Dmitrij Siemens

Pion-Nucleon Scattering in Chiral Perturbation Theory

in collaboration with V. Bernard, E. Epelbaum, A. Gasparyan, M. Hoferichter, J. Gegeila,
H. Krebs, B. Kubis, U.-G. Meißner, J. Elvira de Ruiz, D. Yao

- Δ-less formulation
- Δ-ful formulation
- Subthreshold matching

Motivation and Methodology

Aim	Theoretical description of πN→πN and πN→ππN above threshold
Problem I	QCD is non-perturbative for low energies
Solution I	Effective Field Theory ⇒ Chiral Perturbation Theory
Problem II	Resonances play an important role
Solution II	Inclusion of the most dominant resonance Δ(1232) as an explicit degree of freedom

Chiral Approaches

BχPT

- EFT of Standard Model
- Relies upon chiral symmetry of QCD
- DOF are mesons and baryons instead of quarks
- Breakdown scale of theory: Λ_b

HBχPT

- Non-relativistic limit of χPT
- Inclusion of $1/m_N$ expansion into power counting
- HB-πN : $q/m_N \sim q/\Lambda_b$ HB-NN : $q/m_N \sim (q/\Lambda_b)^2$
- Original motivation: calculations beyond tree-level

Formal Aspects

BχPT & HBχPT

Effective Lagrangian

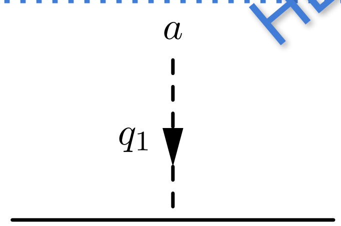
$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

$$Q = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right\}$$

χPT

HBχPT

Examples



Effective Lagrangian

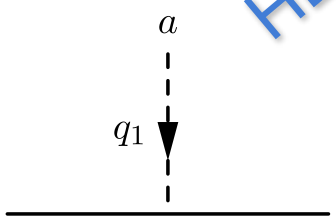
$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N}$$

$$Q = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right\}$$

Examples

χPT

$$-\frac{g_A}{2F_\pi} \not{q}_1 \gamma_5 \, \tau^a$$



HBχPT

$$-\frac{g_A}{F_\pi} S \cdot q_1 \, \tau^a$$

BχPT & HBχPT

Effective Lagrangian

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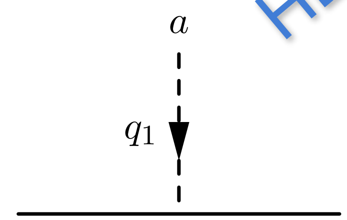
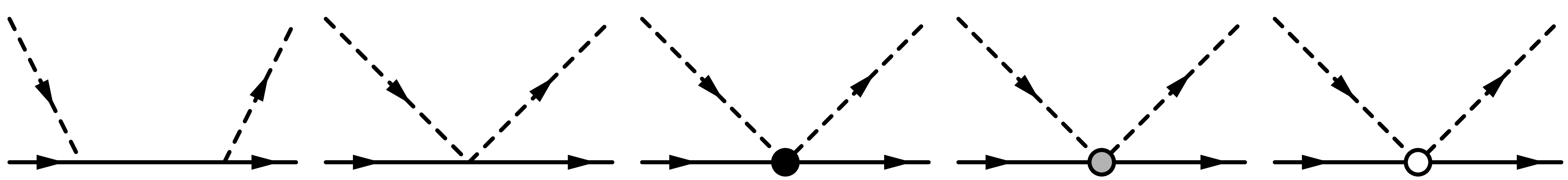


HBχPT

$$-\frac{g_A}{F_\pi} S \cdot q_1 \tau^a$$

Tree Graphs



BχPT & HBχPT

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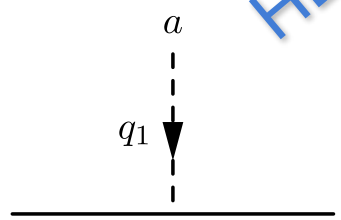

HBχPT

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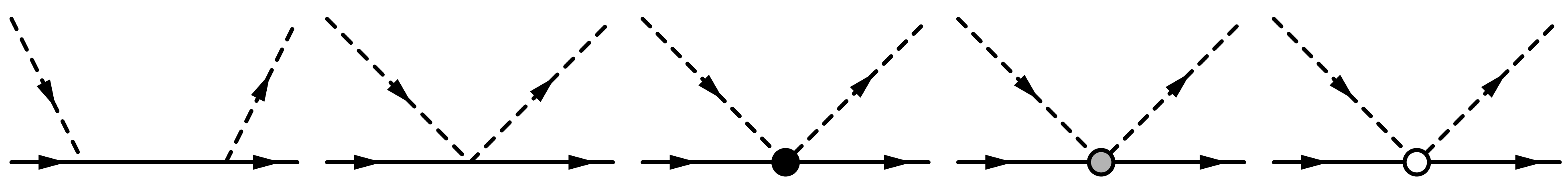


~ $c_{1,2,3,4}$

BχPT & HBχPT

Effective Lagrangian

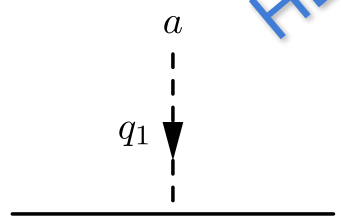
$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \boxed{\mathcal{L}^{(3)}_{\pi N}} + \mathcal{L}^{(4)}_{\pi N}$$

$$Q = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b} \right\}$$

Examples

χPT

$$-\frac{g_A}{2F_\pi} \not{q}_1 \gamma_5 \tau^a$$



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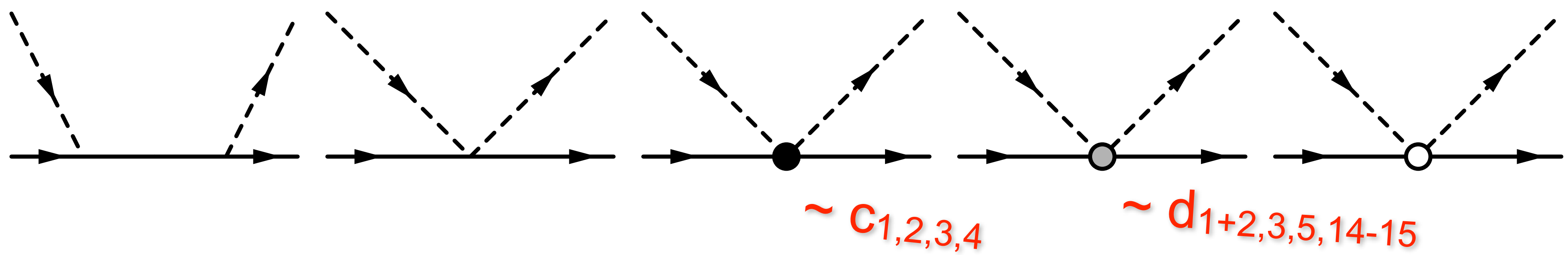

BχPT & HBχPT

Effective Lagrangian

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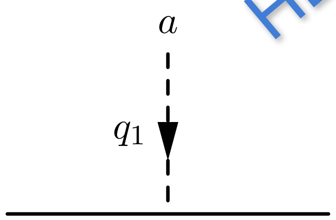
Examples

χPT

HBχPT

$$-\frac{g_A}{2F_\pi} \not{q}_1 \gamma_5 \tau^a$$



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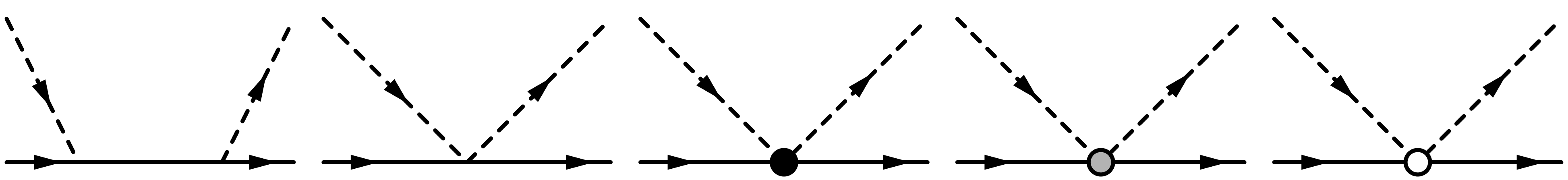
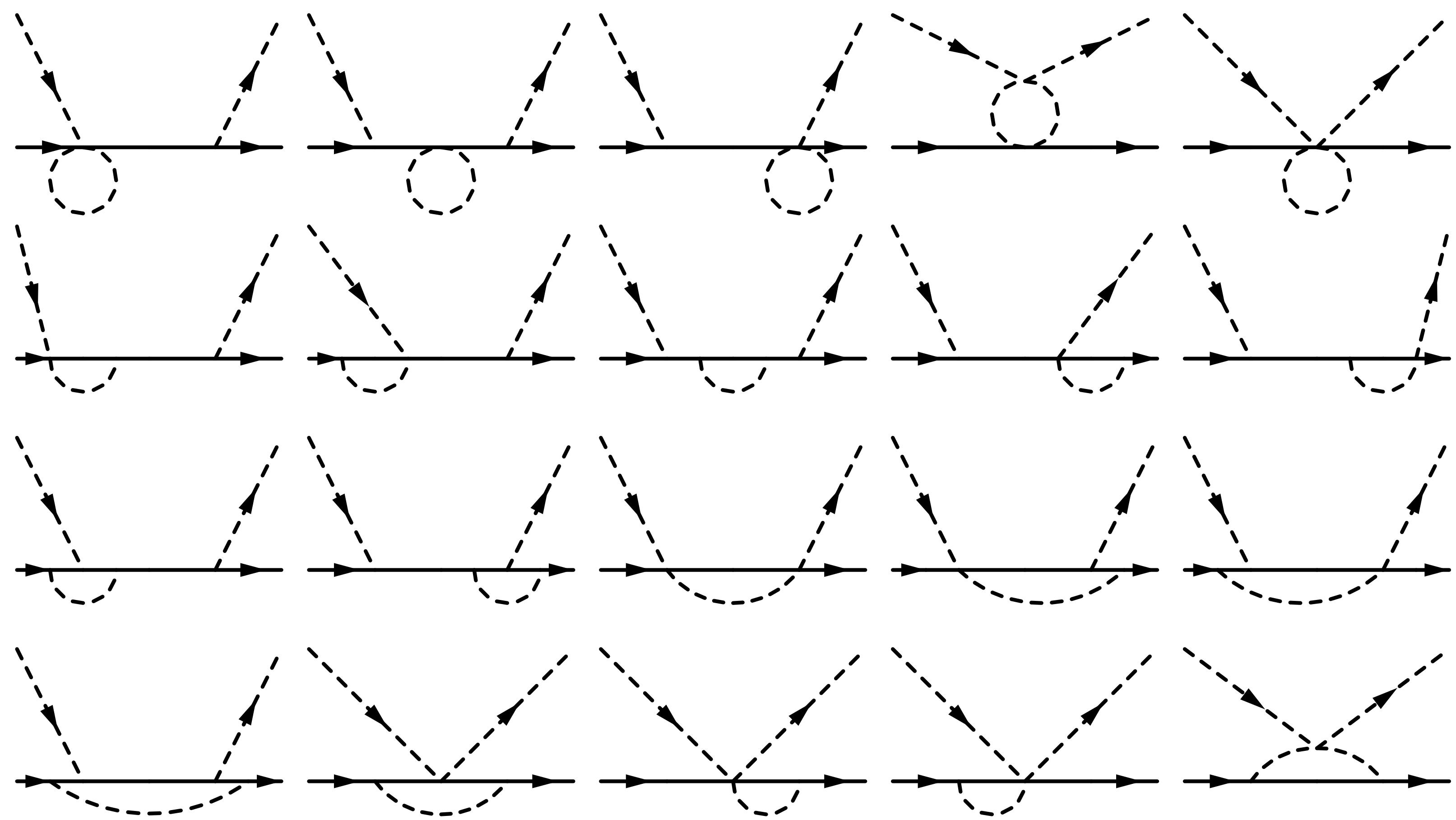
Tree Graphs

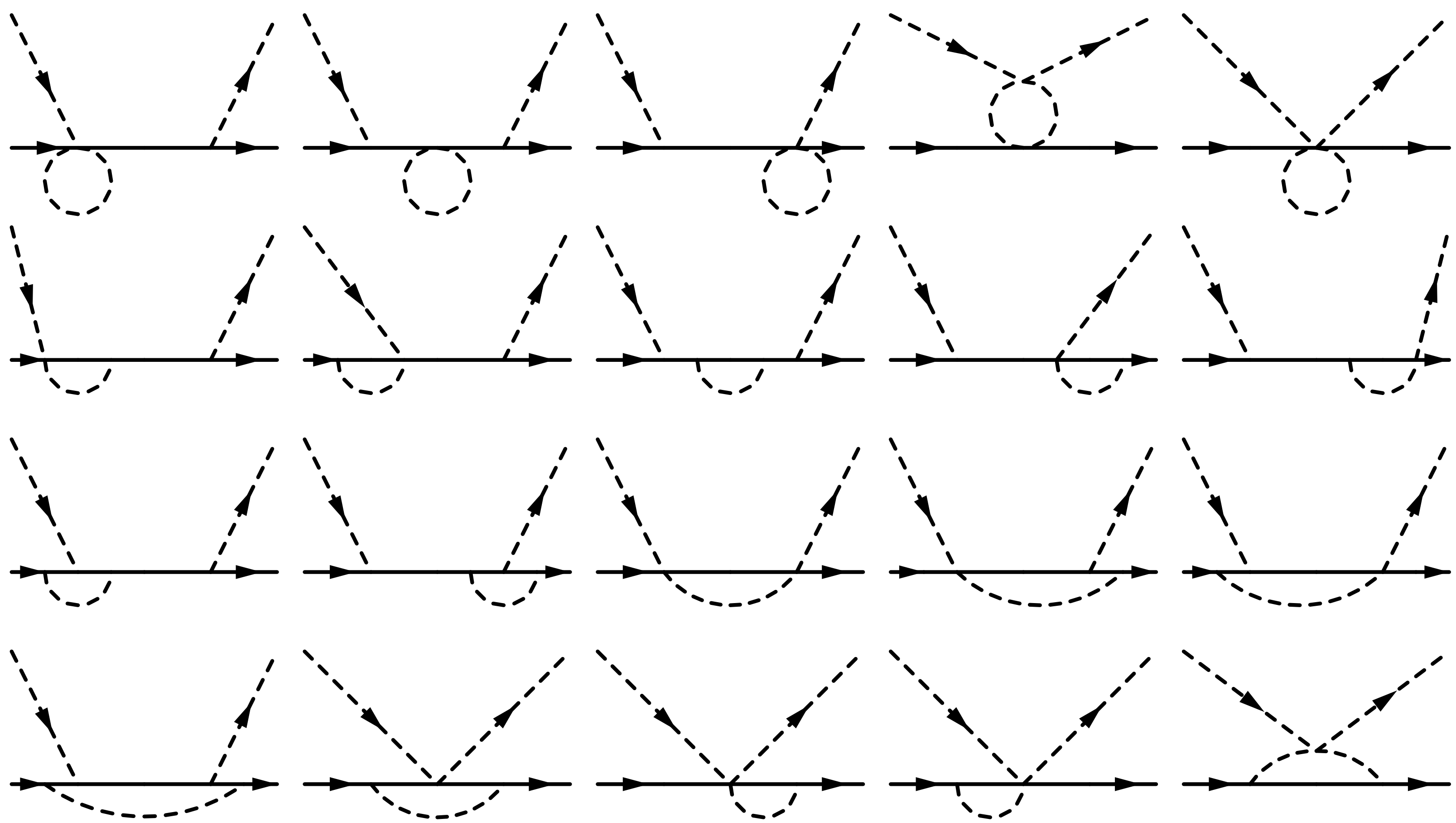


~ $c_{1,2,3,4}$ ~ $d_{1+2,3,5,14\text{-}15}$ ~ $e_{14,15,16,17,18}$

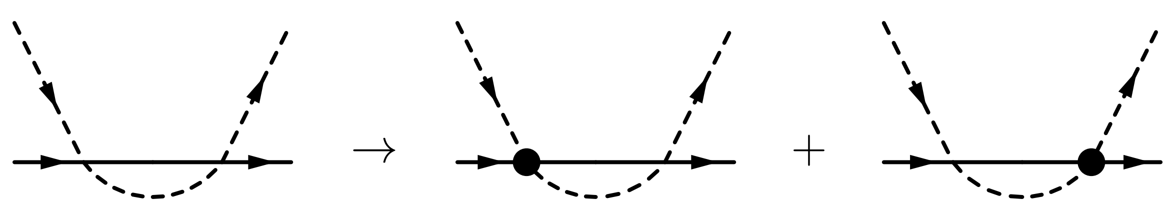
Loop Graphs



Loop Graphs



Transition from LO loops to NLO loops

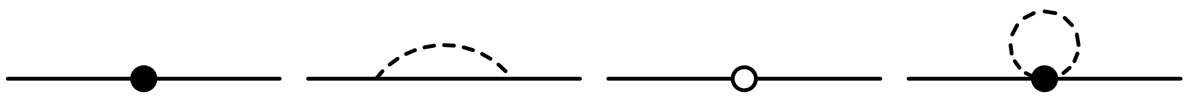


Renormalization I

Meson Sector

$$M^2 = M_\pi^2 + \delta M^{(4)}$$
$$Z_\pi = 1 + \delta Z_\pi^{(4)}$$
$$F = F_\pi + \delta F_\pi^{(4)}$$

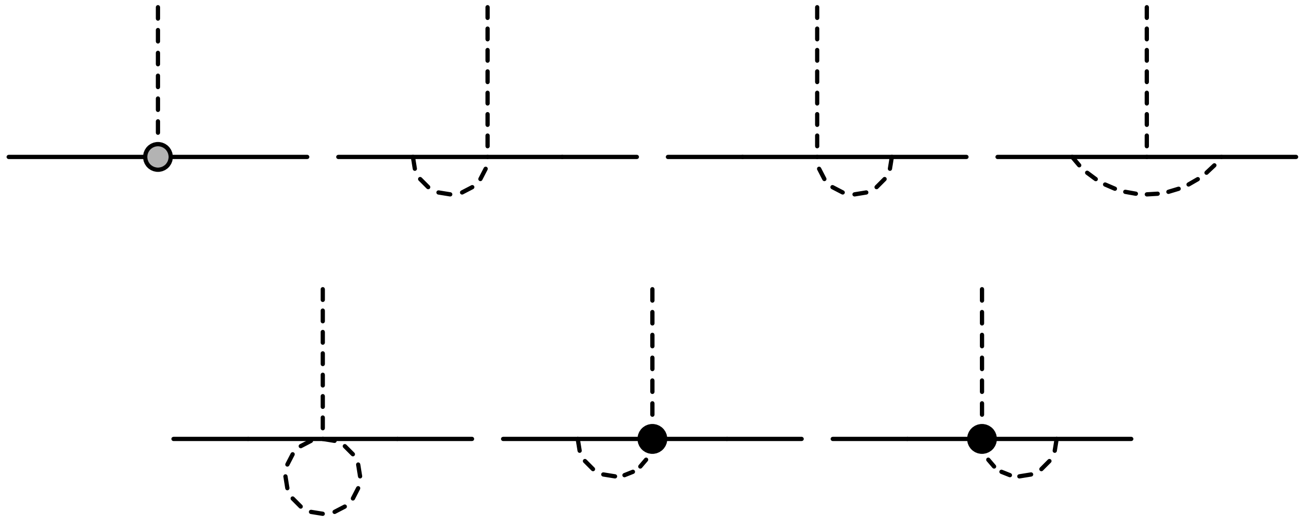
Nucleon Self Energy



$$m = m_N + \delta m^{(2)} + \delta m^{(3)} + \delta m^{(4)}$$

$$Z_N = 1 + \delta Z_N^{(3)} + \delta Z_N^{(4)}$$

Axial-coupling constant



$$g = g_A + \delta g^{(3)} + \delta g^{(4)}$$

GTR $$\frac{g_{\pi NN} F_\pi}{m_N} = g_A - 2M_\pi^2 d_{18} + \mathcal{O}(Q^5)$$

Linear Combinations

$$\bar{c}_1 \to \bar{c}_1 + 2M_\pi^2(\bar{e}_{22} - 4\bar{e}_{38} + \bar{c}_1 \beta_{l_3} \bar{l}_3/(32\pi^2 F_\pi^2))$$
$$\bar{c}_2 \to \bar{c}_2 - 8M_\pi^2(\bar{e}_{20} + \bar{e}_{35})$$
$$\bar{c}_3 \to \bar{c}_3 - 4M_\pi^2(2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36})$$
$$\bar{c}_4 \to \bar{c}_4 - 4M_\pi^2(2\bar{e}_{21} - \bar{e}_{37})$$

Renormalization II

Meson Sector

$$l_i = \frac{\beta_{l_i}}{32\pi^2}\bar{l}_i + \beta_{l_i}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$\bar{\lambda} = \frac{1}{16\pi^2}\left(\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi)\right)$$

HB approach

$$d_i = \bar{d}_i + \delta d_i = \bar{d}_i + \frac{\beta_{d_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$e_i = \bar{e}_i + \delta e_i = \bar{e}_i + \frac{\beta_{e_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

Covariant “modified“ EOMS scheme

$$c_i = \bar{c}_i + \delta c_i^{(3)} + \delta c_i^{(4)}$$

$$d_i = \bar{d}_i + \delta d_i + \delta d_i^{(3)} + \delta d_i^{(4)}$$

$$e_i = \bar{e}_i + \delta e_i + \delta e_i^{(4)}$$

$$x \in \{c, d, e\}$$

$$\delta x_i^{(n)} = \frac{\delta \bar{x}_{i,f}^{(n)}}{F_\pi^2} + \frac{\beta_{x_i,B}^{(n)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{m_N^2}{\mu^2}\right)\right)$$

Fits to Experimental Data

Phase Shifts

$$T^{ba} = \chi^{\dagger}_{N'} \left(\delta^{ab} T^{+} + \mathrm{i}\epsilon^{bac} \tau_c T^{-} \right) \chi_N$$

