SUSY and the Alignment Limit

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Outline

- The alignment limit: approaching the SM Higgs boson
- Alignment in the CP-conserving 2HDM
- Alignment in the MSSM Higgs sector
- The MSSM Higgs sector in light of precision Higgs data
- Impact of the leading two-loop corrections
- The alignment limit revealed in an MSSM parameter scan
- Adding a Higgs singlet to the 2HDM
- The alignment limit in the NMSSM
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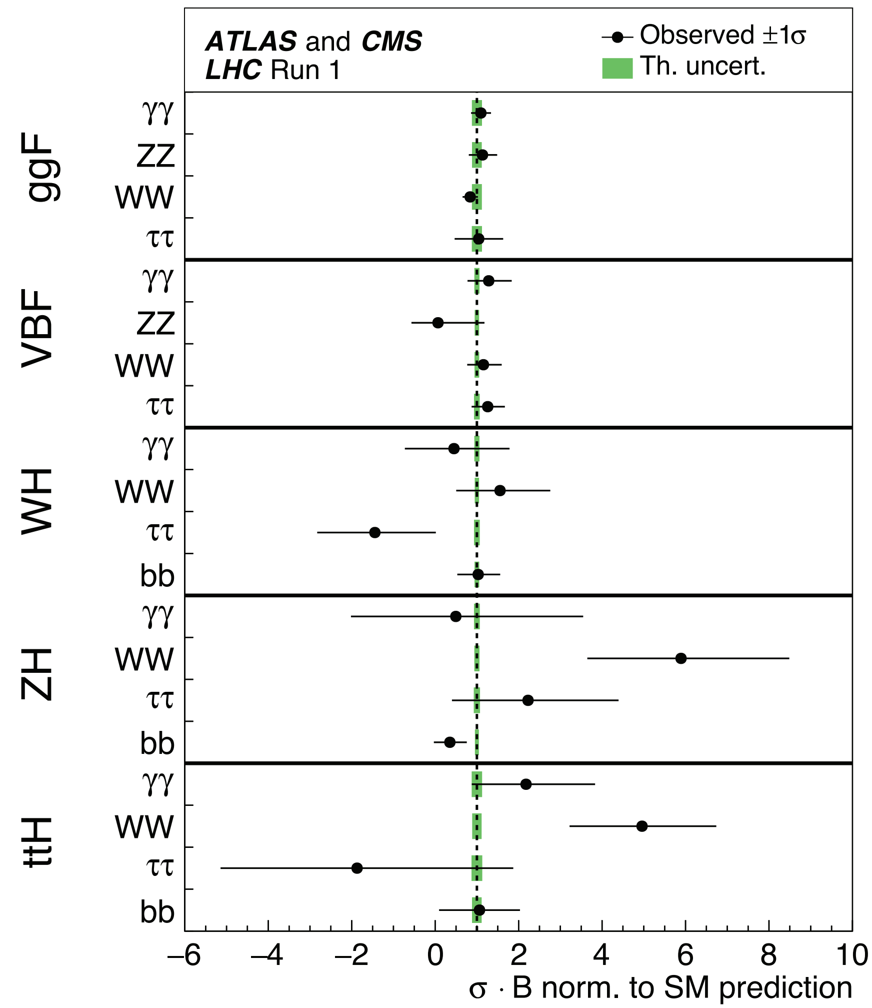
Implications of a SM-like Higgs boson

We already know that the observed Higgs boson is SM-like.

Thus, the LHC Higgs data constrain any model of new physics beyond the Standard Model that incorporates a non-minimal Higgs sector.

In models of an extended Higgs sector, a SM-like Higgs boson can be achieved in a particular limit of the model called the alignment limit.



The alignment limit: approaching the SM Higgs boson

Consider an extended Higgs sector with n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i.

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vacuum expectation values (in order to preserve $\mathrm{U}(1)_{\mathrm{EM}}$),

$$\langle \Phi_i^0 \rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle \phi_j^0 \rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\mathrm{GeV})^2$.

The Higgs basis

Define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^* \Phi_i \,, \qquad\qquad \langle H_1^0 \rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations of doublet scalar fields such that $\langle H_i^0 \rangle = 0$ (for $i = 2, 3, \ldots, n$).

That is H_1^0 is aligned in field space with the direction of the Higgs vacuum expectation value (vev). Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson. This is the exact alignment limit.

A SM-like Higgs boson

In general, $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ is not a mass-eigenstate due to mixing with other neutral scalars. Nevertheless, a SM-like Higgs boson exists if either:

- the diagonal squared masses of the other Higgs basis scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

and/or

- the elements of the neutral scalar squared-mass matrix that govern the mixing of $\sqrt{2}\,\mathrm{Re}(H_1^0)-v$ with other neutral scalars are suppressed.

Alignment without decoupling*

The alignment limit is most naturally achieved in the decoupling regime. However, in this case the additional Higgs boson states are very heavy and may be difficult to observe at the LHC.

In the case of approximate alignment without decoupling (due to suppressed scalar mixing), non-SM-like Higgs boson states need not be very heavy and thus more easily accessible at the LHC.

REMARK: In some theories, alignment without decoupling can be achieved by a symmetry (e.g., the inert doublet model). In most cases, approximate alignment is an accidental (fine-tuned?) region of the model parameter space.

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*J.F. Gunion and H.E. Haber, hep-ph/0207010; N. Craig, J. Galloway and S. Thomas, arXiv:1305.2424.

Alignment in the CP-conserving 2HDM

The Higgs sector of the MSSM is a CP-conserving two-Higgs doublet model (2HDM). Consider the Higgs basis fields,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v},$$

such that $\langle H_1^0 \rangle = v/\sqrt{2}$ and $\langle H_2^0 \rangle = 0$. The most relevant terms of the Higgs basis scalar potential are:

$$\mathcal{V} \ni \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \left\{\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.}\right\}.$$

For a CP-conserving scalar potential, Z_5 and Z_6 are real.

Remark: Generically, the Z_i are $\mathcal{O}(1)$ parameters.

We identify the CP-odd Higgs boson as $A = \sqrt{2}\,\mathrm{Im}\, H_2^0$, with squared-mass m_A^2. The CP-even Higgs squared-mass matrix with respect to the Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\ H_1^0 - v\,, \sqrt{2}\,\mathrm{Re}\ H_2^0\}$ is given by,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix} .$$

The CP-even Higgs bosons are h and H with $m_h \leq m_H$. The couplings of $\sqrt{2}\,\mathrm{Re}\ H_1^0 - v$ coincide with those of the SM Higgs boson. Alignment arises two limiting cases:

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A \sim m_H \sim m_{H^\pm} \gg m_h \simeq (Z_1 v^2)^{1/2}$.

2. $|Z_6| \ll 1$. Then, h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, H is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\,H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the original basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Since the SM-like Higgs boson must be approximately $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, it follows that

- h is SM-like if $|c_{\beta-\alpha}| \ll 1$ (alignment with or without decoupling, depending on the magnitude of m_A),
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$ (alignment without decoupling).

The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$Z_1 v^2 = m_h^2 s^2_{\beta-\alpha} + m_H^2 c^2_{\beta-\alpha}\,,$$

$$Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha}\,,$$

$$Z_5 v^2 = m_H^2 s^2_{\beta-\alpha} + m_h^2 c^2_{\beta-\alpha} - m_A^2\,.$$

If h is SM-like, then $m_h^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|c_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,,$$

If H is SM-like, then $m_H^2 \simeq Z_1 v^2$ (i.e., $Z_1 \simeq 0.26$) and

$$|s_{\beta-\alpha}| = \frac{|Z_6|v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6|v^2}{m_H^2 - m_h^2} \ll 1\,.$$

The MSSM Higgs Sector at tree-level

The MSSM Higgs sector is a CP-conserving 2HDM. At tree level, the Higgs basis parameters of interest are fixed by SUSY:

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 \,, \qquad Z_5 v^2 = m_Z^2 s_{2\beta}^2 \,, \qquad Z_6 v^2 = -m_Z^2 s_{2\beta} c_{2\beta} \,.$$

It follows that,

$$\cos^2(\beta - \alpha) = \frac{m_Z^4 \, s_{2\beta}^2 c_{2\beta}^2}{(m_H^2 - m_h^2)(m_H^2 - m_Z^2 c_{2\beta}^2)} \,.$$

The decoupling limit is achieved when $m_H \gg m_h$ as expected. Alignment without decoupling is (naively) possible at tree-level when $Z_6 = 0$, which yields $\sin 4\beta \simeq 0$. However, this limit is not phenomenologically viable. In any case, radiative corrections are required to obtain the observed Higgs mass of 125 GeV.

