Exploring the Matrix Element Method and its Application to $t\bar{t}$ Events

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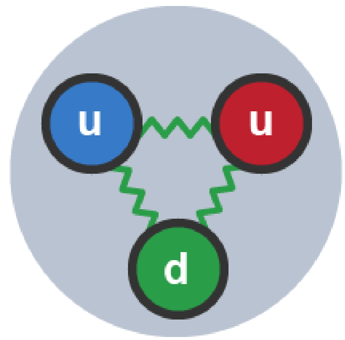
The Top Quark and its Uniqueness

- Heaviest Elementary Particle

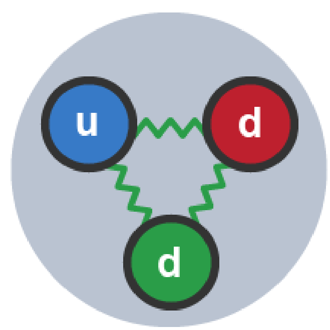
172.99 +/- 0.48 (stat) +/- 0.78 (syst) GeV $/c^2$ (ATLAS 7 TeV data)

"66 Millionths… of a billionth… of a billionth… of a pound"

=



79 Protons
+
118 Neutrons



= 183.43 GeV $/c^2$ (HyperPhysics)

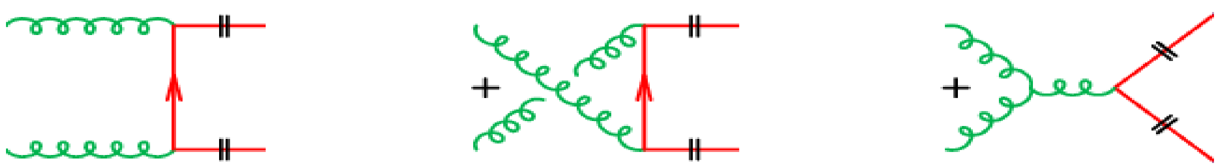
The Top Quark and its Uniqueness

- Heaviest Elementary Particle

172.99 +/- 0.48 (stat) +/- 0.78 (syst) GeV (ATLAS 7 TeV data)

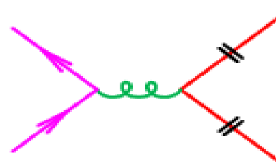
- Natural laboratory for studying Higgs interaction with matter.

- High Mass -> Short lifetime: 5×10^{-25} s.



$gg \rightarrow t\bar{t}$

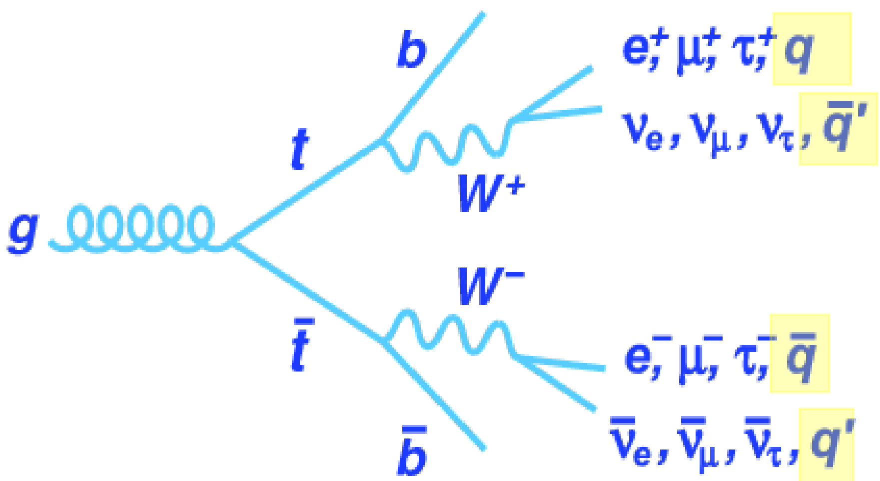
$q\bar{q} \rightarrow t\bar{t}$

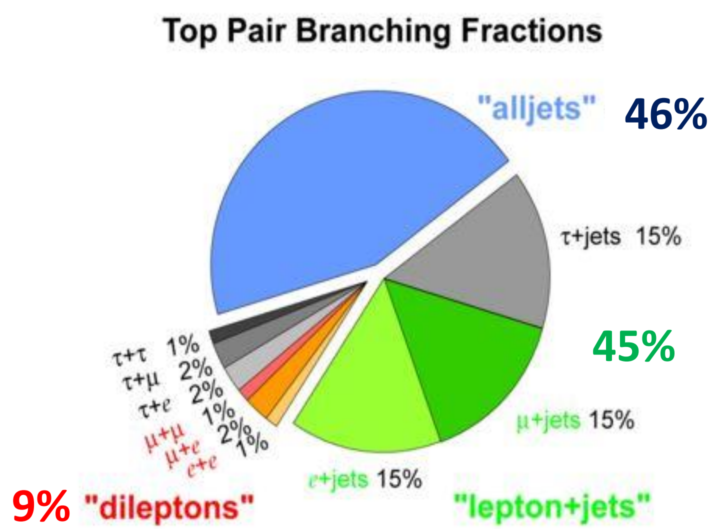


Hard Scattering Production Processes from proton-proton collisions.