χPT

$$T^{\pm} = \bar{u}^{(s')} \left(A^{\pm} + \not{q} B^{\pm} \right) u^{(s)}$$

$$f^I_{l\pm}(s) = \frac{1}{16\pi\sqrt{s}} \Big((E + m_N) \left(A^I_l(s) + (\sqrt{s} - m_N) B^I_l(s) \right) + (E - m_N) \left(-A^I_{l\pm}(s) + (\sqrt{s} + m_N) B^I_{l\pm} \right) \Big)$$

$$X^I_l(s) = \int_{-1}^{1} \mathrm{d}z \, X^I(s,t) P_l(z)$$

$$X \in \{A, B\}$$

HBχPT

$$T^{\pm} = \bar{u}^{(s')}_v \left(g^{\pm} + 2\mathrm{i}\, S \cdot q \times q' h^{\pm} \right) u^{(s)}_v$$

$$f^I_{l\pm}(s) = \frac{E + m_N}{16\pi\sqrt{s}} \int_{-1}^{1} \mathrm{d}z \, \left(g^I P_l(z) + \boldsymbol{q}^2 h^I (P_{l\pm}(z) - z P_l(z)) \right)$$

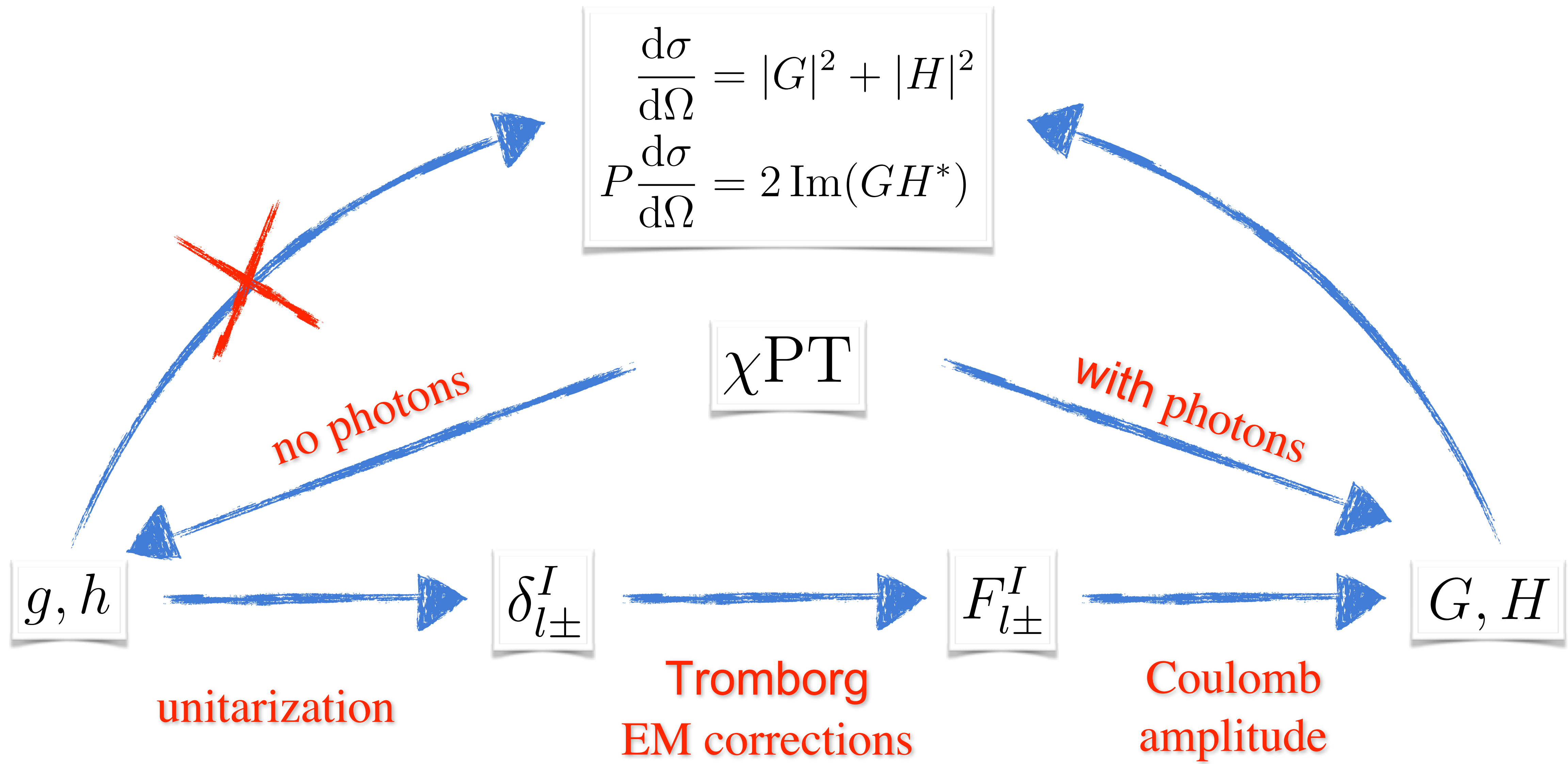
Isospin basis

$$X^{I=1/2} = X^+ + 2X^- \,, \quad X^{I=3/2} = X^+ - X^-$$

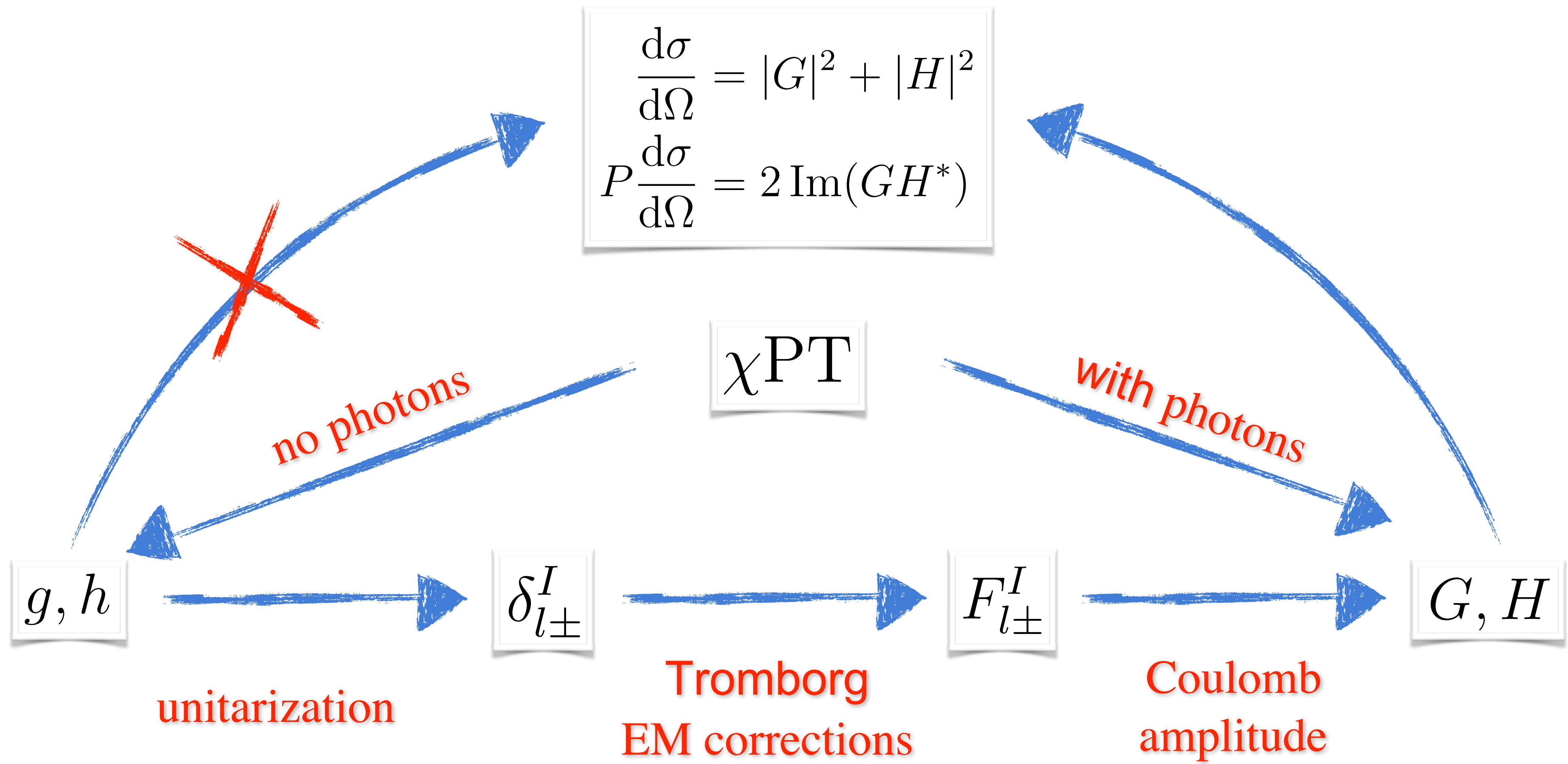
Unitarization prescription

$$\delta^I_{l\pm}(s) = \arctan(|\boldsymbol{q}| \mathrm{Re}\, f^I_{l\pm}(s))$$

Experimental Data



Experimental Data



Electromagnetic corrections to πN scattering

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The Niels Bohr Institute, Copenhagen, Denmark

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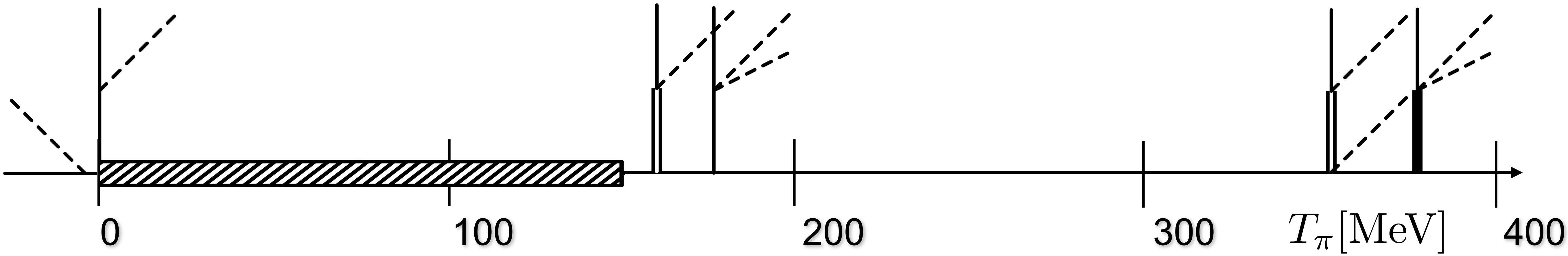
Institute of Physics, University of Trondheim, NLHT, Trondheim, Norway

(Received 27 October 1976)

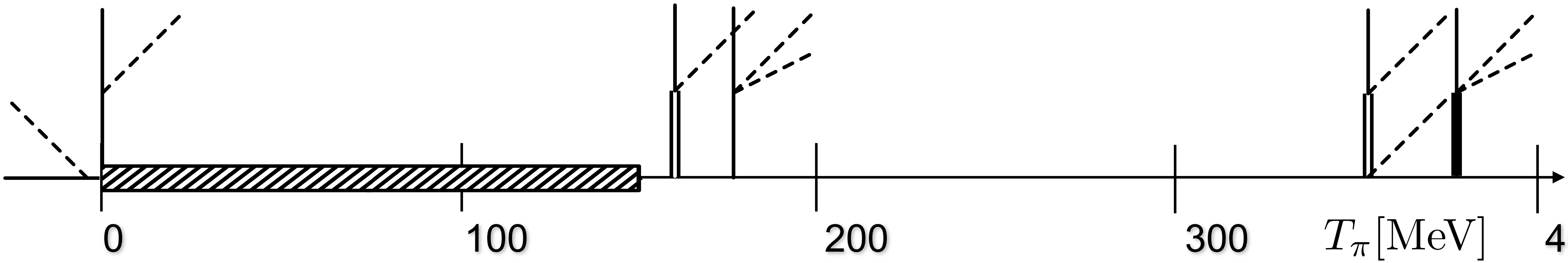
Numerical results are presented for the electromagnetic corrections to the *S*- and *P*-wave phase shifts and inelasticities in $\pi^+ p$ and $\pi^- p$ scattering. A discussion is given of how to apply the corrections in practical data analysis.

used in PWAs of GW and KH

Fitting Procedure



Fitting Procedure



Least Squares Fit

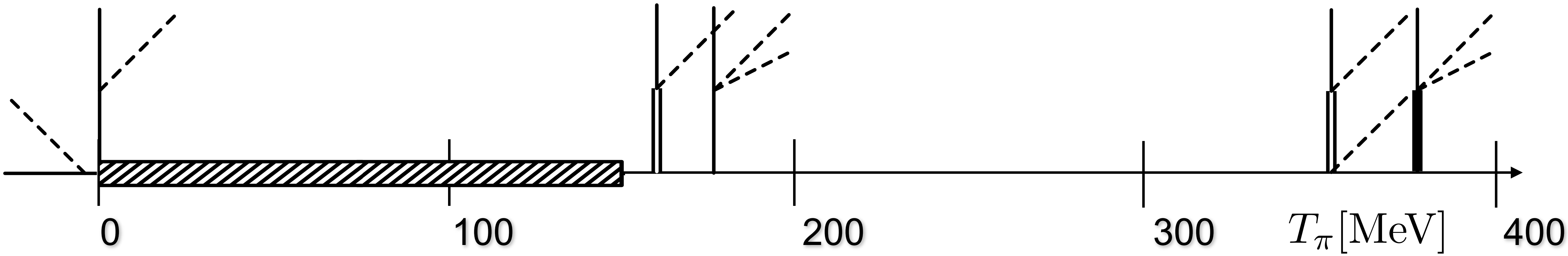
$$\chi^2 = \sum_i \left(\frac{\mathcal{O}_i^{exp} - N_i \mathcal{O}_i^{(n)}}{\delta \mathcal{O}_i} \right)^2$$

$\mathcal{O}_i^{exp}, \delta\mathcal{O}_i^{exp}, N_i$
from GW data base

$$\delta\mathcal{O}_i = \sqrt{(\delta\mathcal{O}_i^{exp})^2 + (\delta\mathcal{O}_i^{(n)})^2}$$

Workman et al. - Phys. Rev. C 86 (2012)

Fitting Procedure



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from GW data base

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Workman et al. - Phys. Rev. C 86 (2012)

Theoretical Error

convergence behavior

$$\delta\mathcal{O}_i^{(n)} = \max(|\mathcal{O}_i^{(1)}|Q^n, \{|\mathcal{O}_i^{(k)} - \mathcal{O}_i^{(j)}|Q^{n-j}\}) \quad j < k \leq n$$

actual higher order contributions

$$\delta\mathcal{O}_i^{(n)} \geq \max(\{|\mathcal{O}_i^{(k)} - \mathcal{O}_i^{(j)}|\}) \quad n \leq j < k$$

$$Q = \frac{\omega_{\mathrm{CMS}}}{\Lambda_b}$$

$$\Lambda_b = 600\ \mathrm{MeV}$$

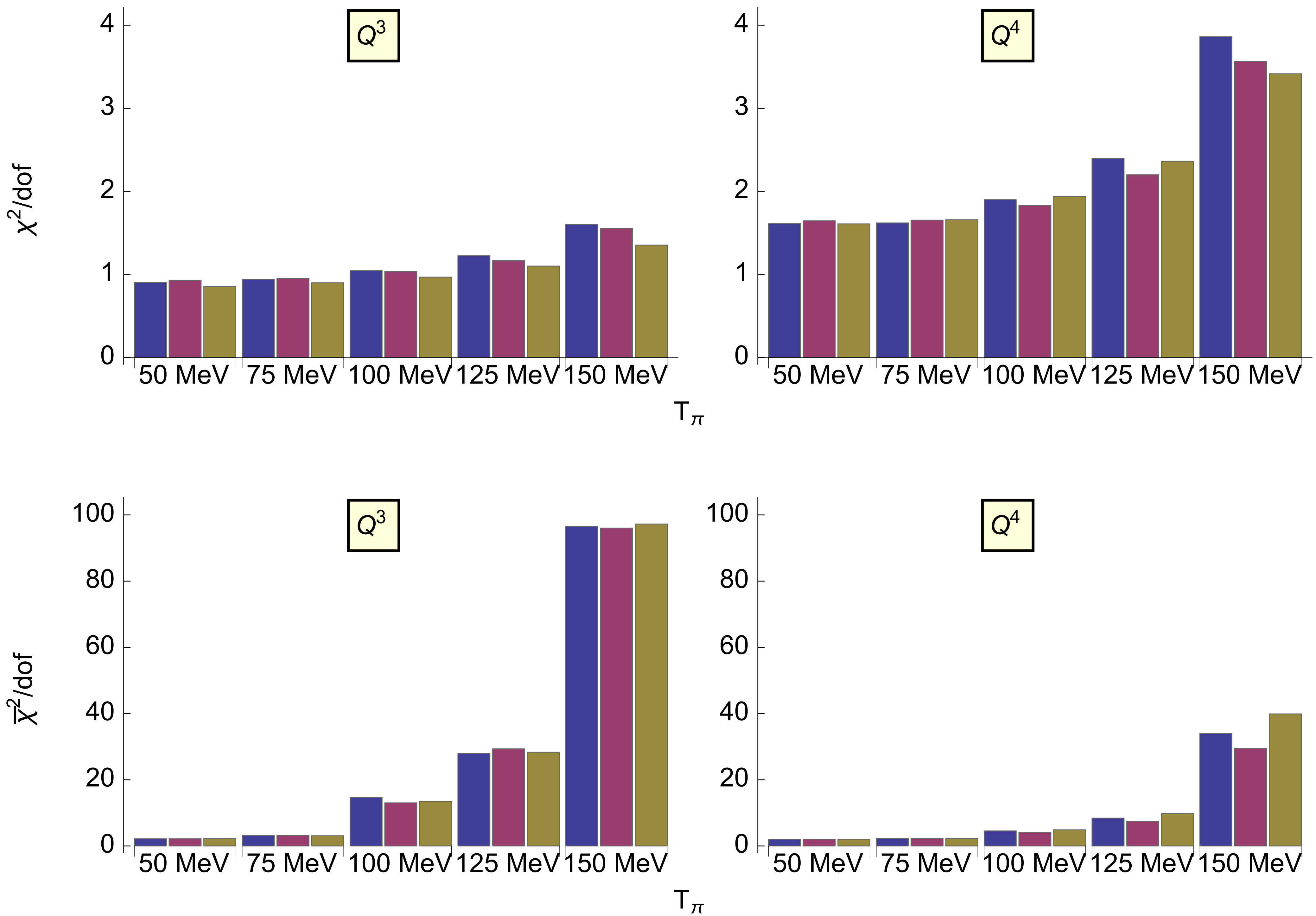
Epelbaum et al. - Eur. Phys. J. A 51 (2015)