Tree-level MSSM Higgs couplings to quarks and squarks

The MSSM employs the Type–II Higgs–fermion Yukawa couplings. It is convenient to write these couplings using the more common MSSM notation, $H_D^i \equiv \epsilon_{ij}\Phi_1^{j\,*}$ and $H_U^i = \Phi_2^i$,

$$-\mathscr{L}_{\rm Yuk} = \epsilon_{ij}\left[h_b\bar{b}_R H_D^i Q_L^j + h_t\bar{t}_R Q_L^i H_U^j\right] + {\rm h.c.}\,,$$

which yields $m_b = h_b v c_\beta/\sqrt{2}$ and $m_t = h_t v s_\beta/\sqrt{2}$.

The leading terms in the coupling of the Higgs bosons to third generation squarks are proportional to the Higgs–top quark Yukawa coupling h_t, and depend on the SUSY parameters μ, A_t,

$$\mathscr{L}_{\rm int} \ni h_t\big[\mu^*(H_D^\dagger\widetilde{Q})\widetilde{U}+A_t\epsilon_{ij}H_U^i\widetilde{Q}^j\widetilde{U}+{\rm h.c.}\big]-h_t^2\left[H_U^\dagger H_U(\widetilde{Q}^\dagger\widetilde{Q}+\widetilde{U}^*\widetilde{U})-|\widetilde{Q}^\dagger H_U|^2\right],$$

where $\widetilde{Q} = \begin{pmatrix}\widetilde{t}_L\\ \widetilde{b}_L\end{pmatrix}$ and $\widetilde{U} \equiv \widetilde{t}_R^*$.

In terms of the Higgs basis fields H_1 and H_2,

$$\begin{aligned}\mathscr{L}_{\rm int} \ni\ & h_t\epsilon_{ij}\big[(\sin\beta X_t H_1^i + \cos\beta Y_t H_2^i)\widetilde{Q}^j\widetilde{U} + {\rm h.c.}\big] \\ & -h_t^2\left\{\left[s_\beta^2|H_1|^2 + c_\beta^2|H_2|^2 + \sin\beta\cos\beta(H_1^\dagger H_2 + {\rm h.c.})\right](\widetilde{Q}^\dagger\widetilde{Q} + \widetilde{U}^*\widetilde{U}) \right. \\ & \left. \qquad -s_\beta^2|\widetilde{Q}^\dagger H_1|^2 - c_\beta^2|\widetilde{Q}^\dagger H_2|^2 - \sin\beta\cos\beta\big[(\widetilde{Q}^\dagger H_1)(H_2^\dagger\widetilde{Q}) + {\rm h.c.}\big]\right\},\end{aligned}$$

where

$$X_t \equiv A_t - \mu^*\cot\beta\,, \qquad\qquad Y_t \equiv A_t + \mu^*\tan\beta\,.$$

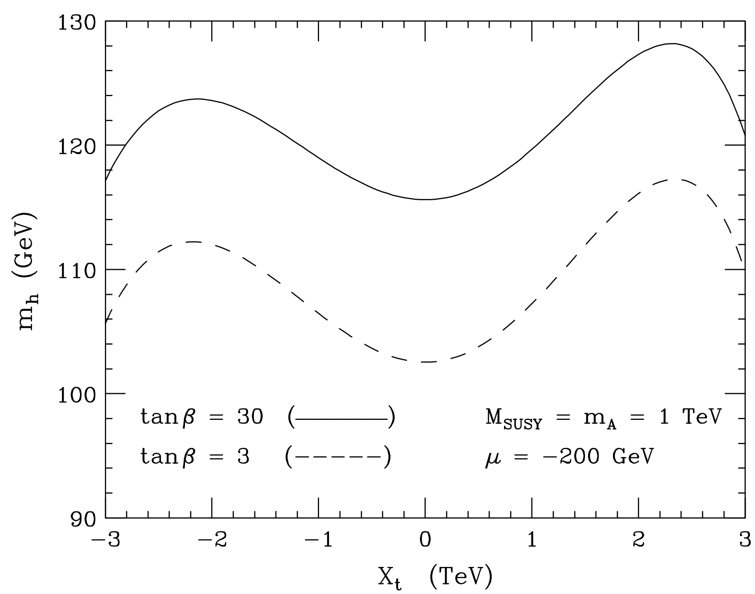
Assuming CP-conservation for simplicity, we shall henceforth take μ, A_t real.

By convention, $\tan\beta$ is (real and) positive.

The radiatively corrected MSSM Higgs Sector

The leading effects due to radiative corrections can be illustrated in the limit where m_h, m_A, m_H, $m_{H^\pm} \ll M_S$, where M_S is the SUSY-breaking scale. In this case, we can formally integrate out the squarks and generate a low-energy effective 2HDM Lagrangian (which is no longer of the tree-level MSSM form).

Large radiative corrections can easily accommodate the observed Higgs mass of 125 GeV (in some regions of the MSSM parameter space).



The dominant one-loop corrected expressions for Z_1 and Z_6 are given by[†]

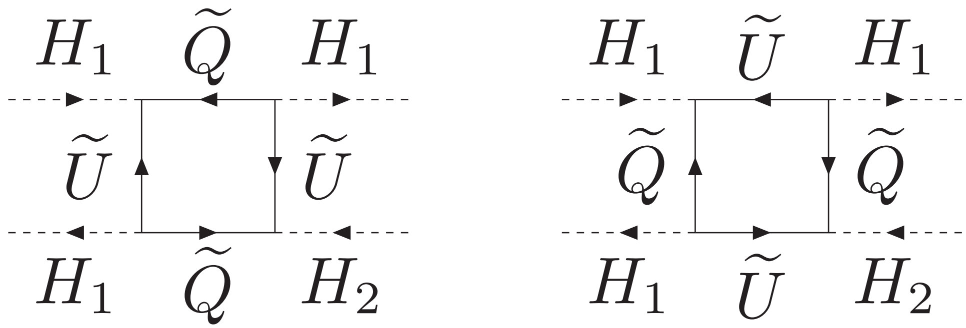
$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\},$$

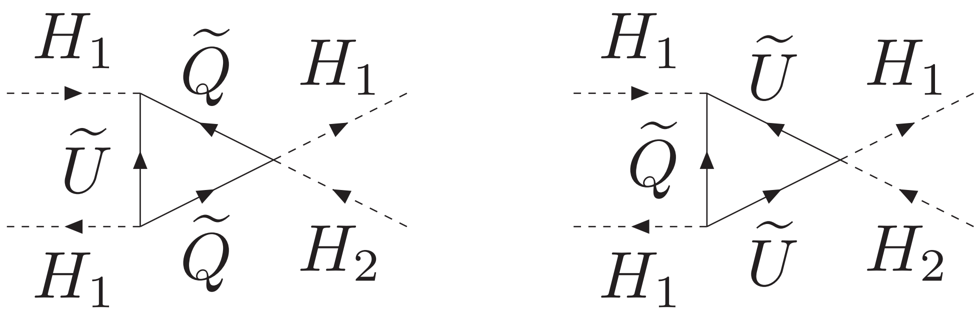
where $M_S^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu\cot\beta$ and $Y_t = A_t + \mu\tan\beta$.

Note that $m_h^2 \simeq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for M_S (as a function of $\tan\beta$ and X_t). Exact alignment (i.e., $Z_6 = 0$) can now be achieved due to an accidental cancellation between tree-level and loop contributions.

