

Flavor anomalies and possible manifestations in kaon decays

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Collaboration with Crivellin, A Hoferichter, M and Tunstall, L

Phys.Rev. D 2016

Collaboration with Coluccio-Leskow, E. Greynat,D. and Nath,A.

Phys.Rev. D 2016

Collaboration with Cappiello,Luigi Cata, Oscar and Gao, Droneng

EPJ . C72 (2012) 1872

Outline

- LFUV from B decays to Kaon decays
- BBG approach: can we test it in rare decays? I like to understand the method!
- $K^+ \rightarrow \pi^+ l^+ l^- / K_S \rightarrow \pi^0 l^+ l^-$ form factors
- L9 in progress
- $K^+ \rightarrow \pi^+ \pi^0 l^+ l^-$ and other decays channels
- Conclusions

Why testing Lepton Flavor Universality Violation (LFUV) in Kaon decays?

- Several anomalies in B-physics

- LFUV from LHCb

$$R(K) = \frac{\text{Br}[B \rightarrow K \mu^+ \mu^-]}{\text{Br}[B \rightarrow K e^+ e^-]} = 0.745_{-0.074}^{+0.090} \pm 0.036$$

[Bobeth, Hiller, Piranishvili (07)]

- P'_5 angular observable in $B \rightarrow K^* \mu^+ \mu^-$

[Descotes-Genon et al. (13 & 14); Altmannshofer & Straub (15); Jäger & Martin Camalich (16)]

- Also in semi-leptonic B-decays $B \rightarrow D(D^*) \tau \nu$ see FLAG

- maybe a global 3 sigma effect

[Altmannshofer & Straub (15); Descotes-Genon et al. (15)]

Timelines

Aoki

	13	14	15	16	17	18	19	20	21	22	23	24	25
MEG, MEG II	10^{-13}												
Mu2e													10^{-17}
COMET								10^{-15}					10^{-17}
DeeMe							10^{-13}						
Mu3e								10^{-15}					
g-2@FNAL													
g-2@J-PARC													
Belle II													10^{-9}

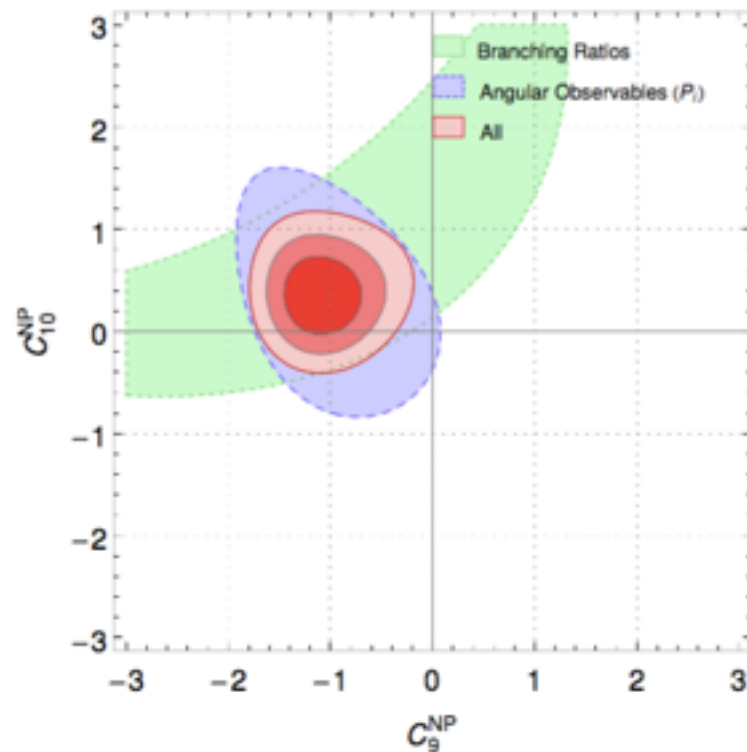
Addressing LFUV in kaon decays

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i^B(\mu) Q_i^B(\mu)$$

$$Q_9^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1-\gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell],$$

$$Q_{10}^B = \frac{e^2}{32\pi^2} [\bar{s}\gamma^\mu(1-\gamma_5)b] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell].$$

[Figure from Descotes-Genon et al. (15)]