Fits - χ^2 over T_π

with
theo. error

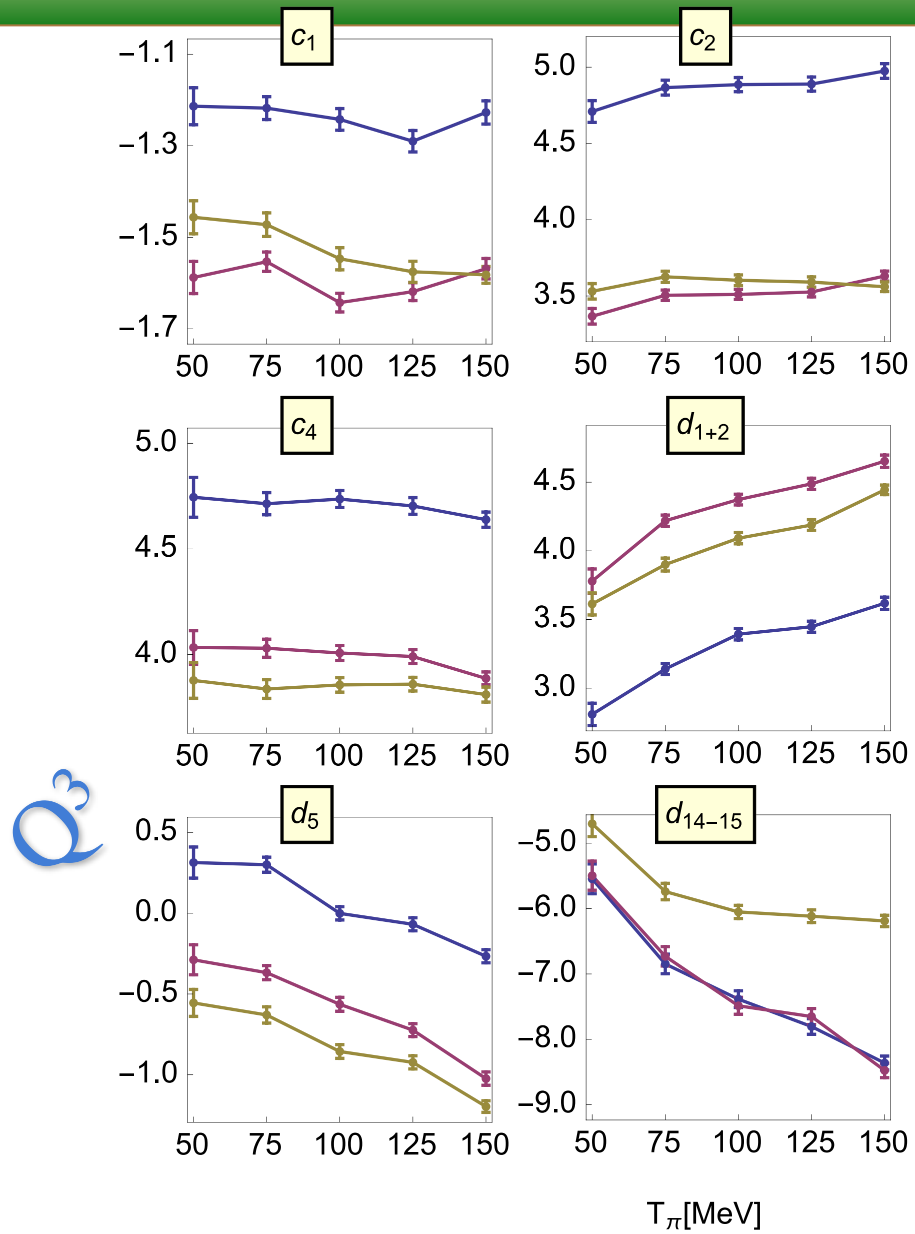
- HB−NN
- HB−πN
- Cov

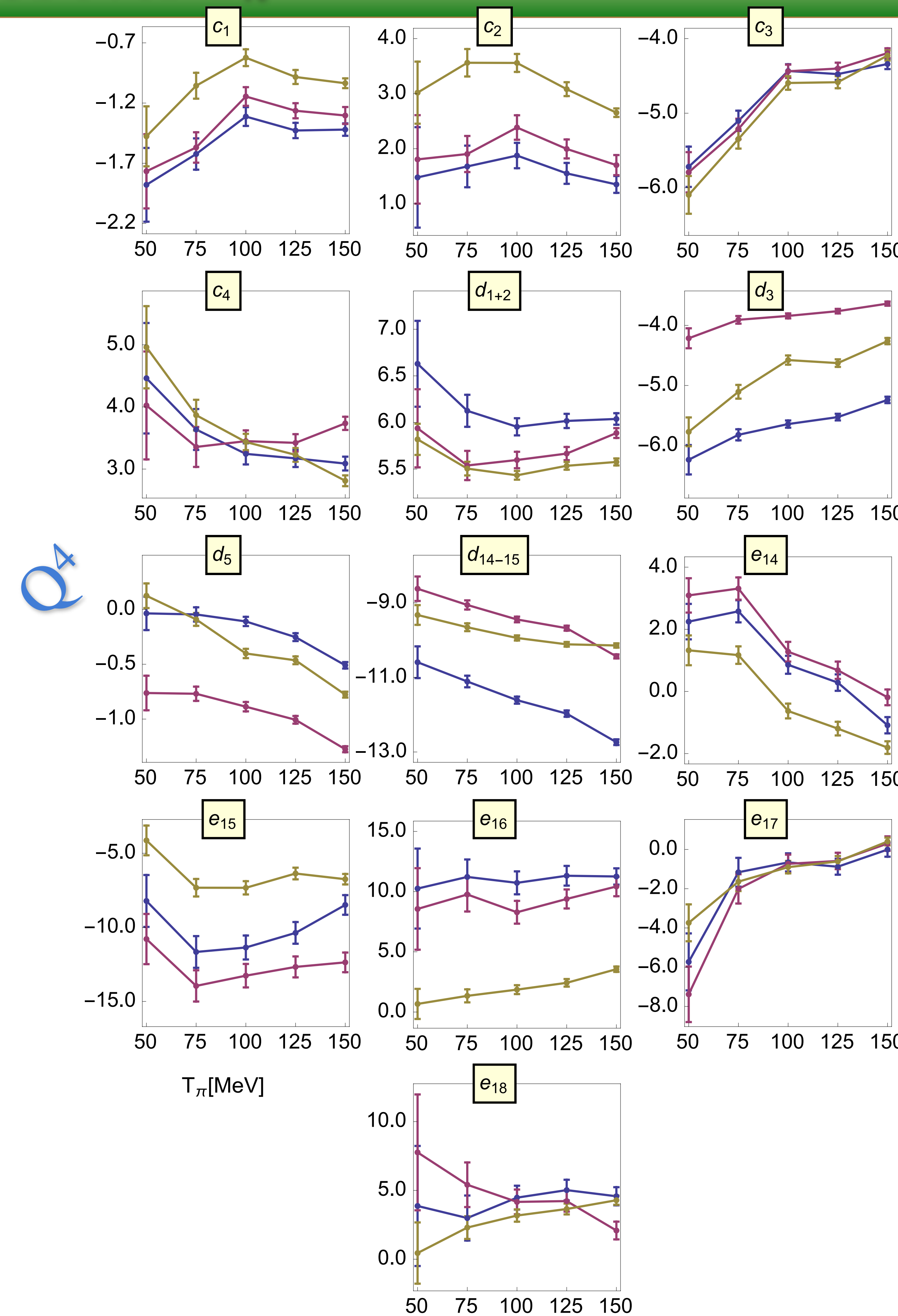
without
theo. error



$T_\pi < \{50, 75, 100, 125, 150\}$ MeV $\widehat{=}$ $\{1035, 1368, 1704, 1854, 2177\}$ data points

Fits - LECs over T_π





- HB−NN
- HB−πN
- Cov

Representative Fits - $T_\pi < 100$ MeV

Input

m_N	M_π	F_π	g_A
938.27	139.57	92.2	1.289

MeV

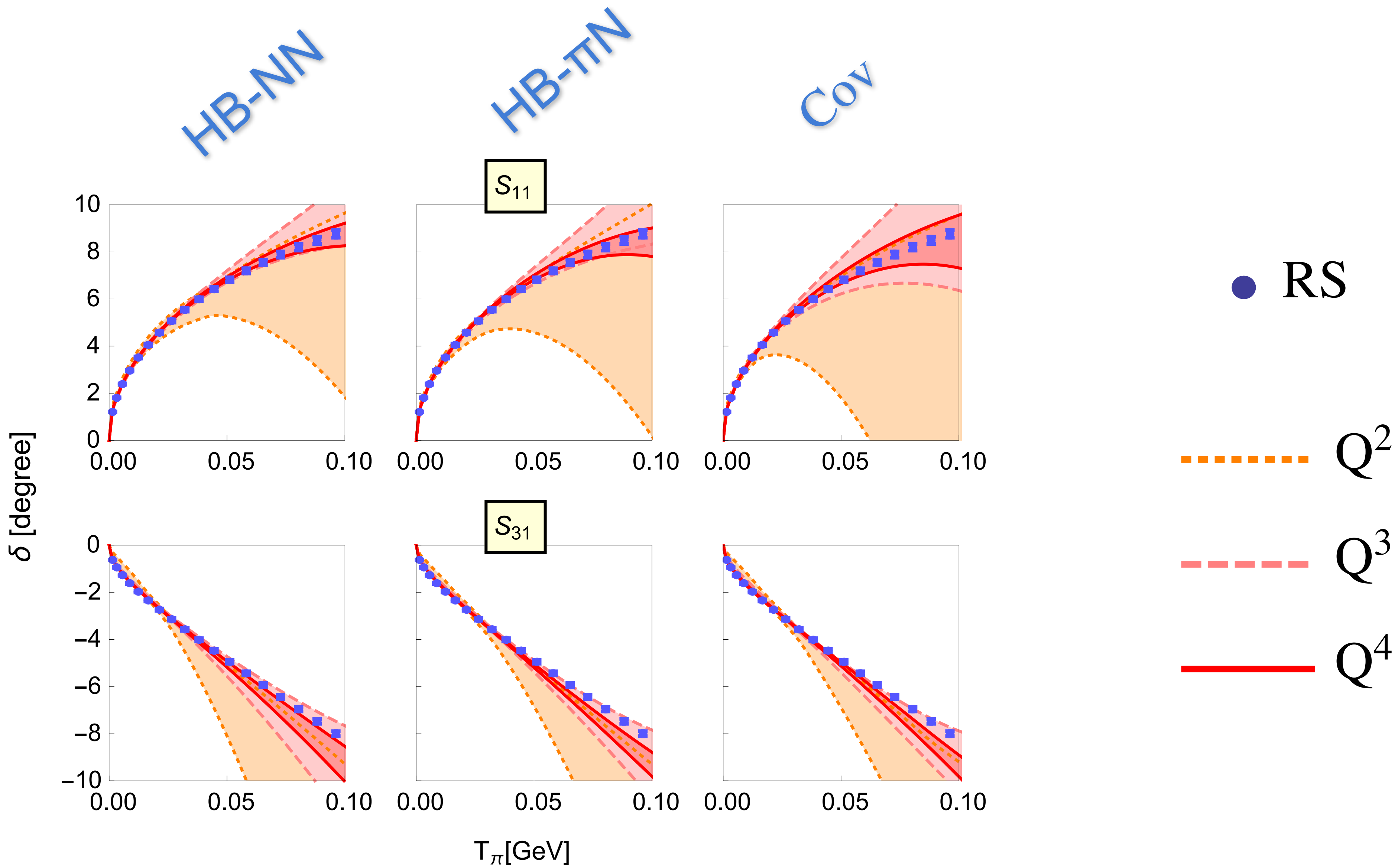
Q^3	HB-NN	HB-πN	Cov
c_1	-1.24(2)	-1.64(2)	-1.55(2)
c_2	4.89(5)	3.51(3)	3.60(4)
c_3	-7.25(2)	-6.63(2)	-6.54(2)
c_4	4.74(4)	4.01(4)	3.86(3)
d_{1+2}	3.39(4)	4.37(4)	4.09(4)
d_3	-3.47(7)	-3.34(7)	-2.50(4)
d_5	0.00(4)	-0.56(4)	-0.86(4)
d_{14-15}	-7.39(13)	-7.49(13)	-6.05(10)
$\chi^2_{\pi N}/\mathrm{dof}$	1.04	1.03	0.97
$\bar{\chi}^2_{\pi N}/\mathrm{dof}$	14.6	13.0	13.5

Q^4	HB-NN	HB-πN	Cov
c_1	🟠 -1.31(8)	🟠 -1.15(8)	🟠 -0.82(7)
c_2	🟠 1.88(23)	🟠 2.39(22)	🟠 3.56(16)
c_3	-4.43(9)	-4.44(9)	-4.59(9)
c_4	🟢 3.24(17)	🟢 3.45(17)	3.44(13)
d_{1+2}	🟢 5.95(9)	🟢 5.60(9)	5.43(5)
d_3	-5.64(6)	-3.84(4)	-4.58(8)
d_5	-0.11(4)	-0.89(4)	-0.40(4)
d_{14-15}	-11.61(9)	-9.45(8)	-9.94(7)
e_{14}	0.86(29)	1.28(32)	-0.63(24)
e_{15}	-11.36(81)	-13.26(79)	-7.33(45)
e_{16}	🟠 10.73(95)	🟠 8.29(95)	1.86(37)
e_{17}	-0.66(46)	-0.73(47)	-0.90(32)
e_{18}	4.47(87)	4.17(90)	3.17(45)
$\chi^2_{\pi N}/\mathrm{dof}$	1.90	1.83	1.94
$\bar{\chi}^2_{\pi N}/\mathrm{dof}$	4.5	4.1	4.9

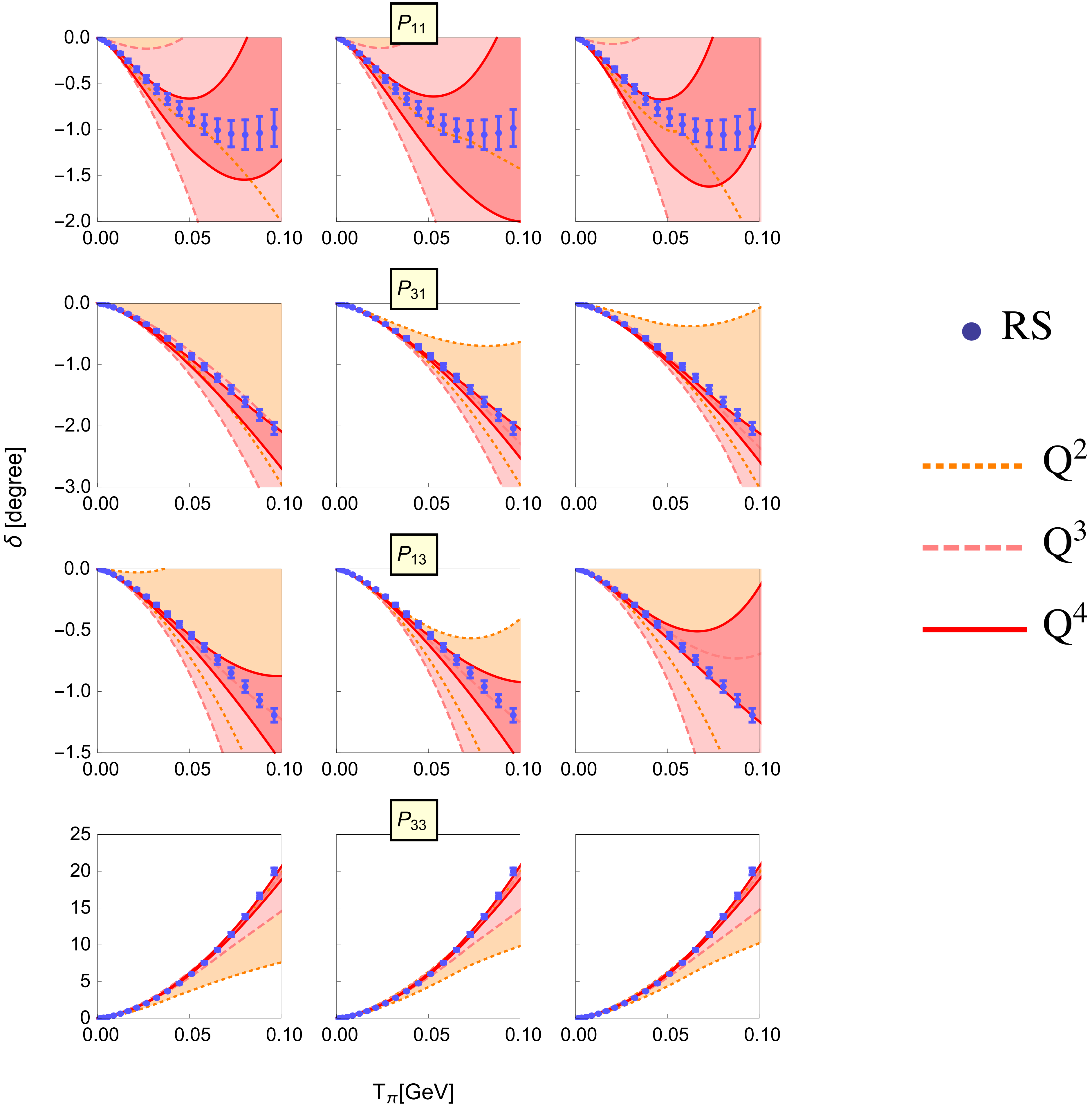
correlations: 🟠 🟢

Predictions

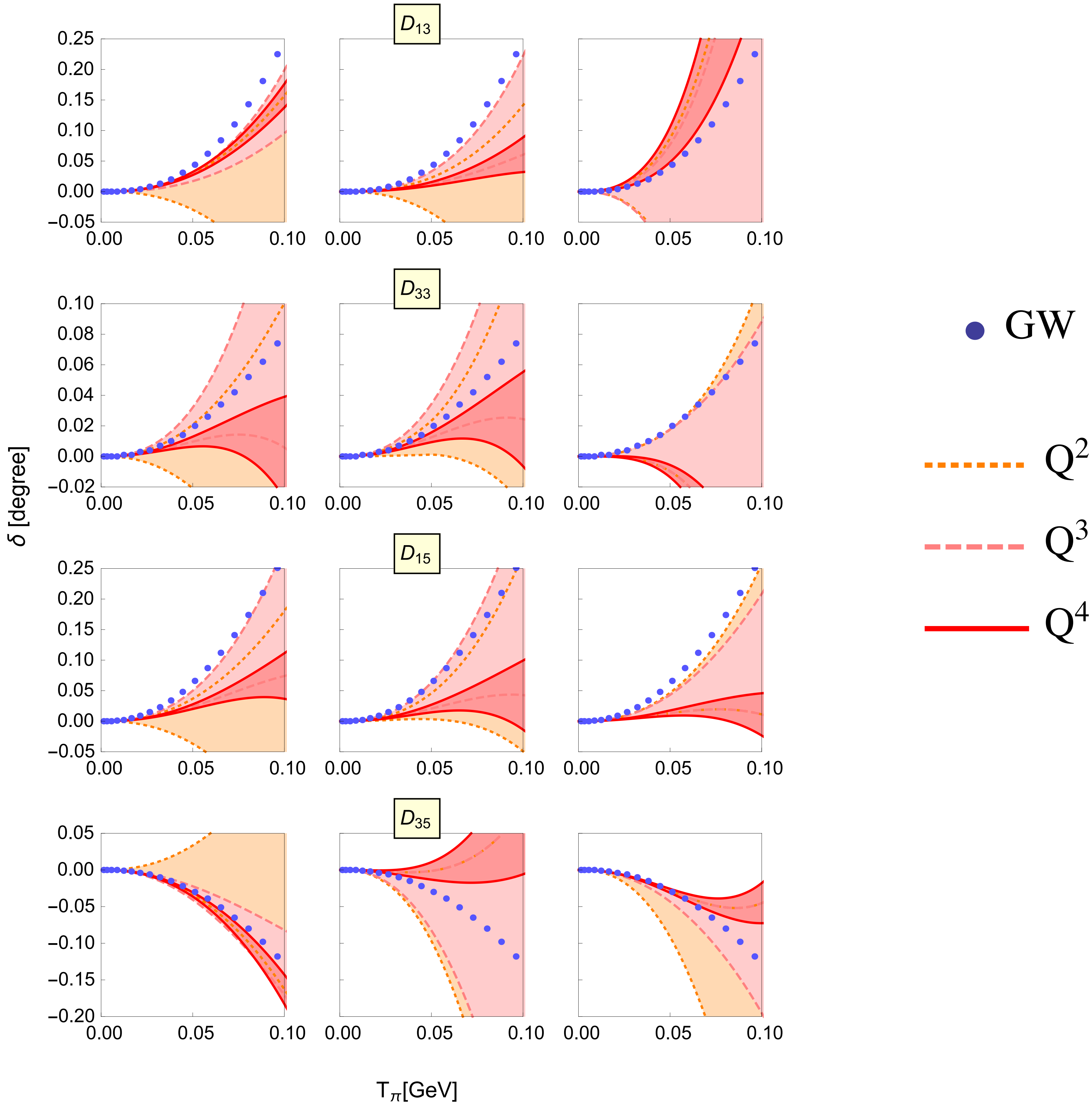
S-Waves Theo. Error



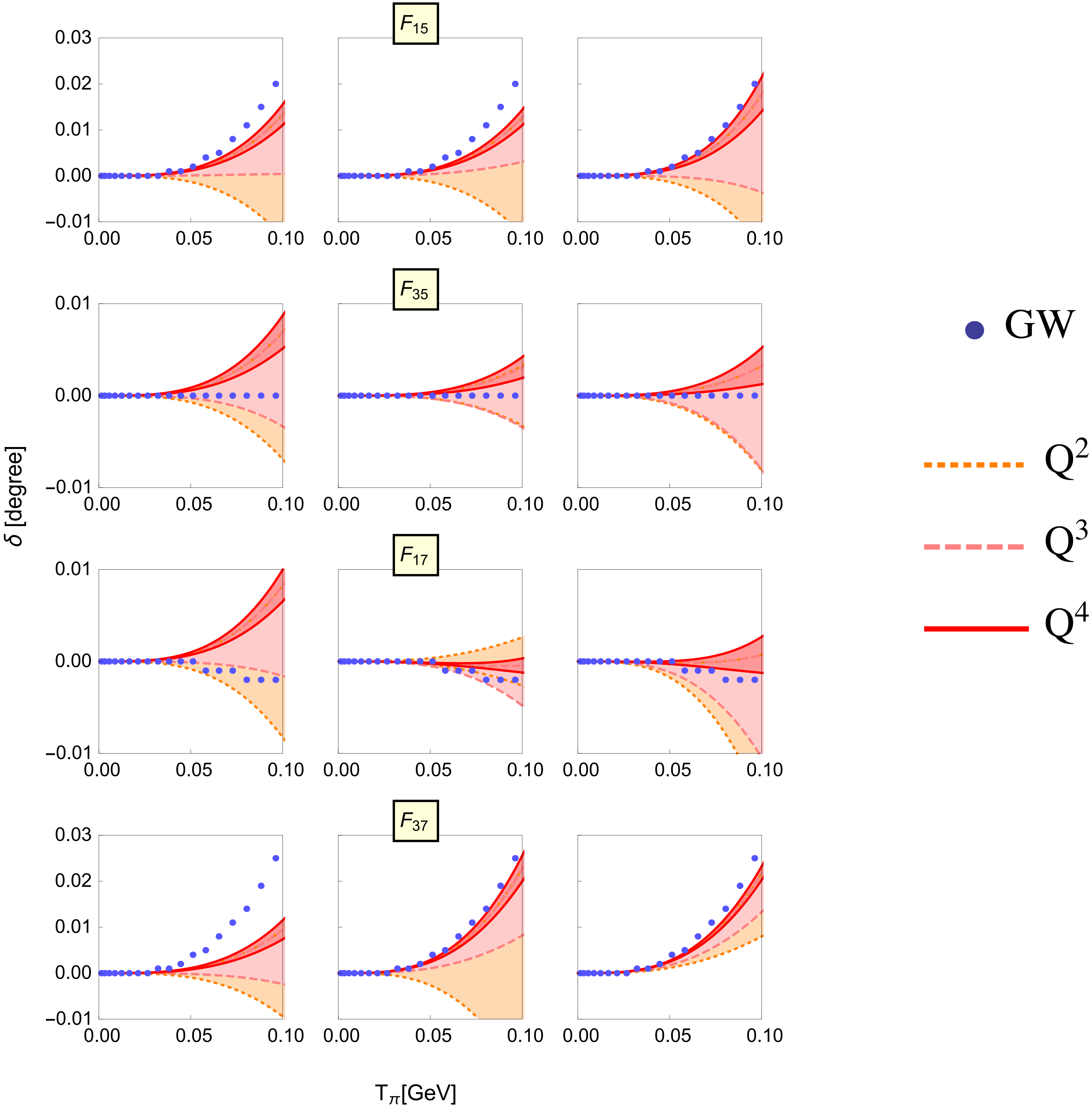
P-Waves Theo. Error



D-Waves Theo. Error



F-Waves Theo. Error



Summary

Good description of $\pi N \rightarrow \pi N$ data up to 100 MeV

- agreement with RS S- and P-waves
- disagreement with some GW D- and F-waves
- almost no differences between the counting schemes
- χ^2/dof increases for energies above 100 MeV
- deviations from plateau-like behavior for LECs above 100 MeV

Theoretical error underestimated for T_π>100 MeV

- Λ_b < 600 MeV
- $\Delta(1232)$ is not included explicitly

Including $\Delta(1232)$

Effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi\Delta}^{(1)} + \mathcal{L}_{\pi\Delta}^{(2)} + \mathcal{L}_{\pi\Delta}^{(4)} + \mathcal{L}_{\pi N\Delta}^{(1)} + \mathcal{L}_{\pi N\Delta}^{(2)} + \mathcal{L}_{\pi N\Delta}^{(3)} + \mathcal{L}_{\pi N\Delta}^{(4)}$$