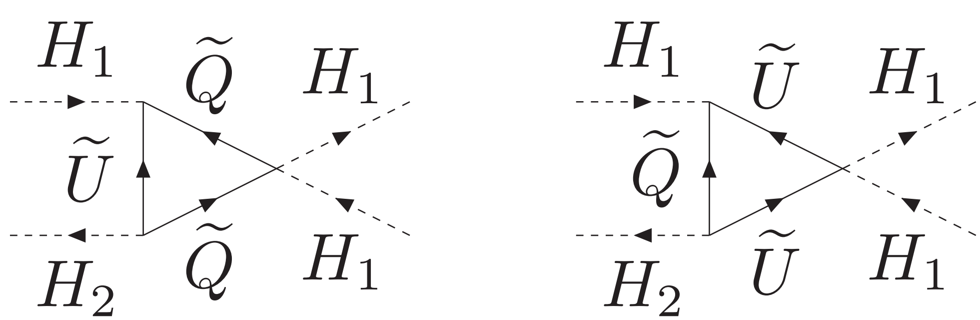
[†]CP-violating phases that could appear in the MSSM parameters such as μ and A_t are neglected. The expression for Z_6 exhibited above first appears in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, arXiv:1410.4969.



$$\propto s_\beta^3 c_\beta X_t^3 Y_t$$



$$\propto s_\beta^3 c_\beta X_t^2$$



$$\propto s_\beta^3 c_\beta X_t Y_t$$

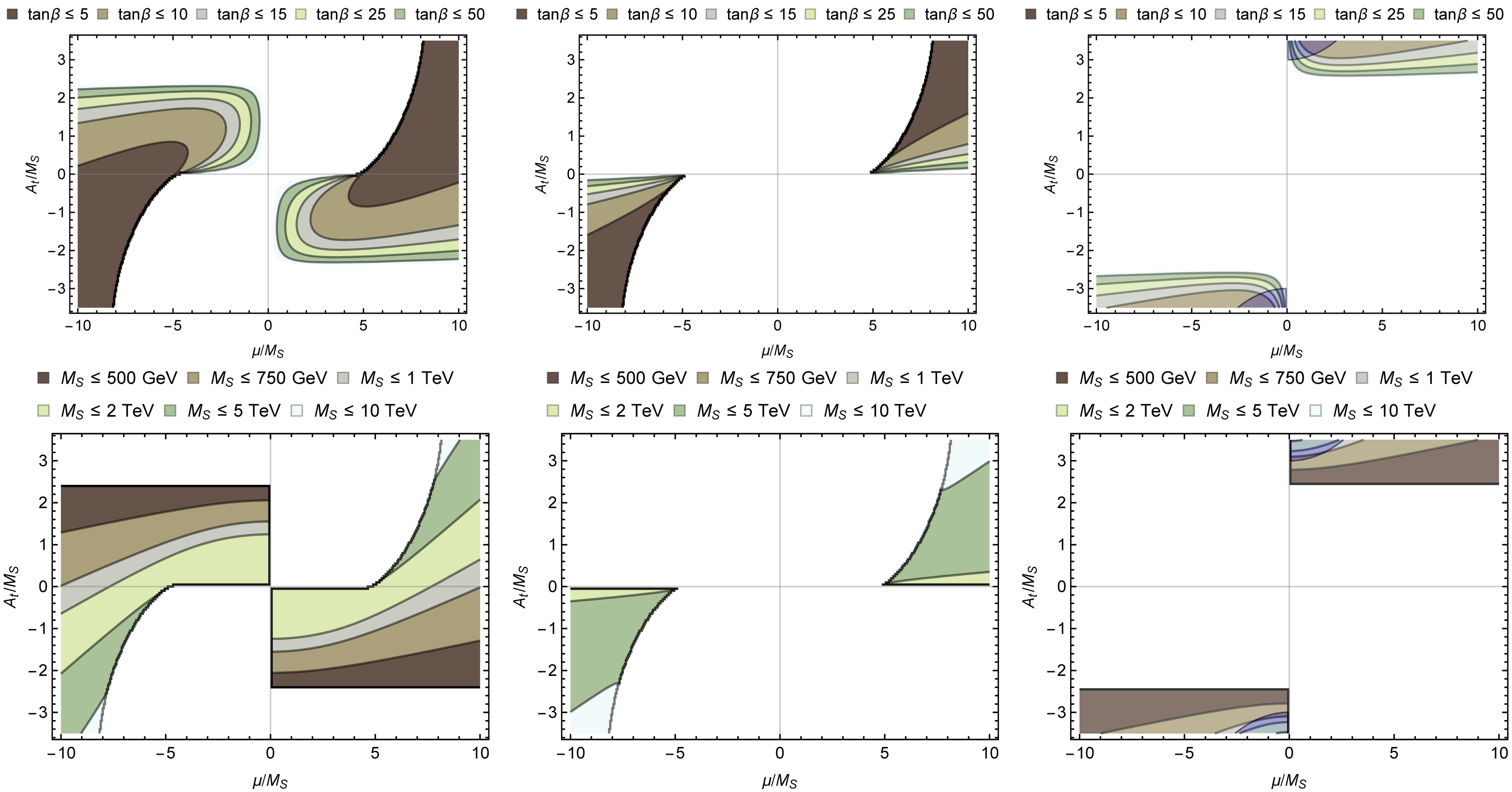
Example: One-loop threshold corrections to Z_6

Setting $Z_6 = 0$, one obtains a 7th order polynomial equation for $t_\beta \equiv \tan\beta$ as a function of $\widehat{A}_t \equiv A_t/M_S$ and $\widehat{\mu} \equiv \mu/M_S$,

$$m_Z^2 t_\beta^4(1-t_\beta^2) - Z_1 v^2 t_\beta^4(1+t_\beta^2) + \frac{3m_t^4\widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu})(1+t_\beta^2)^2}{4\pi^2 v^2}\left[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\right] = 0\,.$$

which can be solved numerically for real positive solutions.

Typically, we identify h as the SM-like Higgs boson. However, in the alignment limit there exist parameter regimes, corresponding to the case of $m_A^2 + (Z_5 - Z_1)v^2 < 0$ (where the radiatively corrected Z_1 and Z_5 are employed), in which H is the SM-like Higgs boson. In either case, $Z_1 v^2$ is the (approximate) squared mass of the SM-like Higgs boson.



Top panels: Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane, in the one-loop approximation. Z_1 is adjusted to give the correct Higgs mass. Taking the three top panels together, one can immediately discern the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

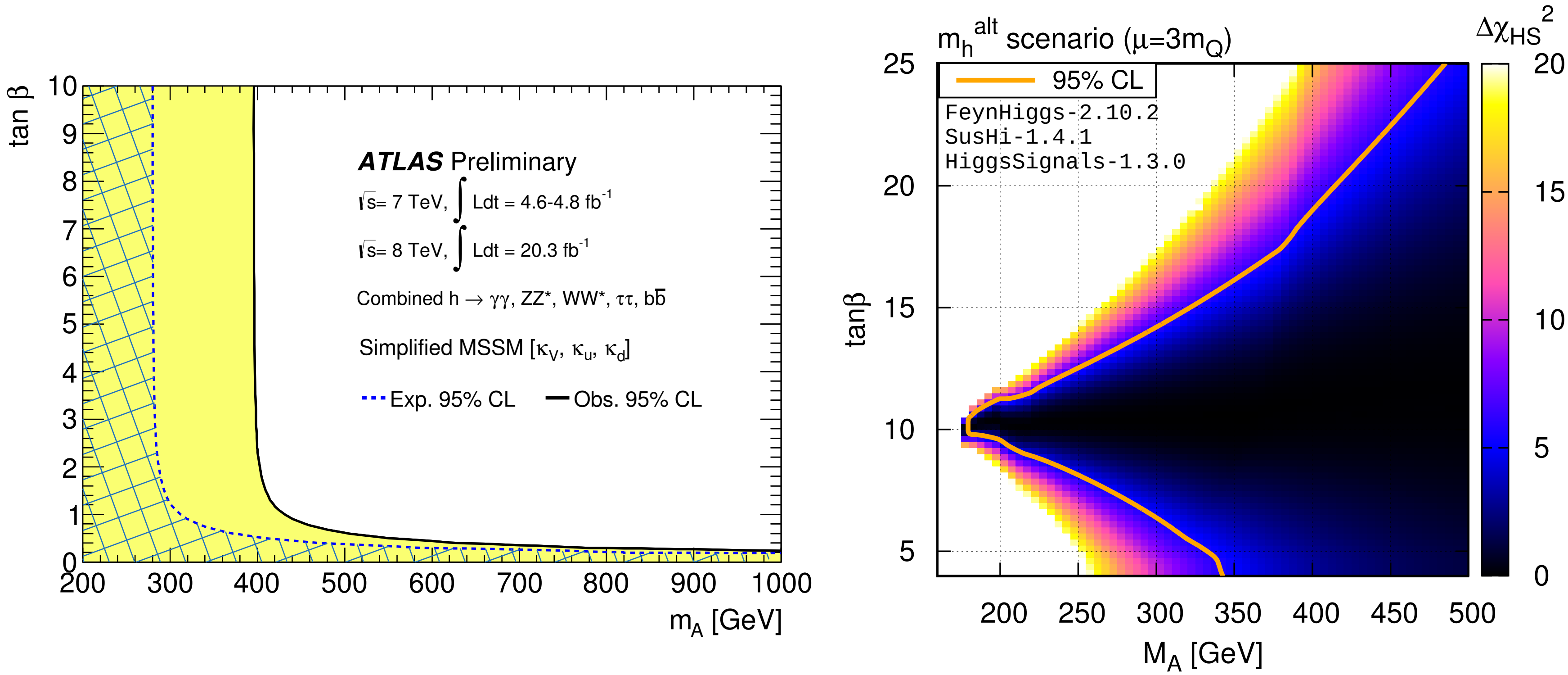
Bottom panels: Contours of the top squark mass parameter M_S, which depends on the values of μ/M_S and A_t/M_S, needed to obtain the correct Higgs squared-mass in the alignment limit, $Z_1 v^2 = 125$ GeV. The three figures correspond to the three $\tan\beta$ solutions of exact alignment previously exhibited.