Decay Processes (Leading Order)

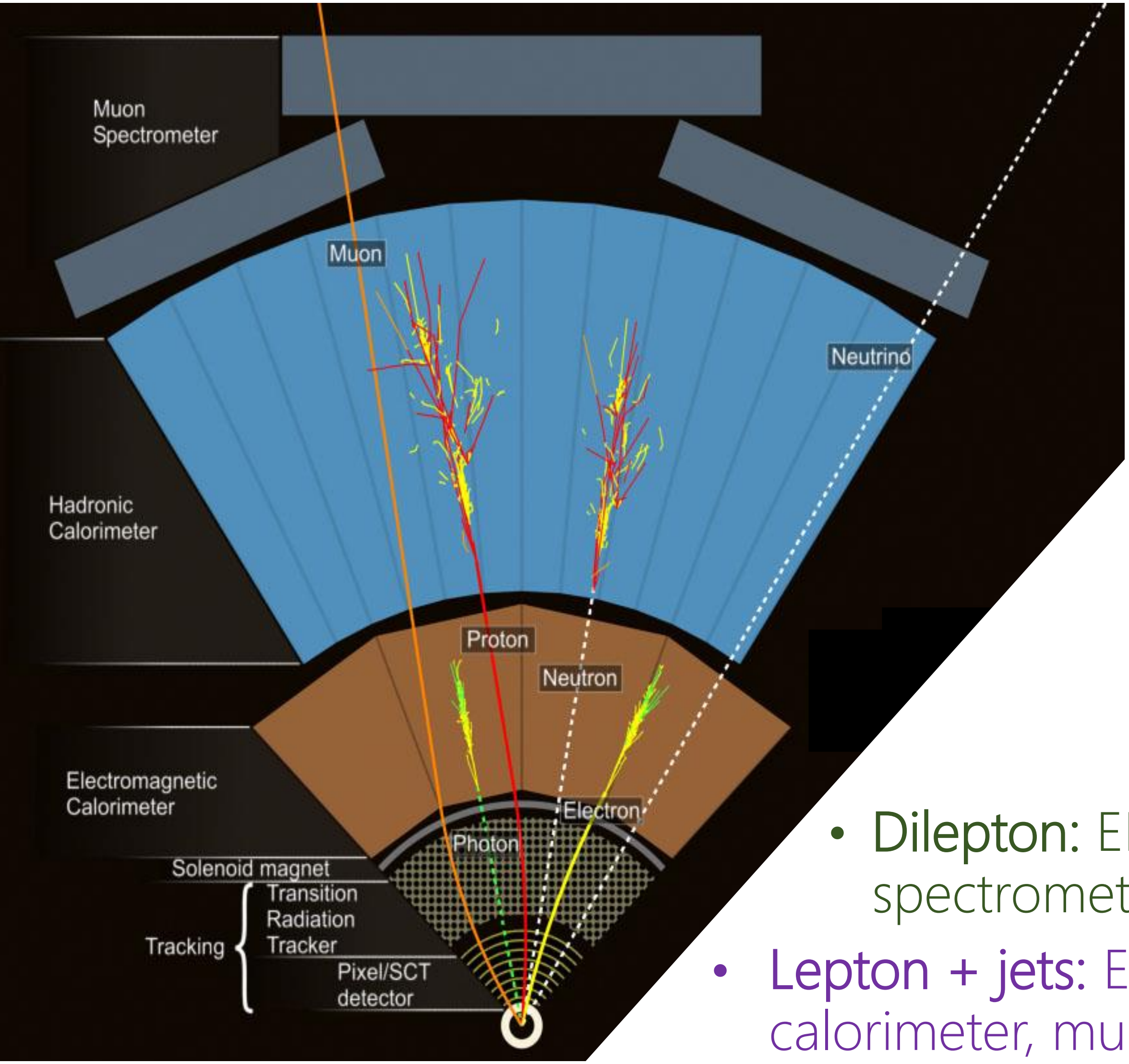
- Top quarks can decay in different ways, each with its own likelihood
- Goal: Reconstruct kinematic properties of the top quark using the decay products
- Three categories: "Dilepton" vs. " Lepton + Jets " vs. " Alljets"





