Quarkonium Physics

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Quarkonium Physics I

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15:30 ~ 17:00

Introduction

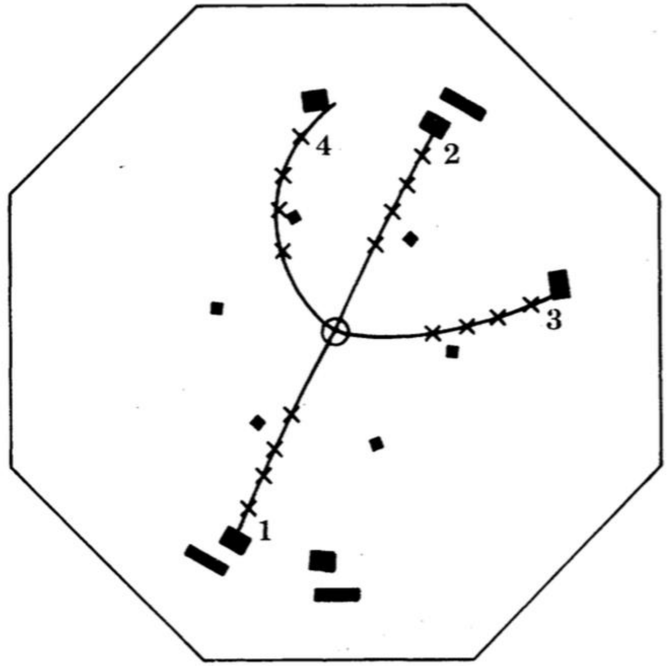


FIG. 3. An example of the decay $\psi(3684) \rightarrow \pi^+ + \pi^- + \psi(3095)$, where $\psi(3095) \rightarrow e^+ + e^-$, from an off-line reconstruction of the data. The event is seen in the x–y projection where z is the beam (and magnetic field) direction. Also shown are the trigger and shower counters which detected the tracks. Tracks 3 and 4 are the slow pions and tracks 1 and 2 are the two leptons from $\psi(3095)$ decay.

$$\psi(2S) \rightarrow J/\psi + \pi^+ \pi^-$$

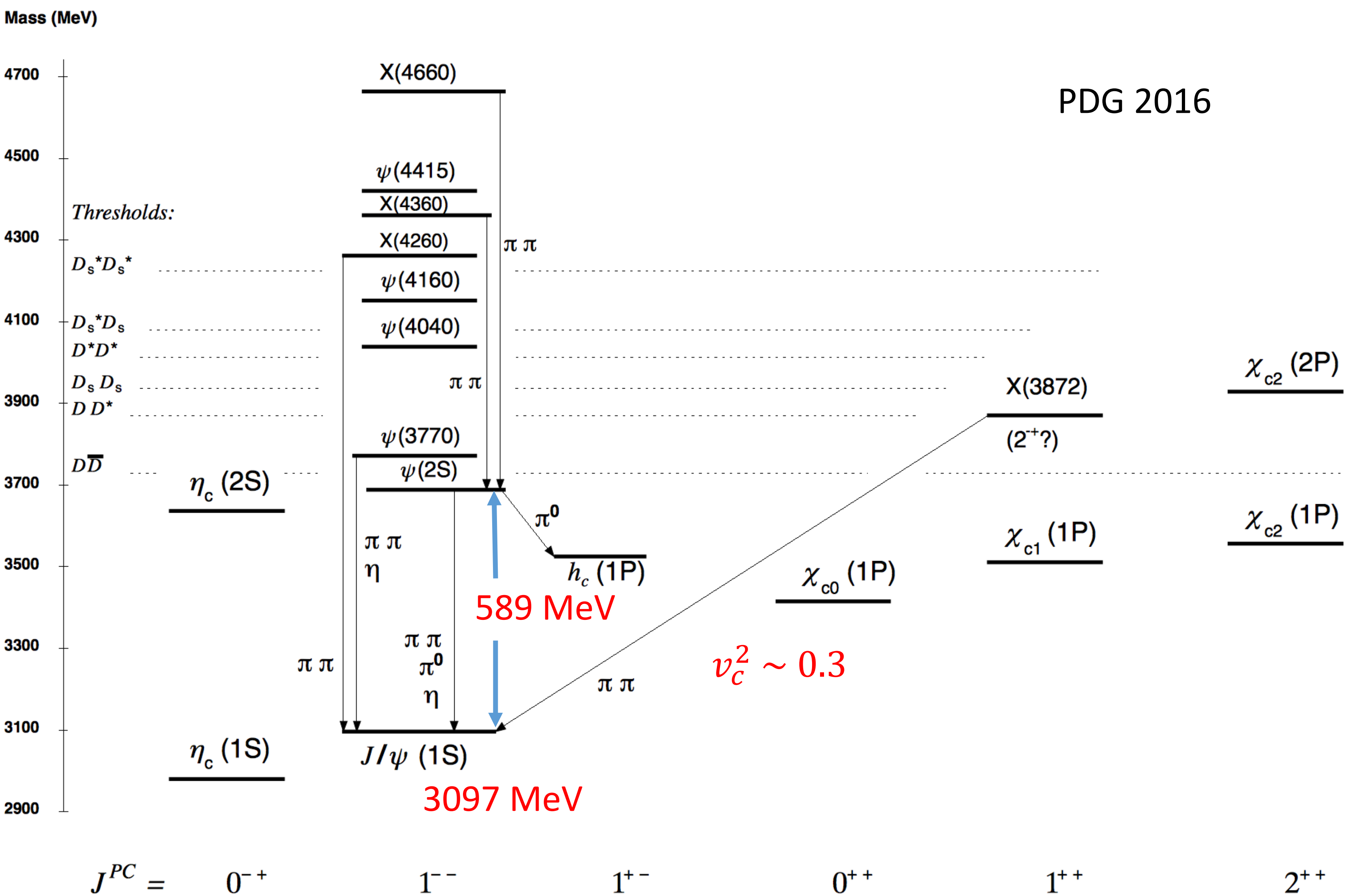
(PRL 34, 1181)

What is heavy quarkonium?

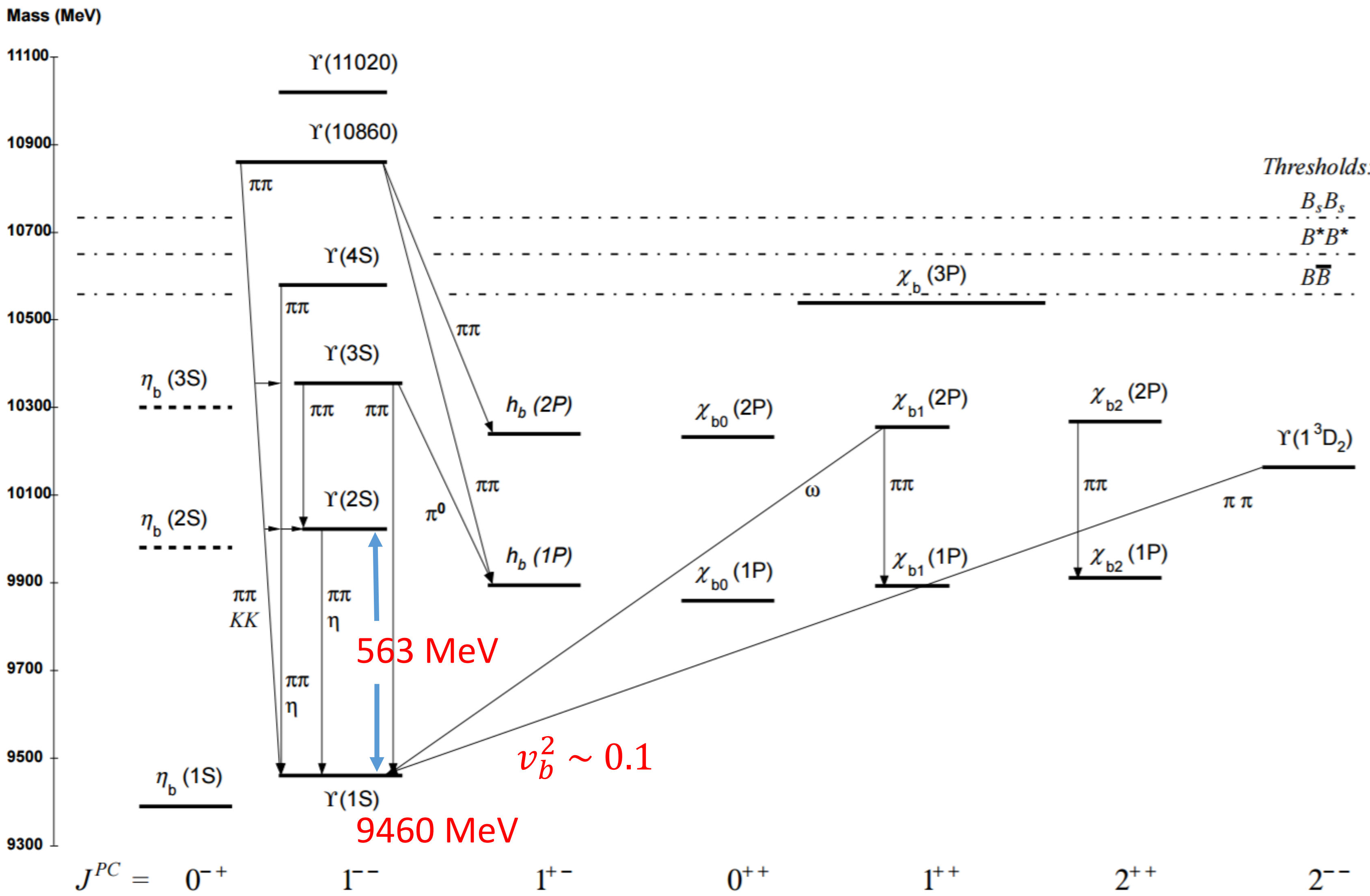
- Positronium (e^+e^-)
- Heavy Quarkonium

	S-wave		P-wave	
${}^{2S+1}L_J$	${}^1S_0$	${}^3S_1$	${}^1P_1$	${}^3P_{J(J=0,1,2)}$
Charmonium	η_c	J/ψ	h_c	χ_{cJ}
Bottomonium	η_b	Υ	h_b	χ_{bJ}
$c\bar{b}$	B_c	B_c^*		

THE CHARMONIUM SYSTEM



THE BOTTOMONIUM SYSTEM



Parity & Charge Conjugation

- Discrete Symmetry
  - Spin:
    - $S = 0$ symmetric
    - $S = 1$ antisymmetric $\Rightarrow (-1)^{S+1}$
  - Parity:
    - Orbital Angular Momentum $\Rightarrow (-1)^L$
    - Intrinsic parity:
      - Q(fermion)=+1, $\bar{Q}$(anti-fermion)= -1 $\Rightarrow -1$

$$P = (-1)^{L+1}$$

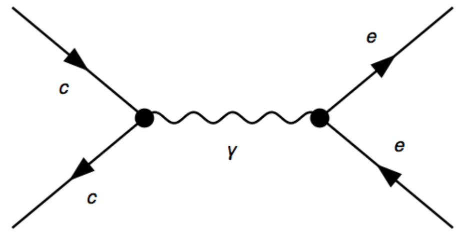
- Interchange of space and spin is equivalent to charge conjugation.

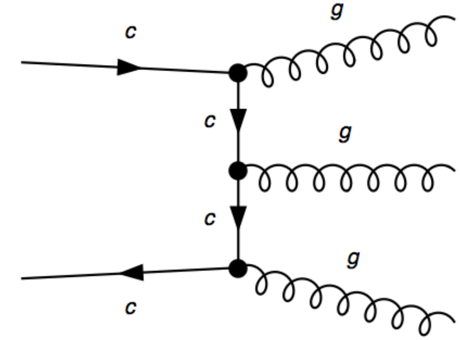
$$C = (-1)^{S+1}P = (-1)^{L+S}$$

	S-wave		P-wave	
$^{2S+1}L_J$	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_{J(J=0,1,2)}$
Charmonium	η_c	J/ψ	h_c	χ_{cJ}
Bottomonium	η_b	Υ	h_b	χ_{bJ}
S	0	1	0	1
L	0	0	1	1
J	0	1	1	$J = 0, 1, 2$
$P = (-1)^{L+1}$	$-$	$-$	$+$	$+$
$C = (-1)^{L+S}$	$+$	$-$	$-$	$+$
J^{PC}	0^{-+}	1^{--}	1^{+-}	J^{++}

J/ψ width is narrow

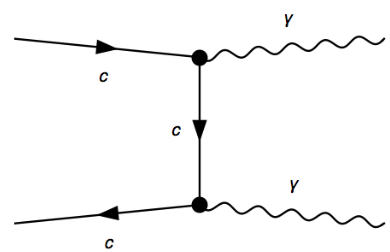
- J/ψ $(J^{PC} = 1^{--})$

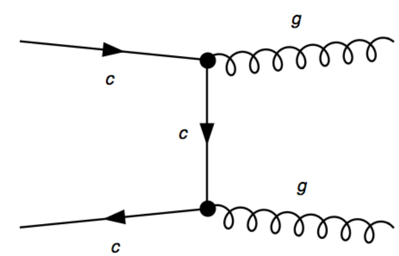




$C(g) = -1$

- η_c (0^{-+})
  χ_{cJ} (J^{++})





	Γ_{total}	Br(leptonic)	Br(hadronic)
J/ψ	93 keV	$2 \times 6\%$	88%
η_c	32 MeV	1.6×10^{-4}	~ 1
χ_{c0}	11 MeV	2.2×10^{-4}	~ 1

Narrow-width Resonance

1974, BNL

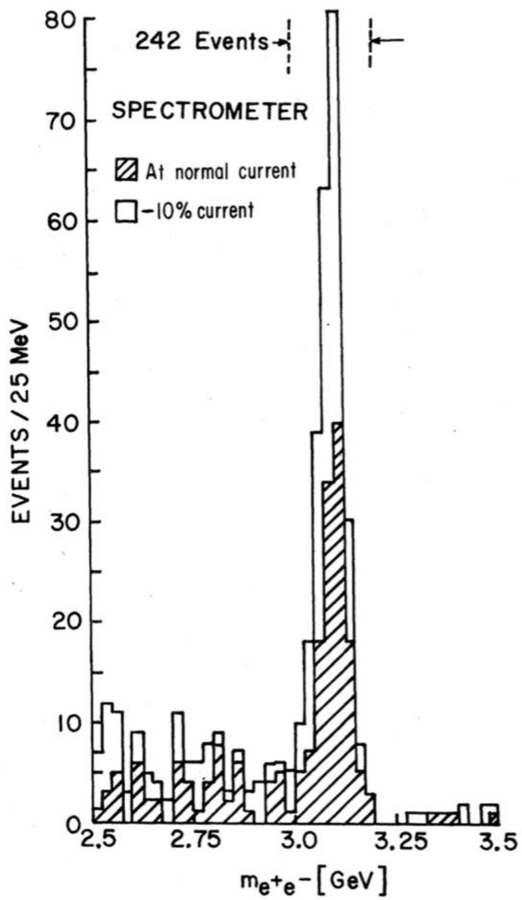


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

(PRL 33, 1404)