$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

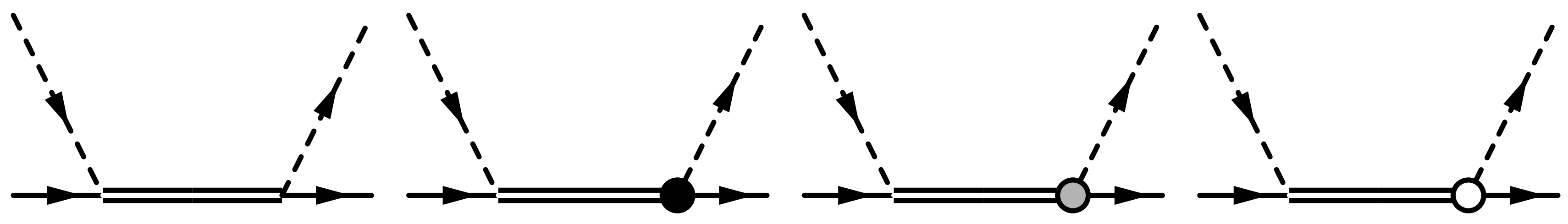
BχPT & HBχPT

Effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} \\ &+ \mathcal{L}^{(1)}_{\pi\Delta} + \mathcal{L}^{(2)}_{\pi\Delta} + \mathcal{L}^{(4)}_{\pi\Delta} \\ &+ \mathcal{L}^{(1)}_{\pi N\Delta} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \mathcal{L}^{(4)}_{\pi N\Delta}\end{aligned}$$

$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



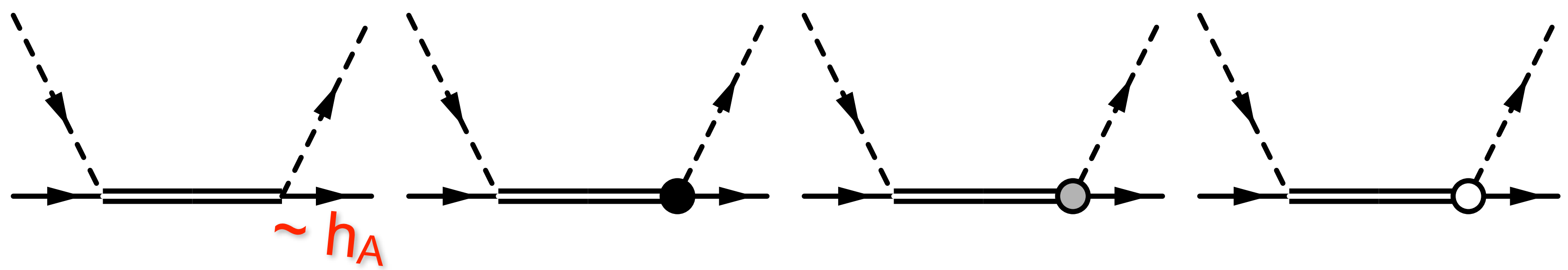
BχPT & HBχPT

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \mathcal{L}^{(1)}_{\pi\Delta} + \mathcal{L}^{(2)}_{\pi\Delta} + \mathcal{L}^{(4)}_{\pi\Delta} + \mathcal{L}^{(1)}_{\pi N\Delta} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \mathcal{L}^{(4)}_{\pi N\Delta}$$

$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi\Delta}^{(1)} + \mathcal{L}_{\pi\Delta}^{(2)} + \mathcal{L}_{\pi\Delta}^{(4)} + \mathcal{L}_{\pi N\Delta}^{(1)} + \boxed{\mathcal{L}_{\pi N\Delta}^{(2)}} + \mathcal{L}_{\pi N\Delta}^{(3)} + \mathcal{L}_{\pi N\Delta}^{(4)}$$

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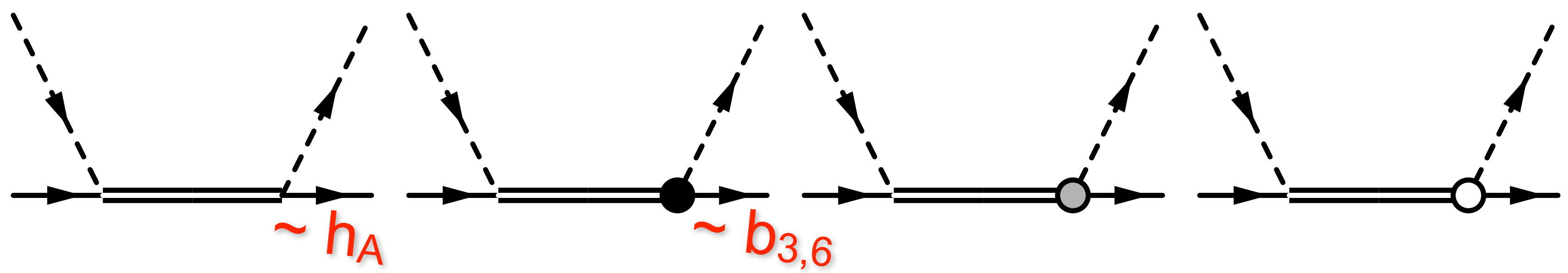
Δ Tree Graphs



Effective Lagrangian

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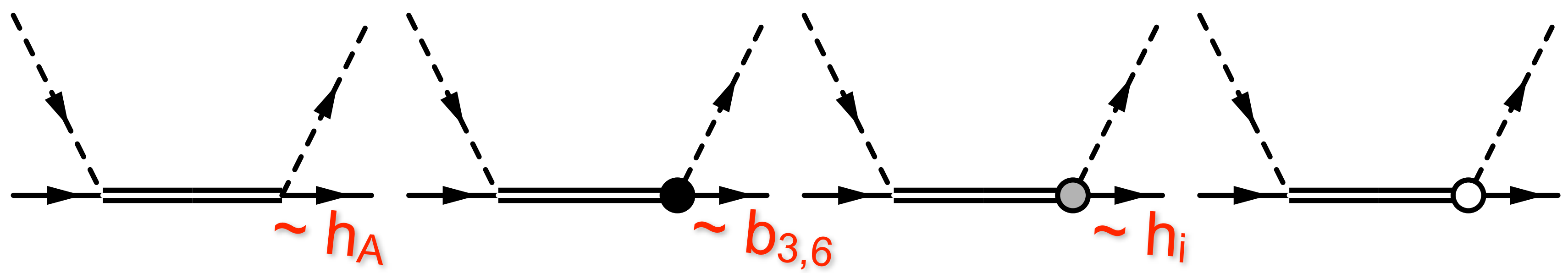
Δ Tree Graphs



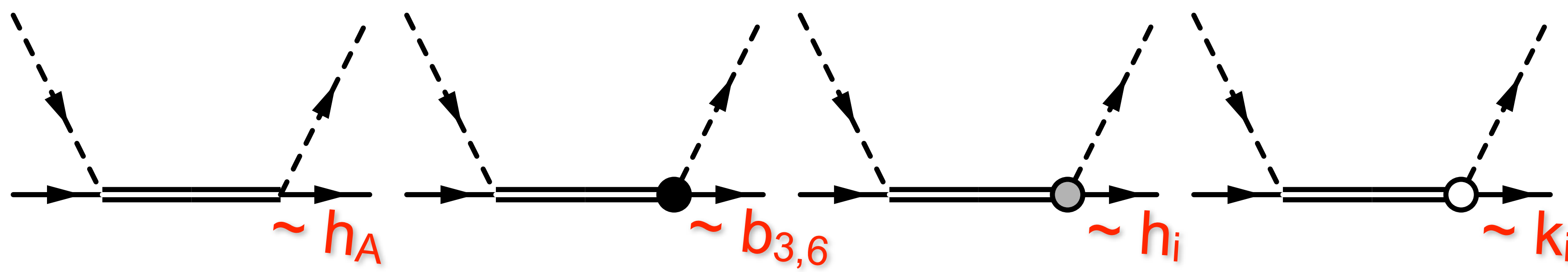
BχPT & HBχPT

Effective Lagrangian

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \mathcal{L}^{(1)}_{\pi\Delta} + \mathcal{L}^{(2)}_{\pi\Delta} + \mathcal{L}^{(4)}_{\pi\Delta} + \mathcal{L}^{(1)}_{\pi N\Delta} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \boxed{\mathcal{L}^{(4)}_{\pi N\Delta}}$$

$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



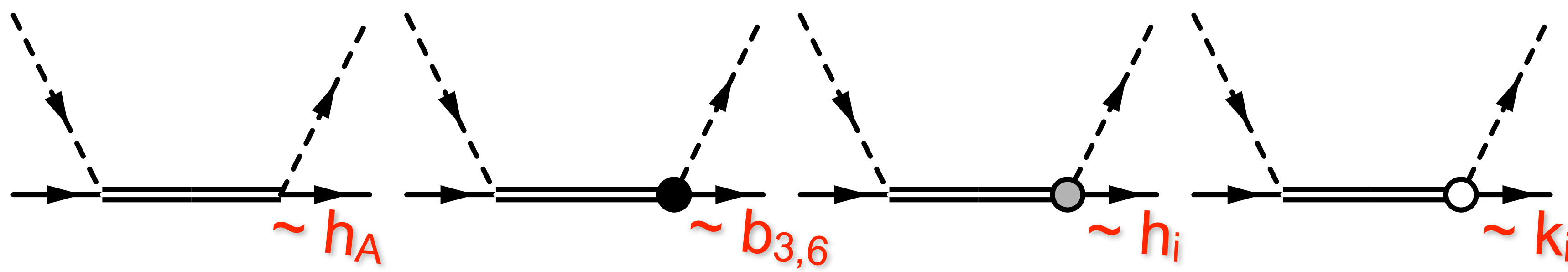
BχPT & HBχPT

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \mathcal{L}_{\pi\Delta}^{(1)} + \mathcal{L}_{\pi\Delta}^{(2)} + \mathcal{L}_{\pi\Delta}^{(4)} + \mathcal{L}_{\pi N\Delta}^{(1)} + \mathcal{L}_{\pi N\Delta}^{(2)} + \mathcal{L}_{\pi N\Delta}^{(3)} + \mathcal{L}_{\pi N\Delta}^{(4)}$$

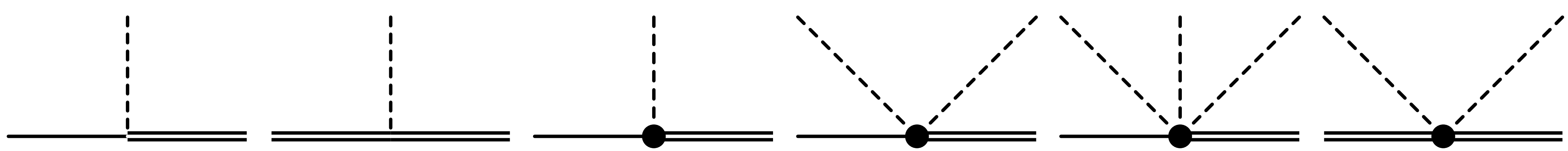
$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



Δ Loop Vertices

ε^3 ε^4

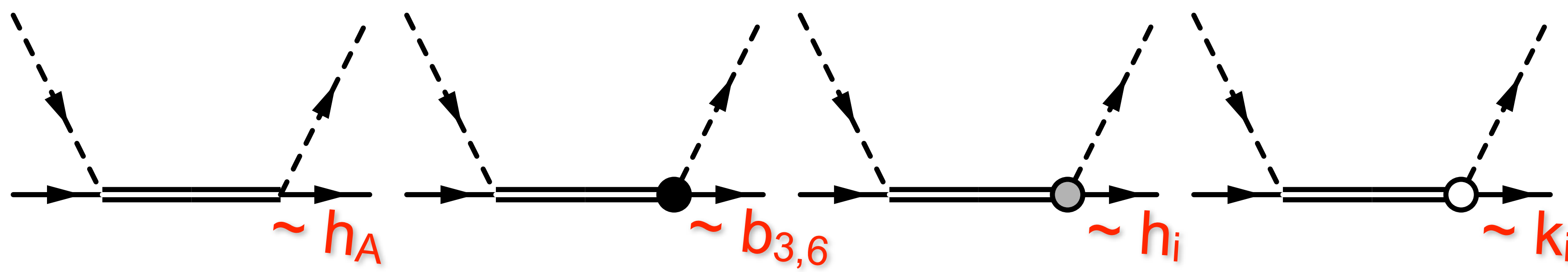


Effective Lagrangian

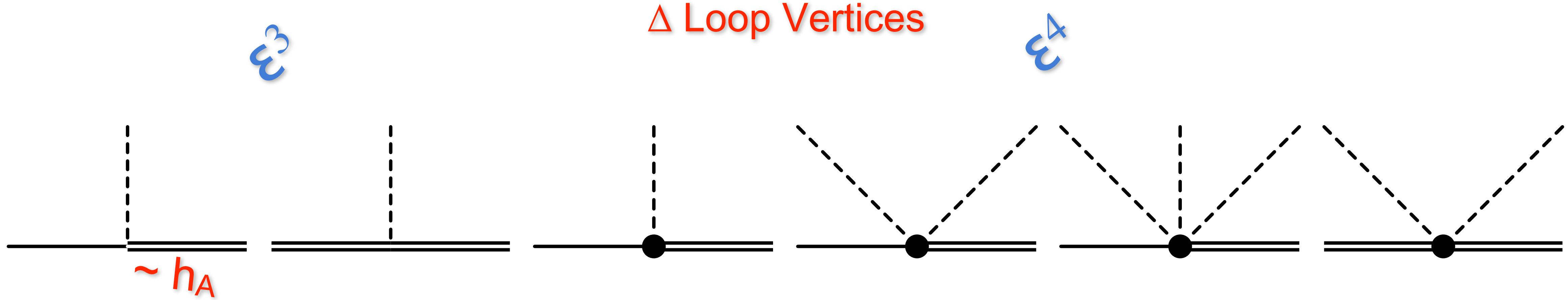
$$
\begin{aligned}
\mathcal{L}_{\text{eff}} = {} & \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} \\
& + \mathcal{L}^{(1)}_{\pi \Delta} + \mathcal{L}^{(2)}_{\pi \Delta} + \mathcal{L}^{(4)}_{\pi \Delta} \\
& + \boxed{\mathcal{L}^{(1)}_{\pi N\Delta}} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \mathcal{L}^{(4)}_{\pi N\Delta}
\end{aligned}
$$

$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



Δ Loop Vertices



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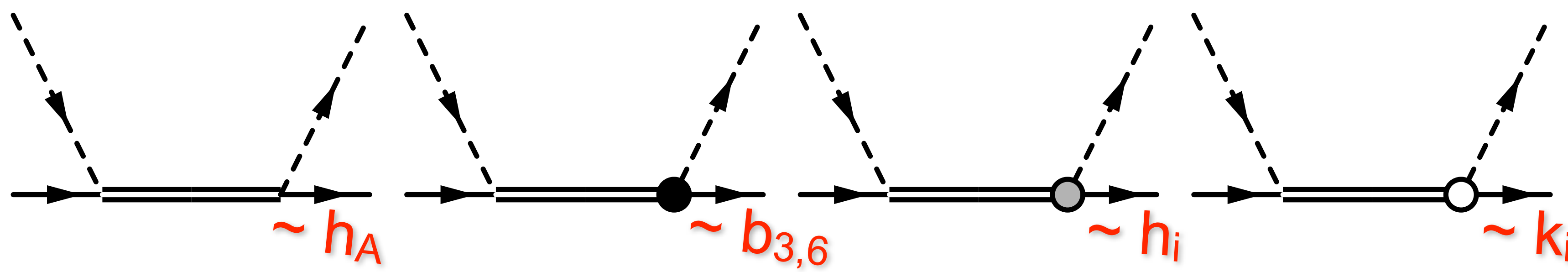
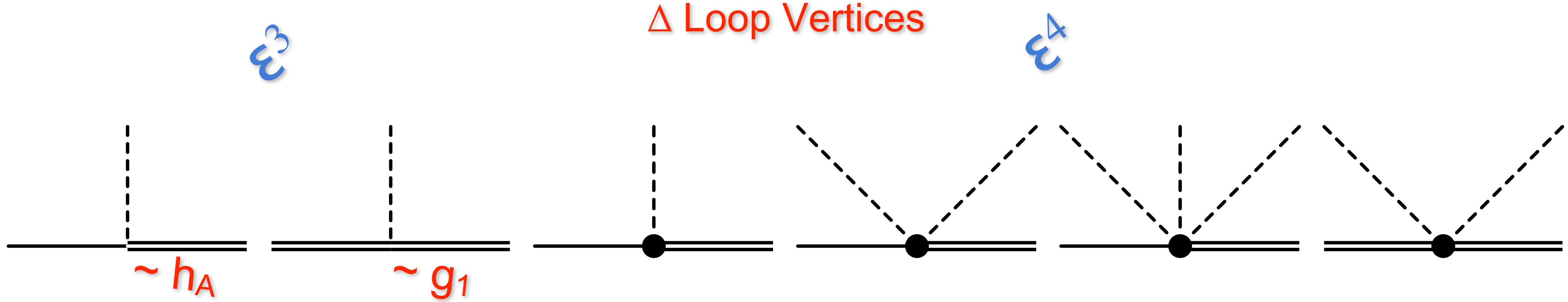
Δ Tree Graphs



Δ Loop Vertices



BχPT & HBχPT

Effective Lagrangian

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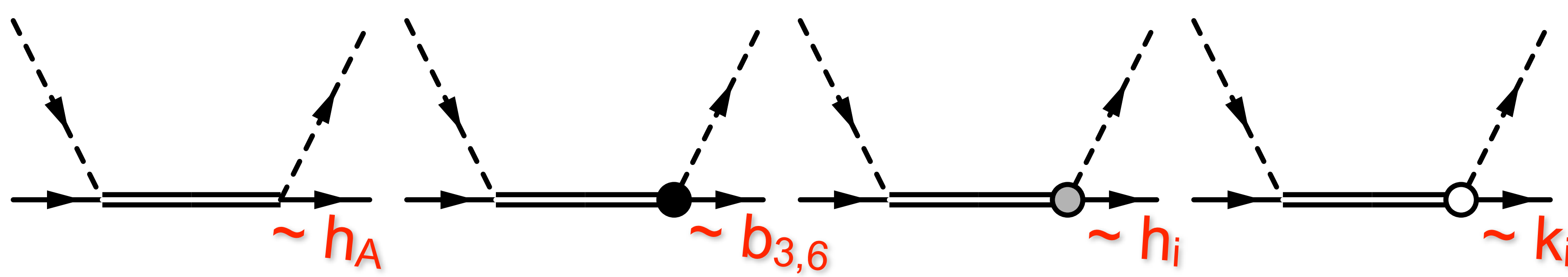
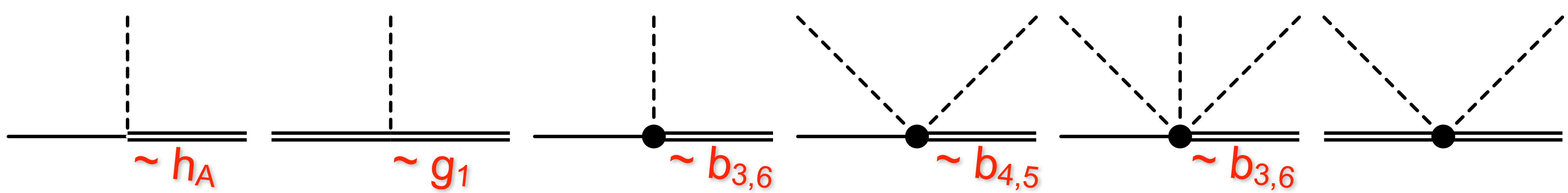
Δ Tree Graphs



Δ Loop Vertices

ε^3 ε^4

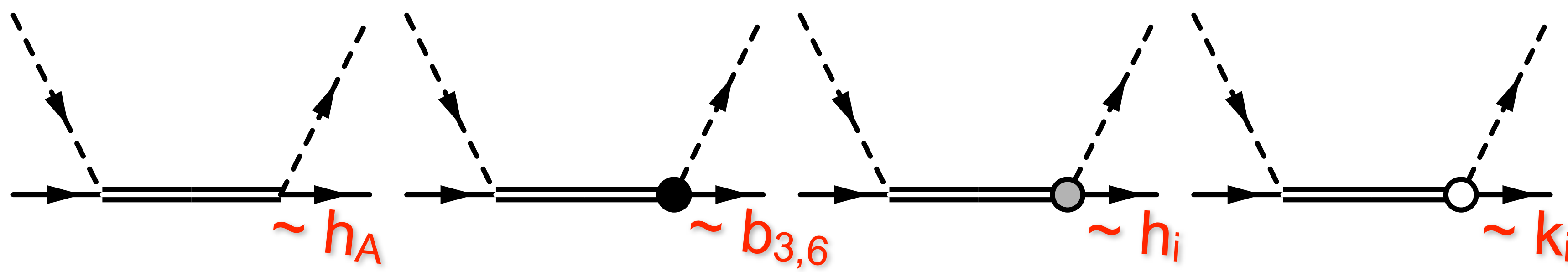


Effective Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} \\
&+ \mathcal{L}^{(1)}_{\pi\Delta} + \boxed{\mathcal{L}^{(2)}_{\pi\Delta}} + \mathcal{L}^{(4)}_{\pi\Delta} \\
&+ \mathcal{L}^{(1)}_{\pi N\Delta} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \mathcal{L}^{(4)}_{\pi N\Delta}
\end{aligned}
$$

$$
\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}
$$

Δ Tree Graphs



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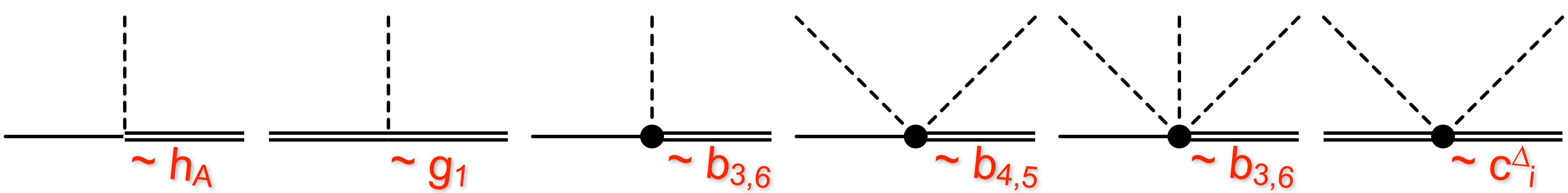

