Basics of QCD

Lecture III

Fabio Maltoni
Center for Particle Physics and Phenomenology (CP3)
Université Catholique de Louvain

Plan

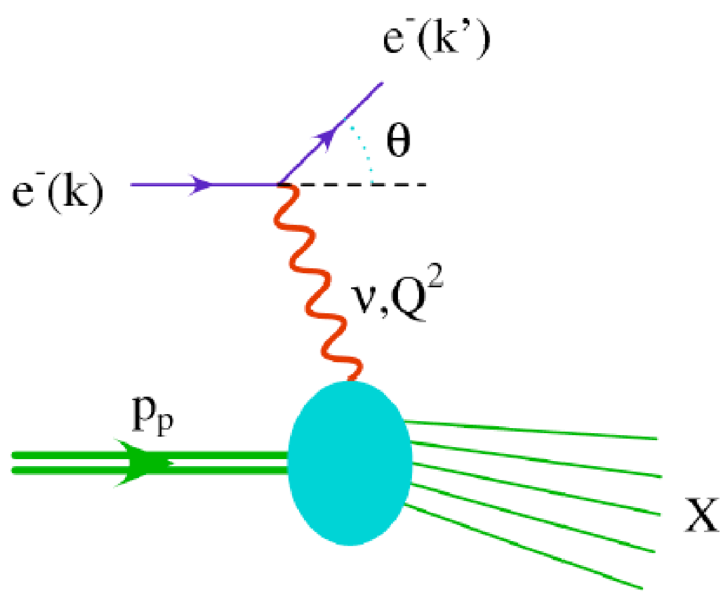
Three lectures:

1. Intro and QCD fundamentals
2. QCD in the final state : e+ e- collisions
3. QCD in the initial state : p p collisions

QCD in the initial state

1. DIS: from the parton model to pQCD
2. Q^2 Evolution and PDF's
3. pp collisions : a glimpse

DIS: The parton model



$$s = (P+k)^2$$ cms energy²

$$Q^2 = -(k-k')^2$$ momentum transfer²

$$x = Q^2/2(P\cdot q)$$ scaling variable

$$\nu = (P\cdot q)/M = E - E'$$ energy loss

$$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$$ rel. energy loss

$$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x} Q^2$$ recoil mass

"deep inelastic" : $Q^2 >> 1$ GeV^2

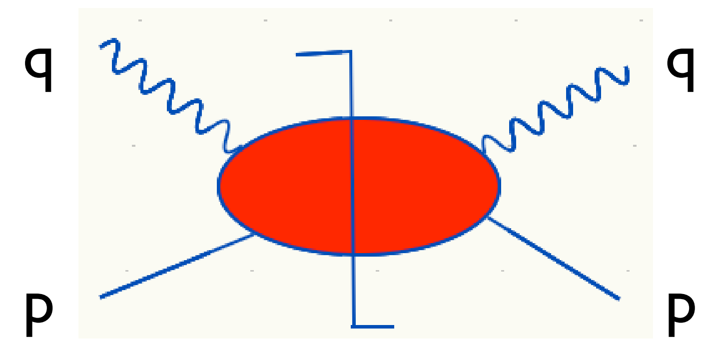
"scaling limit": $Q^2 \to \infty$, x fixed

The idea is that by measuring all the kinematics variables of the outgoing electron one can study the structure of the proton in terms of the probe characteristics, Q2,x,y... Very inclusive measurement from the QCD point of view.

DIS: The parton model

* Divide phase-space factor into a leptonic and a hadronic part:

$$d\Phi = \frac{d^3k'}{(2\pi)^3 2E'} d\Phi_X = \frac{ME}{8\pi^2} y\, dy\, dx\, d\Phi_X$$



* Separate also the square of the Feynman amplitude, by defining:

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} h_{X\mu\nu}$$

* The leptonic tensor can be calculated explicitly:

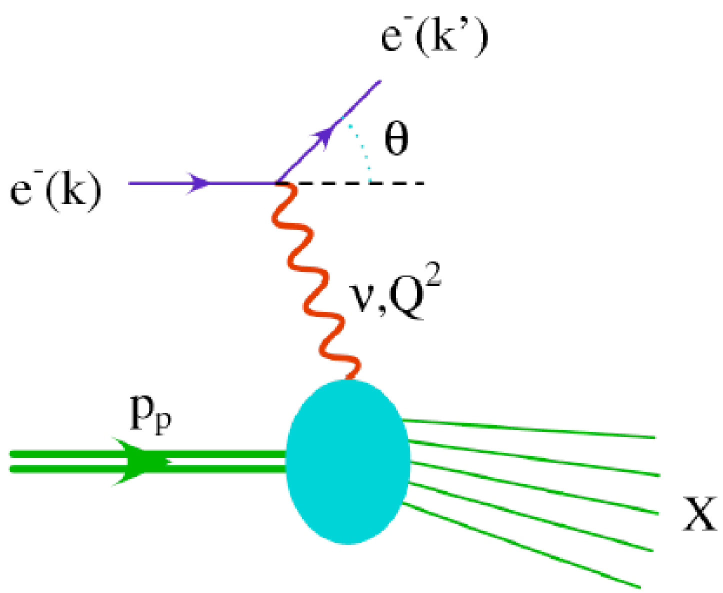
$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k \cdot k'$$

* Combine the hadronic part of the amplitude and phase space into "hadronic tensor" and use just Lorentz symmetry and gauge invariance to write

$$W^{\mu\nu} = \sum_X \int d\Phi_X h_{X\mu\nu}$$

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{p \cdot q} F_2(x,Q^2)$$

DIS: The parton model



$$\sigma^{ep\to eX} = \sum_X \frac{1}{4ME} \int d\Phi \, \frac{1}{4} \sum_{\rm spin} |\mathcal{M}|^2$$

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left\{ [1+(1-y)^2] F_1(x,Q^2) + \frac{1-y}{x} \left[F_2(x,Q^2) - 2xF_1(x,Q^2) \right] \right\}$$

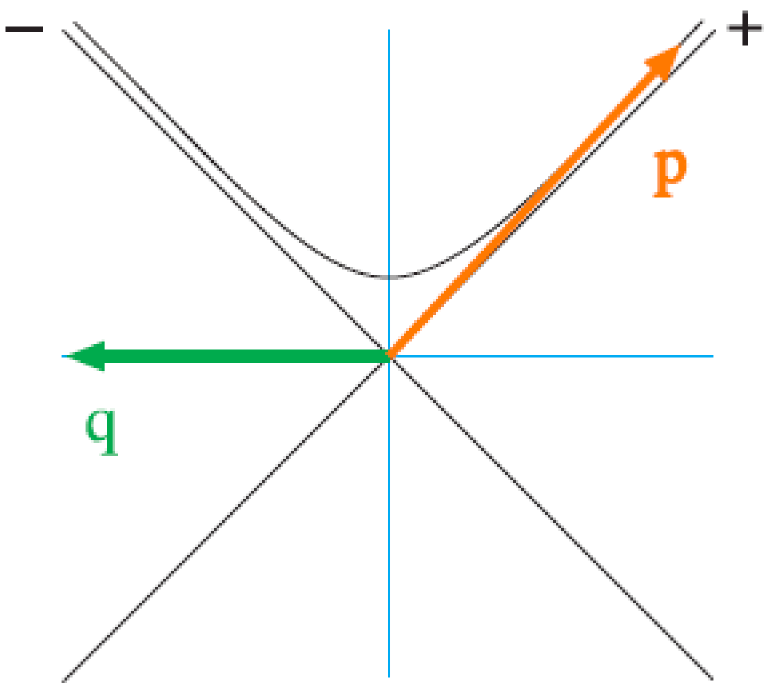
Comments:

* Different y dependence can differentiate between F_1 and F_2
* The first term represents the absorption of a transversely polarized photon, the second of a longitudinal one.
* Bjorken scaling $\Rightarrow$ F_1 and F_2 obey scaling themselves, i.e. they do not depend on Q.

A look from the Breit frame

We want to "watch" the scattering from a frame where the physics is clear. Feynman suggested that what happens can be best understood in a reference frame where the proton moves very fast and $Q \gg m_h$ is large.

4-vector	hadron rest frame	Breit frame
$(p^+, p^-, \vec{p}_T)$	$\frac{1}{\sqrt{2}}(m_h, m_h, \vec{0})$	$\frac{1}{\sqrt{2}}(\frac{Q}{x}, \frac{x m_h^2}{Q}, \vec{0})$
$(q^+, q^-, \vec{q}_T)$	$\frac{1}{\sqrt{2}}(-m_h x, \frac{Q^2}{m_h x}, \vec{0})$	$\frac{1}{\sqrt{2}}(-Q, Q, \vec{0})$

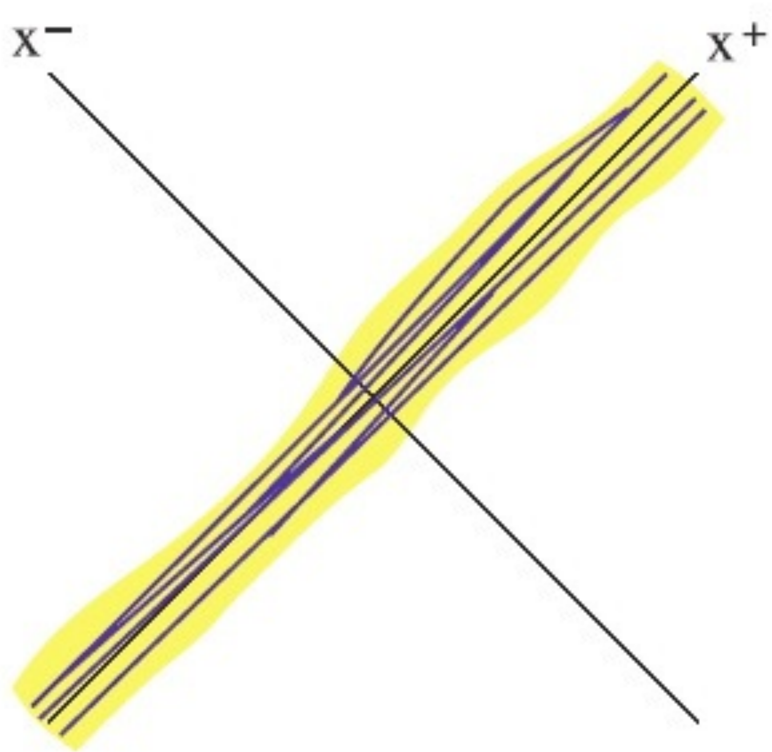


You can check that a Lorentz transformation acts on a light-cone formulation of the four-momentum:

$$(a^+, a^-, \vec{a}) \to (e^{\omega} a^+, e^{-\omega} a^-, \vec{a}) \quad \text{with} \quad e^{\omega} = Q/(x m_h)$$

A look from the Breit frame

Now let’s see how the proton looks in this frame, and in the light-cone space coordinates (suitable for describing relativistic particles).



Lorentz transformation divides out the interactions. Hadron at rest has separation of order:

$\Delta x^+ \sim \Delta x^- \sim 1/m$,

while in the moving hadron has:

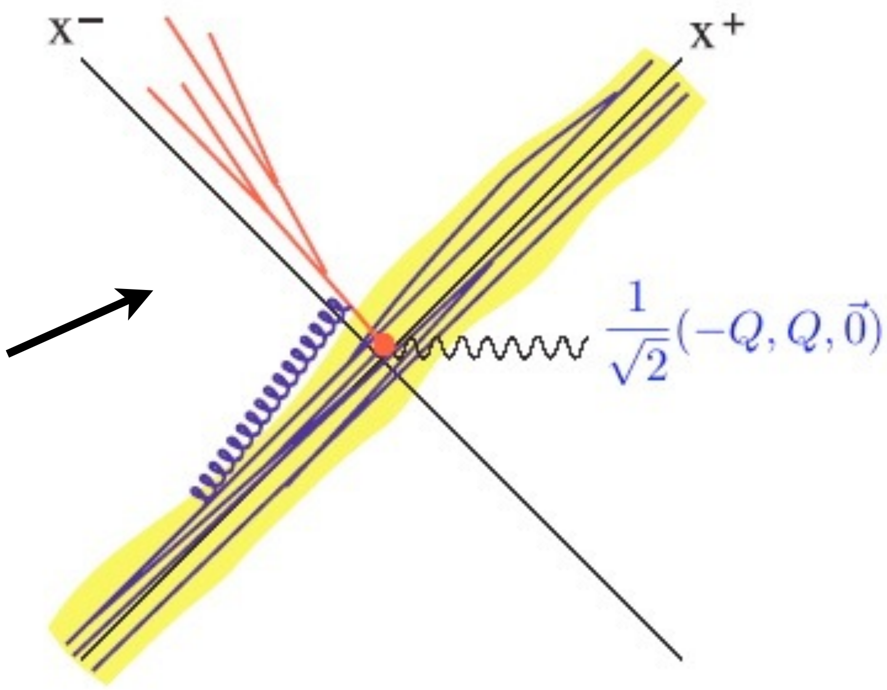
$\Delta x^+ \sim 1/m \times Q/m = Q/m^2$ LARGE

$\Delta x^- \sim 1/m \times m/Q = 1/Q$, SMALL

A look from the Breit frame

And now let the virtual photon hit the fast moving hadron:

Struck quark kicked into the x- direction



Moving hadron has:

Δx+~Q/m2,

interaction with photon q-~Q is localized within

Δx+ ~ 1/Q,

thus quarks and gluons are like partons and effectively free.

In this frame the time scale of a typical parton-parton interaction is much larger than the hard interaction time.

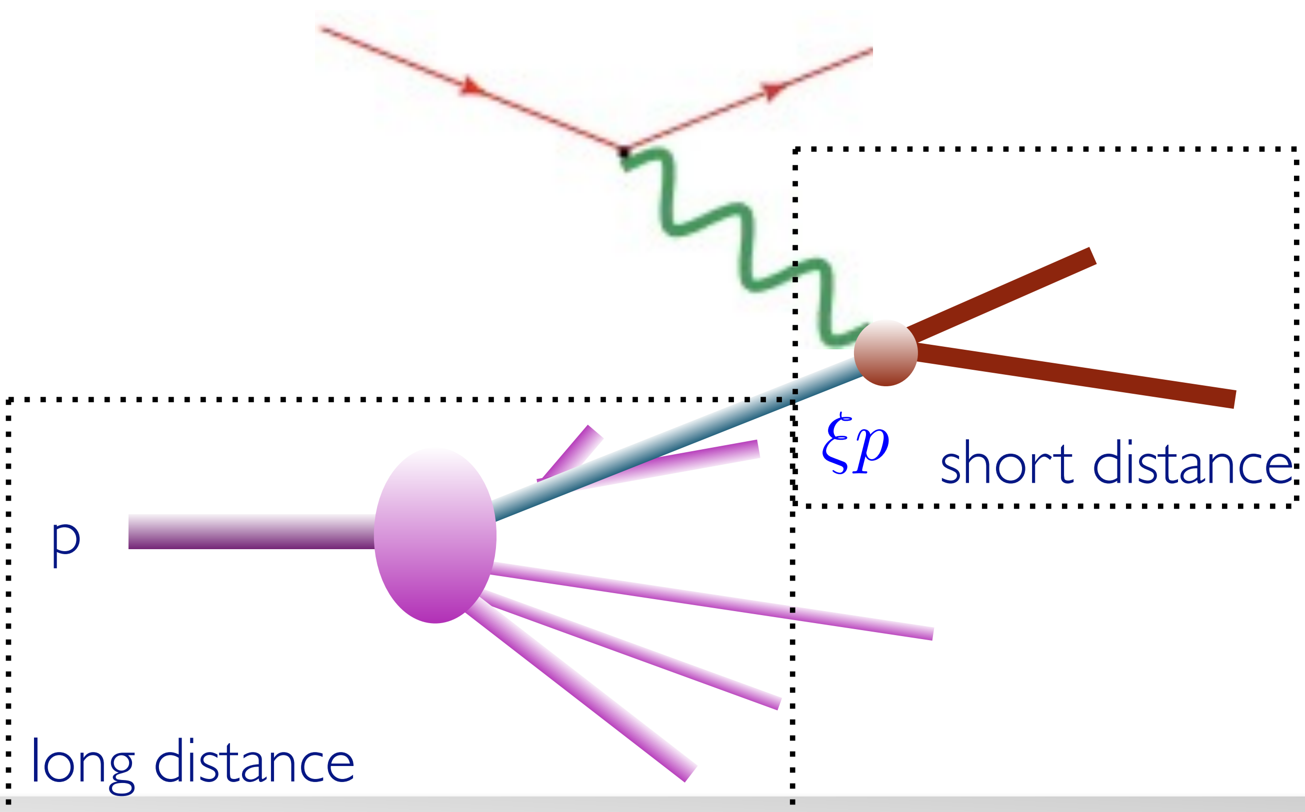
So we can picture the hadron as an incoherent flux of partons (p+,p-,p⊥)i , each carrying a fraction 0<ξi = pi+/p+<1 of the total available momentum.

DIS: The parton model

The space-time picture suggests the possibility of separating short- and long-distance physics ⇒ factorization! Turned into the language of Feynman diagrams DIS looks like:

$$\frac{d^2\sigma}{dx dQ^2} = \int_0^1 \frac{d\xi}{\xi} \sum_i f_i(\xi) \frac{d^2\hat{\sigma}}{dx dQ^2}\left(\frac{x}{\xi}, Q^2\right)$$

where



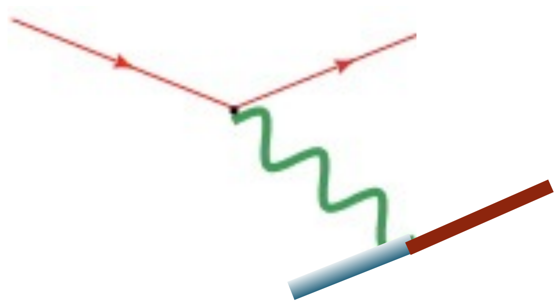
$f_{i/h}(\xi)$ is the probability to find a parton with flavor i in an hadron h carrying a light-cone momentum ξp+

$$\frac{d^2\hat{\sigma}}{dx dQ^2}$$ is the cross section for electron-parton scattering

DIS: The parton model

We can now explain scaling within the parton model:

Let’s take the LO computation we performed for e+e- → qq, cross it (which also mean to be careful with color), and use it the DIS variables to express the differential cross section in dQ2



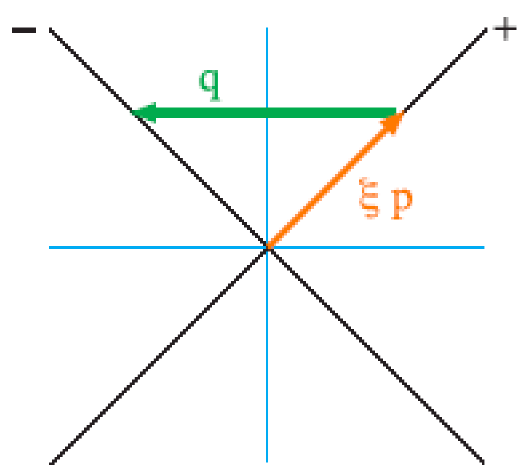
$$\frac{d\hat{\sigma}}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1+(1-y)^2\right]$$

Notice that the outgoing quark is on its mass shell:

$$\xi p^+ + q^+ = 0$$

$$p^+ = Q/(x\sqrt{2})$$

$$q^+ = -Q/\sqrt{2}$$



$$\frac{d^2\hat{\sigma}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right]\delta(x-\xi)$$

This implies that $\xi = x$ at LO!

DIS: The parton model

We can now compare with our "inclusive" description of DIS in terms of structure functions (which, BTW, are physical measurable quantities),

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{[1+(1-y)^2]F_1(x,Q^2) + \frac{1-y}{x}\left[F_2(x,Q^2) - 2xF_1(x,Q^2)\right]\right\}$$

with our parton model formulas:

with $\frac{d^2\hat{\sigma}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{1}{2}\left[1+(1-y)^2\right] e_q^2\, \delta(x-\xi)$

we find (be careful to distinguish x and ξ)

* So we find the scaling is true: no dependence on Q2.
* q and qbar enter together : no way to distinguish them with NC. Charged currents are needed.
* FL(x) = F2(x) - 2 F1(x) vanishes at LO (Callan-Gross relation), which is a test that quarks are spin 1/2 particles! In fact if the quarks where scalars we would have had F1(x) = 0 and F2=FL .

DIS: The parton model

Probed at scale Q, sea contains all quarks flavours with mq less than Q.
For Q ~1 we expect

$$\begin{aligned} u(x) &= u_V(x) + \bar{u}(x) \\ d(x) &= d_V(x) + \bar{d}(x) \\ s(x) &= \bar{s}(x) \end{aligned}$$

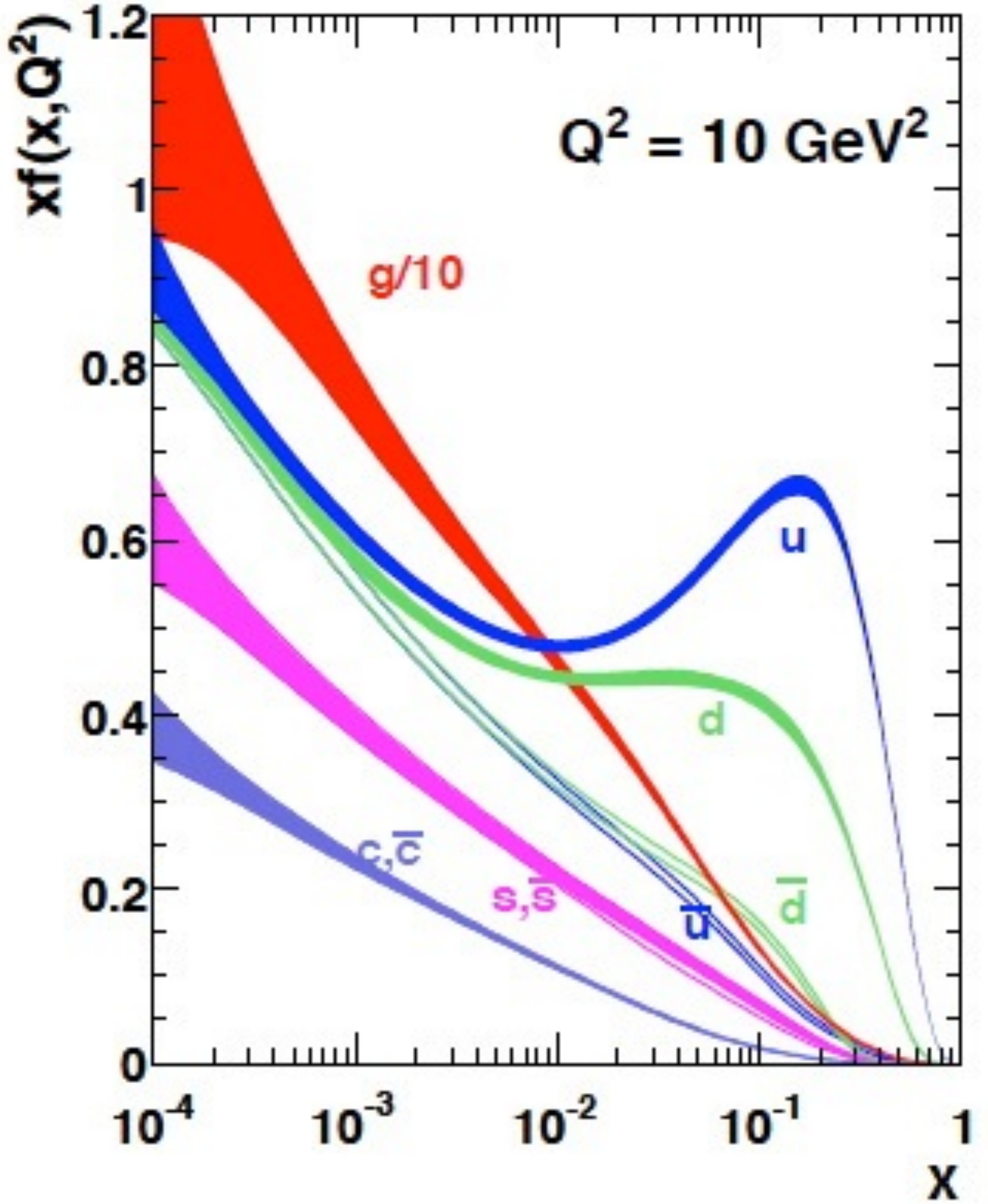
$$\int_0^1 dx\, u_V(x) = 2\,, \quad \int_0^1 dx\, d_V(x) = 1\,.$$

And experimentally one finds

$$\sum_q \int_0^1 dx\, x[q(x) + \bar{q}(x)] \simeq 0.5\,.$$

Thus quarks carry only about 50% of proton's momentum. The rest is carried by gluons. Although not directly measured in DIS, gluons participate in other hard scattering processes such as large-pt and prompt photon production.

Quark and gluon distribution functions



Comments:

The sea is NOT SU(3) flavor symmetric.

The gluon is huge at small x

There is an asymmetry between the ubar and dbar quarks in the sea.

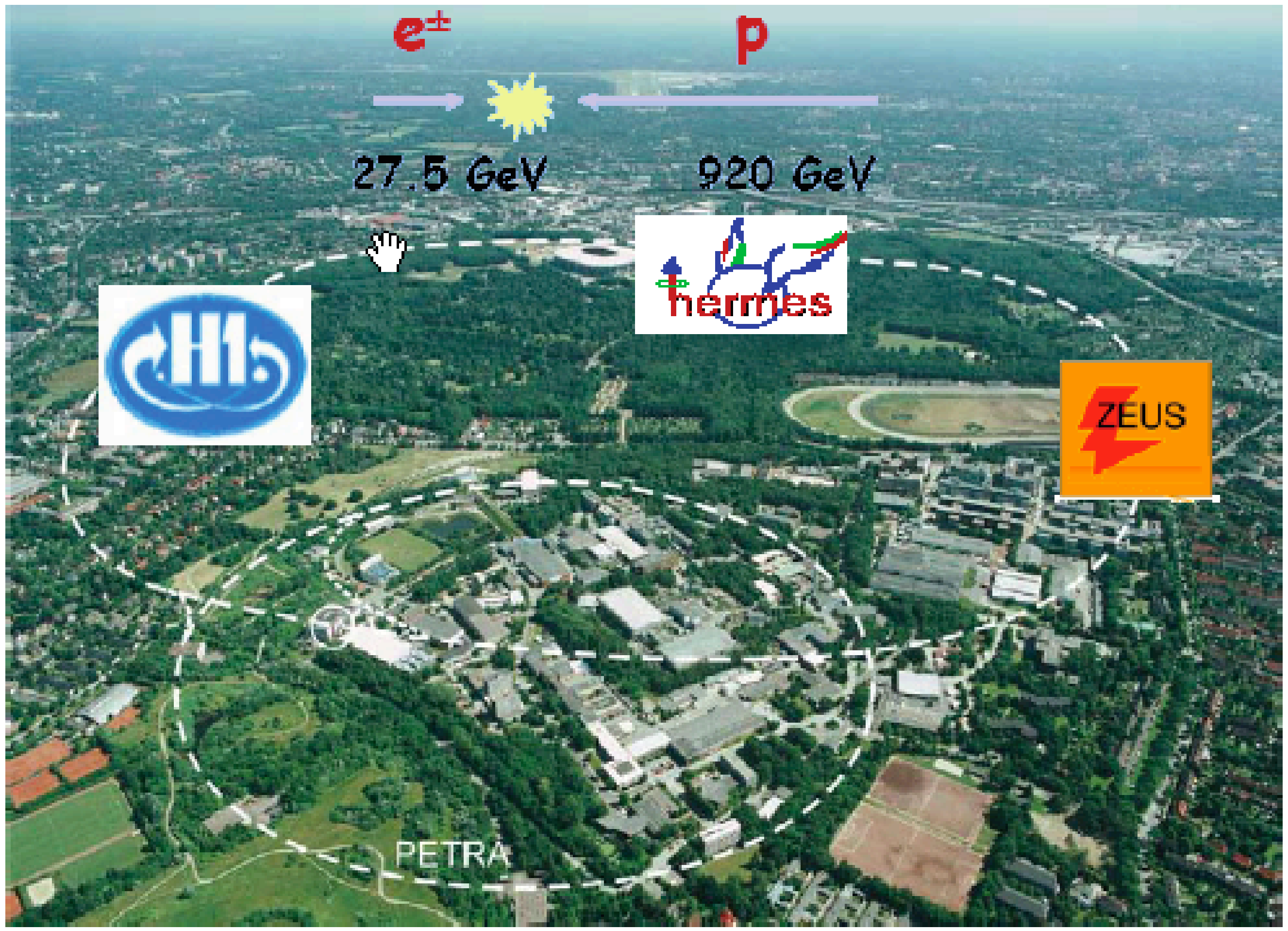
Note that there are uncertainty bands!!