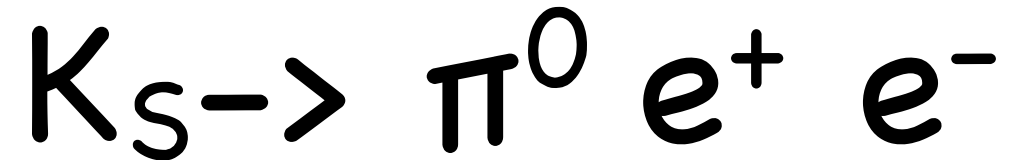
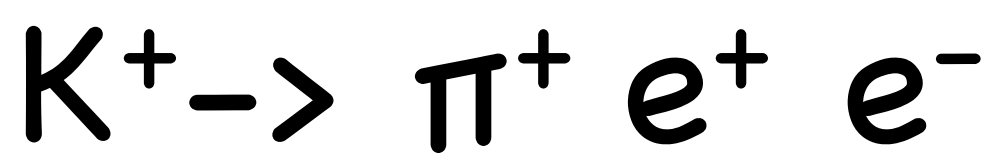
$$C_{9,10}^{NP} \sim \mathcal{O}(1)$$

Assuming MFV: what we expect in kaon decays ?

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{13} C_i(\mu) Q_i(\mu)$$

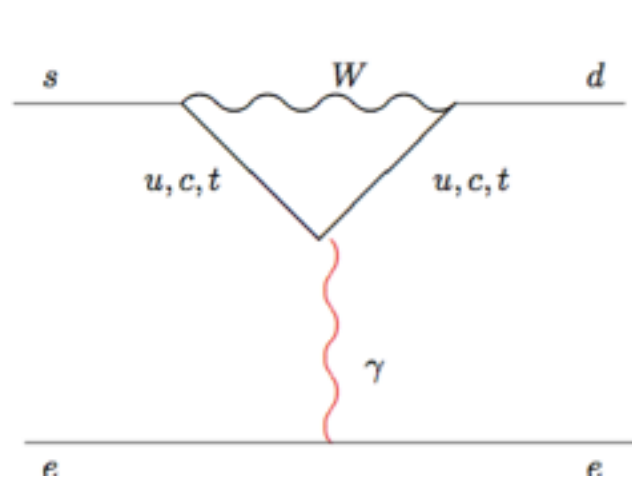
$$Q_{11} \equiv Q_{7V} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell],$$

$$Q_{12} \equiv Q_{7A} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell].$$

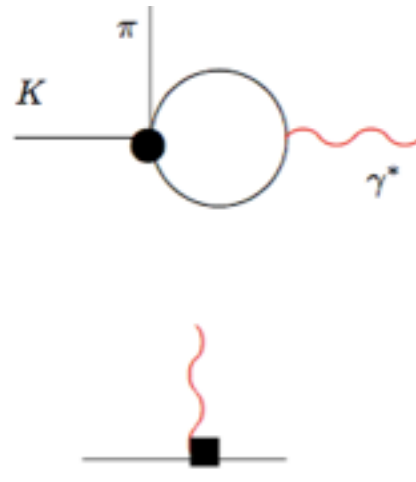


- gauge+Lorentz inv. \Rightarrow 1 ff $W^{+/-,S}$

CHPT



SD



CHPT

$$W^i = G_F m_K^2 (a_i + b_i z) + W_{\pi\pi}^i(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1),$$

$$z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)$, $\Gamma(K^+ \rightarrow \pi^+ \mu \bar{\mu})$, slopes

- $a_i \sim O(p^4)$ $a_+ \sim N_{14} - N_{15}$, $a_S \sim 2N_{14} + N_{15}$

Ecker, Pich, de Rafael

- $b_i \sim O(p^6)$

G.D., Ecker, Isidori, Portole

- a_+, b_+ in general not related to a_S, b_S

Recent lattice determinations Christ et al.

averaging flavour

$$a_+^{\text{exp.}} = -0.578 \pm 0.016$$

$$b_+^{\text{exp.}} = -0.779 \pm 0.066$$

LFUV: Kaons

Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\text{NP}} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* * C_{7V}^{\text{NP}}$$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*} \xrightarrow{\text{MFV}} C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td} V_{ts}^*} = -19 \pm 79$$

NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2

Also analyzed LFV Kaon decays and $K_L \rightarrow \mu\mu$ (C_{10}^{NP})

$$K^+ \rightarrow \pi^+ e^+ e^- \quad K_S \rightarrow \pi^0 e^+ e^-$$

form factor calculation

- Bardeen Buras Gerard
- Recent lattice determinations Christ et al.
- ϵ' ?

