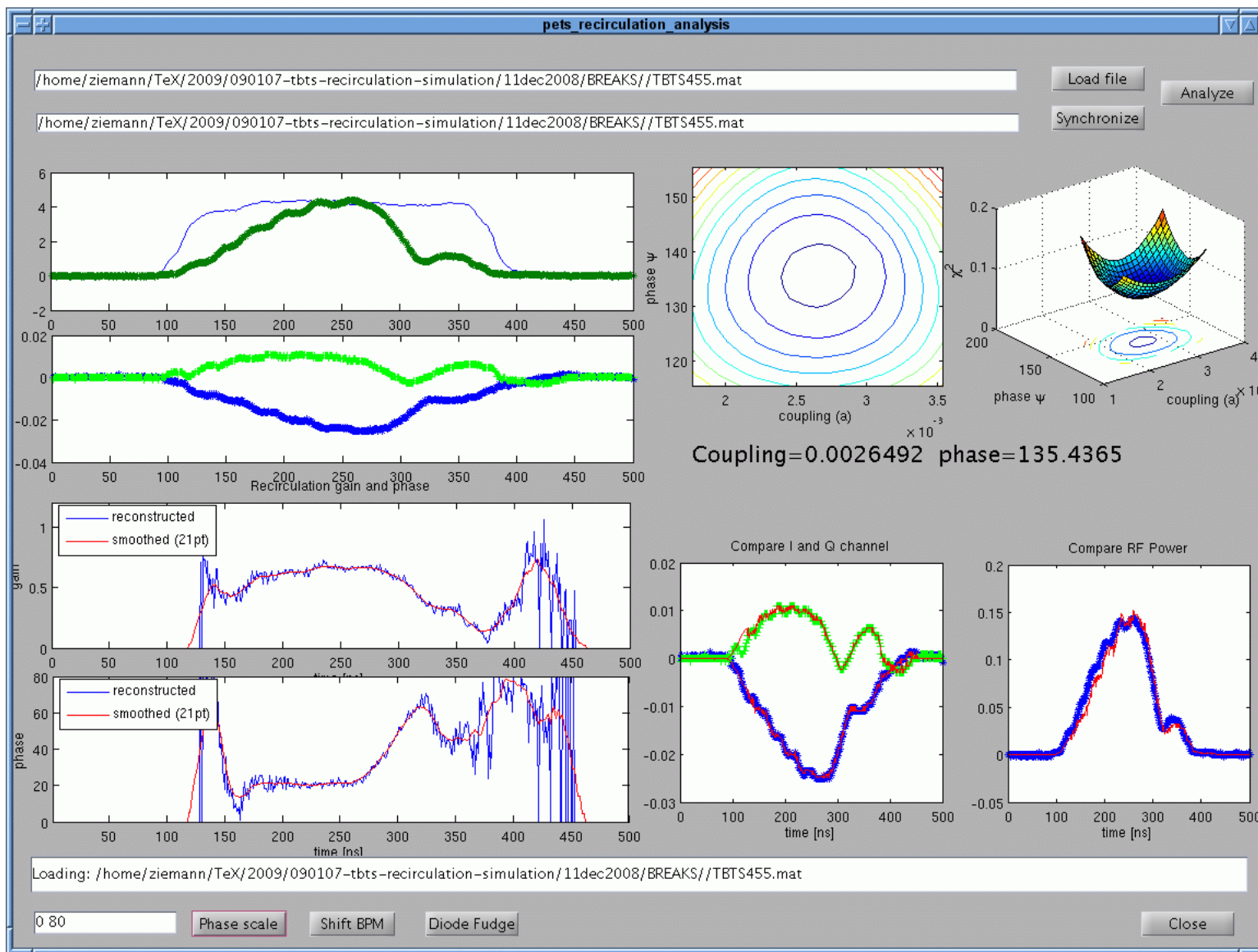


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# Software





# Breakdown

- The event on the previous slide shows
  - shortened RF-pulse
  - gain-drop
  - phase shift
- Interpretation attempt
  - free electrons absorb RF power, thus the gain drop
  - and change the dielectric properties and therefore the refractive index, thus the phase shift



# Conclusion

- Simple model for PETS recirculation
  - with constant recirculation parameters
  - Works well for power, poor for IQ
- Extension for varying parameters: can reconstruct the phase and gain along the pulse
  - now both power and IQ signals fit well
- Breakdown
- Software



# Another data set