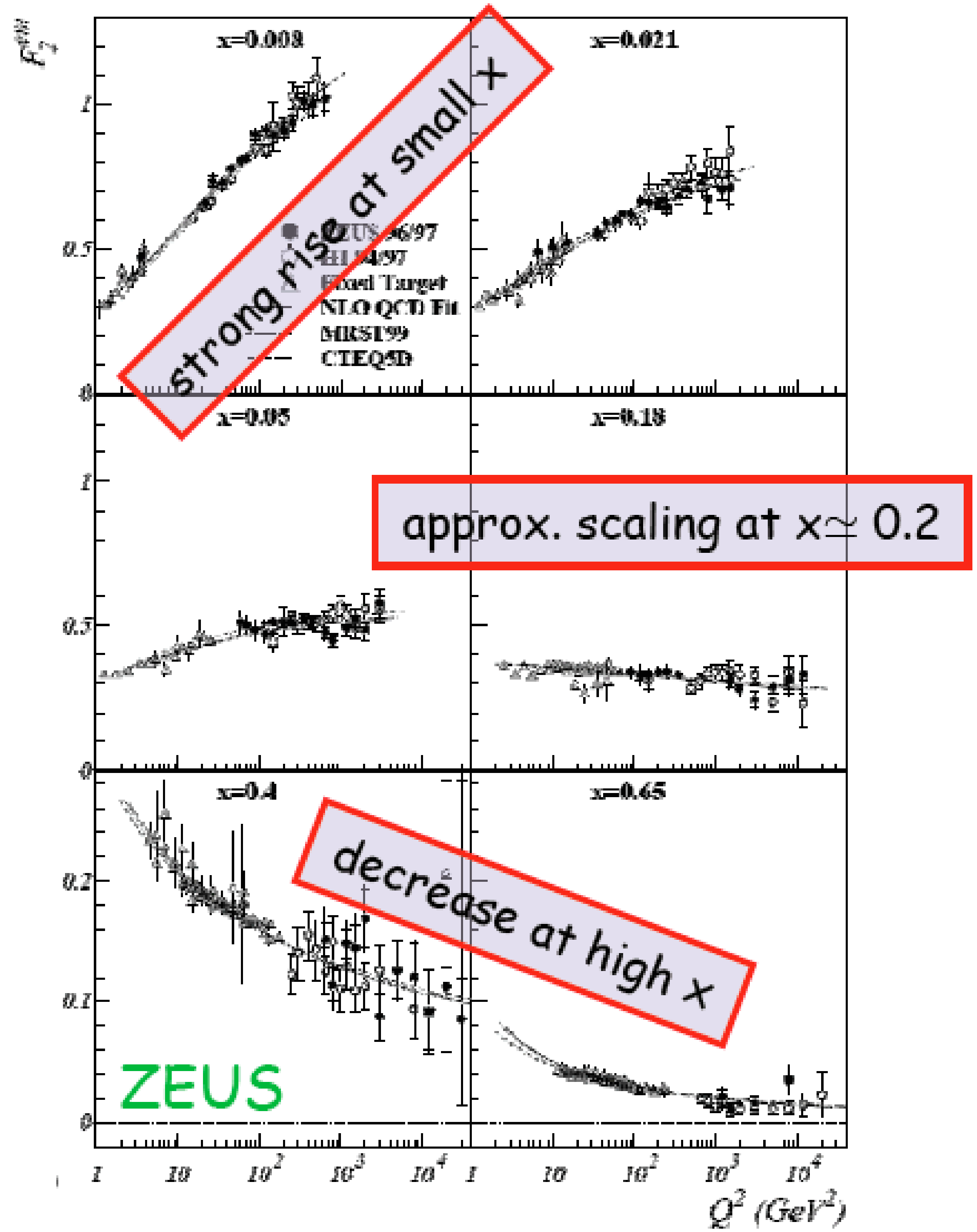
Questions:

1. What has QCD to say about the naïve parton model?
2. Is the picture unchanged when higher order corrections are included?
3. Is scaling exact?

Scaling violations

first ep collider



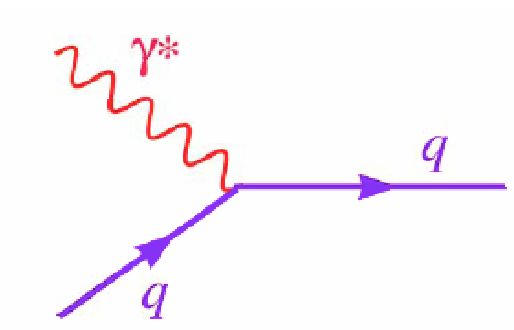


At HERA scaling violations were observed!

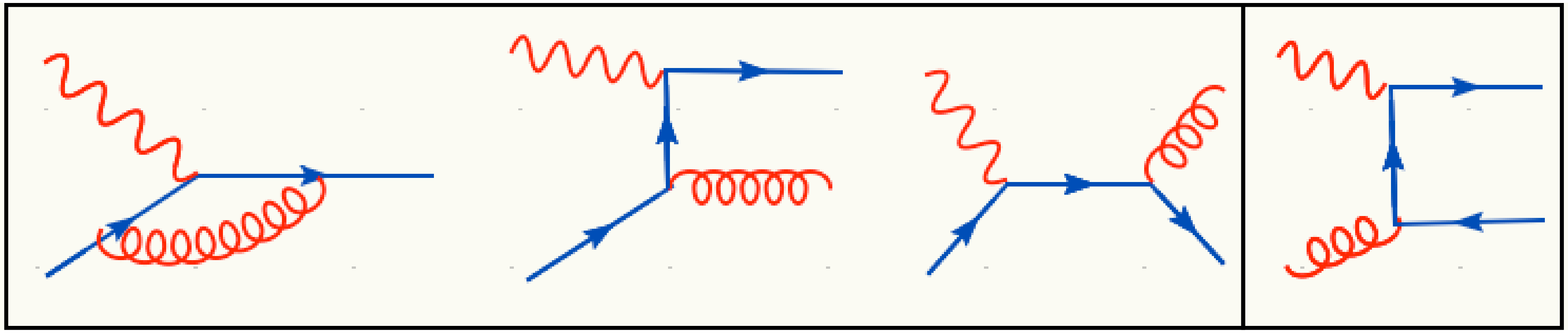
DIS in QCD

We got a long way without even invoking QCD. Let's do it now.

The first diagram to consider is the same as in the parton model:



At NLO we find again both real and virtual corrections:



α_S corrections to the LO process photon-gluon fusion

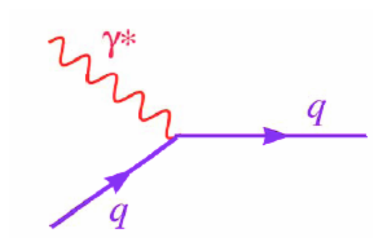
Our experience so far: have to expect IR divergences!
In order to make the intermediate steps of the calculation finite, we introduce a regulator, which will be removed at the end.

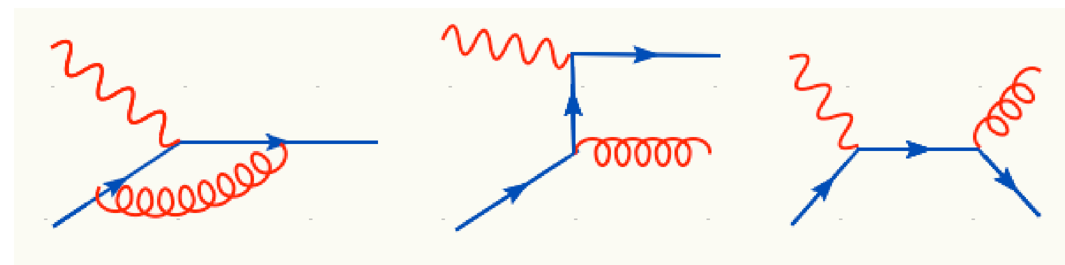
Dimensional regularization is the best choice to perform serious calculations.
However for illustrative purposes other regulators (that cannot be easily used beyond NLO) are better suited. We'll use here a small quark/gluon mass.

DIS in QCD

Once we compute the diagrams we indeed find that UV and soft divergences all cancel, but for a collinear divergence arising when the emitted gluon becomes collinear to the incoming quark:

$$\frac{d^2\hat{\sigma}}{dxdQ^2}\Big|_{F_2} \equiv \hat{F}_2^q$$



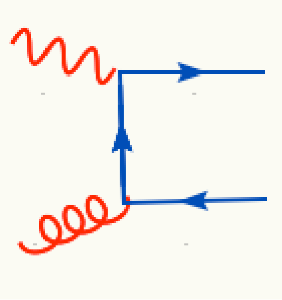


$$= e_q^2 x \left[\delta(1-x) + \frac{\alpha_S}{4\pi}\left[P_{qq}(x)\log\frac{Q^2}{m_g^2} + C_2^q(x)\right]\right]$$

$$\frac{d^2\hat{\sigma}}{dxdQ^2}\Big|_{F_2} \equiv \hat{F}_2^g$$

IR cutoff

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_S}{4\pi}\left[P_{qg}(x)\log\frac{Q^2}{m_q^2} + C_2^g(x)\right]\right]$$



The presence of large logs is a clear sign that we have a residual infrared sensitivity that we have to deal with!

DIS in QCD

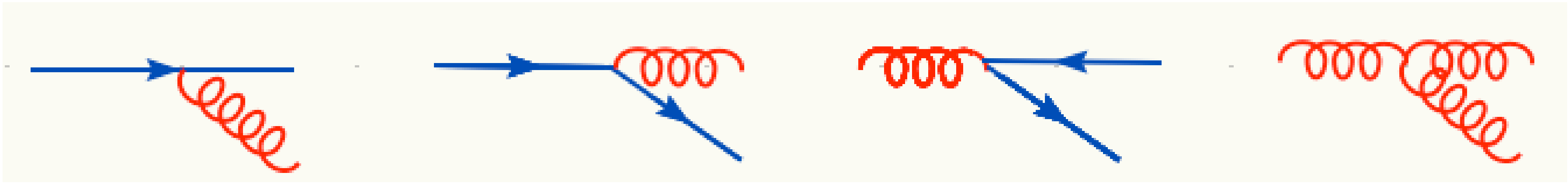
Important observations:

1. Large logarithms of Q2/m2 or (1/ε in dim reg) incorporate ALL the RESIDUAL long-distance physics left after summing over all real and virtual diagram. This terms are of a collinear nature.

2. The coefficients Pij(x) that multiply the log's are UNIVERSAL and calculable in perturbative QCD.

They are called SPLITTING FUNCTIONS and their physical meaning is easy to give:

Pij(x) give the probability that a parton j splits collinearly into a parton i + something else carrying a momentum fraction x of the original parton j.



Pqq(x)   Pgq(x)   Pqg(x)   Pgg(x)

DIS in QCD

So the natural question is: what is it that is going wrong? Do we have IR sensitiveness in a physical observable? Well not yet!!

To obtain the physical cross section we have to convolute our partonic results with the parton densities, as we have learned from the parton model.

For instance:

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \left[f_{i,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{i,0}(\xi) \left[P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{m_g^2} + C_2^q(\frac{x}{\xi}) \right] \right]$$

And now comes the magic: as long as the divergences are universal and do not depend on the hard scattering functions but only on the partons involved in the splitting, we can reabsorb the dependence on the IR cutoff (once for all!) into fq,0(x):

$$f_q(x,\mu_f) \equiv f_{q,0}(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) P_{qq}(\frac{x}{\xi}) \log \frac{\mu_f^2}{m_g^2} + z_{qq}$$

“Renormalized” parton densities: we have factorized the IR collinear physics into a quantity that we cannot calculate but it is universal. So how does the final result looks like?

Factorization

The structure function is a MEASURABLE object, therefore, at all orders, it cannot depend on the choice of scales.
This will lead exactly to the same concepts of renormalization group invariance that we found in the UV.

The final result depends of course also on α_S and therefore to the choice of the renormalization scale.

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[\underline{P_{qq}(\frac{x}{\xi}) \log\frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi})}\right]\right]$$

Long distance physics is universally factorized into the parton distribution functions. These cannot be calculated in pQCD. They depend on μ_f in the exact way so as to cancel the overall dependence at all orders.

Short-distance (Wilson coefficient), perturbative calculable and finite. It depends on the factorization scale. It also depends on finite terms which define the factorization scheme.

Factorization

$$F_2^q(x,Q^2) = x \sum_{i=q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} f_i(\xi,\mu_f^2) \left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_S(\mu_r)}{2\pi} \left[P_{qq}(\frac{x}{\xi}) \log \frac{Q^2}{\mu_f^2} + (C_2^q - z_{qq})(\frac{x}{\xi}) \right] \right]$$

Questions:

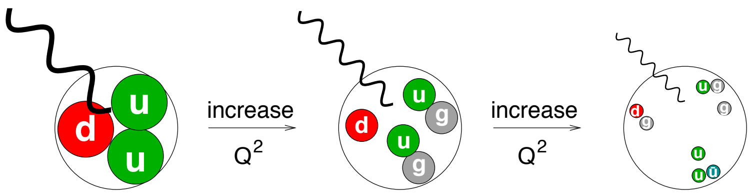
1. Can we exploit the fact that physical quantities have to be scale independent to gain information on the pdfs?

2. What exactly have we gained in hiding the large logs in the redefined pdf's? Aren't we just hiding the problem?

QCD in the initial state

1. DIS: from the parton model to pQCD
2. Q^2 Evolution and PDF's
3. pp collisions : a glimpse

Evolution



$$F_2(x,Q^2) \sim \sum_i f_i(x,\mu_f) \otimes \hat{F}_2(x,\frac{Q}{\mu_f})$$

As a first step it is very convenient to transform the nasty convolution into a simple product. This can be done with the help of a Mellin transform:

$$f(N) \equiv \int_0^1 dx x^{N-1} f(x)$$

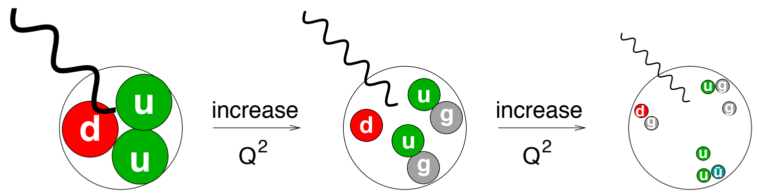
small/large x ⇔small/large N

Let us show that a Mellin transform turns a convolution into a simple product:

$$\int_0^1 dx x^{N-1} \left[\int_x^1 \frac{dy}{y} f(y) g(\frac{x}{y}) \right] = \int_0^1 dx x^{N-1} \int_0^1 dy \int_0^1 dz \delta(x-zy) f(y) g(z)$$

$$= \int_0^1 dy \int_0^1 dz (zy)^{N-1} f(y) g(z) = f(N) g(N)$$

Evolution



$$F_2(x,Q^2) \sim \sum_i f_i(x,\mu_f) \otimes \hat{F}_2(x,\frac{Q}{\mu_f})$$

Let's now apply it to F2

$$\frac{dF_2(x,Q^2)}{d\log\mu_f} = 0$$

we get:

$$\frac{dq(N,\mu_f)}{d\log\mu_f}\hat{F}_2(N,\frac{\mu_f}{Q}) + q(N,\mu_f)\frac{d\hat{F}_2(N,\frac{\mu_f}{Q})}{d\log\mu_f} = 0$$

$$\frac{d\log\hat{F}_2(N,\frac{Q}{\mu_f})}{d\log\frac{Q}{\mu_f}} = \frac{d\log q(N,\mu_f)}{d\log\mu_f} = k$$

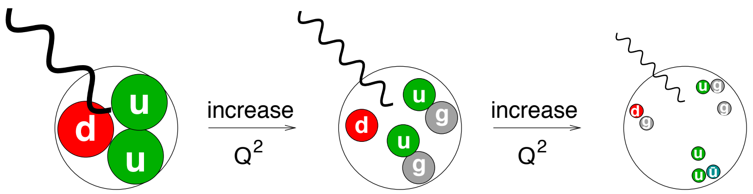
These are called anomalous dimensions and are just the Mellin transform of the corresponding splitting function

whose solution is:

$$q(N,\mu) = q(N,\mu_0)e^{k\log(\frac{\mu_f}{\mu_0})}$$

The pdf "evolves" with the scale!

Evolution



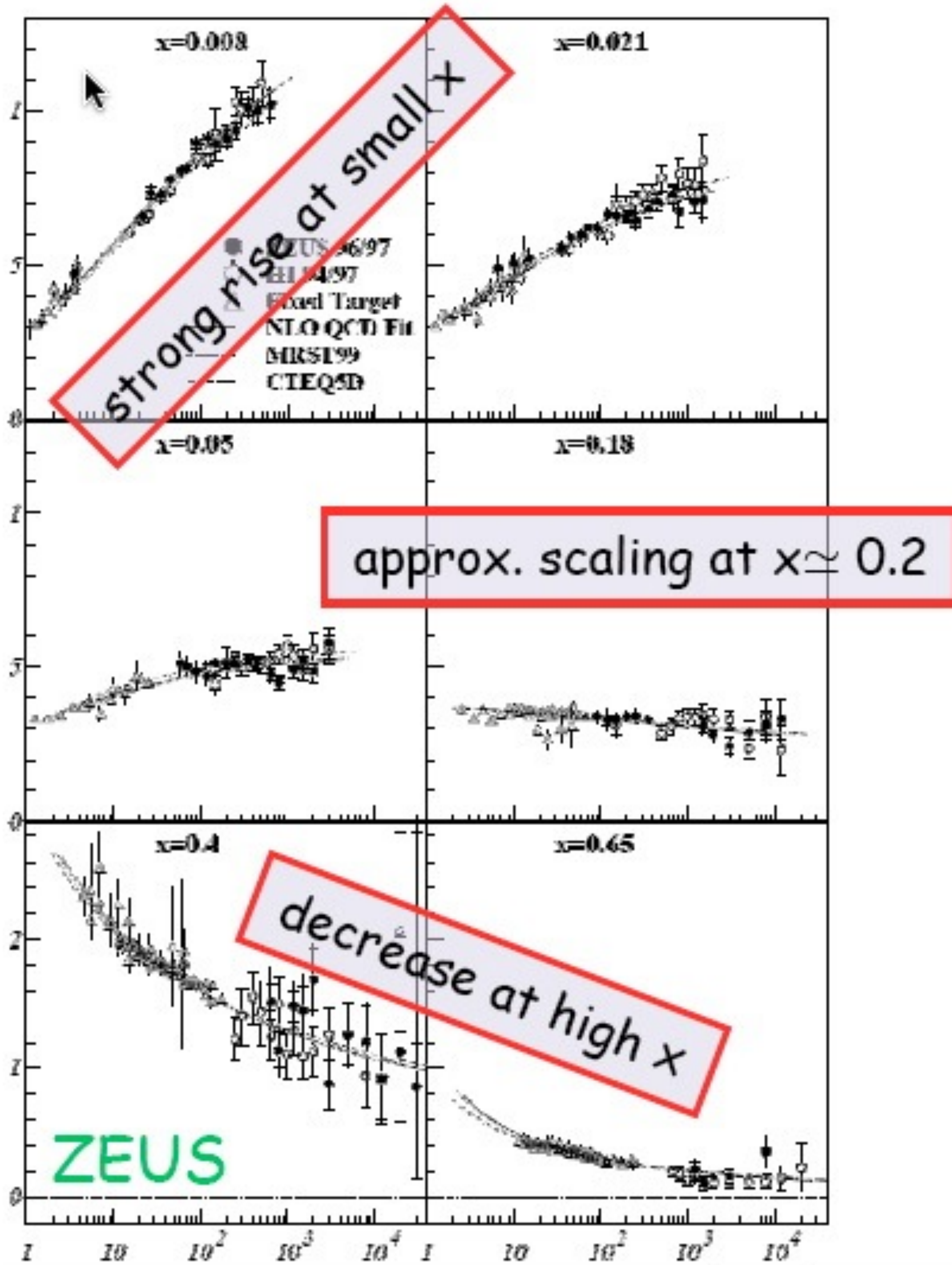
The solution for V can be rewritten in terms of t and αS as follows:

$$q(N,t) = q(N,t_0)\left(\frac{\alpha_S(t_0)}{\alpha_S(t)}\right)^{d_{qq}(N)}$$

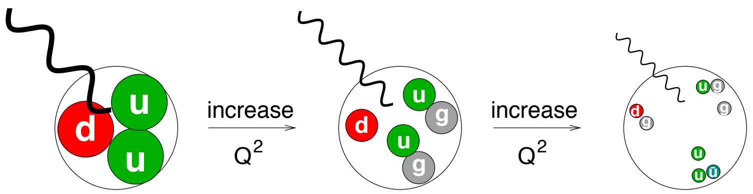
where

$$d_{qq}(N) = \gamma_{qq}^{(0)}/2\pi b_0$$

Now dqq(1)=0 and dqq(N) <0 for N>1. Thus as t increases V decreases at large x and increases at small x. Physically this is due to an increase in the phase space for gluon emission by quarks as t increases, leading to a loss of momentum.



Evolution



In fact the equations are a bit more complicated as quarks and gluons do mix.
It is convenient to introduce two linear combinations, the singlet **Σ** and the non-singlet q^{NS} to separate the piece that mixes with that that does not:

$$\Sigma(x,Q^2) = \sum_{i=1}^{n_f} (q_i(x,Q^2) + \bar{q}_i(x,Q^2))$$

this is coupled to the gluon

$$q^{\mathrm{NS}}(x,Q^2) = q_i(x,Q^2) - \bar{q}_j(x,Q^2)$$

these evolve independently

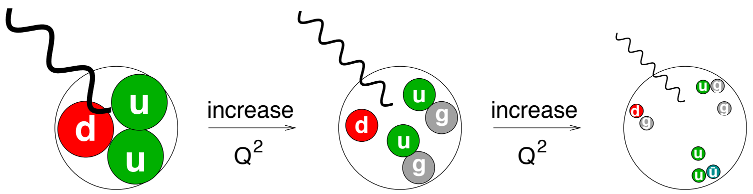
The complete evolution equations (in Mellin space) to solve are:

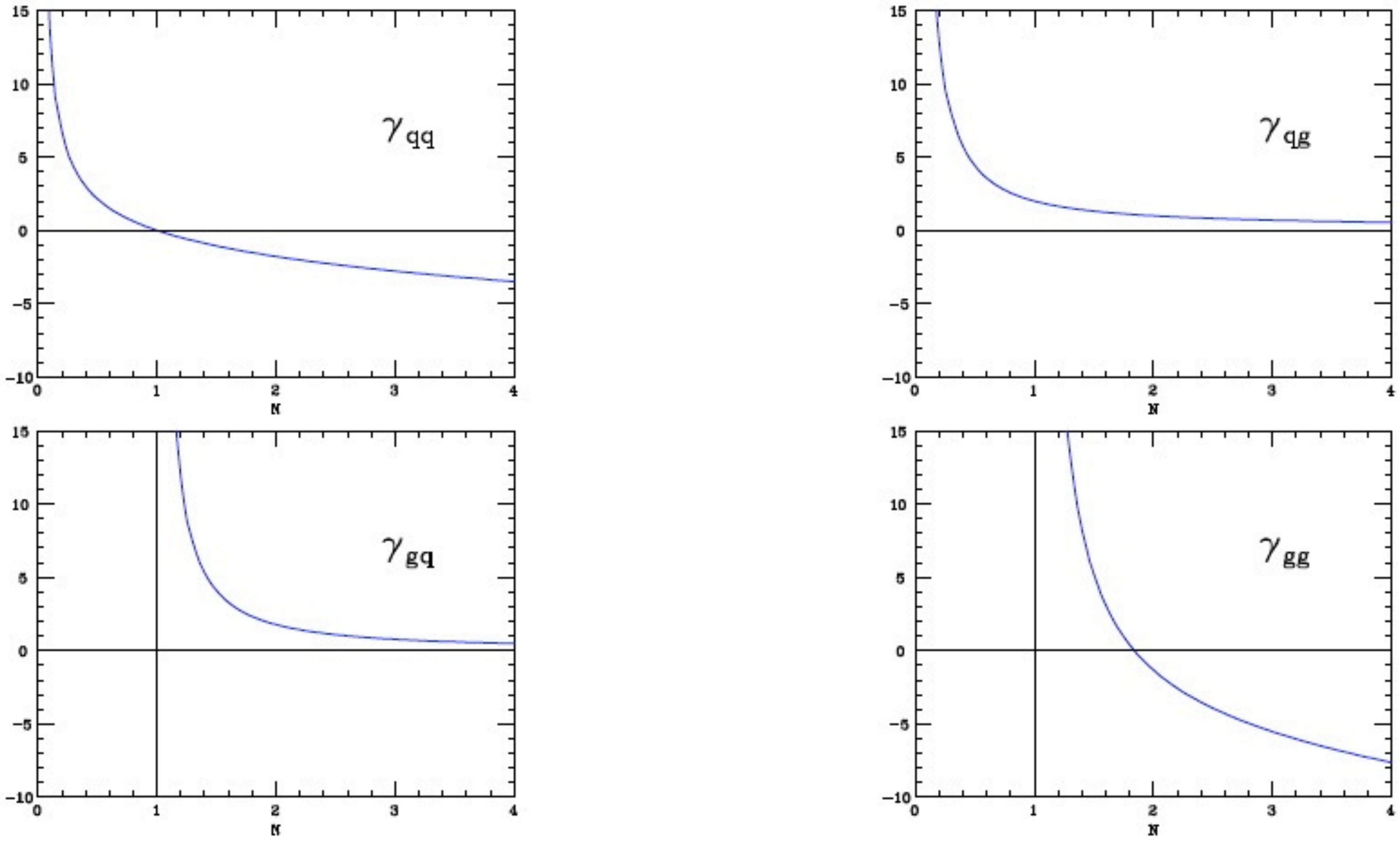
$$\frac{d}{dt}\Delta q^{\mathrm{NS}}(N,Q^2) = \frac{\alpha_S(t)}{2\pi}\gamma_{qq}^{\mathrm{NS}}(N,\alpha_S(t))\Delta q^{\mathrm{NS}}(N,Q^2)$$

$$\frac{d}{dt}\begin{pmatrix} \Delta\Sigma(N,Q^2) \\ \Delta g(N,Q^2) \end{pmatrix} = \frac{\alpha_S(t)}{2\pi}\begin{pmatrix} \gamma_{qq}^{\mathrm{S}} & 2n_f\gamma_{qg}^{\mathrm{S}} \\ \gamma_{gq}^{\mathrm{S}} & \gamma_{gg}^{\mathrm{S}} \end{pmatrix}\begin{pmatrix} \Delta\Sigma(N,Q^2) \\ \Delta g(N,Q^2) \end{pmatrix}$$

StefanoForte®

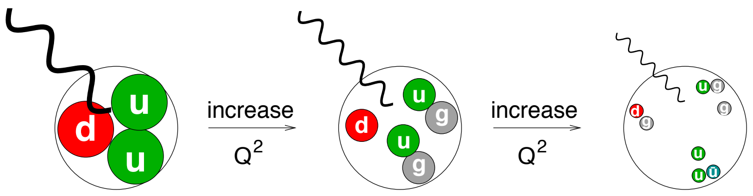
Evolution





- As Q^2 increases, pdf's decrease at large x and increase at small x due to radiation and momentum loss.
- Gluon singularity at N=1 $\Rightarrow$ it grows more at small x.
- $\gamma_{qq}(1)=0$ $\Rightarrow$ number of quarks conserved.