1974, SLAC

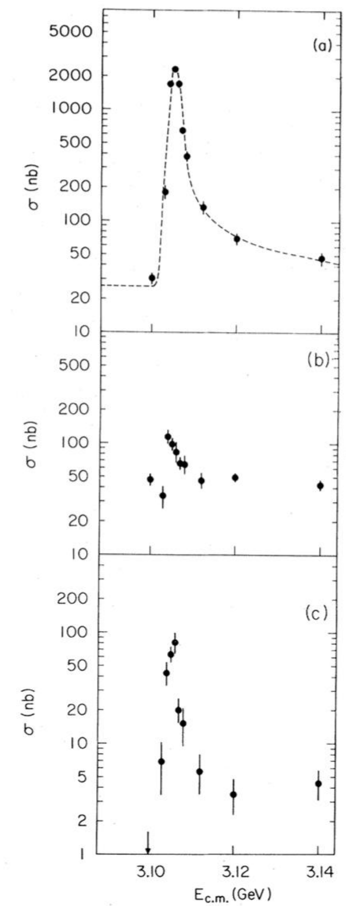
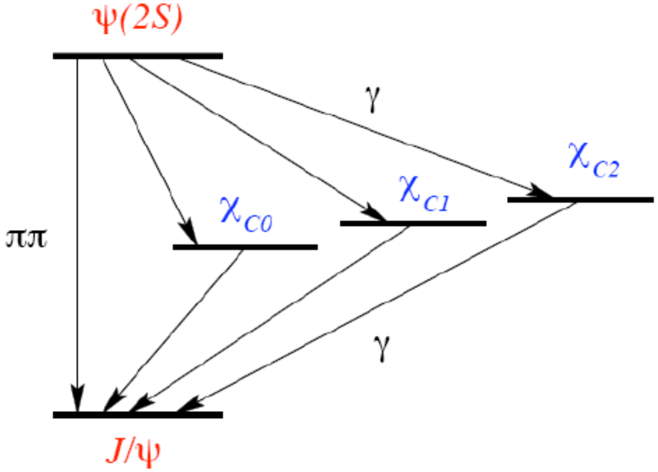


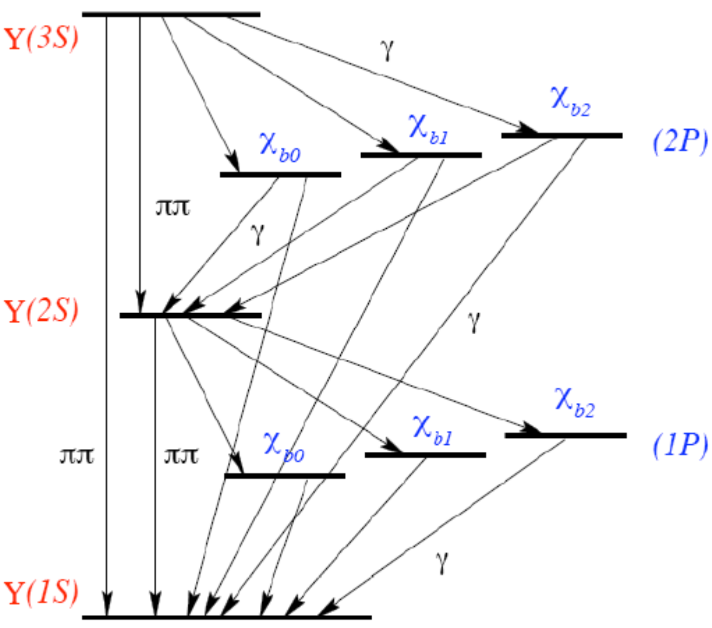
FIG. 1. Cross section versus energy for (a) multi-hadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\pi^+\pi^-$, and K^+K^- final states. The curve in (a) is the expected shape of a δ-function resonance folded with the Gaussian energy spread of the beams and including radiative processes. The cross sections shown in (b) and (c) are integrated over the detector acceptance. The total hadron cross section, (a), has been corrected for detection efficiency.

(PRL 33, 1406)

Prompt vs. Non-prompt

- Non-prompt = from B decay
  - → Large theoretical uncertainties
- Prompt = direct + feed-down





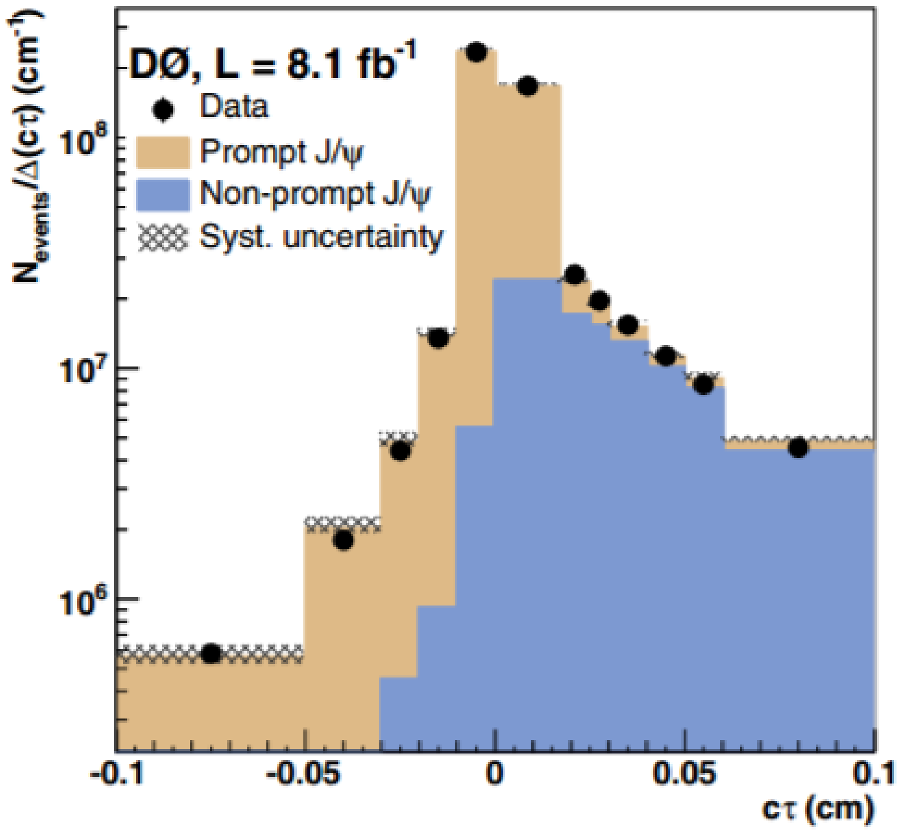


FIG. 1 (color online). The $c\tau$ distribution of background subtracted single J/ψ events after all selection criteria. The distributions for the signal and background templates are shown normalized to their respective fitted fractions. The uncertainty band corresponds to the total systematic uncertainty on the sum of signal and background events.

Quarkonium Theory

Physical Scales

m_Q	$m_Q v$	$m_Q v^2$
heavy quark mass	momentum	kinetic energy

$$\Lambda_{\mathrm{QCD}} \ll m_Q v^2 \ll m_Q v \ll m_Q$$

soft hard

$$m_c v_c^2 \sim m_b v_b^2 \sim 500\ \mathrm{MeV}$$

$$v_c^2 \sim 0.3$$

$$v_b^2 \sim 0.1$$

Two aspects of QCD

- Perturbative QCD (hard process):
  - $\alpha_s(m_Q)$ is small.
  - Creation/Annihilation of $Q\bar{Q}$ pair is perturbative.
- Nonperturbative QCD (soft process):
  - $\alpha_s(m_Q v), \alpha_s(m_Q v^2)$ are not small.
  - Wavefunction of $Q\bar{Q}$ pair is nonperturbative.
- We require a theory that respects both contributions simultaneously.
- Separation of hard and soft scales
  → Factorization: $\Gamma(H)$ or $\sigma(H)$ = hard × soft

Color-singlet model

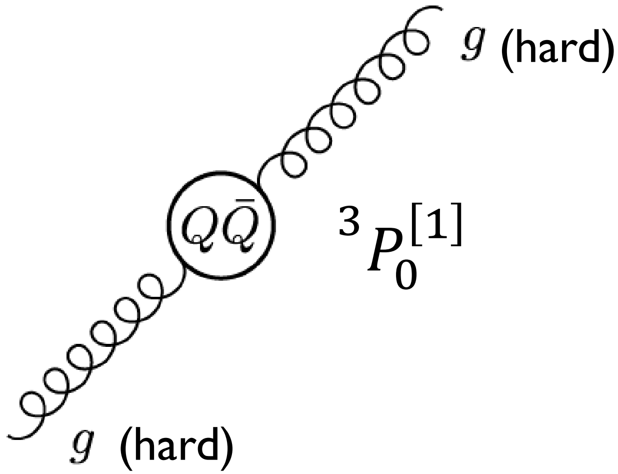
Factorization Assumption

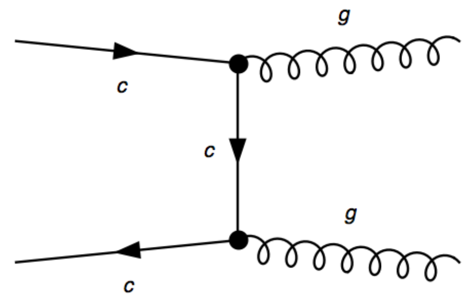
- $\Gamma(J/\psi \to e^+e^-) = c[c\bar{c}(^3S_1^{[1]}) \to e^+e^-] \times |R(0)|^2$
- $\Gamma(\chi_0 \to \text{hadrons}) = c[c\bar{c}(^3P_0^{[1]}) \to gg] \times |R'(0)|^2$

- c: perturbative expansion of α_s(hard)
- $R(0)$: radial wave function at the origin which can be determined by potential model, experiments, etc. (soft)

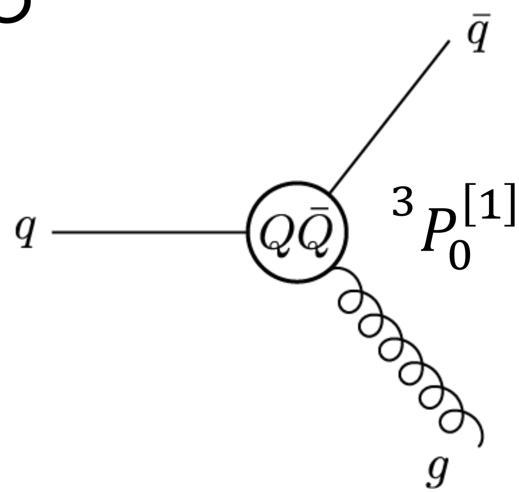
Failure in factorization at NLO

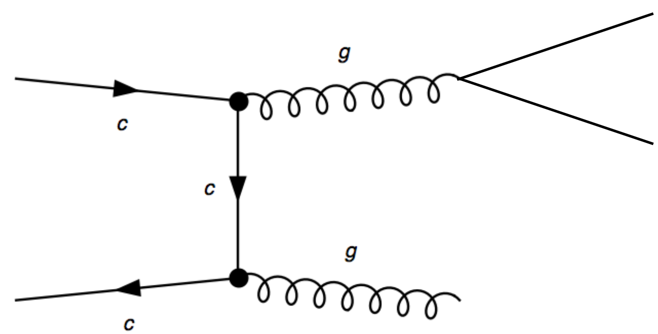
LO





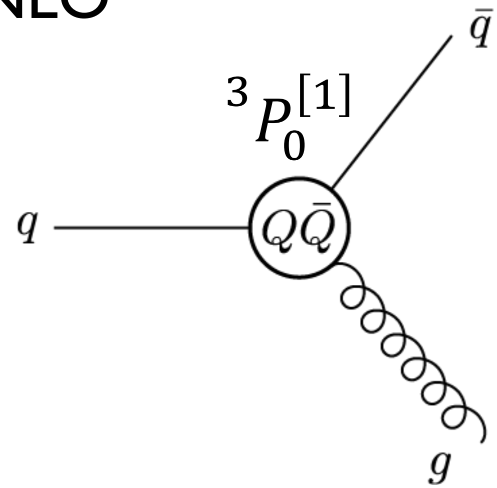
NLO





Failure in factorization at NLO

NLO



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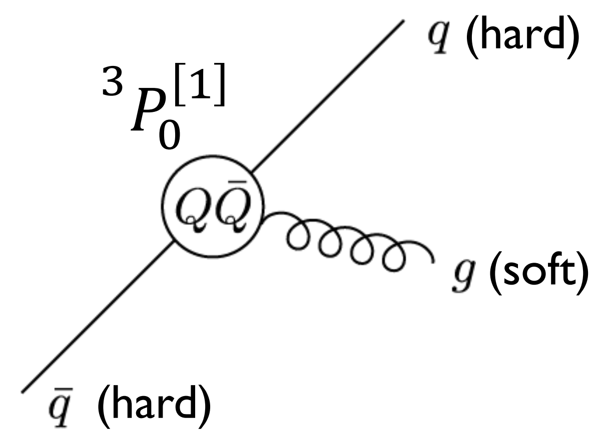


- $\Gamma(\chi_{c0} \to \text{hadrons}) = \left[6\alpha_s^2 + \left(\frac{8n_f}{9\pi}\log\frac{m_c}{\mu} + \cdots\right)\alpha_s^3\right] \times \frac{|R'(0)|^2}{m_c^4}$

- The short-distance factor is infrared sensitive at NLO.
  $\rightarrow$ CSM factorization fails.

PRD 46, R1914(R)

NRQCD

Fock-state of χ_{c0}

- $|\chi_{c0}\rangle = |c\bar{c}(^3P_0^{[1]})\rangle$: color-singlet state only (CSM)
- $|\chi_{c0}\rangle = |c\bar{c}(^3P_0^{[1]})\rangle + |c\bar{c}(^3S_1^{[8]}) + g_{\text{soft}}\rangle + \cdots$
  in NRQCD higher Fock states are included.
  $c\bar{c}$ states different from $^3P_0^{[1]}$ may contribute.
- Higher Fock states are suppressed in powers of v.
- If the short-distance coefficients are enhanced, for example, by powers of $1/\alpha_s$, then higher Fock states may contribute significantly.

