AEPSHEP2016 Lecture

Field theory and the Electroweak Standard Model

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Plan:

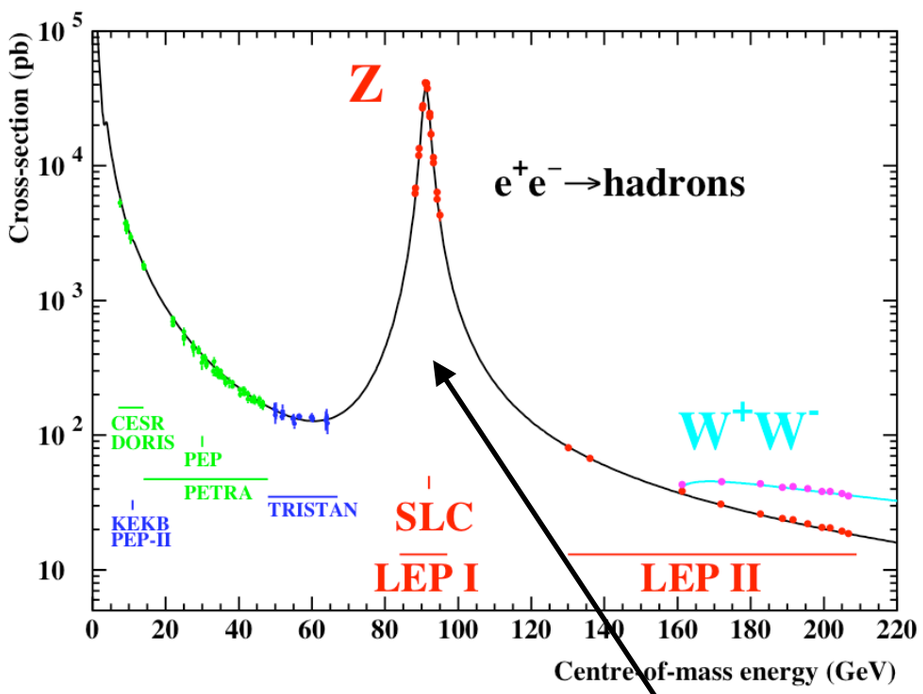
Lecture 1: Fields and Particles

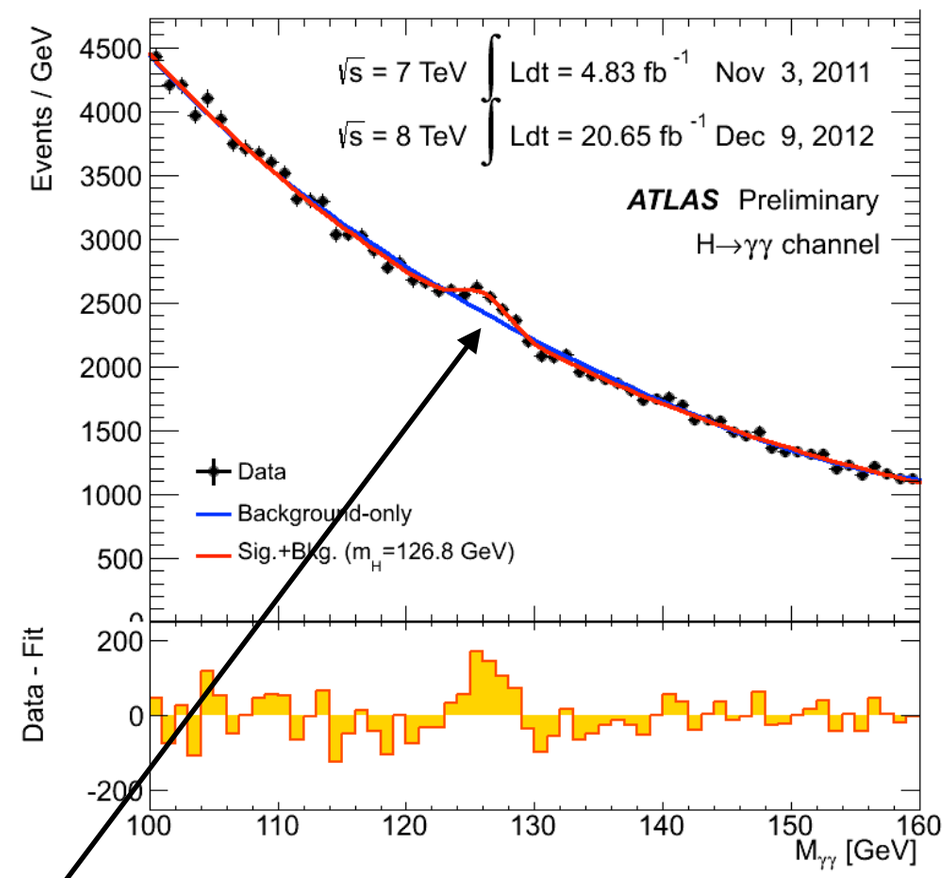
Lecture 2: Symmetry Breaking and the Higgs mechanism

Lecture 3: Electroweak Standard Model

Lecture 1

Fields and Particles





Particles

Let's try to understand why these peaks are called "**particles.**"

Fields (classical)

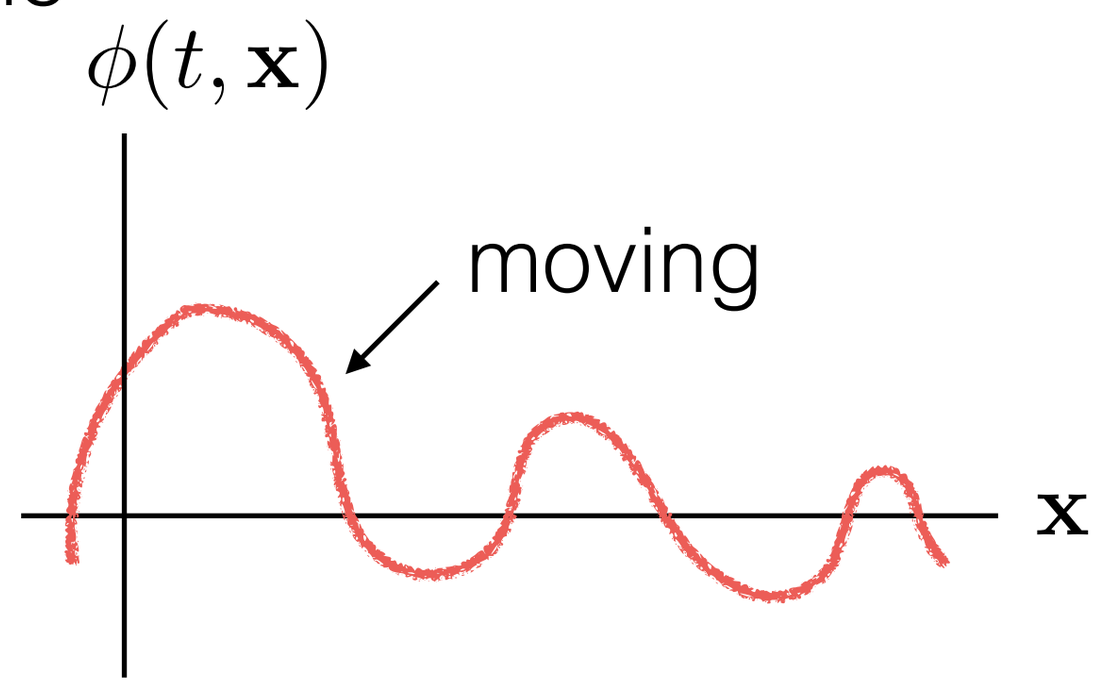
Fields are functions of space-time

scalar field: $\phi(t, \mathbf{x})$

spinor field: $\psi_\alpha(t, \mathbf{x})$

vector field: $A_\mu(t, \mathbf{x})$

.....



They can be classified by how they transform under the Lorentz transformation.

Quantum Field Theory

quantum mechanics:

$$q(t) \to \hat{q}(t)$$

coordinate $\longrightarrow$ operator

quantum field theory:

$$\phi(t, \mathbf{x}) \to \hat{\phi}(t, \mathbf{x})$$

note here that the coordinate x is a label. **Not an operator**!

It's just a collection of Q.M.

$$\phi(t, \mathbf{x_1}) \quad \phi(t, \mathbf{x_2}) \quad \cdots\cdots$$

Particles

$|\mathbf{P}\rangle$

one-particle state with three-momentum **P**=(p_x,p_y,p_z)

special relativity says $E = \sqrt{|\mathbf{P}|^2 + m^2}$

energy mass

In QFT, there are also states that describes many particles:

$$|\mathbf{P}_1, \mathbf{P}_2, \cdots\rangle$$

(Note: $\hbar = c = 1$)

Wave functions

The relation between the wave functions in the QM and the state in QFT is

$$\varphi(x) = \langle 0|\hat{\phi}(x)|\mathbf{P}\rangle \qquad (x = (t, \mathbf{x}))$$

vacuum (the lowest energy state)

The functional form can be fixed by the Lorentz covariance.

$$\varphi(x) = \sqrt{Z}e^{-ip\cdot x}$$

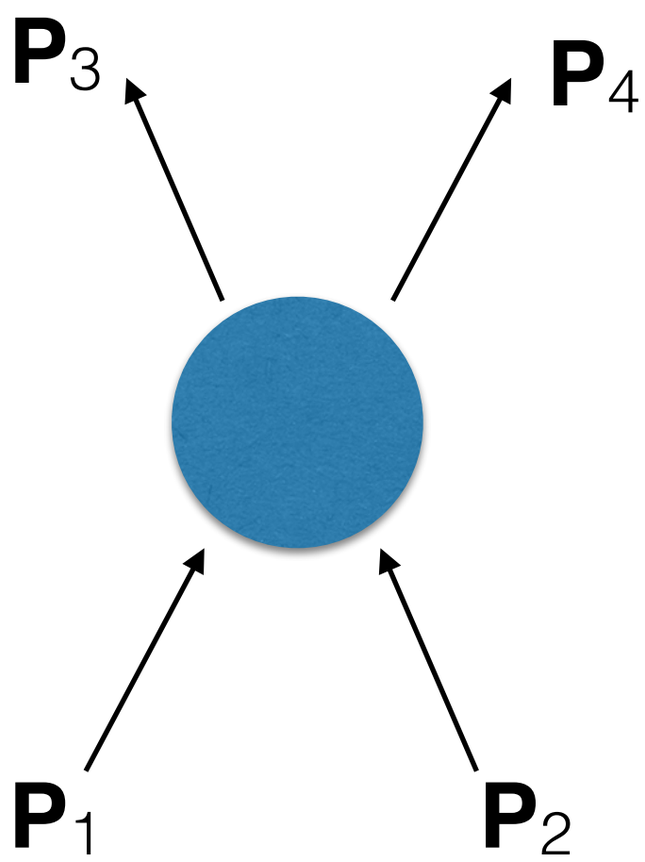
$$\langle 0|\hat{\psi}_\alpha(x)|\mathbf{P},\sigma\rangle = \sqrt{Z}u(\mathbf{P},\sigma)e^{-ip\cdot x}$$

$$\langle 0|\hat{A}_\mu(x)|\mathbf{P},\sigma\rangle = \sqrt{Z}\epsilon_\mu(\mathbf{P},\sigma)e^{-ip\cdot x}$$

these are solutions of wave equations. (e.g. Klein-Gordon, Dirac, Maxwell eq.)

Scattering amplitudes

$$\mathcal{M} = {}_{\text{out}}\langle \mathbf{P}_3,\ \mathbf{P}_4 | \mathbf{P}_1,\ \mathbf{P}_2 \rangle_{\text{in}}$$



normalization

$$\langle \mathbf{P}' | \mathbf{P} \rangle = (2\pi)^3 2E \delta^3(\mathbf{P} - \mathbf{P}')$$

$$\int \frac{d^3\mathbf{P}}{(2\pi)^3 2E} |\mathbf{P}\rangle\langle\mathbf{P}| = 1$$

scattering cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}|\mathcal{M}|^2$$

$$s = (p_1 + p_2)^2$$

Correlation functions

two-point function:

$$\langle 0|\mathbf{T}\hat{\phi}(x)\hat{\phi}(y)|0\rangle = \langle 0|\hat{\phi}(x)\hat{\phi}(y)|0\rangle\theta(x^0 - y^0)$$
$$+\langle 0|\hat{\phi}(y)\hat{\phi}(x)|0\rangle\theta(y^0 - x^0)$$

Time ordered product

three-point function:

$$\langle 0|\mathbf{T}\hat{\phi}(x)\hat{\phi}(y)\hat{\phi}(z)|0\rangle = \langle 0|\hat{\phi}(x)\hat{\phi}(y)\hat{\phi}(z)|0\rangle$$
$$\times\theta(x^0 - y^0)\theta(y^0 - z^0)$$
$$+\cdots$$

Particles and Poles

$$\int d^4x \langle 0|\mathbf{T}\hat{\phi}(x)\hat{\phi}(y)\cdots|0\rangle e^{ip\cdot x}$$

$$= \frac{i\sqrt{Z}}{p^2 - m^2 + i\epsilon}\langle \mathbf{P}|\mathbf{T}\hat{\phi}(y)\cdots|0\rangle + \text{non pole terms.}$$

(derive this on the board.)

