# Heavy baryon decay form factors from lattice QCD

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Production fraction of  $\Lambda_b$ Production fraction of B<sup>0</sup> 0.9 0.8°, V 0.7 <u>LHCD</u> 0.6 0.5 0.4 0.3 0.2 0.1 0  $p_{\rm T}^{30}$  [GeV/c] 30 10 20

[LHCb Collaboration, JHEP 08, 143 (2014)]

- $1 \quad |V_{ub}| \text{ and } |V_{cb}|$
- $b \rightarrow c \tau^- \bar{\nu}$
- $b \rightarrow s \mu^+ \mu^-$



[Particle Data Group, November 2015]



#### LHCb result:

$$\frac{\int_{15 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \to p \ \mu^- \overline{\nu}_\mu)}{dq^2} dq^2}{\int_{7 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \to \Lambda_c \ \mu^- \overline{\nu}_\mu)}{dq^2} dq^2} = (1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$$
$$(q = p - p').$$



To extract  $|V_{ub}/V_{cb}|$  from this, need

$$\begin{array}{l} \left\langle p \mid \bar{u}\gamma^{\mu}b \mid \Lambda_{b} \right\rangle, \ \left\langle p \mid \bar{u}\gamma^{\mu}\gamma_{5}b \mid \Lambda_{b} \right\rangle, \\ \left\langle \Lambda_{c} \mid \bar{c}\gamma^{\mu}b \mid \Lambda_{b} \right\rangle, \ \left\langle \Lambda_{c} \mid \bar{c}\gamma^{\mu}\gamma_{5}b \mid \Lambda_{b} \right\rangle \end{array}$$

from lattice QCD.

Helicity form factors:

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$$\begin{split} \langle F | \overline{q} \gamma^{\mu} b | \Lambda_{b} \rangle &= \overline{u}_{F} \left[ (m_{\Lambda_{b}} - m_{F}) \frac{q^{\mu}}{q^{2}} f_{0} \\ &+ \frac{m_{\Lambda_{b}} + m_{F}}{s_{+}} \left( p^{\mu} + p'^{\mu} - (m_{\Lambda_{b}}^{2} - m_{F}^{2}) \frac{q^{\mu}}{q^{2}} \right) f_{+} \\ &+ \left( \gamma^{\mu} - \frac{2m_{F}}{s_{+}} p^{\mu} - \frac{2m_{\Lambda_{b}}}{s_{+}} p'^{\mu} \right) f_{\perp} \right] u_{\Lambda_{b}}, \end{split}$$

$$F | \overline{q} \gamma^{\mu} \gamma_{5} b | \Lambda_{b} \rangle = -\overline{u}_{F} \gamma_{5} \left[ (m_{\Lambda_{b}} + m_{F}) \frac{q^{\mu}}{q^{2}} g_{0} \\ &+ \frac{m_{\Lambda_{b}} - m_{F}}{s_{-}} \left( p^{\mu} + p'^{\mu} - (m_{\Lambda_{b}}^{2} - m_{F}^{2}) \frac{q^{\mu}}{q^{2}} \right) g_{+} \\ &+ \left( \gamma^{\mu} + \frac{2m_{F}}{s_{-}} p^{\mu} - \frac{2m_{\Lambda_{b}}}{s_{-}} p'^{\mu} \right) g_{\perp} \right] u_{\Lambda_{b}}. \end{split}$$

$$F = p, \Lambda_c, \quad \bar{q} = \bar{u}, \bar{c}, \quad s_{\pm} = (m_{\Lambda_b} \pm m_F)^2 - q^2$$

[T. Feldmann and M. Yip, PRD 85, 014035 (2012)]

## $\Lambda_b \to p \, \ell^- \, \bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c \, \ell^- \, \bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks

[W. Detmold, C. Lehner, S. Meinel, PRD 92, 034503 (2015)]

• Gauge field configurations generated by the RBC and UKQCD collaborations

[Y. Aoki et al., PRD 83, 074508 (2011)]