Color-singlet NRQCD long-distance matrix elements (ME)

- $\langle O_1 \rangle_H \equiv \langle H | O[^{2S+1}L_J^{[1]}] | H \rangle, \quad H$: quarkonium

- $\langle J/\psi | O[^3S_1^{[1]}] | J/\psi \rangle \sim |R(0)|^2$
- $\langle \eta_c | O[^1S_0^{[1]}] | \eta_c \rangle \sim |R(0)|^2$
- $\langle \chi_{c0} | O[^3P_0^{[1]}] | \chi_{c0} \rangle \sim |R'(0)|^2$

Color-octet NRQCD long-distance ME

- $\langle O_8[n] \rangle_H \equiv \langle H | O[^{2S+1}L_J^{[8]}] | H \rangle, \quad H$: quarkonium

- $\langle J/\psi | O[^3S_1^{[8]}] | J/\psi \rangle, \dots$
- $\langle \eta_c | O[^1P_1^{[8]}] | \eta_c \rangle, \dots$
- $\langle \chi_{c0} | O[^3S_1^{[8]}] | \chi_{c0} \rangle, \dots$

NRQCD operators

$$\mathcal{O}_1(^1S_0) = \psi^\dagger\chi\,\chi^\dagger\psi,$$
$$\mathcal{O}_1(^3S_1) = \psi^\dagger\boldsymbol{\sigma}\chi\cdot\chi^\dagger\boldsymbol{\sigma}\psi,$$
$$\mathcal{O}_8(^1S_0) = \psi^\dagger T^a\chi\,\chi^\dagger T^a\psi,$$
$$\mathcal{O}_8(^3S_1) = \psi^\dagger\boldsymbol{\sigma}T^a\chi\cdot\chi^\dagger\boldsymbol{\sigma}T^a\psi.$$

$$\mathcal{O}_1(^1P_1) = \psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}})\chi\cdot\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}})\psi,$$
$$\mathcal{O}_1(^3P_0) = \frac{1}{3}\,\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\chi\,\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}\cdot\boldsymbol{\sigma})\psi,$$
$$\mathcal{O}_1(^3P_1) = \frac{1}{2}\,\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})\chi\cdot\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\mathbf{D}}\times\boldsymbol{\sigma})\psi,$$

Operator	Estimate	Description
α_s	v	effective quark-gluon coupling constant
ψ	$(Mv)^{3/2}$	heavy-quark (annihilation) field
χ	$(Mv)^{3/2}$	heavy-antiquark (creation) field
D_t	Mv^2	gauge-covariant time derivative
$\mathbf{D}$	Mv	gauge-covariant spatial derivative
$g\mathbf{E}$	M^2v^3	chromoelectric field
$g\mathbf{B}$	M^2v^4	chromomagnetic field
$g\phi$ (in Coulomb gauge)	Mv^2	scalar potential
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3	vector potential

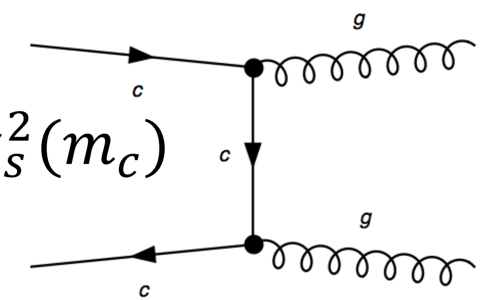
PRD 51, 1126

NRQCD factorization

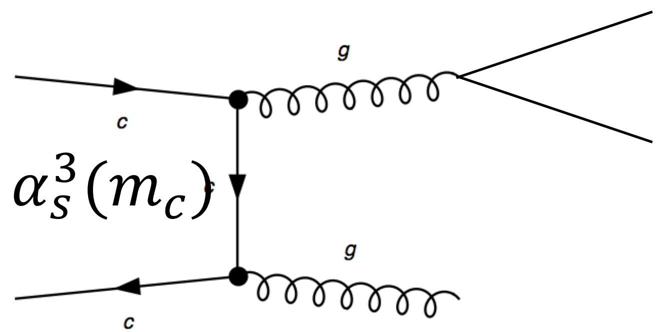
- $\Gamma(\chi_{c0} \to \text{hadrons}) = c_1 \langle \chi_{c0} | O[{}^3P_0^{[1]}] | \chi_{c0} \rangle$
  $+ c_8 \langle \chi_{c0} | O[{}^3S_1^{[8]}] | \chi_{c0} \rangle + \ \ldots$

- c_1

$\alpha_s^2(m_c)$

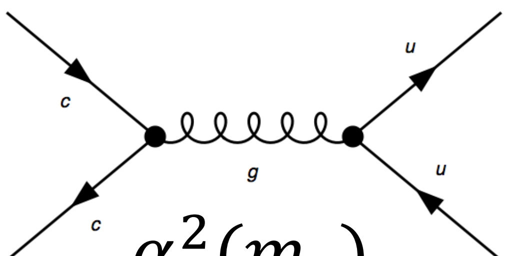


$+ \ \alpha_s^3(m_c)$



$+ \ \ldots$

- c_8



$\alpha_s^2(m_c)$

IR divergence of c_1 is originated from the 1-loop correction to $\langle \chi_{c0} | O[{}^3S_1^{[8]}] | \chi_{c0} \rangle$
$\to$ completely cancels after renormalizing the octet matrix element at NLO
$\to$ **c_1 is then free of IR divergence**

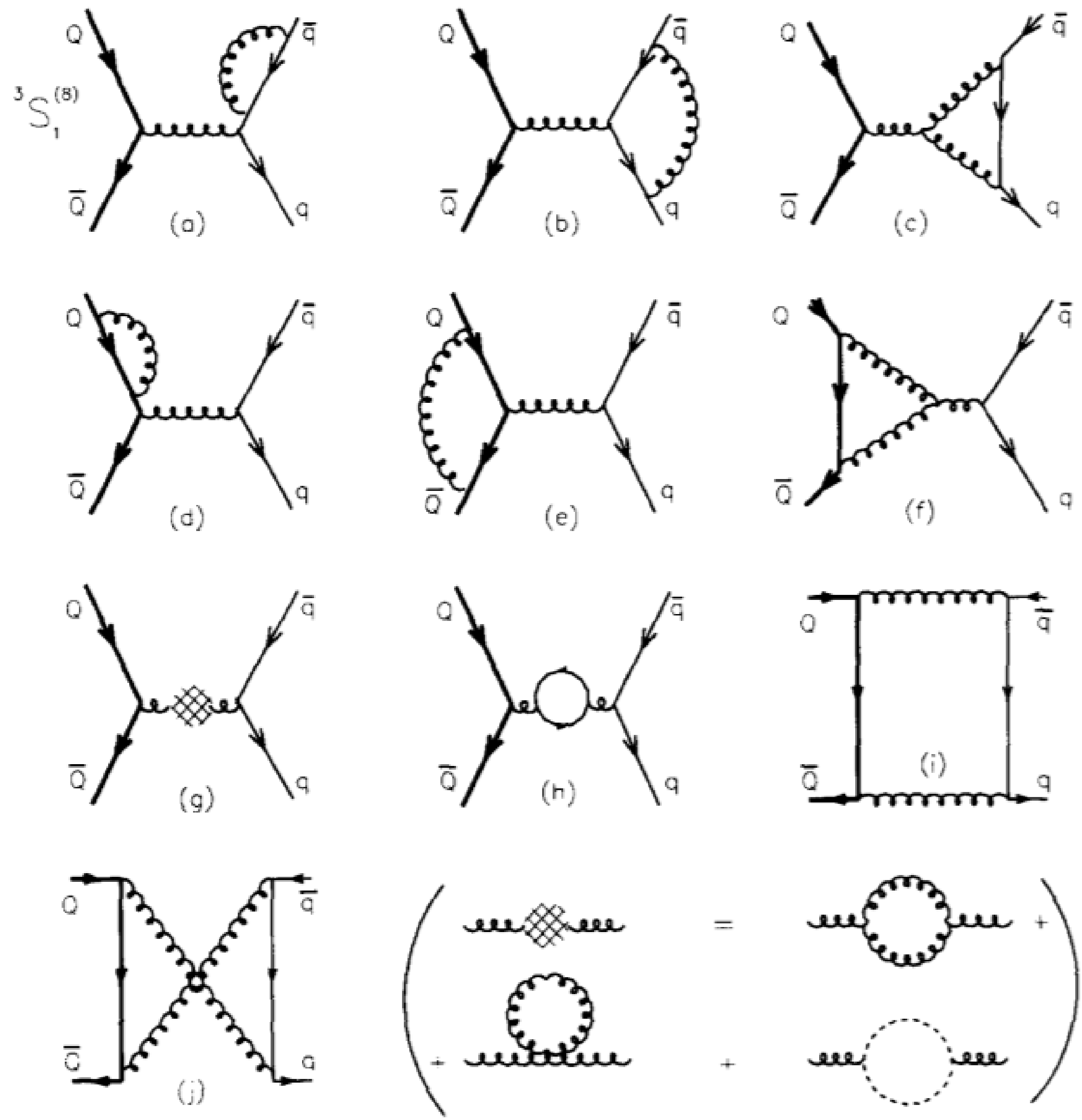


Fig. 1. Virtual Feynman diagrams contributing to the process $Q\overline{Q}[^3S_1^{(8)}] \to q\bar{q}$.

QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{light}} + \overline{\Psi}\left(i\gamma^{\mu} D_{\mu} - m_{Q}\right)\Psi$$

$$\mathcal{L}_{\text{light}} = -\frac{1}{2}\text{tr}\, G_{\mu\nu}G^{\mu\nu} + \sum \bar{q}\, i\not{D} q,$$

Ψ: 4-component Dirac spinor for $Q, \bar{Q}$

Parameters: $\alpha_s, m_q \approx 0, m_Q$

Degrees of freedom: gluons, $q, \bar{q}, Q, \bar{Q}$

NRQCD Lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}} = \mathcal{L}_{\mathrm{light}} + \mathcal{L}_{\mathrm{heavy}} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\mathrm{heavy}} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi,$$

$$\delta\mathcal{L} = \delta\mathcal{L}_{\mathrm{bilinear}} + \delta\mathcal{L}_{4-\mathrm{fermion}}$$

- Hard scale of order m_Q or higher integrated out
- Light degrees of freedom are identical to full QCD
- Nonrelativistic Q is annihilated by Pauli spinor field ψ
- Nonrelativistic $\bar{Q}$ is created by Pauli spinor field χ

Gauge-covariant derivative

$$D^{\mu} = \partial^{\mu} + igA^{\mu} = (D_t, -\boldsymbol{D})$$

$$D_t = \frac{\partial}{\partial t} + ig\phi \text{ and } \boldsymbol{D} = \boldsymbol{\nabla} - ig\boldsymbol{A}$$

$$A^{\mu} = A_a^{\mu} T^a$$

$$igG^{\mu\nu} = [D^{\mu}, D^{\nu}]$$

$$T^a = \frac{1}{2}\lambda^a : \text{SU(3) generators } (a = 1, \ldots, 8)$$

$$\left[T^a, T^b\right] = if^{abc}T^c$$

$$\mathrm{tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}$$

$\boldsymbol{\delta}\mathcal{L}_{\mathbf{bilinear}}$

- $\mathcal{L}_{\mathbf{heavy}}$: $\psi^\dagger \ldots \psi, \chi^\dagger \ldots \chi$ terms of LO in v
- $\boldsymbol{\delta}\mathcal{L}_{\mathbf{bilinear}}$: $\psi^\dagger \ldots \psi, \chi^\dagger \ldots \chi$ terms of higher orders in v

$$
\begin{aligned}
\delta\mathcal{L}_{\mathrm{bilinear}} = & \frac{c_1}{8M^3}\left[\psi^\dagger(\mathbf{D}^2)^2\psi \;-\; \chi^\dagger(\mathbf{D}^2)^2\chi\right] \\
& + \frac{c_2}{8M^2}\left[\psi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\psi \;+\; \chi^\dagger(\mathbf{D}\cdot g\mathbf{E} - g\mathbf{E}\cdot\mathbf{D})\chi\right] \\
& + \frac{c_3}{8M^2}\left[\psi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\psi \;+\; \chi^\dagger(i\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times i\mathbf{D})\cdot\boldsymbol{\sigma}\chi\right] \\
& + \frac{c_4}{2M}\left[\psi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\psi \;-\; \chi^\dagger(g\mathbf{B}\cdot\boldsymbol{\sigma})\chi\right],
\end{aligned}
$$

NRQCD Feynman Rules

Diagrammatic element	Feynman rule
quark propagator	$i/[\pm p_0 - \mathbf{p}^2/(2m) + i\epsilon]$
A_0 vertex	$\mp ig\, t_a$
$\boldsymbol{\nabla}\cdot\mathbf{A}_i$ vertex	$ig(p_i + p_i')t_a/(2m)$
$\mathbf{A}_i\cdot\mathbf{A}_j$ seagull vertex	$-ig^2(\delta_{ij}\, t_b t_a + \mathrm{perm})/(2m)$
$\boldsymbol{\sigma}\cdot\mathbf{B}$ spatial-gluon vertex	$g\,\epsilon_{ijk} l_{1j}\sigma_k\, t_a/(2m)$
$\mathbf{D}\cdot\mathbf{E}$ temporal-gluon vertex	$\pm ig\mathbf{l}_1^2\, t_a/(8m^2)$
$\mathbf{D}\cdot\mathbf{E}$ spatial-gluon vertex	$\mp ig\, l_{1i} l_{10}\, t_a/(8m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ temporal-gluon vertex	$\pm g\,\epsilon_{ijk} p_i' p_j \sigma_k\, t_a/(4m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-gluon vertex	$\mp g\,\epsilon_{ijk}(p+p')_j \sigma_k l_{10}\, t_a/(8m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-temporal seagull vertex	$\pm g^2(\epsilon_{ijk} l_{1j}\sigma_k\, t_b t_a + \mathrm{perm})/(4m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-spatial seagull vertex	$\pm g^2(\epsilon_{ijk}\sigma_k l_{10}\, t_b t_a + \mathrm{perm})/(4m^2)$
$\mathbf{D}^4/(8m^3)$ quark-propagator correction	$i\mathbf{p}^4/(8m^3)$
$\mathbf{D}^4/(8m^3)$ spatial-gluon vertex	$-ig(\mathbf{p}^2 + \mathbf{p}'^2)(p+p')_i\, t_a/(8m^3)$
$\mathbf{D}^4/(8m^3)$ spatial-gluon seagull vertex	$ig^2\{[(2p'-l_2)_j(2p+l_1)_i + (\mathbf{p}^2+\mathbf{p}'^2)\delta_{ij}]\, t_b t_a + \mathrm{perm}\}/(8m^3)$
$\mathbf{D}^4/(8m^3)$ 3-spatial-gluon vertex	$-ig^3\{[(2p'-l_3)_k\delta_{ij} + (2p+l_1)_i\delta_{kj}]t_c t_b t_a + \mathrm{perm}\}/(8m^3)$
$\mathbf{D}^4/(8m^3)$ 4-spatial-gluon vertex	$ig^4(\delta_{ij}\delta_{km}\, t_d t_c t_b t_a + \mathrm{perm})/(8m^3)$

$Q\bar{Q}_n \rightarrow$ Quarkonium

$$
{}^3S_1^{[1,8]} \xrightarrow{E1} {}^3P_J^{[8]} \xrightarrow{E1} \boxed{\begin{matrix} {}^3S_1^{[1]} \\ J/\psi \end{matrix}} \xleftarrow{M1} {}^1S_0^{[8]}
$$

$$
{}^3P_J^{[1,8]} \xrightarrow{E1} {}^3S_1^{[8]} \xrightarrow{E1} \boxed{\begin{matrix} {}^3P_J^{[1]} \\ \chi_{cJ} \end{matrix}} \xleftarrow{M1} {}^1P_1^{[8]}
$$

$\boldsymbol{\delta\mathcal{L}_{4-\mathbf{fermion}}}$

- $\boldsymbol{\delta\mathcal{L}_{4-\mathbf{fermion}}}$ involves $Q\bar{Q}$ annihilation decay

$$\delta\mathcal{L}_{\text{4-fermion}} = \sum_n \frac{f_n(\Lambda)}{M^{d_n-4}} \mathcal{O}_n(\Lambda),$$

$$(\delta\mathcal{L}_{\text{4-fermion}})_{d=6} = \frac{f_1(^1S_0)}{M^2}\,\mathcal{O}_1(^1S_0) + \frac{f_1(^3S_1)}{M^2}\,\mathcal{O}_1(^3S_1) + \frac{f_8(^1S_0)}{M^2}\,\mathcal{O}_8(^1S_0) + \frac{f_8(^3S_1)}{M^2}\,\mathcal{O}_8(^3S_1),$$

$$\begin{aligned}(\delta\mathcal{L}_{\text{4-fermion}})_{d=8} = {} & \frac{f_1(^1P_1)}{M^4}\,\mathcal{O}_1(^1P_1) + \frac{f_1(^3P_0)}{M^4}\,\mathcal{O}_1(^3P_0) + \frac{f_1(^3P_1)}{M^4}\,\mathcal{O}_1(^3P_1) \\ & + \frac{f_1(^3P_2)}{M^4}\,\mathcal{O}_1(^3P_2) + \frac{g_1(^1S_0)}{M^4}\,\mathcal{P}_1(^1S_0) + \frac{g_1(^3S_1)}{M^4}\,\mathcal{P}_1(^3S_1) \\ & + \frac{g_1(^3S_1,{}^3D_1)}{M^4}\,\mathcal{P}_1(^3S_1,{}^3D_1) + \cdots .\end{aligned}$$

Velocity-scaling rules (VSR)

- $\Delta p\, \Delta x \sim 1, \Delta p \sim mv \to \int d^3x \sim 1/(mv)^3$
- $\int d^3x\, \psi^\dagger(x)\psi(x) = 1 \to \psi^\dagger(x)\psi(x) \sim (mv)^3$
- $\nabla\psi(x) \sim (mv)\psi(x)$
- Equation of motion

$$\left(i\partial_t - g\phi(x) + \frac{\nabla^2}{2m}\right)\psi(x) = 0$$

The virial theorem implies that

$$\partial_t\psi \sim g\phi\psi \sim \frac{\nabla^2}{2m}\psi \sim mv^2\psi$$

- Therefore, $g\phi \sim mv^2$
- In a similar manner, VS of an NRQCD operator can be estimated.