Contribution from one-**particle** states to correlation functions

=

poles!

location of the pole = **mass²** of the particle

Repeating this procedure

$$_{\text{out}}\langle \mathbf{P}_3,\ \mathbf{P}_4 | \mathbf{P}_1,\ \mathbf{P}_2 \rangle_{\text{in}} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

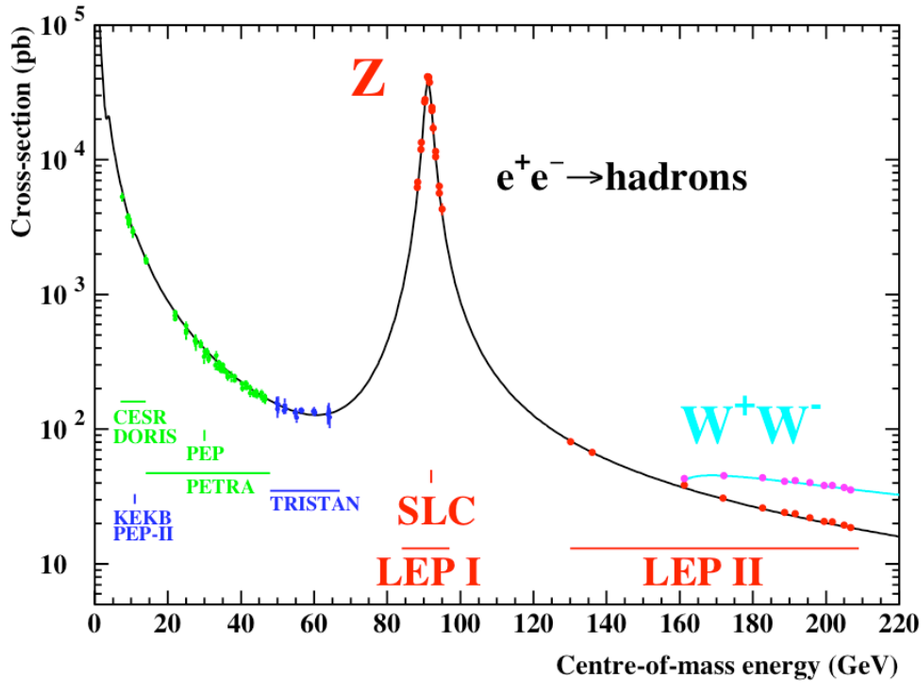
$$= \left(\frac{i\sqrt{Z}}{p_1^2 - m^2 + i\epsilon} \right)^{-1} \left(\frac{i\sqrt{Z}}{p_2^2 - m^2 + i\epsilon} \right)^{-1} \left(\frac{i\sqrt{Z}}{p_3^2 - m^2 + i\epsilon} \right)^{-1} \left(\frac{i\sqrt{Z}}{p_4^2 - m^2 + i\epsilon} \right)^{-1}$$

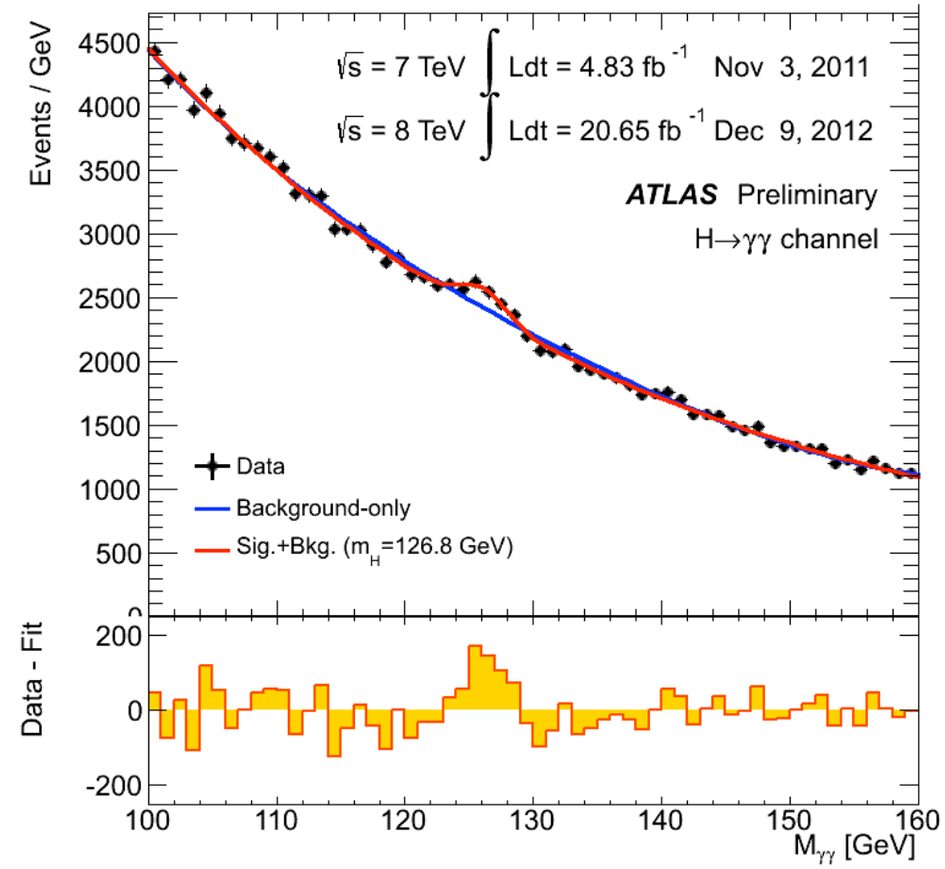
x $\langle 0 | \mathbf{T} \hat{\phi}(x_1) \hat{\phi}(x_2) \hat{\phi}(x_3) \hat{\phi}(x_4) | 0 \rangle |_{\text{fourier transform.}}$

scattering amplitude

removing poles for initial and final state particles

Now,





If there are contributions from "**intermediate**" one-particle state, the scattering amplitude has a pole at $p^2 = m^2$.

$$\langle 0|\mathbf{T}\hat{\phi}(x_1)\hat{\phi}(x_2)|\mathbf{P}\rangle\langle\mathbf{P}|\mathbf{T}\hat{\phi}(x_3)\hat{\phi}(x_4)|0\rangle \neq 0$$

peak = particle!

remember, that

We haven't specified the theory. It is general (and actually that's the definition) that

particles	=	**poles**!
mass²	=	location of the pole $\frac{1}{p^2 - m^2 + i\epsilon}$

How to compute the correlation functions

$$\langle 0|\mathbf{T}\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)|0\rangle$$

$$= \frac{1}{Z}\int [d\phi]\phi(x_1)\cdots\phi(x_n)e^{iS[\phi]}$$

$$Z = \int [d\phi]e^{iS[\phi]}$$

path integral

action

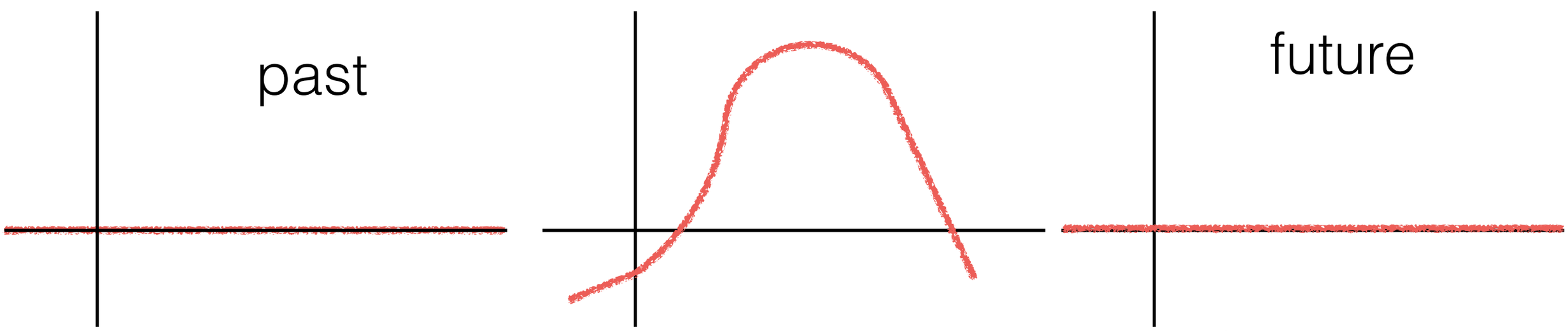
$\phi(t = -\infty + i\epsilon, \mathbf{x})$

past

middle

$\phi(t = \infty - i\epsilon, \mathbf{x})$

future



path integral = integrate over all the possible functions

action sets the theory

$$S[\phi] = \int d^4x \mathcal{L}(\phi)$$

functional of fields

Lagrangian density
(Lorentz invariant real function of fields)

For example,

we'll see that this term represents
the mass of a particle. (mass term)

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \cdots$$

This factor can be chosen to be 1/2
by field rescaling. (kinetic term)

interaction term

Free theory

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

$$\int d^4x \langle 0|\mathbf{T}\hat{\phi}(x)\hat{\phi}(0)|0\rangle e^{ip\cdot x} = \frac{i}{p^2 - m^2 + i\epsilon}$$

(derive this on the board.)

1. we see a pole at m^2. $\longrightarrow$ particle with mass m!
2. the numerator is “i”. $\longrightarrow$ Z factor is unity.

   this is why we choose this normalization.
3. fields = particles in free theories.

Feynman diagrams

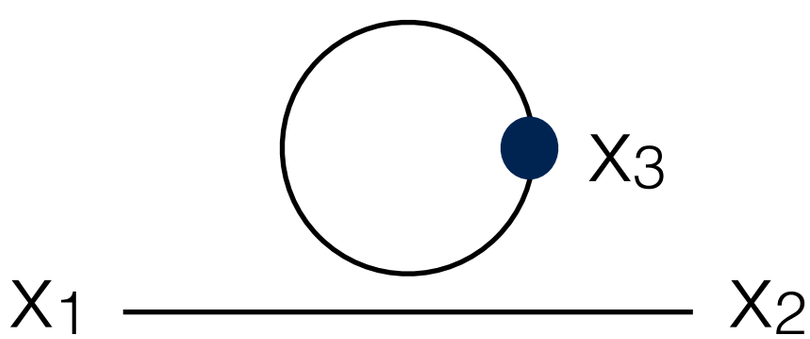
For example, (still free theory)

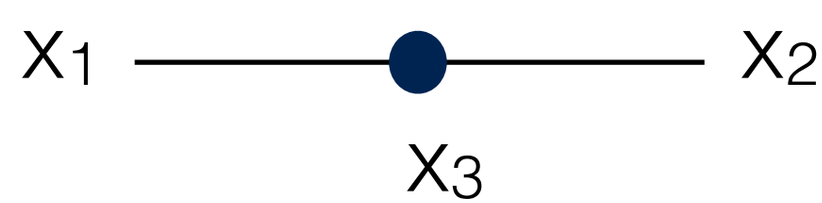
$$\langle 0|\mathbf{T}\phi(x_1)\phi(x_2)\phi(x_3)^2|0\rangle$$

$$= \frac{1}{Z[0]}\frac{\delta}{\delta iJ(x_1)}\frac{\delta}{\delta iJ(x_2)}\frac{\delta}{\delta iJ(x_3)}\frac{\delta}{\delta iJ(x_3)}Z[J]\Big|_{J=0}$$

$$= \frac{\delta}{\delta iJ(x_1)}\frac{\delta}{\delta iJ(x_2)}\frac{\delta}{\delta iJ(x_3)}\frac{\delta}{\delta iJ(x_3)}e^{-(i/2)J\cdot D^{-1}J}\Big|_{J=0}$$

$$= i(D^{-1})_{x_1x_2}i(D^{-1})_{x_3x_3} + i(D^{-1})_{x_1x_3}i(D^{-1})_{x_3x_2}$$





Perturbation theory

Let's consider

$$\mathcal{L}(\phi) = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2}_{\text{free}} \underbrace{- \frac{1}{4!}\lambda\phi^4}_{\text{interaction}}$$

One can calculate the correlation functions as a series expansion of λ.

$$Z[J] = \int [d\phi] e^{iS[\phi] + i\int d^4x J(x)\phi(x)}$$

$$= \int [d\phi] \left(1 - i\int d^4x \frac{\lambda}{4!}\phi^4 + \cdots\right) e^{iS_{\text{free}}[\phi] + i\int d^4x J(x)\phi(x)}$$

Each terms can be evaluated in the free theory.