Courtesy of D0 Collaboration

The ATLAS Detector- Observables



- 4 vectors of all leading order decay products can be detected, except neutrinos.

$$\not{E}_x = p_{\nu_x} + p_{\bar{\nu}_x}$$
$$\not{E}_y = p_{\nu_y} + p_{\bar{\nu}_y}$$
$$E_\nu^2 = p_{\nu_x}^2 + p_{\nu_y}^2 + p_{\nu_z}^2 \to p_\nu^2 = 0$$
$$E_{\bar{\nu}}^2 = p_{\bar{\nu}_x}^2 + p_{\bar{\nu}_y}^2 + p_{\bar{\nu}_z}^2 \to p_{\bar{\nu}}^2 = 0$$
$$m_{W^+}^2 = (p_{l^+} + p_\nu)^2$$
$$m_{W^-}^2 = (p_{l^-} + p_{\bar{\nu}})^2$$
$$m_t^2 = (p_{l^+} + p_\nu + p_b)^2$$
$$m_{\bar{t}}^2 = (p_{l^-} + p_{\bar{\nu}} + p_{\bar{b}})^2.$$

- **Dilepton:** EM calorimeter and muon spectrometer.
- **Lepton + jets:** EM calorimeter, hadronic calorimeter, muon spectrometer.

ReadFromSherpa – A Simple Example

- Use algorithms and operational definitions to reconstruct a tt-bar event.
- Determine P(event) using simple, 2D profile histogram as a **"look-up"** table.

Loop over all events

- Use observable theory (b and lepton selection) to identify all possible "configurations" (combinations of three particles, a top candidate)
- Determine if Δ variable algorithm chose the correct configuration
- Create and fill maps of configuration properties (true and algorithm)

Normalize histograms (Σweights / Nentries) = <weight>

Loop over all events

Loop over all configurations

- Calculate probability P(variable1,…,variableN) using <weight>

- Select configuration with highest probability

Determine performance of Δvariable and probability algorithms

ReadFromSherpa

Current performance (10,000 dilepton entries)
Δ relative mass: 97.48% Accurate
P(x1, Δ rel.mass): 98.30% Accurate

Previously

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Determine performance of Δvariable and probability algorithms

New Goal: Calculate P(event) using n-dimensional data structure.

A Computationally Expensive Problem

- n-dimensional data structure, b bins per dimension.
- Simple binning methods get computationally intensive very, very quickly.

-> You need a binning that is fine enough to catch fine structure in your event distribution.

(you need **lots** of bins)

- The *intrinsic dimensionality* of your problem is reduced because variables may be correlated,
- Thus your problem lives in a subsection of your data structure and many bins are scarcely populated.

How can we be efficient?

One Solution…

- Self-adaptive binning. Non-equidistant binning in n-dimensions.
  …a "Foam" of cells.

- For a given d-dimensional analytically known distribution, the Foam algorithm creates a hyper-rectangular " foam of cells" , which is more dense around the peaks of the distribution and less dense in areas where the distribution is only slowly varying (or flat).

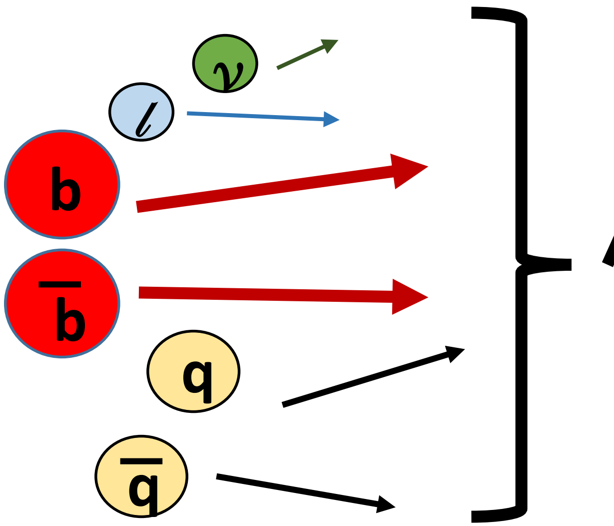
- Iteratively produced using a binary-split algorithm for the cells acting on samplings of the input distribution within the cell boundaries.
- The number of cells is a predefined free parameter.

- A priori, this approach only limited by the amount of available computer memory.

- The optimal number of cells depends on the number of training samples (events used to build the foam).