Evolution


d
u
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increase
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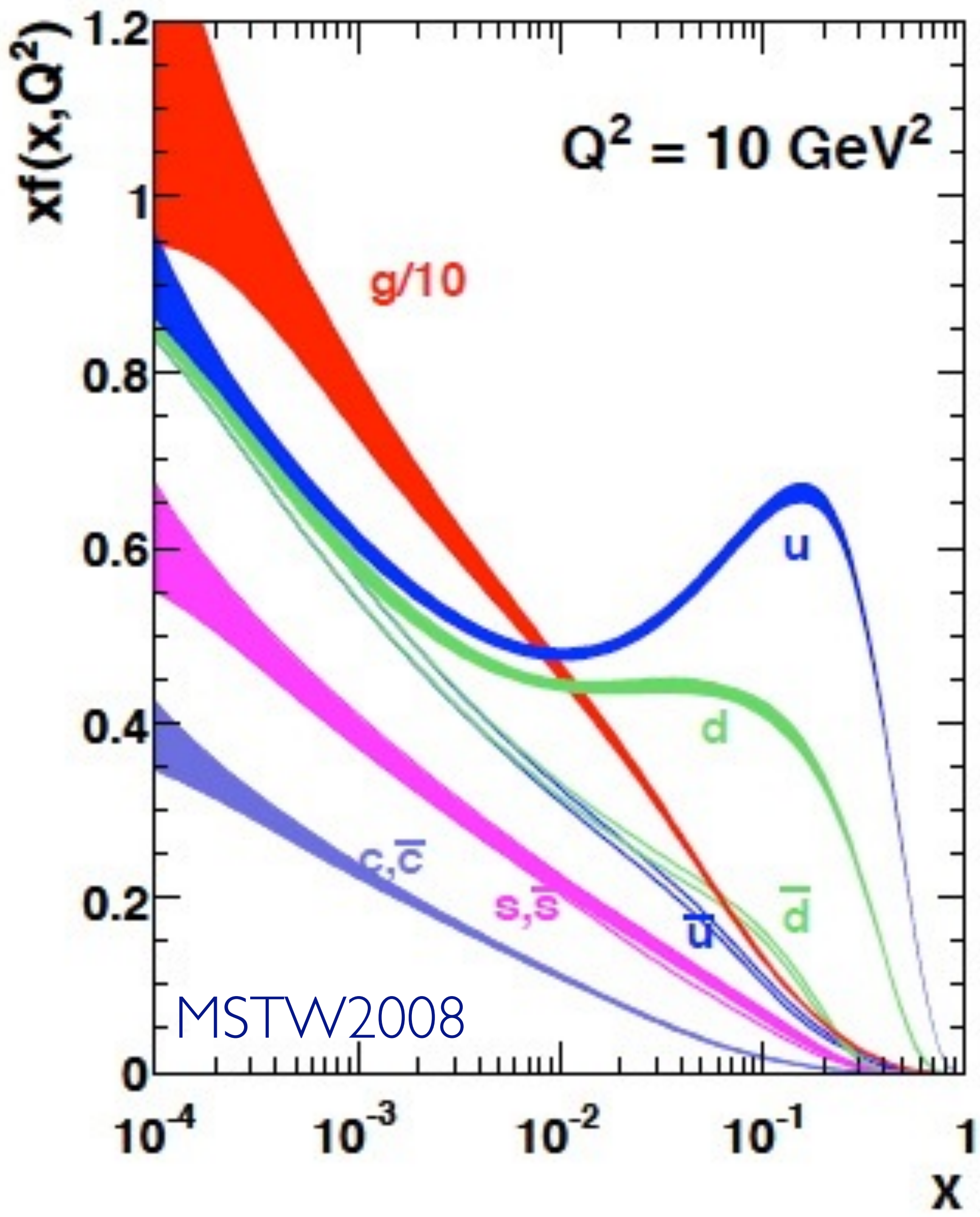
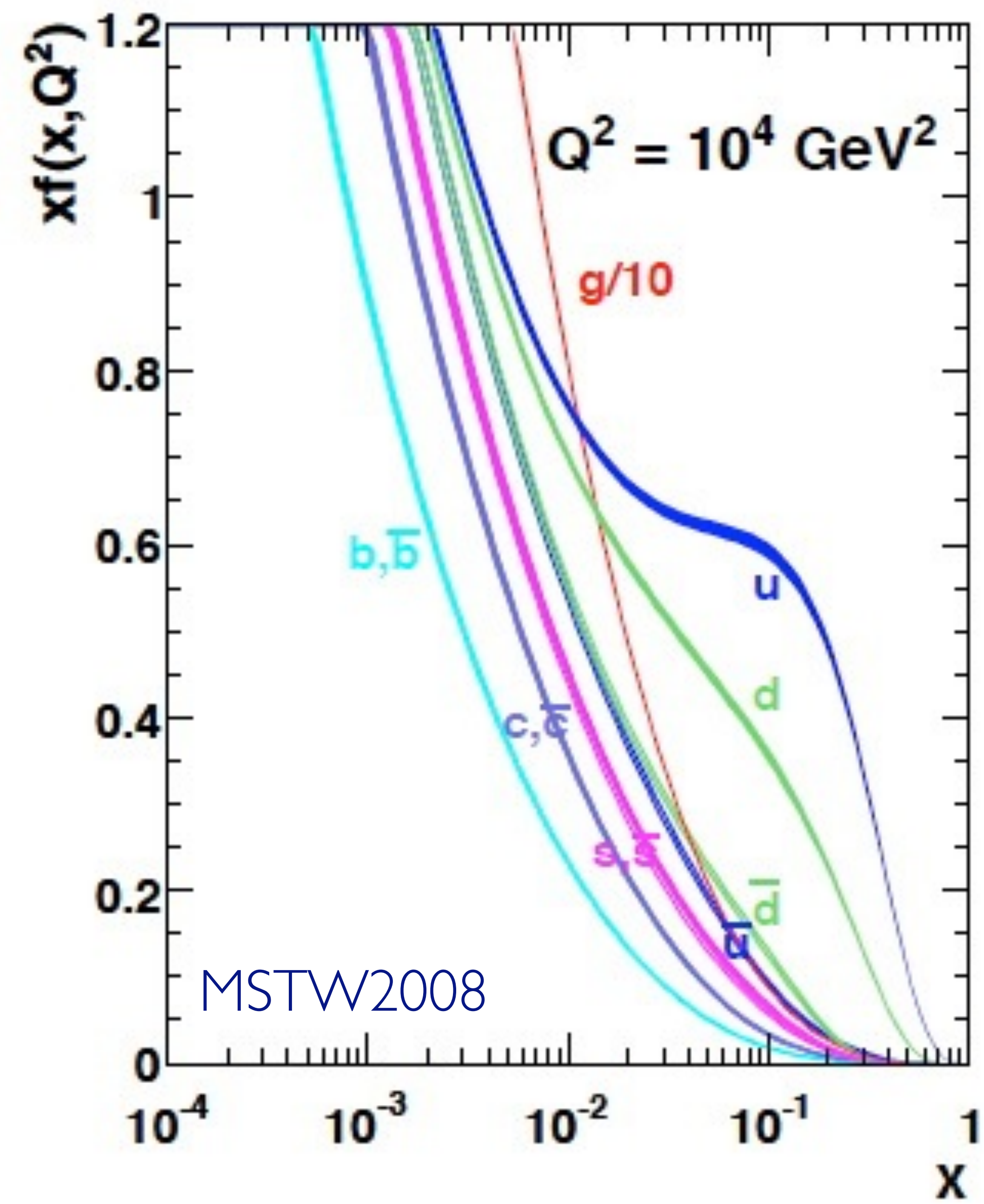
u
g
g
increase
Q^2


$xf(x,Q^2)$
$Q^2 = 10\ \mathrm{GeV}^2$
g/10
u
d
$\bar{d}$
$\bar{u}$
$s,\bar{s}$
$c,\bar{c}$
MSTW2008
0
0.2
0.4
0.6
0.8
1
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x


$xf(x,Q^2)$
$Q^2 = 10^4\ \mathrm{GeV}^2$
g/10
$b,\bar{b}$
u
d
$c,\bar{c}$
$s,\bar{s}$
$\bar{d}$
$\bar{u}$
MSTW2008
0
0.2
0.4
0.6
0.8
1
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Final strategy for QCD predictions

We now have a strategy to get a reliable result in perturbation theory:

1. Calculate the short distance coefficient in pQCD corresponding to an observable. All divergences will cancel except those due to the collinear splitting of initial partons.

2. Re-absorbe such divergences in the pdf's and introduce a factorization scale.

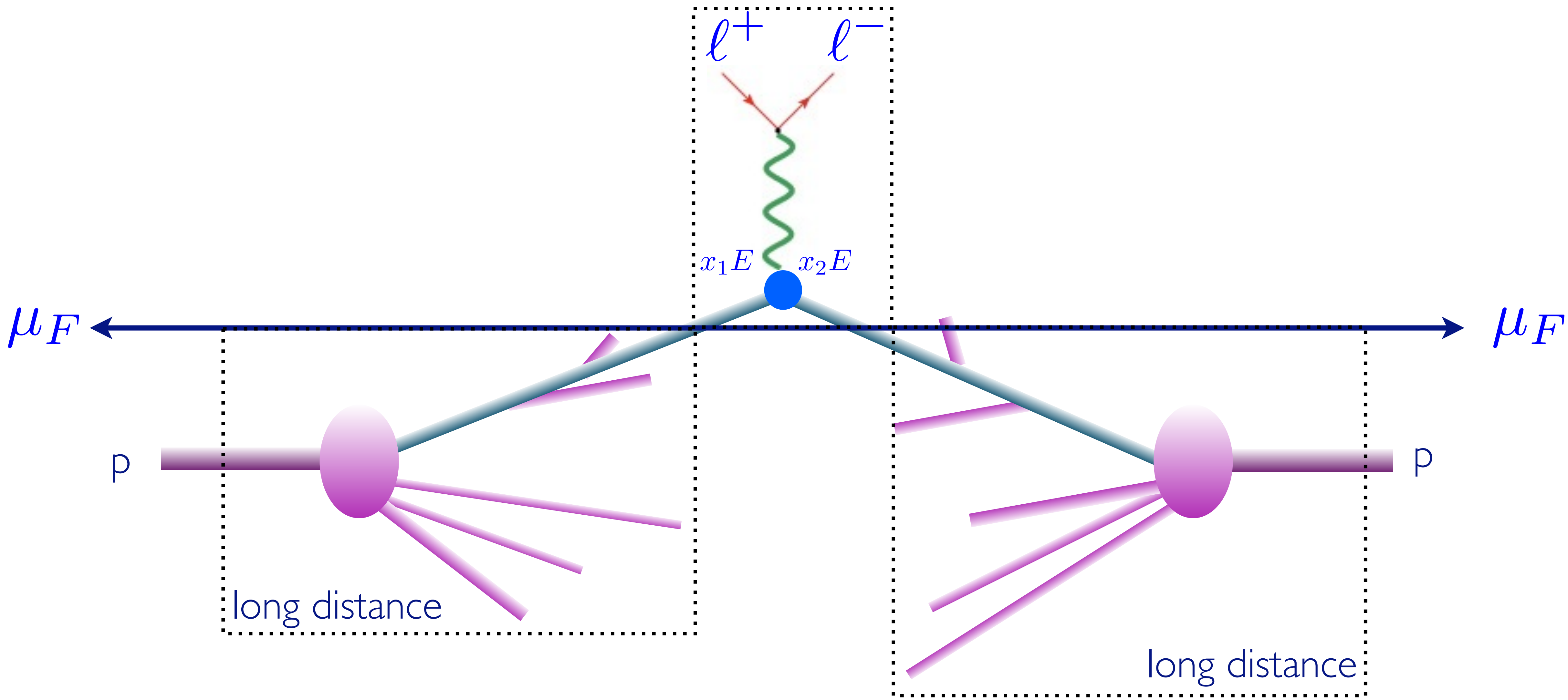
3. Extract from experiment the initial condition for the pdf's at a given reference scale.

4. Evolve the pdf's at the scale of the process we are interested it. In so doing all large logs of the factorization scale over a small scale are resummed.

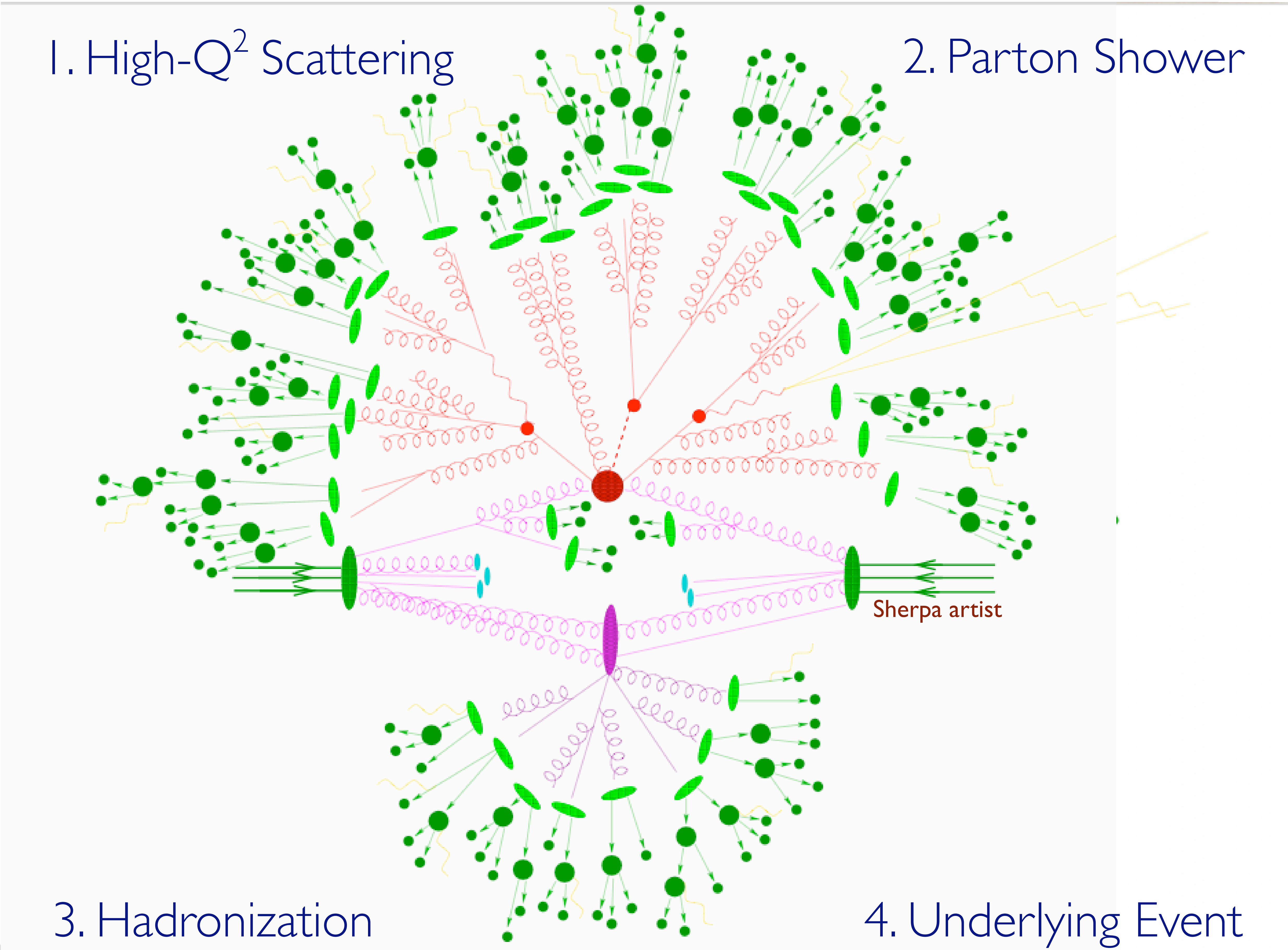
QCD in the initial state

1. DIS: from the parton model to pQCD
2. Q^2 Evolution and PDF's
3. pp collisions : a glimpse

LHC master formula

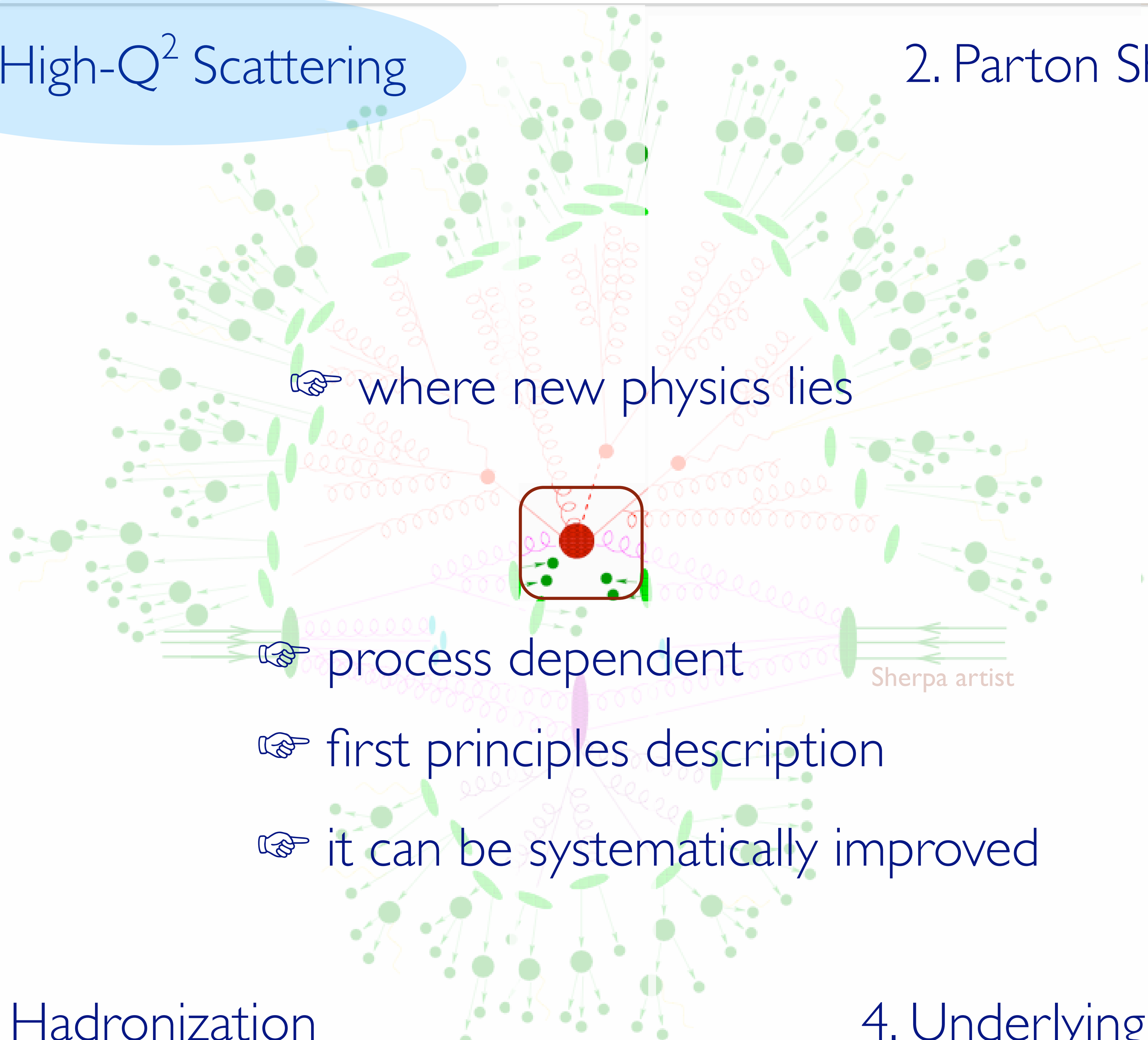


$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$


1. High-Q^2 Scattering
2. Parton Shower
Sherpa artist
3. Hadronization
4. Underlying Event

I. High-Q^2 Scattering

2. Parton Shower



☞ where new physics lies

☞ process dependent

☞ first principles description

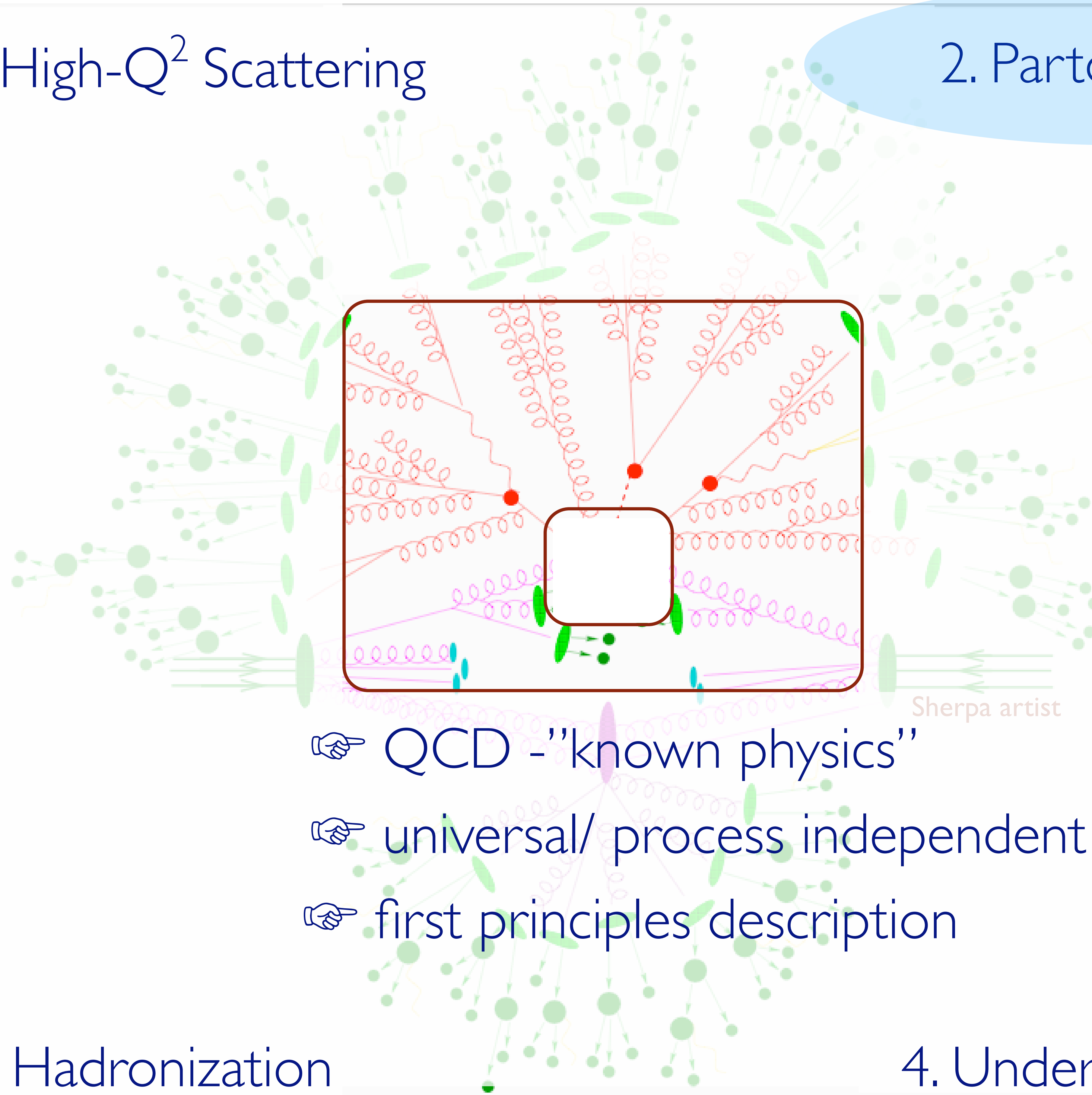
☞ it can be systematically improved

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



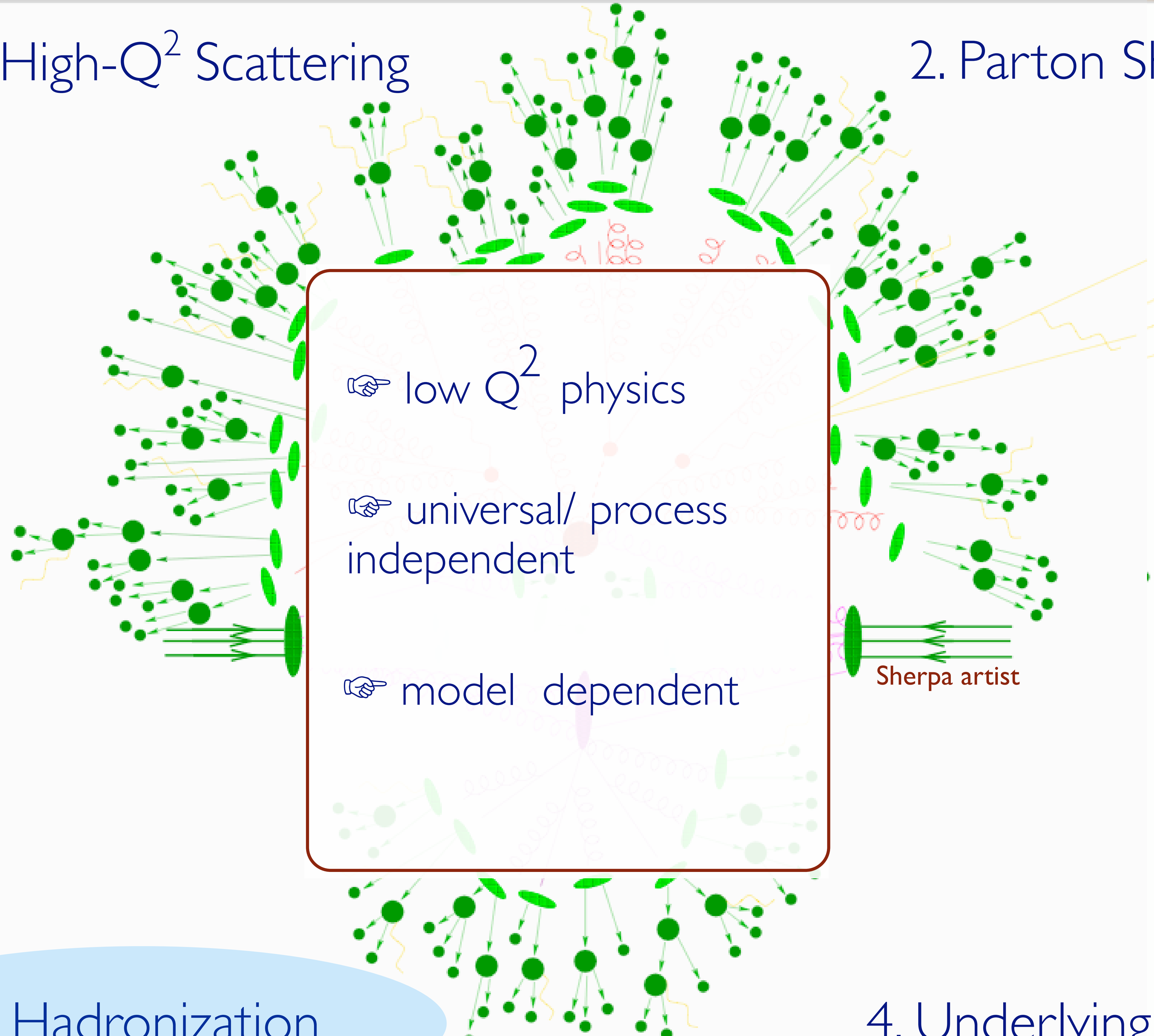
- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



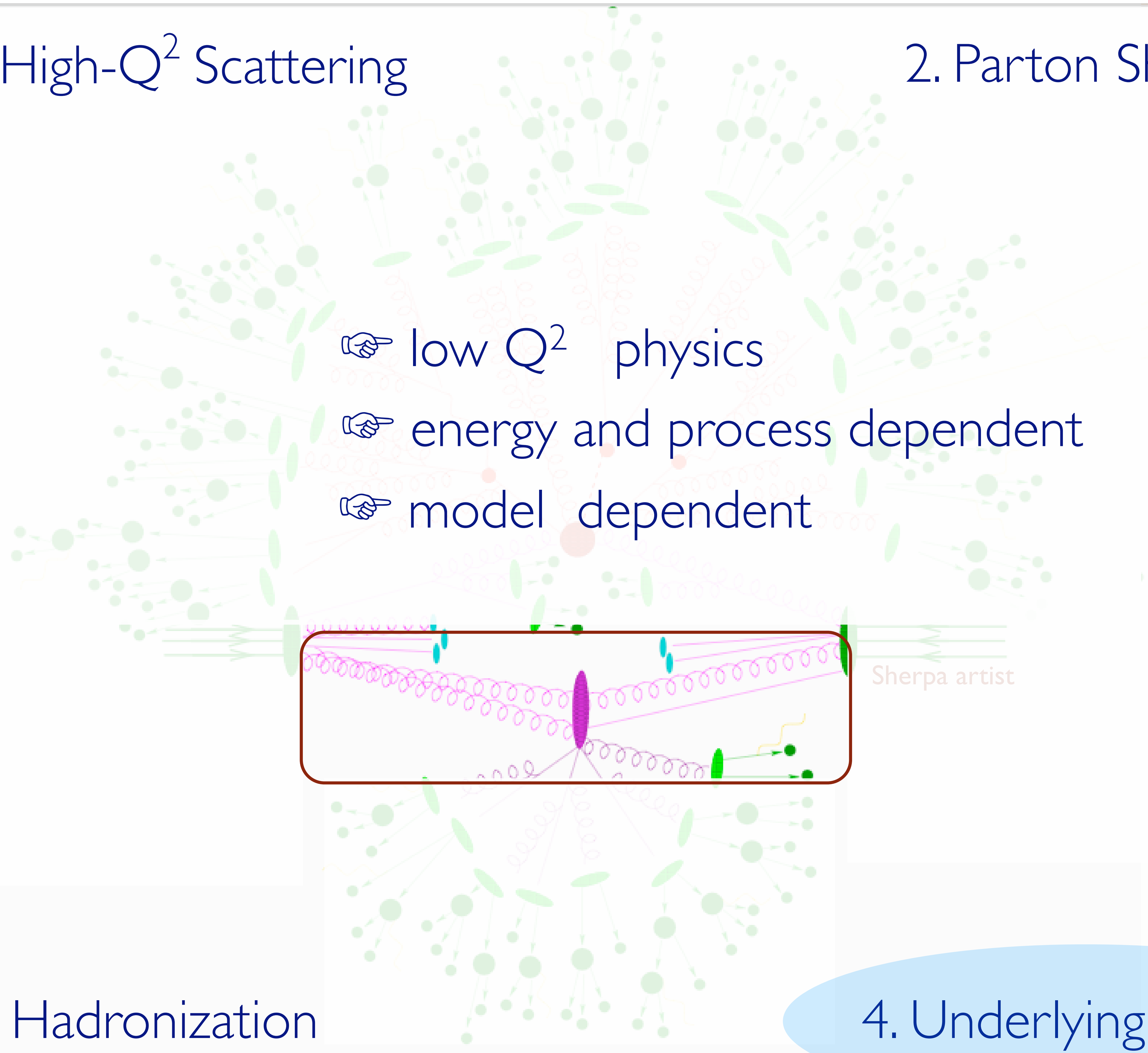
- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model dependent

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

2. Parton Shower



- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model dependent

3. Hadronization

4. Underlying Event

LHC master formula

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1,x_2,\alpha_S(\mu_R^2),\frac{Q^2}{\mu_F^2},\frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution Functions (from exp, but evolution from th).

2. Short distance coefficients as an expansion in α_S (from th).

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

PDFs

Non-perturbative information that is fitted from a wealth of experimental data

- The pdf is parametrised at a given low scale in terms of an analytic or NN function and momentum sum rules are imposed.