For example,

$$\langle 0|\mathbf{T}\phi(x_1)\phi(x_2)|0\rangle$$

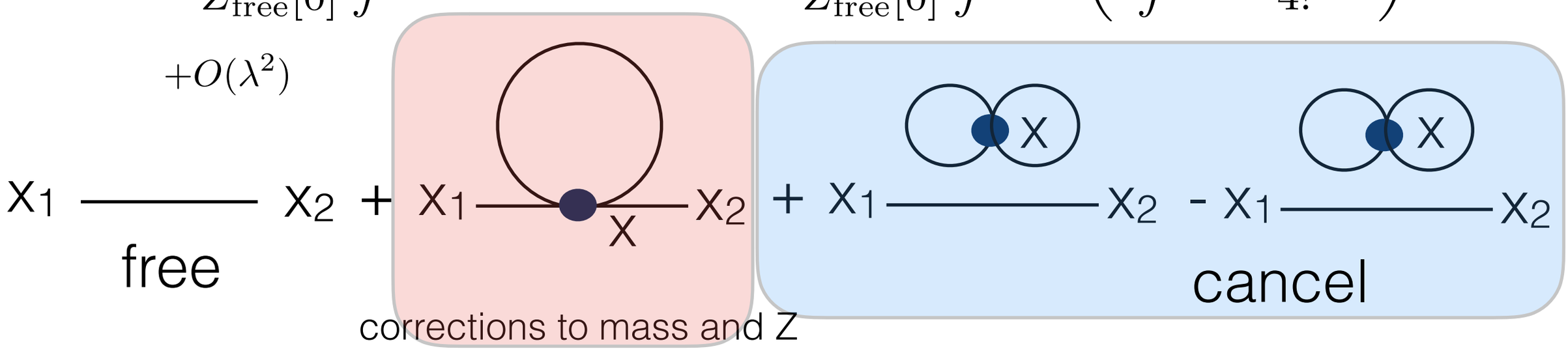
$$= \frac{1}{Z[0]}\int [d\phi]\phi(x_1)\phi(x_2)e^{iS_{\text{free}}+i\int d^4x\left(-\frac{\lambda}{4!}\phi(x)^4\right)}$$

$$= \frac{1}{Z_{\text{free}}[0]}\int [d\phi]\phi(x_1)\phi(x_2)e^{iS_{\text{free}}}$$

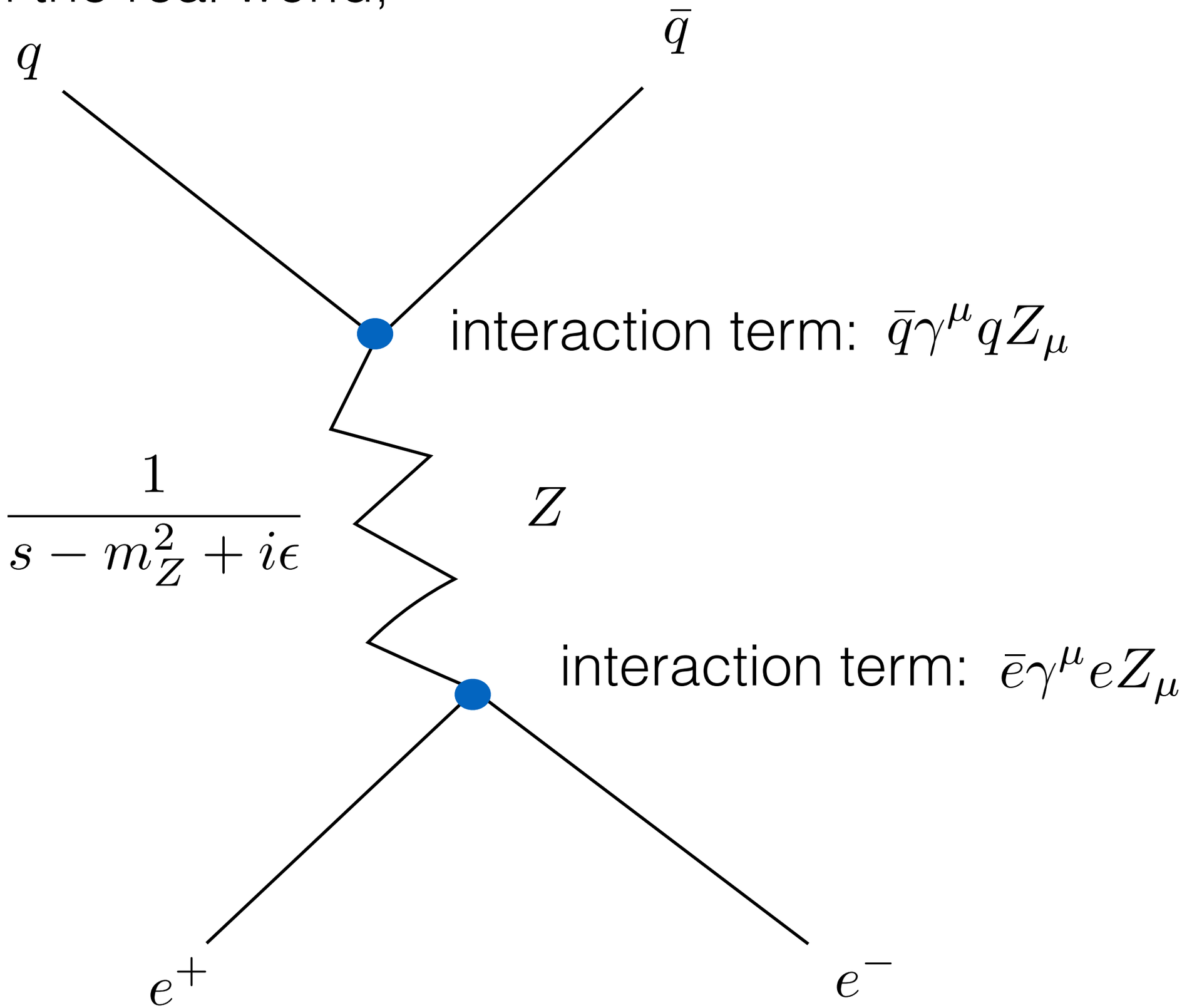
$$+\frac{1}{Z_{\text{free}}[0]}\int [d\phi]\phi(x_1)\phi(x_2)\left(i\int d^4x\frac{-\lambda}{4!}\phi^4\right)e^{iS_{\text{free}}}$$

$$-\frac{1}{Z_{\text{free}}[0]}\int [d\phi]\phi(x_1)\phi(x_2)e^{iS_{\text{free}}}\frac{1}{Z_{\text{free}}[0]}\int [d\phi]\left(i\int d^4x\frac{-\lambda}{4!}\phi^4\right)e^{iS_{\text{free}}}$$

$$+O(\lambda^2)$$



In the real world,



We will learn this soon.

Lecture 2

gauge theory

theory to describe "**massless**" spin-1 particles

(e.g. photon)

photon states $|\mathbf{P}, \pm\rangle$

vector field $A_\mu(x) \to \hat{A}_\mu(x)$

wave function $\langle 0|\hat{A}_\mu(x)|\mathbf{P}, \pm\rangle = \sqrt{Z}\epsilon_\mu^\pm(\mathbf{P})e^{-ip\cdot x}$

polarization vector

A_μ is a combination of **four** independent functions.
but, there are only **two** degrees of freedom.

Lagrangian for Aμ

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

field strength
(electric and magnetic fields)

gauge invariance

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \theta(x)$$

$$F_{\mu\nu}(x) \to F_{\mu\nu}(x)$$

Action is invariant under this transformation.

$\longrightarrow$ The **same** physics is described by

A_μ and $A_\mu + \partial_\mu \theta$

physical degrees of freedom

Let A_μ be a field configuration and $\partial^\mu A_\mu(x) = c(x)$.

The **same** physical system can be described by

$$A'_\mu = A_\mu + \partial_\mu \theta$$

for an arbitrary scalar function $\theta(x)$

$$\longrightarrow \quad \partial^\mu A'_\mu = c(x) + \Box\theta(x) \ = 0$$

by choosing $\theta(x)$ such that $\Box\theta(x) = -c(x)$.

One can restrict ourselves that

$$\partial^\mu A_\mu = 0 \quad \text{(Lorentz condition)}$$

One can still describe **all** the physical system.

Now we consider the wave function:

$$\langle 0|\hat{A}_\mu(x)|\mathbf{P}\rangle = \epsilon_\mu e^{-ip\cdot x} \qquad p^2 = 0$$

(massless)

Lorentz condition $\quad p^\mu \epsilon_\mu = 0$

Yet unfixed gauge $\quad \hat{A}_\mu \to \hat{A}_\mu + \partial_\mu \hat{\theta}$

$$\epsilon^\mu \to \epsilon^\mu - iCp^\mu$$

for arbitrary C, the new ϵ^μ satisfies the Lorentz condition.

$\longrightarrow$ $\epsilon_\mu \propto p_\mu$ part can be zero.

(ϵ_μ shifted by p_μ describes the same physics)

Therefore,

$$p^\mu = \begin{pmatrix} p \\ 0 \\ 0 \\ p \end{pmatrix}$$

$\epsilon^\mu p_\mu \neq 0$

zero by Lorentz condition

(unphysical)

$$\epsilon^\mu = c_L \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} + c_S \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$\epsilon_\mu \propto p_\mu$

one can set this zero

(unphysical)

transverse modes

$$+c_+ \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix} + c_- \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}$$

two physical polarizations

massive spin-1 particle

(e.g. W-boson, Z-boson)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad + \frac{m^2}{2}A_\mu A^\mu$$

mass term

not gauge invariant anymore.

eq. of motion

$$\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) + m^2 A^\nu = 0$$

$$\partial_\nu \left(\boxed{\partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu)} + m^2 A^\nu \right) = 0$$

identically zero

$$\longrightarrow \quad \partial^\mu A_\mu = 0$$ (Lorentz condition)

massive case

$$p^\mu = \begin{pmatrix} E \\ 0 \\ 0 \\ p \end{pmatrix}$$

zero by Lorentz condition
(unphysical)

$$\epsilon^\mu = c_L \begin{pmatrix} p \\ 0 \\ 0 \\ E \end{pmatrix} + c_S \begin{pmatrix} p \\ 0 \\ 0 \\ -E \end{pmatrix} + c_+ \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix} + c_- \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}$$

3 d.o.f. in total.

Longitudinal mode
(physical)

transverse polarizations

We will learn soon that

Higgs mechanism:

massless spin-1: 2 d.o.f.

feeding 1 d.o.f.
by Higgs fields

massive spin-1: 3 d.o.f.

Symmetry and symmetry breaking

Let's consider a model:

potential term

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi)$$

$$V(\phi) = m^2 |\phi|^2 + \frac{\lambda}{4} |\phi|^4$$

U(1) global symmetry

$$\phi \to e^{i\theta} \phi \quad \text{(U(1) transformation)}$$

Here, θ is an arbitrary real number (not a function!)

$$\mathcal{L} \to \mathcal{L}$$

Lagrangian is invariant under the U(1) transformation.

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) \qquad \phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$

$$= \frac{1}{2}\partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{1}{2}\partial_\mu \phi_2 \partial^\mu \phi_2$$

$$-\frac{m^2}{2}(\phi_1^2 + \phi_2^2) \; -\frac{\lambda}{16}(\phi_1^2 + \phi_2^2)^2$$

$\longrightarrow$ spectrum of the theory is

2 massive spin-0 d.o.f. with the **same** mass “m”

(real part and the imaginary part.)

Also, there is a conserved charge “ϕ” number.

Symmetry in the Lagrangian

$\longrightarrow$ Symmetry in the spectrum?

… actually, not necessarily the case.

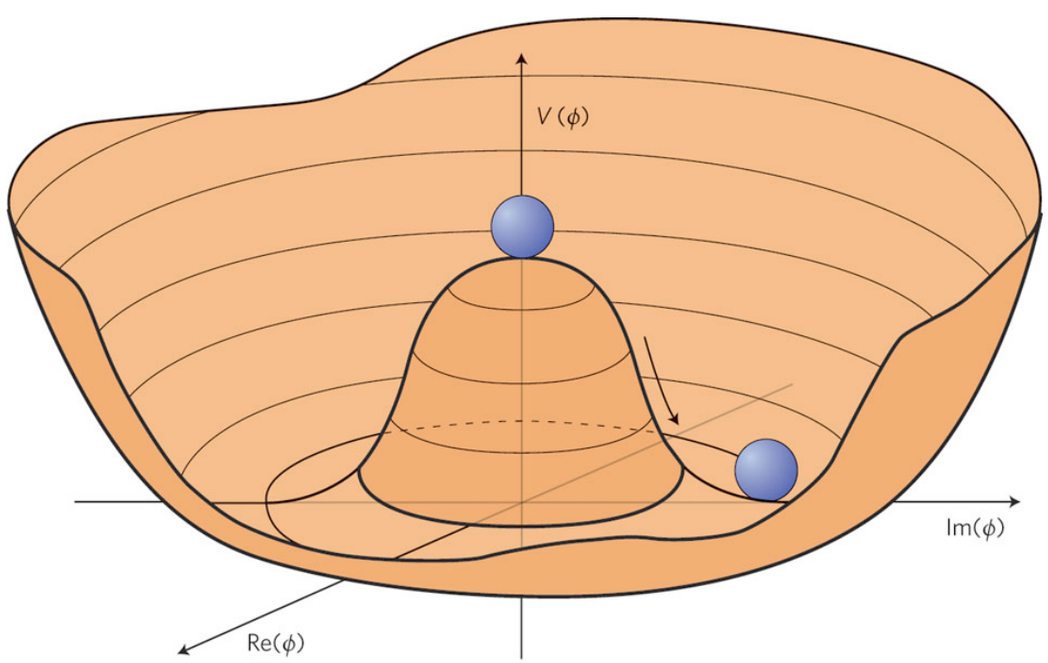
Spontaneous symmetry breaking

Consider the case with

$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

Lagrangian is still U(1) invariant.