BχPT & HBχPT

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$$+ \mathcal{L}^{(1)}_{\pi\Delta} + \mathcal{L}^{(2)}_{\pi\Delta} + \mathcal{L}^{(4)}_{\pi\Delta}$$
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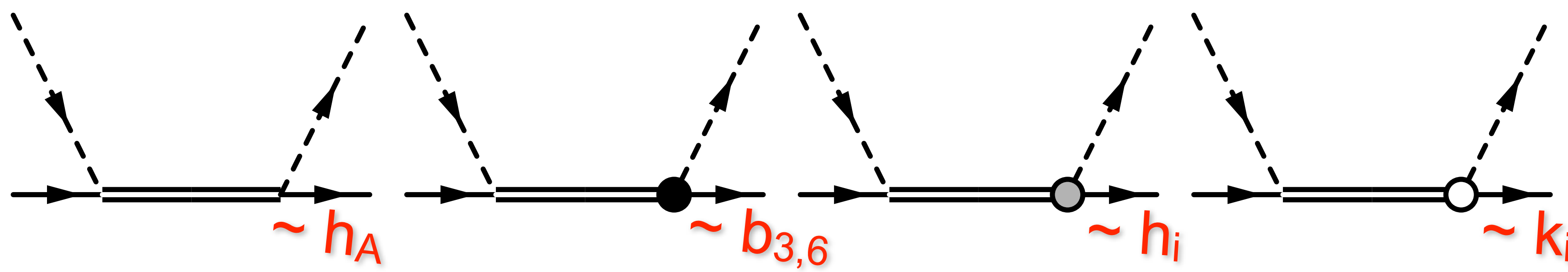
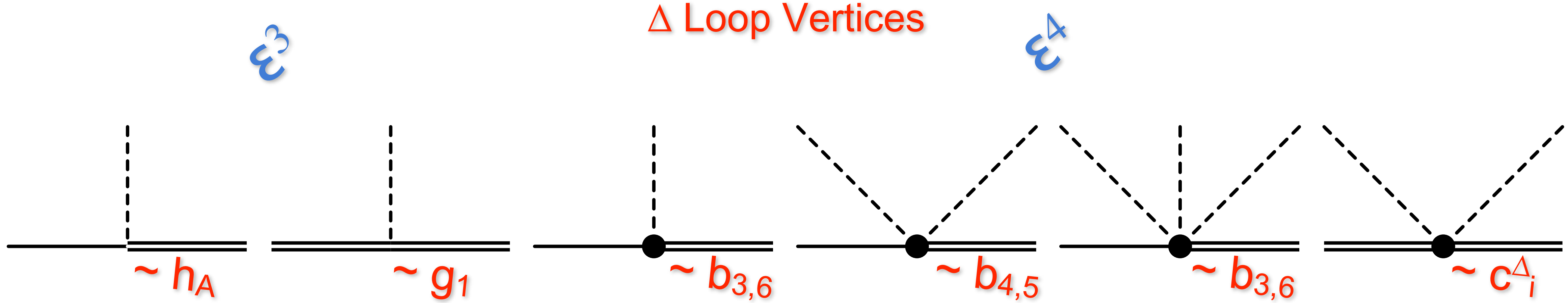
Δ Tree Graphs



Δ Loop Vertices

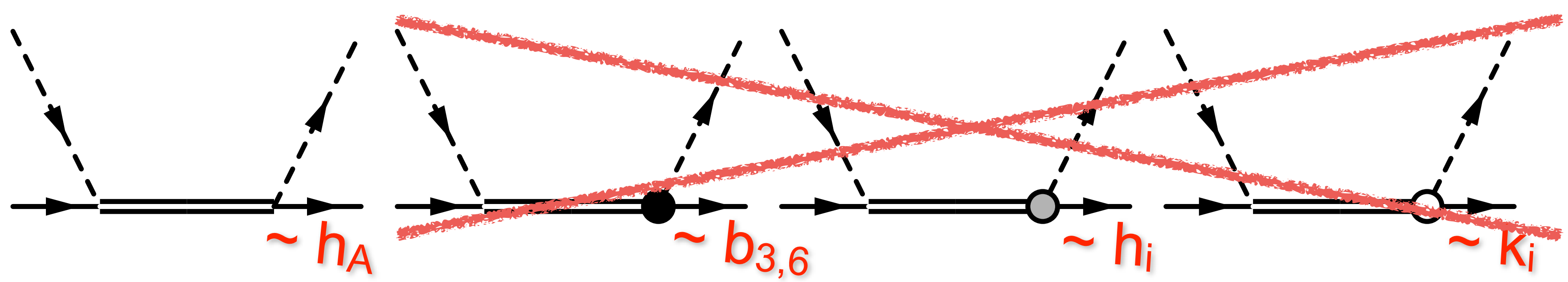


Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \mathcal{L}^{(1)}_{\pi\Delta} + \mathcal{L}^{(2)}_{\pi\Delta} + \mathcal{L}^{(4)}_{\pi\Delta} + \mathcal{L}^{(1)}_{\pi N\Delta} + \mathcal{L}^{(2)}_{\pi N\Delta} + \mathcal{L}^{(3)}_{\pi N\Delta} + \mathcal{L}^{(4)}_{\pi N\Delta}$$

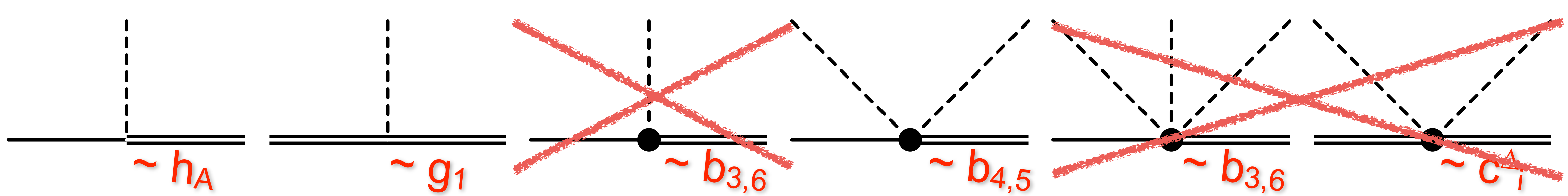
$$\varepsilon = \left\{ \frac{q}{\Lambda_b}, \frac{M_\pi}{\Lambda_b}, \frac{\Delta}{\Lambda_b} \right\}$$

Δ Tree Graphs



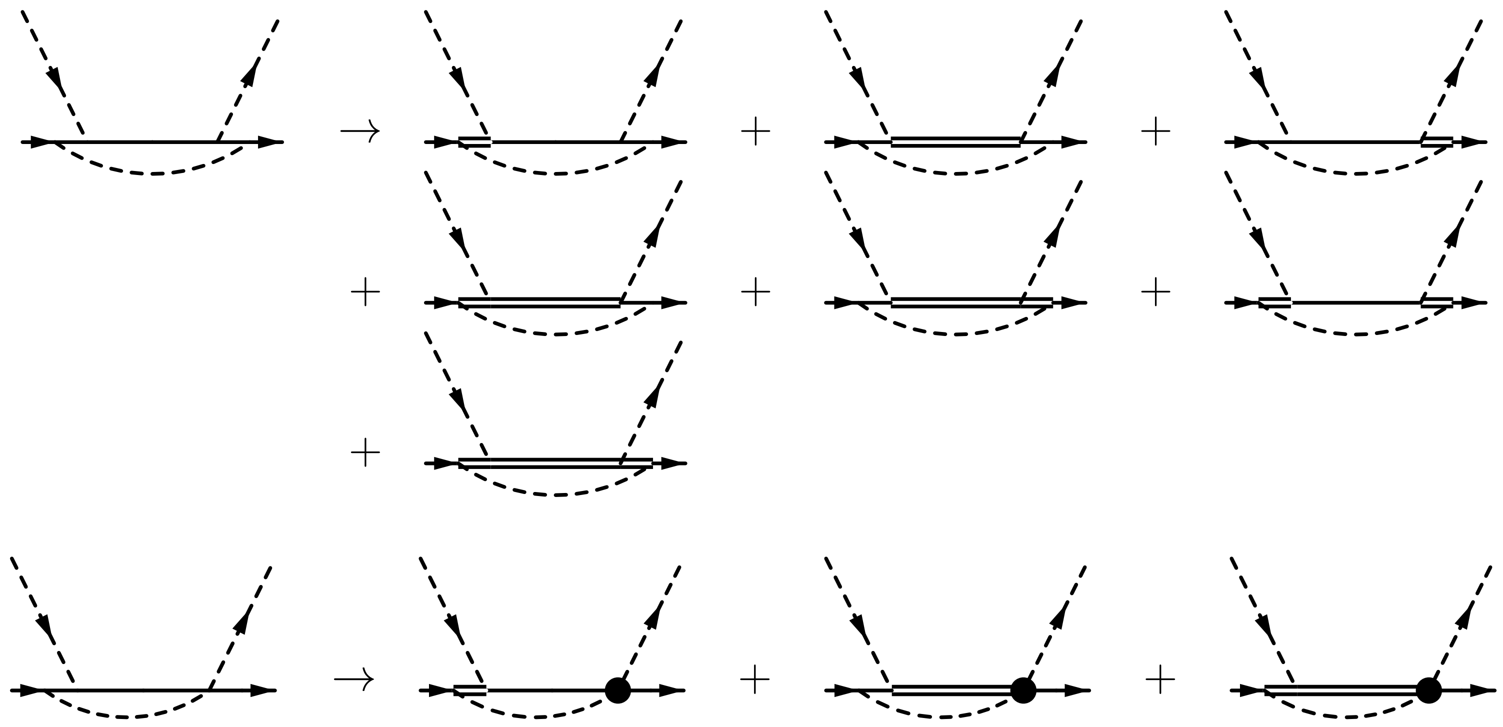
Δ Loop Vertices

ε^3 ε^4



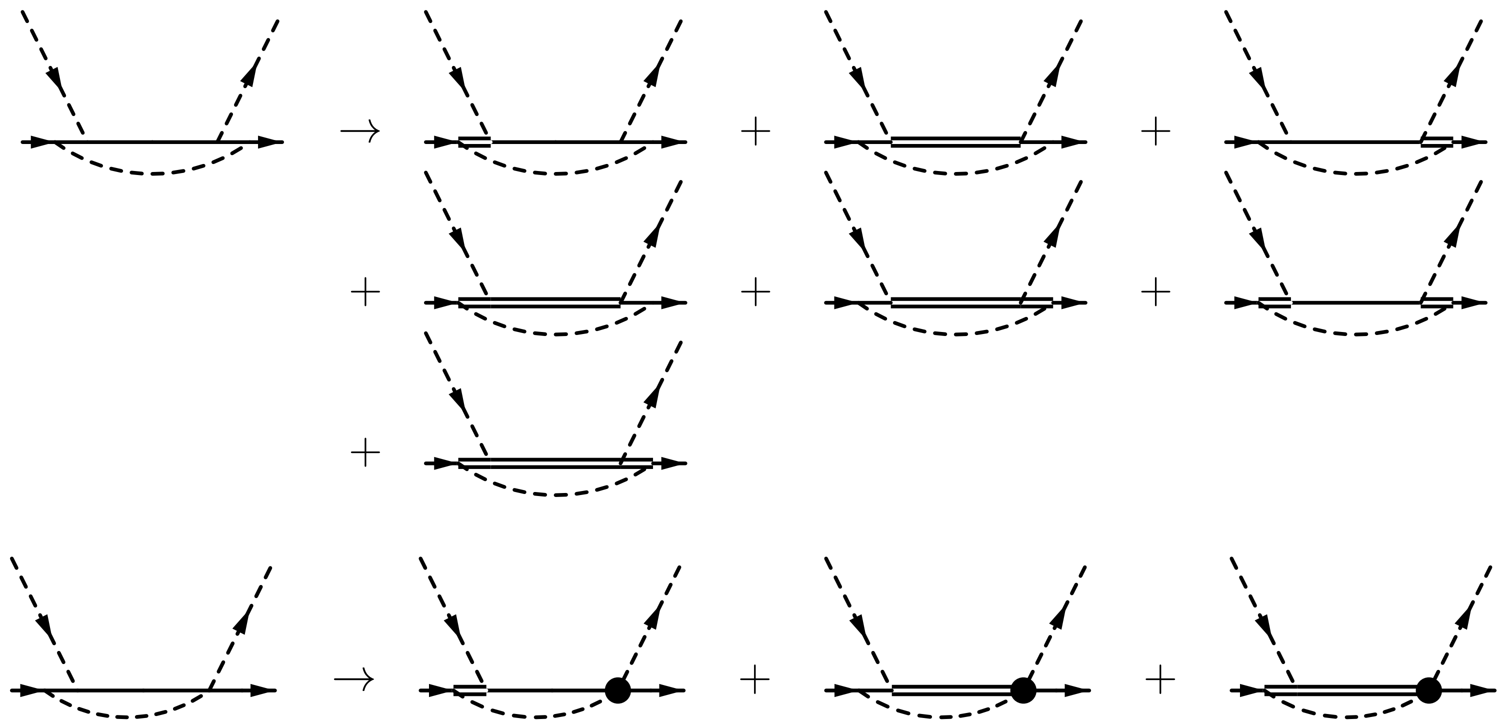
Renormalization I

Transition to Δ-ful loops



Renormalization I

Transition to Δ-ful loops



Nucleon Sector

$$m = m_N + \delta m^{(2)} + \delta m^{(3)} + \delta m^{(3,\Delta)} + \delta m^{(4)} + \delta m^{(4,\Delta)}$$

$$Z_N = 1 + \delta Z_N^{(3)} + \delta Z_N^{(3,\Delta)} + \delta Z_N^{(4)} + \delta Z_N^{(4,\Delta)}$$

$$g = g_A + \delta g^{(3)} + \delta g^{(3,\Delta)} + \delta g^{(4,\Delta)}$$

on mass-shell

Δ Sector

$$\mathbf{m} = m_\Delta + \delta \mathbf{m}^{(2)} + \delta \mathbf{m}^{(3)} + \delta \mathbf{m}^{(4)}$$

$$Z_\Delta = 1 + \delta Z_\Delta^{(3)} + \delta Z_\Delta^{(4)}$$

$$h = h_A + \delta h^{(3)} + \delta h^{(4)}$$

Renormalization II

HB approach

$$c_i = \bar{c}_i + \delta c_i^{(3,\Delta)} + \delta c_i^{(4,\Delta)}$$
$$d_i = \bar{d}_i + \delta d_i + \delta d_i^{(3,\Delta)} + \delta d_i^{(4,\Delta)}$$
$$e_i = \bar{e}_i + \delta e_i + \delta e_i^{(4,\Delta)}$$

$$\delta x_i = \frac{\beta_{x_i} + \beta_{x_i}^{\Delta}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$
$$\delta x_i^{(n,\Delta)} = \frac{\delta \bar{x}_{i,f}^{(n,\Delta)}}{F_\pi^2} + \frac{\beta_{x_i}^{(n,\Delta)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{16\pi^2}\ln\left(\frac{2\Delta}{\mu}\right)\right)$$

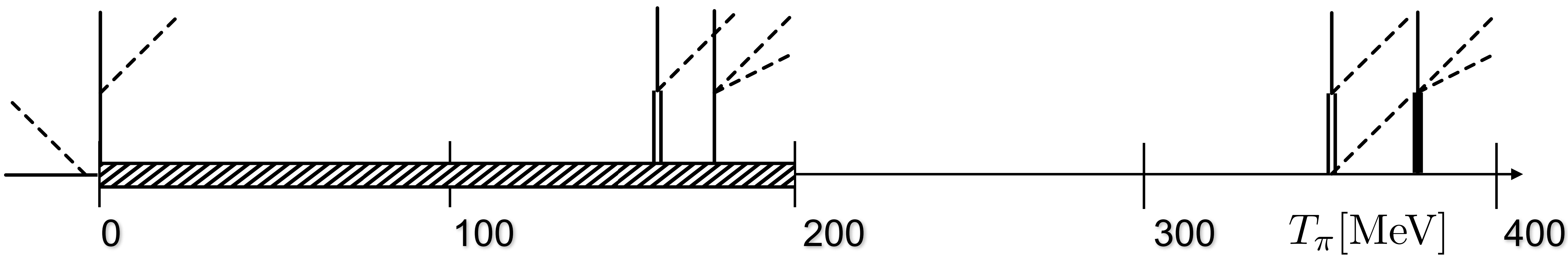
Covariant “modified“ EOMS scheme

$$c_i = \bar{c}_i + \delta c_i^{(3)} + \delta c_i^{(3,\Delta)} + \delta c_i^{(4)} + \delta c_i^{(4,\Delta)}$$
$$d_i = \bar{d}_i + \delta d_i + \delta d_i^{(3)} + \delta d_i^{(3,\Delta)} + \delta d_i^{(4)} + \delta d_i^{(4,\Delta)}$$
$$e_i = \bar{e}_i + \delta e_i + \delta e_i^{(4)} + \delta e_i^{(4,\Delta)}$$

$$x \in \{c, d, e\}$$

$$\delta x_i = \frac{\beta_{x_i} + \beta_{x_i}^{\Delta}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\ln\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$
$$F_\pi^2\,\delta x^{(n)} = a_0 + a_1 A_0(m_N^2)$$
$$\begin{aligned} F_\pi^2\,\delta x^{(n,\Delta)} &= a_0 + a_1 A_0(m_N^2) + a_2 A_0(m_\Delta^2) + b_1 B_0(m_N^2, 0, m_\Delta^2) + b_2 B_0(m_\Delta^2, 0, m_N^2) \\ &+ c_1 C_0(m_N^2, 0, m_\Delta^2, 0, m_N^2, m_N^2) + c_2 C_0(m_N^2, 0, m_\Delta^2, 0, m_\Delta^2, m_\Delta^2) \\ &+ c_3 C_0(m_\Delta^2, 0, m_N^2, 0, m_N^2, m_\Delta^2) + c_4 C_0(m_N^2, 0, m_\Delta^2, 0, m_N^2, m_\Delta^2) \end{aligned}$$

Fitting Procedure



Least Squares Fit

$$\chi^2 = \chi^2_{\pi N} + \chi^2_C$$

$$\hat{\chi}^2 = \chi^2_{\pi N} + \chi^2_{\rm RS} + \chi^2_C$$

incl. 8 leading subthreshold parameters

$$\chi^2_C = \sum_i \left(\frac{a_i^2 - \bar{a}_i^2}{\delta a_i^2} \right)^2$$

$$\boldsymbol{a} = \{g_1, b_4, b_5\}$$

Theoretical Error

convergence behavior

$$\delta \mathcal{O}_i^{(n)} = \max(|\mathcal{O}_i^{(1)}| Q^n, \{|\mathcal{O}_i^{(k)} - \mathcal{O}_i^{(j)}| Q^{n-j}\}) \qquad j < k \le n$$

actual higher order contributions

$$\delta \mathcal{O}_i^{(n)} \ge \max(\{|\mathcal{O}_i^{(k)} - \mathcal{O}_i^{(j)}|\}) \qquad n \le j < k$$

$$Q = \frac{\omega_{\rm CMS}}{\Lambda_b}$$

$$\Lambda_b = 700\,\text{MeV}$$

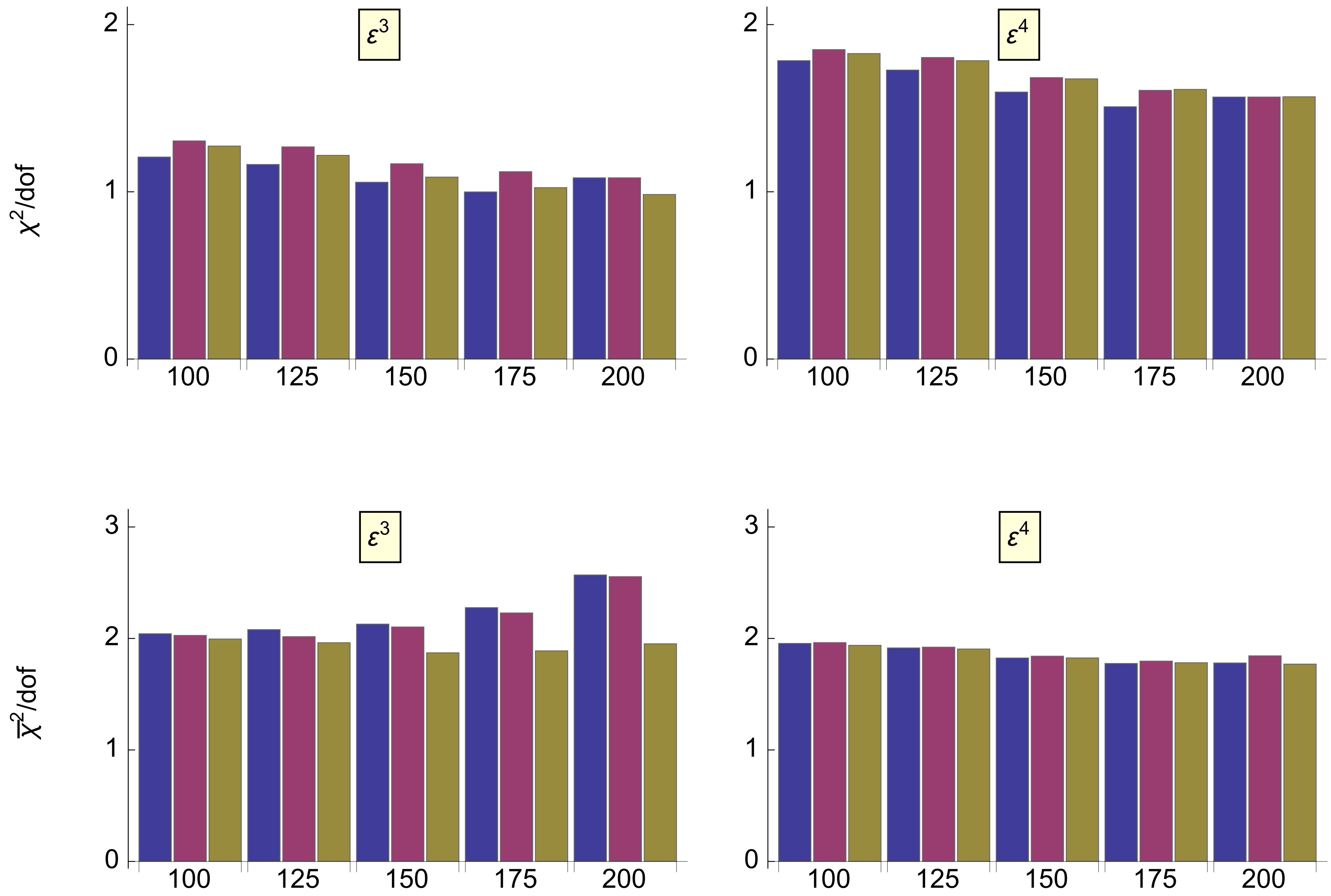
Epelbaum et al. - Eur. Phys. J. A 51 (2015)