QCD at work: Short Distance expansion for weak interaction

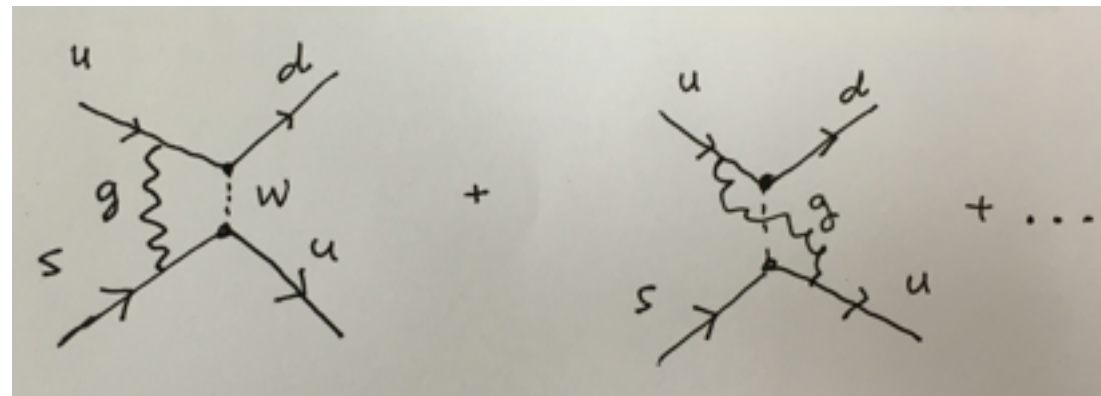
- Fermi lagrangian: description of the $\Delta S=1$ weak lagrangian, in particular the explanation of $\Delta I = 1/2$ rule

$$\frac{A(K^+ \rightarrow \pi^+ \pi^0)}{A(K_S \rightarrow \pi^+ \pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\bar{s}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



QCD at work, theoretical tools

- analytic calculation 't Hooft, large N_c (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena
- G. Parisi, '80s lattice: can we predict from QCD the proton mass at 10% level?
- Precise calculation of low energy QCD?

Bardeen Buras Gerard approach to $K \rightarrow \pi\pi$

Also evaluated $\Delta S=2$ transitions, ϵ' (Buras) and $\pi^+ - \pi^0$ mass diff.

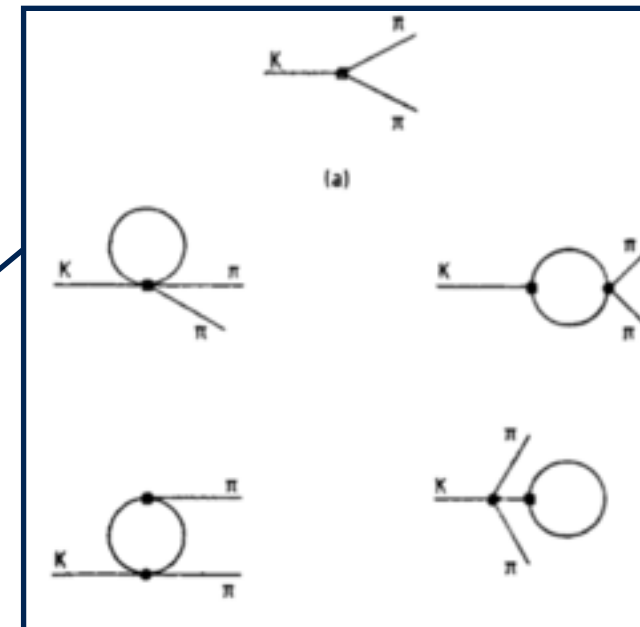
Main idea: phys. amplitudes scale independent