But the lowest energy solution to the eq. of motion is



$$-\Box\phi + m^2\phi - \frac{\lambda}{2}|\phi|^2\phi = 0$$

$$\longrightarrow \quad \phi = \sqrt{\frac{2m^2}{\lambda}}e^{i\eta}$$

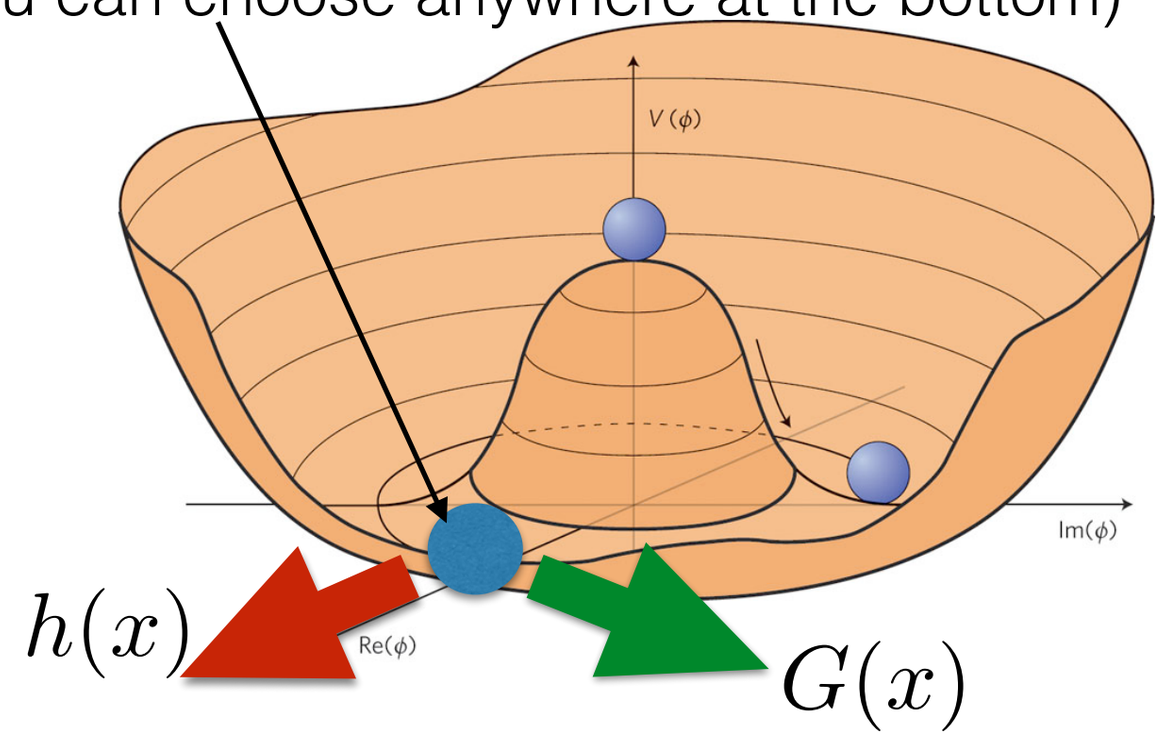
arbitrary phase

Let's choose

$$\phi = \sqrt{\frac{2m^2}{\lambda}} \equiv v \qquad V = \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2 + \text{const.}$$

(this choice is not special. The phase rotation leaves the Lagrangian invariant.)

our choice
(you can choose anywhere at the bottom)



Now, we rename the fields

$$\phi(x) = (v + \frac{h(x)}{\sqrt{2}})e^{iG(x)/\sqrt{2}v}$$

$\sqrt{\frac{2m^2}{\lambda}}$

radial direction

phase direction

mass splitting, NG boson

$$\mathcal{L} = \partial_\mu \left(\left(v + \frac{h}{\sqrt{2}} \right) e^{iG/\sqrt{2}v} \right)^* \partial^\mu \left(\left(v + \frac{h}{\sqrt{2}} \right) e^{iG/\sqrt{2}v} \right)$$

$$+ \frac{\lambda}{4} \left(\left(v + \frac{h}{\sqrt{2}} \right)^2 - v^2 \right)^2 + \text{const.}$$

$$= \frac{1}{2} \partial_\mu h \partial^\mu h - \frac{1}{2} \lambda v^2 h^2 + \dots$$

mass term with the correct sign

$$+ \frac{1}{2} \left(1 + \frac{h}{\sqrt{2}v} \right) \partial_\mu G \partial^\mu G$$

no mass term for G

spectrum: one **massive** spin-0 boson $m_h^2 = \lambda v^2$

one **massless** spin-0 boson

(Nambu-Goldstone boson)

no symmetry in the spectrum

Nambu-Goldstone theorem

(# of broken symmetry) = (# of massless NG boson)

We saw it in a U(1) example at the classical level,

but

this is true at the **quantum level**.

e.g. pions in QCD

Couple to gauge theory

Let's couple the gauge field A_μ to the scalar field.

remember that gauge invariance
is necessary for consistency
(reducing d.o.f.)

Gauge transformation

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \theta(x)$$
$$\phi \to e^{ie\theta(x)}\phi$$

$$\begin{aligned}(\partial_\mu - ieA_\mu)\phi \quad &\to e^{ie\theta}\partial_\mu\phi + ie\partial_\mu\theta e^{ie\theta}\phi \\ &\quad -ieA_\mu e^{ie\theta}\phi - ie\partial_\mu\theta e^{ie\theta}\phi \\ &= e^{ie\theta}(\partial_\mu - ieA_\mu)\phi \\ &\equiv e^{ie\theta}D_\mu\phi\end{aligned}$$

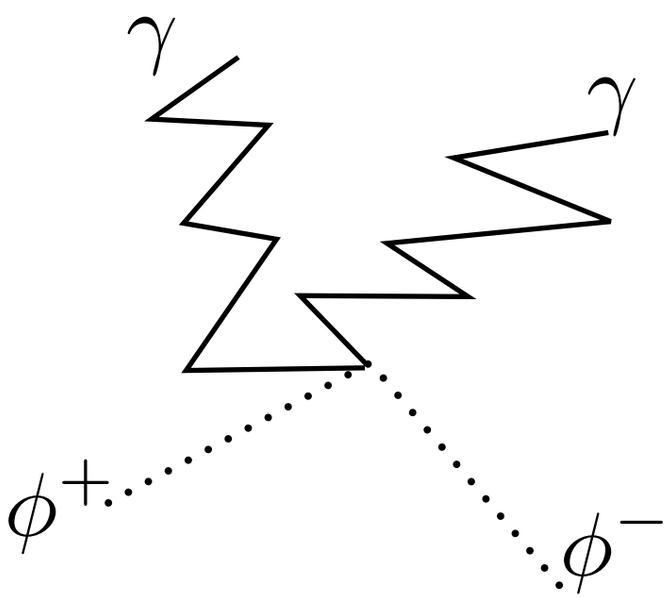
cancel

covariant derivative

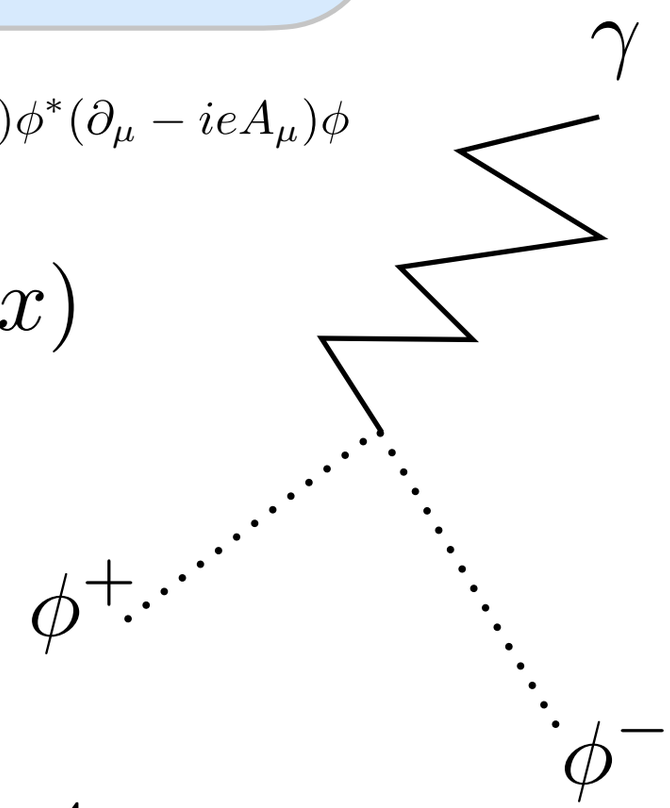
gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad +|D_\mu\phi|^2 - V(|\phi|)$$

$(\partial_\mu + ieA_\mu)\phi^*(\partial_\mu - ieA_\mu)\phi$



$$A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x)$$
$$\phi \to e^{ie\theta(x)}\phi$$

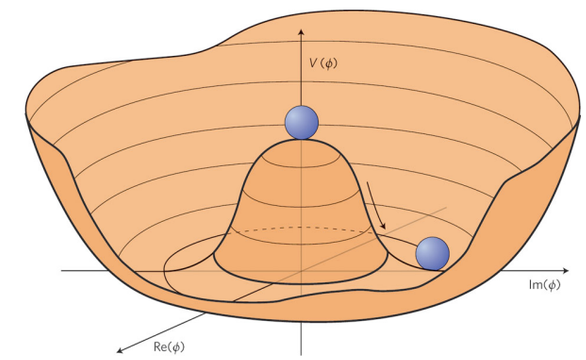


For $V(\phi) = m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$,

this Lagrangian describes physics of a **charged** spin-0 particle coupled to the **massless** photon.

broken phase

Now, consider the case with



$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

$$= \frac{\lambda}{4}\left(|\phi|^2 - \frac{2m^2}{\lambda}\right)^2 + \text{const.}$$

$$\phi(x) = (v + \frac{h(x)}{\sqrt{2}})e^{iG(x)/\sqrt{2}v}$$

Now, by gauge transformation:

$$\phi \to \phi e^{-iG(x)/\sqrt{2}v} = v + \frac{h(x)}{\sqrt{2}}$$

$$A_\mu \to A_\mu - \frac{1}{\sqrt{2}ev}\partial_\mu G(x) \equiv A'_\mu$$

New Lagrangian

$$|D_\mu\phi|^2 = |\partial\phi|^2 + \boxed{eA_\mu(i\phi^*\partial^\mu\phi - i\partial_\mu\phi^*\phi)}$$

this term vanishes

$$+e^2|\phi|^2 A_\mu A^\mu$$

$$= \frac{1}{2}\partial_\mu h\partial^\mu h$$

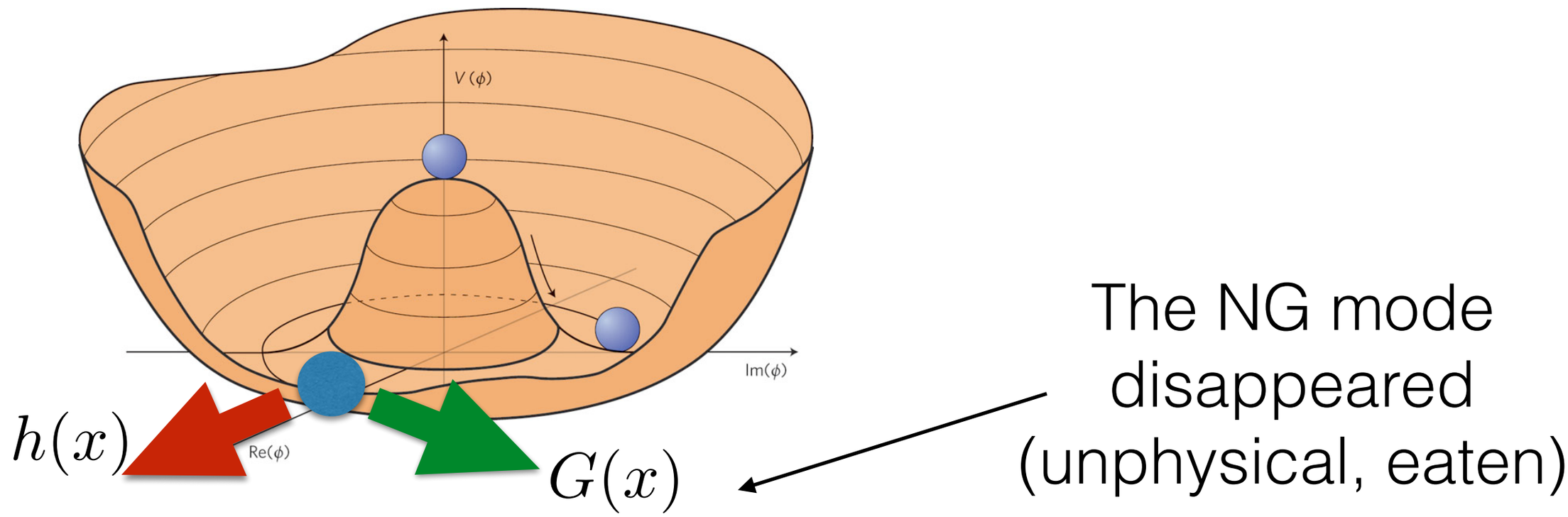
$$+e^2\left(v + \frac{h}{\sqrt{2}}\right)^2 A'_\mu A'^\mu$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} \qquad F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu$$

$$V(\phi) = \frac{1}{2}(\lambda v^2)h^2 + \cdots$$

NG boson, G(x), **disappeared**!

The Higgs mechanism



instead, the mass term for the gauge boson appeared.

$$\mathcal{L} = \cdots + e^2 v^2 A'_\mu A'^\mu + \cdots$$

This Lagrangian describes physics of
a **massive** spin-1 particle and a neutral **Higgs** boson.

Symmetric phase

$$V(\phi) = m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

massless gauge boson + charged particle

Higgs phase

$$V(\phi) = -m^2|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

massive gauge boson + Higgs boson

We will learn next that we are in the Higgs phase!

Lecture 3

A little bit of history

We knew there is an approximate symmetry in strong interactions

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix} \rightarrow e^{i\sigma^a\theta^a} \begin{pmatrix} p \\ n \end{pmatrix}$$

isospin symmetry

Yang-Mills theory

in 1954, Yang and Mills proposed a theory where force between **isospins** as an analogy of force between charges in E&M.

This theory (the non-abelian gauge theory) predicts **massless** spin-1 particle.

But… there isn't such a particle in the theory of strong interactions…

Nambu

In 1961, Nambu and Jona-Lasinio proposed a theroy of **spontaneous symmetry breaking**.

$$\psi(x) = \begin{pmatrix} p \\ n \end{pmatrix}$$

Proton and neutron masses come from spontaneous symmetry breaking.

Nambu-Goldstone bosons are identified with the pions.

theorists have thought that

Yang-Mills theory : **massless** spin-1 bosons

spontaneous symmetry breaking: **massless** spin-0 bosons

↑

this may be a good tool for approximate symmetries
such as isospin symmetry

No application to real physics?

Higgs mechanism

In 1964, Higgs and independently by Brout, Englert, Guralnik, Hagen and Kibble have realized that

in the Higgs phase of the gauge theory,
there is **no** massless gauge bosons or Nambu-Goldstone bosons!

In the paper by Higgs, it is mentioned that
the model contains a scalar boson, now it is called
“the Higgs boson,” in such a theory.

Theorists have thought that the mechanism can be applied to the theory of **strong** interactions.

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The

about the "vacuum" solution $\varphi_1(x)=0$, $\varphi_2(x)=\varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1)-e\varphi_0 A_\mu\}=0, \tag{2a}$$

$$\{\partial^2-4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2)=0, \tag{2b}$$

. . . .