VSR Table

Operator	Estimate
α_s	v
ψ	$(mv)^{3/2}$
χ	$(mv)^{3/2}$
D_t (acting on ψ or χ)	mv^2
$\mathbf{D}$ (acting on ψ or χ)	mv
$g\mathbf{E}$	m^2v^3
$g\mathbf{B}$	m^2v^4
$g\phi$ (in Coulomb gauge)	mv^2
$g\mathbf{A}$ (in Coulomb gauge)	mv^3

$$\langle \mathcal{O}_1^{J/\psi}(^3S_1)\rangle \sim v^0,$$
$$\langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle \sim v^3,$$
$$\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle \sim v^4,$$
$$\langle \mathcal{O}_8^{J/\psi}(^3P_0)\rangle \sim v^4.$$

NRQCD Factorization Formula

for annihilation decay rate of charmonium H

$$\Gamma[H] = \sum_n \hat{\Gamma}[c\bar{c}(n)]\ \langle H|\mathcal{O}_n|H\rangle$$

- sum over color/angular momentum channels

  1 or 8 $\ {}^1S_0,\ {}^3S_1,\ {}^1P_1,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2, \ldots$

- hard factors: annihilation rate into partons for $c\bar{c}$ at threshold

  expand in powers of $\alpha_s(m_c)$

- soft factors: NRQCD matrix element $\langle H \mid O_n \mid H\rangle$

  probability density at the origin

  for Fock state with $c\bar{c}$ in state n

  scales as definite power of v

- rigorous factorization formula

  double expansion in $\alpha_s(m_c)$ and v

NRQCD Factorization Formula

Annihilation decay rate of charmonium *H*

$$\Gamma[H] = \sum_n \hat{\Gamma}[c\bar{c}(n)]\ \langle H|\mathcal{O}_n|H\rangle$$

- velocity scaling of NRQCD matrix elements

$$J/\psi: \quad \boxed{\langle \underline{1}\ {}^3S_1\rangle} \sim v^3$$

$$\langle \underline{8}\ {}^3P_J\rangle, \langle \underline{8}\ {}^1S_0\rangle, \langle \underline{8}\ {}^3S_1\rangle \sim v^7$$

CSM

$$\chi_{cJ}: \quad \boxed{\langle \underline{1}\ {}^3P_J\rangle}, \langle \underline{8}\ {}^3S_1\rangle \sim v^5$$

- solves infrared divergence problem for P-waves
- spin symmetry relates J/ψ, η_c
  χ_{c0}, χ_{c1}, χ_{c2}, h_c

NRQCD Factorization

for inclusive hard production of charmonium H

- Inclusive: cross section for $H + X$
  summed over all possible hadronic states X

- hard: involves large momentum transfer
  (creation of $c\bar{c}$ pair is not be enough)

Hadron collisions:

forward (diffractive) production can create cc pair
without large momentum transfer
NRQCD factorization will not apply

production with large transverse momentum p_T
should be sufficient to use NRQCD factorization
but how large does p_T need to be?
$p_T >> \Lambda_{QCD}$? $p_T >> Mv$? $p_T >> M$?

NRQCD Factorization Formula

Inclusive differential cross section for charmonium *H*

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)]\ \langle \mathcal{O}_n^H \rangle$$

- conjectured factorization formula
  motivated by perturbative QCD factorization theorems
- sum over color/angular momentum channels
  1 or 8 $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$, ...
- hard factors: parton cross sections for creating $c\bar{c}$
  expand in powers of $\alpha_s(m_c)$
- soft factors: NRQCD matrix element $\langle O_n^H \rangle \equiv \langle n \rangle$
  probability density at origin for $c\bar{c}$ pair in state *n*
  to bind into charmonium *H* plus soft gluons
  scale as definite powers of *v*

NRQCD Factorization

Inclusive production of charmonium *H*

$$d\sigma[H] = \sum_n d\hat{\sigma}[c\bar{c}(n)] \, \langle \mathcal{O}_n^H \rangle$$

- velocity scaling of NRQCD matrix elements

CSM

$$J/\psi : \qquad \boxed{\langle \underline{1}\ ^3S_1 \rangle} \sim v^3$$

$$\chi_{cJ} : \qquad \boxed{\langle \underline{1}\ ^3P_J \rangle}, \langle \underline{8}\ ^3S_1 \rangle \sim v^5$$

- solves infrared divergence problem for P-waves

- vacuum saturation approximation
  relates CSM matrix elements for production
  to those for annihilation decays

Matching

- Determination of short-distance coefficients (SDC) in the (squared) amplitude $A^{H}_{\mathrm{NRQCD}}(m_Q, \alpha_s, c_i)$
- SDC must be insensitive to long-distance (LD) effects: $A^{Q\bar{Q}}_{\mathrm{NRQCD}}(m_Q, \alpha_s, c_i)$ and $A^{H}_{\mathrm{NRQCD}}(m_Q, \alpha_s, c_i)$ must share identical SDC
- Compute corresponding $Q\bar{Q}$ matrix elements using perturbative NRQCD: $A^{Q\bar{Q}}_{\mathrm{NRQCD}}(m_Q, \alpha_s, c_i)$
- Compute the scattering amplitude using perturbative QCD: $A^{Q\bar{Q}}_{\mathrm{QCD}}(m_Q, \alpha_s)$
- Determine SDC from the matching condition:

$$A^{Q\bar{Q}}_{\mathrm{NRQCD}}(m_Q, \alpha_s, c_i) = A^{Q\bar{Q}}_{\mathrm{QCD}}(m_Q, \alpha_s)$$

Example of matching

- Tree-level matching of the pole in heavy quark propagator.

- full QCD $$E = \sqrt{m_Q^2 + \boldsymbol{p}^2} - m_Q = \frac{\boldsymbol{p}^2}{2m_Q} - \frac{\boldsymbol{p}^4}{8m_Q^3} + \cdots .$$

- NRQCD $$E = \frac{\boldsymbol{p}^2}{2m} - c_1 \frac{\boldsymbol{p}^4}{8m^3} + \cdots .$$

- then $m = m_Q$ and $c_1 = 1$.

Determination of NRQCD ME

- $\langle Q\bar{Q}|O[^{2S+1}L_J^{[1 \text{ or } 8]}]|Q\bar{Q}\rangle$ is calculable perturbatively
- $\langle H|O[^{2S+1}L_J^{[1 \text{ or } 8]}]|H\rangle$ is nonperturbative
  - Color-singlet case: some ME is calculable on the lattice or predictable using potential models.
  - Color-octet case: some ME is calculable on the lattice (difficult). Potential model is not applicable.
  - Comparing the factorization formula with empirical values, one can fit the data to determine ME. The number of measurables is limited → one must terminate the series for NRQCD factorization formula at a certain order in v.

Quarkonium Physics 2

Oct. 24, 2016

15:30 ~ 17:00

$J/\psi \to e^+e^-$

Relativistic Corrections to $J/\psi \to \ell^+\ell^-$ at order α_s

PRD 74, 014014 (2006)
PRD 77, 094017 (2008)
PRD 79, 014007(2009)

Jungil Lee
(Korea University)

In collaboration with Geoffrey Bodwin (Argonne),
Hee Sok Chung and Chaehyun Yu (Korea U.)

Outline

- Charmonium
- NRQCD
- 1-loop corrections to $J/\psi \to \ell^+\ell^-$
  at leading and sub-leading orders in v
- Renormalization of NRQCD operator
- Matching
- Summary

Charmonium

- Simplest bound state of quarks:
  bound state of a charm quark and an charm antiquark.
- The lightest mesons in $^3S_1$ and $^1S_0$ states are the J/ψ and the η_c.

Nonrelativistic QCD

- NRQCD is an effective theory of QCD.
- NRQCD factorization separates short-distance perturbative effects from long-distance non-perturbative effects.
- NRQCD involves a series expansion in v, where mv is the typical heavy-quark momentum at the meson rest frame.

Phenomenological importance of $\boldsymbol{J/\psi \to \ell^+\ell^-}$

- One method of determining long-distance non-perturbative effects is to use the leptonic decay rate.
- Leptonic decay rate is one of the most accurate measurements involving J/ψ.

$$\Gamma[J/\psi \to e^+e^-] = 5.55 \pm 0.14 \pm 0.02\ \mathrm{keV}$$ (PDG2016)

NRQCD factorization formula for $\boldsymbol{\Gamma[J/\psi \to \ell^+\ell^-]}$

- Comparing the theoretical expression with experimental data, we can determine the NRQCD matrix element. At leading order,

$$\Gamma[J/\psi \to \ell^+\ell^-] = \frac{8\pi e_c^2\alpha^2}{3(M_{J/\psi}^2)}\langle O_1\rangle_{J/\psi}$$

$$\langle O_1\rangle_{J/\psi} = |\langle 0|\chi^\dagger\boldsymbol{\sigma}\psi|J/\psi\rangle|^2$$

- Determination of the NRQCD matrix element is very important in processes involving the J/ψ.

Impact of the resummation of relativistic corrections

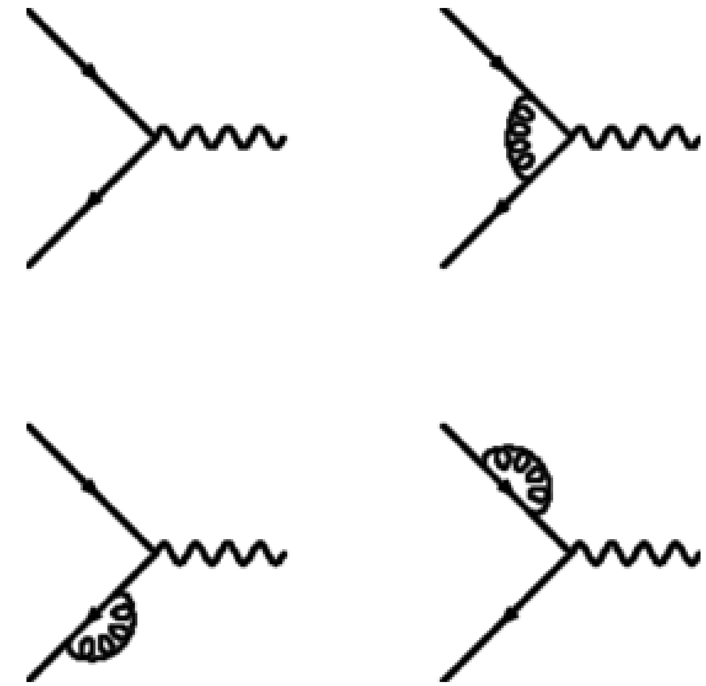
- Leading-order NRQCD ME fails to predict $e^+e^- \to J/\psi + \eta_c$ at B factories
- Relativistic and α_s corrections resolve the problem.
- Relativistic corrections are resummed to all orders in v.

Impact of the resummation of relativistic corrections

- In a previous work, the J/ψ matrix element was determined by using the Cornell potential model and the resummation of relativistic corrections.
  (Bodwin, Kang, Lee, PRD74, 014014,
  Bodwin, Chung, Kang, Lee, Yu, arXiv:0710.0994 [hep-ph])

- In the previous work, the order-α_s correction has only been included at leading order in v.
- For a more accurate determination of the NRQCD matrix element, we evaluate the relativistic corrections at order α_s.
- UV and IR singularities are dimensionally regularized with $d = 4 - 2\epsilon$.