Match SD with LD with a precise prescription for CT

CHPT+Large N_c

$$H_{\text{eff}} = \sum_i C_i(\mu) Q_i(\mu)$$

SD



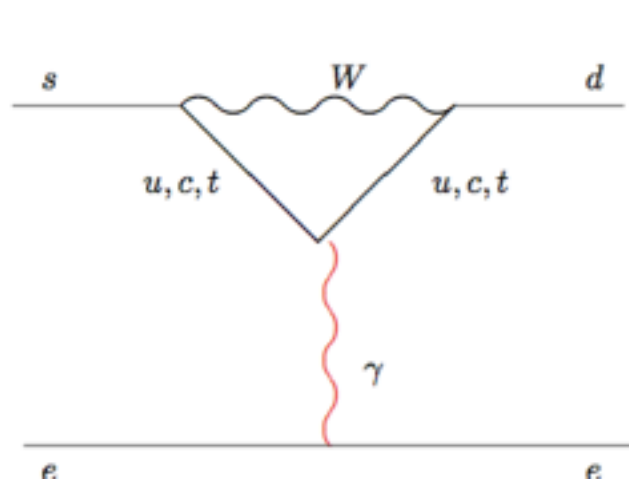
Can we test somewhere else
the Bardeen Buras Gerard
(BBG) approach?

Coluccio-Leskow, Estefania, GD, Greynat, David and Nath, Atanu

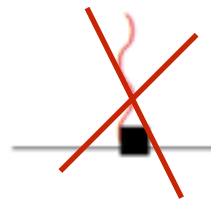
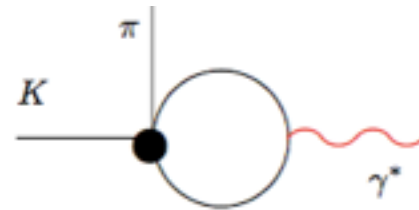


Coluccio-Leskow, E. G.D., Greynat, D and Nath, A

BBG approach



SD



NO CT IN BGG

CHPT

$$W^i = G_F m_K^2 (a_i + b_i z) + W_{\pi\pi}^i(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \quad z = \frac{q^2}{m_K^2}$$

Quadratic divergences in $K \rightarrow 3\pi$ interaction matched to the subleading log in $C_7(\mu)$

$$Q_{7V} = \bar{s} \gamma^\mu (1 - \gamma_5) d \bar{\ell} \gamma_\mu \ell$$

$$\mathcal{H}_{eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left[\sum_{i=1}^{6,7V} (z_i(\mu) + \tau y_i(\mu)) Q_i(\mu) + \tau y_{7A}(M_W) Q_{7A}(M_W) \right]$$