Fits - χ^2 over T_π

with
theo. error

● HB−NN
● HB−πN
● Cov

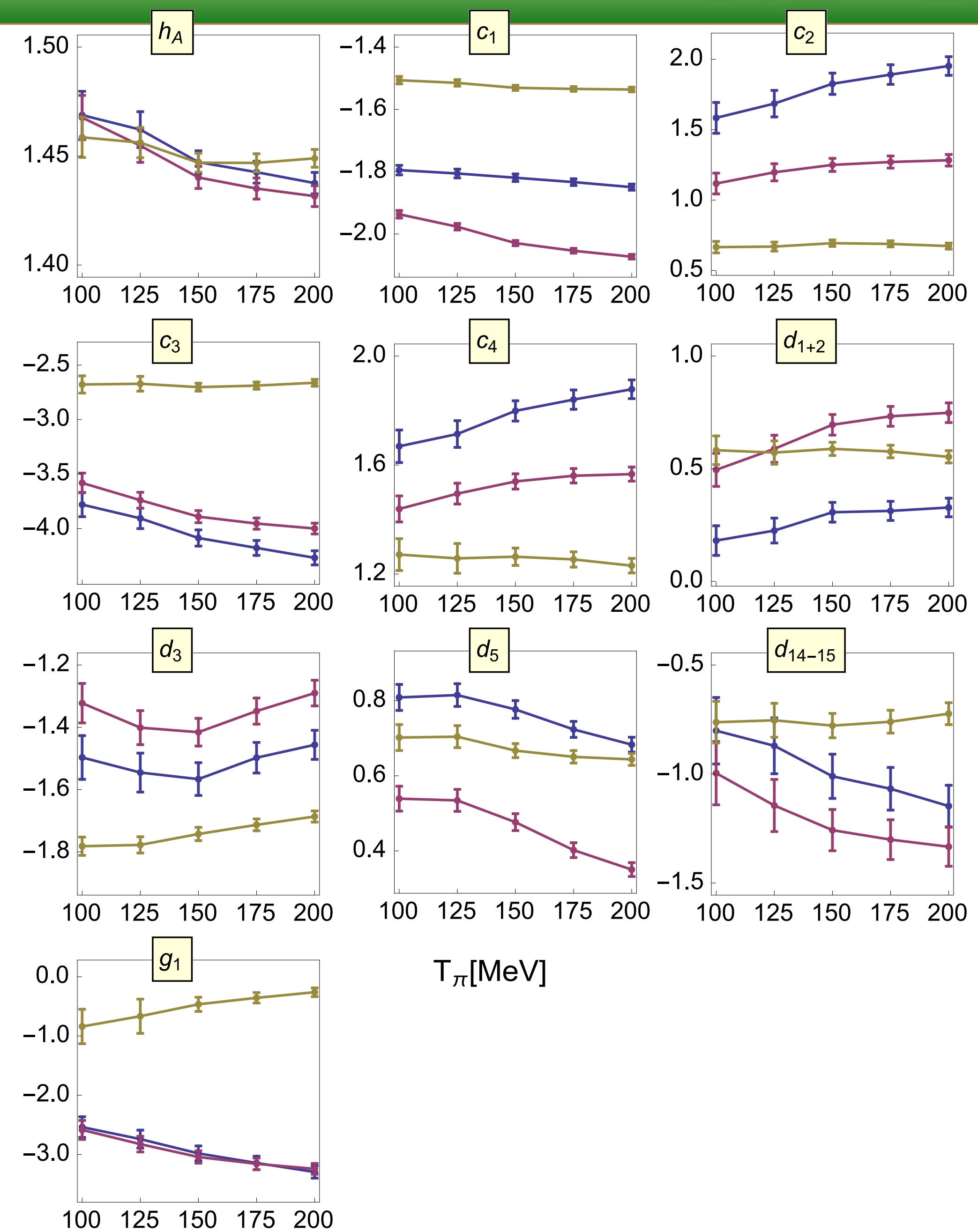
without
theo. error



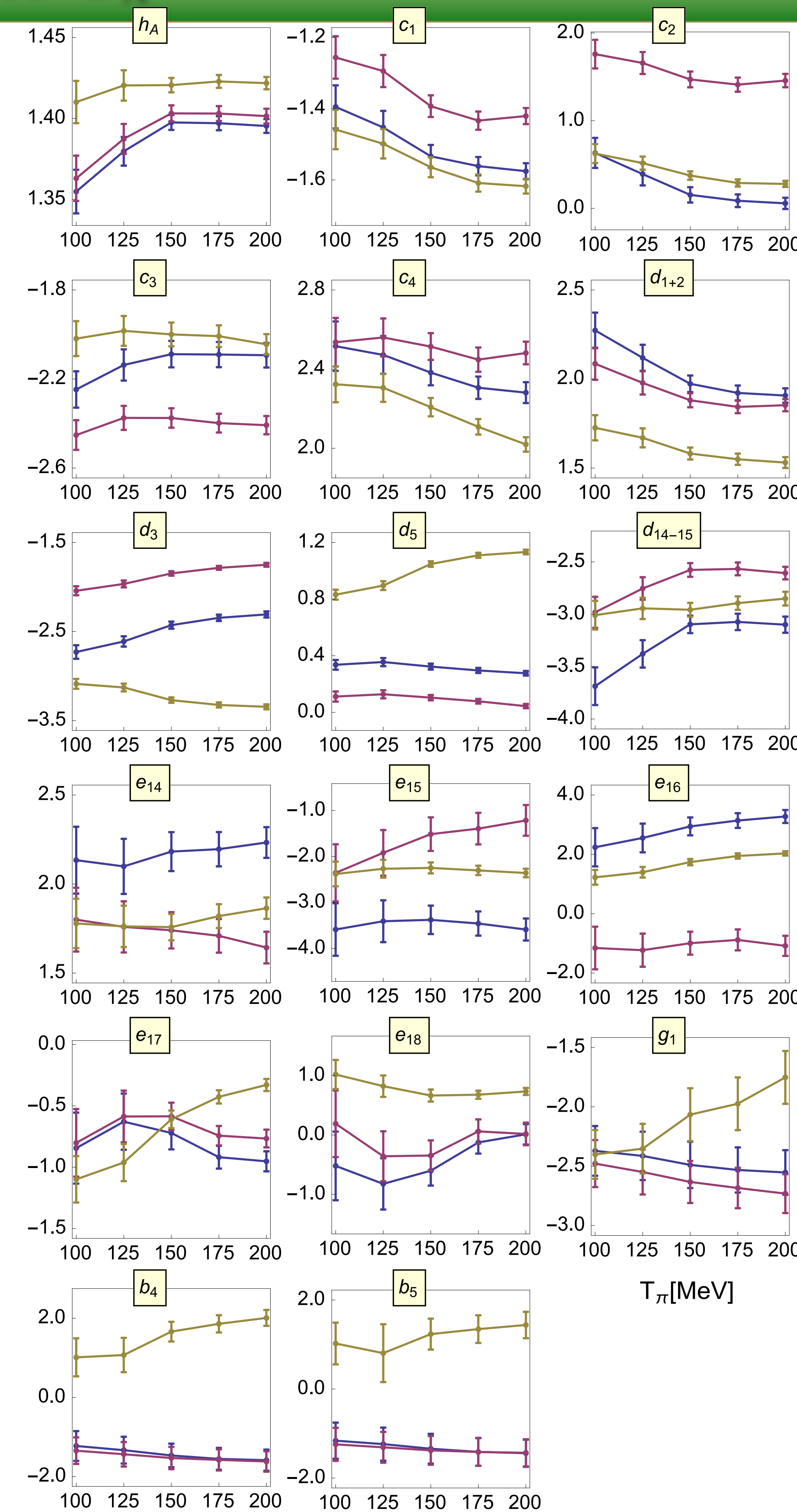
$T_\pi < \{100, 125, 150, 175, 200\}$ MeV $\widehat{=}$ $\{1704, 1854, 2176, 2399, 2564\}$ data points

Fits - LECs over T_π

ε^3



ε^4



- HB$-$NN
- HB$-\pi$N
- Cov

Representative Fits - $T_\pi < 125$ MeV

Input

m_N	M_π	F_π	m_Δ	g_A
938.27	139.57	92.2	1232	1.289

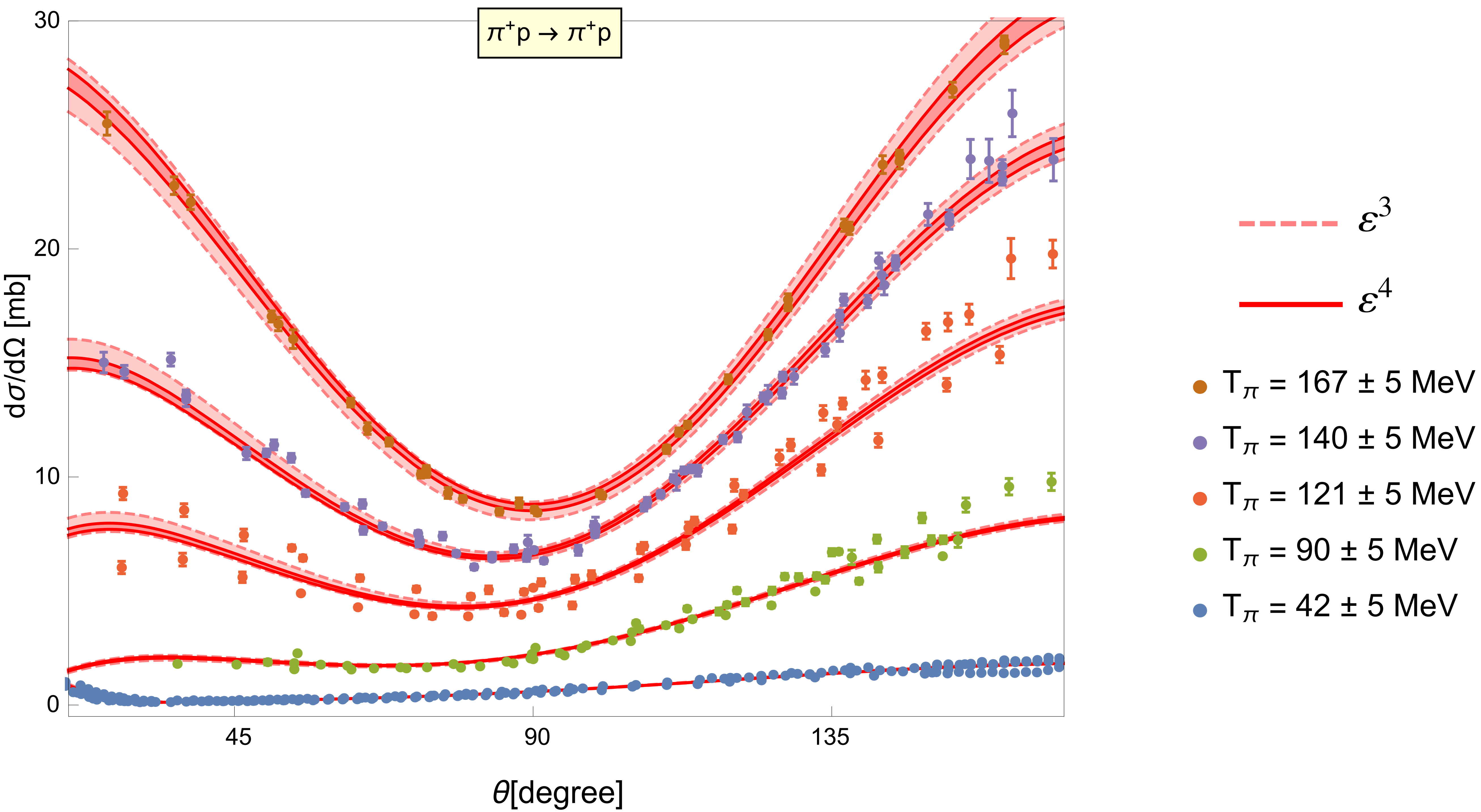
MeV

correlations: 🟠 🟢

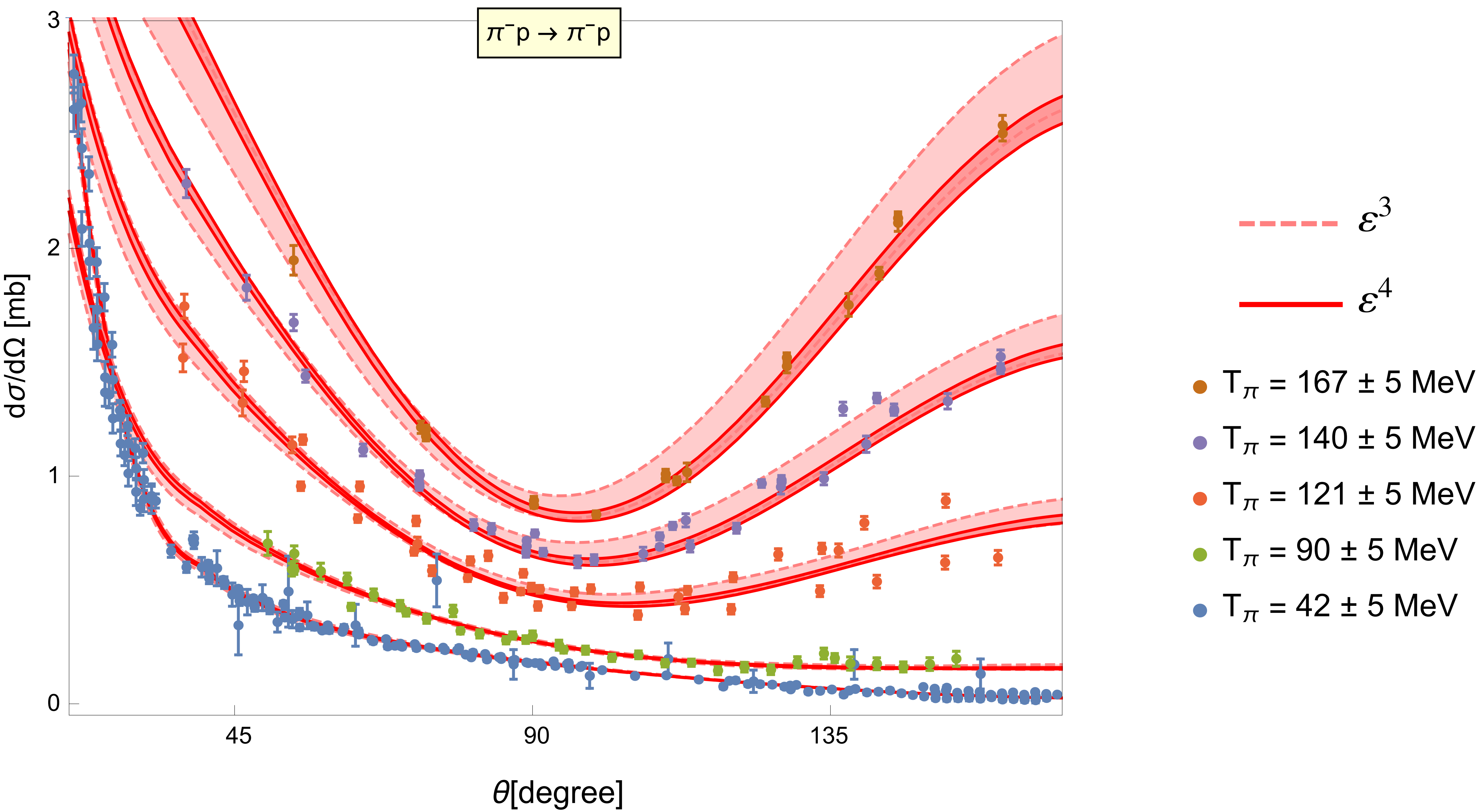
ε^4	HB-NN πN	HB-NN πN+RS	HB-πN πN	HB-πN πN+RS	Cov πN	Cov πN+RS
h_A	🟠 1.38(1)	🟠 1.37(1)	🟠 1.39(1)	🟠 1.38(1)	🟠 1.42(1)	🟠 1.40(1)
c_1	-1.45(5)	-1.39(3)	🟢 -1.29(5)	🟢 -1.30(4)	🟢 -1.50(4)	-1.32(3)
c_2	0.39(13)	0.51(10)	🟢 1.66(13)	🟢 1.61(10)	🟢 0.52(7)	0.86(5)
c_3	-2.14(7)	-2.12(6)	-2.37(5)	-2.34(5)	-1.98(7)	-1.98(6)
c_4	2.47(10)	2.29(5)	2.56(10)	2.43(6)	2.31(7)	2.28(4)
d_{1+2}	2.12(7)	🟠 2.07(6)	1.98(7)	🟠 1.94(6)	1.67(5)	1.74(5)
d_3	-2.61(6)	-2.62(5)	-1.97(4)	-1.96(4)	-3.13(4)	-3.07(4)
d_5	0.36(3)	0.39(3)	0.13(3)	0.15(3)	0.90(3)	0.81(3)
d_{14-15}	🟠 -3.38(13)	🟠 -3.53(12)	🟠 -2.75(11)	🟠 -2.76(10)	🟠 -2.94(10)	🟠 -3.16(9)
e_{14}	2.10(15)	2.30(13)	1.76(14)	1.92(12)	1.76(12)	1.61(10)
e_{15}	-3.41(45)	-4.13(26)	🟢 -1.92(50)	-2.61(31)	-2.27(19)	-2.50(17)
e_{16}	2.55(48)	2.70(28)	🟢 -1.23(56)	🟢 -0.65(37)	1.40(18)	0.88(9)
e_{17}	-0.63(23)	-0.53(20)	-0.59(21)	-0.71(19)	-0.96(15)	-0.87(14)
e_{18}	-0.82(43)	-0.11(15)	-0.36(42)	0.30(21)	0.82(18)	1.03(10)
g_1	-2.41(20)	-2.52(19)	-2.55(19)	-2.60(17)	-2.35(21)	-2.32(20)
b_4	-1.33(34)	-1.45(29)	-1.44(31)	-1.56(28)	1.07(43)	1.55(28)
b_5	-1.24(37)	-1.39(32)	-1.31(35)	-1.39(32)	0.81(65)	1.35(32)
$\chi^2_{\pi N}/\text{dof}$	1.73	1.73	1.80	1.80	1.78	1.80
$\bar{\chi}^2_{\pi N}/\text{dof}$	1.91	1.92	1.92	1.92	1.91	1.93