1-loop corrections at leading order in v



- The QCD amplitude is

$$iA^{\mu}_{\mathrm{QCD}} = Z_Q(1+\Lambda)\bar{v}(p_2)\gamma^{\mu}u(p_1)$$

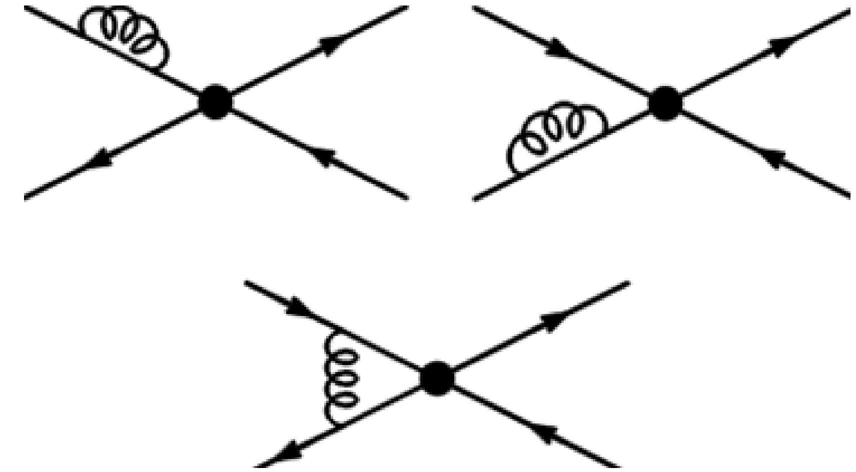
$$Z_Q = 1 + \frac{\alpha_s C_F}{4\pi}\left[-\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{2}{\epsilon_{\mathrm{IR}}} - 3\log\left(\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) - 4\right]$$

$$\Lambda = \frac{\alpha_s C_F}{4\pi}\left\{\frac{1}{\epsilon_{\mathrm{UV}}} + \frac{2}{\epsilon_{\mathrm{IR}}} + 3\log\left(\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) - 4 + \frac{1}{v}\left[\pi^2 - \frac{i\pi}{\epsilon_{\mathrm{IR}}} - i\pi\log\left(\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right]\right\}$$

- UV and IR poles cancel.

$$Z_Q(1+\Lambda) = 1 + \frac{\alpha_s C_F}{4\pi}\left\{-8 + \frac{1}{v}\left[\pi^2 - \frac{i\pi}{\epsilon_{\mathrm{IR}}} - i\pi\log\left(\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right]\right\}$$

1-loop corrections at leading order in v



- The remaining Coulomb terms are canceled by the loop corrections in NRQCD.

$$
\begin{aligned}
Z_Q^{\mathrm{NRQCD}} &= 1, \\
\Lambda^{\mathrm{NRQCD}} &= \frac{\alpha_s C_F}{4\pi}\frac{1}{v}\left[\pi^2 - \frac{i\pi}{\epsilon_{\mathrm{IR}}} - i\pi\log\left(\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right]
\end{aligned}
$$

1-loop corrections at sub-leading orders in v

- In QCD loop corrections, IR divergence remains uncanceled.

$$i\mathcal{A}^{\mu}_{\rm QCD} = \bar{v}(p_2)\left[Z_Q(1+\Lambda)\gamma^\mu + Bq^\mu\right]u(p_1)$$

IR divergence

$$\begin{aligned} Z_Q(1+\Lambda) &= 1 + \frac{\alpha_s C_F}{4\pi}\left\{\frac{8v^2}{3}\left[\frac{1}{\epsilon_{\rm IR}} + \log\left(\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right)\right] - 8 + \frac{2v^2}{9}\right. \\ &\quad \left. + \left(\frac{1}{v} + \frac{3v}{2}\right)\left[\pi^2 - \frac{i\pi}{\epsilon_{\rm IR}} - i\pi\log\left(\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right)\right] - 3i\pi v + O(v^3)\right\} \\ B &= -\frac{\alpha_s C_F}{4\pi}\frac{1}{m}\left(\frac{i\pi}{v} - 2 + O(v)\right) \end{aligned}$$

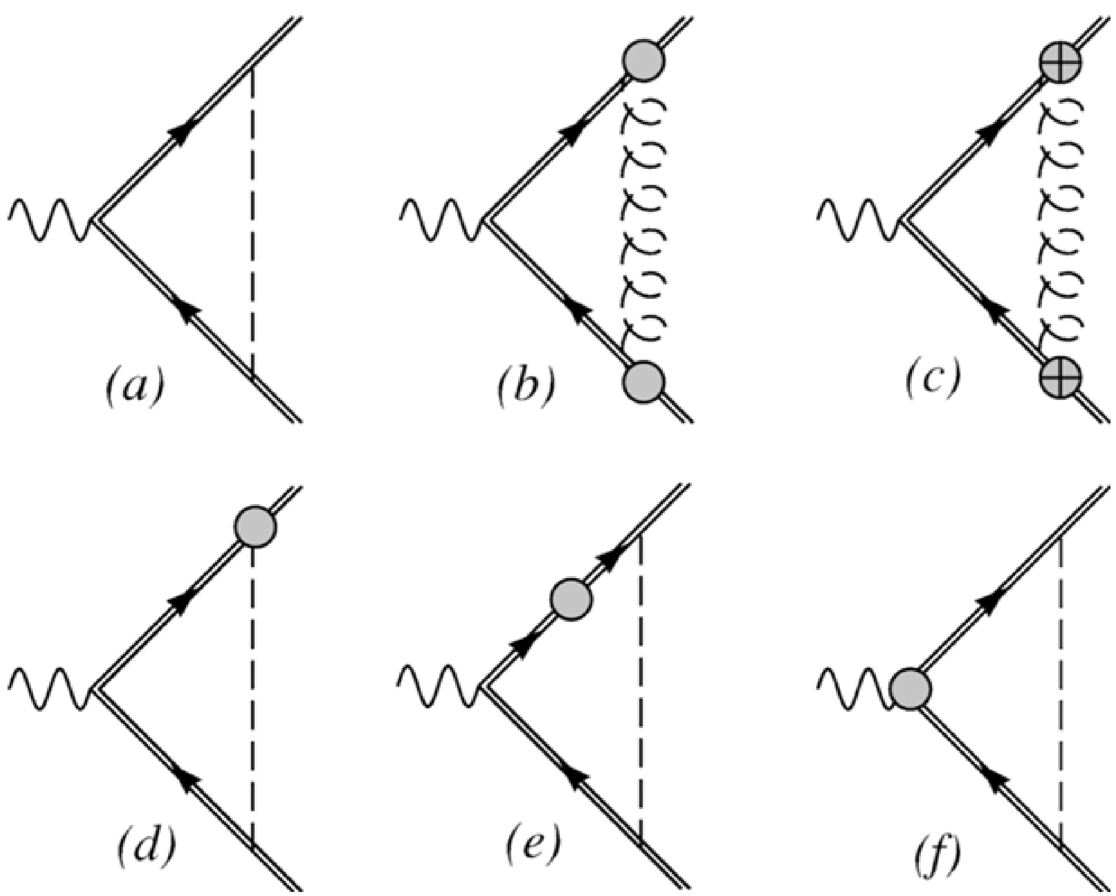
Coulomb interaction

1-loop corrections at sub-leading orders in v

- The remaining IR poles and non-analytic dependence on $\boldsymbol{q}^2$ in perturbative QCD calculations must be canceled by NRQCD counterparts.

1-loop corrections to NRQCD matrix elements

- The following diagrams contribute at order-$\alpha_s v^2$



Corrections to $\langle 0|\chi^\dagger \boldsymbol{\sigma}^i \psi|Q\bar{Q}\rangle$
(a) A_0 temporal
(b) $\boldsymbol{\nabla}\cdot\boldsymbol{A}$ spatial
(c) $\boldsymbol{\sigma}\cdot\boldsymbol{B}$ spatial
(d) $\boldsymbol{D}\cdot\boldsymbol{E}$ temporal
(e) $\boldsymbol{D}^4$ quark propagator correction

Corrections to
$\langle 0|\chi^\dagger \left(\overrightarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overrightarrow{\boldsymbol{\nabla}}^i + \overleftarrow{\boldsymbol{\nabla}}\cdot\boldsymbol{\sigma}\overleftarrow{\boldsymbol{\nabla}}^i\right)\psi|Q\bar{Q}\rangle$
(f) A_0 temporal

Diagrammatic element	Feynman rule
quark propagator	$i/[\pm p_0 - \mathbf{p}^2/(2m) + i\epsilon]$
A_0 vertex	$\mp ig\, t_a$
$\boldsymbol{\nabla}\cdot\mathbf{A}_i$ vertex	$ig(p_i + p'_i)t_a/(2m)$
$\mathbf{A}_i\cdot\mathbf{A}_j$ seagull vertex	$-ig^2(\delta_{ij}\, t_b t_a + \text{perm})/(2m)$
$\boldsymbol{\sigma}\cdot\mathbf{B}$ spatial-gluon vertex	$g\epsilon_{ijk} l_{1j}\sigma_k\, t_a/(2m)$
$\mathbf{D}\cdot\mathbf{E}$ temporal-gluon vertex	$\pm ig\mathbf{l}_1^2\, t_a/(8m^2)$
$\mathbf{D}\cdot\mathbf{E}$ spatial-gluon vertex	$\mp ig l_{1i} l_{10}\, t_a/(8m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ temporal-gluon vertex	$\pm g\epsilon_{ijk} p'_i p_j \sigma_k\, t_a/(4m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-gluon vertex	$\mp g\epsilon_{ijk}(p+p')_j\sigma_k l_{10}\, t_a/(8m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-temporal seagull vertex	$\pm g^2(\epsilon_{ijk} l_{1j}\sigma_k\, t_b t_a + \text{perm})/(4m^2)$
$\mathbf{D}\times\mathbf{E}\cdot\boldsymbol{\sigma}$ spatial-spatial seagull vertex	$\pm g^2(\epsilon_{ijk}\sigma_k l_{10}\, t_b t_a + \text{perm})/(4m^2)$
$\mathbf{D}^4/(8m^3)$ quark-propagator correction	$i\mathbf{p}^4/(8m^3)$
$\mathbf{D}^4/(8m^3)$ spatial-gluon vertex	$-ig(\mathbf{p}^2 + \mathbf{p}'^2)(p+p')_i\, t_a/(8m^3)$
$\mathbf{D}^4/(8m^3)$ spatial-gluon seagull vertex	$ig^2\{[(2p'-l_2)_j(2p+l_1)_i + (\mathbf{p}^2+\mathbf{p}'^2)\delta_{ij}]\, t_b t_a + \text{perm}\}/(8m^3)$
$\mathbf{D}^4/(8m^3)$ 3-spatial-gluon vertex	$-ig^3\{[(2p'-l_3)_k\delta_{ij} + (2p+l_1)_i\delta_{kj}]t_c t_b t_a + \text{perm}\}/(8m^3)$
$\mathbf{D}^4/(8m^3)$ 4-spatial-gluon vertex	$ig^4(\delta_{ij}\delta_{km}\, t_d t_c t_b t_a + \text{perm})/(8m^3)$

Bodwin and Chen, PRD 60 054008

1-loop corrections to NRQCD matrix elements

- Integral involving real parts of IR poles:

$$\int \frac{d^N k}{(2\pi)^N} \frac{1}{|\boldsymbol{k}|^3} = \frac{1}{4\pi^2} \left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}} \right)$$

- Integral involving imaginary parts of IR poles:

$$\int \frac{d^N k}{(2\pi)^N} \frac{1}{\boldsymbol{k}^2(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon)} = \frac{1}{16\pi|\boldsymbol{q}|} \left[\pi - \frac{i}{\epsilon_{\mathrm{IR}}} \left(\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} \right)^{\epsilon} \right]$$

- Typical integral involving Coulomb interaction:

$$\int \frac{d^N k}{(2\pi)^N} \frac{1}{\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon} = \frac{i}{4\pi} |\boldsymbol{q}|$$

Renormalization of NRQCD matrix elements

- The UV pole associated with the real part of the IR pole must be subtracted using renormalization of the NRQCD operator.
- In the $\overline{\text{MS}}$ scheme,

$$\chi^\dagger \sigma^i \psi = (\chi^\dagger \sigma^i \psi)^{(\Lambda)} - \frac{(4\pi e^{-\gamma_E})^\epsilon}{\epsilon_{\text{UV}}} \frac{2\alpha_s C_F}{3\pi m^2} \; \chi^\dagger \sigma^i (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2 \psi$$

Λ: NRQCD factorization scale

Matching

- Matching the NRQCD amplitude onto the QCD amplitude, we see that the remaining IR poles and the Coulomb interaction terms exactly cancel.

Matching

- The amplitude can now be written as

$$i\mathcal{A}(Q\bar{Q} \to \gamma^*) =$$

$$c_0 \langle 0|\chi^\dagger \sigma^i \psi|Q\bar{Q}_1\rangle + c_1 \frac{1}{4m^2} \langle 0|\chi^\dagger \left(\overrightarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \overrightarrow{\boldsymbol{\nabla}}^i + \overleftarrow{\boldsymbol{\nabla}} \cdot \boldsymbol{\sigma} \overleftarrow{\boldsymbol{\nabla}}^i \right) \psi|Q\bar{Q}_1\rangle$$

$$+ c_2 \frac{1}{m^2} \langle 0|\chi^\dagger \sigma^i (-\tfrac{i}{2}\boldsymbol{\nabla})^2 \psi|Q\bar{Q}_1\rangle + O(v^4)$$

with the short-distance coefficients

$$\begin{aligned} c_0 &= 1 - \frac{8\alpha_s}{3\pi} \\ c_1 &= 1 - \frac{4\alpha_s}{3\pi} \\ c_2 &= \frac{\alpha_s}{9\pi}\left(\frac{2}{3} + 8\log\frac{\Lambda^2}{m^2}\right) \end{aligned}$$

Matching

- Averaging over the angles of the relative momentum, we pull out the S-wave contribution.

$$
\begin{aligned}
i\mathcal{A}(Q\bar{Q}({}^3S_1) \to \gamma^*) \quad &= \quad \left(1 - \frac{8\alpha_s}{3\pi}\right) \langle 0|\chi^\dagger \sigma^i \psi | Q\bar{Q}_1({}^3S_1)\rangle \\
&+ \left[-\frac{1}{6} + \frac{4\alpha_s}{3\pi}\left(\frac{2}{9} + 8\log\frac{\Lambda^2}{m^2}\right)\right] \frac{1}{m^2} \langle 0|\chi^\dagger \sigma^i (-\tfrac{i}{2}\boldsymbol{\nabla})^2 \psi | Q\bar{Q}_1({}^3S_1)\rangle + O(v^4)
\end{aligned}
$$

$\Gamma[J/\psi \to \ell^+\ell^-]$

- The decay rate is

$$\Gamma[J/\psi \to \ell^+\ell^-] \quad = \quad \frac{8\pi e_c^2\alpha^2}{3(M_{J/\psi}^2)} \left\{1 - \frac{8\alpha_s}{3\pi} \right.$$

$$\left. +\langle v^2\rangle_{J/\psi} \left[-\frac{1}{6} + \frac{4\alpha_s}{3\pi}\left(\frac{2}{9} + 8\log\frac{\Lambda^2}{m^2}\right)\right]\right\}^2 \langle O_1\rangle_{J/\psi}$$

$$\langle O_1\rangle_{J/\psi} = |\langle 0|\chi^\dagger \boldsymbol{\sigma}\psi|J/\psi\rangle|^2$$

$$\langle v^2\rangle_{J/\psi} = \frac{1}{m^2}\frac{\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi|J/\psi\rangle}{\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle}$$

References for $c_n^{(i)}$

	v^0	v^2	v^4	v^{all}
α_s^0	[1]	[1]	[2]	[8]
α_s^1	[3], [4]	[5]	[9]	[9]
α_s^2	[6], [7]			
α_s^3	[10]			