The MSSM Higgs sector in light of precision Higgs data

The observed Higgs boson at 125 GeV is SM-like (to within roughly an accuracy of 20%). The common wisdom is that this observation implies that additional Higgs states of the MSSM Higgs sector must be rather heavy (corresponding to the decoupling limit).

Indeed, ATLAS has claimed to rule out $m_A \lesssim 400$ GeV based on Run 1 precision Higgs data. But, one needs to be careful about the underlying assumptions...

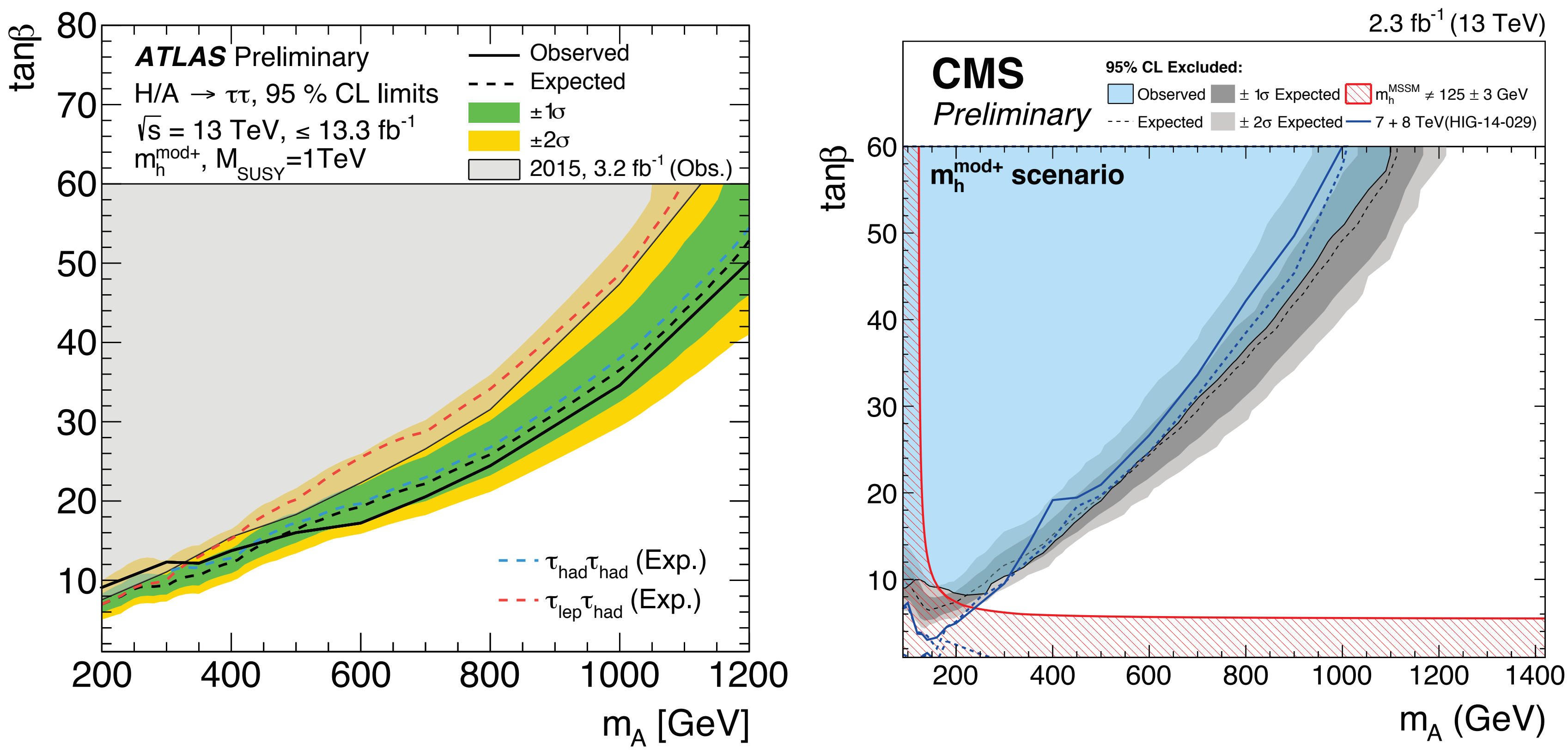
For example, in the so called MSSM $m_h^{\rm alt}$ benchmark scenario introduced in M. Carena et al., arXiv:1410.4969, the Run 1 precision Higgs data places virtually no bound on m_A if $\tan\beta \sim 10$. This is a consequence of the alignment limit without decoupling, which is achieved in the $m_h^{\rm alt}$ benchmark scenario when $\tan\beta \simeq 10$.



Left panel: Regions of the $(m_A, \tan\beta)$ plane excluded in a simplified MSSM model via fits to the measured rates of the production and decays of the SM-like Higgs boson h. Taken from ATLAS-CONF-2014-010.

Right panel: Likelihood distribution, $\Delta\chi^2_{\rm HS}$ obtained from testing the signal rates of‘h against a combination of Higgs rate measurements from the Tevatron and LHC experiments, obtained with `HiggsSignals`, in the alignment benchmark scenario of Carena et al. (op. cit.). From P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, arXiv:1507.06706.

Direct searches for the additional Higgs states also suggest that these states must be heavy, although the sensitivity of these searches are limited if $\tan\beta \lesssim 10$.



The observed and expected 95% CL limits on $\tan\beta$ as a function of m_A in the MSSM $m_h^{\rm mod+}$ benchmark scenario. Left panel: ATLAS results taken from ATLAS-CONF-2016-085. Right panel: CMS results taken from CMS-PAS-HIG-16-006.

Leading two-loop corrections of $\mathcal{O}(\alpha_s h_t^2)$[‡]

Leading two-loop corrections of $\mathcal{O}(\alpha_s h_t^2)$ can be obtained from the leading one-loop corrected results by replacing h_t with $h_t(\lambda)$, where $\lambda \equiv \left[m_t(m_t)M_S\right]^{1/2}$ in the one-loop leading log pieces and $\lambda \equiv M_S$ in the leading threshold corrections.[§] Imposing $Z_6 = 0$ now leads to an 11th order polynomial equation in t_β.

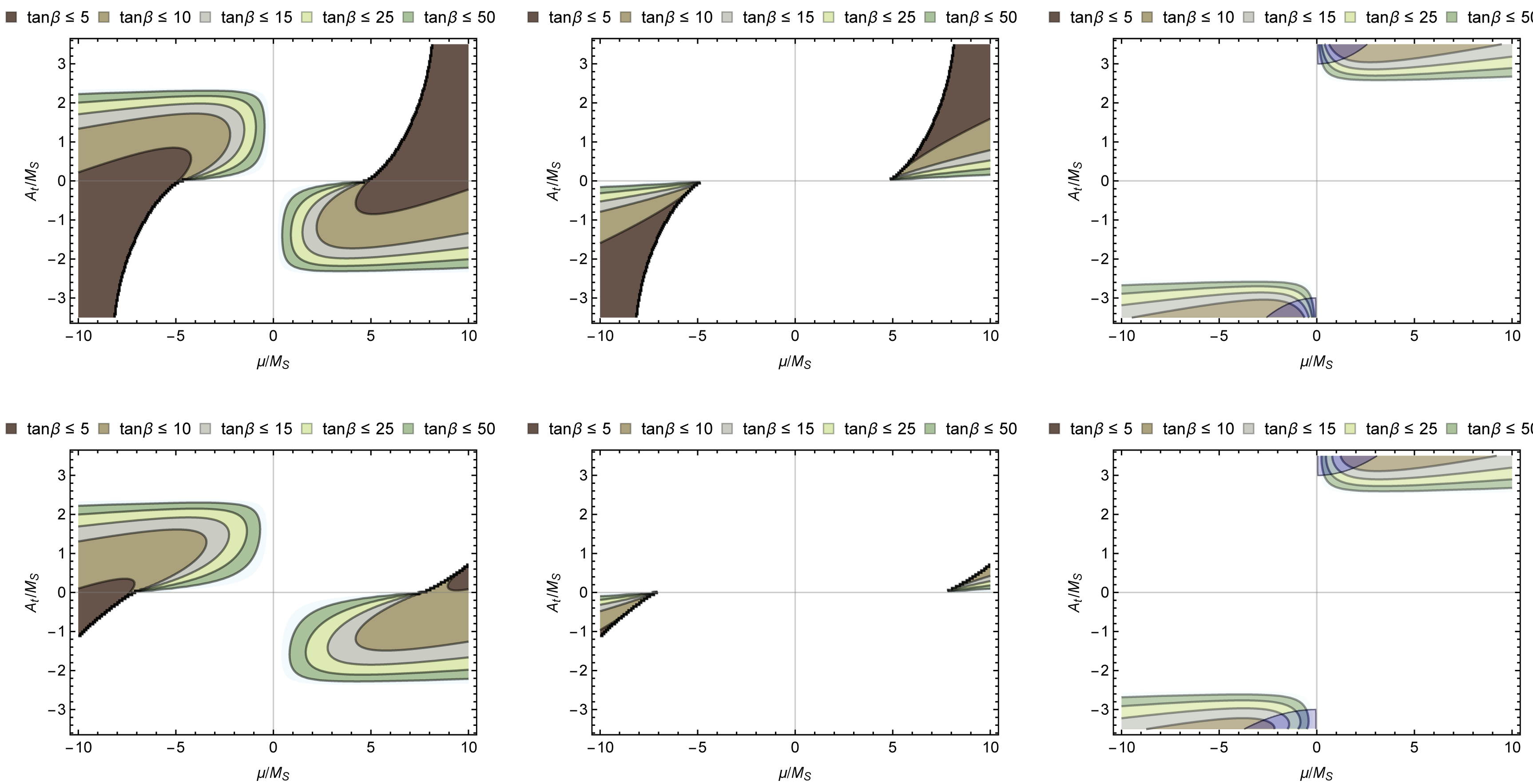
In the region of interest in the $(\mu/M_S, A_t/M_S)$ plane, we find that the previous one-loop real $\tan\beta$ solutions are still present (appropriately perturbed at the two-loop level).[¶]