Predictions

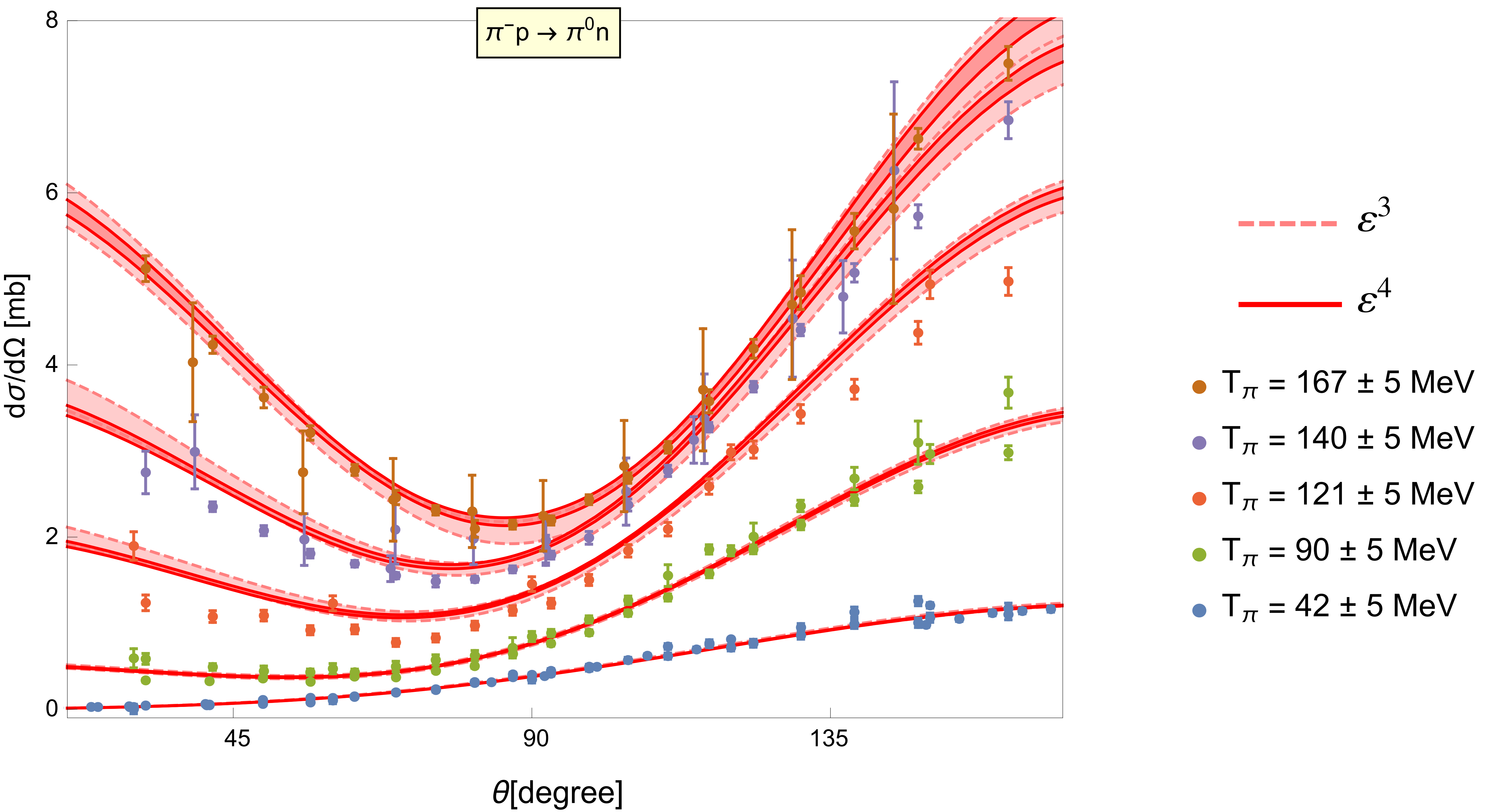
Diff. Cross Sections Theo. Error

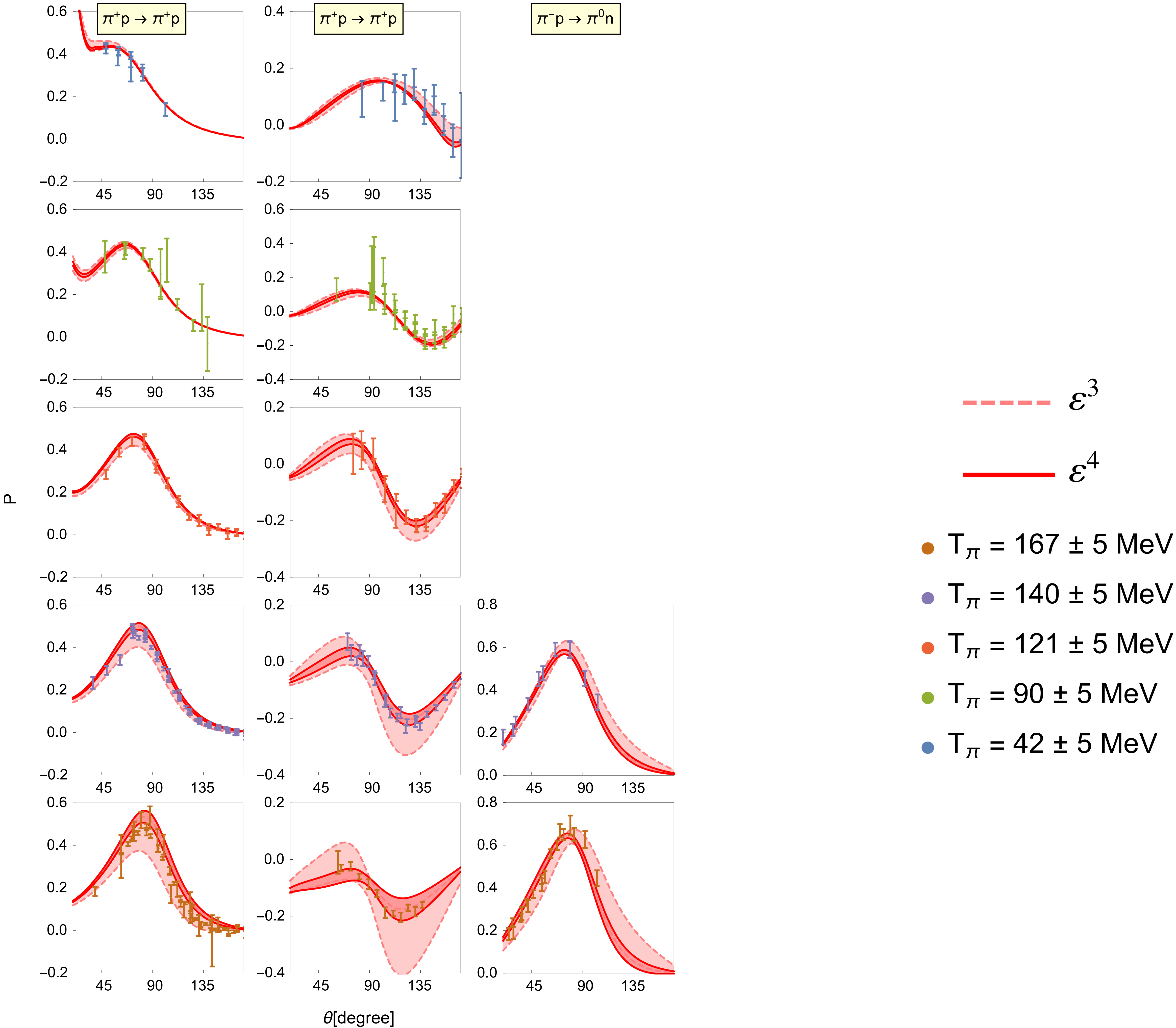


Diff. Cross Sections Theo. Error



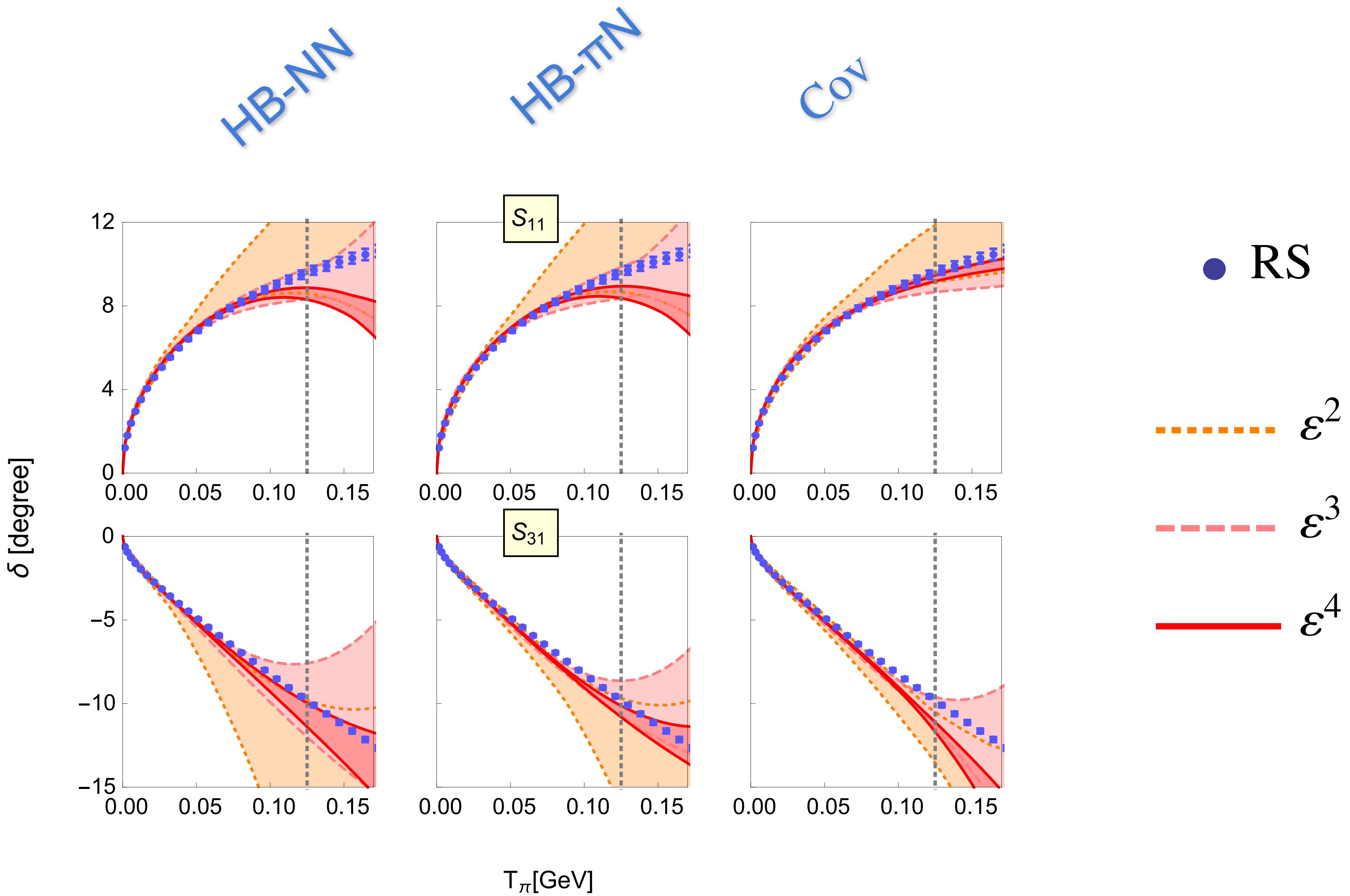
Diff. Cross Sections Theo. Error



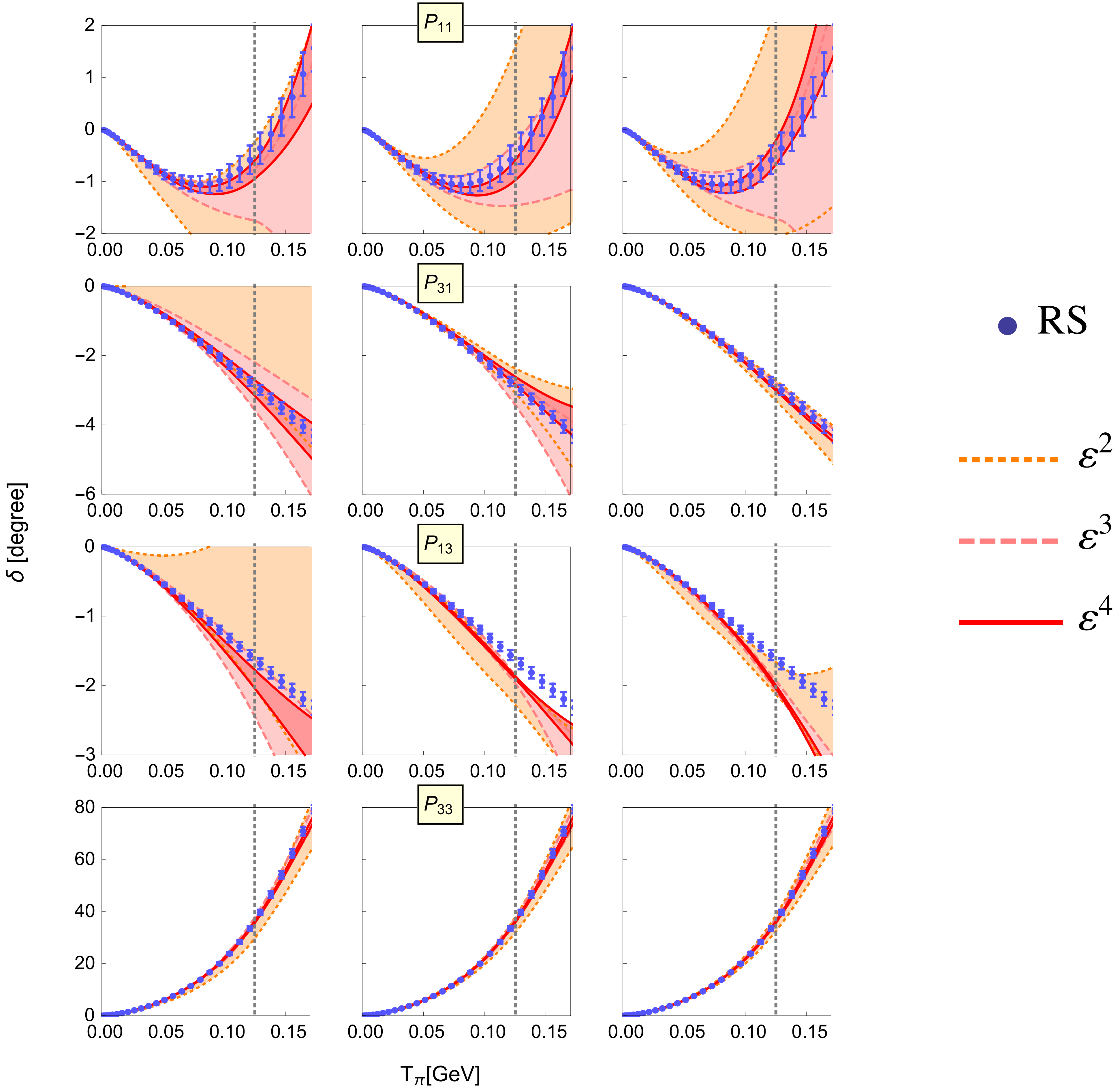


$\pi^+p \to \pi^+p$
$\pi^+p \to \pi^+p$
$\pi^-p \to \pi^0n$
P
θ[degree]
ε^3
ε^4
T_π = 167 ± 5 MeV
T_π = 140 ± 5 MeV
T_π = 121 ± 5 MeV
T_π = 90 ± 5 MeV
T_π = 42 ± 5 MeV

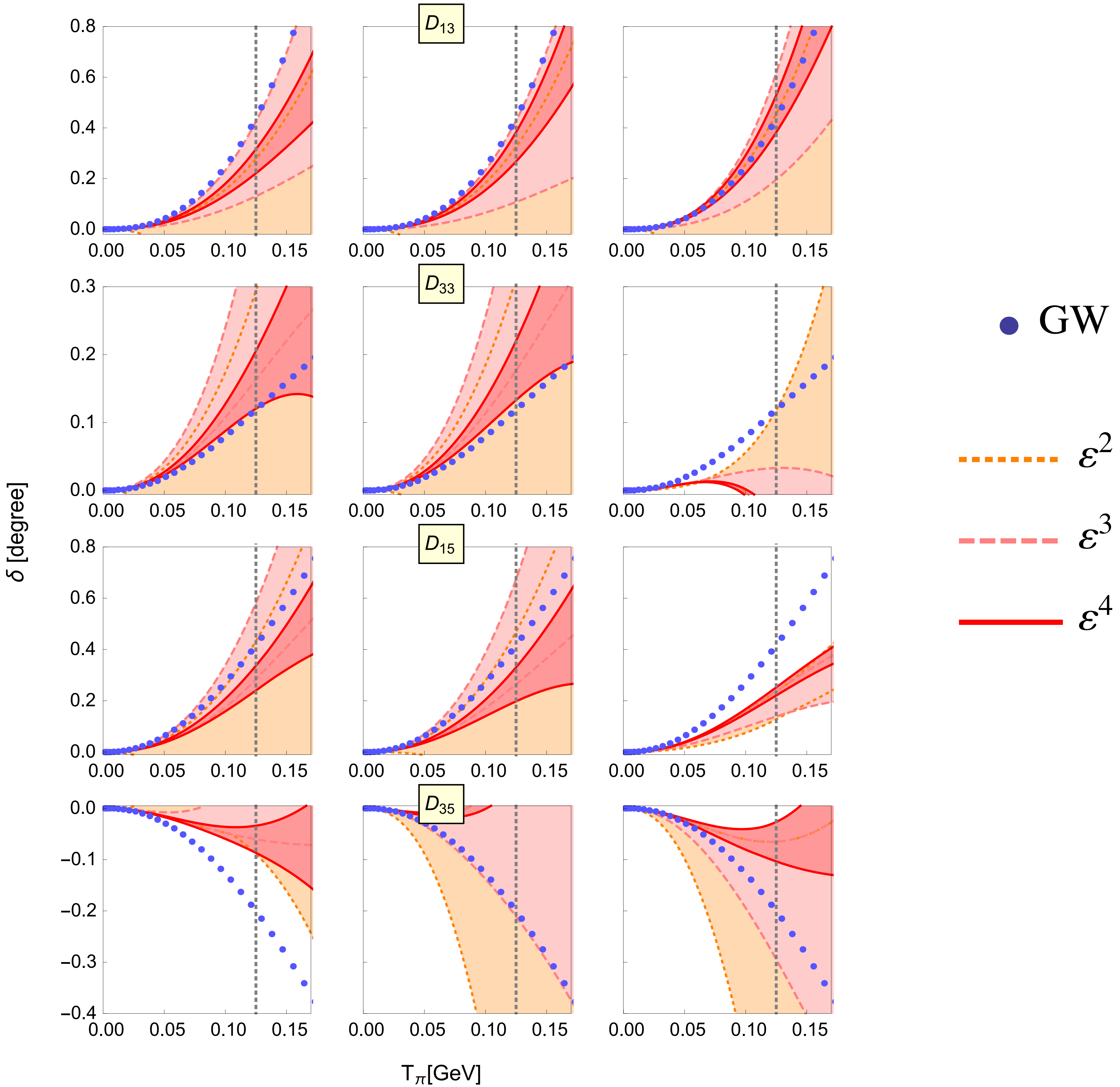
S-Waves Theo. Error



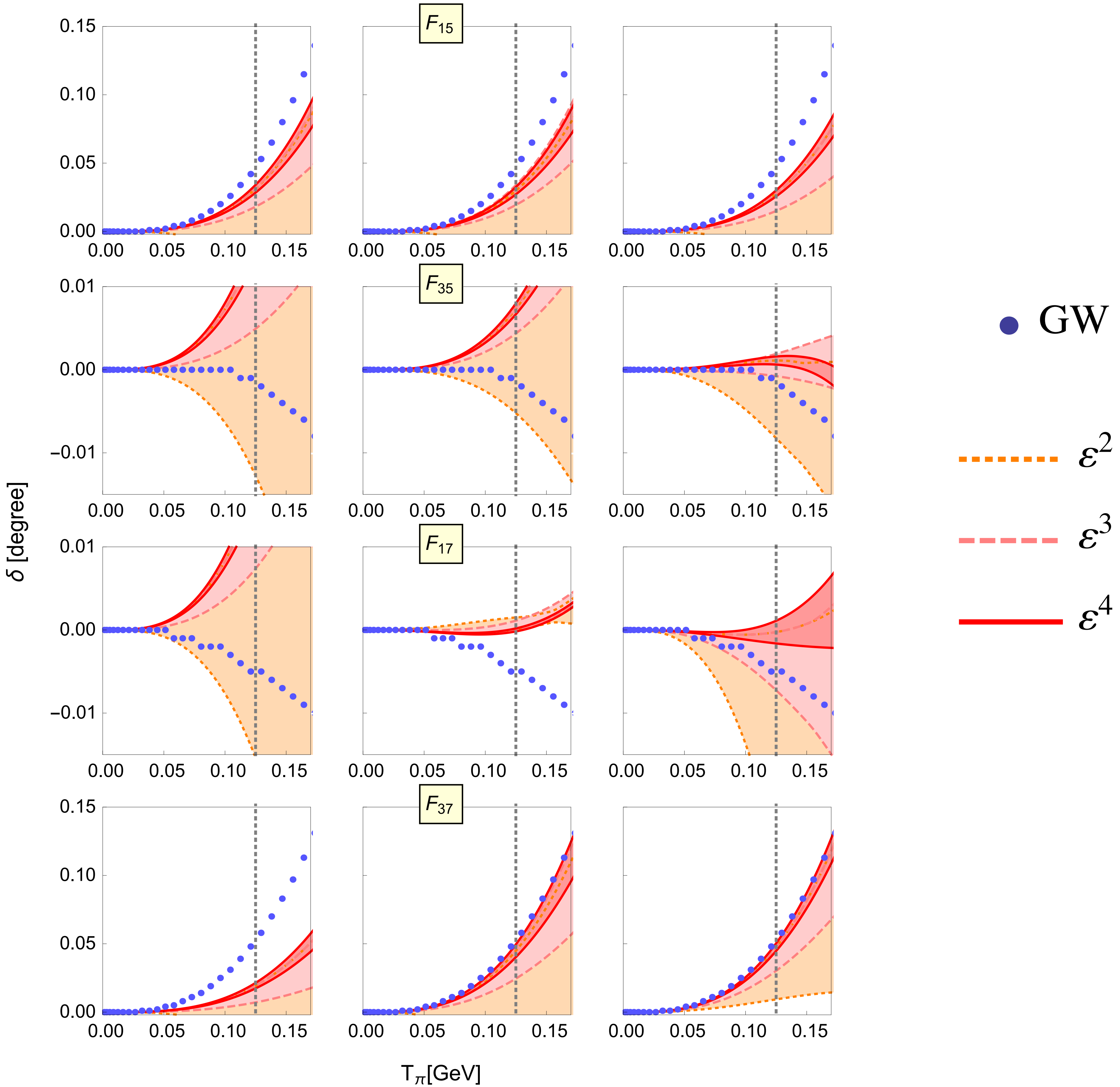
P-Waves Theo. Error



D-Waves Theo. Error



F-Waves Theo. Error



Summary

Good description of $\pi N \rightarrow \pi N$ data up to 170 MeV

- agreement with exp. scattering data
- agreement with RS S- and P-waves
- problems with some GW D- and F-waves
- almost no differences between the counting schemes
- χ^2/dof stays constant for energies above 100 MeV
- limited by applicability of K-matrix unitarization
- correlations between LECs

Extensions

- Complex mass approach
- consistent combined fits of $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi\pi N$ exp. data
    - Q^4
    - ε^4

Subthreshold Parameters

Matching RS to χPT

RS analysis

d^+_{00} $[M_\pi^{-1}]$	$-1.36(3)$	d^-_{00} $[M_\pi^{-2}]$	$1.41(1)$
d^+_{10} $[M_\pi^{-3}]$	$1.16(2)$	d^-_{10} $[M_\pi^{-4}]$	$-0.159(4)$
d^+_{01} $[M_\pi^{-3}]$	$1.16(2)$	d^-_{01} $[M_\pi^{-4}]$	$-0.141(5)$
d^+_{20} $[M_\pi^{-5}]$	$0.196(3)$	b^-_{00} $[M_\pi^{-2}]$	$10.49(11)$
d^+_{11} $[M_\pi^{-5}]$	$0.185(3)$	b^-_{10} $[M_\pi^{-4}]$	$1.00(3)$
d^+_{02} $[M_\pi^{-5}]$	$0.0336(6)$	b^-_{01} $[M_\pi^{-4}]$	$0.21(2)$
b^+_{00} $[M_\pi^{-3}]$	$-3.45(7)$		

Hoferichter, Ruiz de Elvira, Kubis,
Meißner - Phys.Rev.Lett. 115 (2015)

χPT

$$T^I(\nu,t) = \bar{u}(p')\left\{D^I(\nu,t) - \frac{[\not{q}',\not{q}]}{4m_N}B^I(\nu,t)\right\}u(p)$$

$$\bar{D}^\pm(\nu,t) = \binom{1}{\nu}\sum_{n,m=0}^{\infty} d^\pm_{mn}\nu^{2m}t^n$$

$$\bar{B}^\pm(\nu,t) = \binom{\nu}{1}\sum_{n,m=0}^{\infty} b^\pm_{mn}\nu^{2m}t^n$$

Matching RS to χPT

RS analysis

d^+_{00} $[M_\pi^{-1}]$	$-1.36(3)$	d^-_{00} $[M_\pi^{-2}]$	$1.41(1)$
d^+_{10} $[M_\pi^{-3}]$	$1.16(2)$	d^-_{10} $[M_\pi^{-4}]$	$-0.159(4)$
d^+_{01} $[M_\pi^{-3}]$	$1.16(2)$	d^-_{01} $[M_\pi^{-4}]$	$-0.141(5)$
d^+_{20} $[M_\pi^{-5}]$	$0.196(3)$	b^-_{00} $[M_\pi^{-2}]$	$10.49(11)$
d^+_{11} $[M_\pi^{-5}]$	$0.185(3)$	b^-_{10} $[M_\pi^{-4}]$	$1.00(3)$
d^+_{02} $[M_\pi^{-5}]$	$0.0336(6)$	b^-_{01} $[M_\pi^{-4}]$	$0.21(2)$
b^+_{00} $[M_\pi^{-3}]$	$-3.45(7)$		

Hoferichter, Ruiz de Elvira, Kubis, Meißner - Phys.Rev.Lett. 115 (2015)

χPT

$$T^I(\nu,t) = \bar{u}(p')\left\{D^I(\nu,t) - \frac{[\not{q}',\not{q}]}{4m_N}B^I(\nu,t)\right\}u(p)$$

$$\bar{D}^\pm(\nu,t) = \begin{pmatrix}1\\ \nu\end{pmatrix}\sum_{n,m=0}^{\infty} d^\pm_{mn}\nu^{2m}t^n$$

$$\bar{B}^\pm(\nu,t) = \begin{pmatrix}\nu\\ 1\end{pmatrix}\sum_{n,m=0}^{\infty} b^\pm_{mn}\nu^{2m}t^n$$