The "Matrix Element" Method

- Introduced by D0 Collaboration, Fermilab, for precision measurement of top quark mass (arXiv: 0406031)
- Creates a theoretical matrix element for a decay process in order to compute a probability that this decay was observed for a given set of parameters (I. Volobouev, arXiv: 1101.2259)
- Foam input: A value that uniquely represents the kinematic properties of this event in nature. (Event cross section).



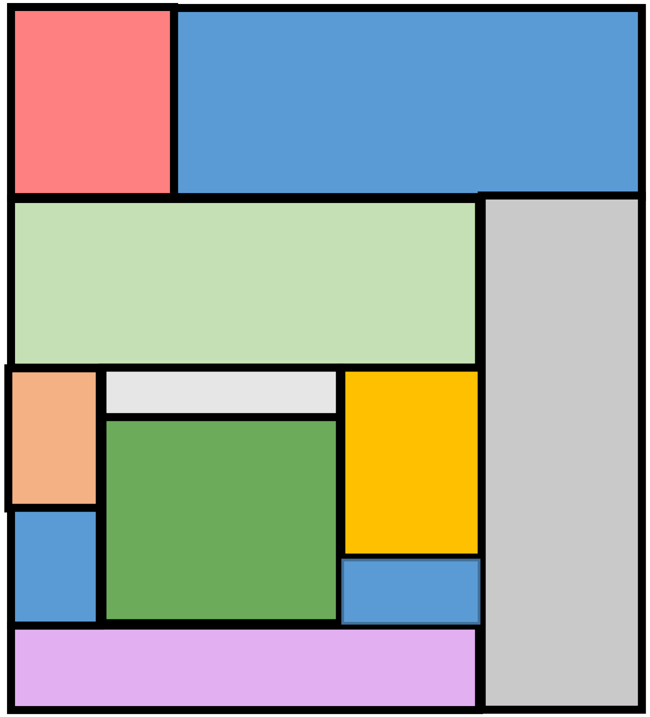
$$P_i(\mathrm{y}|\mathrm{a}) = \frac{\Omega(\mathrm{y})}{\sigma_i(\mathrm{a})A_i(\mathrm{a})} \int_{X_i} W_i(\mathrm{y}|\mathrm{x},\mathrm{a})\,\epsilon_i(\mathrm{x},\mathrm{a})\,|M_i(\mathrm{x},\mathrm{a})|^2\,T_i(\mathrm{x},\mathrm{a})\,d\mathrm{x}$$

True
Event
Cross-Section
(Probability)

- Strong potential for new physics searches (AMVA4 Workshop, Venice)

PDE Foam

ROOT::PDEFoam is a classification tool that subdivides an n-dim space in bins of constant density. based on ROOT::TFoam for MC integration of analytical functions.



For more information on PDE Foam: D. Dannheim *et al.*, arXiv: 0812.0922

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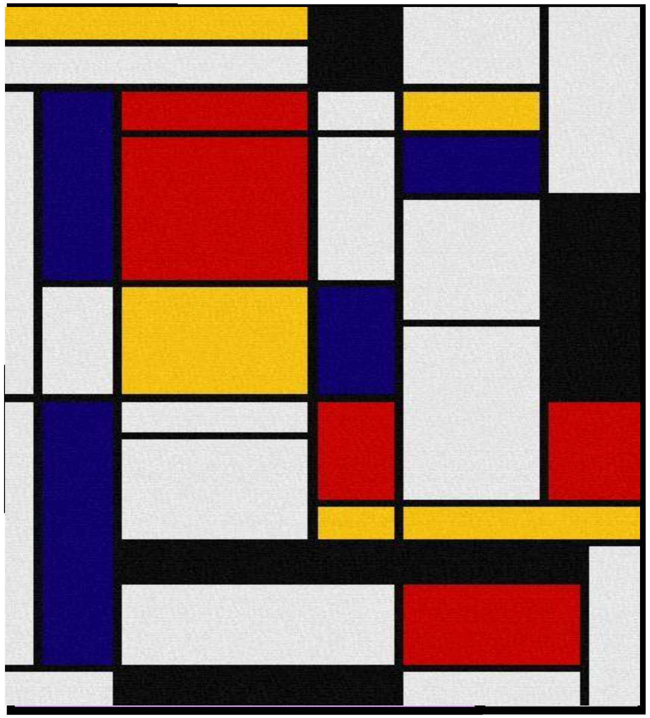