- They are evolved through the DGLAP equations:

$$Q^2 \frac{\partial f_a(x,Q^2)}{\partial Q^2} = \int_x^1 \frac{dz}{z} P_{ab}(\alpha_S(Q^2), z) f_b(x/z, Q^2)$$

$$P_{ab}(\alpha_S, z) = \frac{\alpha_S}{2\pi} P_{ab}^{(0)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^2 P_{ab}^{(1)}(z) + \left(\frac{\alpha_S}{2\pi}\right)^3 P_{ab}^{(2)}(z) + \ldots..$$

LO (1974) NLO (1980) NNLO (2004)

PDFs

Global fits: recent progress in methodology and data sets:

- NNPDF3.0 1410.8849
- MMHTCT14 1412.3989
- CT14 1506.07443

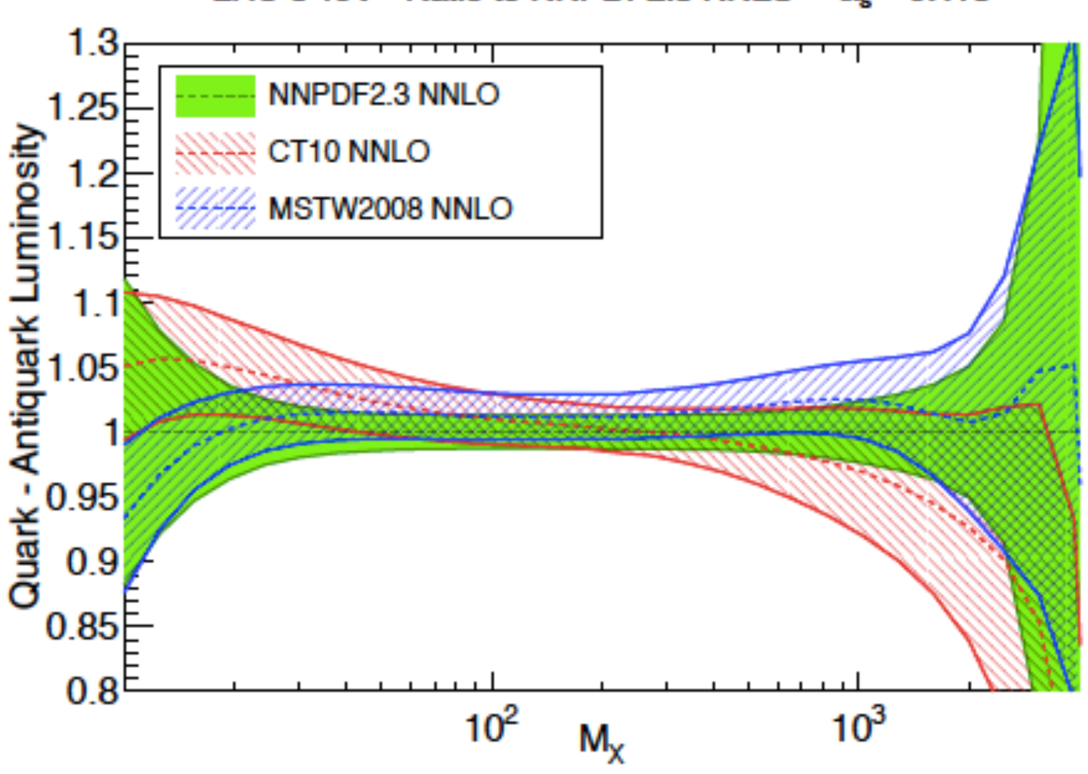
StefanoForte®

	NNPDF3.0	MMHT14	CT14
No. of fitted PDFs	7	7	6
Parametrization	neural nets	$x^a(1-x)^b\times$ Chebyschev	$x^a(1-x)^b\times$Bernstein
Free parameters	259	37	30-35
Uncertainties	Replicas	Hessian	Hessian
Tolerance	None	Dynamical	Dynamical
Closure test	✔	✗	✗
Reweighting	replicas	eigenvectors	eigenvectors

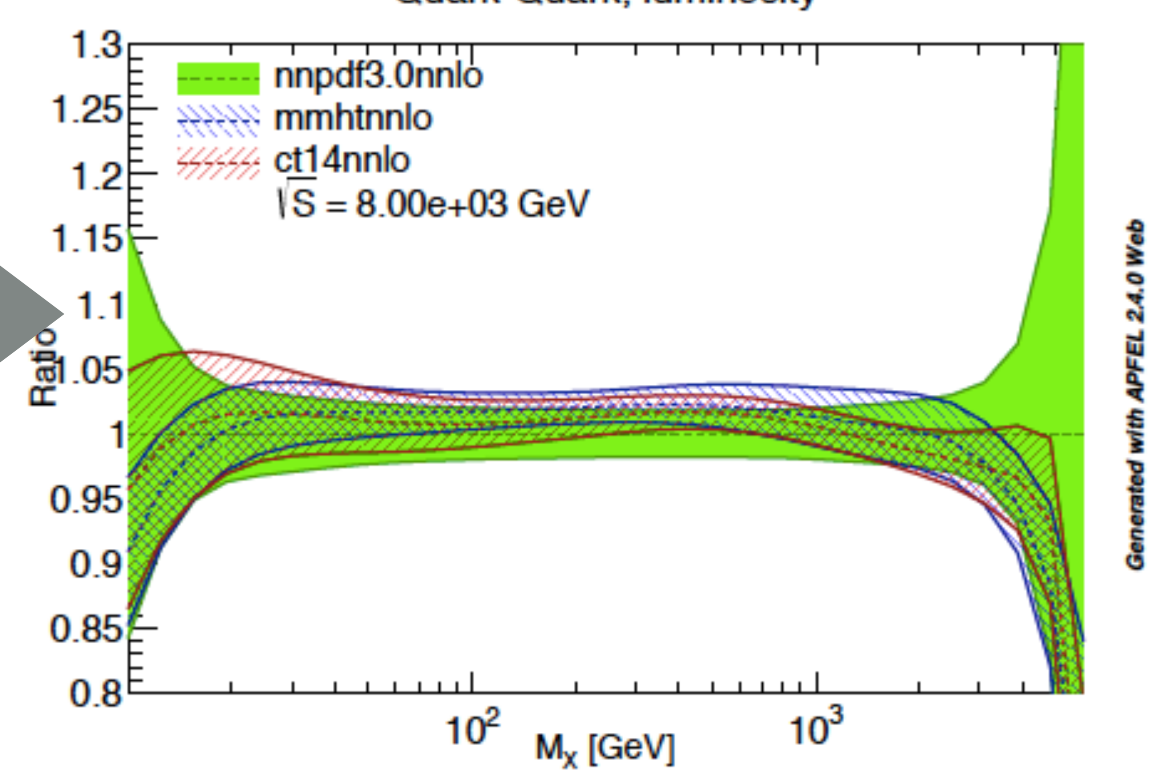
Other non-global sets: HeraPDF, ABM14, GJR

QUARK-QUARK

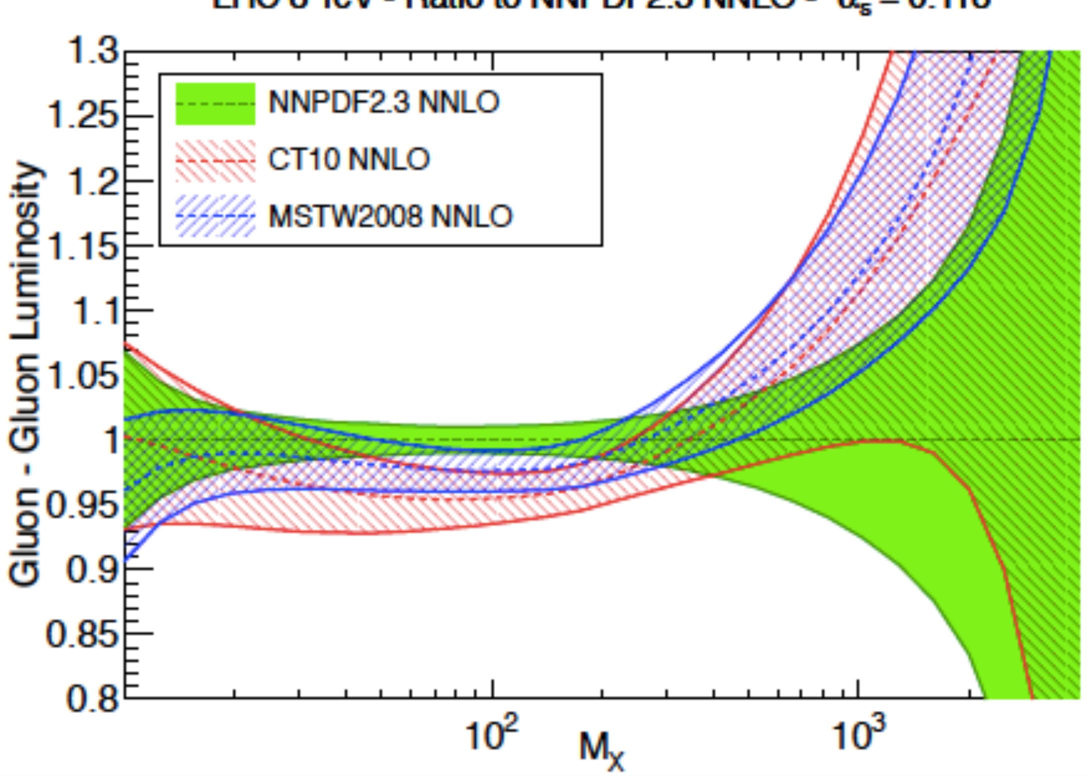
2012

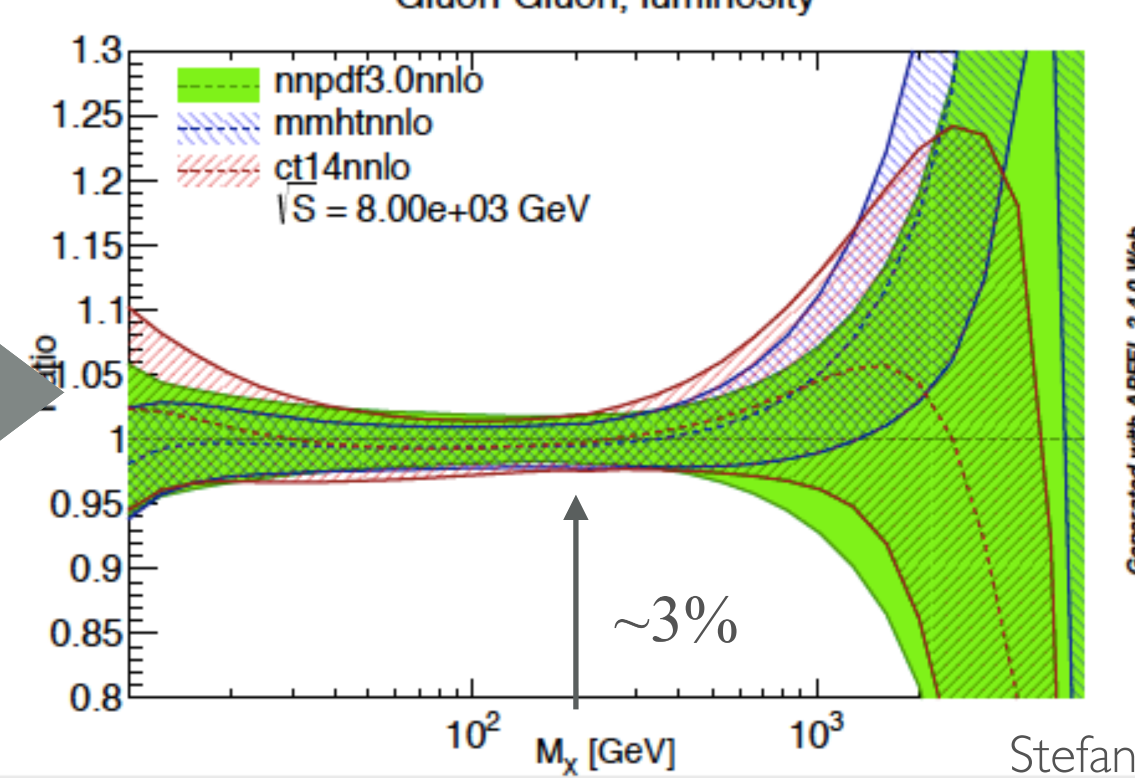


2015



GLUON-GLUON





StefanoForte

Perturbative expansion

$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ Parton-level cross section

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

LO predictions

NLO corrections

NNLO corrections

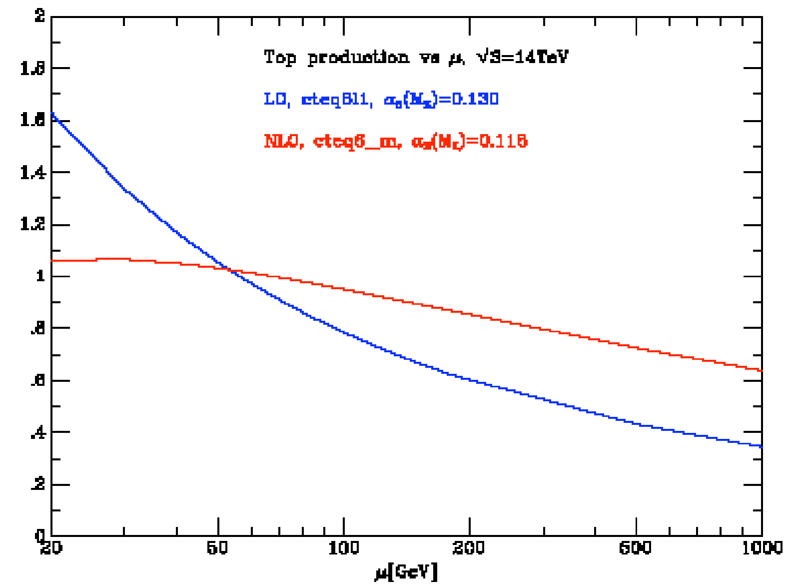
NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties: improvement in accuracy and precision.

Perturbative expansion

- Leading order (LO) calculations typically give only the order of magnitude of cross sections and distributions

  - the scale of α_s is not defined
  - jets partons: jet structure starts to appear only beyond LO
  - Born topology might not be leading at the LHC



- To obtain reliable predictions at least NLO is needed

- NNLO allows to quantify uncertainties

Furthermore:

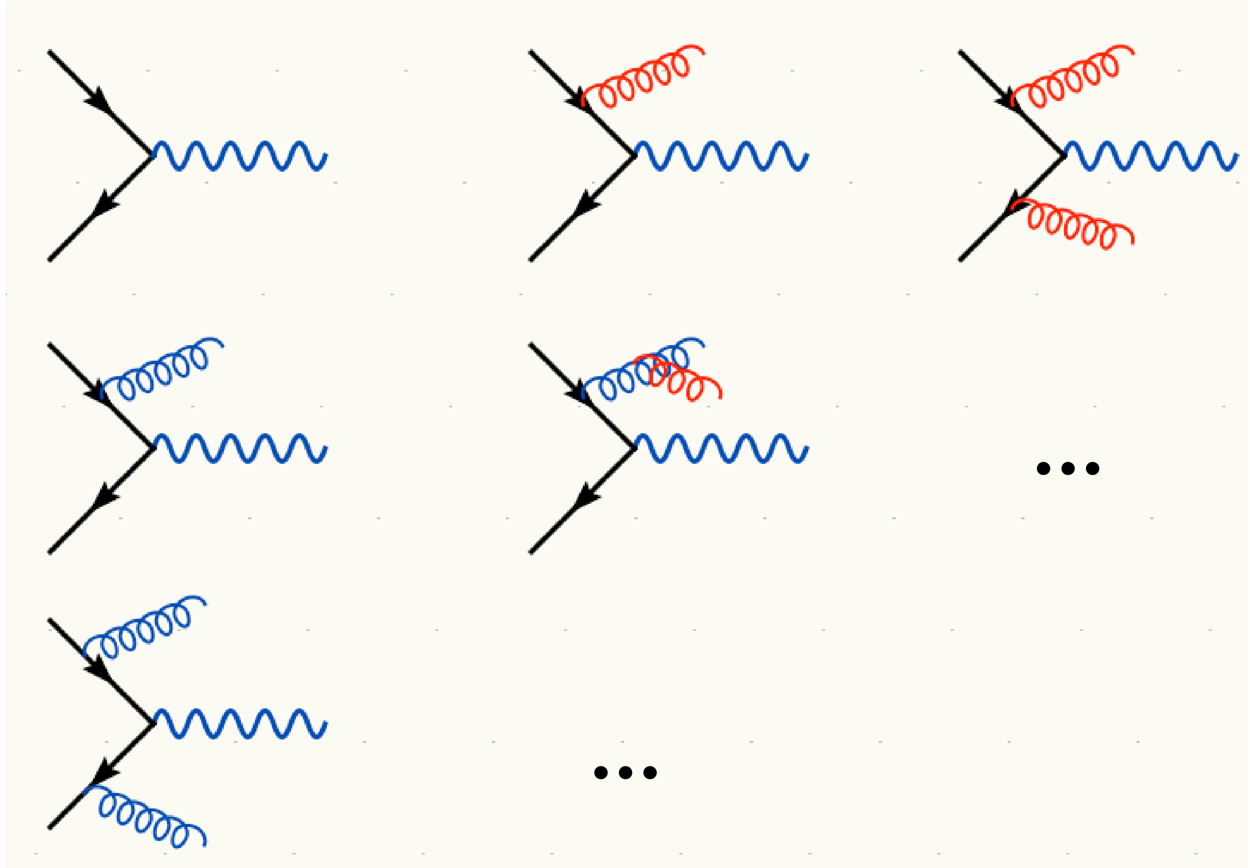
- Resummation of the large logarithmic terms at phase space boundaries
- NLO ElectroWeak corrections ($\alpha_s^2 = \alpha_W$)
- Fully exclusive predictions available in terms of event simulation that can be used in experimental analysis

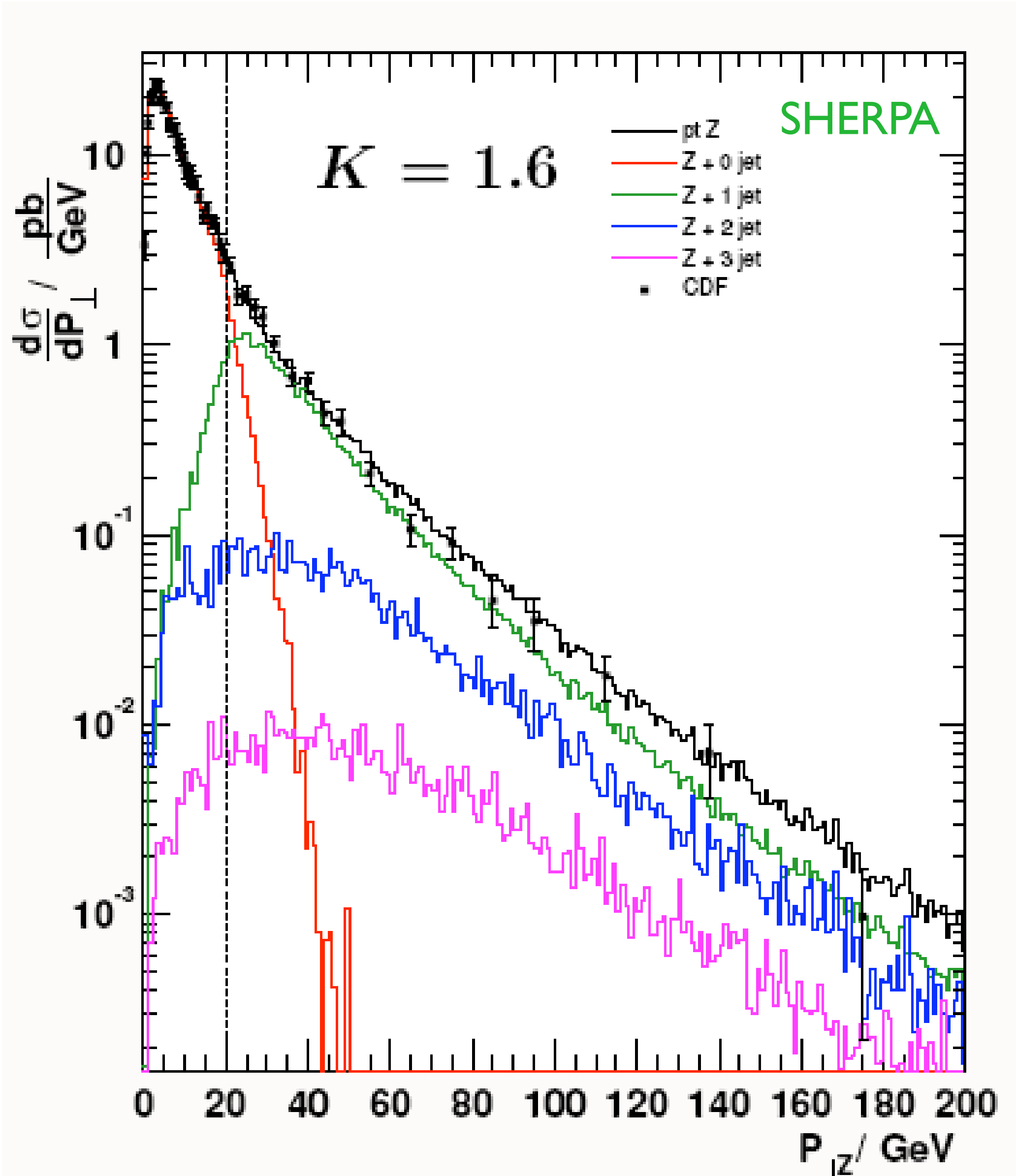
Merging fixed order with PS: LO

[Mangano]
[Catani, Krauss, Kuhn, Webber]

PS →

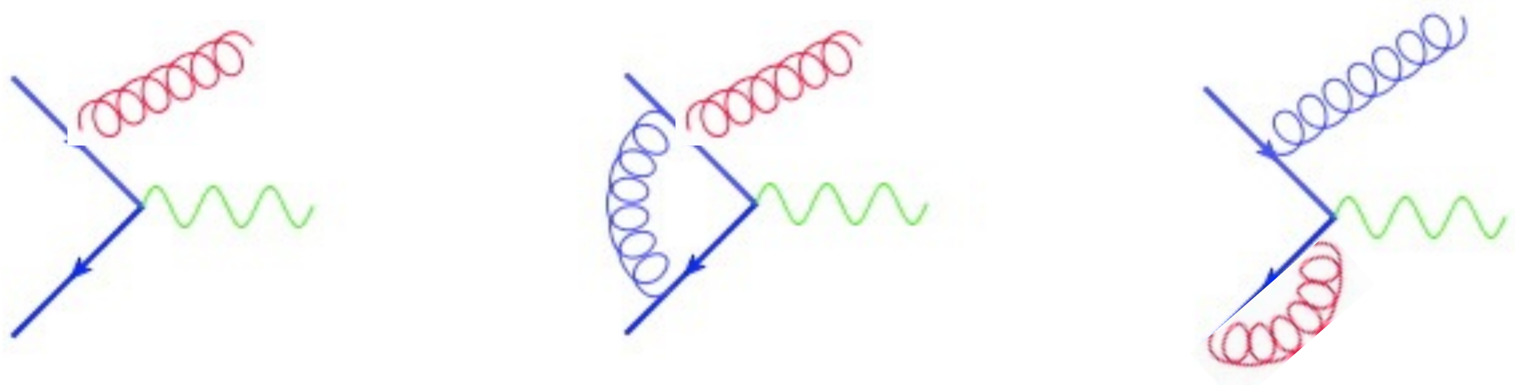
ME ↓





Double counting of configurations that can be obtained in different ways (histories). All the matching algorithms (CKKW, MLM,...) apply criteria to select only one possibility based on the hardness of the partons. As the result events are exclusive and can be added together into an inclusive sample. Distributions are accurate but overall normalization still "arbitrary".

Merging fixed order with PS: NLO



$$d\sigma^{\mathrm{NLOwPS}}_{\mathrm{NAIVE}} = \left[d\Phi_B(B(\Phi_B) + V + S^{\mathrm{int}}_{\mathrm{ct}})\right] I^n_{\mathrm{MC}} + \left[d\Phi_B d\Phi_{R|B}(R - S_{ct})\right] I^{n+1}_{\mathrm{MC}}$$

This simple approach does not work:

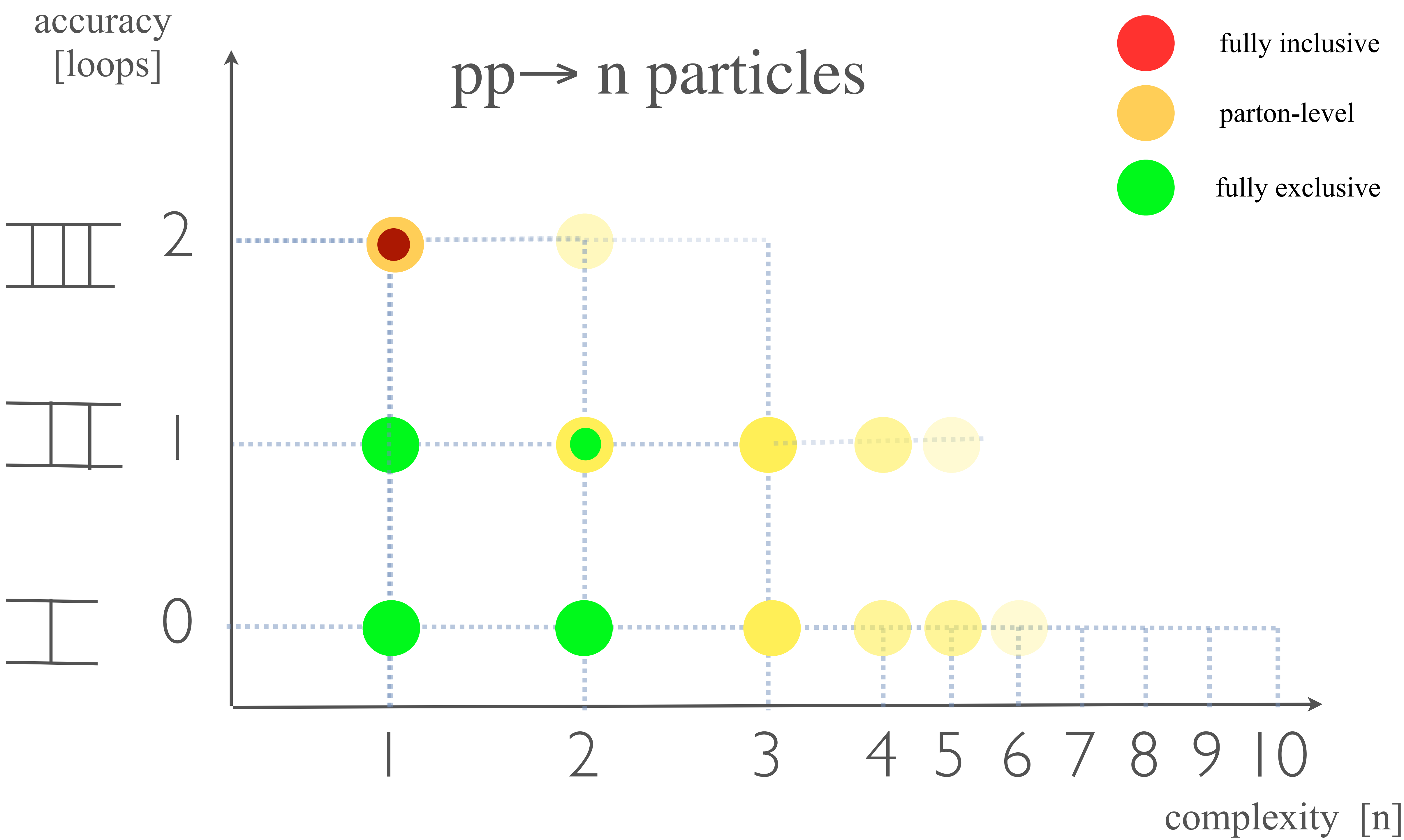
• Instability: weights associated to I^n_{MC} and I^{n+1}_{MC} are divergent pointwise (infinite weights).

• Double counting: $d\sigma^{naive}_{NLOwPS}$ expanded at NLO does not coincide with NLO rate. Some configurations are dealt with by both the NLO and the PSMC.

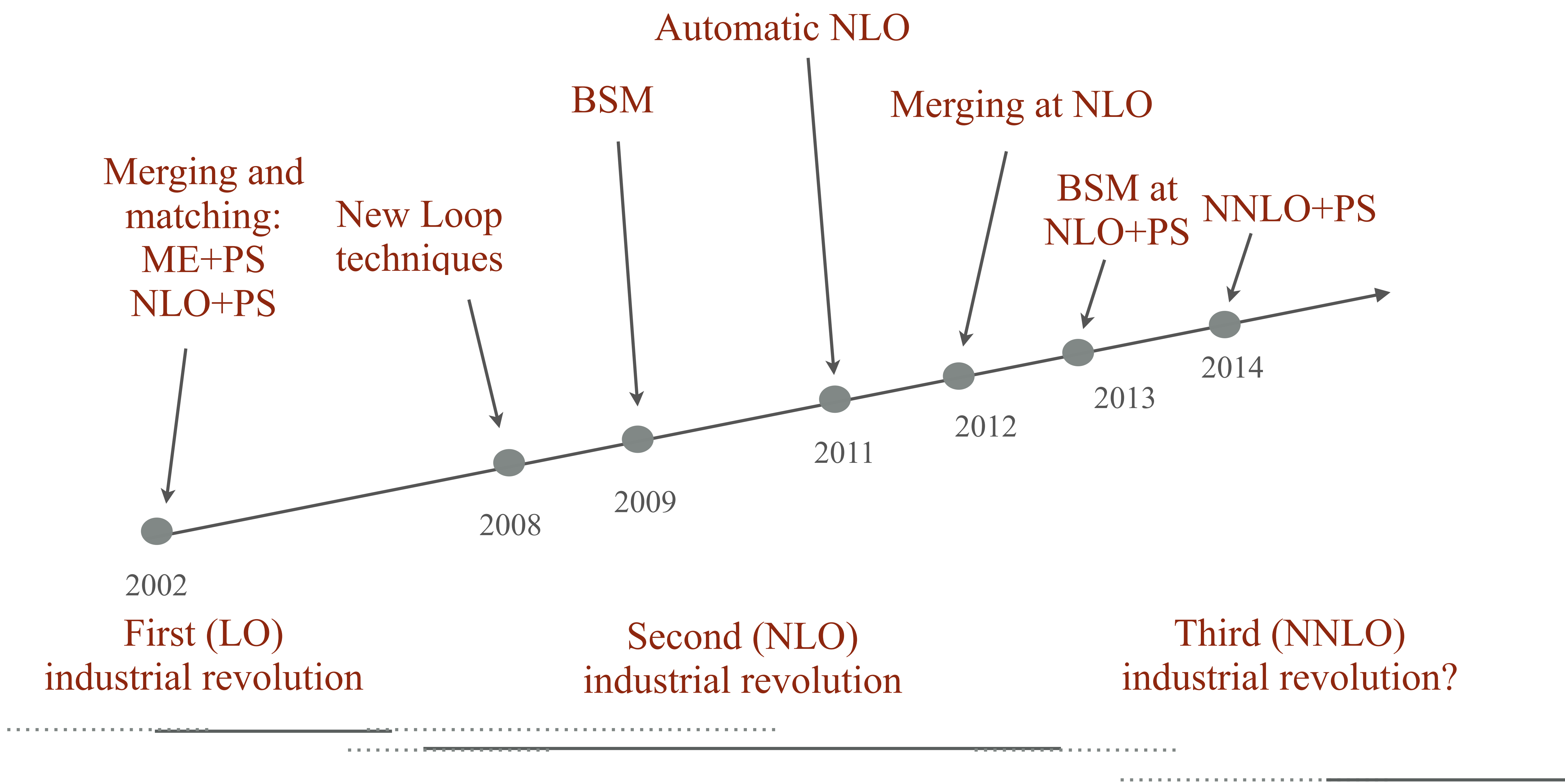
Two solutions available

The **MC@NLO** and **POWHEG** methods allow to combine NLO calculations with existing shower/hadronisation programs such as PYTHIA8, HW7, SHERPA….