- *u*, *d*, *s* quarks: domain-wall action
   [D. Kaplan, PLB 288, 342 (1992); V. Furman and Y. Shamir, NPB 439, 54 (1995)]
- c, b quarks: "relativistic heavy-quark action"
   [A. El-Khadra, A. Kronfeld, P. Mackenzie, PRD 55, 3933 (1997); Y. Aoki et al., PRD 86, 116003]
- "Mostly nonperturbative" renormalization [A. El-Khadra *et al.*, PRD **64**, 014502 (2001)]
- $a \approx 0.11 \, \mathrm{fm}, \, 0.085 \, \mathrm{fm}$
- 230 MeV  $\leq m_\pi \leq$  350 MeV
- Three-point functions with 12 source-sink separations



t = source-sink separation t' = current insertion time



$\mathbf{H}_{\mathbf{H}} = 0.112 \text{ fm}, \ m_{\pi} = 336 \text{ MeV}$	$H_{\pm}$ $a=0.085$ fm, $m_{\pi}=352$ MeV
Here $a=0.112$ fm, $m_{\pi}=270$ MeV	$\mathbf{F}_{\mathbf{H}}$ $a=0.085~\mathrm{fm},~m_{\pi}=295~\mathrm{MeV}$
$\mathbf{H}_{\mathbf{H}} a = 0.112 \text{ fm}, \ m_{\pi} = 245 \text{ MeV}$	$\mathbf{H}_{\mathbf{T}}^{\mathbf{T}}$ $a=0.085~\mathrm{fm},~m_{\pi}=227~\mathrm{MeV}$



### Combined chiral/continuum/kinematic extrapolation using modified z-expansion

[C. Bourrely, I. Caprini, L. Lellouch, PRD 79, 013008 (2009)]

$$z^{f}(q^{2}) = rac{\sqrt{t_{+}^{f}-q^{2}}-\sqrt{t_{+}^{f}-t_{0}}}{\sqrt{t_{+}^{f}-q^{2}}+\sqrt{t_{+}^{f}-t_{0}}},$$

"Nominal fit"

$$\begin{split} f(q^2) &= \frac{1}{1 - q^2 / (m_{\text{pole}}^f)^2} \bigg[ a_0^f \bigg( 1 + c_0^f \frac{m_\pi^2 - m_{\pi,\text{phys}}^2}{\Lambda_\chi^2} \bigg) + a_1^f \, z^f(q^2) \bigg] \\ &\times \bigg[ 1 + \frac{b^f}{(\pi/a)^2} + \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + \frac{d^f}{(\pi/a)^2} \bigg], \end{split}$$

"Nominal fit" in physical limit a = 0,  $m_{\pi} = m_{\pi,\text{phys}}$ :

$$f(q^2) = \frac{1}{1 - q^2 / (m_{pole}^f)^2} \left[ a_0^f + a_1^f z^f(q^2) \right]$$

Here $a=0.112$ fm, $m_{\pi}=336$ MeV	$H_{\pm}$ $a=0.085~{ m fm},~m_{\pi}=352~{ m MeV}$	$a = 0, \ m_{\pi} = 135 \text{ MeV}$
$\mathbf{F}_{\mathbf{T}} = 0.112 \ fm, \ \ m_{\pi} = 270 \ MeV$	${f H}_{2}$ $a=0.085~{ m fm},~m_{\pi}=295~{ m MeV}$	
Herefore $a=0.112~{ m fm},~m_{\pi}=245~{ m MeV}$	ਸਤੂੰਸ $a=0.085~{ m fm},~m_{\pi}=227~{ m MeV}$	



Gray band = statistical uncertainty.