Matching a la BBG for $K^+ \rightarrow \pi^+ e^+ e^-$

Coluccio-Leskow, E. G.D , Greynat, D and Nath, A

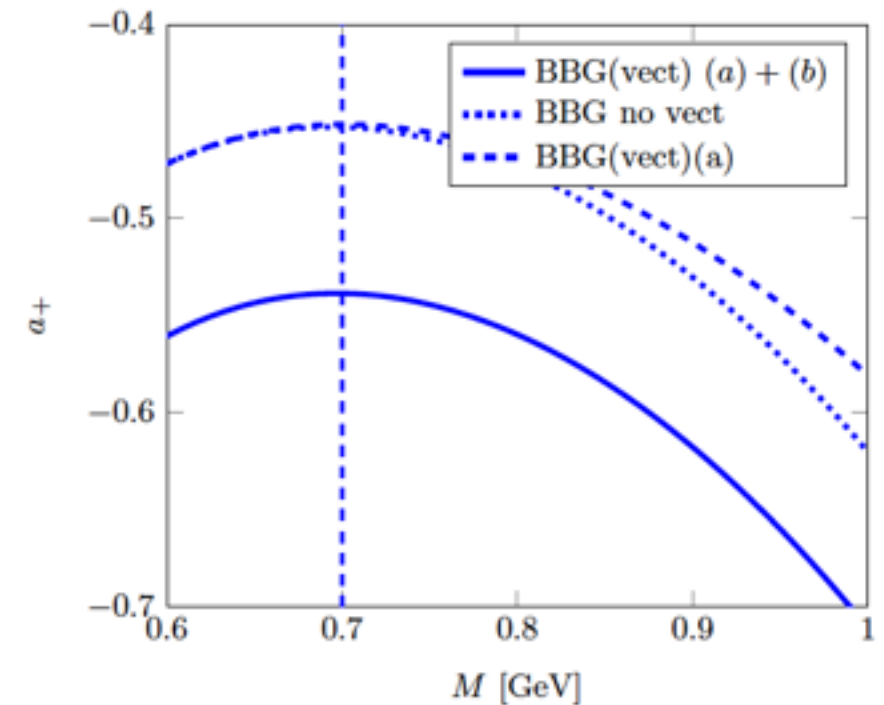
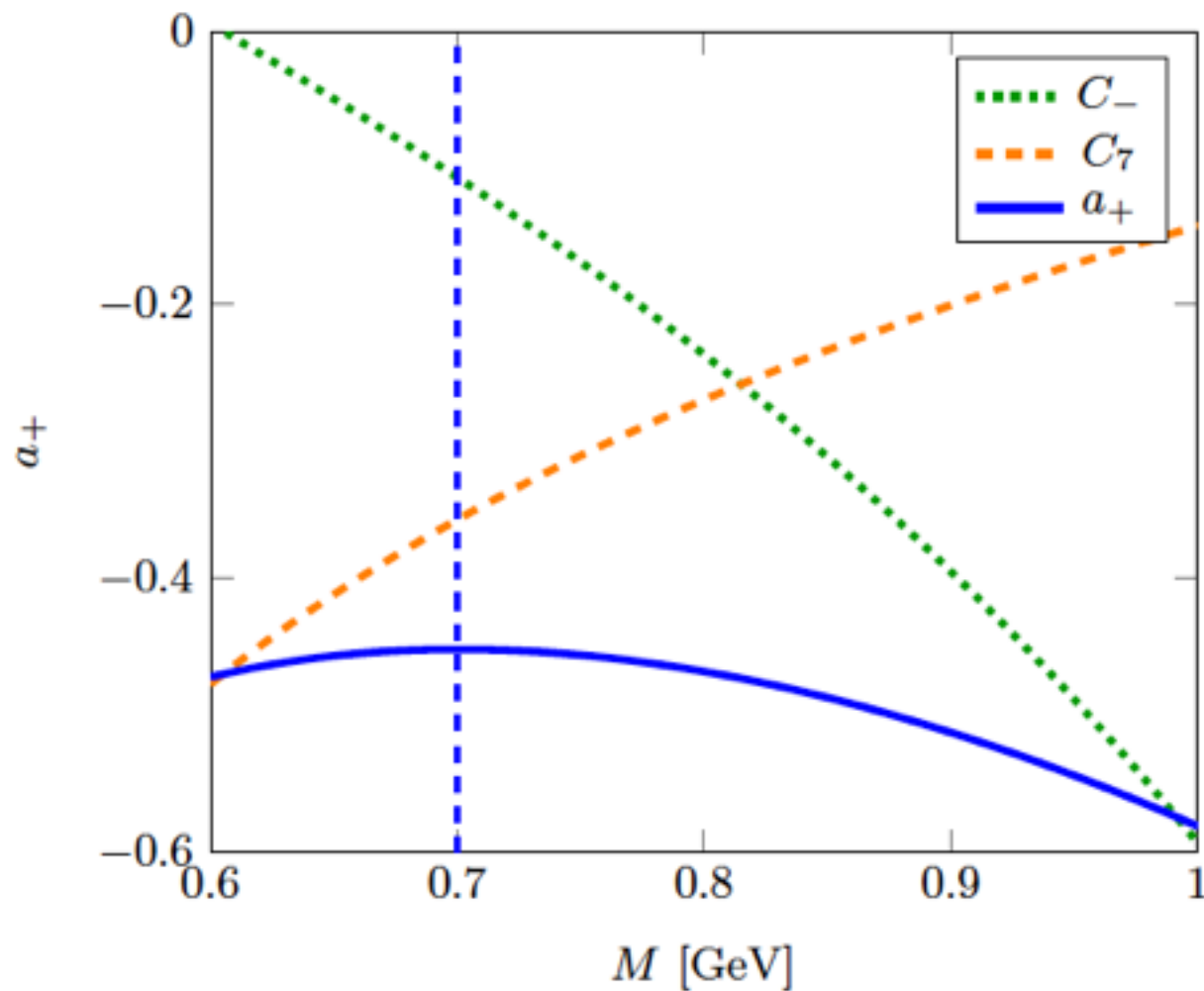


FIG. 5. a_+ as a function of M in the three different frameworks: 'BBG no vect.' where vectors are not included, 'BBG(vect)(a)' represents the contribution coming only from diagrams (a) in Fig. 4 and 'BBG(vect) (a) + (b)' is the case where both (a) and (b) diagrams were included. The vertical line indicates the value $M = 0.7$ GeV.