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[8]Tentative proposals that incomplete SU(3) octets of scalar particles exist have been made by a number of people. Such a rôle, as an isolated $Y=\pm1$, $I=\frac{1}{2}$ state, was proposed for the κ meson (725 MeV) by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963). More recently the possibility that the σ meson (385 MeV) may be the $Y=I=0$ member of an incomplete octet has been considered by L. M. Brown, Phys. Rev. Letters 13, 42 (1964).

The Standard Model

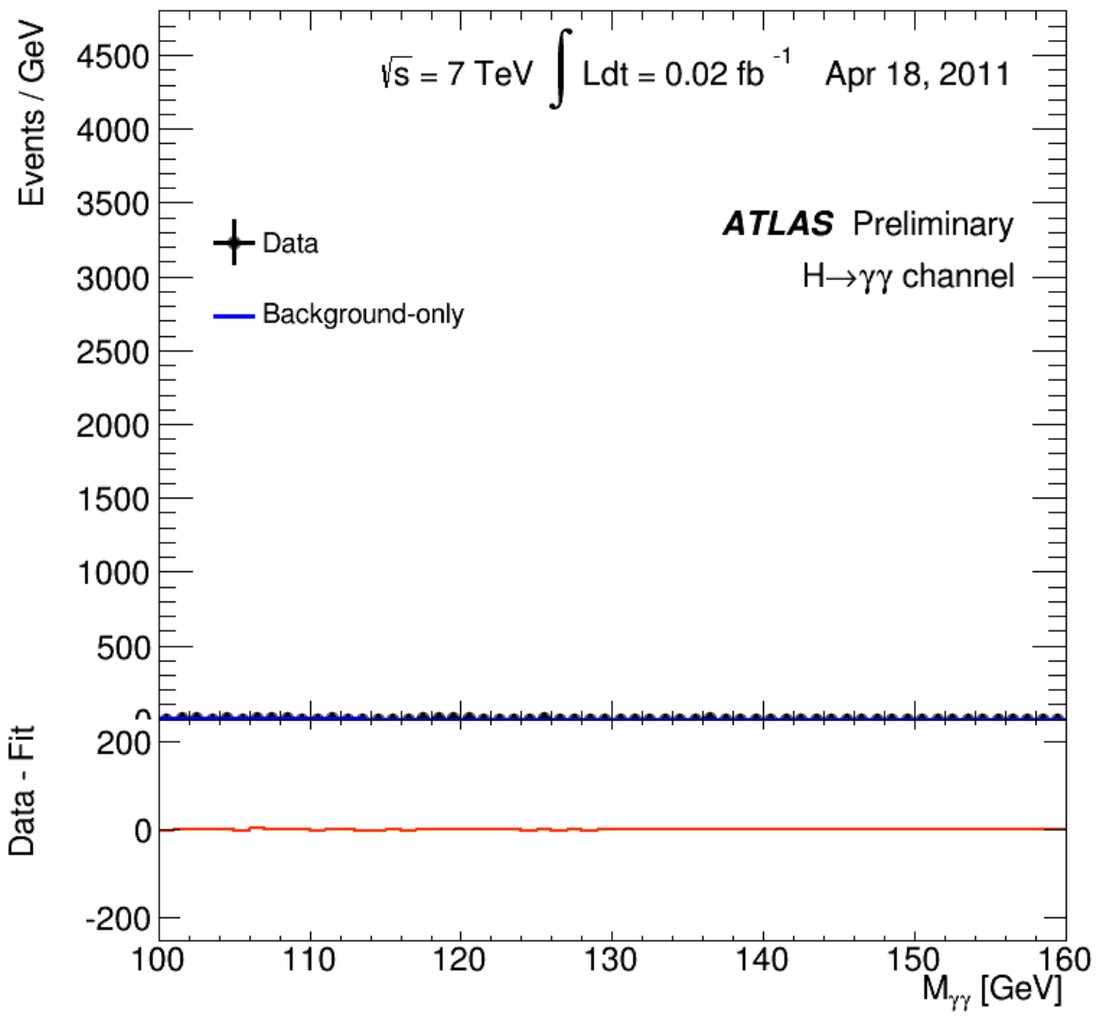
In 1967, Weinberg has realized that
the Higgs mechanism can actually be applied by
the theory of **weak** interactions.

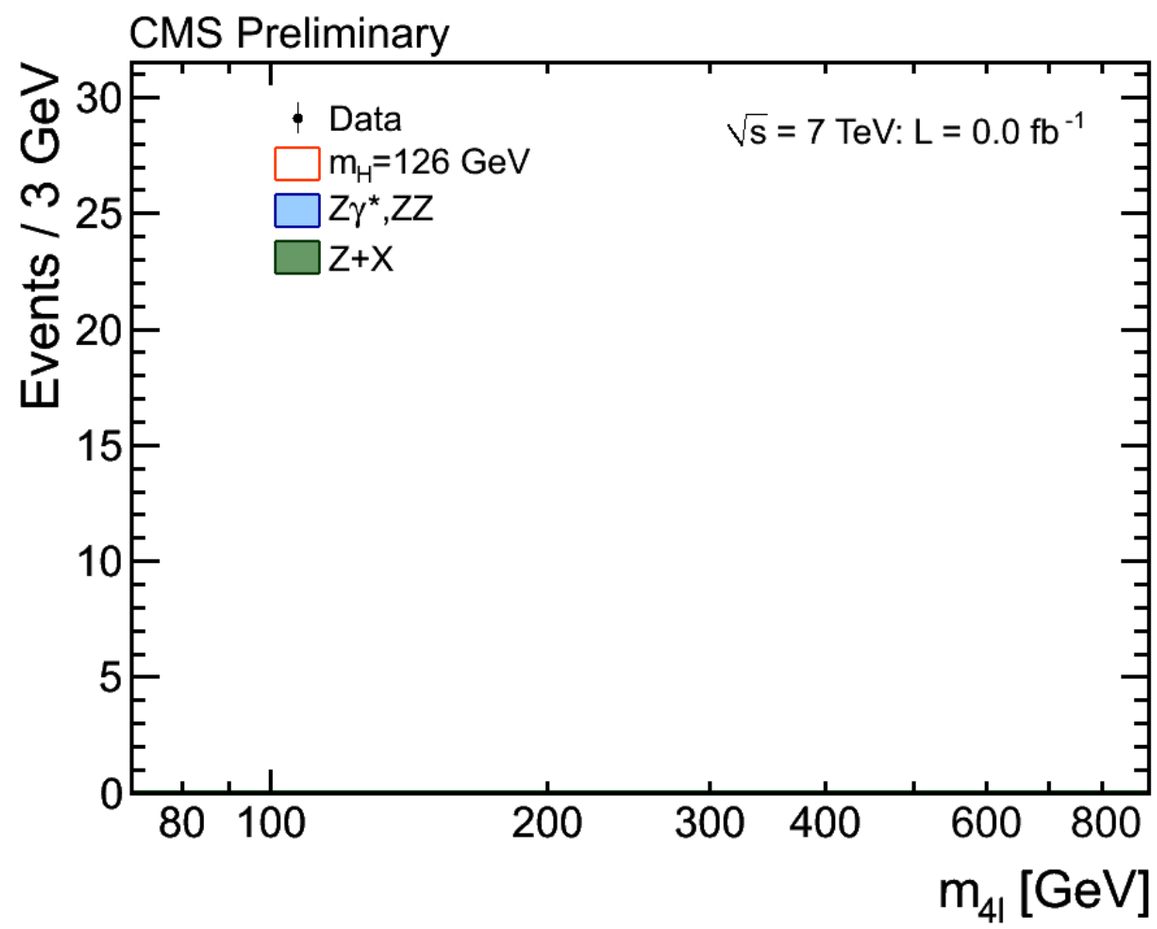
Yang-Mills+Nambu+Higgs

somehow, the theory developed from different motivations
turns out to the kernel of the electroweak theory.

Surprisingly, the theory of strong interaction turns out to be
also the gauge theory in a yet another phase,
the confining phase.

and then,


Events / GeV
√s = 7 TeV ∫ Ldt = 0.02 fb⁻¹ Apr 18, 2011
ATLAS Preliminary
H→γγ channel
Data
Background-only
Data - Fit
M_γγ [GeV]


CMS Preliminary
Events / 3 GeV
Data
m_H=126 GeV
Zγ*,ZZ
Z+X
√s = 7 TeV: L = 0.0 fb⁻¹
m_4l [GeV]

The Standard Model

$SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

electroweak interaction

strong interaction

SU(3): 3x3 special unitary matrix

Gell-Mann matrices

$$U^\dagger U = \mathbf{1} \qquad \det U = 1 \longrightarrow U = e^{i\theta^a T^a}$$

8 dimensional group

SU(2): 2x2 special unitary matrix

Pauli matrices

$$V^\dagger V = \mathbf{1} \qquad \det V = \mathbf{1} \longrightarrow V = e^{i\theta^A \sigma^A/2}$$

3 dimensional group

gauge fields

gluon (a=1,…,8)

$$g_\mu = g_\mu^a T^a \to U g_\mu U^\dagger + \frac{i}{g_3} U \partial_\mu U^\dagger$$

(3x3 matrix)

SU(2) gauge boson (A=1,2,3)

$$A_\mu = A_\mu^A \sigma^A/2 \to V A_\mu V^\dagger + \frac{i}{g_2} V \partial_\mu V^\dagger$$

(2x2 matrix)

U(1) gauge boson

$$B_\mu \to B_\mu + \frac{i}{g_Y} \partial_\mu \theta$$

all massless at this stage

Quark fields

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad (\mathbf{3}, \mathbf{2})_{1/6}$$

SU(3): $q \to Uq$

SU(2): $q \to Vq$

U(1): $q \to e^{i\theta/6} q$

there are three of them: $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

$u^c \qquad (\bar{\mathbf{3}}, \mathbf{1})_{-2/3}$

SU(3): $u^c \to u^c U^\dagger$

SU(2): $u^c \to u^c$

U(1): $u^c \to e^{-2i\theta/3} u^c$

$d^c \qquad (\bar{\mathbf{3}}, \mathbf{1})_{1/3}$

SU(3): $d^c \to d^c U^\dagger$

SU(2): $d^c \to d^c$

U(1): $d^c \to e^{i\theta/3}$

All **massless** at this stage.

Lepton fields

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad (\mathbf{1},\mathbf{2})_{-1/2}$$

SU(3): $l \to l$

SU(2): $l \to Vl$

U(1): $l \to e^{-i\theta/2} l$

three generations $\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

$$e^c \qquad (\mathbf{1},\mathbf{1})_1$$

SU(3): $e^c \to e^c$

SU(2): $e^c \to e^c$

U(1): $e^c \to e^{i\theta} e^c$

All **massless** at this stage

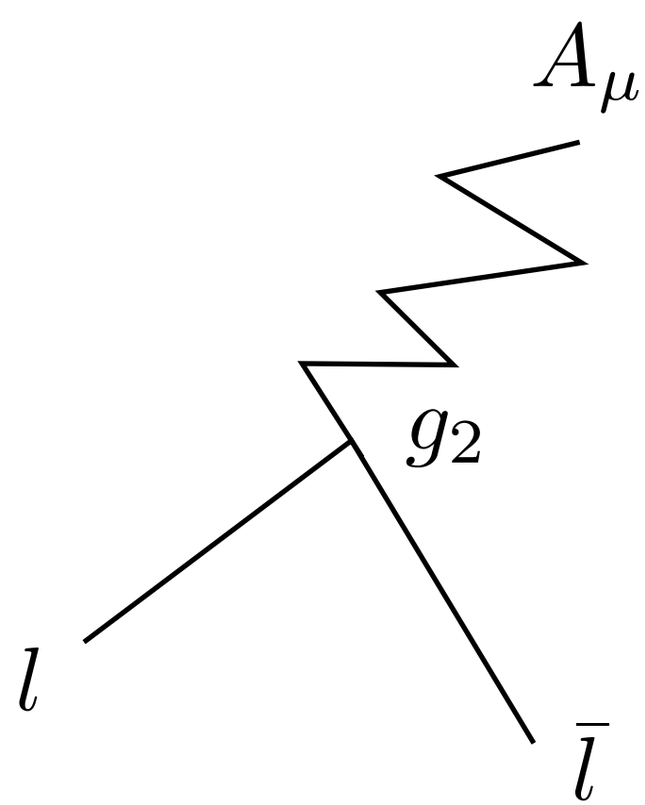
(no way to write down mass terms in a gauge invariant way)

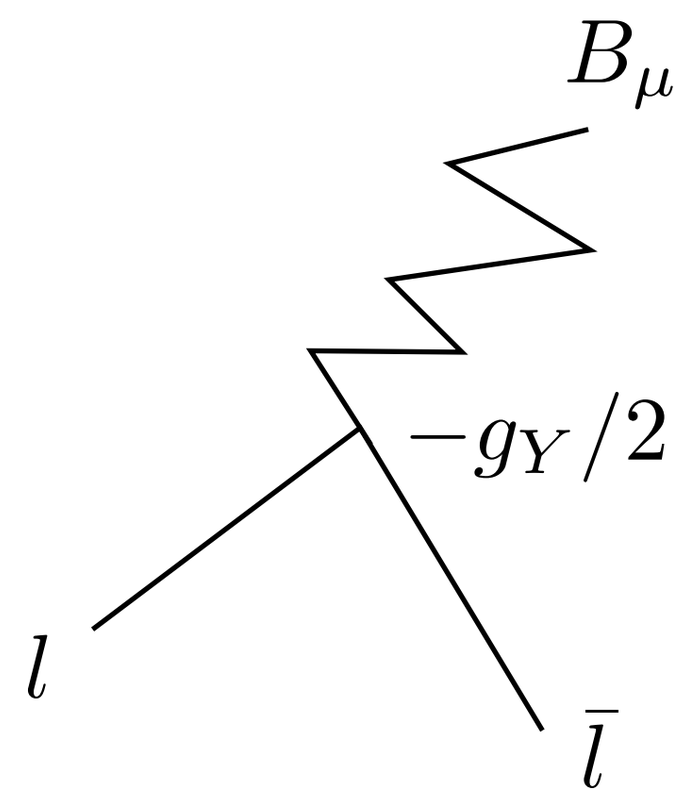
gauge interactions

for example, for lepton doublets,
gauge invariant kinetic term is

$$\mathcal{L}_{\rm kin} = \bar{l} i \gamma^\mu \left[\partial_\mu - i g_2 A_\mu - i g_Y (-1/2) B_\mu\right] l$$

covariant derivative





Higgs field

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad (\mathbf{1},\mathbf{2})_{1/2}$$