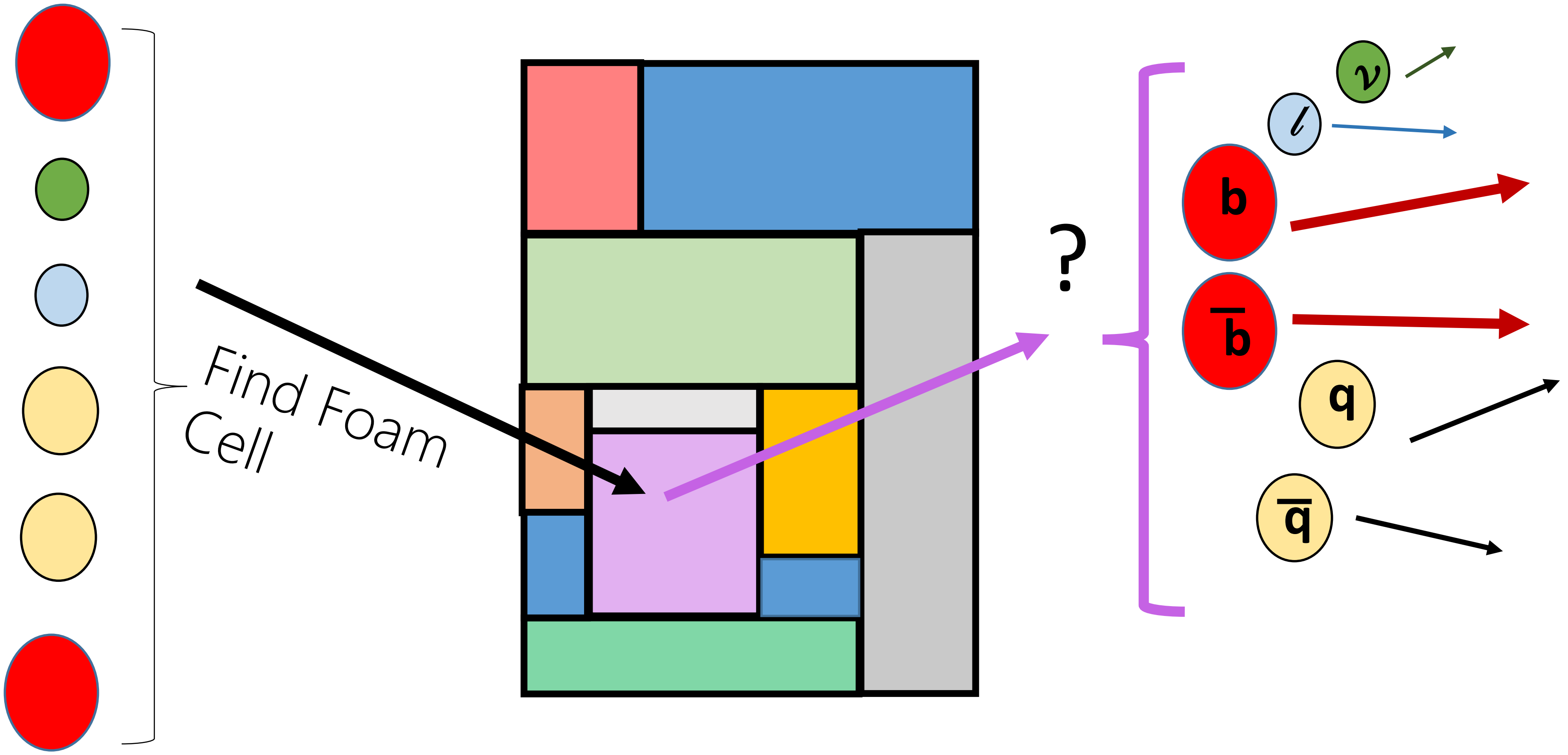
Mondriaan was definitely onto something…

…Who says art can't inspire science?