"Higher-order fit":

$$\begin{split} f_{\text{HO}}(q^2) &= \frac{1}{1 - q^2 / (m_{\text{pole}}^f)^2} \left[ a_0^f \left( 1 + c_0^f \frac{m_{\pi}^2 - m_{\pi,\text{phys}}^2}{\Lambda_{\chi}^2} + \tilde{c}_0^f \frac{m_{\pi}^3 - m_{\pi,\text{phys}}^3}{\Lambda_{\chi}^3} \right) \\ &+ a_1^f \left( 1 + c_1^f \frac{m_{\pi}^2 - m_{\pi,\text{phys}}^2}{\Lambda_{\chi}^2} \right) z^f(q^2) + a_2^f \left[ z^f(q^2) \right]^2 \right] \\ &\times \left[ 1 + b^f \frac{|\mathbf{p}'|^2}{(\pi/a)^2} + d^f \frac{\Lambda_{\text{QCD}}^2}{(\pi/a)^2} + \tilde{b}^f \frac{|\mathbf{p}'|^3}{(\pi/a)^3} + \tilde{d}^f \frac{\Lambda_{\text{QCD}}^3}{(\pi/a)^3} \right. \\ &+ j^f \frac{|\mathbf{p}'|^2 \Lambda_{\text{QCD}}}{(\pi/a)^3} + k^f \frac{|\mathbf{p}'| \Lambda_{\text{QCD}}^2}{(\pi/a)^3} \right] \end{split}$$

Higher-order fit parameters constrained with Gaussian priors to be natural-sized. Modified data correlation matrix to include other sources of uncertainty. "Higher-order fit" in physical limit a = 0,  $m_{\pi} = m_{\pi,\text{phys}}$ :

$$f_{\rm HO}(q^2) = \frac{1}{1 - q^2 / (m_{\rm pole}^f)^2} \left[ a_0^f + a_1^f z^f(q^2) + a_2^f [z^f(q^2)]^2 \right]$$





Compute systematic uncertainty of any observable O using

$$\sigma_{O,\text{syst.}} = \max\left(|O_{\text{HO}} - O|, \sqrt{|\sigma_{\text{HO}}^2 - \sigma_O^2|}\right)$$









$$rac{{
m d}\Gamma/{
m d}q^2}{|V_{ub}|^2}~({
m ps}^{-1}~{
m GeV}^{-2})$$



$$rac{{
m d}\Gamma/{
m d}q^2}{|V_{ub}|^2}~({
m ps}^{-1}~{
m GeV}^{-2})$$



$$\frac{1}{|V_{ub}|^2} \int_{15 \text{ GeV}^2}^{q^2_{\text{max}}} \frac{d\Gamma(\Lambda_b \to p \ \mu^- \bar{\nu}_\mu)}{dq^2} dq^2$$
$$= (12.31 \pm 0.76_{\text{stat}} \pm 0.77_{\text{syst}}) \text{ ps}^{-1}$$

$$rac{{
m d}\Gamma/{
m d}q^2}{|V_{cb}|^2}~({
m ps}^{-1}~{
m GeV}^{-2})$$



$$\frac{{\rm d}\Gamma/{\rm d}q^2}{|V_{cb}|^2} \,\, ({\rm ps}^{-1}\,{\rm GeV}^{-2})$$



$$\begin{aligned} &\frac{1}{|\mathbf{V}_{cb}|^2} \int_{7 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c \ \mu^- \bar{\nu}_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2 \\ &= (8.37 \pm 0.16_{\text{stat}} \pm 0.34_{\text{syst}}) \text{ ps}^{-1} \end{aligned}$$

$$\frac{|\boldsymbol{V_{cb}}|^2}{|\boldsymbol{V_{ub}}|^2} \frac{\int_{15 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \to p \ \mu^- \bar{\nu}_\mu)}{dq^2} dq^2}{\int_{7 \text{ GeV}^2}^{q_{\text{max}}^2} \frac{d\Gamma(\Lambda_b \to \Lambda_c \ \mu^- \bar{\nu}_\mu)}{dq^2} dq^2}$$

$$= 1.471 \pm 0.095_{
m stat.} \pm 0.109_{
m syst.}$$

Systematic uncertainties in the ratio of decay rates:

z expansion	1.8 % 1.3 %
	1.8%
Scale setting	
Isospin breaking/QED	2.0%
Matching & improvement	2.1 %
RHQ parameters	2.3%
Chiral extrapolation	2.6 %
Continuum extrapolation	2.8%
Finite volume	4.9%