Predictions in QCD: before the LHC

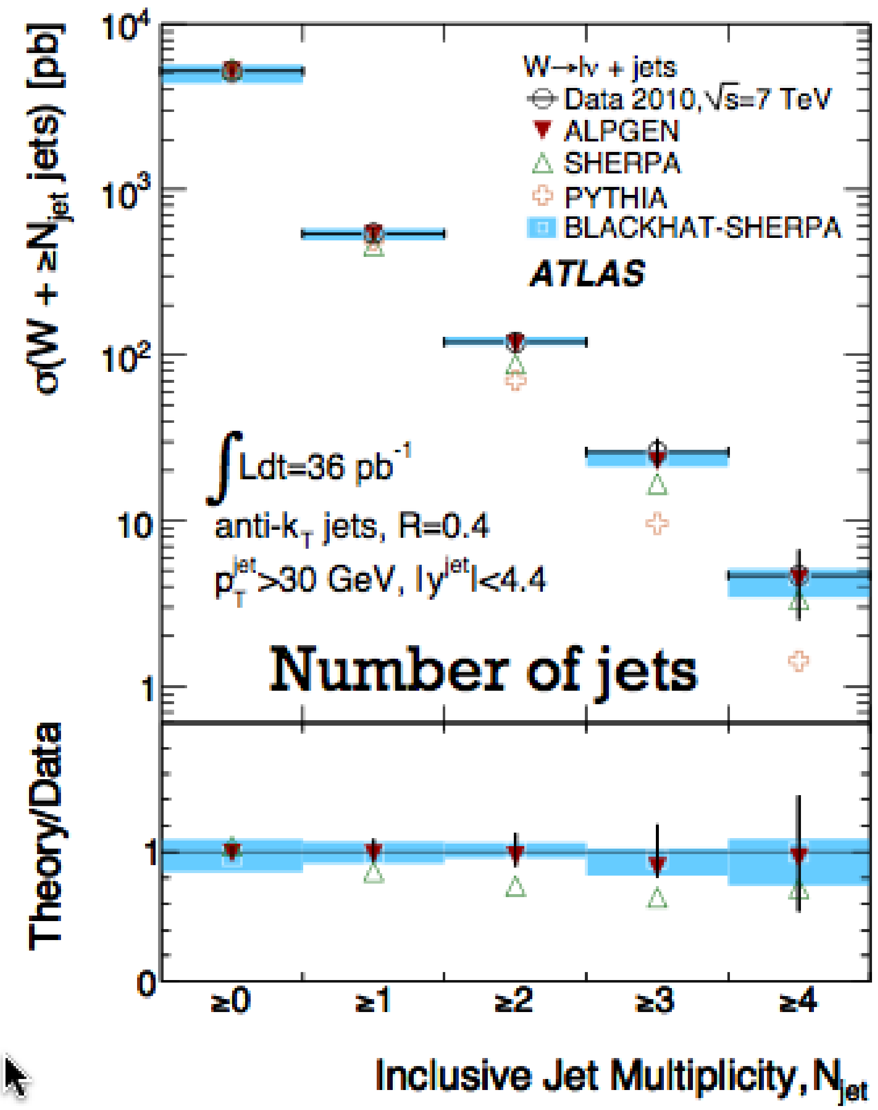


Predictive MC (simplified) progress



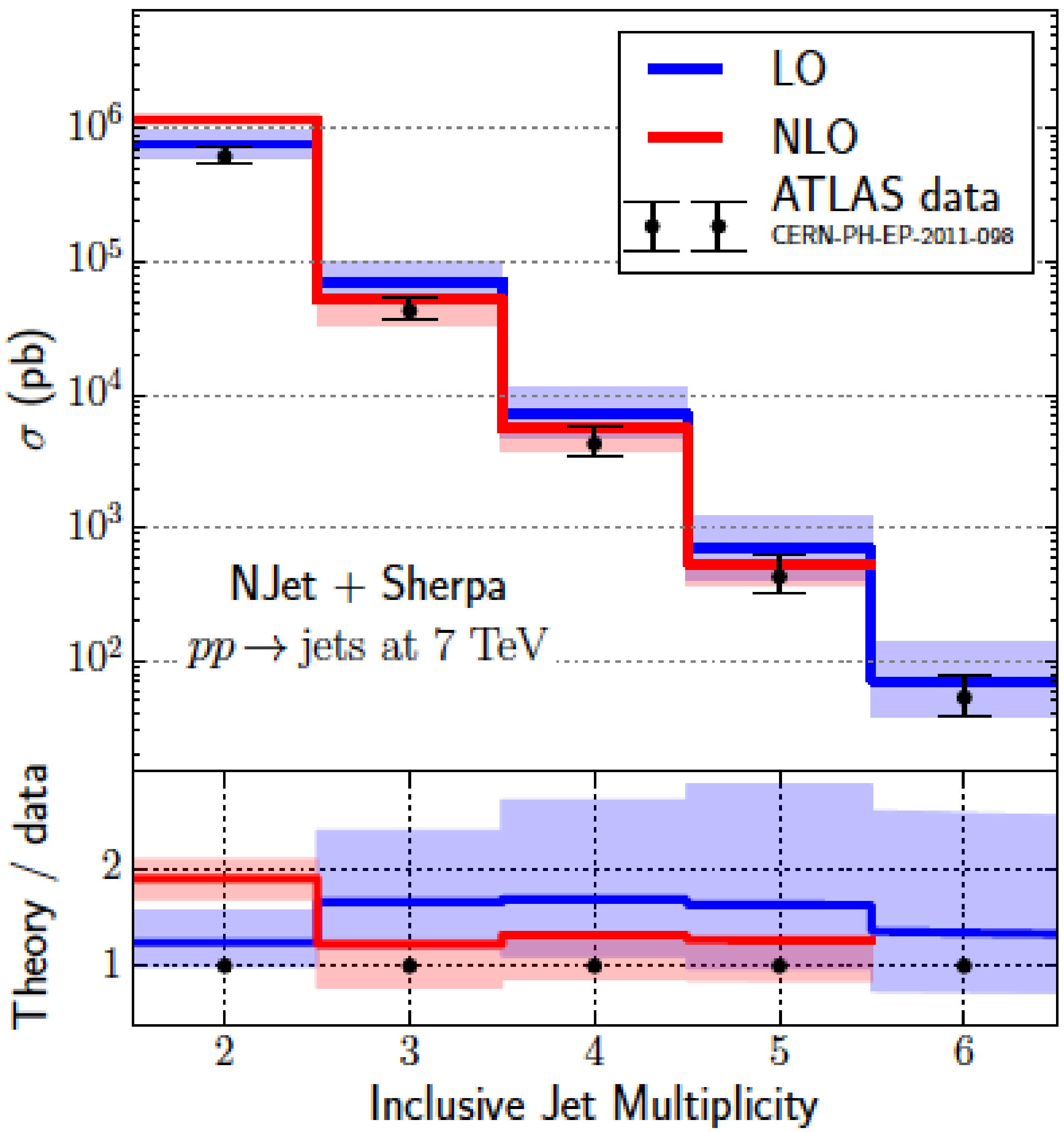
The NLO Guinness World Records

p p →W + 5 jets



[Bern et al., 1304.1253]

p p →5 jets



[Badger et al. 1309.6585]

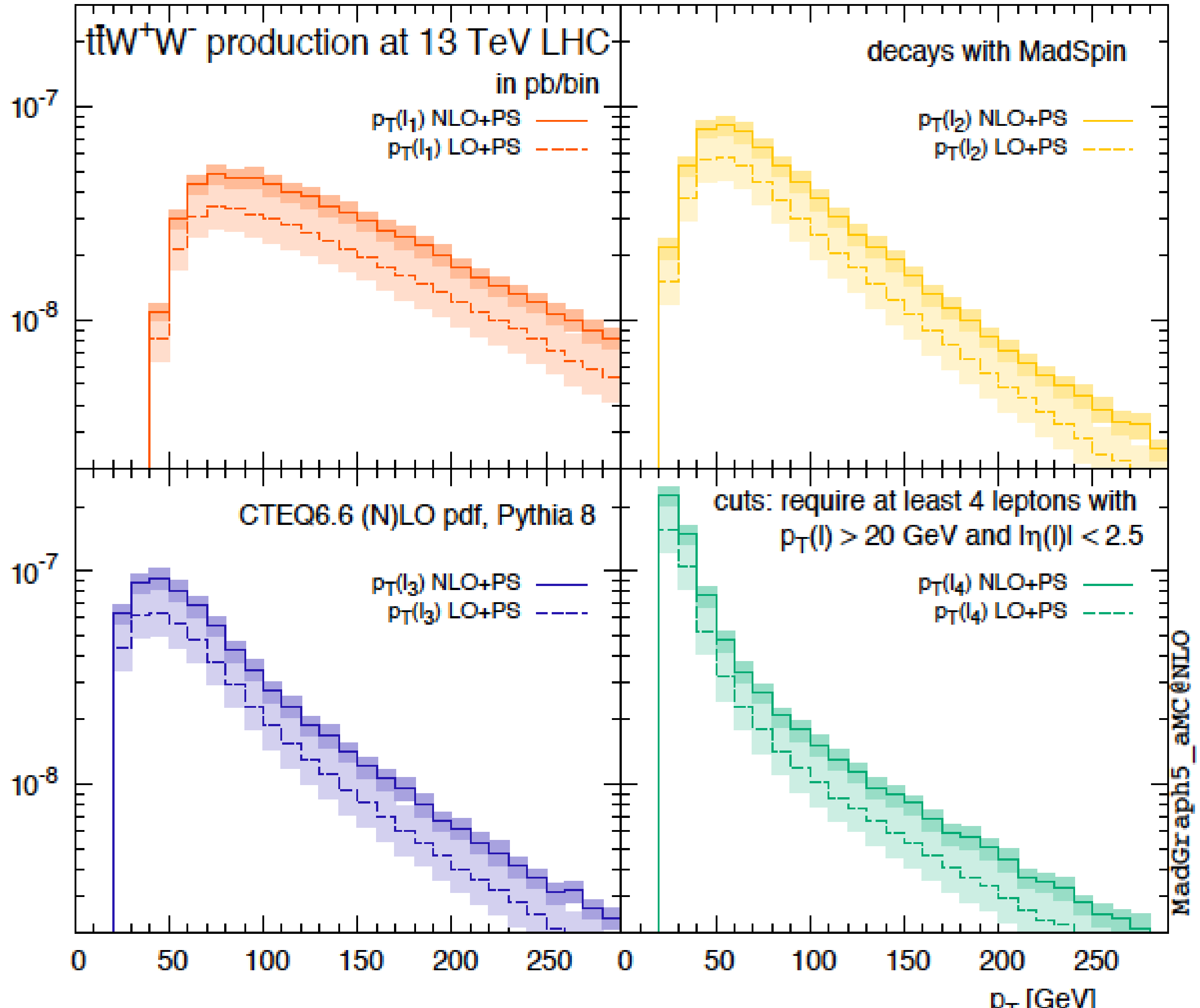
NLO+PS Automation

For example, the level of automation is as follows:

```
./bin/mg5_aMC
> generate p p > t t~ W+ W- [QCD]
> output ttww
> launch
```

Uncertainties from scale variation and pdfs are automatically computed (at no extra cost) and associated to each of the unweighted events (=any distribution will have the corresponding uncertainty band). Short-distance events ready to be "dressed" by PS and hadronisation.

[Alwall, et al. 1505.0301]



Virtually unlimited set of LHC processes available at NLO

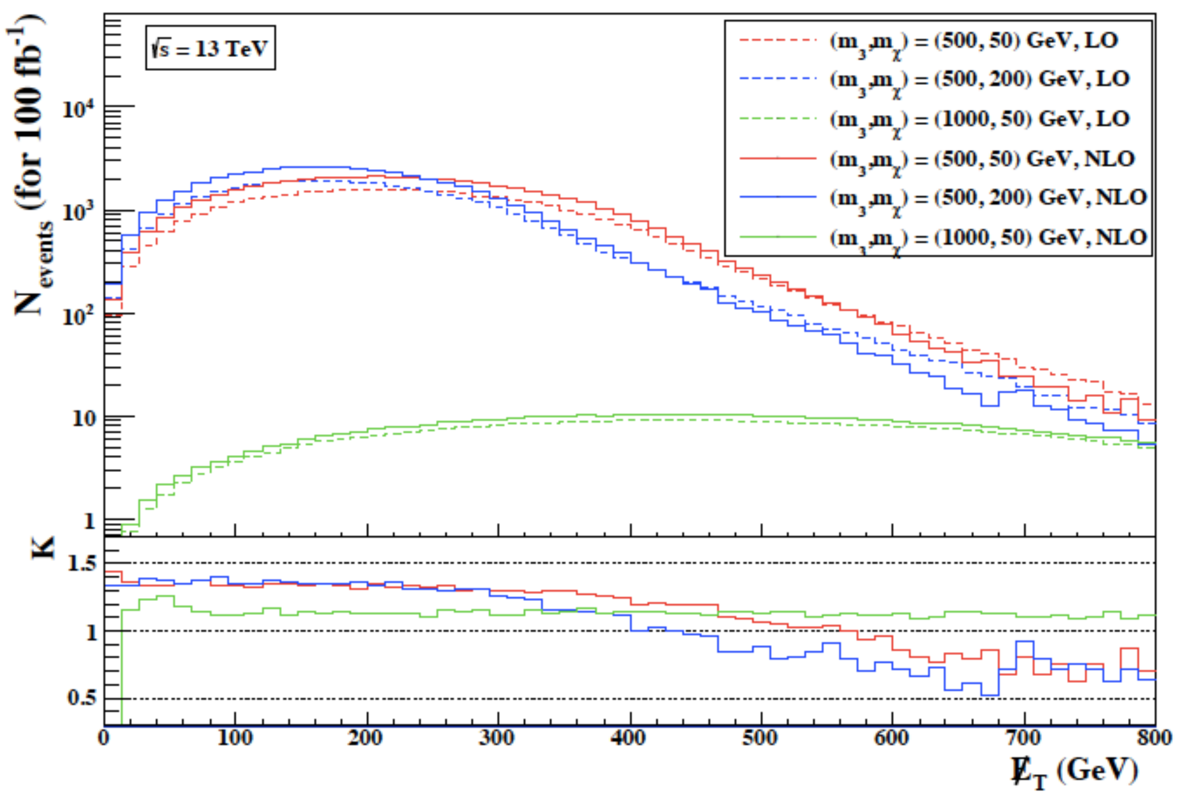
NLO+PS Automation

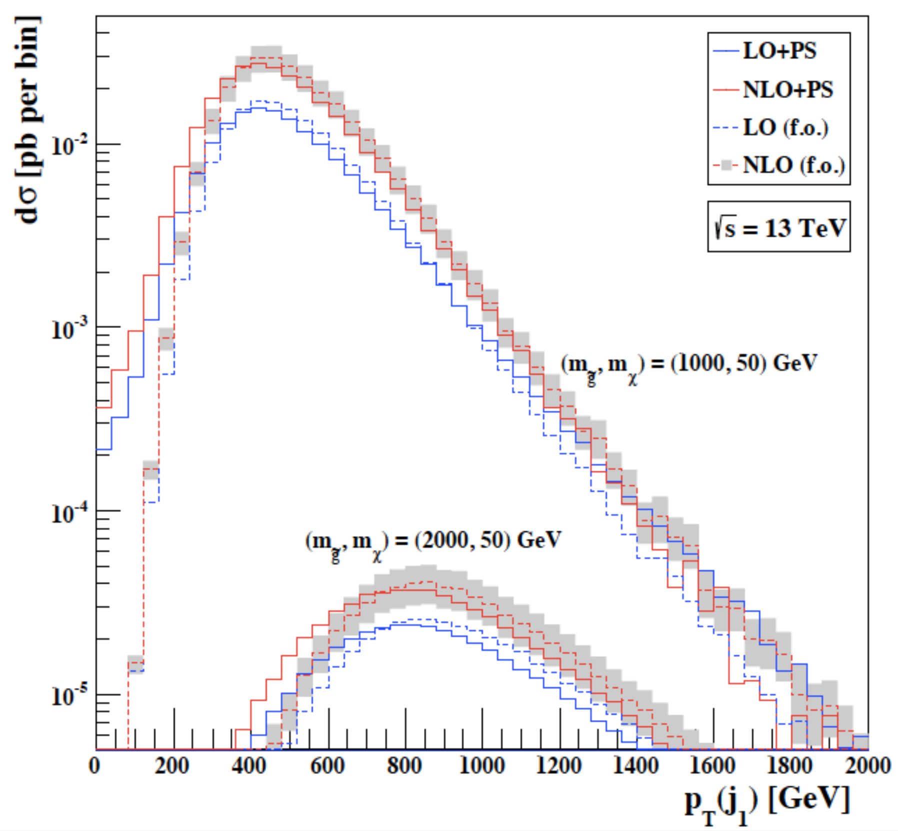
The same level of automation is being achieved for BSM:

[HuaSheng Shao et al. , 1412.5589, 1510.00391]

```
./bin/mg5_aMC
> import model SUSYQCD
> generate p p > t1 t1~ [QCD]
> output StopPair
> launch
```

```
./bin/mg5_aMC
> import model SUSYQCD
> generate p p > gl gl [QCD]
> output GluinoPair
> launch
```





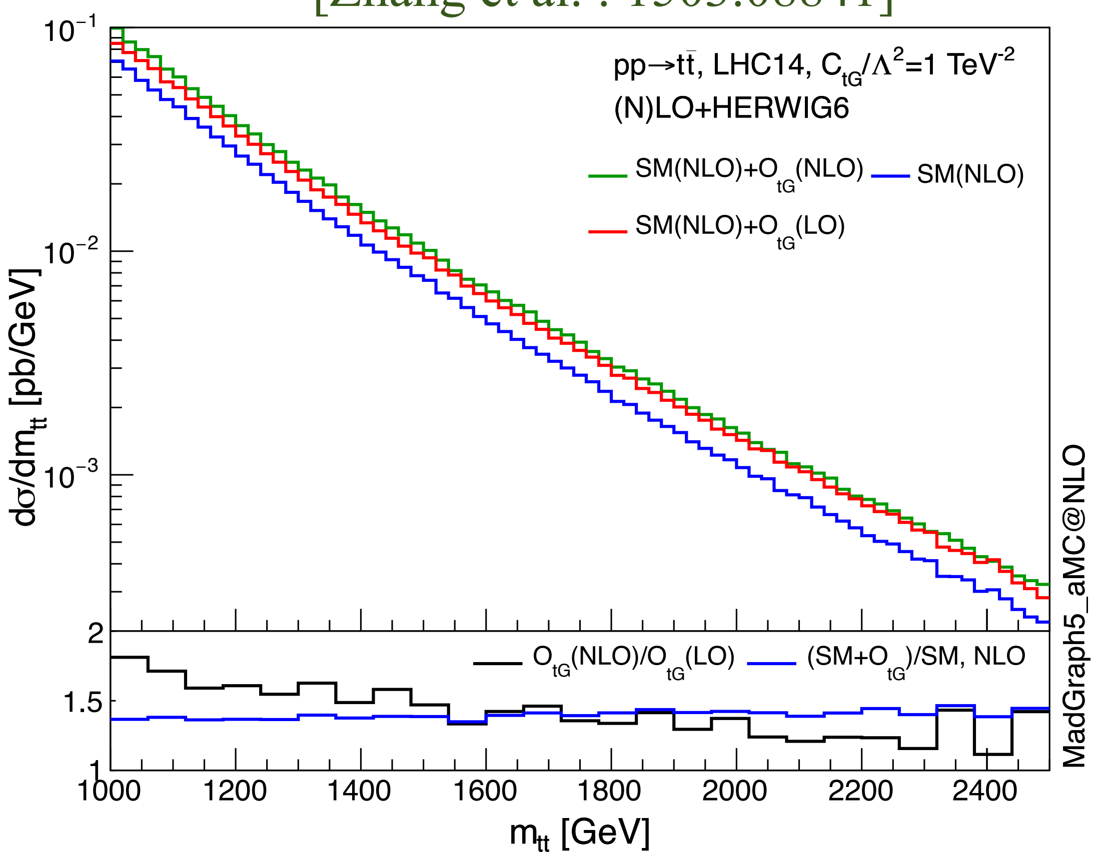
NLO+PS Automation

The same level of automation is being achieved for EFT's:

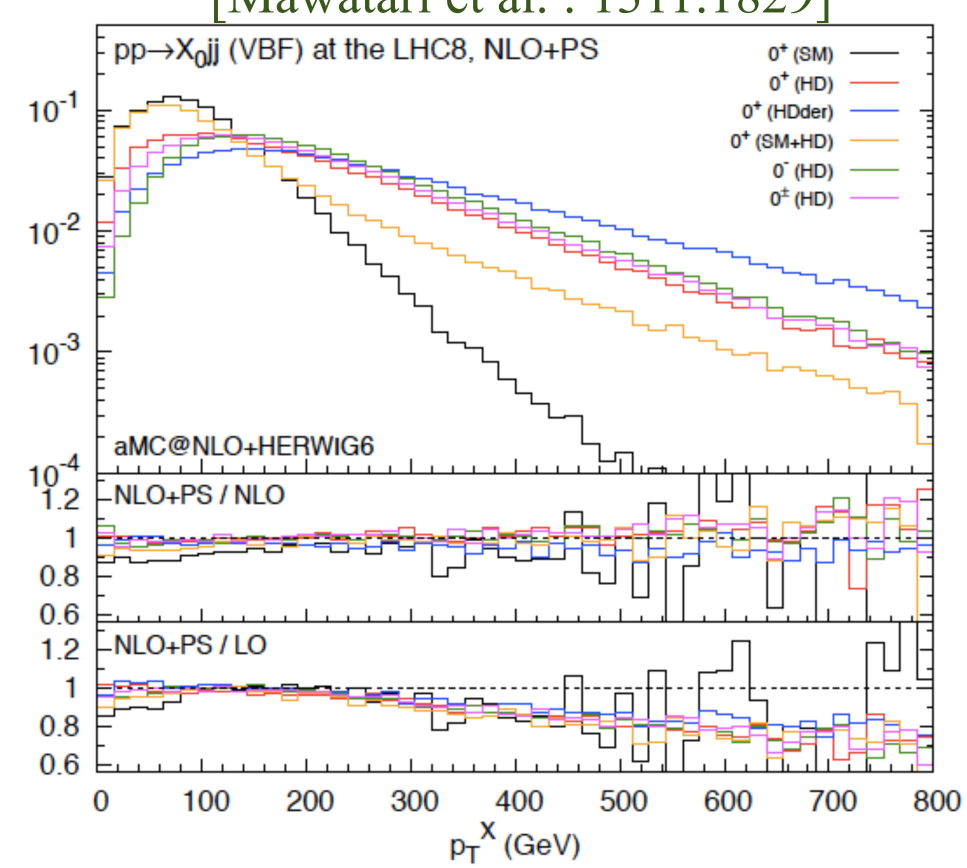
```
./bin/mg5_aMC
> import model TopEFT
> generate p p > t t~ , NP=1 [QCD]
> output Chromott
> launch
```

```
./bin/mg5_aMC
> import model HC
> generate p p > X0 j j [QCD]
> output VBFdim6
> launch
```

[Zhang et al. : 1503.08841]



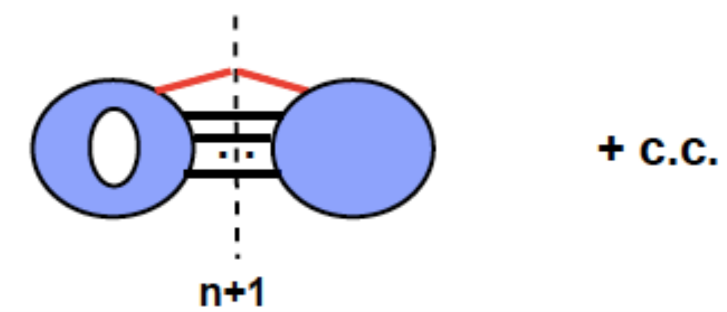
[Mawatari et al. : 1311.1829]



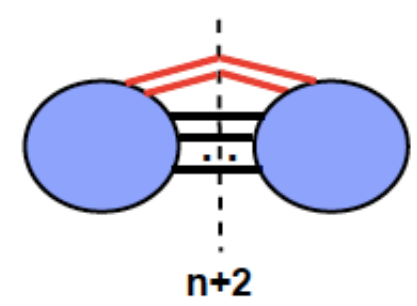
Ingredients of NNLO calculations

Double virtual contribution
with n resolved partons

Real-virtual contribution with 1 unresolved parton



Double-real contribution with 2 unresolved partons



Each of the three contributions is divergent, yet the sum is finite (KLN theorem).
How to deal with IR singularities ?

The NNLO era

NNLO calculations important at least for the following cases:

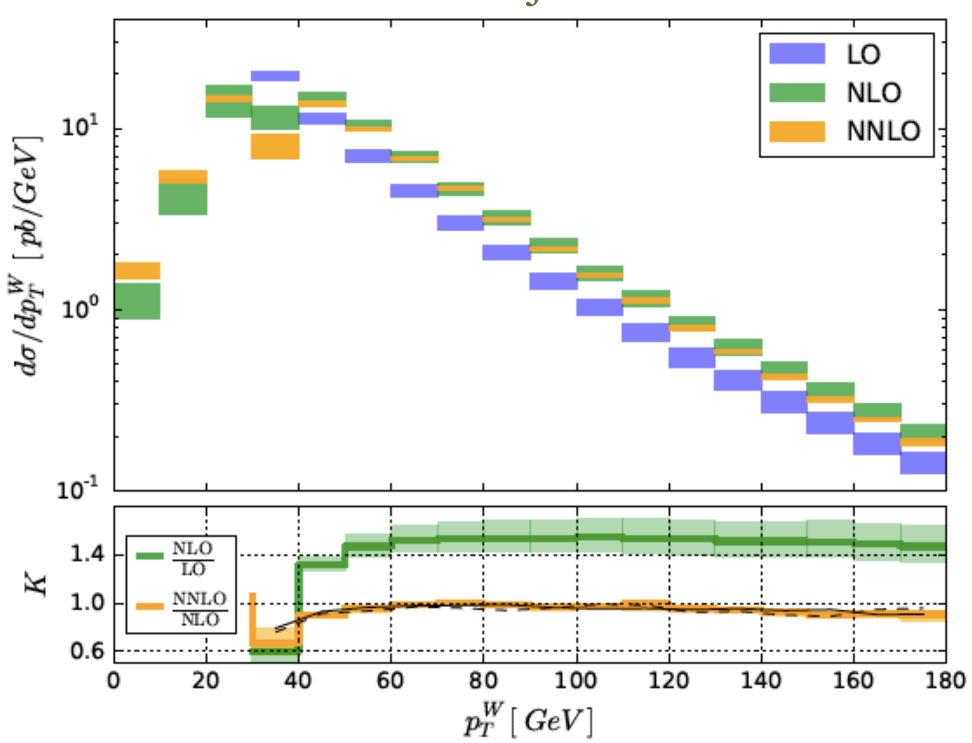
1) Benchmark processes measured with high precision	$e^+e^- \to$ 3 jets	✓
	pp $\to$ W, Z	✓
	pp $\to$ 2 jets	partial
	pp $\to$ t tbar	✓
2) Processes with large NLO corrections (eg, new channels)	pp$\to$H (EFT)	✓
	pp$\to$H+jet (EFT)	✓
	pp$\to$HH (EFT)	✓
3) Important/Irreducible backgrounds for Higgs or NP searches	pp $\to$ t tbar	✓
	pp$\to$VV' (W,γ,Z)	✓
	pp$\to$W/Z j	✓

In addition it is essential to provide codes that are able to deal with final state selections (at the parton level) so that fiducial cross sections and distributions can be directly compared with data.