For more information on PDE Foam: D. Dannheim *et al.*, arXiv: 0812.0922

Kinematic Reconstruction

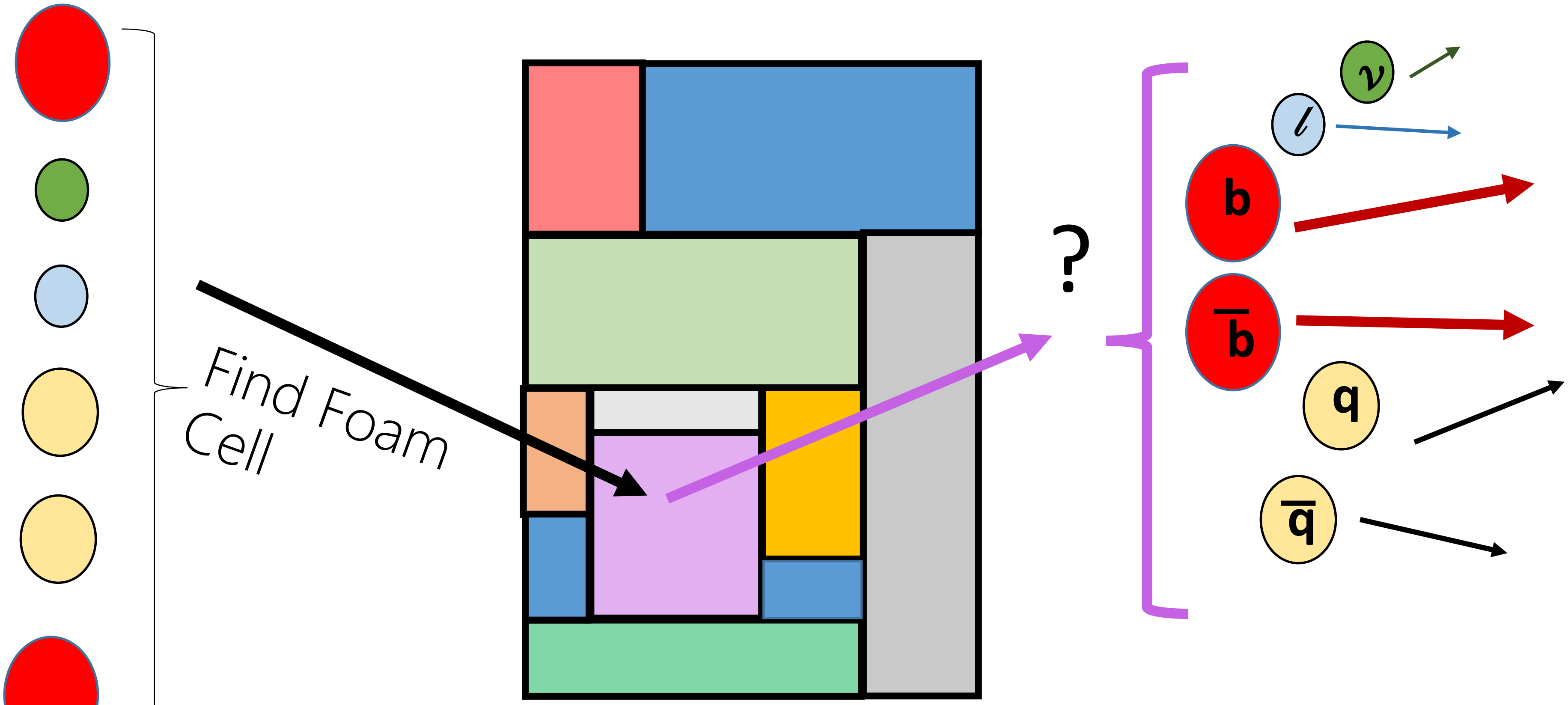
New data set…



New data set…

Kinematic Reconstruction

New data set…



" Nature (or that which I see) inspires me…but I want to approach the truth as closely as possible, abstracting everything until I come to the foundation…. "

•.In a letter to H. P. Bremmer, Paris 29 January 1914; ; as quoted in *Mondrian, - The Art of Destruction*, Carel Blotkamp,

Trial Run… 1 Dimension

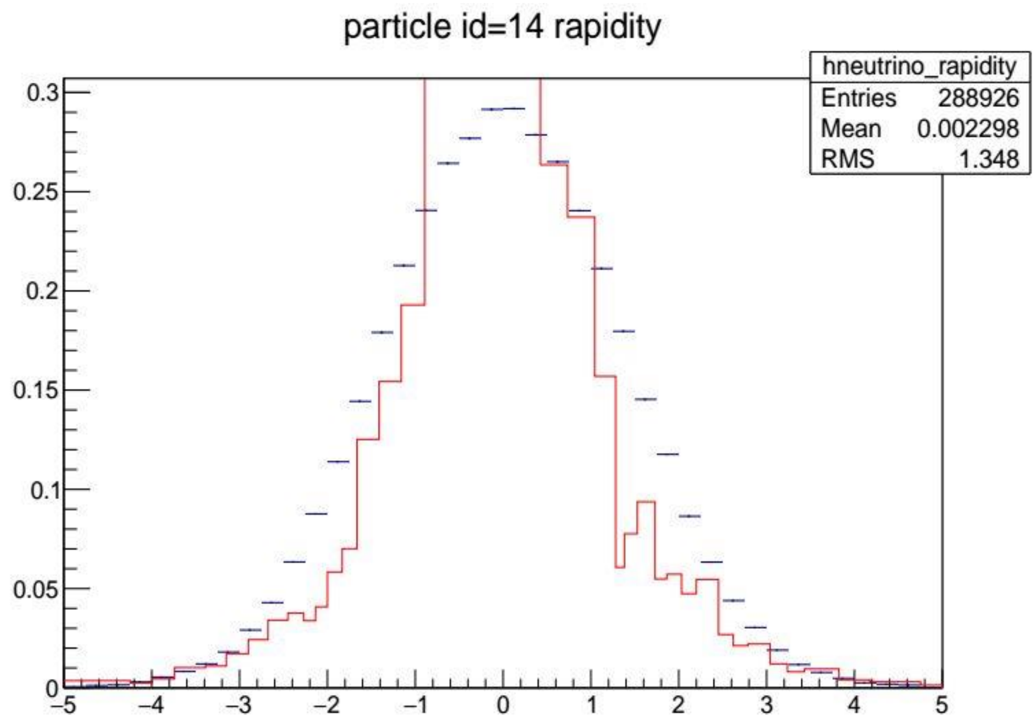
- Investigations have started using Sherpa in fixed order QCD mode
- (Only using leading order (LO) for the moment)