Note: the combined uncertainty takes into account the correlations between the individual uncertainties

Combine with LHCb measurement:

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004_{\text{expt}} \pm 0.004_{\text{lat}}$$

[LHCb Collaboration, Nature Physics 11, 743-747 (2015)]





### $b \rightarrow c \tau^- \bar{\nu}$

### $b \rightarrow s \mu^+ \mu^-$





[http://www.slac.stanford.edu/xorg/hfag/]

$$rac{{
m d}\Gamma/{
m d}q^2}{|V_{cb}|^2}~({
m ps}^{-1}~{
m GeV}^{-2})$$



$$R[\Lambda_c] = \frac{\Gamma(\Lambda_b \to \Lambda_c \ \tau^{-} \overline{\nu}_{\tau})}{\Gamma(\Lambda_b \to \Lambda_c \ \mu^{-} \overline{\nu}_{\mu})} = 0.3328 \pm 0.0074 \pm 0.0070$$

[W. Detmold, C. Lehner, S. Meinel, PRD 92, 034503 (2015)]

- $|V_{ub}|$  and  $|V_{cb}|$
- $b \rightarrow c \tau^- \bar{\nu}$

### $b \rightarrow s \mu^+ \mu^-$



Fits of  $C_i^{\text{NP}} = C_i - C_i^{\text{SM}}$ to experimental data for mesonic  $b \to s \mu^+ \mu^-$  decays





[S. Descotes-Genon, L. Hofer, J. Matias, J. Virto, JHEP **1606**, 092 (2016)] Complementary information can be obtained from  $\Lambda_b o \Lambda \mu^+ \mu^-$ 



Combines the best aspects of  $B \to K^* \mu^+ \mu^-$  and  $B \to K \mu^+ \mu^-$ :  $\Lambda$  has nonzero spin and is stable under strong interactions.

 $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$  form factors, differential branching fraction, and angular observables from lattice QCD with relativistic *b* quarks [W. Detmold, C. Lehner, S. Meinel, PRD **92**, 034503 (2015)]







For unpolarized  $\Lambda_b$ :

$$\frac{d^{4}\Gamma}{dq^{2} d\cos\theta_{\ell} d\cos\theta_{\Lambda} d\phi} = \frac{3}{8\pi} \left[ (K_{1ss} \sin^{2}\theta_{\ell} + K_{1cc} \cos^{2}\theta_{\ell} + K_{1c} \cos\theta_{\ell}) + (K_{2ss} \sin^{2}\theta_{\ell} + K_{2cc} \cos^{2}\theta_{\ell} + K_{2c} \cos\theta_{\ell}) \cos\theta_{\Lambda} + (K_{3sc} \sin\theta_{\ell} \cos\theta_{\ell} + K_{3s} \sin\theta_{\ell}) \sin\theta_{\Lambda} \sin\phi + (K_{4sc} \sin\theta_{\ell} \cos\theta_{\ell} + K_{4s} \sin\theta_{\ell}) \sin\theta_{\Lambda} \cos\phi \right]$$
  
$$\Rightarrow \frac{d\Gamma}{dq^{2}} = 2K_{1ss} + K_{1cc}$$

[T. Gutsche et al., PRD 87, 074031 (2013); P. Böer, T. Feldmann, D. van Dyk, JHEP 1501, 155 (2015)]



Hint of an excess at high  $q^2$  – contrary to mesonic  $b 
ightarrow s \mu^+ \mu^-$  decays.



 $3\sigma$  discrepancy at high  $q^2$ .





This is nonzero because  $\Lambda \to p^+\pi^-$  is a parity-violating weak decay.