V+jet at NNLO

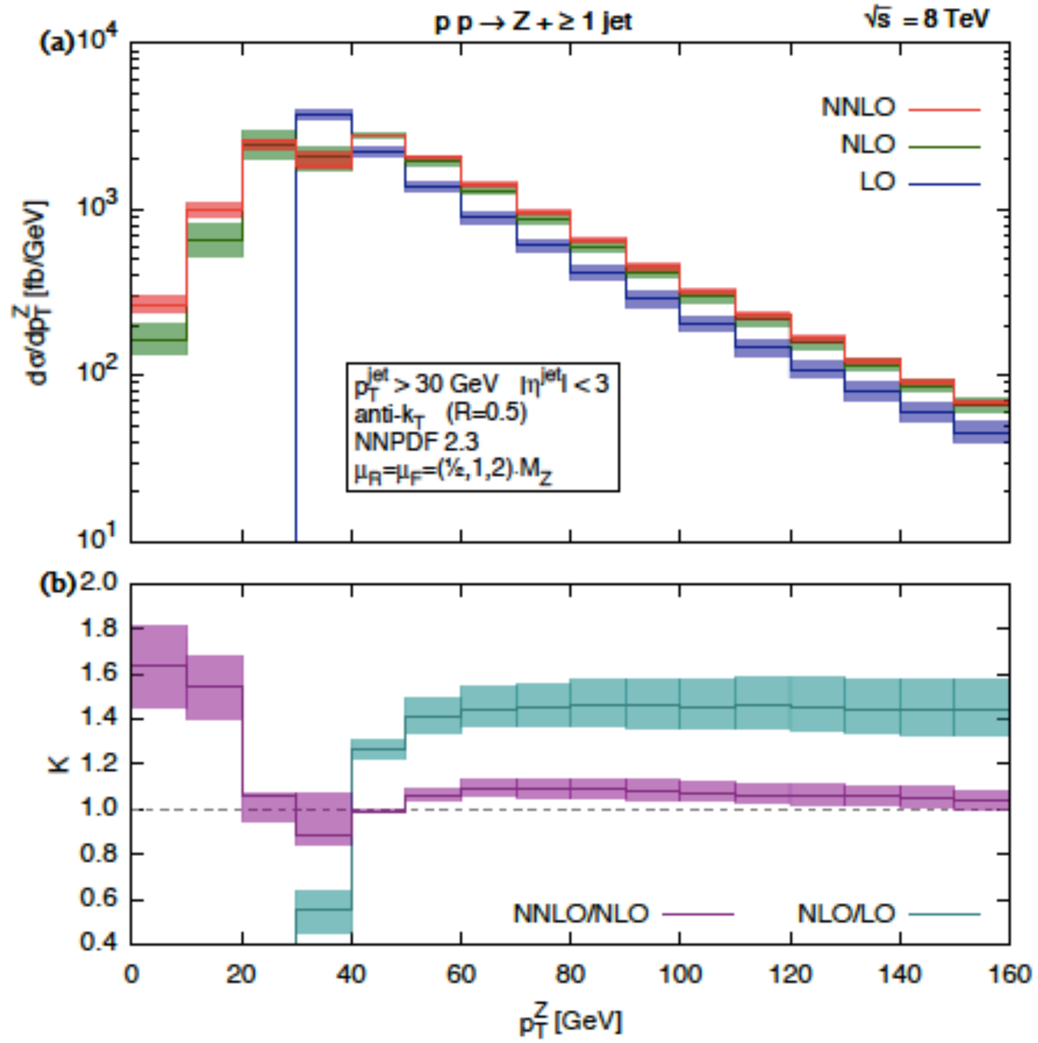
[R.Boughezal, C.Focke,X.Liu, F.Petriello (2015)]

W+jet



Small NNLO effect and significant reduction of scale uncertainties. First application of new "N-jettiness" method: relatively flat NNLO correction.

[A and T. Gehrmann, N. Glover, T.Morgan, A.Huss (2015)]



Similar effects for Z+jet: antenna subtraction (large Nc approximation for the dominant channels)

H+jet at NNLO (in the EFT)

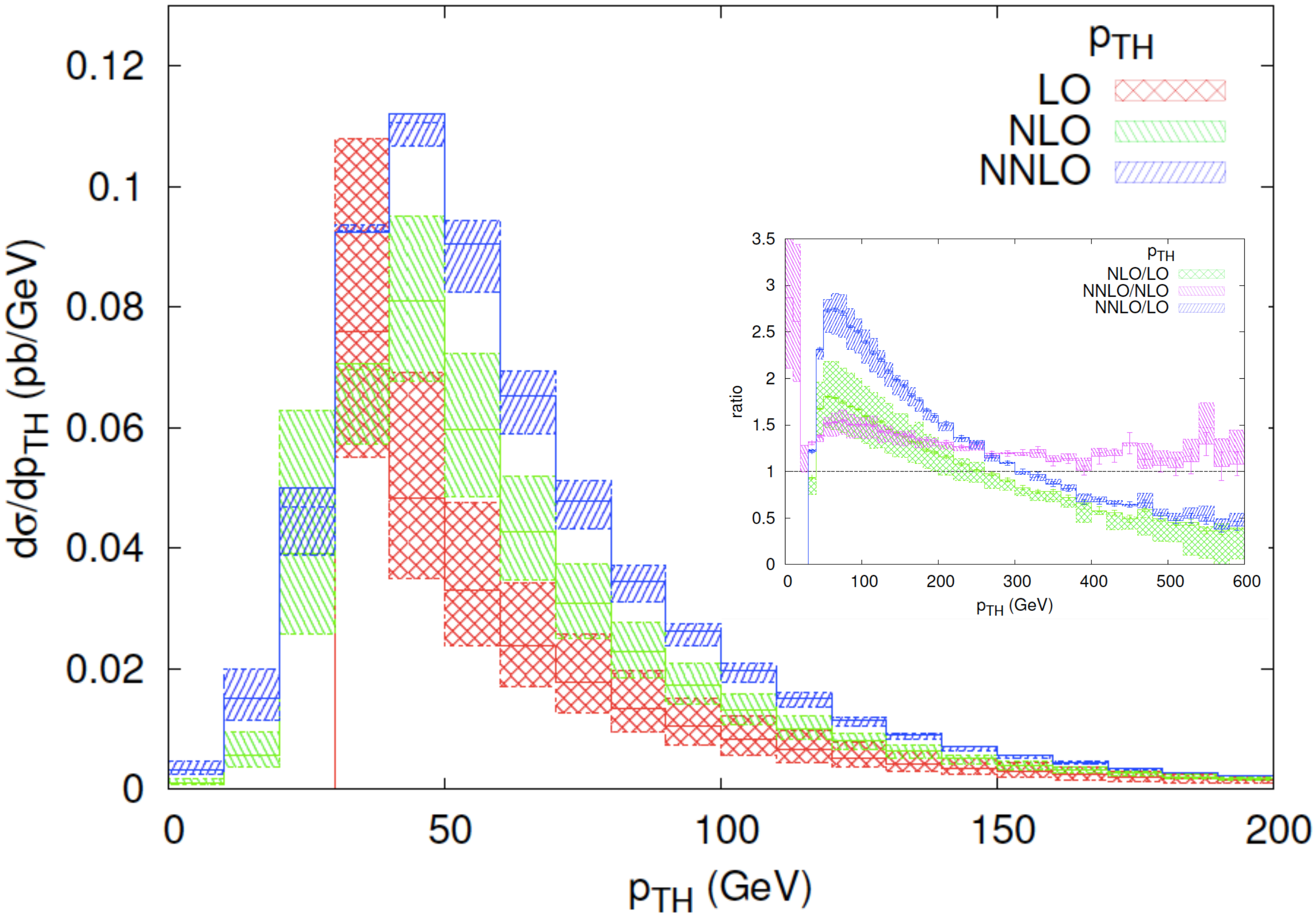
NNLO calculation carried out with three independent methods (antenna subtraction, subtraction+sector, N-jettiness)

X. Chen, T. Gehrmann, N. Glover, M. Jaquier (2014)

R.Boughezal, F.Caola, K.Melnikov, ,F.Petriello, M.Schulze (2015)

R.Boughezal, C.Focke, W.Giele ,X.Liu, F.Petriello (2015)

Quantitative effect smaller than previously anticipated from gg only: at the 20% level (μ=m_H)



VBF at NNLO

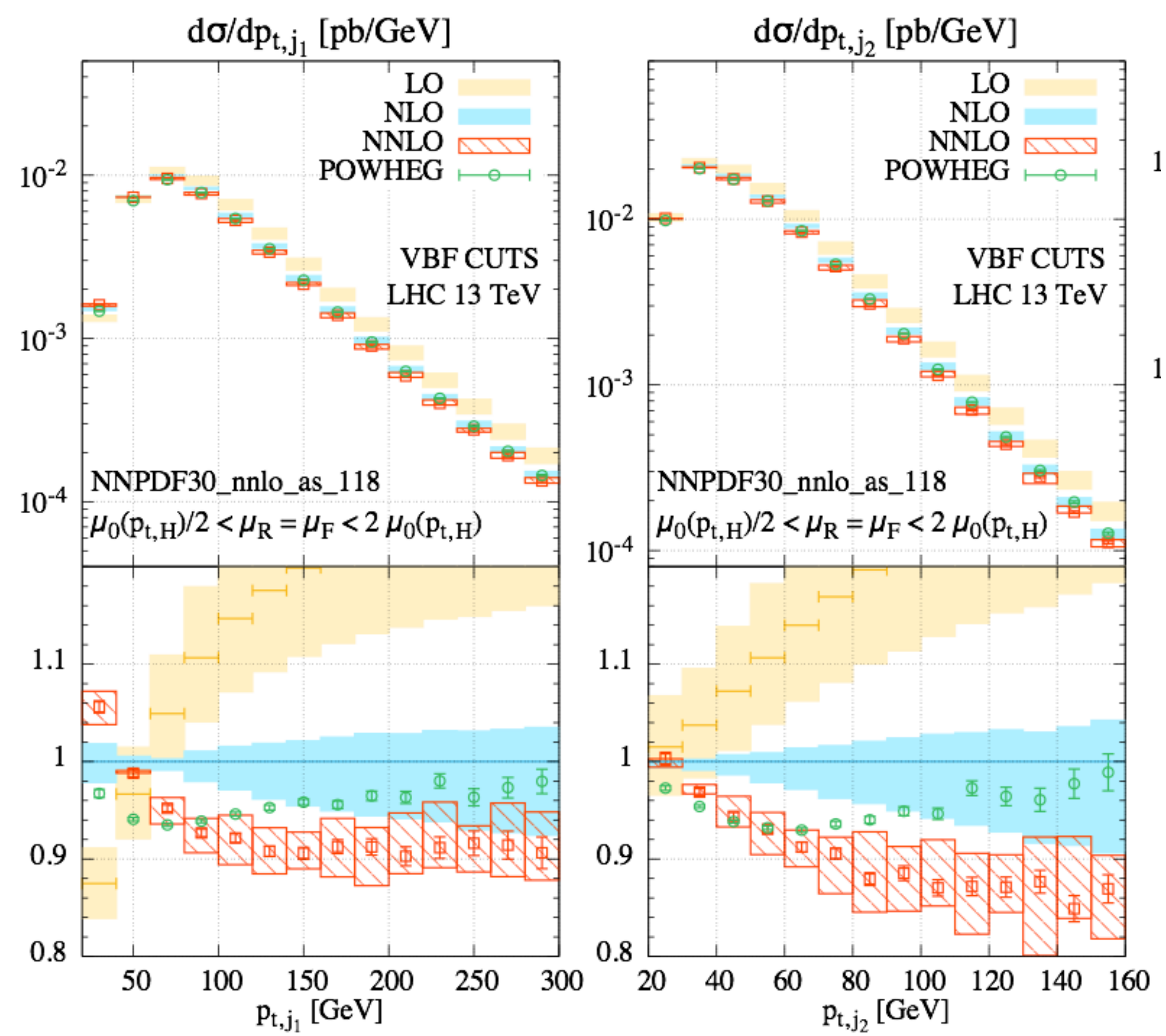
Vector boson fusion (VBF) is an important production channel for the Higgs boson: distinctive signature with little hadronic activity in the central rapidity region.

Fully inclusive NNLO corrections known since quite some time [P.Bolzoni, F.M,S.Moch,M.Zaro (2010)] in the structure function approach: O(1%) effect.

Fully exclusive NNLO computation recently completed (still neglecting color exchanges between quark lines) [M.Cacciari, F.Dreyer, A.Karlberg, G.Salam,G.Zanderighi (2015)]

NNLO corrections make pT spectra softer larger impact when VBF cuts are applied

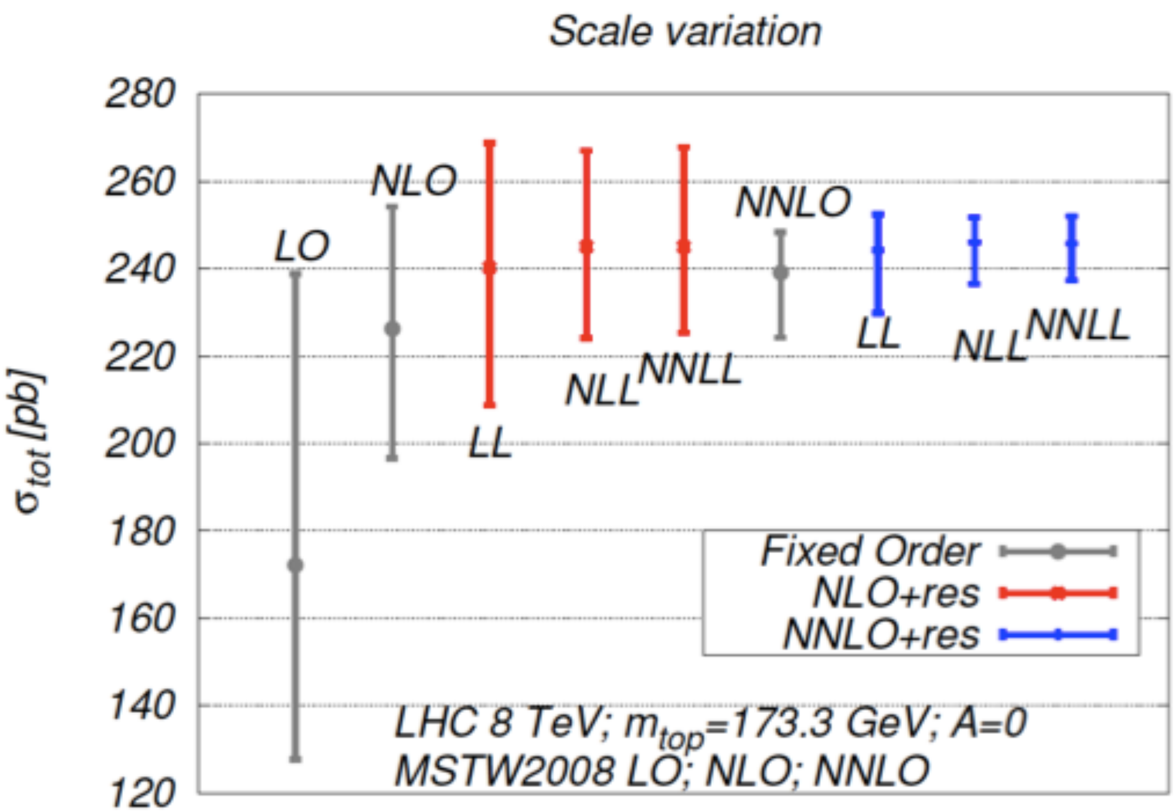
	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032\,^{+0.057}_{-0.069}$	$0.957\,^{+0.066}_{-0.059}$
NLO	$3.929\,^{+0.024}_{-0.023}$	$0.876\,^{+0.008}_{-0.018}$
NNLO	$3.888\,^{+0.016}_{-0.012}$	$0.826\,^{+0.013}_{-0.014}$

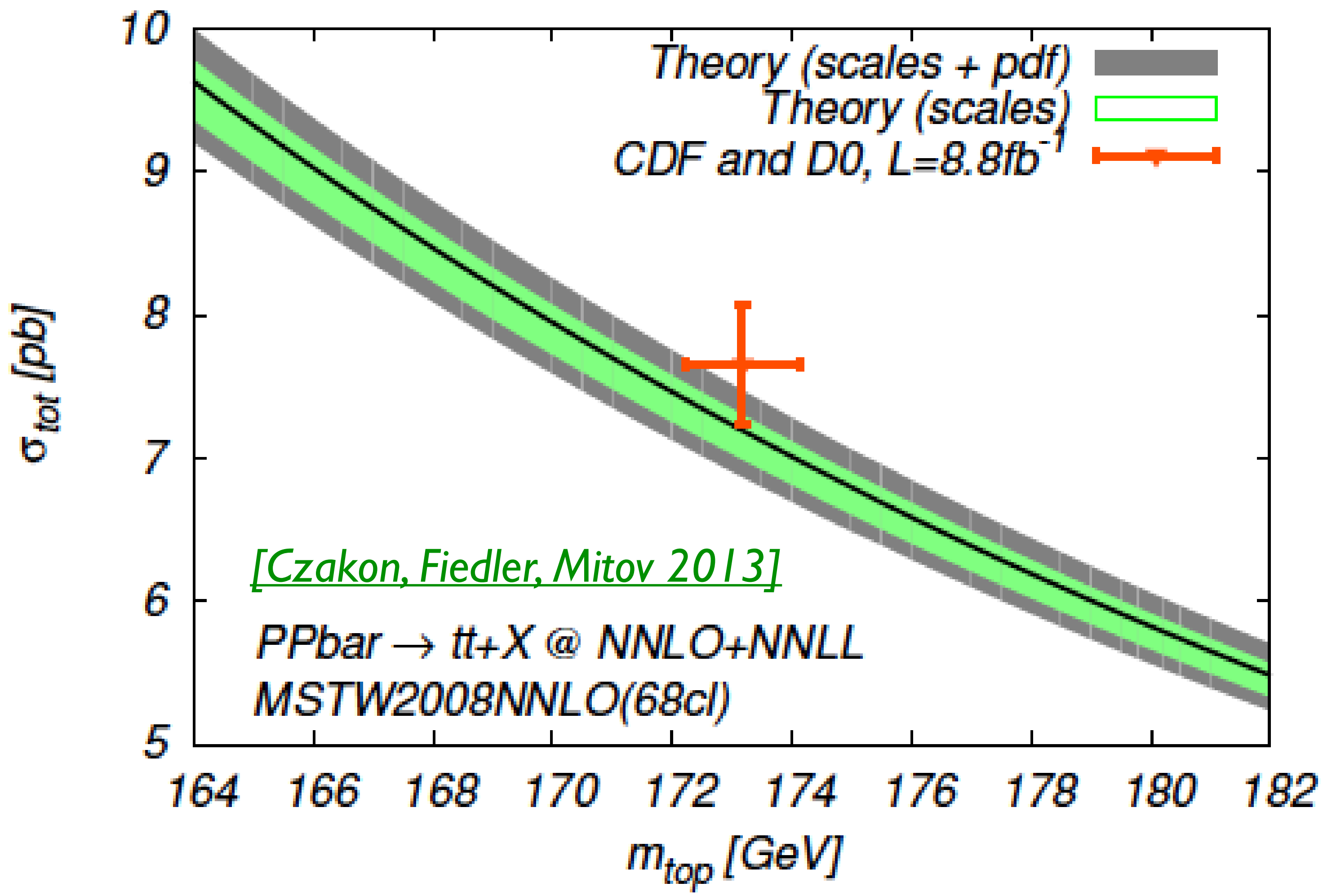


$t\bar{t}$ cross section at NNLO

Monumental MILESTONE in perturbative QCD:

[Bärnreuther, Czakon, Mitov 2012]
[Czakon, Mitov 2012]
[Czakon, Mitov 2012]
[Czakon, Fiedler, Mitov 2013]





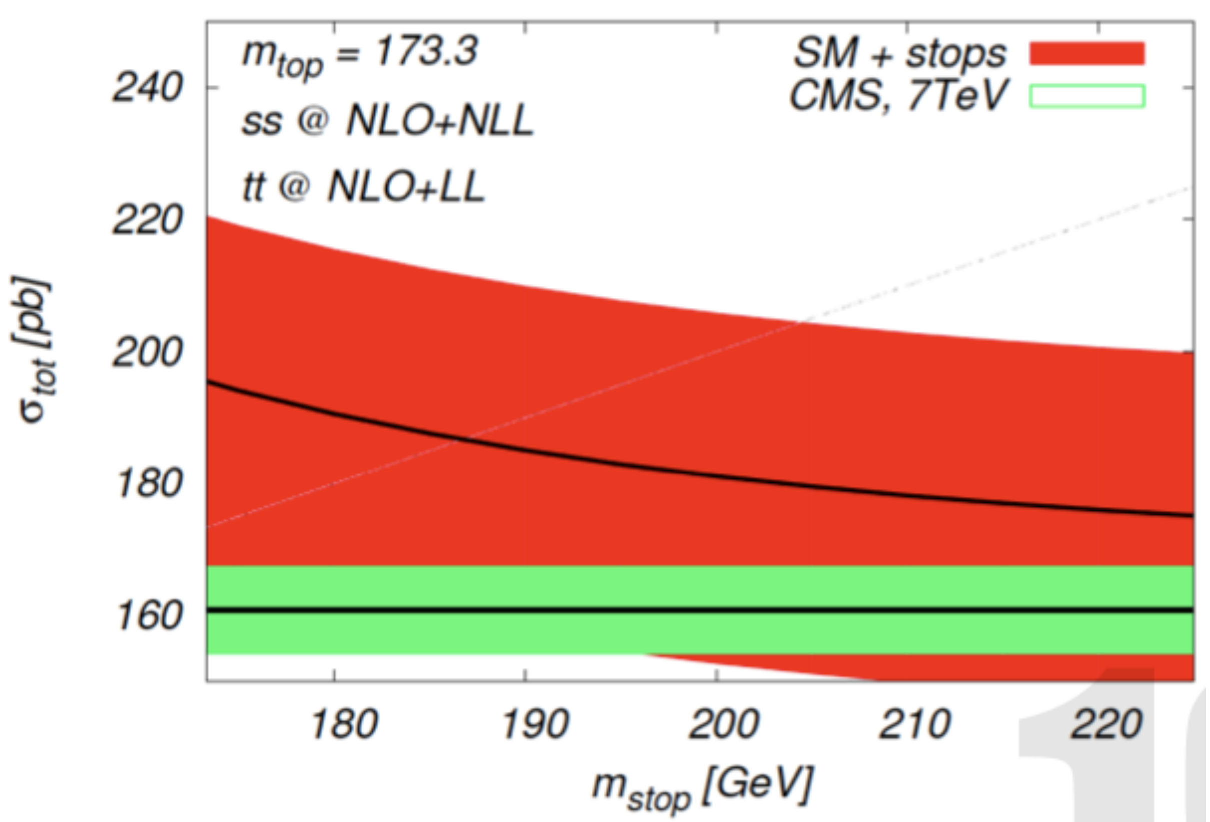
- Two loop hard matching coefficient extracted and included
- Very weak dependence on unknown parameters (sub 1%): gg NNLO, A, etc.
- ~ 50% scales reduction compared to the NLO+NNLL analysis

$t\bar{t}$ cross section at NNLO

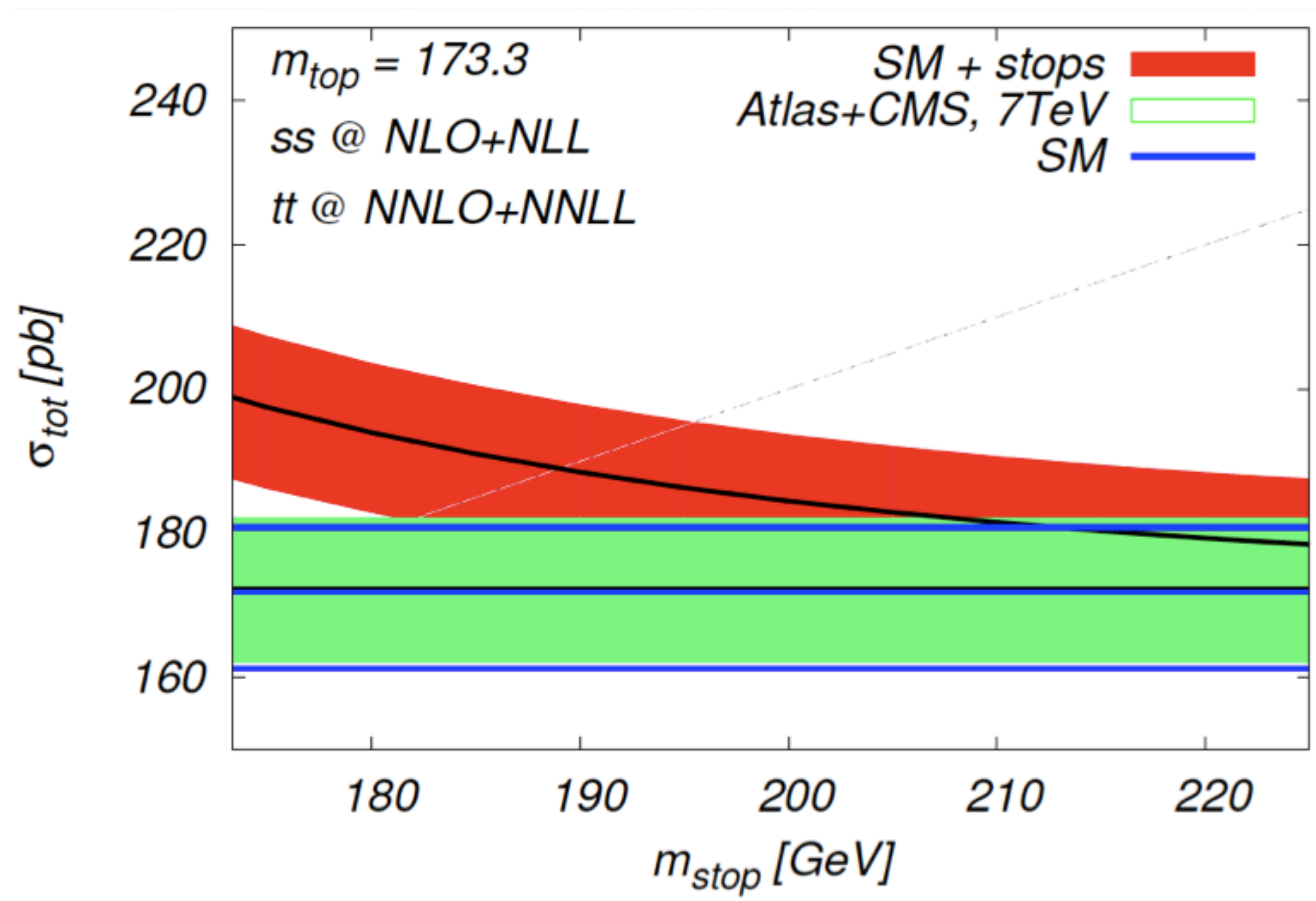
Having a NNLO prediction opens the door to new possibilities.