SU(3): $H \to H$

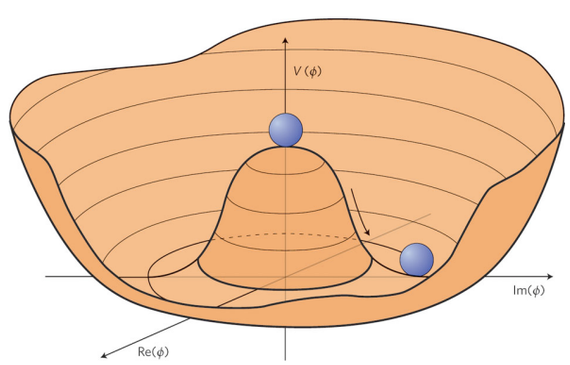
SU(2): $H \to VH$

U(1): $H \to e^{i\theta/2} H$

SU(2) doublet complex scalar field

kinetic term $\mathcal{L}_{\rm kin} = |(\partial_\mu - i g_2 A_\mu - i g_Y/2 B_\mu) H|^2$

Higgs potential $V = \frac{\lambda}{4}\left(|H|^2 - v^2\right)^2$



$$|H|^2 = H^\dagger H = |H^+|^2 + |H^0|^2$$

vacuum

One can choose $H = \begin{pmatrix} 0 \\ v \end{pmatrix}$

remember that other choices are all equivalent to this.

since H is charged under electroweak gauge group,

$SU(2)_L \times U(1)_Y$ is broken

Higgs mechanism for gauge boson masses.

broken gauge group and unbroken gauge group

gauge transformation

$$H \to H = \begin{pmatrix} 0 \\ v + h/\sqrt{2} \end{pmatrix} \qquad \begin{aligned} A_\mu &\to A'_\mu \\ B_\mu &\to B'_\mu \end{aligned}$$

One can eliminate **three** NG bosons.

why three not four?

$$e^{i\sigma_3/2\theta} e^{i\theta/2} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{pmatrix}$$

One combination of SU(2)xU(1) leaves $H = \begin{pmatrix} 0 \\ v \end{pmatrix}$ invariant. **There is an unbroken U(1).**

gauge boson masses

$$\mathcal{L}_{\rm kin} = |(\partial_\mu - ig_2 A_\mu - ig_Y/2 B_\mu)H|^2 \qquad H = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\to \left| \left(g_2 \frac{\sigma^A}{2} A^A_\mu + g_Y \frac{1}{2} B_\mu \mathbf{1} \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{1}{4} g_2^2 v^2 \left| A^1_\mu - i A^2_\mu \right|^2 + \frac{1}{4} v^2 (g_2 A^3_\mu - g_Y B_\mu)^2$$

$$= \frac{1}{2} g_2^2 v^2 W^+_\mu W^{\mu -} + \frac{1}{2} \frac{g_2^2 + g_Y^2}{2} v^2 Z^2_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \mp A^2_\mu) \qquad Z_\mu = \frac{1}{\sqrt{g_Y^2 + g_2^2}} (g_2 A^3_\mu - g_Y B_\mu)$$

$$m_W^2 = \frac{1}{2} g_2^2 v^2 \qquad m_Z^2 = \frac{1}{2} (g_2^2 + g_Y^2) v^2$$

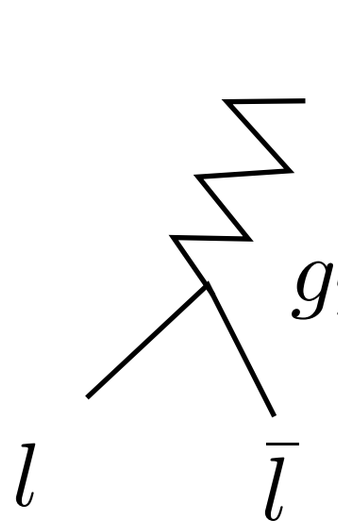
three out of four gauge bosons become **massive**!

photon

there is one gauge boson left massless.

$$\mathcal{A}_\mu = \frac{1}{\sqrt{g_Y^2 + g_2^2}}(g_Y A_\mu^3 + g_2 B_\mu)$$

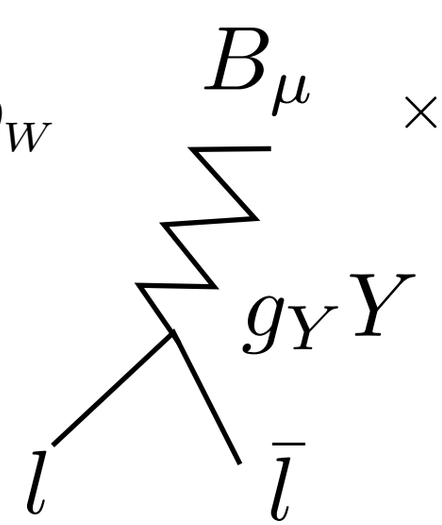
coupling strength to this combination is



$\times \frac{g_Y}{\sqrt{g_Y^2 + g_2^2}} \equiv \sin\theta_W$

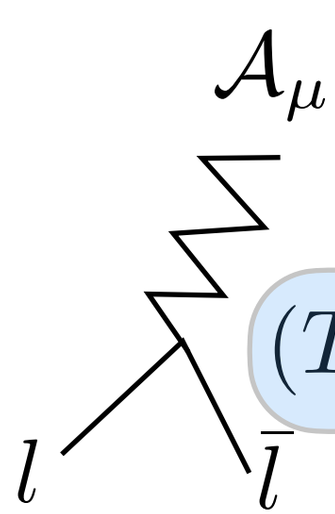
$g_2\sigma^3/2$
$= g_2 T^3$

+



$\times \frac{g_2}{\sqrt{g_Y^2 + g_2^2}} \equiv \cos\theta_W$

=



Q electric charge

$(T^3 + Y)e$

$$e = \frac{g_2 g_Y}{\sqrt{g_Y^2 + g_2^2}}$$

electric charges

$$l = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

$$Q = T^3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} - 1/2 \cdot \mathbf{1} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

neutrino is neutral, whereas electron has charge -1.

The doublet gets separated into two different particles!

$$e^c \qquad (\mathbf{1}, \mathbf{1})_1 \qquad T^3 = 0,\ Y = 1 \quad \longrightarrow \quad Q = 1$$

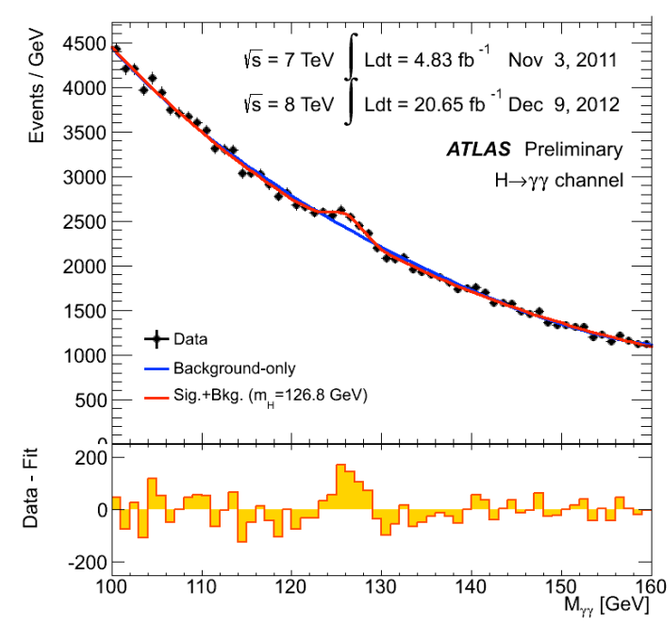
Now, e and e^c can form a mass term. (later)

Higgs boson mass

$$H = \begin{pmatrix} 0 \\ v + h/\sqrt{2} \end{pmatrix}$$

$$V = \frac{\lambda}{4}\left(|H|^2 - v^2\right)^2$$

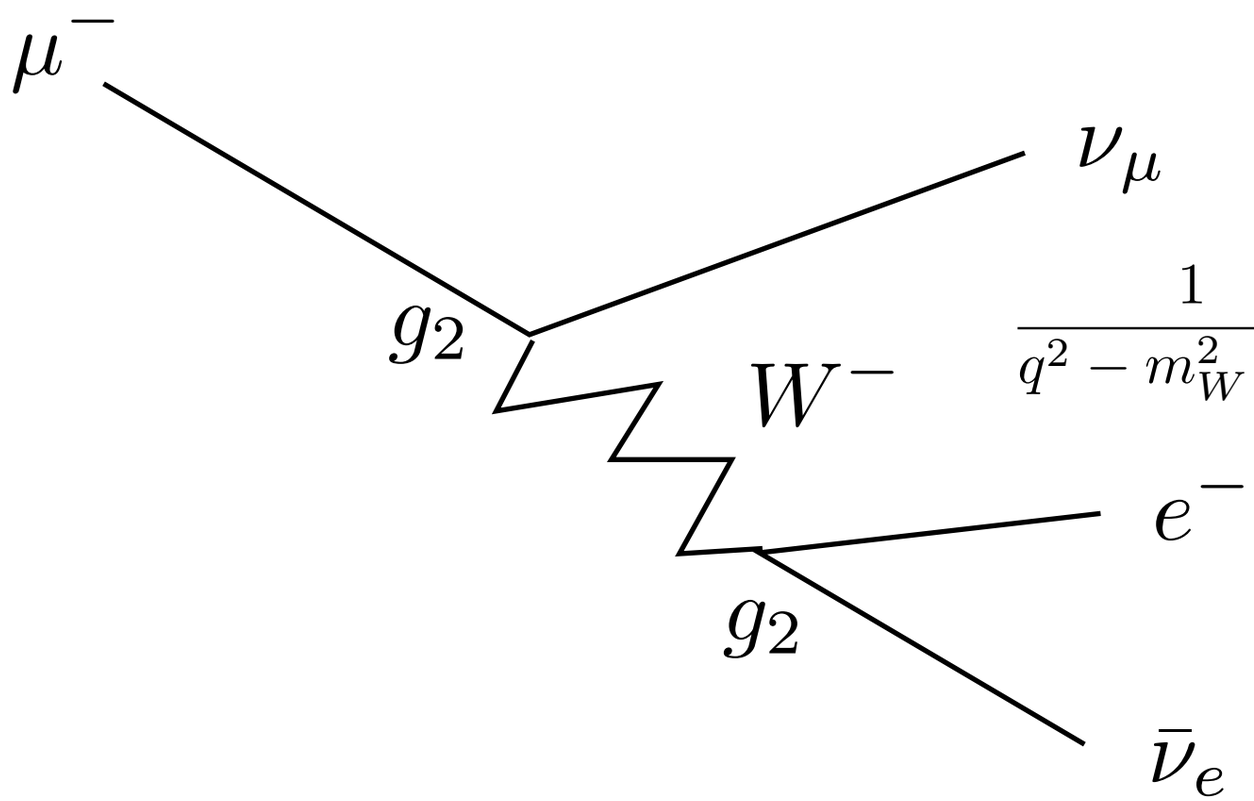
$$= (\lambda v^2)\frac{h^2}{2} + \frac{3\lambda v}{\sqrt{2}}\frac{h^3}{3!} + \frac{3\lambda}{2}\frac{h^4}{4!}$$



$$m_h^2 = \lambda v^2 = (125\ \text{GeV})^2$$

$$\longrightarrow \lambda \sim 0.5$$

v = 174GeV



$$\frac{1}{q^2 - m_W^2 + i\epsilon} \sim -1/m_W^2 = -\frac{2}{g_2^2 v^2}$$

g_2 cancels

The strength of the weak interaction is determined by the Higgs VEV, v.

$$G_F = \frac{\sqrt{2}}{4v^2} \sim 10^{-5}\ \mathrm{GeV}^{-2} \quad \longrightarrow \quad v = 174\ \mathrm{GeV}$$

Yukawa interactions

one can write down terms like

Yukawa coupling constant

$$\mathcal{L}_{\text{Yukawa}} = -f_e^i \tilde{H} \cdot (e_i^c l_i) + \text{h.c.}$$

$$(\tilde{H} = i\sigma^2 H^*, \ A \cdot B = a_1 b_2 - a_2 b_1)$$

and similar terms for quarks.