[1] Bodwin, Braaten, Lepage, PRD 51, 1125

[2] Bodwin, Petrelli, PRD 66, 094011

[3] Barbieri et al., PL 57B, 455

[4] Celmaster, PRD 19, 1517

[5] Luke, Savage, PRD 57, 413

[6] Czarnecki, Melnikov, PRL 80, 2531

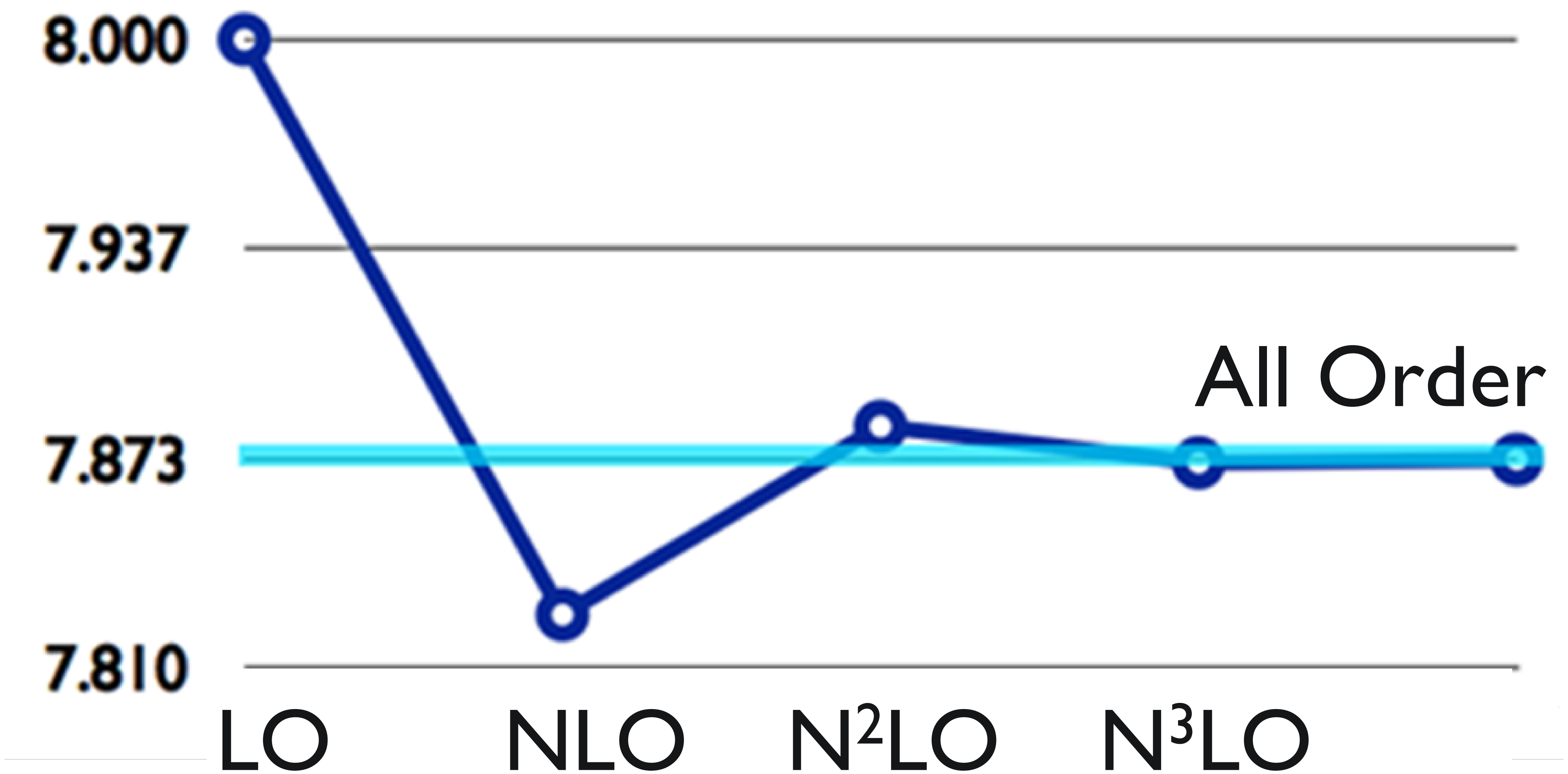
[7] Beneke, Signer, Smirnov, PRL 80, 2535

[8] Bodwin, Chung, Kang, Lee, Yu, PRD 77, 094017

[9] Bodwin, Chung, Lee, Yu, PRD 79, 014007

[10] Beneke et al., PRL 112, 151801

Order-α_s coefficient of EM currents, resumed to v^{2n}



MEs are from Bodwin, Chung, Kang, Lee and Yu (PRD 77, 094017)

Summary

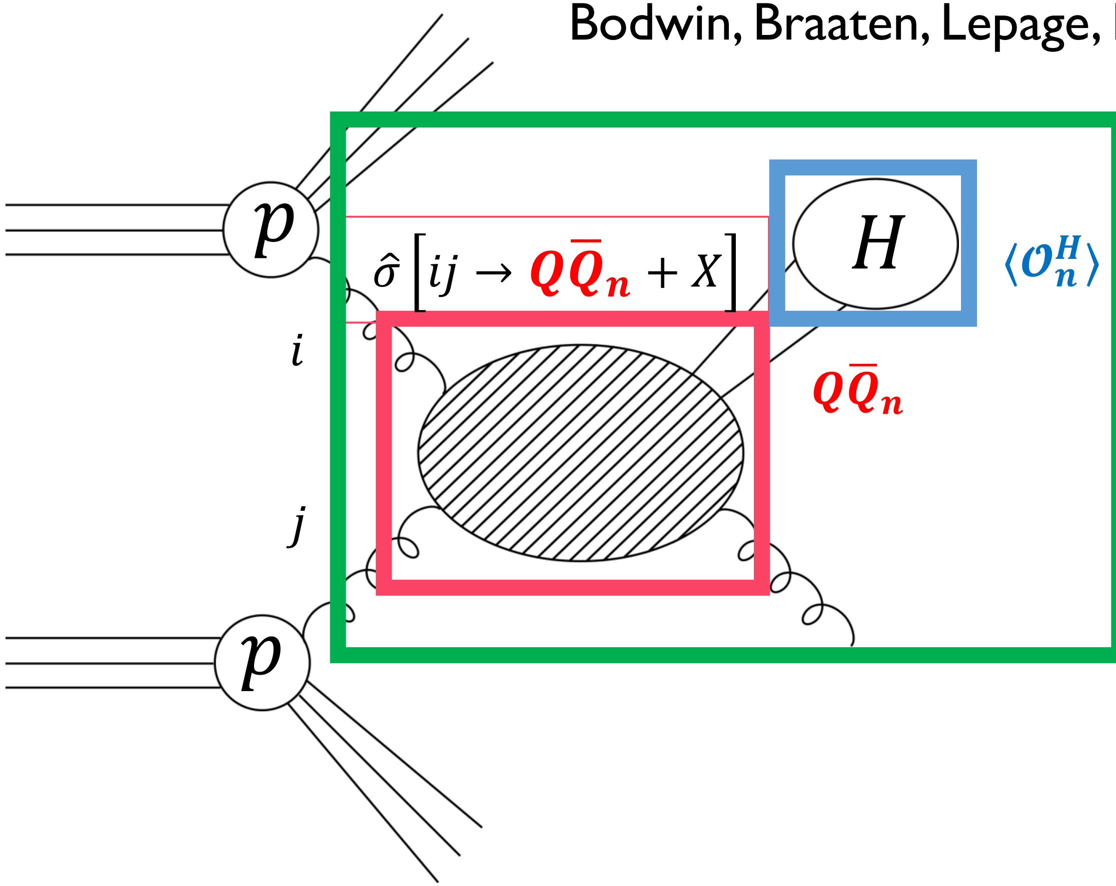
- As a heuristic demonstration of the NRQCD factorization formula that separates the LD and SD factors, we have considered the 1-loop QCD corrections to the spin-triplet S-wave decay into a lepton pair in which relativistic corrections are resummed to all orders.
- The IR divergence and the non-analytic dependence on the momentum are canceled by 1-loop corrections of the matrix elements.
- The UV divergence is subtracted by using the renormalization of the NRQCD operator.

J/ψ hadroproduction

Hadroproduction of J/ψ and NRQCD factorization

NRQCD Factorization

Bodwin, Braaten, Lepage, PRD (1995)



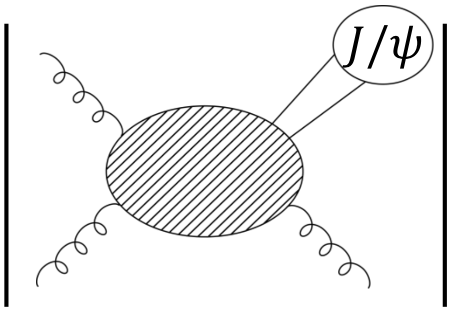
- For a heavy quarkonium process, factorization was proved in inclusive decay and conjectured in production:

$$\hat{\sigma}[ij \to H + X] = \hat{\sigma}[ij \to Q\bar{Q}_n + X] \times \langle O_n^H \rangle$$

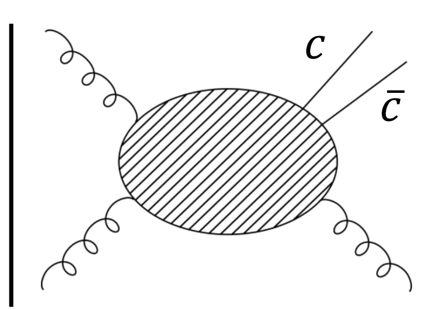
Nonperturbative NRQCD matrix elements (MEs) $\langle O_n^H \rangle$ are determined from experimental data.

J/ψ polarization puzzle

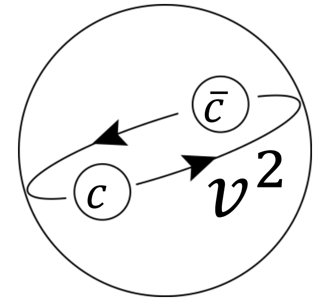
Leading NRQCD MEs in v expansion



$^2 \simeq$



$^2 \otimes$



$\simeq 0.25$

$\langle \mathcal{P}[Q\bar{Q}(n) \to J/\psi] \rangle$

$$\hat{\sigma}(H) \simeq \sum_n \hat{\sigma}[Q\bar{Q}(n)] \otimes \langle \mathcal{O}_n^{J/\psi} \rangle$$

SD$[\alpha_s]$ **LDME**$[v]$**, global**

Double Expansion

n: $Q\bar{Q}$ quantum number

Color singlet: $^3S_1^{[1]}(v^0)$: determined from $J/\psi \to \ell^+ \ell^-$

Color octet: $^3S_1^{[8]}(v^4)$, $^1S_0^{[8]}(v^3)$, $^3P_J^{[8]}(v^4)$: fit from data

$v^2 \simeq 0.25$ for $c\bar{c}$ bound states

NRQCD LDMEs for J/ψ

- v^0: $\langle 0|\mathcal{O}_0^H({}^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger\sigma^i\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i\chi|0\rangle$
- v^2: $\langle 0|\mathcal{O}_2^H({}^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i\chi + \text{H. c.}|0\rangle$
- v^3: $\langle 0|\mathcal{O}_0^H({}^1S_0^{[8]})|0\rangle = \langle 0|\chi^\dagger T^a\psi\;\mathcal{P}_H\;\psi^\dagger T^a\chi|0\rangle$
- v^4(color octet):

$$\langle 0|\mathcal{O}_0^H({}^3S_1^{[8]})|0\rangle = \langle 0|\chi^\dagger\sigma^i T^a\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i T^a\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_0^{[1]})|0\rangle = \frac{1}{d-1}\langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\psi\;\mathcal{P}_H\;\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_1^{[1]})|0\rangle = \langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{[i,}\sigma^{j]})\psi\;\mathcal{P}_H\;\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{[i,}\sigma^{j]})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P_2^{[1]})|0\rangle = \langle 0|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)})\psi\;\mathcal{P}_H\;\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)})\chi|0\rangle$$

$$\langle 0|\mathcal{O}_0^H({}^3P^{[8]})|0\rangle = \sum_{J=0,1,2}\langle 0|\mathcal{O}_0^H({}^3P_J^{[8]})|0\rangle$$

NRQCD LDMEs for J/ψ

- Order-v^4(color singlet)

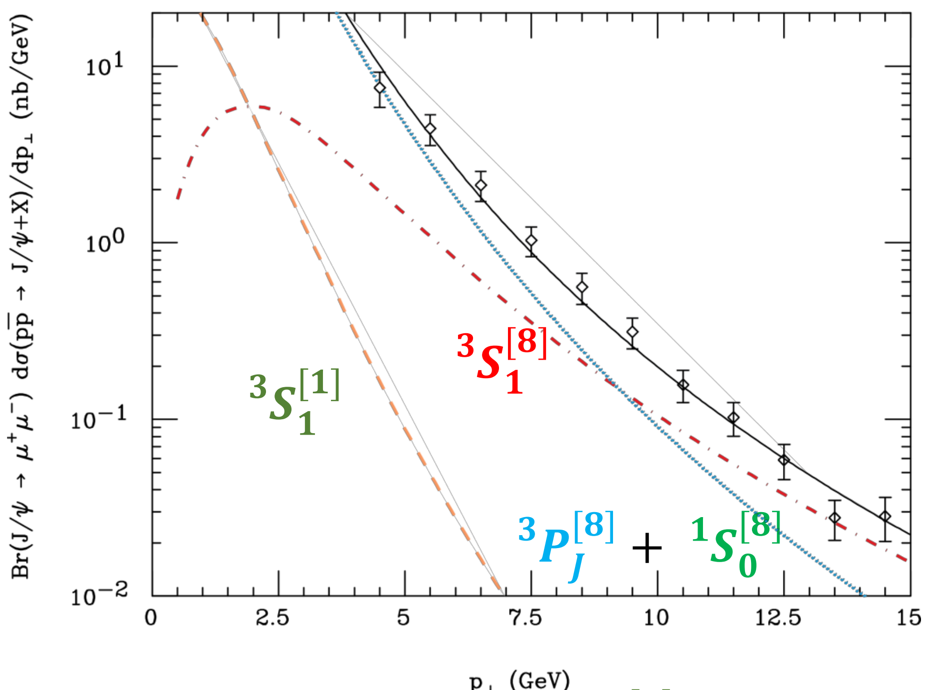
$$\langle 0|\mathcal{O}^H_{4,1}(^3S_1^{[1]})|0\rangle = \langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|0\rangle$$

$$\langle 0|\mathcal{O}^H_{4,2}(^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger\sigma^i(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^4\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i\chi + \mathrm{H.\ c.}|0\rangle$$

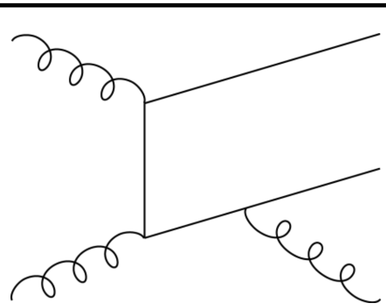
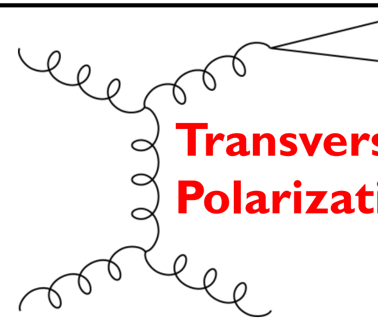
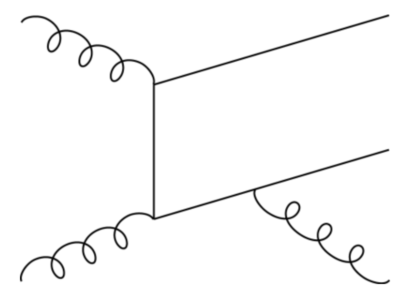
$$\langle 0|\mathcal{O}^H_{4,3}(^3S_1^{[1]})|0\rangle = \frac{1}{2}\langle 0|\chi^\dagger\sigma^i\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i(\overleftrightarrow{\boldsymbol{D}}\cdot g\boldsymbol{E} + g\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\chi$$
$$-\chi^\dagger\sigma^i(\overleftrightarrow{\boldsymbol{D}}\cdot g\boldsymbol{E} + g\boldsymbol{E}\cdot\overleftrightarrow{\boldsymbol{D}})\psi\;\mathcal{P}_H\;\psi^\dagger\sigma^i\chi|0\rangle$$