## Using $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ data within a Bayesian analysis of $|\Delta B| = |\Delta S| = 1$ decays

[S. Meinel and D. van Dyk, PRD 94, 013007 (2016)]

	Scenario		
Constraint	$SM(\nu\text{-only})$	(9,10)	$\left(9,9',10,10'\right)$
$\Lambda_b  o \Lambda \mu^+ \mu^-$	Pull value $[\sigma]$		
$\langle {\cal B}  angle_{15,20}$	+0.86	-0.17	-0.08
$\langle F_L \rangle_{15,20}$	+1.41	+1.41	+1.41
$\langle A^\ell_{FB}  angle_{15,20}$	+3.13	+2.60	+0.72
$\langle A_{FB}^{A} \rangle_{15,20}$	-0.26	-0.24	-1.08
$ar{B}_{s}  ightarrow \mu^{+} \mu^{-}$	Pull value $[\sigma]$		
$\int {\cal B}( au){ m d} au$	-0.72	+0.75	+0.37
$ar{B}  ightarrow X_{s} \ell^{+} \ell^{-}$	Pull value $[\sigma]$		
$\langle \mathcal{B}  angle_{1,6}$ (BaBar)	+0.47	-0.26	-0.10
$\langle \mathcal{B}  angle_{1,6}$ (Belle)	+0.17	-0.35	-0.24
	$\chi^2$ and <i>p</i> -value at best-fit point		
	$\chi^{2} = 13.40$	$\chi^{2} = 9.60$	$\chi^{2} = 3.87$
	<i>p</i> = 0.06	p = 0.09	<i>p</i> = 0.28



blue = without  $\Lambda_b \to \Lambda \mu^+ \mu^-$ , red = with  $\Lambda_b \to \Lambda \mu^+ \mu^-$ **X** = best-fit point with  $\Lambda_b \to \Lambda \mu^+ \mu^-$ :

$$C_9 - C_9^{SM} = +1.6^{+0.7}_{-0.9}$$
  
 $C_{10} - C_{10}^{SM} = +0.7^{+0.5}_{-0.8}$ 

Fits of  $\Lambda_b 
ightarrow \Lambda(
ightarrow 
ho^+ \pi^-) \mu^+ \mu^-$  data only

Fit scenarios:

• SM(*v*-only):

$$p = 0.013$$

• 9:

$$p = 0.015$$

• SM( $\nu$ -only, 100 times wider priors for subleading OPE corrections):

$$p = 0.37$$

#### Overview of exclusive $b \rightarrow s \ell^+ \ell^-$ decays

	Probes all	Final hadron	Charged hadrons from	LQCD
	Dirac structures	QCD-stable	<i>b</i> -decay vertex	Refs.
$B^+  ightarrow K^+ \ell^+ \ell^-$	×	$\checkmark$	$\checkmark$	[1, 2, 3, 4]
$B^0  ightarrow K^{st 0} ( ightarrow K^+ \pi^-) \ell$	$\ell^+\ell^ \checkmark$	×	$\checkmark$	[5, 6, 7]
$B_s  o \phi ( o K^+ K^-) \ell^+ \ell$	√	×	$\checkmark$	[5, 6, 7]
$\Lambda^0_b  o \Lambda^0 ( o p^+ \pi^-)  \ell^+$	ℓ- ✓	$\checkmark$	×	[8, 9, 10]
$\Lambda_b^0 \to \Lambda^{*0} (\to p^+ K^-) \ell$	$^+\ell^ \checkmark$	×	$\checkmark$	[11]

- C. Bouchard et al., PRD 88, 054509 (2013)
- [2] C. Bouchard et al., PRL 111, 162002 (2013)
- [3] J. A. Bailey et al., PRD 93, 025026 (2016)
- [4] D. Du et al., PRD 93, 034005 (2016)
- [5] R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, PRD 89, 094501 (2014)
- [6] R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, PRL 112, 212003 (2014)
- [7] J. Flynn, A. Jüttner, T. Kawanai, E. Lizarazo, O. Witzel, PoS LATTICE2015, 345
- [8] W. Detmold, C.-J. D. Lin, S. Meinel, M. Wingate, PRD 87, 074502 (2013)
- [9] W. Detmold, S. Meinel, PRD 93, 074501 (2016)
- [10] S. Meinel, D. van Dyk, PRD 94, 013007 (2016)
- [11] S. Meinel, G. Rendon, arXiv:1608.08110