[‡] P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638, and in preparation.

[§] M. Carena, H.E. Haber, S. Heinemeyer, W. Hollik, C.E.M. Wagner and G. Weiglein, hep-ph/0001002.

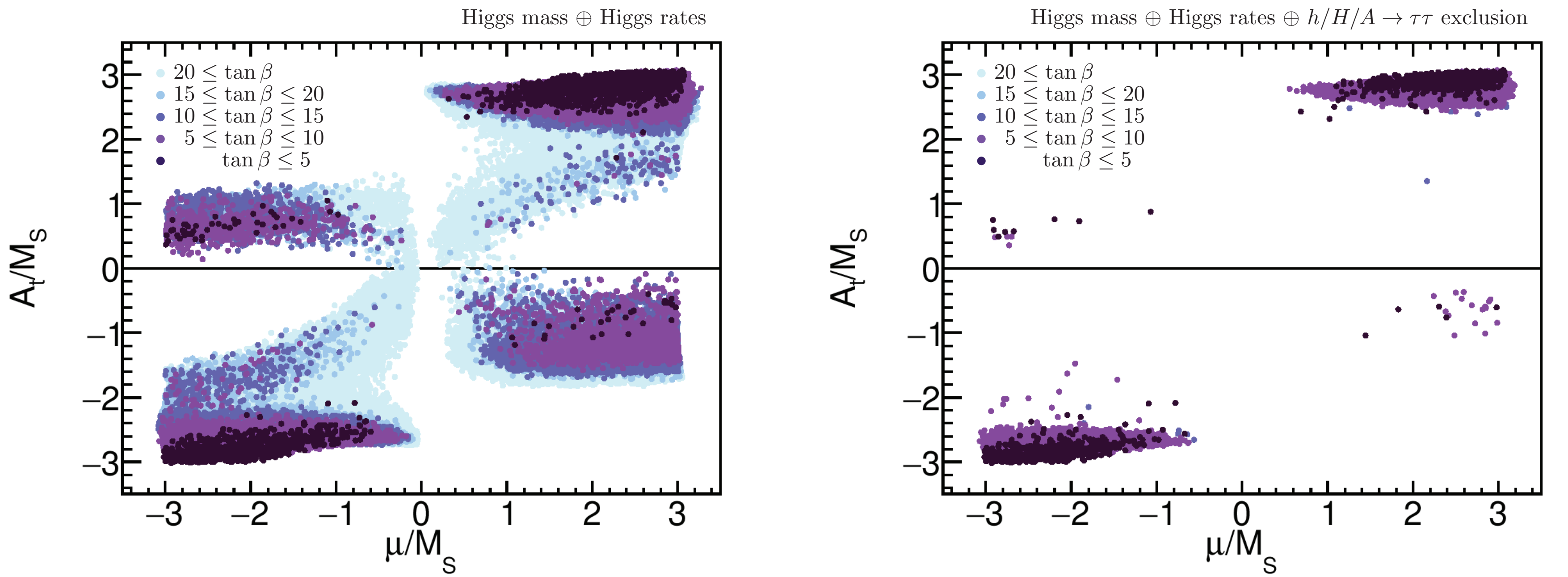
[¶] In addition, another real $\tan\beta$ solution emerges with $|X_t/M_S| \gtrsim 3$, and is therefore discarded.

Comparing approximate one-loop (top panels) and two-loop (bottom panels) results for $\tan\beta$ solutions at exact alignment.



Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane. Z_1 is adjusted to give the correct Higgs mass. Top panels: Approximate one-loop result. Bottom panels: Two-loop improved result. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

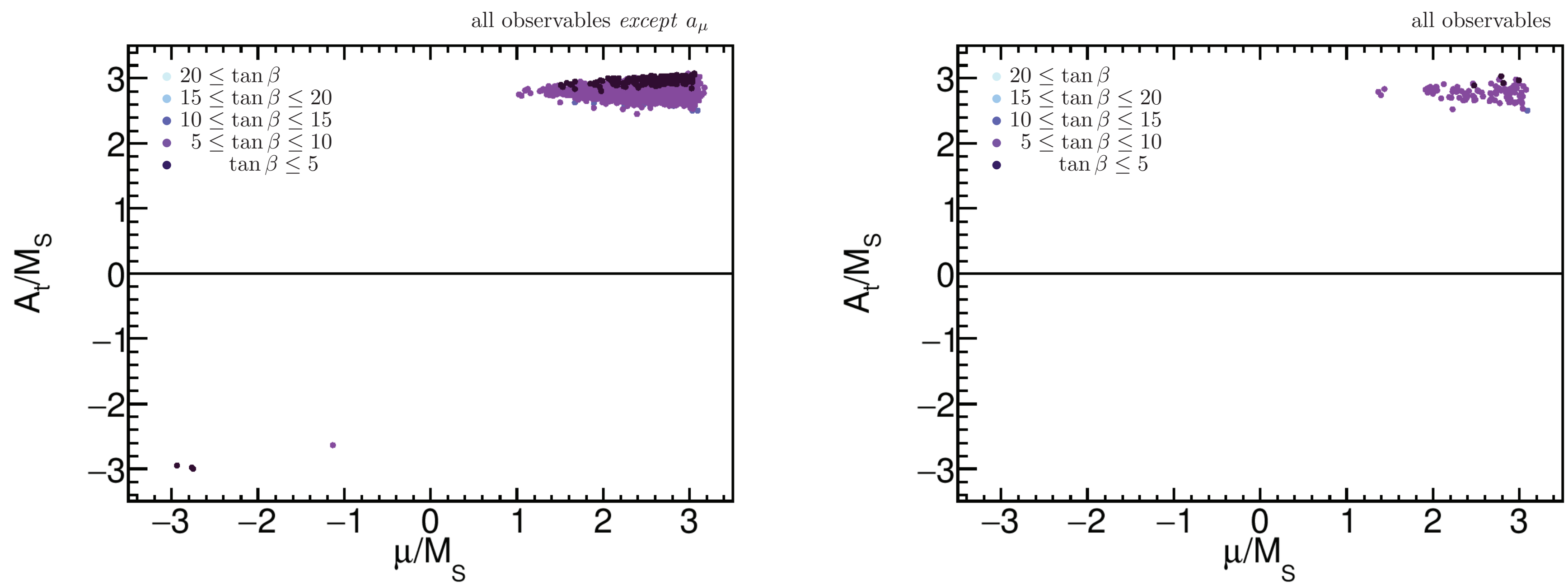
How well do the approximate two-loop results for the exact alignment limit[‖] match a comprehensive scan over the MSSM parameter space? In a recent paper,[**] an 8-parameter pMSSM scan was performed to determine allowed parameter regimes which contain a light CP-odd Higgs boson A. Typically, h is SM-like, although one cannot yet rule out the possibility of a SM-like H.