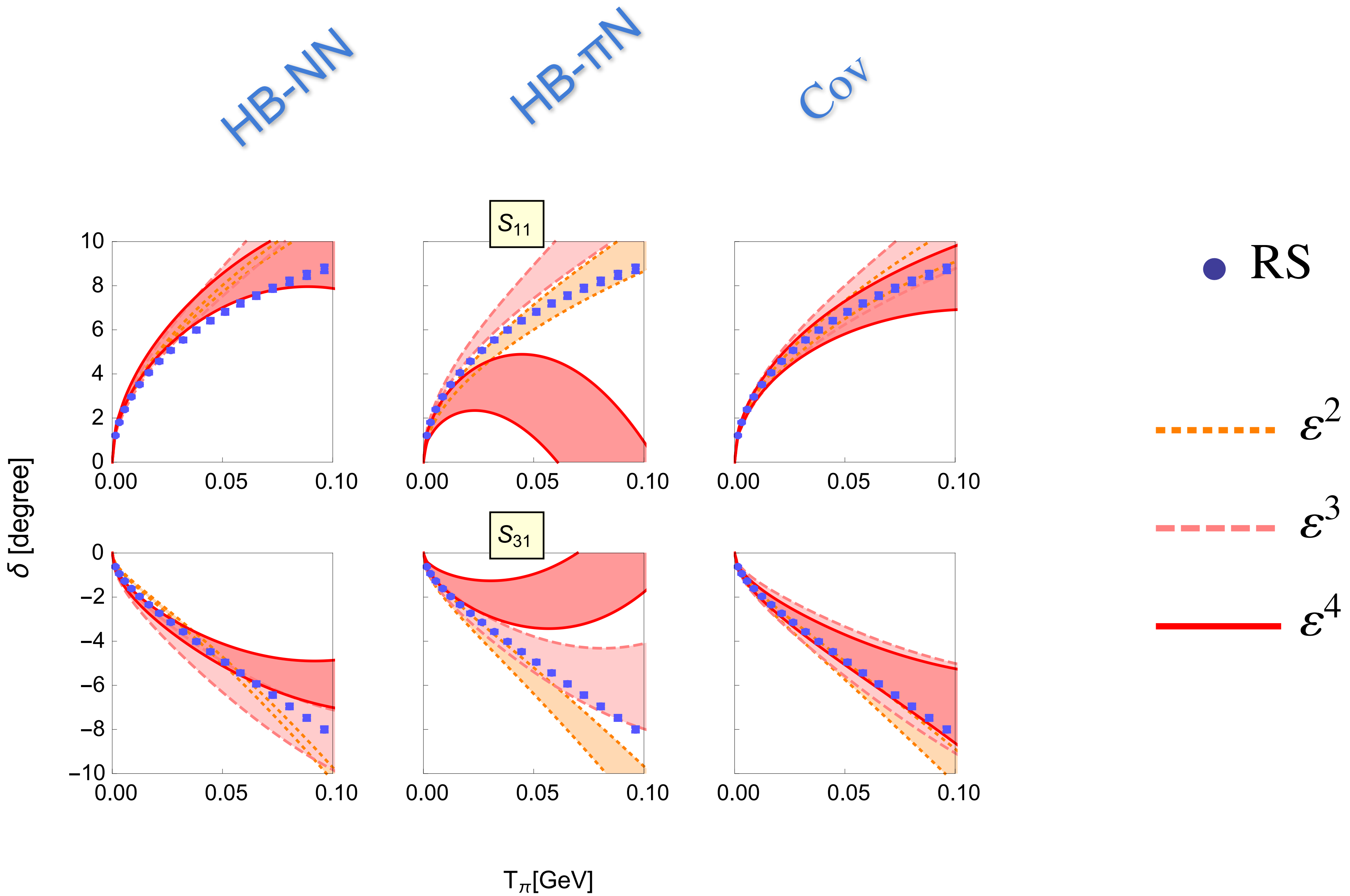
$h_A = 1.40 \pm 0.05$ $\qquad g_1 = b_4 = b_5 = 0 \pm 3$

N^3LO	Q^4	ε^4	Q^4	ε^4	Q^4	ε^4
c_1	-1.11(3)	-1.11(3)	-1.11(3)	-1.11(3)	-1.12(3)	-1.10(3)
c_2	3.61(4)	1.41(38)	3.17(3)	1.28(20)	3.35(3)	1.16(20)
c_3	-5.60(6)	-1.88(45)	-5.67(6)	-2.04(39)	-5.70(6)	-2.10(39)
c_4	4.26(4)	2.03(28)	4.35(4)	2.07(29)	3.97(3)	1.91(27)
d_{1+2}	6.37(9)	1.78(31)	7.66(9)	2.90(30)	4.70(7)	1.78(24)
d_3	-9.18(9)	-3.64(36)	-10.77(10)	-5.91(50)	-5.26(5)	-3.25(14)
d_5	0.87(5)	1.52(7)	0.59(5)	1.03(7)	0.31(5)	0.66(6)
d_{14-15}	-12.56(12)	-4.38(54)	-13.44(12)	-5.17(55)	-8.84(10)	-3.41(41)
e_{14}	1.16(4)	1.64(10)	0.85(4)	1.12(16)	1.17(4)	1.28(11)
e_{15}	-2.26(6)	-4.95(15)	-0.83(6)	-3.30(25)	-2.58(7)	-3.07(13)
e_{16}	-0.29(3)	4.21(16)	-2.75(3)	1.92(43)	-1.77(3)	1.71(17)
e_{17}	-0.17(6)	-0.44(6)	0.03(6)	-0.39(7)	-0.45(6)	-0.51(7)
e_{18}	-3.47(5)	1.34(29)	-4.48(5)	0.67(31)	-1.68(5)	1.30(17)

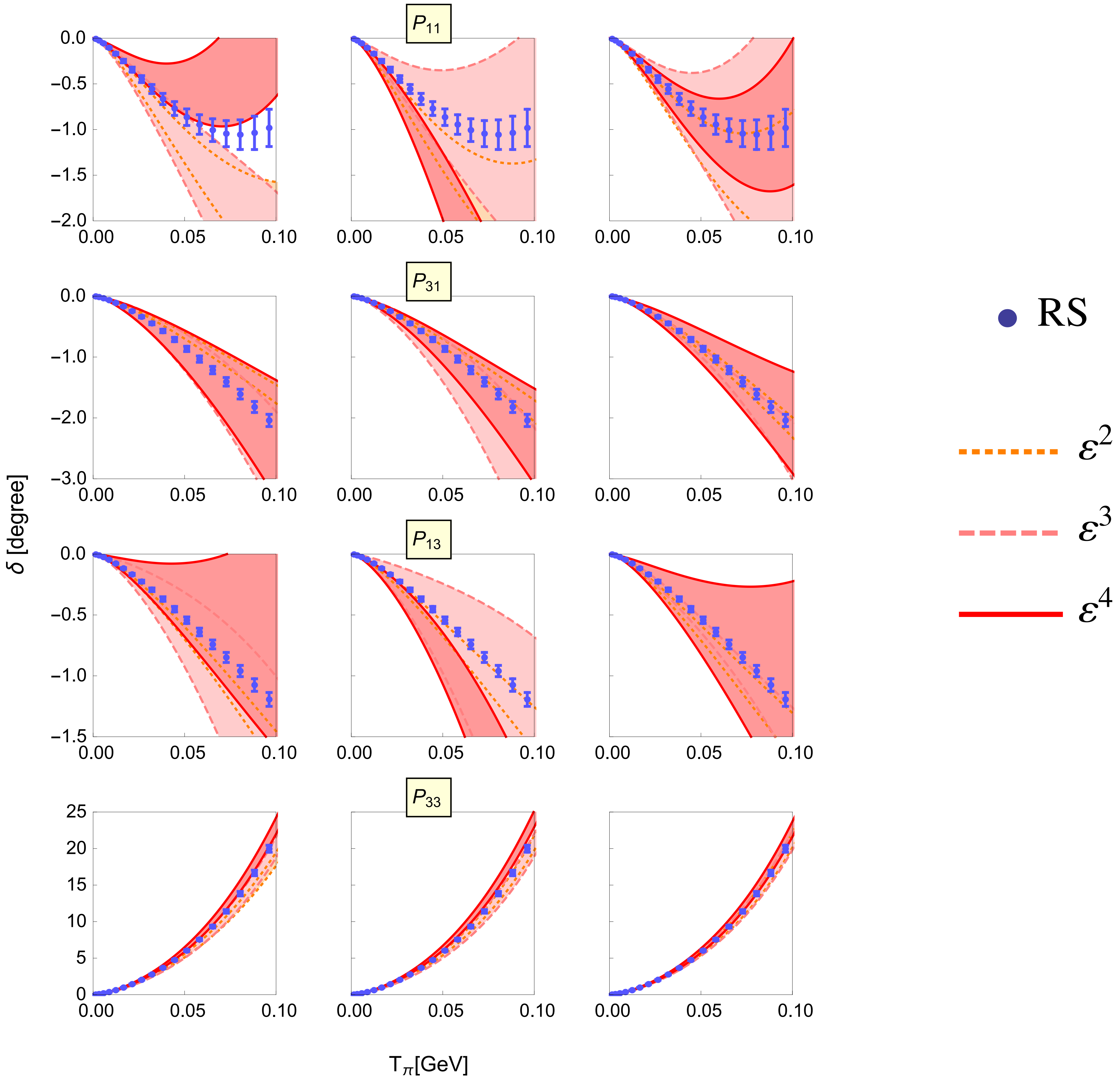
Predictions - Threshold Parameters

N^{3}LO	HB-NN Q^4	HB-NN ε^4	HB-πN Q^4	HB-πN ε^4	Cov Q^4	Cov ε^4	RS
$a^+_{0+}\,[M_\pi^{-1}10^{-3}]$	-1.5	-1.5(8.5)	-8.0	1.2(20.4)	-5.7	-0.8(10.3)	-0.9(1.4)
$a^-_{0+}\,[M_\pi^{-1}10^{-3}]$	68.5	96.3(2.0)	58.6	70.0(3.3)	83.8	83.6(1.9)	85.4(9)
$a^+_{1+}\,[M_\pi^{-3}10^{-3}]$	134.3	136.0(9.7)	132.1	135.2(8.7)	128.0	132.7(9.0)	131.2(1.7)
$a^-_{1+}\,[M_\pi^{-3}10^{-3}]$	-80.9	-80.0(3.4)	-90.1	-86.4(2.7)	-78.1	-81.1(3.6)	-80.3(1.1)
$a^+_{1-}\,[M_\pi^{-3}10^{-3}]$	-55.7	-47.5(10.5)	-73.7	-56.9(7.1)	-53.5	-51.4(7.9)	-50.9(1.9)
$a^-_{1-}\,[M_\pi^{-3}10^{-3}]$	-10.0	-5.6(4.9)	-23.7	-14.4(6.5)	-11.8	-10.4(5.7)	-9.9(1.2)
$b^+_{0+}\,[M_\pi^{-3}10^{-3}]$	-42.2	-31.4(8.1)	-44.5	-32.6(21.3)	-54.7	-33.9(8.5)	-45.0(1.0)
$b^-_{0+}\,[M_\pi^{-3}10^{-3}]$	-31.6	7.1(2.3)	-65.2	-34.1(5.7)	2.3	2.9(2.1)	4.9(8)

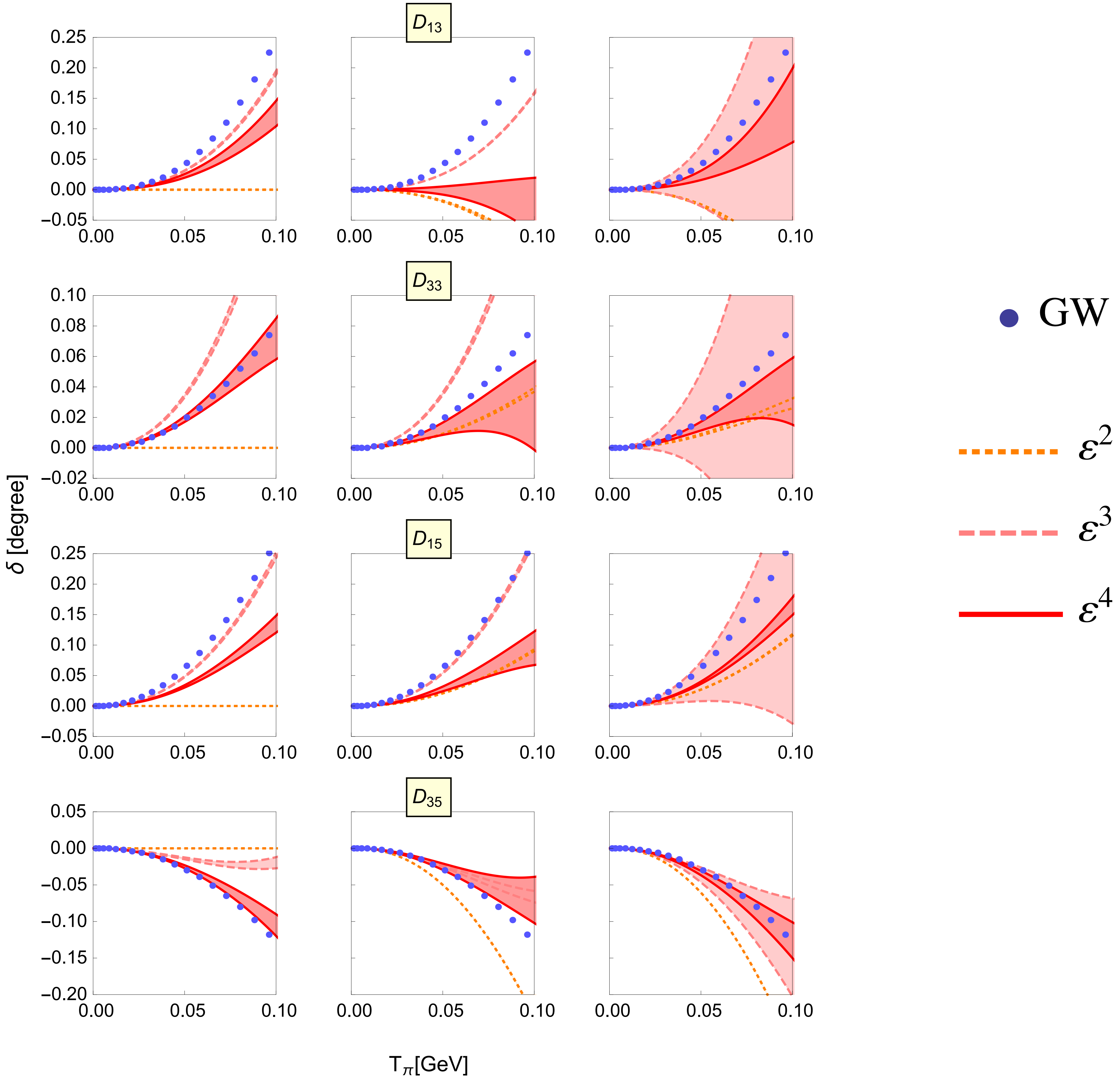
S-Waves Stat. Error



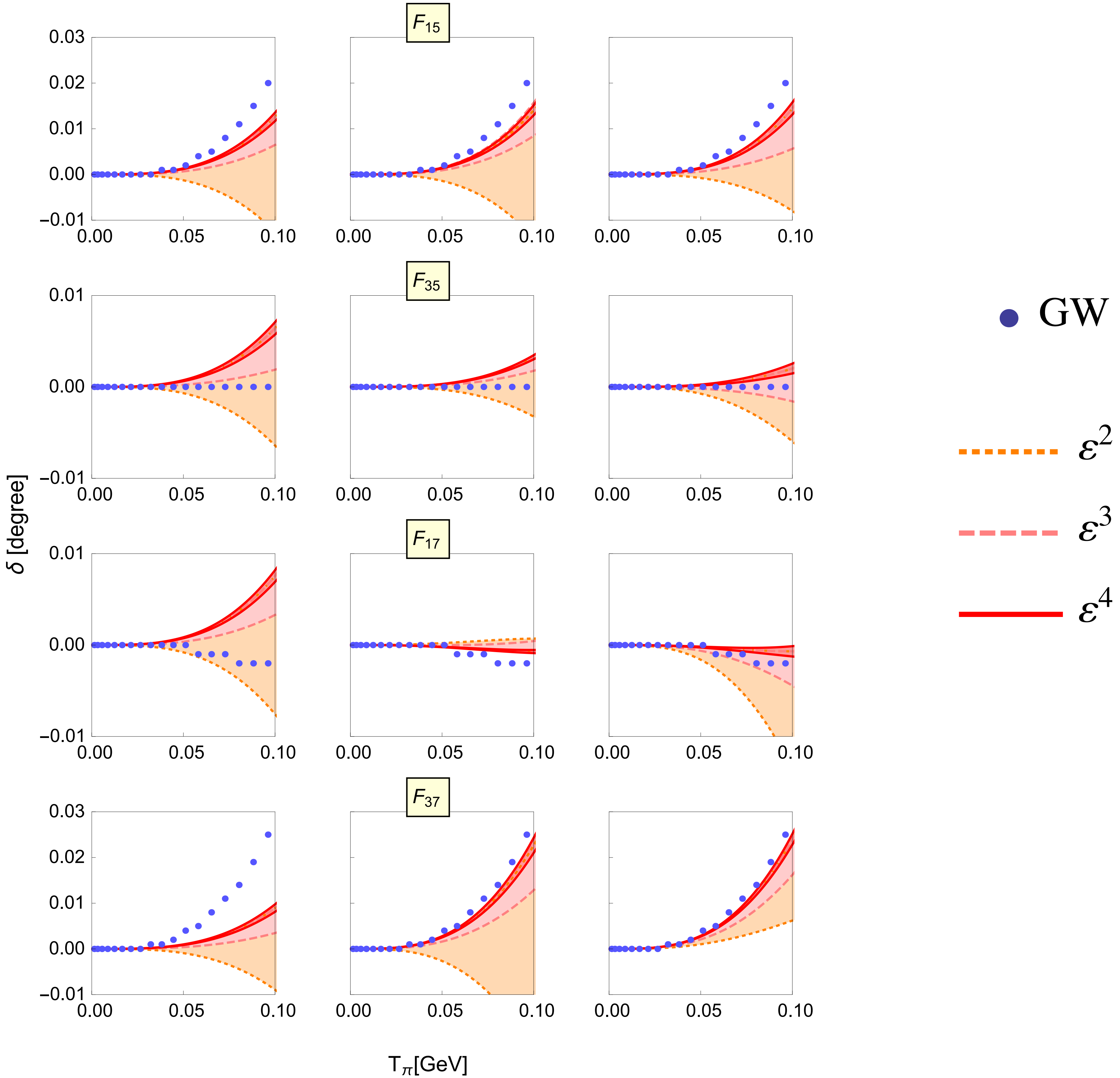
P-Waves Stat. Error



D-Waves Stat. Error



F-Waves Stat. Error



πN-Sigma Term

Hellmann-Feynman theorem

$$\sigma_{\pi N} = M_\pi^2 \frac{\partial m_N}{\partial M_\pi^2}$$

Cov

$\sigma_{\pi N}\,[\mathrm{MeV}]$

Q^2	ε^2
57.8 ± 1.9	53.7 ± 1.9
Q^3	ε^3
58.3 ± 1.9	60.7 ± 3.3
Q^4	ε^4
$64.9 - 0.8 e_1 \pm 2.0$	$63.9 - 0.8 e_1 \pm 2.1$

RS

$$\sigma_{\pi N} = (59.1 \pm 3.5)\ \mathrm{MeV}$$

Thank You !

Backup

$1/m_N$ - Convergence

LECs	$1/m_N$	d^+_{00}	d^+_{10}	d^+_{01}	d^+_{20}	d^+_{11}	d^+_{02}	b^+_{00}	d^-_{00}	d^-_{10}	d^-_{01}	b^-_{00}	b^-_{10}	b^-_{01}
HB	Q^4	-0.48	-0.67	0.70	1.30	0.80	0.052	-1.44	0.71	0.77	-0.06	6.67	6.29	0.47
Cov														
Cov	All	-1.22	0.75	0.97	0.54	0.43	-0.004	-6.05	1.40	-0.21	-0.25	8.03	4.13	0.38
RS		-1.36	1.16	1.16	0.20	0.18	0.034	-3.45	1.41	-0.16	-0.14	10.49	1.00	0.21

$1/m_N$ - Convergence

LECs	$1/m_N$	d_{00}^+	d_{10}^+	d_{01}^+	d_{20}^+	d_{11}^+	d_{02}^+	b_{00}^+	d_{00}^-	d_{10}^-	d_{01}^-	b_{00}^-	b_{10}^-	b_{01}^-
HB	Q^4	-0.48	-0.67	0.70	1.30	0.80	0.052	-1.44	0.71	0.77	-0.06	6.67	6.29	0.47
Cov	Q^4	-1.19	0.69	0.95	0.66	0.51	0.003	-1.85	0.92	0.50	-0.04	6.50	5.62	0.53
Cov	All	-1.22	0.75	0.97	0.54	0.43	-0.004	-6.05	1.40	-0.21	-0.25	8.03	4.13	0.38
RS		-1.36	1.16	1.16	0.20	0.18	0.034	-3.45	1.41	-0.16	-0.14	10.49	1.00	0.21

$1/m_N$ - Convergence

LECs	$1/m_N$	d^+_{00}	d^+_{10}	d^+_{01}	d^+_{20}	d^+_{11}	d^+_{02}	b^+_{00}	d^-_{00}	d^-_{10}	d^-_{01}	b^-_{00}	b^-_{10}	b^-_{01}
HB	Q^4	-0.48	-0.67	0.70	1.30	0.80	0.052	-1.44	0.71	0.77	-0.06	6.67	6.29	0.47
Cov	Q^4	-1.19	0.69	0.95	0.66	0.51	0.003	-1.85	0.92	0.50	-0.04	6.50	5.62	0.53
	Q^5	-1.22	0.73	0.98	0.52	0.38	-0.004	-5.05	1.24	0.21	-0.17	8.49	3.30	0.29
	Q^6	-1.21	0.72	0.97	0.59	0.42	-0.005	-6.24	1.43	-0.33	-0.27	8.06	3.91	0.36
	Q^7	-1.22	0.75	0.97	0.53	0.43	-0.004	-5.96	1.38	-0.19	-0.25	8.00	4.23	0.39
Cov	All	-1.22	0.75	0.97	0.54	0.43	-0.004	-6.05	1.40	-0.21	-0.25	8.03	4.13	0.38
RS		-1.36	1.16	1.16	0.20	0.18	0.034	-3.45	1.41	-0.16	-0.14	10.49	1.00	0.21

odd powers in M_π
enhanced by
π

→ $\arctan(M_\pi/m_N)$

even powers in M_π
enhanced by
$\ln(M_\pi^2/m_N^2)$