The  $K^*(892)$  resonance in  $B^0 \to K^+ \pi^- \mu^+ \mu^-$ 



[LHCb Collaboration, arXiv:1606.04731]

 $\Lambda^*$  resonances in  $\Lambda_b o K^- p^+ \mu^+ \mu^-$  at  $q^2 = m_{J/\psi}^2$ 



[LHCb Collaboration, PRL 115, 072001 (2015)]

∧(1520) 3/2<sup>−</sup>

$$I(J^P) = 0(\frac{3}{2})$$
 Status: \*\*\*\*

#### A(1520) MASS

VALUE (MeV) 1519.5 ±1.0	OUR ESTIMATE	DOCUMENT ID	TECN	COMMENT	
<i>Л</i> (1520) WIDTH					
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT	
15.6 ±1.0 O	UR ESTIMATE				

#### A(1520) DECAY MODES

	Mode	Fraction $(\Gamma_i/\Gamma)$
Г1	NK	(45 ±1 )%
Γ2	$\Sigma \pi$	(42 ±1 )%
Гз	$\Lambda\pi\pi$	(10 ±1 )%
Γ4	$\Sigma(1385)\pi$	
Γ <sub>5</sub>	$\Sigma(1385)\pi( \rightarrow \Lambda\pi\pi)$	
Г <sub>6</sub>	$\Lambda(\pi\pi)_{S-wave}$	
Γ <sub>7</sub>	$\Sigma \pi \pi$	( $0.9 \pm 0.1$ ) %
Г <sub>8</sub>	$\Lambda\gamma$	( 0.85±0.15) %
Го	$\Sigma^0 \gamma$	

Naive treatment as if it were a stable particle in the following.

Helicity form factors for  $\Lambda_b \rightarrow \Lambda(1520)$ 

Vector current:

Similar for axial-vector current  $(g_0, g_+, g_\perp, g_{\perp'})$ and tensor current  $(h_+, h_\perp, h_{\perp'}, \tilde{h}_+, \tilde{h}_\perp, \tilde{h}_{\perp'})$ 

[S. Meinel, G. Rendon, arXiv:1608.08110]

Lattice calculation in  $\Lambda(1520)$  rest frame. Preliminary results at  $\mathbf{p}_{\Lambda_b} = (0,0,3)\frac{2\pi}{L} ~(\approx 1.4 \text{ GeV})$ :



[S. Meinel, G. Rendon, arXiv:1608.08110]



Plan to use moving-NRQCD action for *b* quark to reach higher  $\mathbf{p}_{\Lambda_b}$ . [R. R. Horgan *et al.*, PRD **80**, 074505 (2009)]

## Outlook

 $b \rightarrow u \ell^- \bar{\nu}$  and  $b \rightarrow c \ell^- \bar{\nu}$ :

- $\Lambda_b \rightarrow p$  and  $\Lambda_b \rightarrow \Lambda_c$  form factors directly at physical pion mass (48<sup>3</sup> × 96 RBC/UKQCD ensemble)
- $\Lambda_b \rightarrow \Lambda_c(2595)$  and  $\Lambda_b \rightarrow \Lambda_c(2625)$  form factors

 $b \rightarrow s \ell^+ \ell^-$ :

•  $\Lambda_b \rightarrow \Lambda(1520)$  form factors

 $c 
ightarrow s \ell^+ 
u$ :

•  $\Lambda_c \rightarrow \Lambda$  and  $\Lambda_c \rightarrow \Lambda(1520)$  form factors

 $c \rightarrow u \ell^+ \ell^-$ :

•  $\Lambda_c \rightarrow p$  form factors