Preferred points of the pMSSM-8 scan with low $m_A \leq 350$ GeV for different selections of observables. The points are within the (approximate) 95% CL region, based on the following observables. Left panel: only Higgs mass and signal rates; Right panel: Higgs mass, signal rates and $h/H/A \to \tau^+\tau^-$ exclusion likelihood.

[‖]Of course, the precision Higgs data only require that the condition of alignment is approximately satisfied.
[**]P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638.

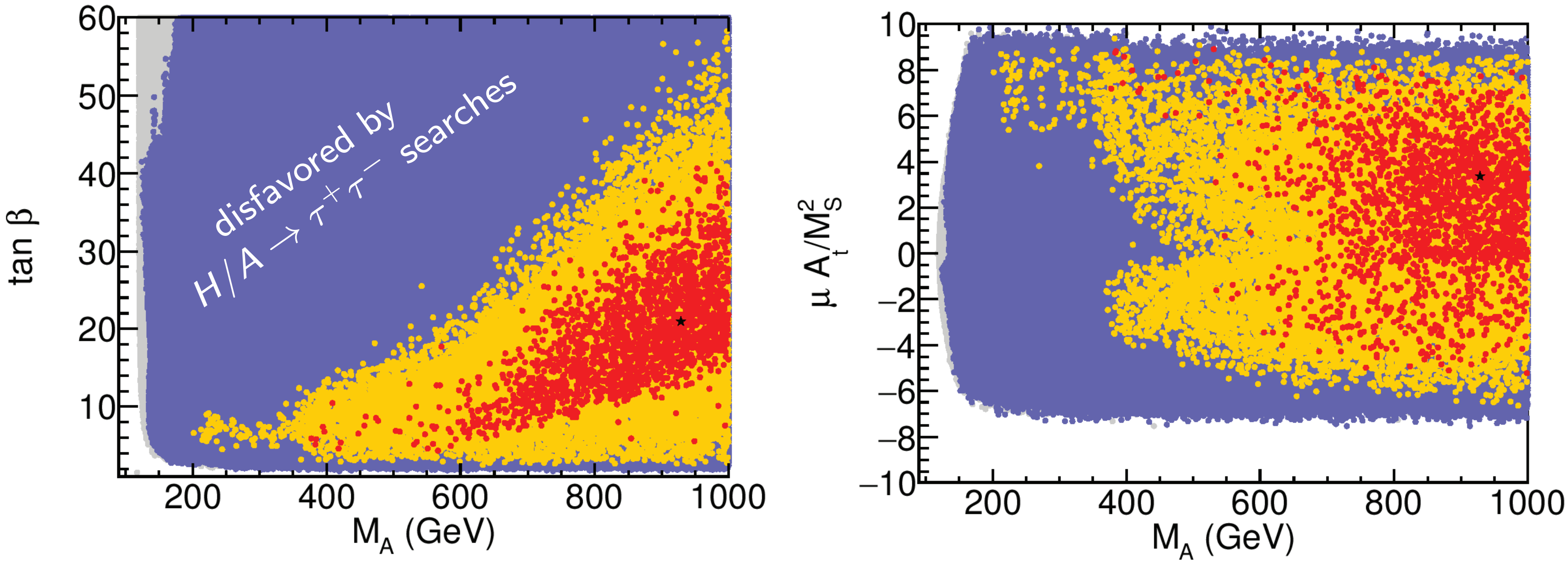
Including additional constraints from SUSY particle searches and the impact of SUSY radiative corrections on SM observables, the allowed parameter regions of the pMSSM-8 scan shrinks further. For example, results from the SuperIso program show that the negative μ region is mostly disfavored by BR($B \to X_s \gamma$), whereas negative A_t is disfavored by BR($B_s \to \mu^+\mu^-$).



Preferred points of the pMSSM-8 scan with low $m_A \leq 350$ GeV for all observables except a_μ (left panel), and for all observables (right panel).

Preferred parameter regions in a pMSSM-8 scan

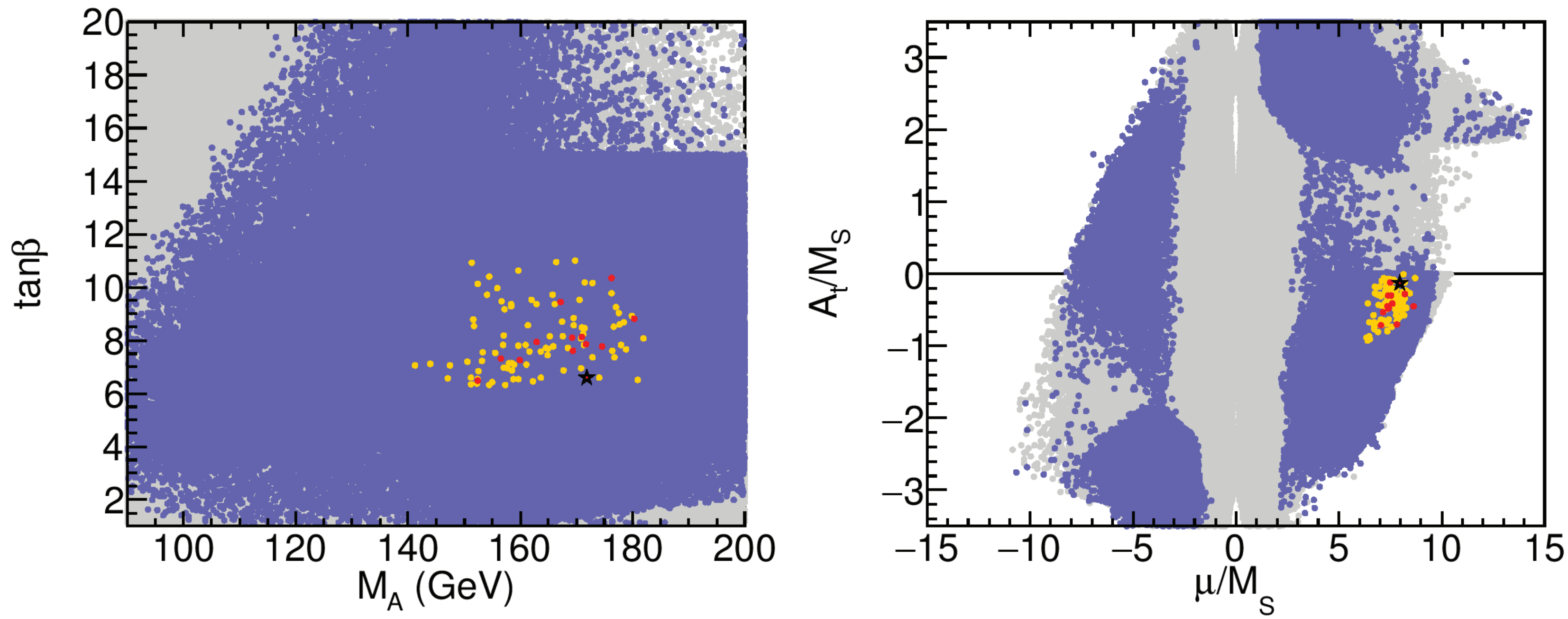
Case 1: h is SM-like



Points that do not pass the direct constraints from Higgs searches from `HiggsBounds` and from LHC SUSY particle searches from `CheckMATE` are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta\chi^2_h < 2.3$, yellow for $\Delta\chi^2_h < 5.99$ and blue otherwise. The best fit point is indicated by a black star.

Bottom line: m_A values as low as 200 GeV are still allowed in the MSSM.

Case 2: H is SM-like

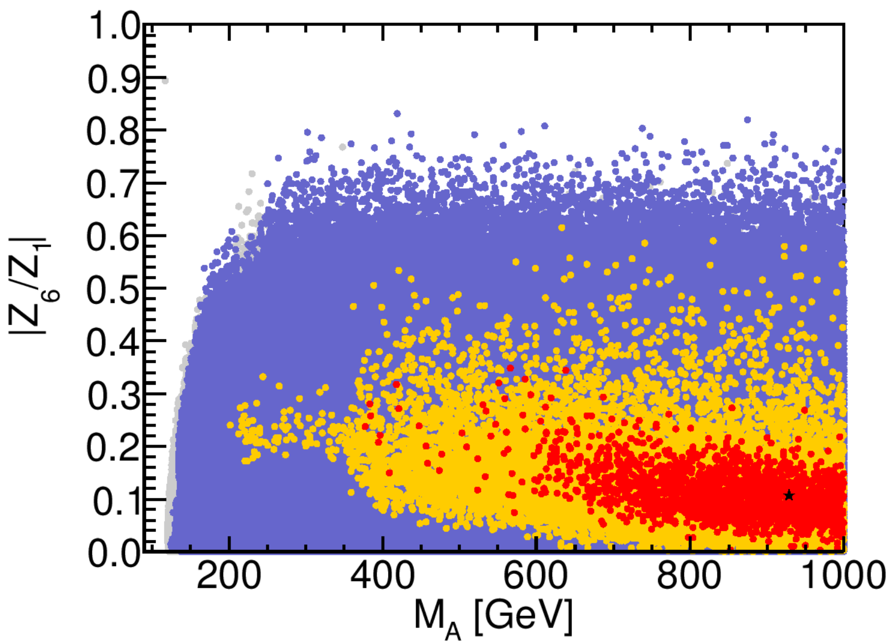


Note: In the preferred region of the pMSSM-8 parameter space with a SM-like H, $X_t \sim -1.5 M_S$ and $150~\text{GeV} \lesssim m_{H^\pm} \lesssim 200~\text{GeV}$ and $m_h \lesssim 100~\text{GeV}$.