Consider the light stop window in a compressed spectrum, that mimicks the normal ttbar production:

[Czakon, Mitov, Papucci, Ruderman, Weiler,2014]



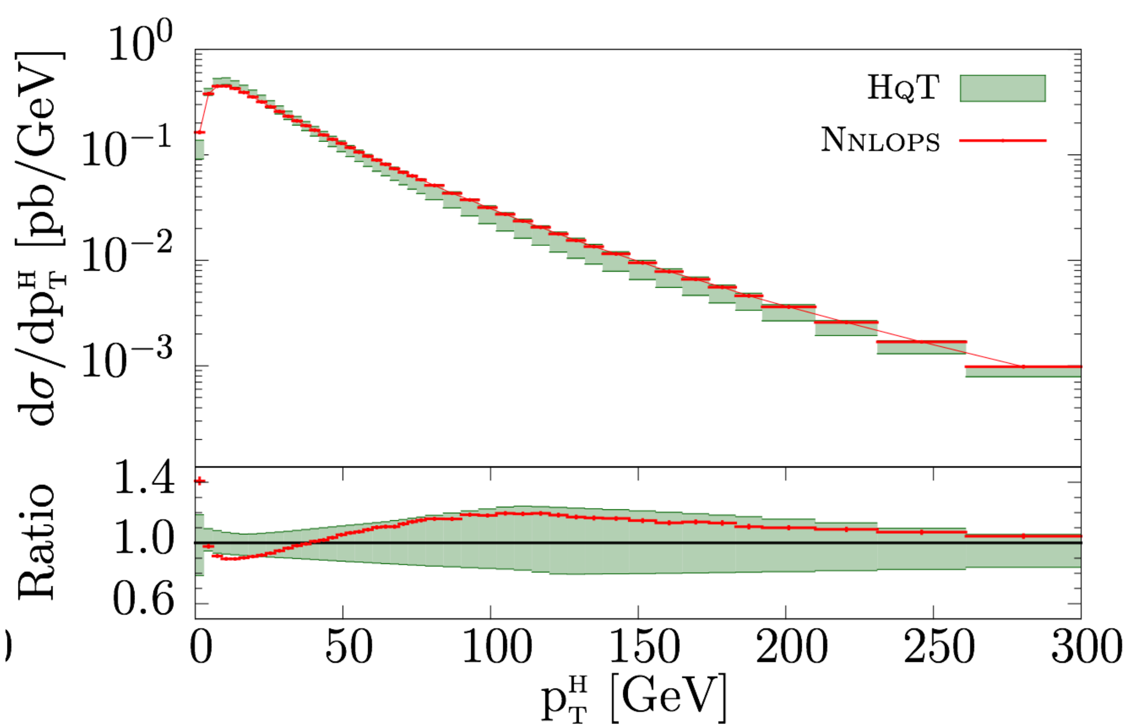
→



NNLO + PS

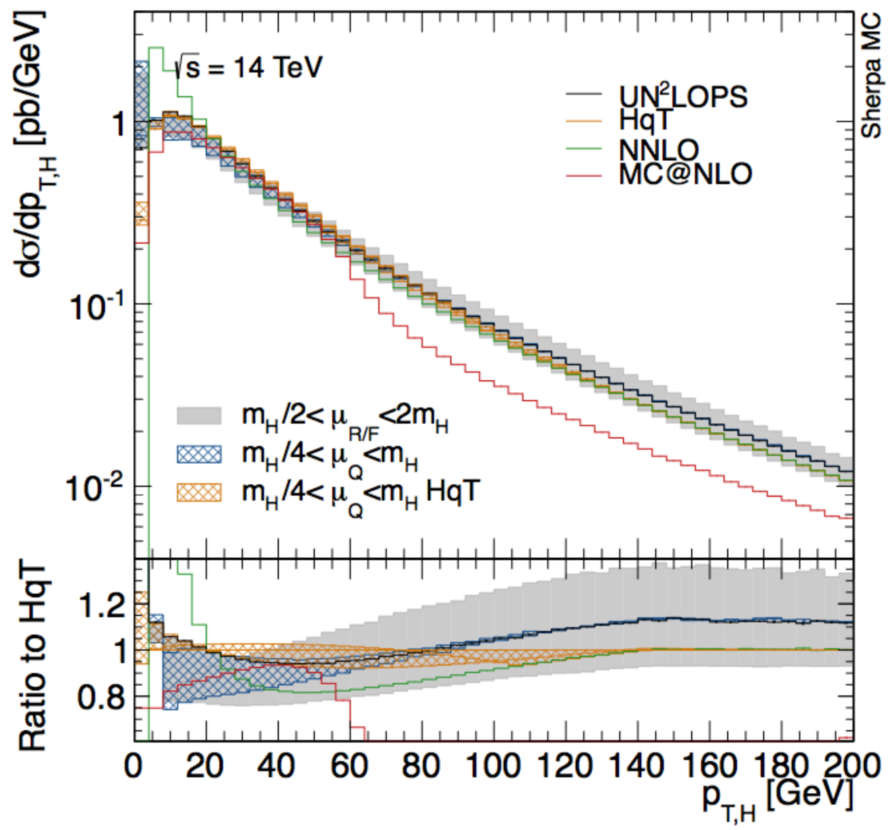
NLO matching well established, while NNLO matching still in its infancy

1) NNLOPS: use MINLO to obtain a NLO generator for both H and H+jet(s)
[K.Hamilton, P.Nason,G.Zanderighi (2014,2015)]



Enforce correct NNLO normalisation by reweighing the inclusive rapidity distribution to the NNLO calculation

2) UN2LOPS: use S-MC@NLO + UNLOPS + q_T slicing
[N.Lavesson, L.Lonnblad (2008), S.Hoeche,Y.Li, S.Prestel (2014)]



NNLO virtual corrections confined in the low pT region while in the POWHEG-MINLO approach they are spread over the whole pT region

The frontier: N3LO

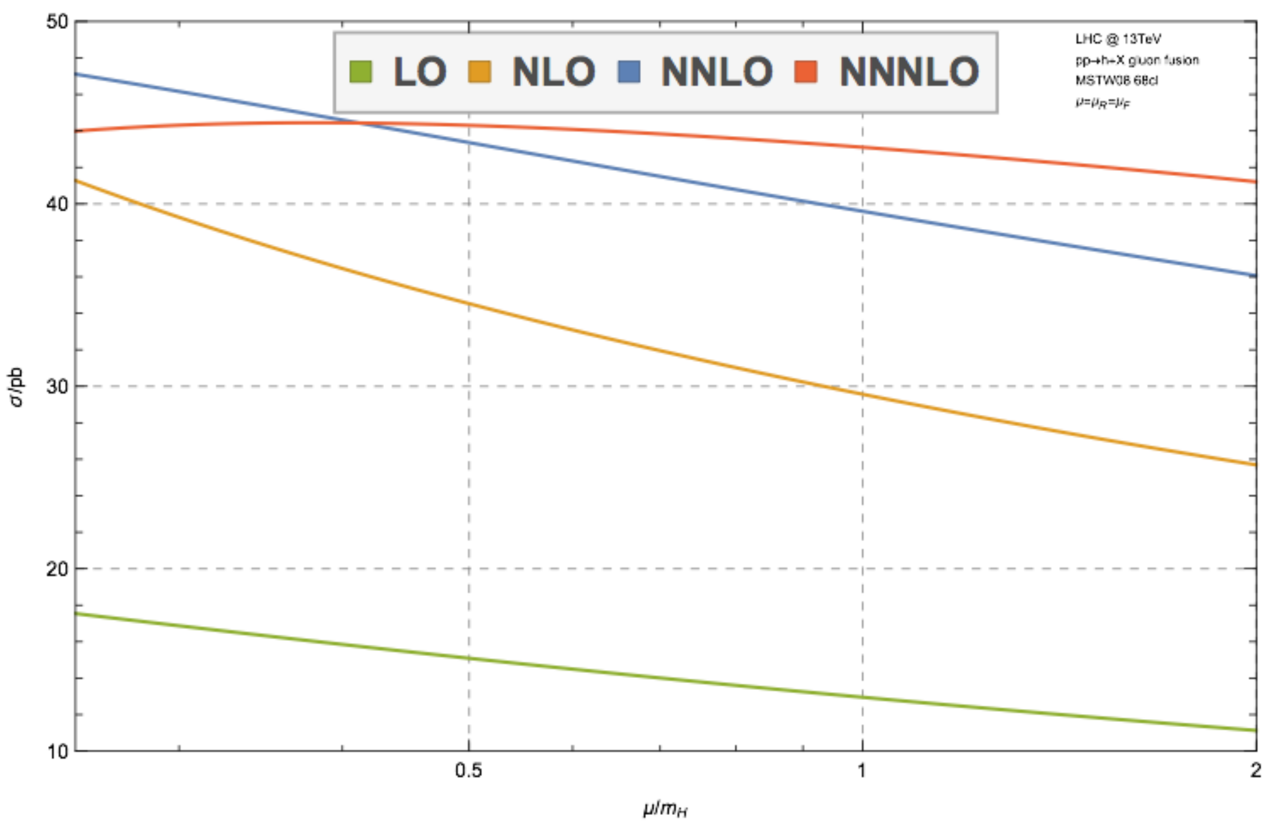
[C.Anastasiou, C.Duhr, F.Dulat, F.Herzog, B.Mistlberger (2015)]

Full calculation for the gg →H completed through the evaluation of 30 terms in the soft-expansion: first ever complete calculation at N3LO in hadronic collisions.

Significant reduction of uncertainties from missing higher orders and PDF+α_s

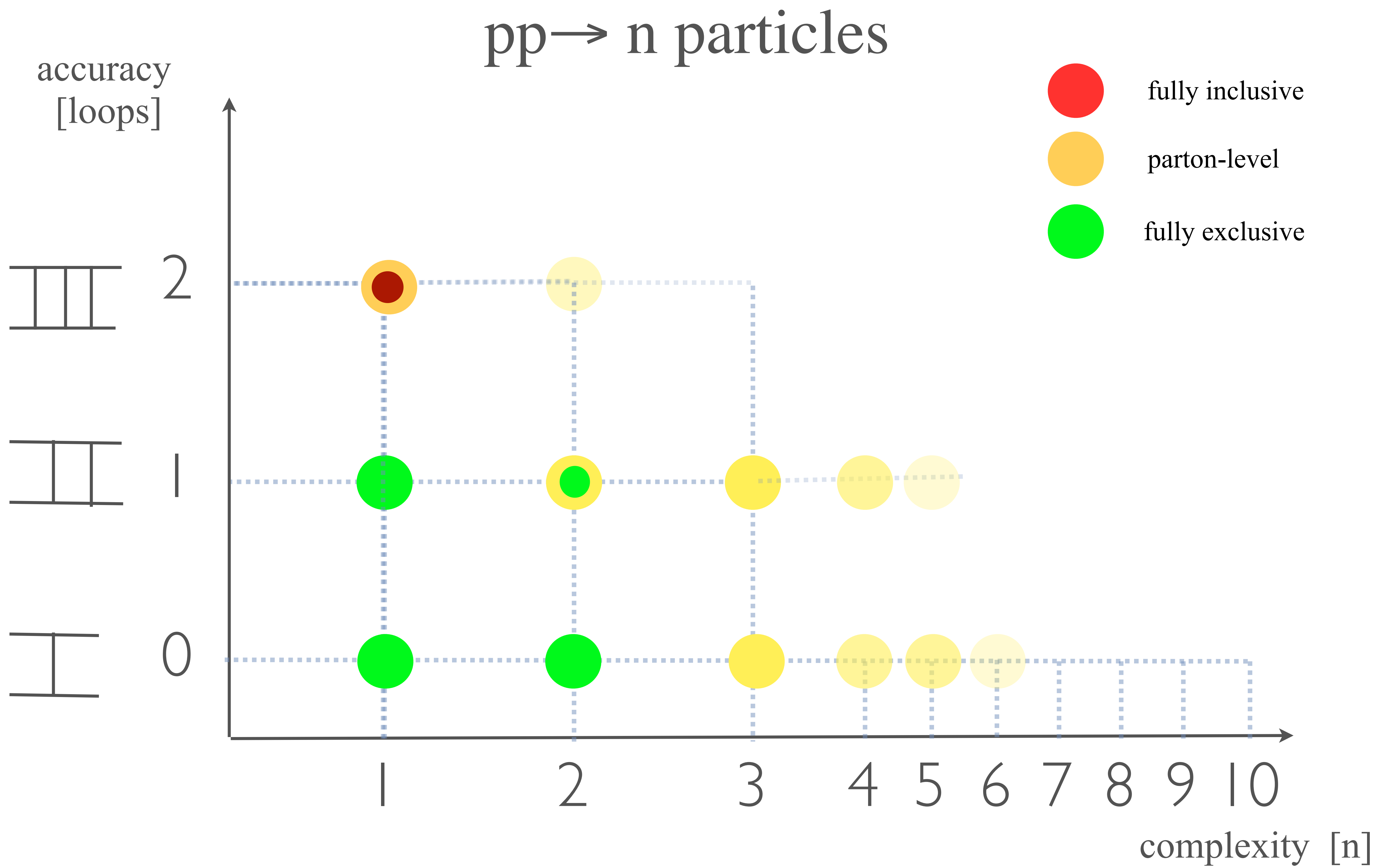
Scale dep. stabilizes around $\mu = m_H/2$

N3LO effect +2.2% at $\mu = m_H/2$

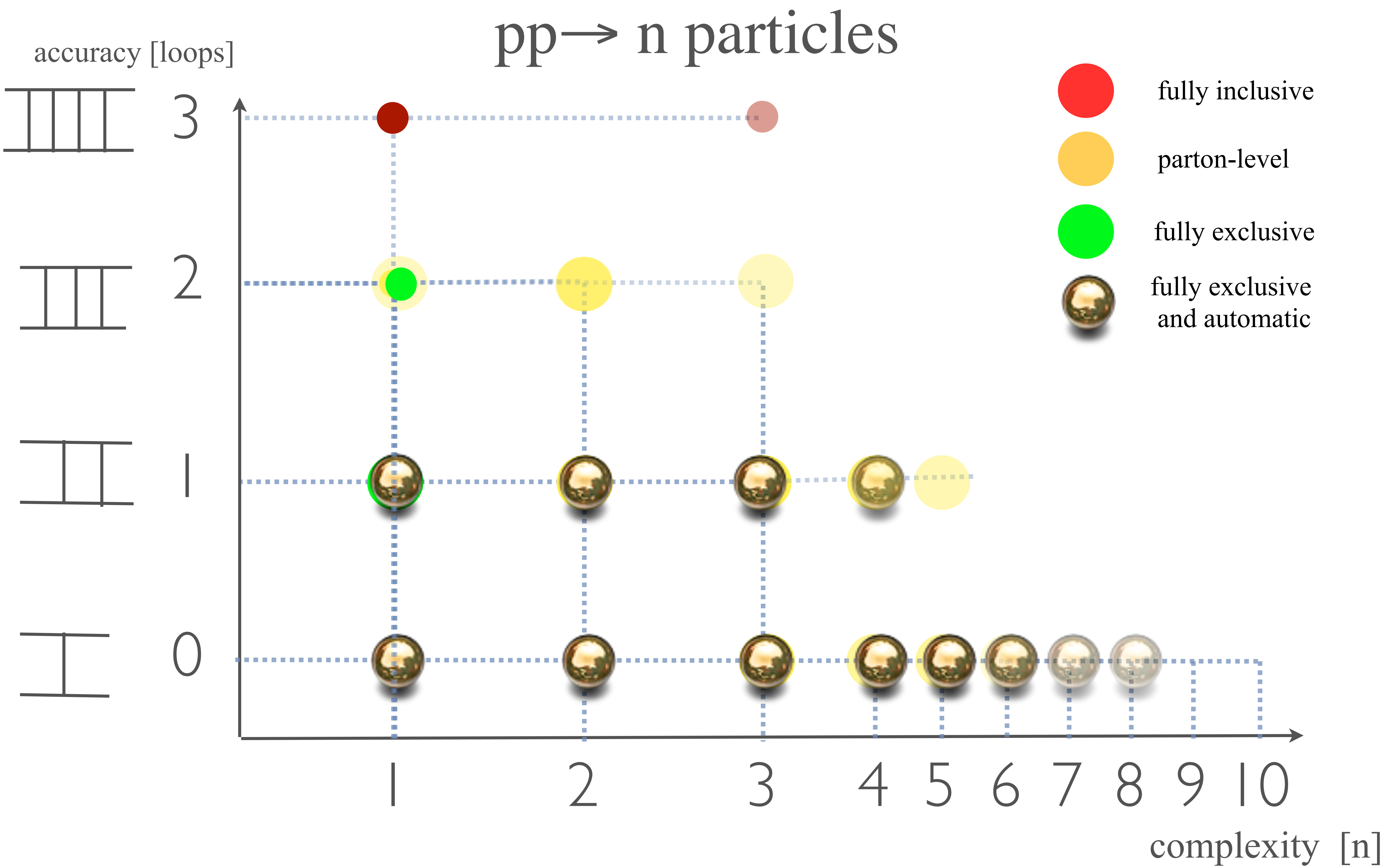


Corresponding new results for the Higgs cross section including mass effects at NLO and the other known corrections at 13 TeV expected soon.

Predictions in QCD: before the LHC



Predictions in QCD for the LHC: status 2016



Summary

- The LHC physics program demands predictions at an unprecedented level of accuracy and precision.
- Rapid and impressive progress in techniques in the last few years has lead to:
  - Full automation of the computation of NLO QCD corrections and their matching/merging with parton shower program: experimental grade predictions are now available for SM and BSM (resonant and in EFTs). Automatic NLO EW is being achieved now.
  - The new era of differential predictions at NNLO in QCD for a every-day increasing set of important SM processes 2→2, such as H+jet, V+jet, VV, t tbar production. In addition first exploration of NNLO+PS for 2→1 process has started.
  - Moving the frontier to N3LO.
- Main outcomes:
  - Progress in understanding of QCD and pp collisions at high Q^2
  - Room for experimentalists to make unprecedented SM and BSM studies

Rapidity and pseudorapidity

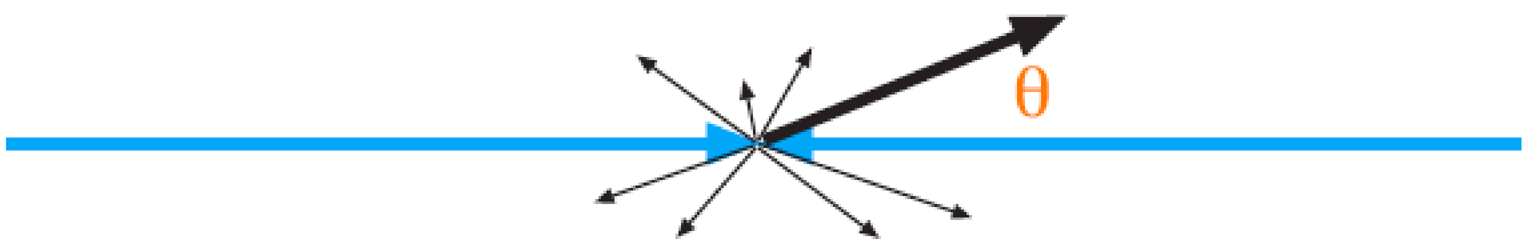
$$y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\log\frac{p^+}{p^-}$$

RAPIDITY

PSEUDORAPIDITY

with

$$\tan\theta = \frac{p_T}{p_z}$$

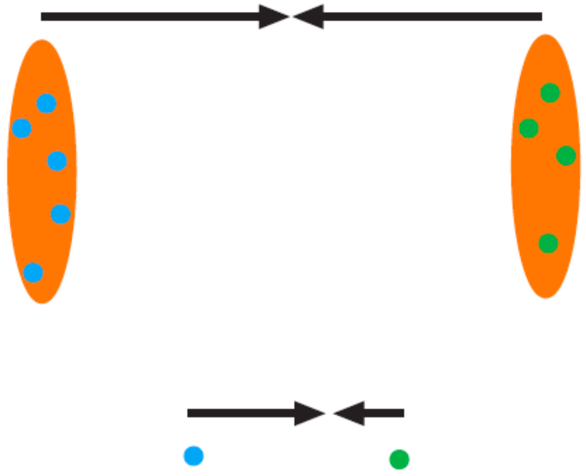


1. Rapidity transforms additively under a Lorentz boost : y→y'=y+ω
2. Rapidity differences are Lorentz invariants : Δy→Δy'
3. Pseudo rapidity has a direct experimental definition but no special properties under the Lorentz boosts.
4. For massless particles rapidity and pseudo rapidity are the same.

pp kinematics

We describe the collision in terms of parton energies

$E_1 = x_1$ Ebeam
$E_2 = x_2$ Ebeam

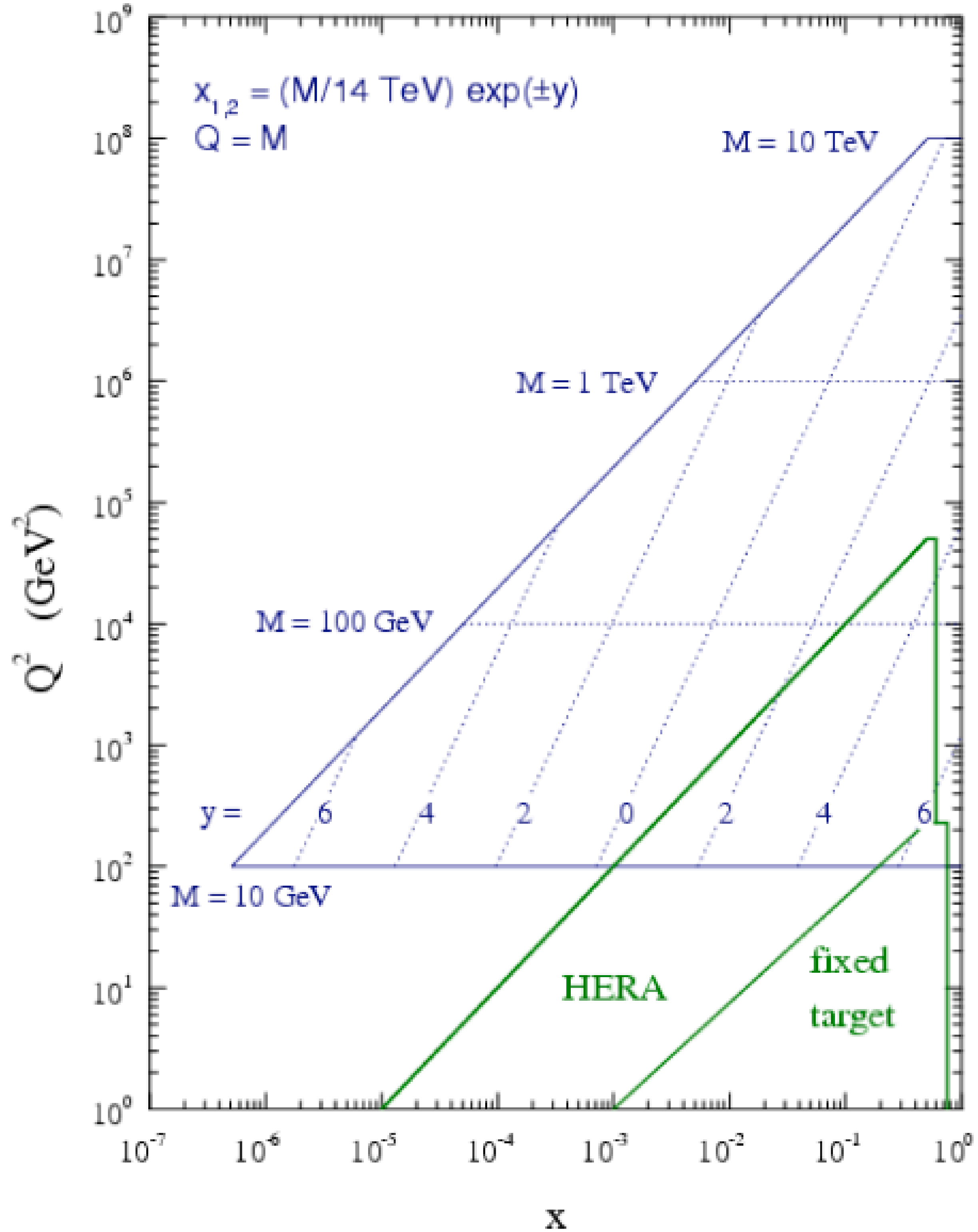


Obviously the partonic c.m.s. frame will be in general boosted. Let us say that the two partons annihilate into a particle of mass M.

$$M^2 = x_1 x_2 S = x_1 x_2 4 E^2_{\mathrm{beam}}$$

$$y = \frac{1}{2} \log \frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{S}} e^{y} \quad x_2 = \frac{M}{\sqrt{S}} e^{-y}$$



Try out a "simple" NLO calculation yourself

Hands-on!

pp→Higgs+x at NLO

- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

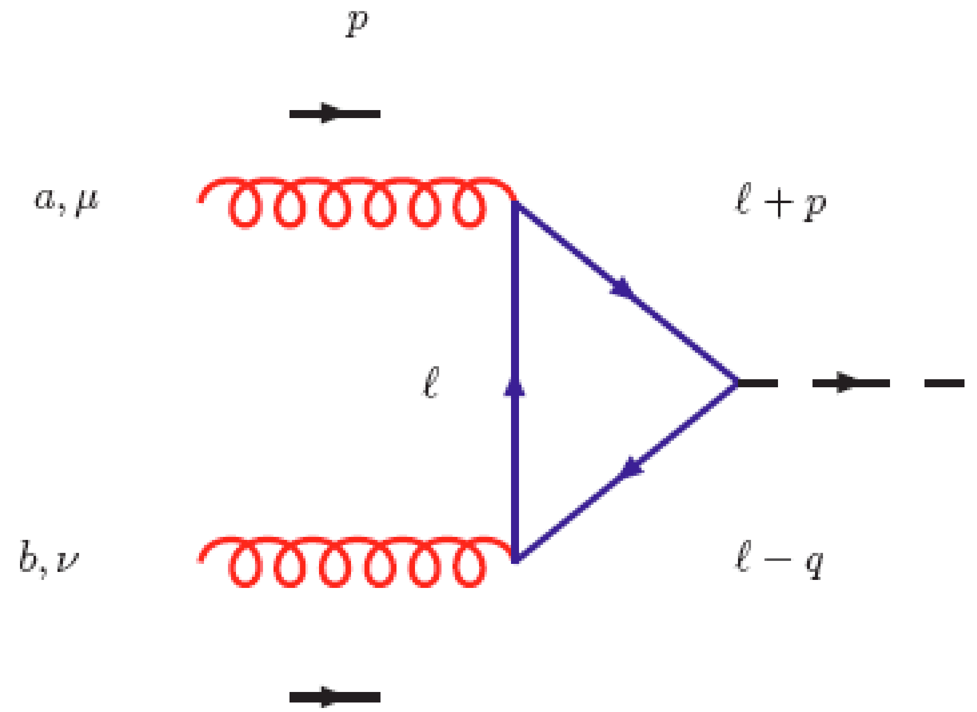
Write-up can be found HERE

pp→H at LO

This is a "simple" 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!
Let's do the calculation!



$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b) \left(\frac{-im_t}{v}\right) \int \frac{d^d\ell}{(2\pi)^n} \frac{T^{\mu\nu}}{\mathrm{Den}} (i)^3 \epsilon_\mu(p) \epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

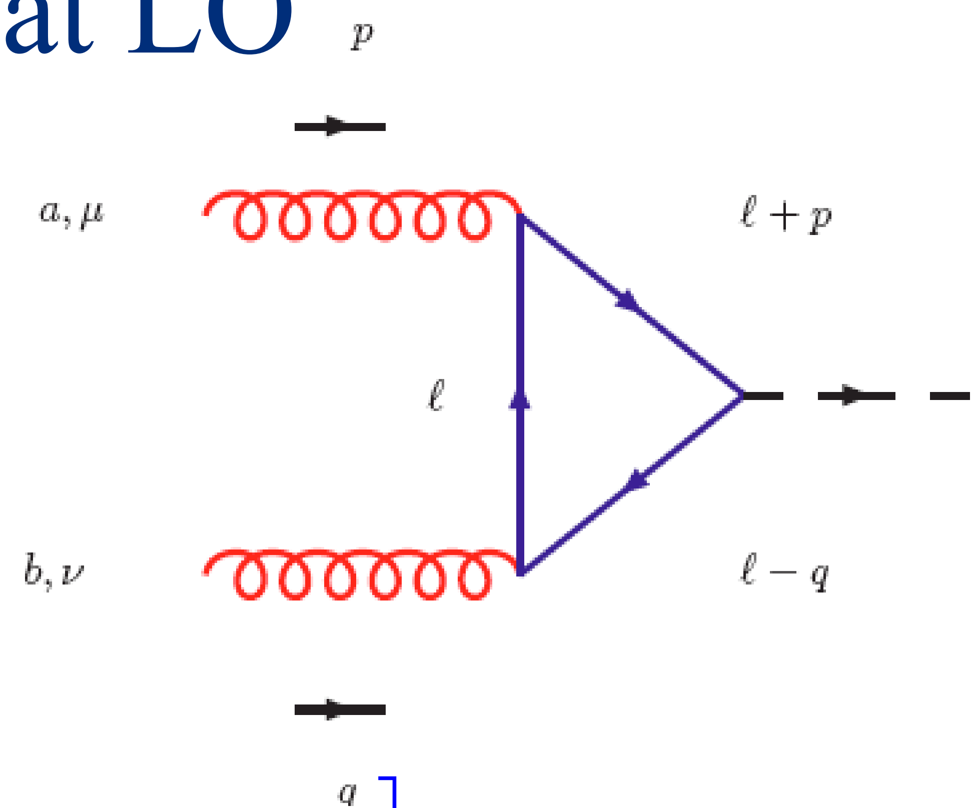
$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell\cdot(px - qy)]^3}.$$

pp→H at LO

We shift the momentum:

$$\ell' = \ell + px - qy$$

$$\frac{1}{\text{Den}} \to 2\int dx\, dy \frac{1}{[\ell'^{\,2} - m_t^2 + M_H^2 xy]^3}.$$



And now the tensor in the numerator:

$$T^{\mu\nu} = \text{Tr}\left[(\ell + m_t)\gamma^\mu(\ell + p + m_t)(\ell - q + m_t)\gamma^\nu)\right]$$

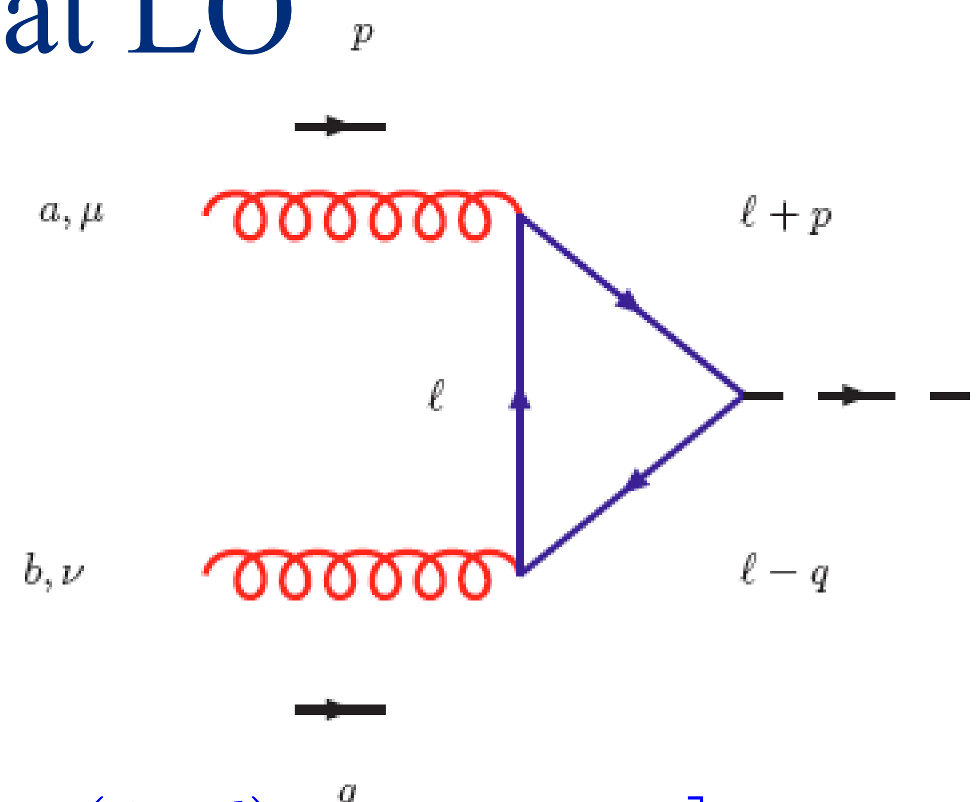
$$= 4m_t\left[g^{\mu\nu}(m_t^2 - \ell^2 - \frac{M_H^2}{2}) + 4\ell^\mu\ell^\nu + p^\nu q^\mu\right]$$

where I used the fact that the external gluons are on-shell. This trace is proportional to mt ! This is due to the spin flip caused by the scalar coupling.