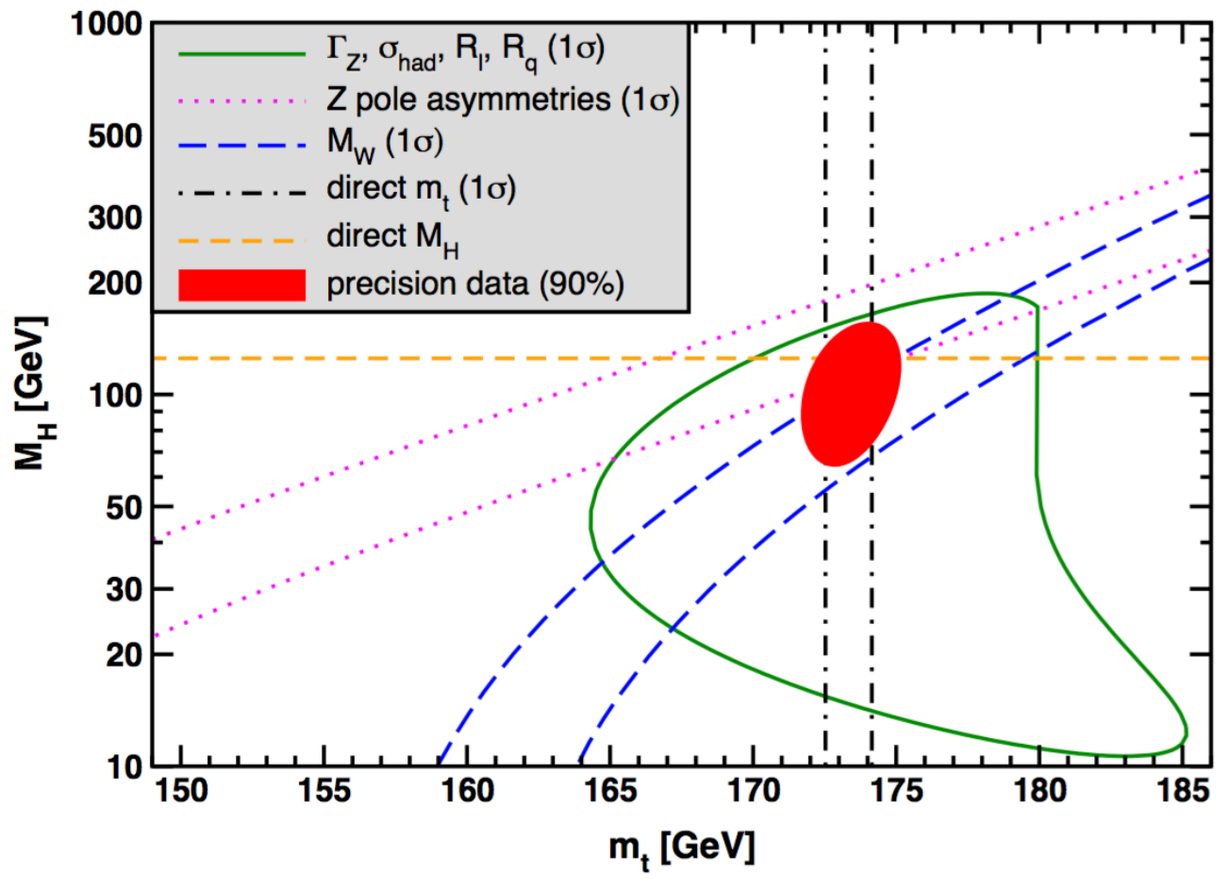
$$H = \begin{pmatrix} 0 \\ v \end{pmatrix} \longrightarrow \tilde{H} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} \to -f_e^i v (e_i^c e_i) + \text{h.c.}$$

masses for charged leptons, but not for neutrinos.

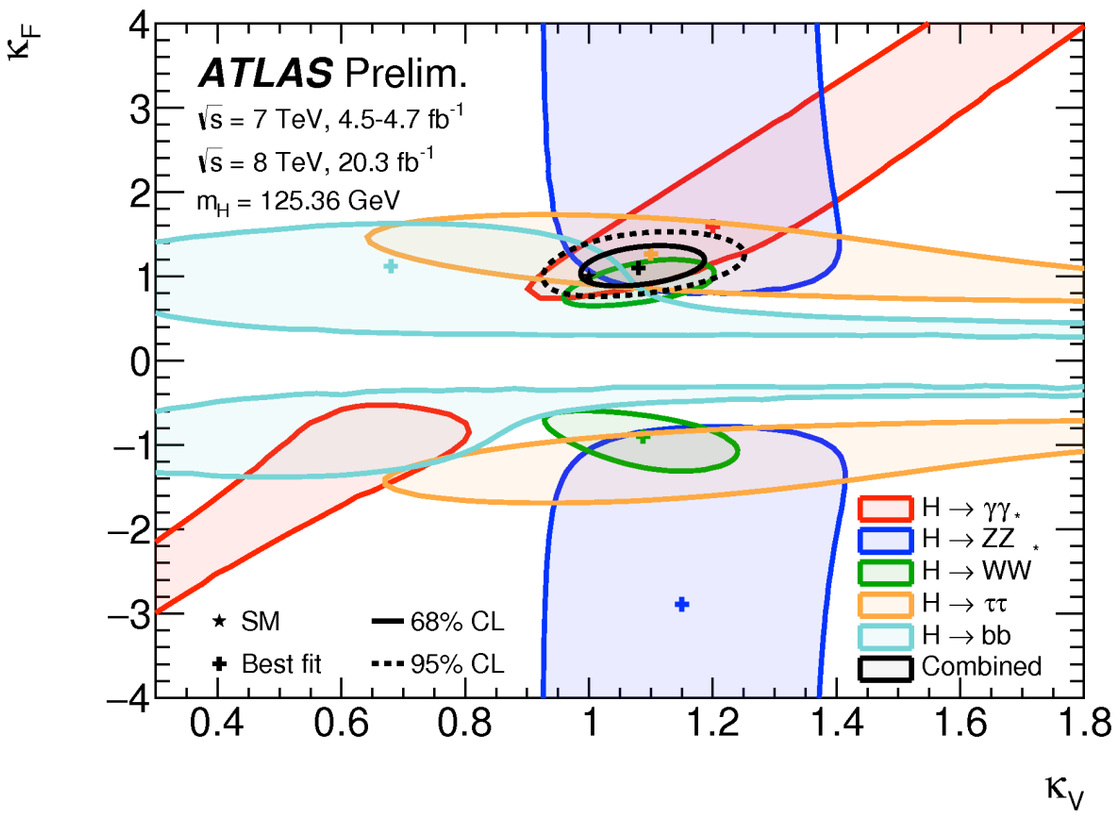
Great success of the Standard Model (1)

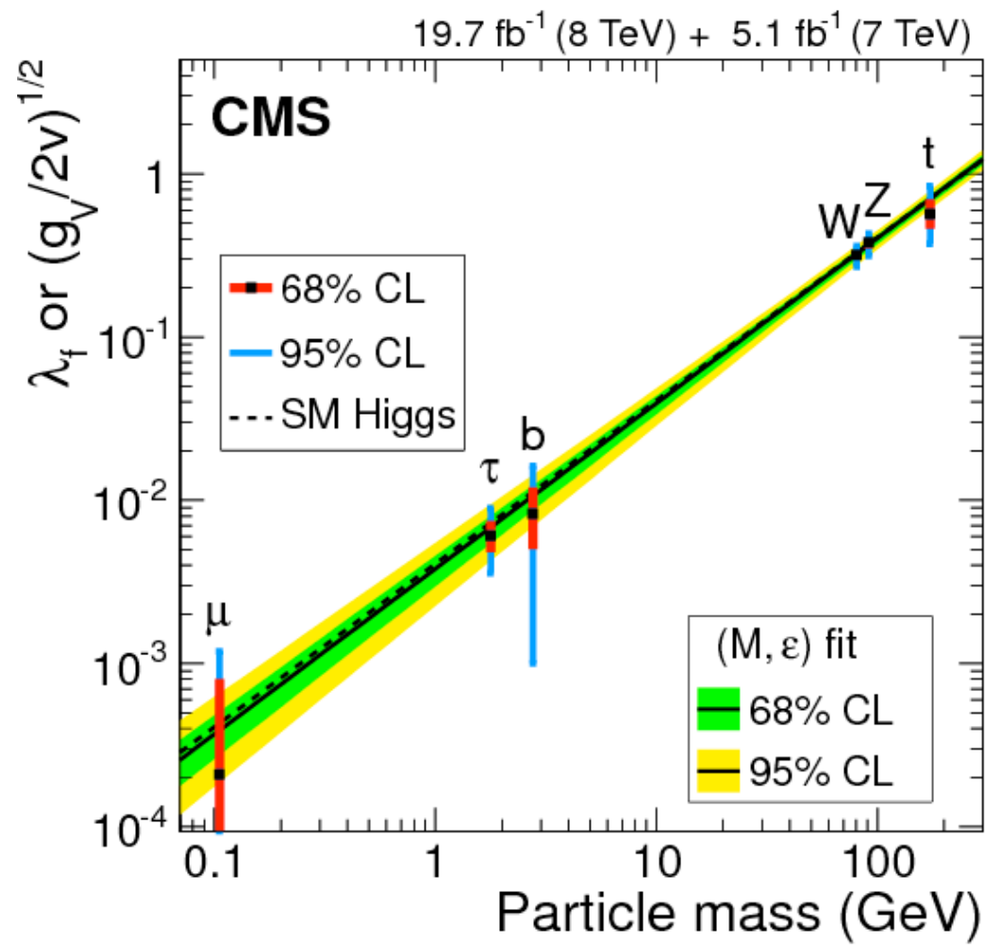
PDG review



$$g_2,\ g_Y,\ \lambda,\ f_t,\ v \quad \Longleftrightarrow \quad m_Z, G_F, \alpha, m_h, m_t$$

Great success of the Standard Model (2)