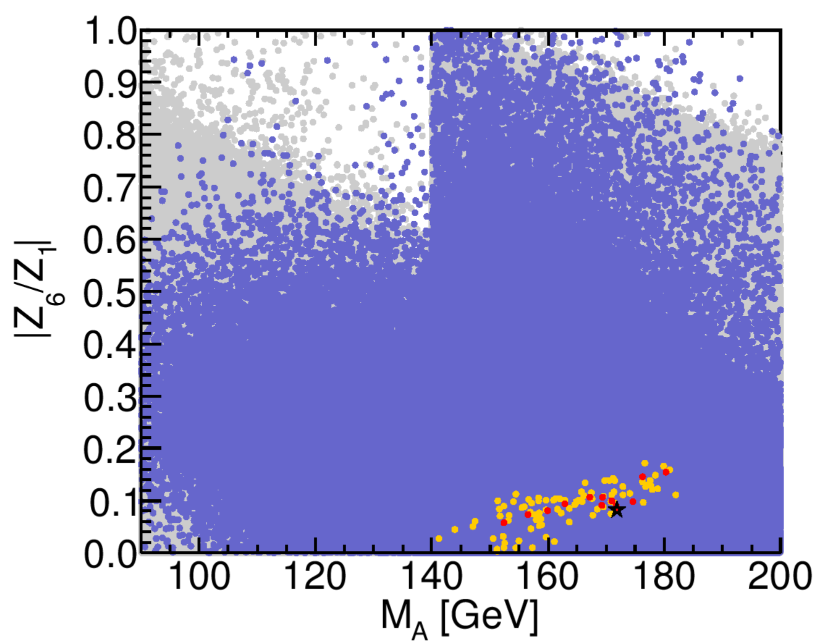
Bottom line: The possibility that the heavier of two CP-even Higgs bosons of the MSSM is the observed 125 GeV Higgs boson is not yet excluded.

[For more details, see Tim Stefaniak's talk in the parallel sessions.]

How fine-tuned is the alignment without decoupling region of the MSSM?



SM-like h



SM-like H

Near the alignment limit, $m_h = 125$ GeV corresponding to $Z_1 \simeq 0.26$. Parameter regions with $Z_6 \sim 0.05$ are compatible with approximate alignment without decoupling (to be compared with $Z_6 = 0$ at exact alignment).

Adding a Higgs singlet to the 2HDM

Consider a Higgs sector that consists of two hypercharge-one complex doublet and a complex neutral singlet S. We can define the doublet fields of the Higgs basis, H_1 and H_2 as before. The relevant scalar potential is more complicated than that of the 2HDM. Here we focus on the terms that are relevant for the scalar squared-mass matrices.

$$
\begin{aligned}
\mathcal{V} \ni \ldots &+ \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \ldots + \left[\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)H_1^\dagger H_2 + \text{h.c.}\right] + \ldots \\
&+ S^\dagger S\left[Z_{s1}H_1^\dagger H_1 + \ldots + (Z_{s3}H_1^\dagger H_2 + \text{h.c.}) + Z_{s4}S^\dagger S\right] \\
&+ \left\{Z_{s5}H_1^\dagger H_1 S^2 + \ldots + Z_{s7}H_1^\dagger H_2 S^2 + Z_{s8}H_2^\dagger H_1 S^2 + Z_{s9}S^\dagger S\, S^2 + Z_{s10}S^4 + \text{h.c.}\right\} \\
&+ \left[C_1 H_1^\dagger H_1 S + \ldots + C_3 H_1^\dagger H_2 S + C_4 H_2^\dagger H_1 S + C_5(S^\dagger S)S + C_6 S^3 + \text{h.c.}\right].
\end{aligned}
$$

For simplicity, we shall assume that the scalar potential is CP-invariant. We then write the squared-mass matrix of the CP-even Higgs bosons with respect to the basis $\{\sqrt{2}\,\text{Re}\ H_1^0 - v\,,\ \sqrt{2}\,\text{Re}\ H_2^0\,,\ \sqrt{2}\,(\text{Re}\ S - v_s)\}$.

The squared-mass matrix for the CP-even scalars is a real symmetric matrix,

$$\mathcal{M}_S^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 & \sqrt{2}\, v\big[C_1 + (Z_{s1} + 2Z_{s5})v_s\big] \\ & \overline{M}_A^2 + Z_5 v^2 & \dfrac{v}{\sqrt{2}}\Big[C_3 + C_4 + 2(Z_{s3} + Z_{s7} + Z_{s8})v_s\Big] \\ & & -C_1\dfrac{v^2}{2v_s} + 3(C_5 + C_6)v_s + 4(Z_{s4} + 2Z_{s9} + 2Z_{s10})v_s^2 \end{pmatrix},$$

where $\overline{M}_A^2$ is the 11 element of the CP-odd squared-mass matrix with respect to the basis $\{\sqrt{2}\,\mathrm{Im}\ H_2^0,\ \sqrt{2}\,\mathrm{Im}\ S\}$.

Exact alignment occurs when $(\mathcal{M}_S^2)_{12} = (\mathcal{M}_S^2)_{13} = 0$. That is,

$$Z_6 = 0\,, \qquad\qquad C_1 + (Z_{s1} + 2Z_{s5})v_s = 0\,.$$

The decoupling limit corresponds to $\overline{M}_A \gg v$ and $v_s \gg v$ and yields approximate alignment.

Approximate alignment can also be achieved with a combination of a subset of the above conditions. For example, $C_1 + (Z_{s1} + 2Z_{s5})v_s \simeq 0$ and $\overline{M}_A \gg v$ [with $Z_6 \sim \mathcal{O}(1)$] yields approximate alignment.

The alignment limit of the Higgs sector of the NMSSM

The NMSSM adds a singlet superfield $\hat{S}$, which couples to itself and to the Higgs superfields $\hat{H}_U$, $\hat{H}_D$ via the superpotential, $W = \lambda \hat{S}\hat{H}_U\hat{H}_D + \frac{1}{3}\kappa\hat{S}^3$.

Including the leading one-loop radiative corrections,

$$Z_1 v^2 = (m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta}^2 + \tfrac{1}{2}\lambda^2 v^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{(m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\}.$$

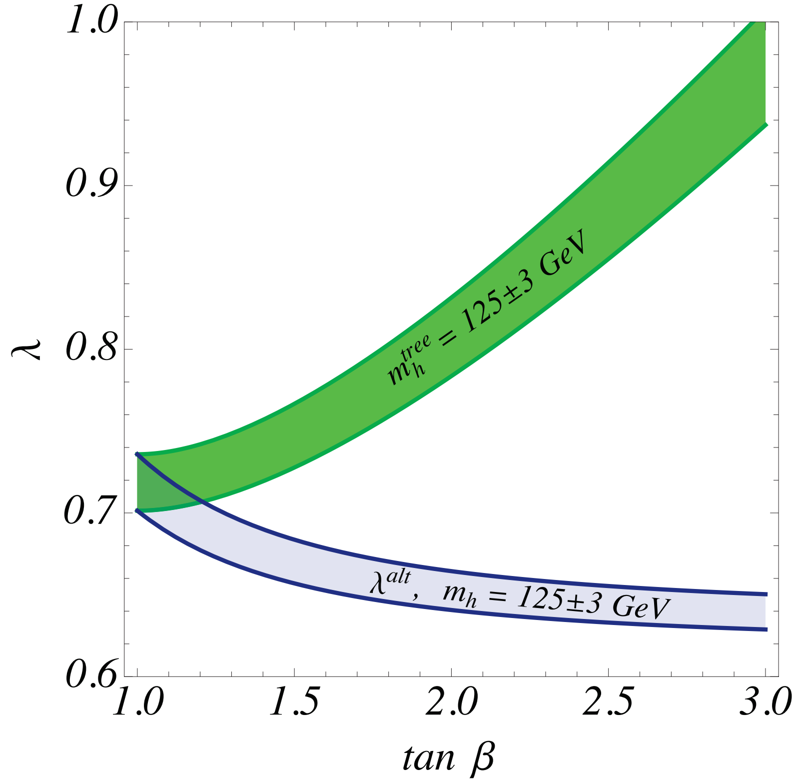
The exact alignment limit requires that $Z_6 = 0$ and $C_1 + (Z_{s1} + 2Z_{s5})v_s = 0$. In the NMSSM, these two conditions yield