Now we shift the loop momentum also here, we drop terms linear in the loop momentum (they are odd and vanish) and

pp→H at LO

We perform the tensor decomposition using:



$$\int d^dk \frac{k^\mu k^\nu}{(k^2-C)^m} = \frac{1}{d} g^{\mu\nu} \int d^dk \frac{k^2}{(k^2-C)^m}$$

So I can write an expression which depends only on scalar loop integrals:

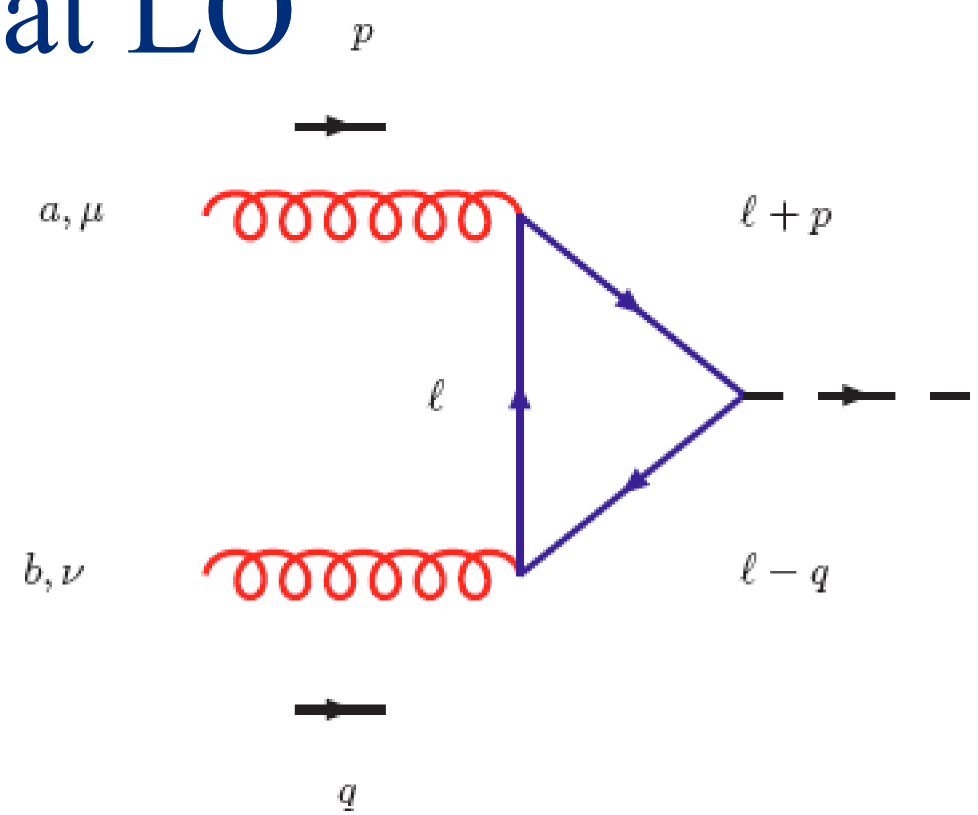
$$i\mathcal{A} = -\frac{2g_s^2 m_t^2}{v}\delta^{ab} \int \frac{d^d\ell'}{(2\pi)^d} \int dxdy \Big\{ g^{\mu\nu}\left[m^2 + \ell'^2\left(\frac{4-d}{d}\right) + M_H^2(xy-\frac{1}{2})\right]$$
$$+p^\nu q^\mu (1-4xy)\Big\} \frac{2dxdy}{(\ell'^2-m_t^2+M_H^2 xy)^3}\epsilon_\mu(p)\epsilon_\nu(q).$$

There's a term which apparently diverges....??

Ok, Let's look the scalar integrals up in a table (or calculate them!)

pp→H at LO

$$\int \frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$
$$\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-C)^3} = -\frac{i}{32\pi^2}(4\pi)^\epsilon\Gamma(1+\epsilon)C^{-1-\epsilon}.$$



where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg\to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2}-p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2-m_H^2xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is m_t^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on m_t and m_h.

pp→H at LO

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1,\mu_f) g(x_2,\mu_f)\, \hat{\sigma}(gg \to H)$$

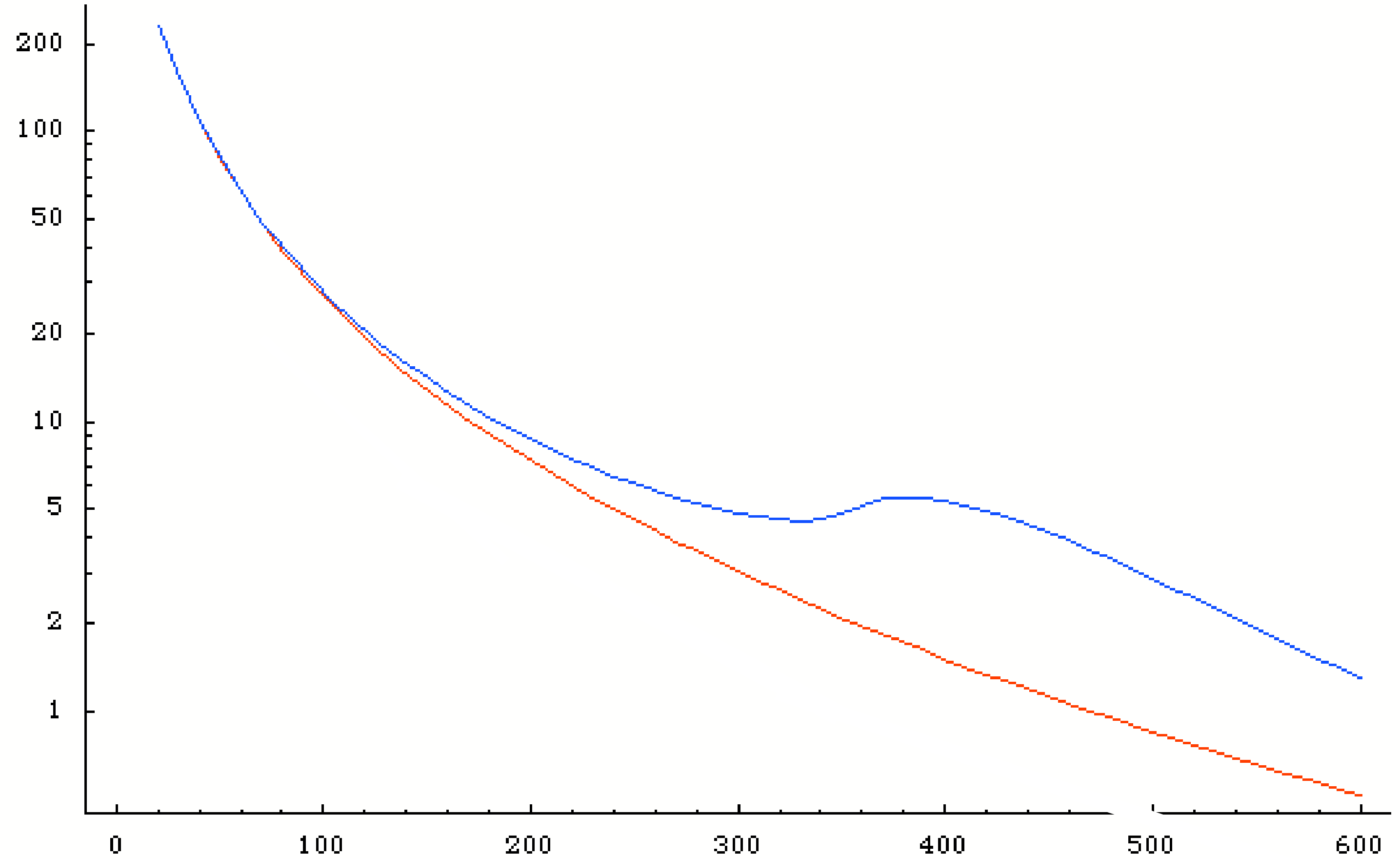
$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \left| I\left(\frac{M_H^2}{m^2}\right) \right|^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy\, g(\sqrt{\tau_0} e^{y}) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at $m_h = 2m_t$.

This causes a bump in the cross section.

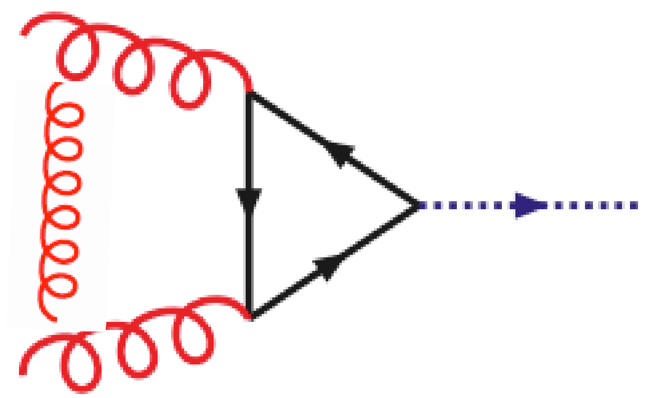


EFT

At NLO we have to include an extra parton (virtual or real).

The virtuals will become a two-loop calculation!!

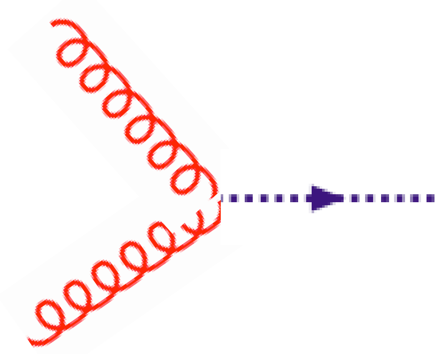
Can we avoid that?



Let's consider the case where the Higgs is light:

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} \quad -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



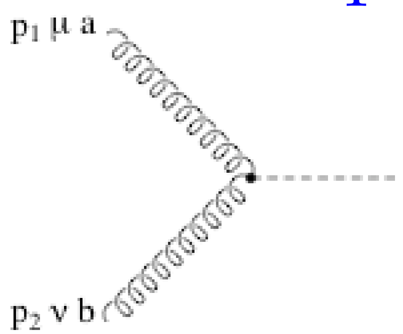
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

EFT

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1-\frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu}G_{\mu\nu}$$

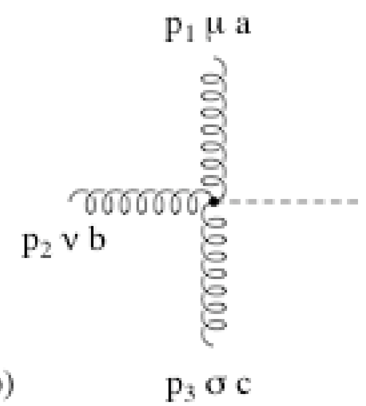
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



(a)

$iA\delta^{ab}H^{\mu\nu}(p_1,p_2)$

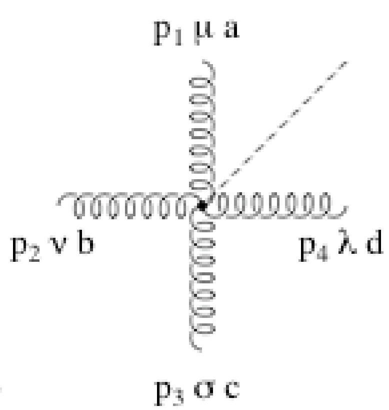
$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu}p_1\cdot p_2 - p_1^\nu p_2^\mu .$$



(b)

$-Agf^{abc}V^{\mu\nu\sigma}(p_1,p_2,p_3)$

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu},$$



(c)

$-iAg^2X^{abcd}_{\mu\nu\sigma\lambda}$

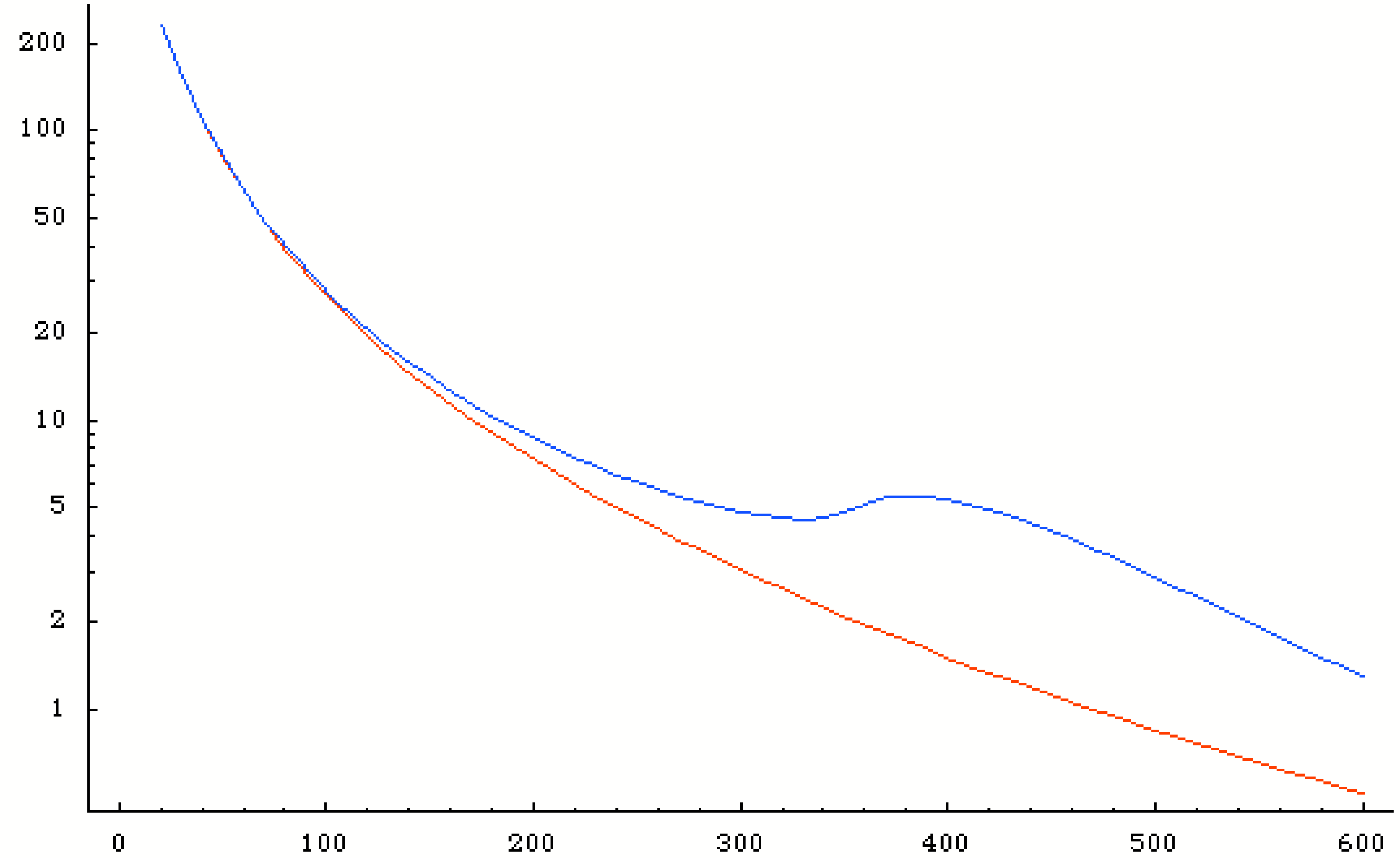
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} = & f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ & + f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

EFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1, \mu_f) g(x_2, \mu_f)\, \hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

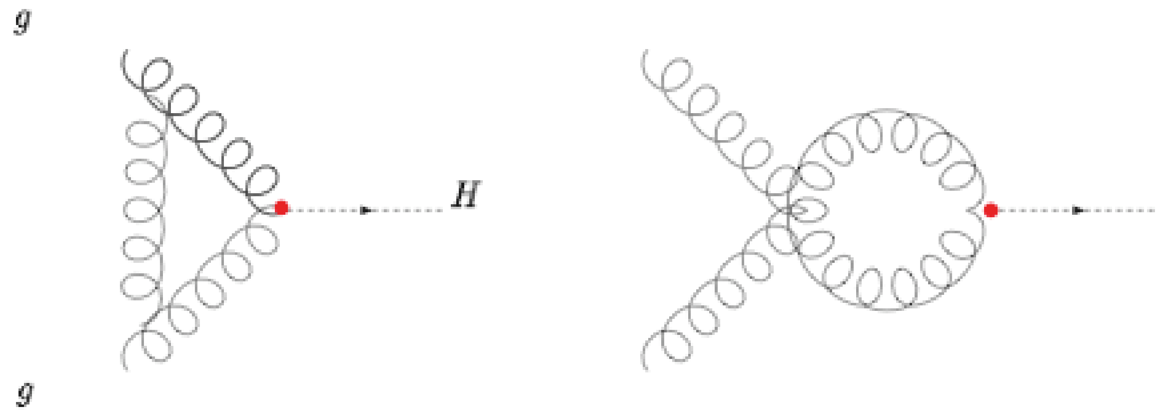
For light Higgs is better than 10%.



So, if we are interested in a light Higgs we use the HEFT and simplify our life. If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can (try to) do it!!

pp→H at NLO in the EFT



Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

Then the integration over the tensor decomposition into scalar integrals and loop integration has to be performed.

$$\mathcal{L}_{\text{eff}}^{\text{NLO}} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu}$$

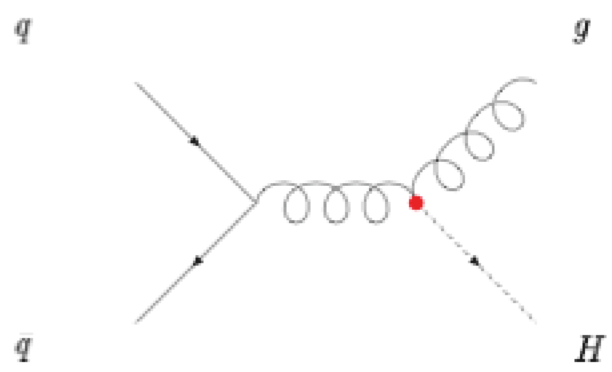
One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

The result is:

$$\sigma_{\text{virt}} = \sigma_0\,\delta(1-z)\,\left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right],$$

$$\sigma_{\text{Born}} = \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1+\epsilon+\epsilon^2)\mu^{2\epsilon}\,\delta(1-z) \equiv \sigma_0\,\delta(1-z) \qquad z = m_H^2/s$$

pp→H at NLO in the EFT



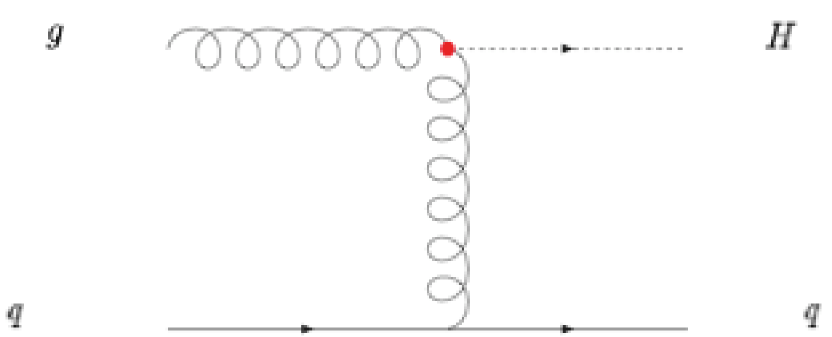
$$\overline{|\mathcal{M}|^2} = \frac{4}{81}\frac{\alpha_S^3}{\pi v^2}\,\frac{(\hat{u}^2+\hat{t}^2)-\epsilon(\hat{u}+\hat{t})^2}{\hat{s}}$$

Integrating over phase space (cms angle theta)

$$\hat{t} = -\hat{s}(1-z)(1-\cos\theta)/2$$
$$\hat{u} = -\hat{s}(1-z)(1+\cos\theta)/2$$

$$\sigma_{\text{real}}(q\bar{q}) = \sigma_0\frac{\alpha_S}{2\pi}\frac{64}{27}\frac{(1-z)^3}{z}$$

finite!



$$\overline{|\mathcal{M}|^2} = -\frac{1}{54(1-\epsilon)}\frac{\alpha_S^3}{\pi v^2}\,\frac{(\hat{u}^2+\hat{s}^2)-\epsilon(\hat{u}+\hat{s})^2}{\hat{t}}$$

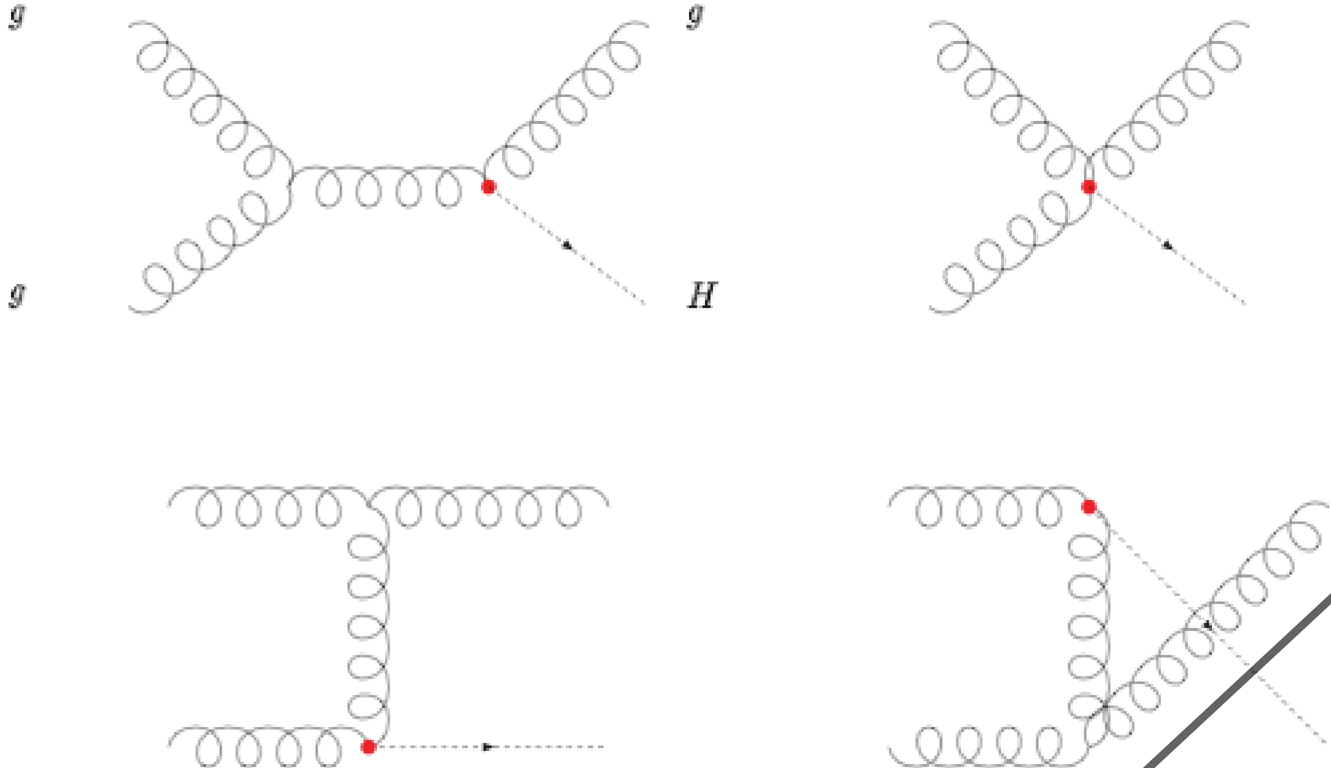
Integrating over the D-dimensional phase space the collinear singularity manifests a pole in 1/eps

$$\sigma_{\text{real}} = \sigma_0\frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon}c_\Gamma\left[-\frac{1}{\epsilon}p_{gq}(z)+\frac{(1-z)(7z-3)}{2z}+p_{gq}(z)\log\frac{(1-z)^2}{z}\right]$$

$$\sigma^{\text{coll.}}_{\text{c.t.}} = \sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gq}(z)\right]$$

$$\begin{aligned}\sigma^{\overline{\text{MS}}}(qg) &= \sigma_{\text{real}}+\sigma^{\text{coll.}}_{\text{c.t.}}\\ &= \sigma_0\frac{\alpha_S}{2\pi}C_F\left[p_{gq}(z)\log\frac{m_H^2}{\mu_F^2}+p_{gq}(z)\log\frac{(1-z)^2}{z}+\frac{(1-z)(7z-3)}{2z}\right]\end{aligned}$$

pp→H at NLO in the EFT



This is the last piece: the result at the end must be finite!

2/eps cancels with the virtual contribution ✓

This is the renormalization of the coulping!!

$$\sigma^{\rm UV}_{\rm c.t.} = 2\,\sigma_{\rm Born}\frac{\alpha_S}{2\pi}\left[-\left(\frac{\mu^2}{\mu^2_{\rm UV}}\right)^{\epsilon} c_\Gamma\frac{b_0}{\epsilon}\right] \checkmark$$

This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma^{\rm coll.}_{\rm c.t.} = 2\,\sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu^2_F}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gg}(z)\right] \checkmark$$

$$\begin{aligned}\sigma_{\rm real} \;=\; & \sigma_0\,\frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^{\epsilon} c_\Gamma\left[\left(\frac{2}{\epsilon^2}+\frac{2}{\epsilon}\frac{b_0}{C_A}-\frac{\pi^2}{3}\right)\delta(1-z)\right. \\ & -\frac{2}{\epsilon}p_{gg}(z)-\frac{11}{3}\frac{(1-z)^3}{z}-4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)}\log z \\ & \left.+\,4\frac{1+z^4+(1-z)^4}{z}\left(\frac{\log(1-z)}{1-z}\right)_+\right].\end{aligned}$$

pp→H at NLO in the EFT

$$\sigma(pp \to H) = \sum_{ij} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_i(x_1,\mu_f) f_j(x_2,\mu_f) \hat{\sigma}(ij)[\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

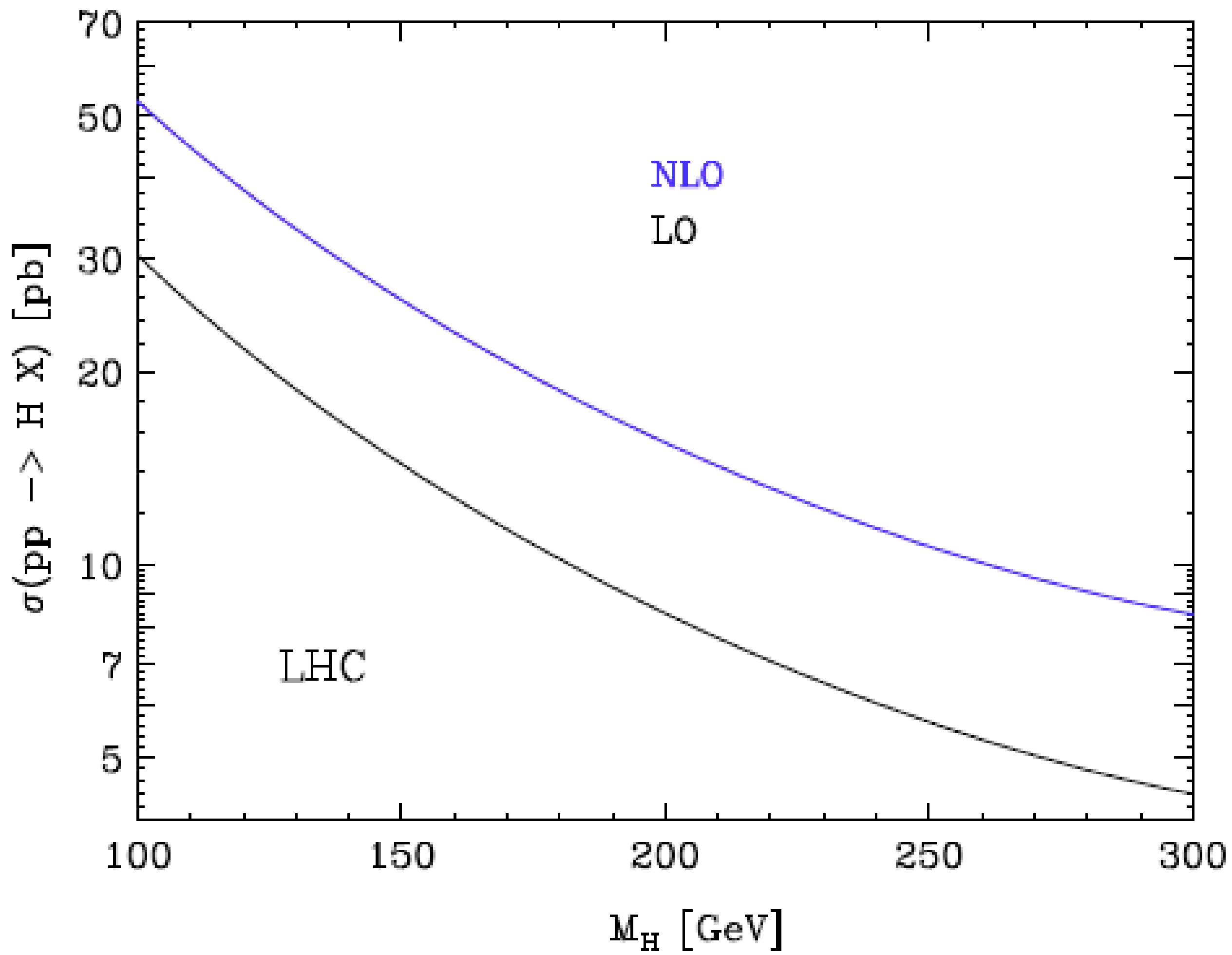
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~2 and scale dependence not really very much improved.

Is perturbation theory valid? NNLO is mandatory...



pp→H at NLO in the EFT

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