Example

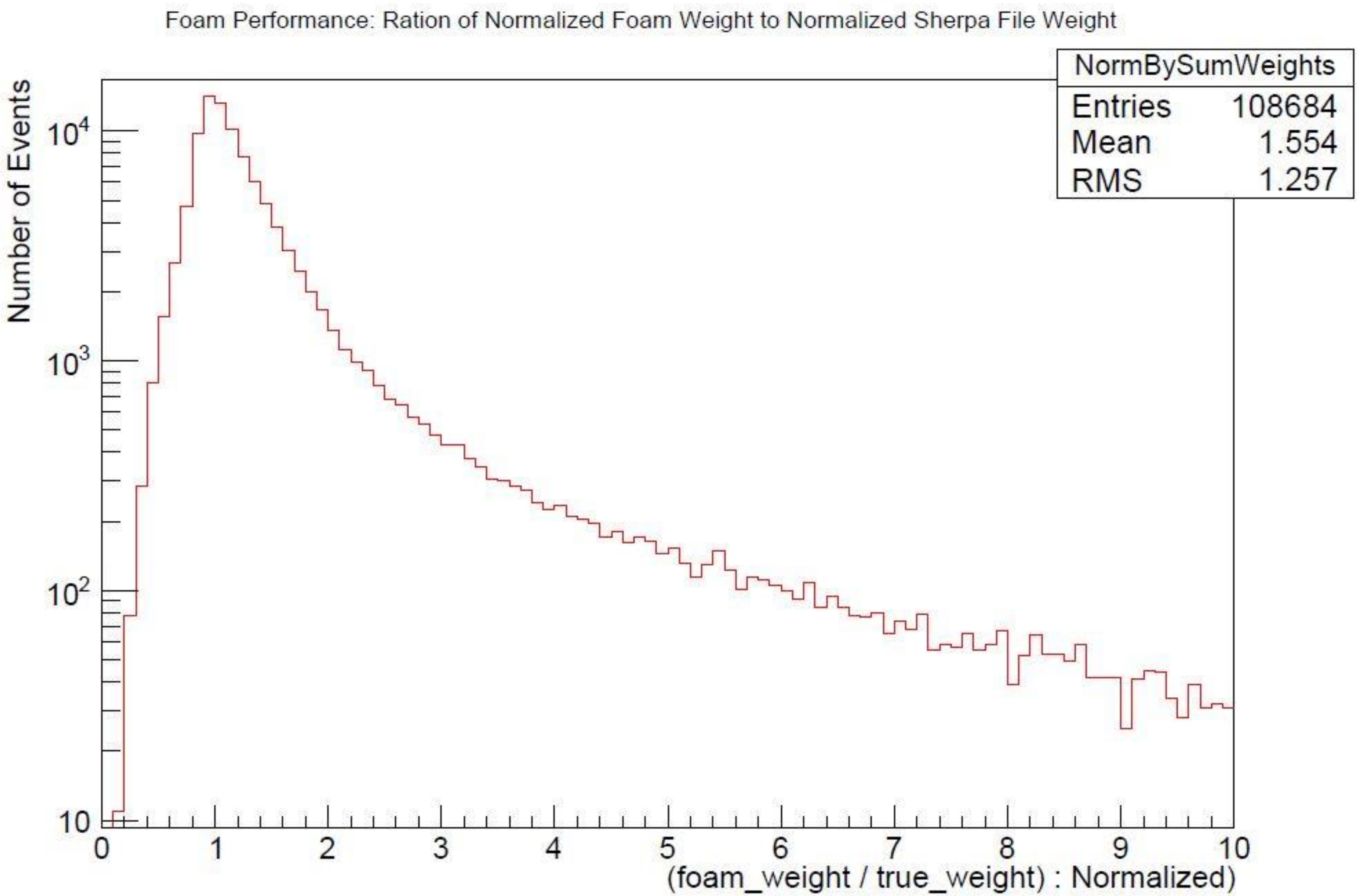
- 5*10^5 events
- 1 Dimension, rapidity of 1 particle