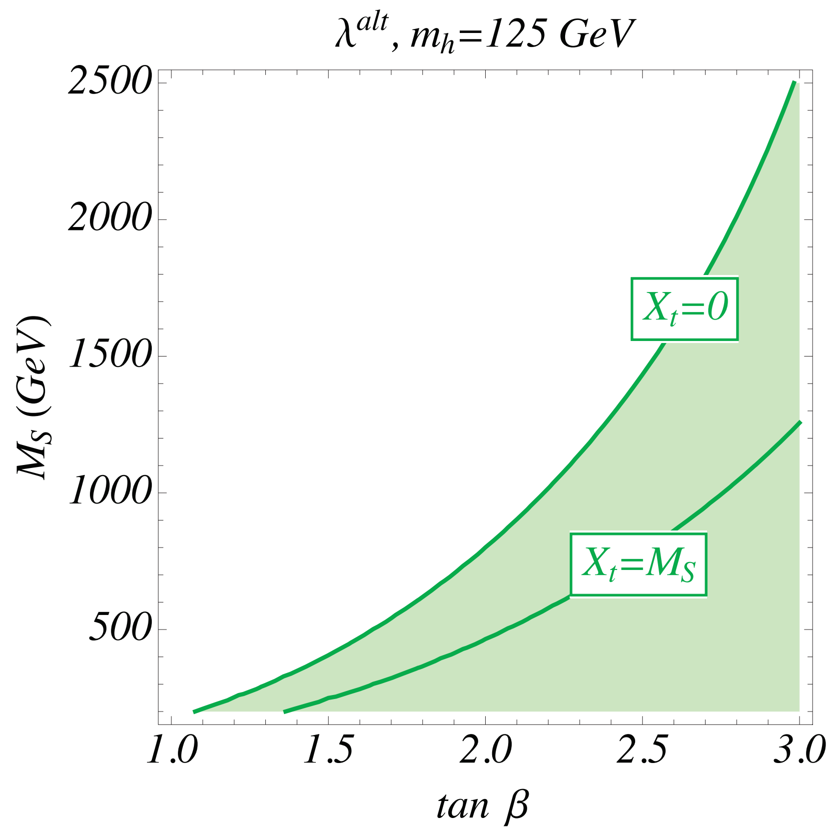
$$\lambda^2 \simeq (\lambda^{\rm alt})^2 \equiv \frac{m_h^2 - m_Z^2 c_{2\beta}}{v^2 s_\beta^2}, \qquad \frac{\overline{M}_A^2 s_{2\beta}^2}{4\mu^2} + \frac{\kappa s_{2\beta}}{2\lambda} = 1,$$

where $\overline{M}_A^2 \equiv 2\mu(A_\lambda + \kappa v_s)/s_{2\beta}$, A_λ is a soft-SUSY-breaking trilinear scalar coupling parameter and $\mu \equiv \lambda v_s$.

In contrast to the MSSM, in the NMSSM one can set $Z_6 = 0$ and obtain $m_h = 125$ GeV, with only small contributions from the one-loop radiative corrections. This leads to a preferred choice of NMSSM parameters,[††]

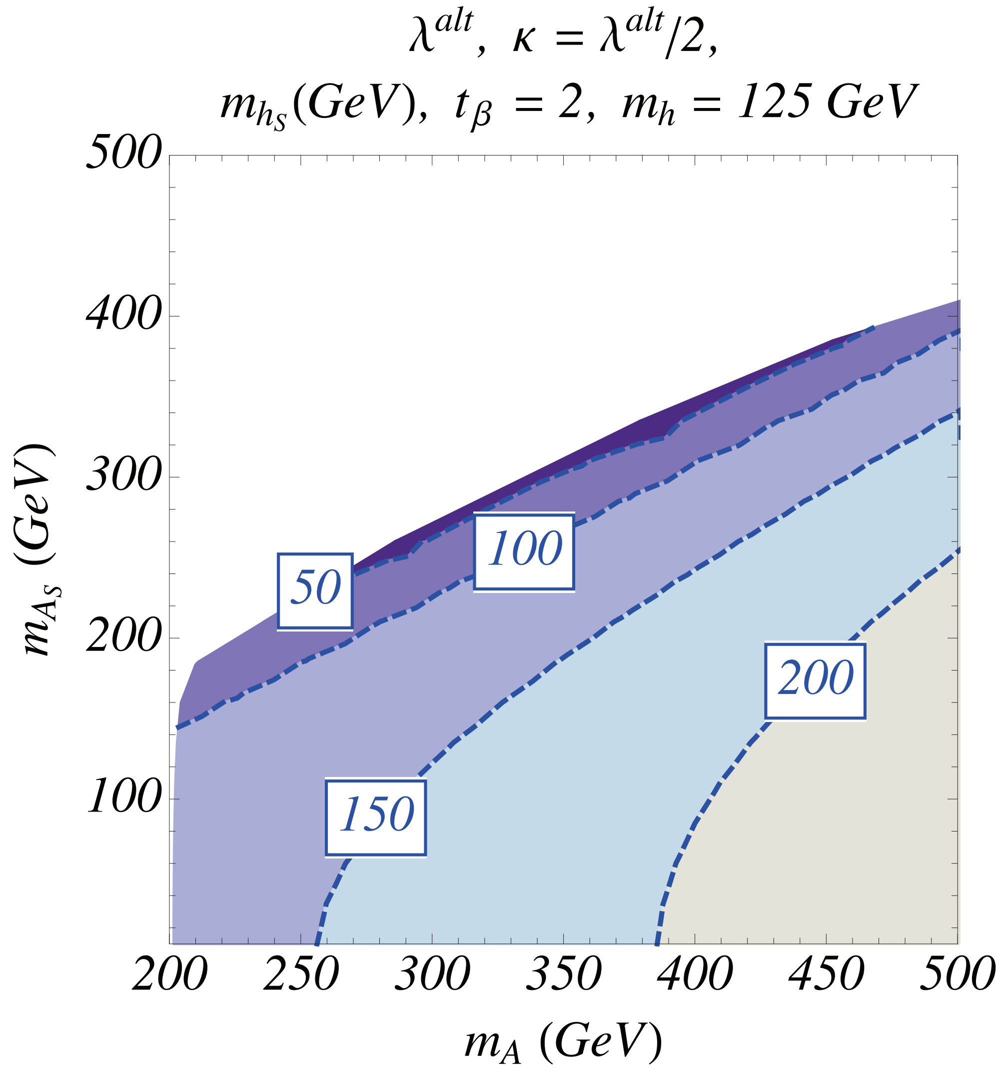
$$\lambda \sim 0.65\,, \qquad\qquad \tan\beta \sim 2\,.$$





[††]See M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, arXiv:1510.09137.

The second alignment limit condition leads to further correlations among the parameters of the NMSSM Higgs sector.



Near the alignment limit, we have $m_A \simeq m_H \simeq \overline{M}_A$.

Conclusions

- In light of the LHC Higgs data, one of the Higgs mass eigenstates is approximately aligned in the direction of the Higgs vev.

- The alignment limit is approximately satisfied in the decoupling regime where $m_A \gg m_h$. But, approximate alignment can also be achieved without decoupling if the Higgs basis parameter $|Z_6| \ll 1$.

- Alignment without decoupling is possible in the MSSM, but it is achieved in a parameter regime in which there is an accidental approximate cancellation between tree-level and loop-level contributions to Z_6.

- Regions of approximate alignment without decoupling must necessarily appear in any comprehensive scan of the MSSM parameter space.

- Using all relevant data to constrain the MSSM Higgs sector, it is still possible that (i) h is SM-like with m_A as low as 200 GeV, or (ii) H (rather than h) is the observed (SM-like) Higgs boson, with m_A, $m_{H^\pm} \lesssim 200$ GeV.

- Alignment without decoupling in the NMSSM can arise in a compelling region of the parameter space, which leads to intriguing correlations among Higgs sector parameters.