- Equation of motion eliminates $\langle 0|\mathcal{O}^H_{4,3}(^3S_1^{[1]})|0\rangle$
- $\langle 0|\mathcal{O}^H_{4,1}(^3S_1^{[1]})|0\rangle = \langle 0|\mathcal{O}^H_{4,2}(^3S_1^{[1]})|0\rangle + \mathcal{O}(v^2)$

LO NRQCD explains $\sigma[p\bar{p} \to J/\psi + X]$ at the Tevatron

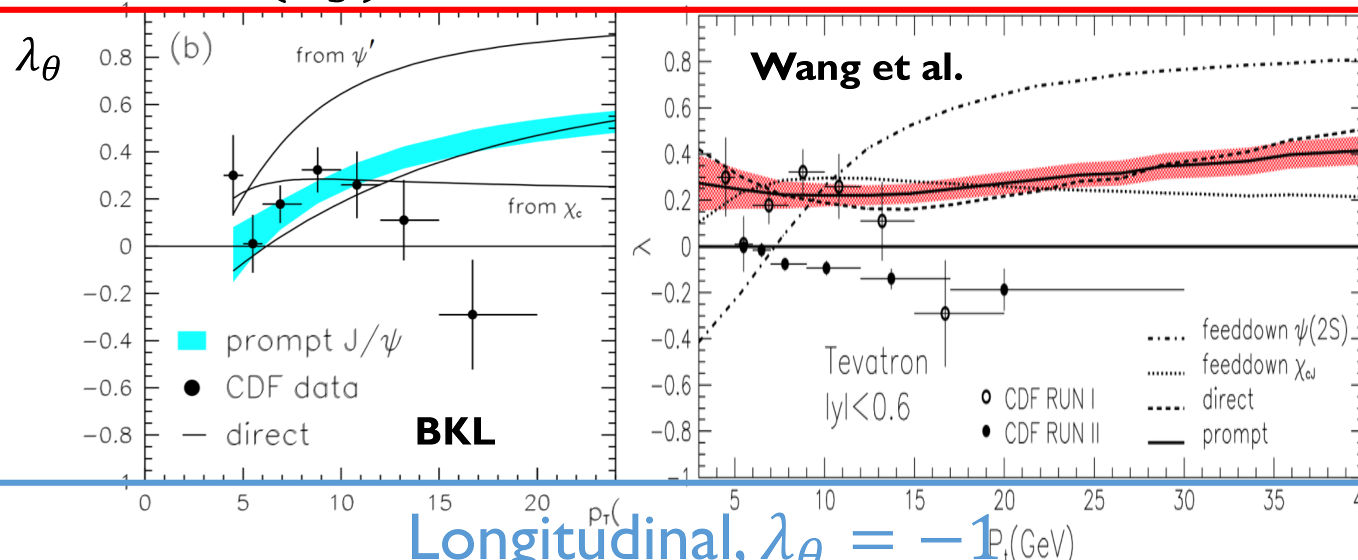


- Because $^3S_1^{[8]}$ **dominates at large p_T** [Braaten and Fleming, PRL (1995)], one can determine $\langle \mathcal{O}^{J/\psi}[\,^3S_1^{[8]}]\rangle$ from large p_T data and then determine $\langle \mathcal{O}^{J/\psi}[\,^3P_J^{[8]}]\rangle$ and $\langle \mathcal{O}^{J/\psi}[\,^1S_0^{[8]}]\rangle$ from lower p_T data.
- **Transverse polarization** is predicted **at large p_T.**
- As an independent test, one can test this with polarization data.

	$^3S_1^{[1]}$	$^3S_1^{[8]}$	$^3P_J^{[8]} + {}^1S_0^{[8]}$
		 **Transverse Polarization**	
$\frac{d\sigma}{dp_T^2} \propto$	$\frac{\alpha_s^3}{p_T^8}$	$\frac{\alpha_s^3 v^4}{p_T^4}$ **Leading Power in p_T^{-1}**	$\frac{\alpha_s^3 v^4}{p_T^8}$, $\frac{\alpha_s^3 v^3}{p_T^8}$

14-year old puzzle of J/ψ polarization at the Tevatron

LO(α_s^3) Transverse, $\lambda_\theta = 1$ NLO(α_s^4)



$p_T \geq 7$ **GeV, prompt**

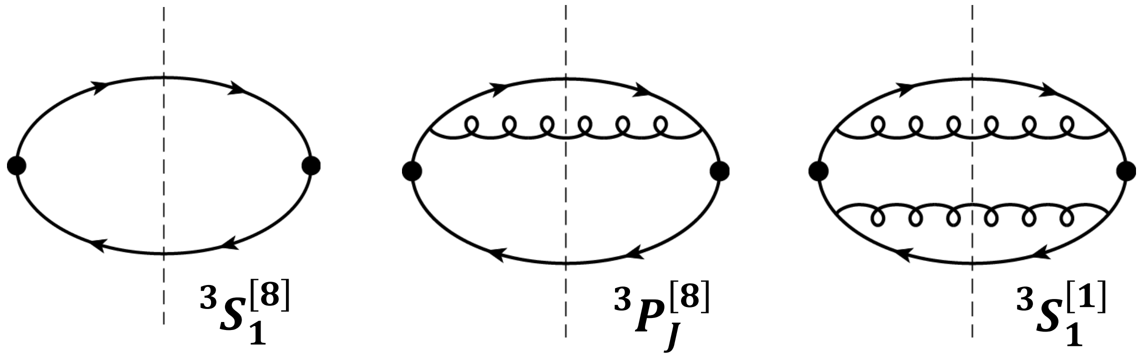
Longitudinal, $\lambda_\theta = -1$

PRD, **2000** PRL, **2013**

- NRQCD predicts transverse polarization at large p_T that confronted CDF data.
- Further predictions with higher-order QCD correction failed to explain the large p_T data.
- **The dominance of ${}^3S_1^{[8]}$ [Braaten, Fleming, PRL(1995)] or NRQCD factorization may FAIL.**

$Q\bar{Q}[\,^3S_1^{[8]}]$ dominance at large p_T

[Braaten, Fleming, PRL(1995)] **may be wrong**



Bodwin, Kim, Lee, JHEP (2012)

- By computing the color-singlet contribution to the **NNLO QCD correction** to the fragmentation function for $g^* \to Q\bar{Q}[\,^3S_1^{[8]}]$, we have found a clue to have a **large cancellation between $Q\bar{Q}[\,^3S_1^{[8]}]$ and $Q\bar{Q}[\,^3P_J^{[8]}]$**.
- **$g^* \to Q\bar{Q}[\,^1S_0^{[8]}]$ dominates at large p_T that replaces previous belief since 1995.**
- $\hat{\sigma}[ij \to Q\bar{Q}_n + X]$ is required to be computed to NNLO in α_s for leading power (LP) contribution.

Leading-power factorization

Leading-power factorization

- LP factorization formula at leading power in $1/p_T^2$ for quarkonium ***H*** production is given by

$$d\sigma_{A+B\to H+X} = \sum_i \int_0^1 dz\, d\hat{\sigma}_{A+B\to i+X}\left(k^+,\mu_f\right) \times D_{i\to H}\left(z=\frac{p^+}{k^+},\mu_f\right)$$

$d\hat{\sigma}_{A+B\to i+X}$: single parton i production cross section (PPCS)

$D_{i\to H}$: single parton fragmentation function, **nonperturbative**

k^+: light-cone momentum of parent parton i

p^+: light-cone momentum of daughter hadron H

μ_f : Factorization scale

NRQCD and LP factorization

- One can apply NRQCD factorization to LP factorization formula,

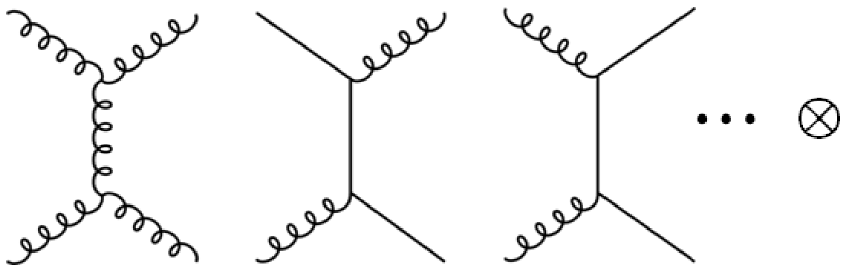
$$d\sigma_{A+B\to H+X} = \sum_{n,i} d\sigma_{A+B\to i+X} \otimes D_{i\to Q\bar{Q}(n)} \langle \mathcal{O}^H(n) \rangle$$

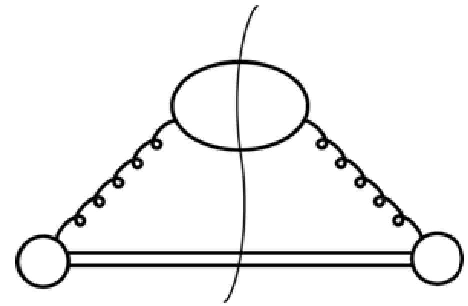
- Here, $D_{i\to Q\bar{Q}(n)}$ is **perturbative**

- By making use of this formula, we evaluated LP contributions to J/ψ production at NNLO in α_s and their all-order leading-log (LL) resummation

LP J/ψ production processes

- Order α_s^3 diagrams:

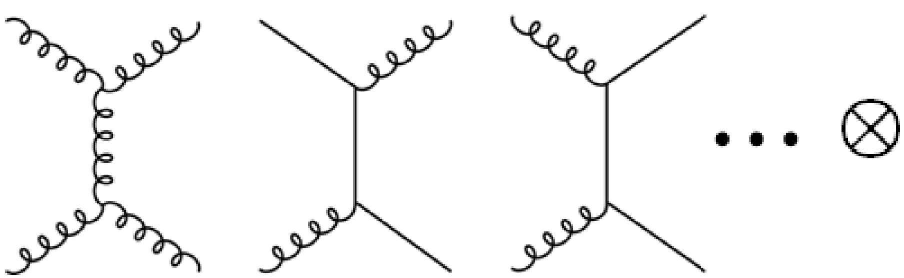


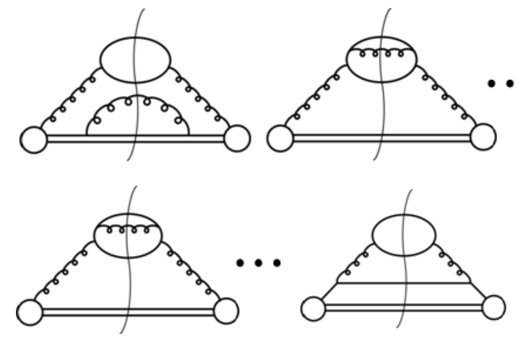


LO PPCS

LO ${}^3S_1^{[8]}$

- Order α_s^4 diagrams:

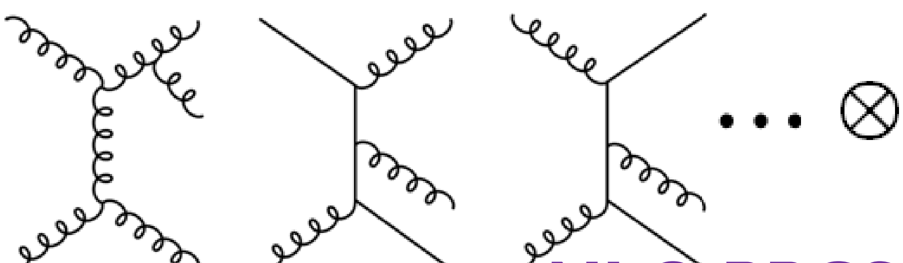


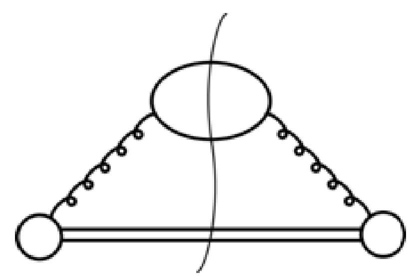


NLO ${}^3S_1^{[8]}$

LO ${}^1S_0^{[8]}$

LO ${}^3P_J^{[8]}$



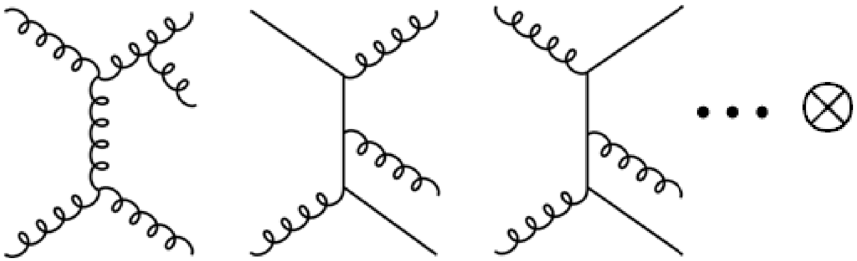


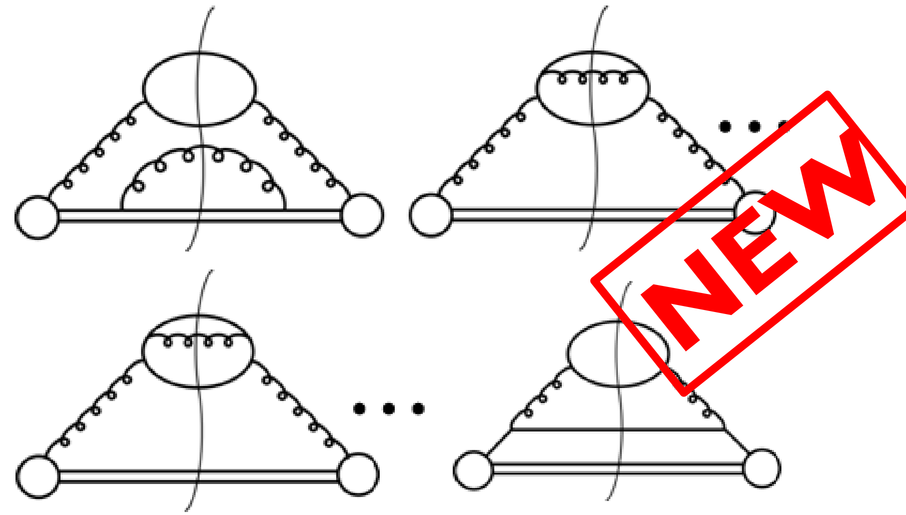
LO ${}^3S_1^{[8]}$

NLO PPCS

LP J/ψ production processes

- Order α_s^5 diagrams:





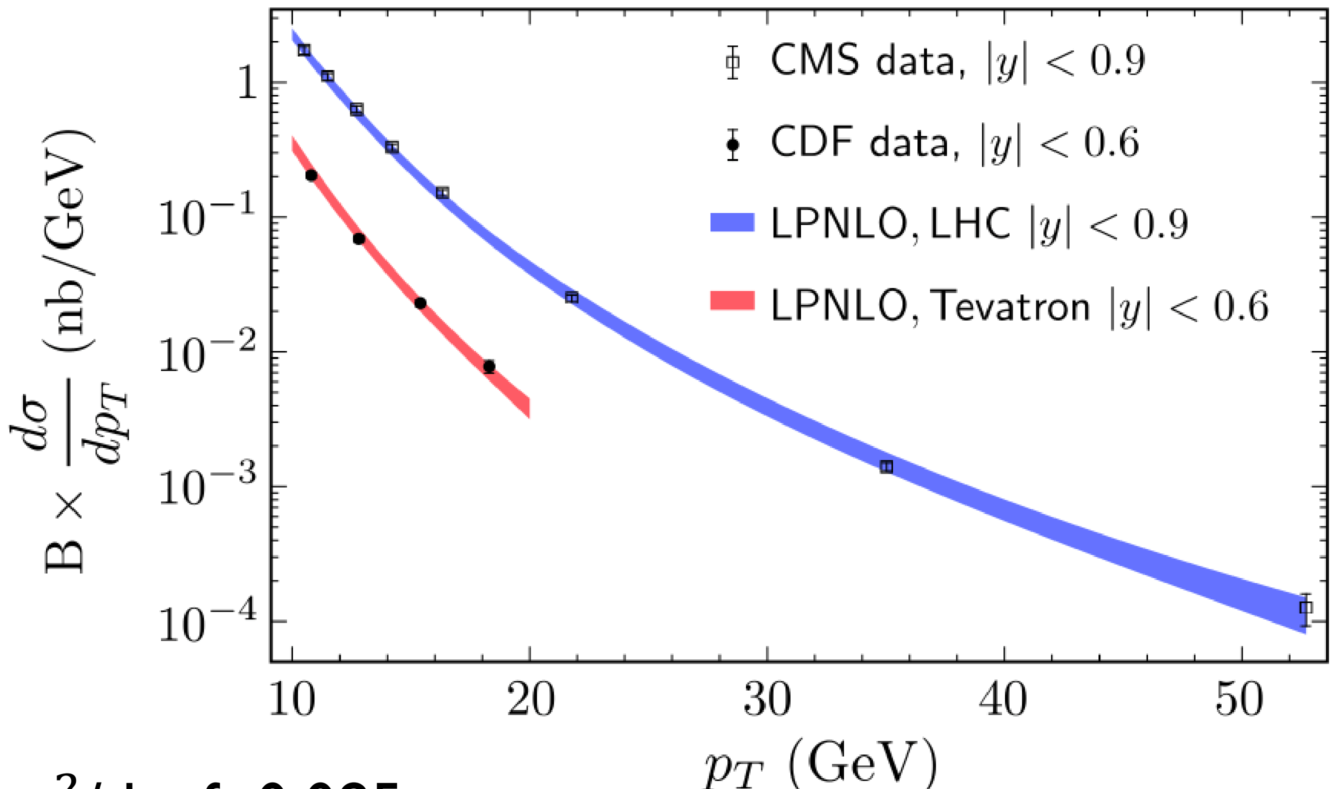
Results	Order		
NLO	$\sim \alpha_s^4$	σ_{NLO}	
Our result	$\sim \alpha_s^4$	σ_{LP}	$\sigma_{\mathrm{overlapped}}$
	$\alpha_s^5 +$ higher (LL resum)		

$$\sigma_{\mathrm{result}} = \sigma_{\mathrm{NLO}} + \boxed{\sigma_{\mathrm{LP}} - \sigma_{\mathrm{overlapped}}}$$

[NEW] NNLO+ higher(LL)

Resolution of the puzzle

J/ψ differential cross section



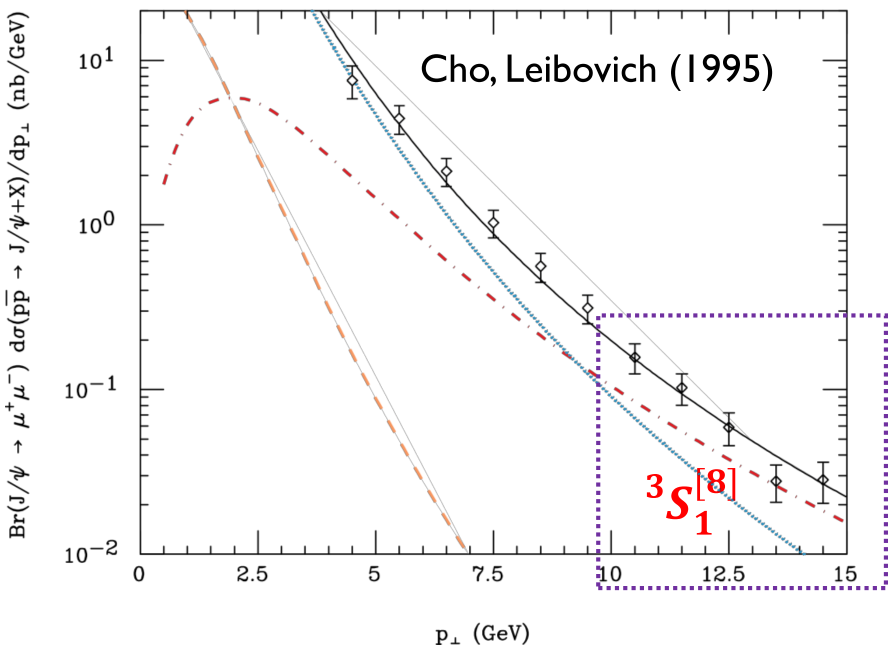
- χ^2/d.o.f=0.085
- CO LDMEs are determined as

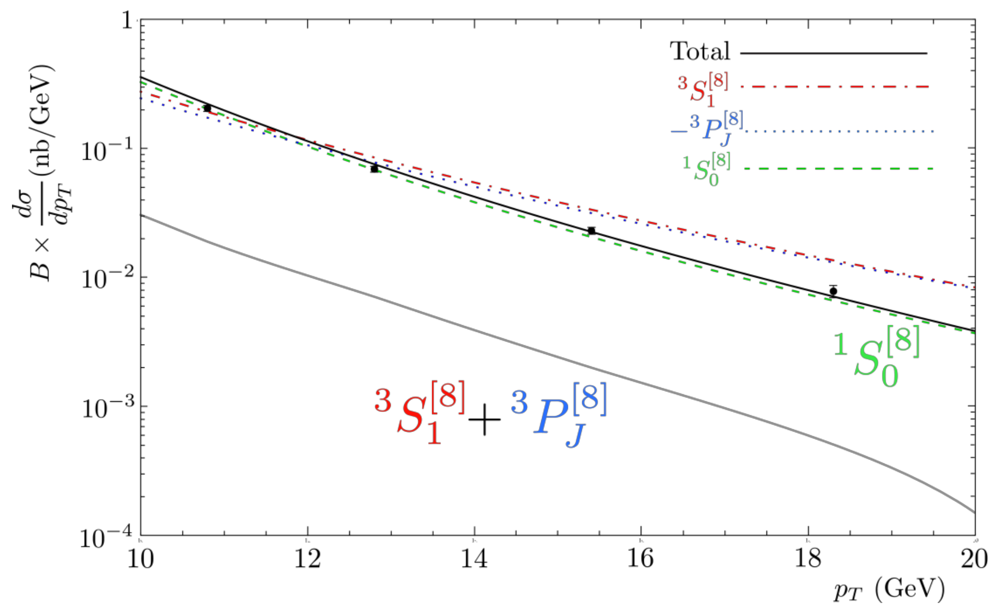
$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$
$0.099 \pm 0.022\ \mathrm{GeV}^3$	$0.011 \pm 0.010\ \mathrm{GeV}^5$	$0.011 \pm 0.010\ \mathrm{GeV}^3$

$Q\bar{Q}[\,{}^1S_0^{[8]}]$ dominates at large p_T

${}^3S_1^{[8]}$ dominates at large p_T **[OLD]** **(1995~)**

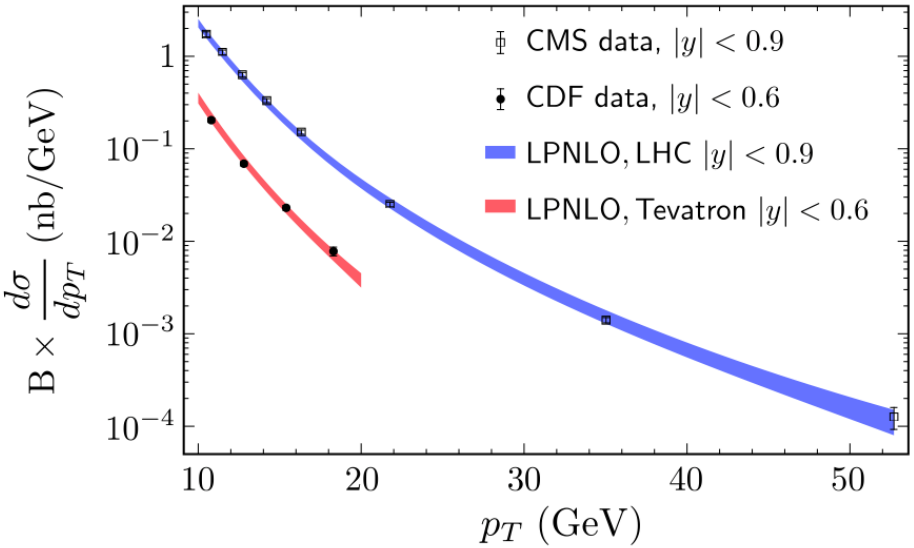
${}^1S_0^{[8]}$ **dominates at large p_T [NEW]**





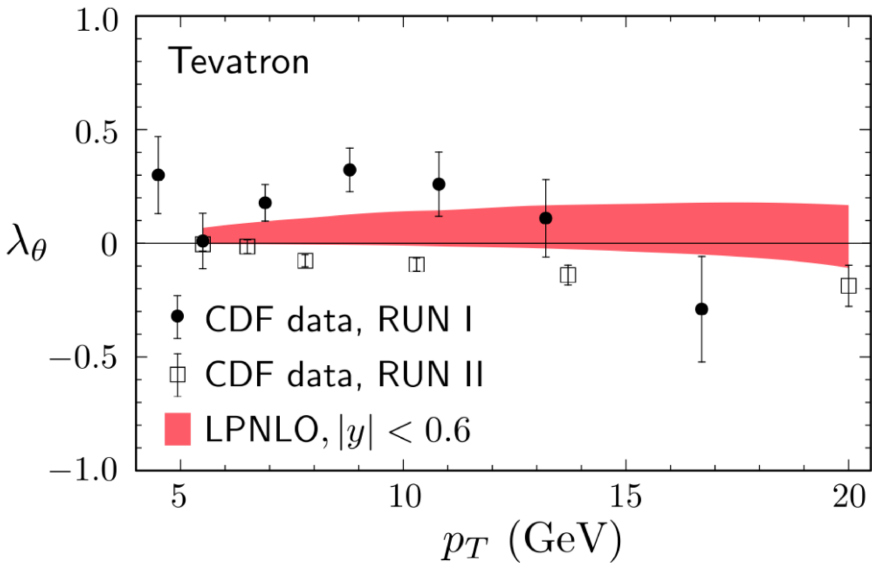
- Due to the large cancellation between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$, ${}^1S_0^{[8]}$ **dominates in J/ψ production at large p_T.**

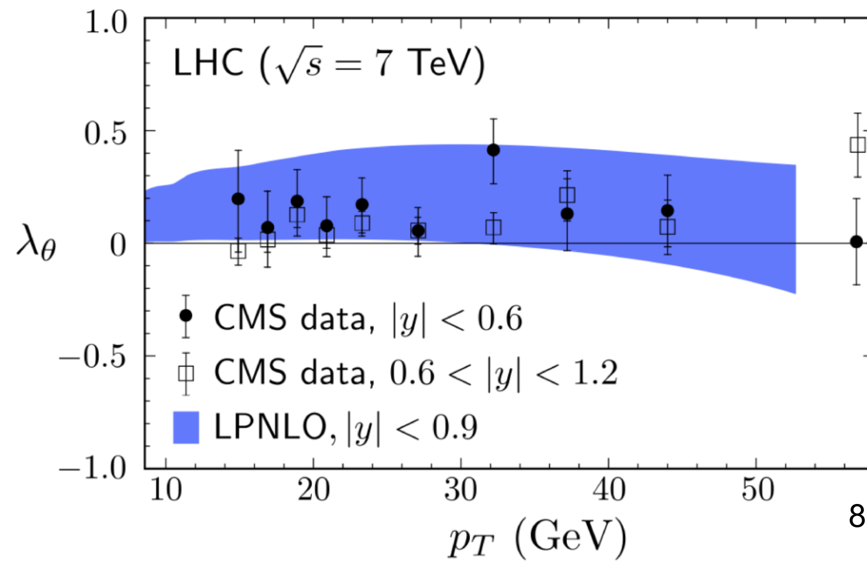
J/ψ polarization puzzle RESOLVED!



At the **Tevatron**,
$p_T \geq 10$ GeV data **fit well**.

At the **LHC**,
$p_T \geq 10$ GeV data **fit perfectly**.





PRL 113, 022001 (2014) PHYSICAL REVIEW LETTERS week ending 11 JULY 2014

Fragmentation Contributions to J/ψ Production at the Tevatron and the LHC

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(Received 28 March 2014; published 7 July 2014)

We compute leading-power fragmentation corrections to J/ψ production at the Tevatron and the LHC. We find that, when these corrections are combined with perturbative corrections through next-to-leading order in the strong coupling constant α_s, we obtain a good fit to high-p_T cross section data from the CDF and CMS Collaborations. The fitted long-distance matrix elements lead to predictions of near-zero J/ψ polarization in the helicity frame at large p_T.

DOI: 10.1103/PhysRevLett.113.022001 PACS numbers: 12.38.Bx, 12.39.St, 14.40.Pq

Much of the current phenomenology of heavy-quarkonium production in high-energy collisions is based

QCD coupling α_s [3–9]. Generally, the NLO calculations, combined with the four-LDME phenomenology, lead

CERN COURIER

Feb 23, 2015

CMS heads towards solving a decades-long quarkonium puzzle

Quarkonia - charm or beauty quark/antiquark bound states - are prototypes of elementary

systems gov

to the large

the quarks,

simpler to

unique insights into the mech

decades, research in the area

hadron collisions has been har

in theoretical calculations and

data, indicating that the bound-state formation through coloured $^1S_0$ pre-resonance is dominant (G Bodwin *et al*. 2014, K-T Chao *et al*. 2012, P Faccioli *et al*. 2014).

Heading towards the solution of a decades-long puzzle, what of the fundamental question: how do quarks and antiquarks interact to form bound states? Future analyses will disclose the complete hierarchy of transitions from pre-resonances with different quantum properties to the family of observed bound states, providing a set of "Kepler" laws for the long-distance interactions between quark and antiquark.

Further reading

G Bodwin *et al*. 2014 *Phys. Rev. Lett.* **113** 022001.

K-T Chao *et al*. 2012 *Phys. Rev. Lett.* **108** 242004.

Summary

Summary

- Replaced the $^3S_1^{[8]}$ (1995~) with $^1S_0^{[8]}$ that dominates J/ψ production at large p_T.
- Resolved **14-year-old** J/ψ polarization puzzle by computing NNLO LP contribution.
- Our results for direct J/ψ were extended to the prompt case that contains feeddowns from higher resonances like χ_{cJ} and $\psi(2S)$.
- Bottomonium like $\Upsilon(nS)$ and $\chi_{bJ}(nP)$ can also be studied.

Thank you