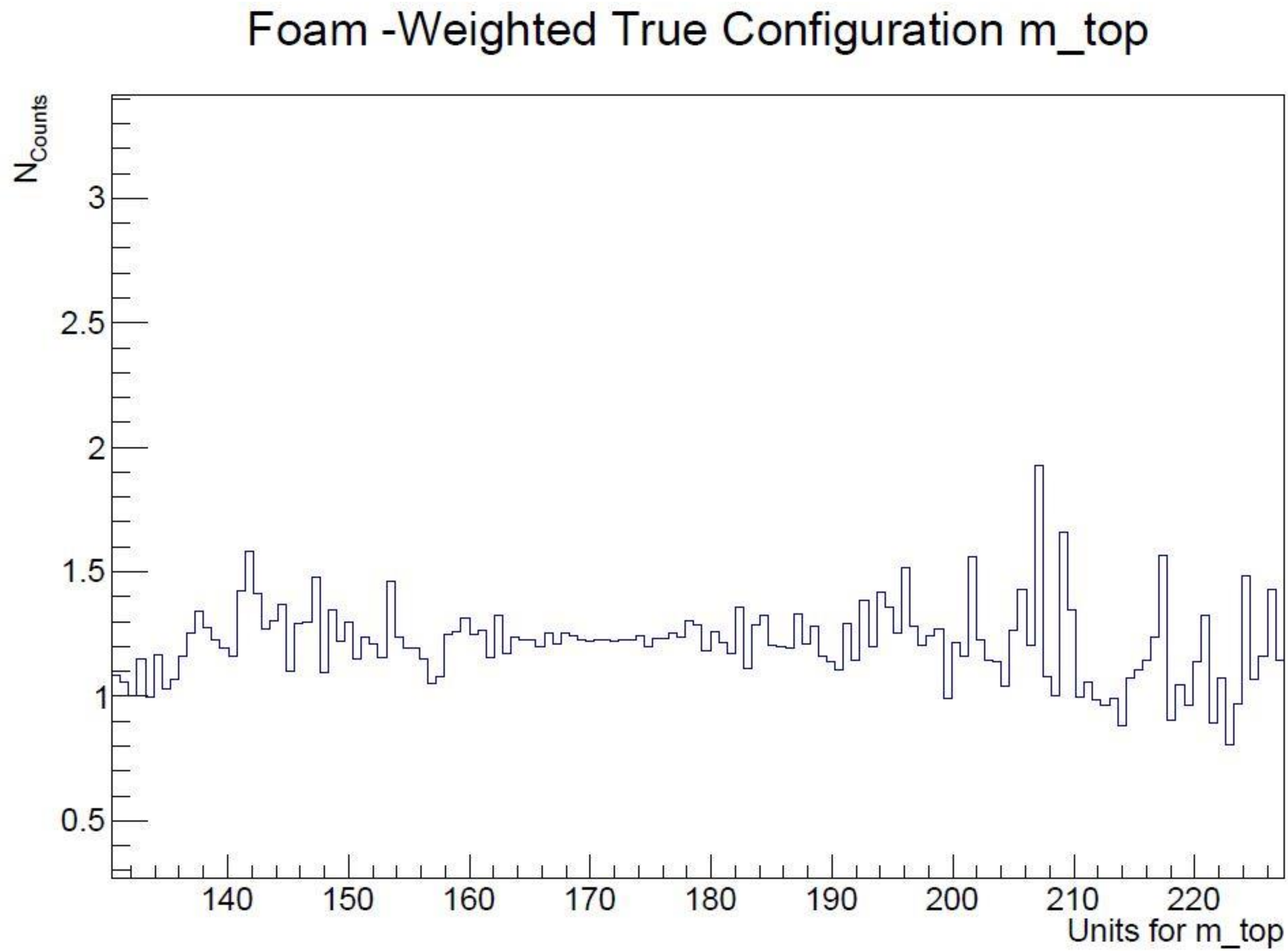
Challenges

- Interpreting foam performance (comparing reference histogram to foam output)
- Correctly dividing by bin width has proven to be non-trivial.



15 Dimension Success




Foam -Weighted True Configuration m_top
N_Counts
3
2.5
2
1.5
1
0.5
140
150
160
170
180
190
200
210
220
Units for m_top