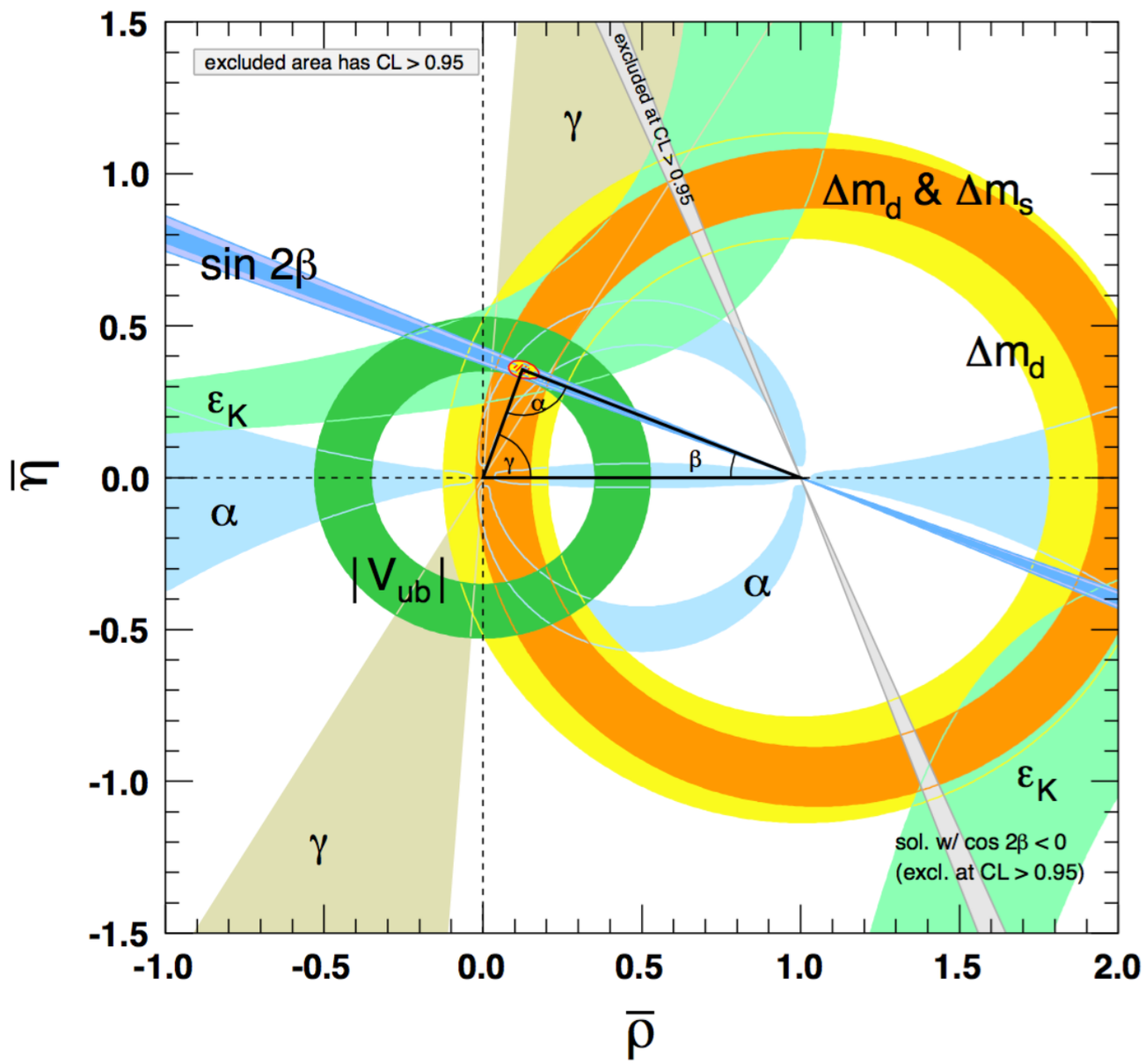


gauge boson mass, fermion mass $\propto v$

$$H = \begin{pmatrix} 0 \\ v + h/\sqrt{2} \end{pmatrix}$$

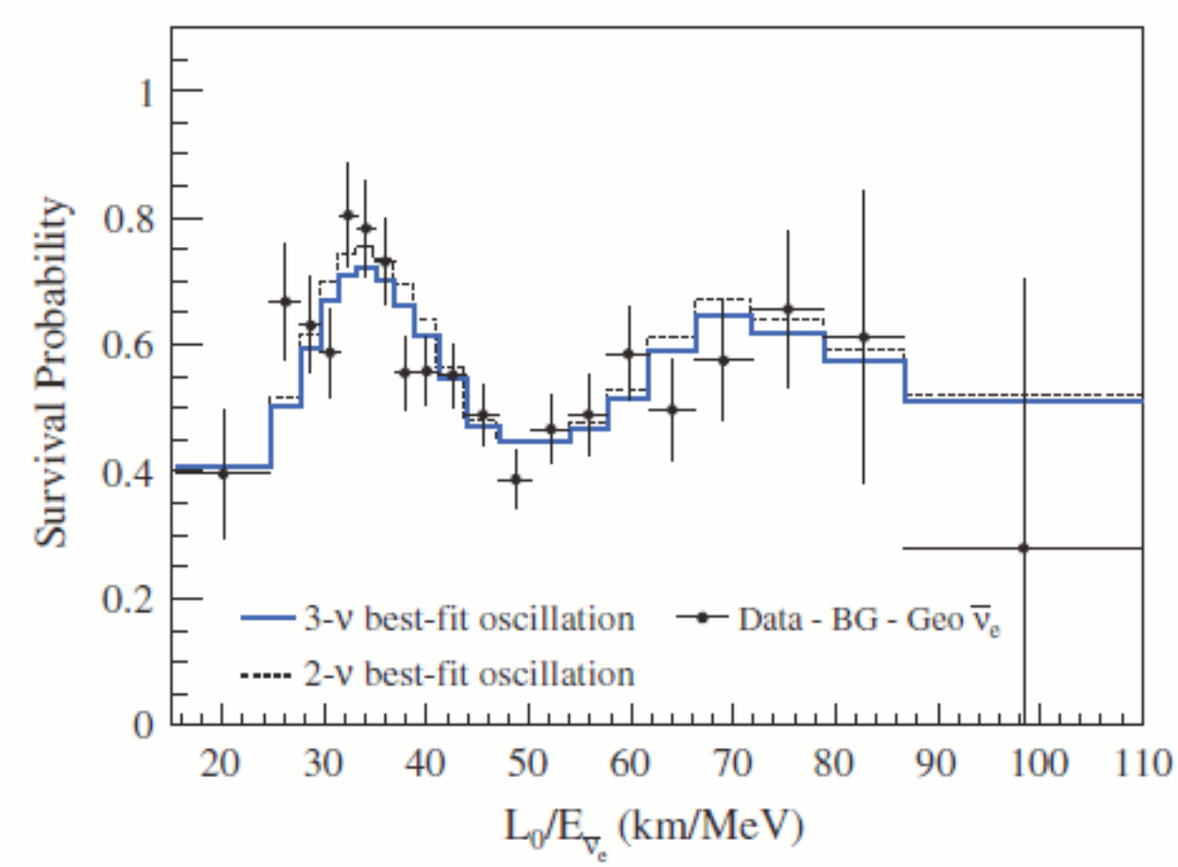
Higgs coupling is proportional to its mass

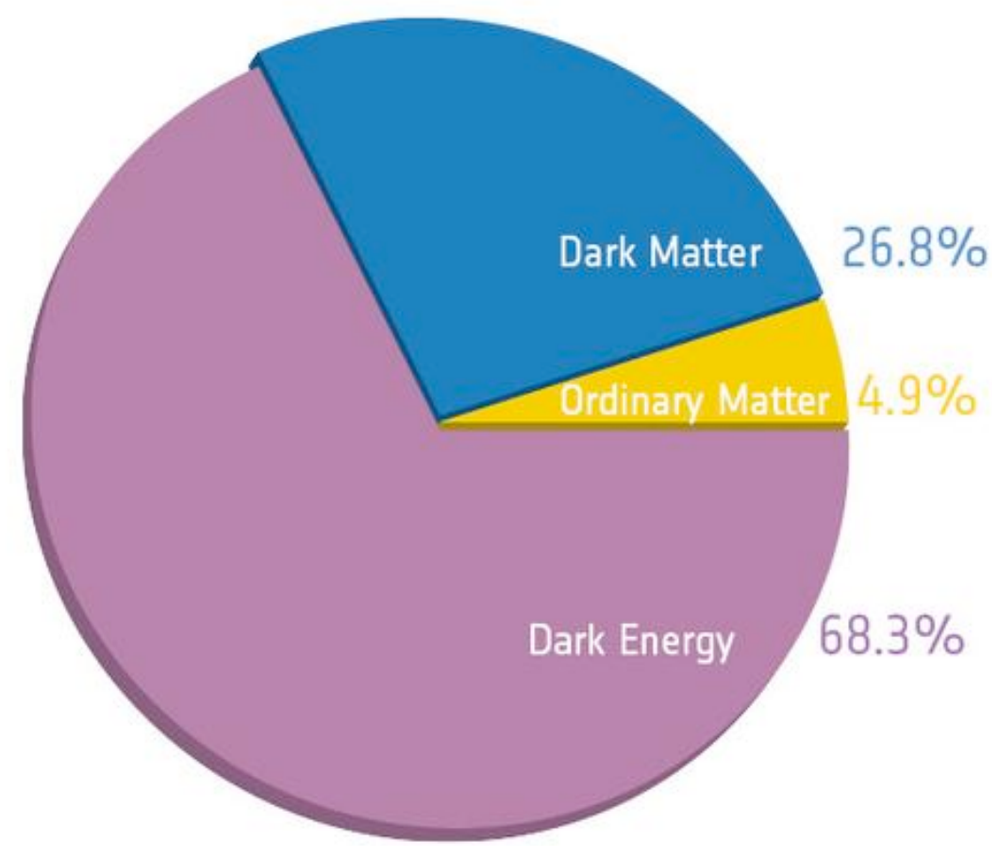
Great success of the Standard Model (3)



You will learn this later.

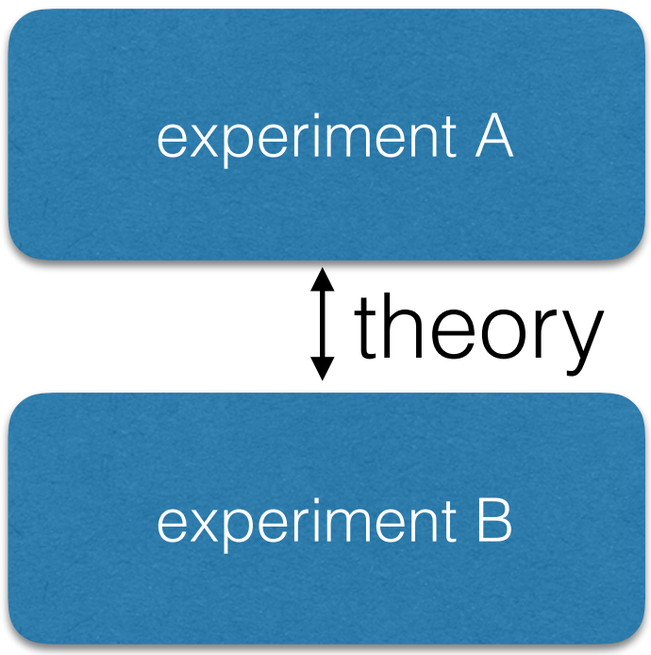
Mysteries of the Standard Model





You will learn this later.

Summary



tool

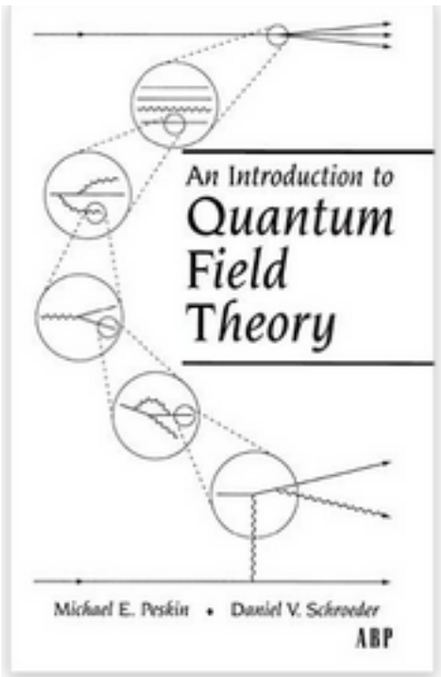
Lagrangian+Path integral

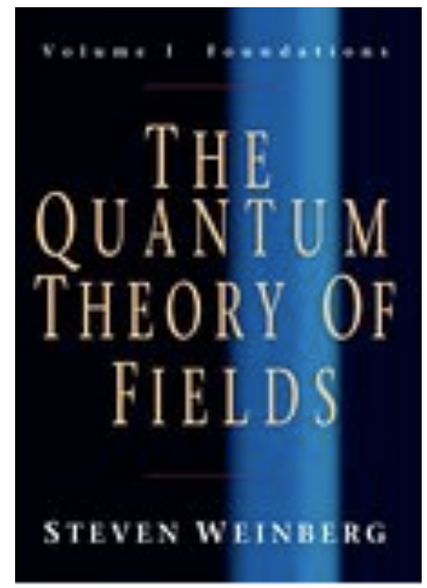
correlation functions

Correlation functions, defined by path integral, can be calculated in the perturbation theory if interactions are sufficiently weak.

The electroweak standard model successfully describes the properties of elementary particles.

text books


An Introduction to
Quantum
Field
Theory
Michael E. Peskin • Daniel V. Schroeder
ABP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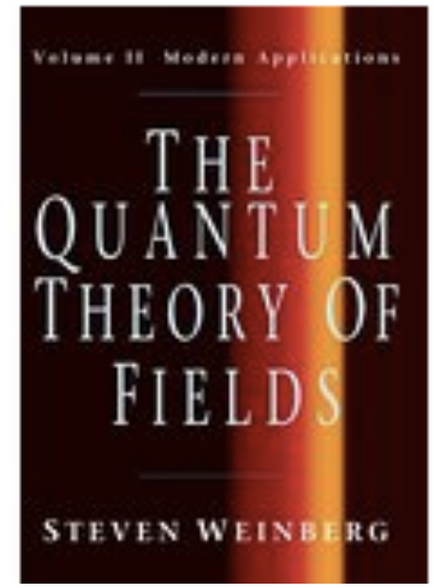
THE QUANTUM THEORY OF FIELDS
STEVEN WEINBERG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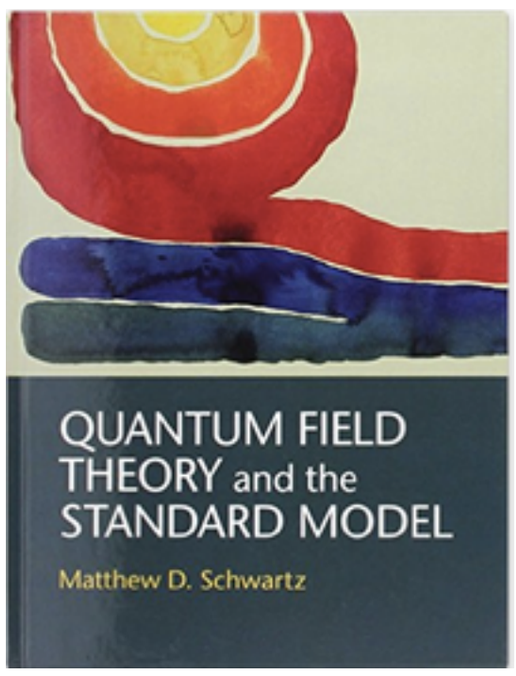
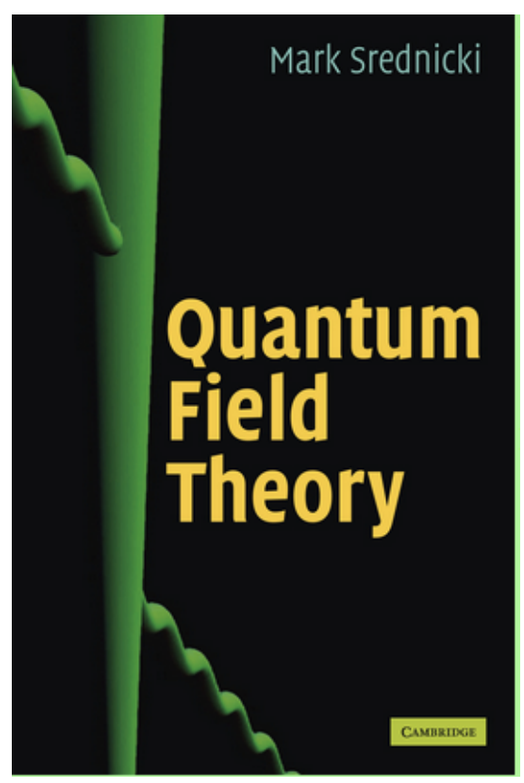
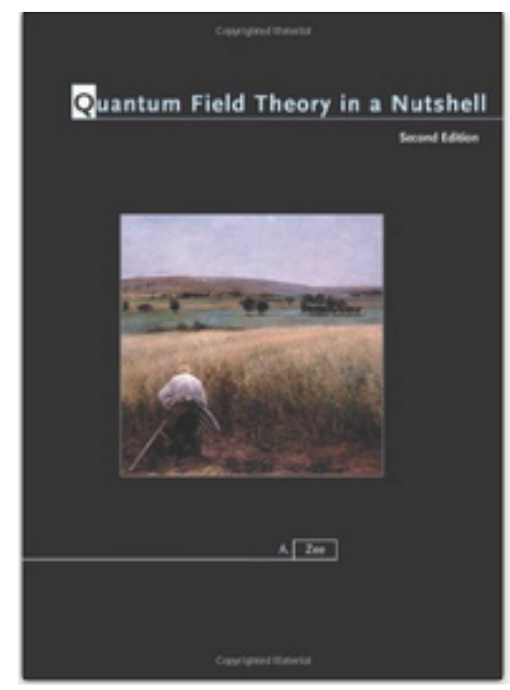

QUANTUM FIELD THEORY and the STANDARD MODEL
Matthew D. Schwartz


Mark Srednicki
Quantum Field Theory
CAMBRIDGE


Quantum Field Theory in a Nutshell
Second Edition
A. Zee