L9

Collaboration with Greynat, David and Nath, Atanu

in progress

$$\mathcal{L} \sim -i L_9 F_{\mu\nu} \langle Q D^\mu U D^\nu U \rangle$$

- No OPE: DIFFERENT from the weak decays!!
- However it must appear at O(p4) to improve matching with QCD

VMD '88: DEGLP, Donoghue et al

Matching the BBG form factor,
M-dependent with phenomenology

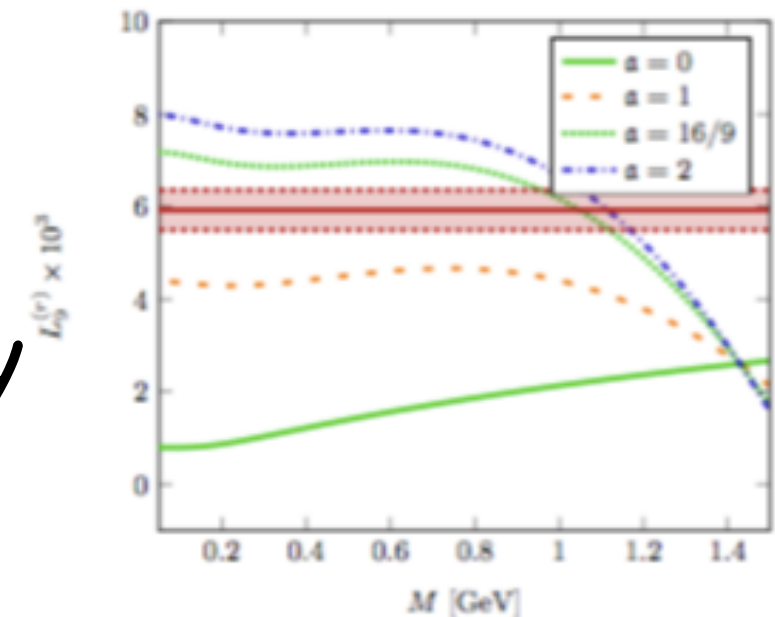


FIG. 9. Variation of $L_9^{(r)+Vec}(M)$ with scale M (in GeV) (calculated in BBG scheme including vectors through hidden local symmetry) given by Eq. (17). Area shaded in pink represents the uncertainty of $\pm 0.43 \times 10^{-3}$ around the phenomenological value 5.93×10^{-3} measured [28] at $m_\rho = 0.77$ GeV.

$$K(p_K) \rightarrow \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance \Rightarrow Electric (E) and Magnetic (M) amplitude

$$A(K \rightarrow \pi\pi\gamma) = F^{\mu\nu} [E \partial_\mu K \partial_\nu \pi + M \varepsilon_{\mu\nu\rho\sigma} \partial^\rho K \partial^\sigma \pi]$$

- Unpolarized photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2\text{Re}(E_{IB}^* E_D) + |E_D|^2$$

↓

$$\text{Low Theorem} \Rightarrow E_{IB} \sim \frac{1}{E_\gamma^*} + c \quad E_D, M \text{ chiral}$$

tests

We need FIGHT $DE/IB \sim 10^{-3}$

	<i>IB</i>	<i>DE_{exp}</i>	
$K_S \rightarrow \pi^+ \pi^- \gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	<i>E1</i>
$K^+ \rightarrow \pi^+ \pi^0 \gamma$	10^{-4} ($\Delta I = \frac{3}{2}$)	$(0.44 \pm 0.07) 10^{-5}$ PDG	<i>M1, E1</i>
$K_L \rightarrow \pi^+ \pi^- \gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07) 10^{-5}$ KTeVnew	<i>M1,</i> VMD

CPV is **only** from IB K_L (also measured in $K_L \rightarrow \pi^+ \pi^- e^+ e^-$)

BUT IB suppressed in K^+ and K_L .

$$K^+ \rightarrow \pi^+ \pi^0 \gamma$$

$$A(K \rightarrow \pi\pi\gamma) = F^{\mu\nu} [E \partial_\mu K \partial_\nu \pi + M \varepsilon_{\mu\nu\rho\sigma} \partial^\rho K \partial^\sigma \pi]$$

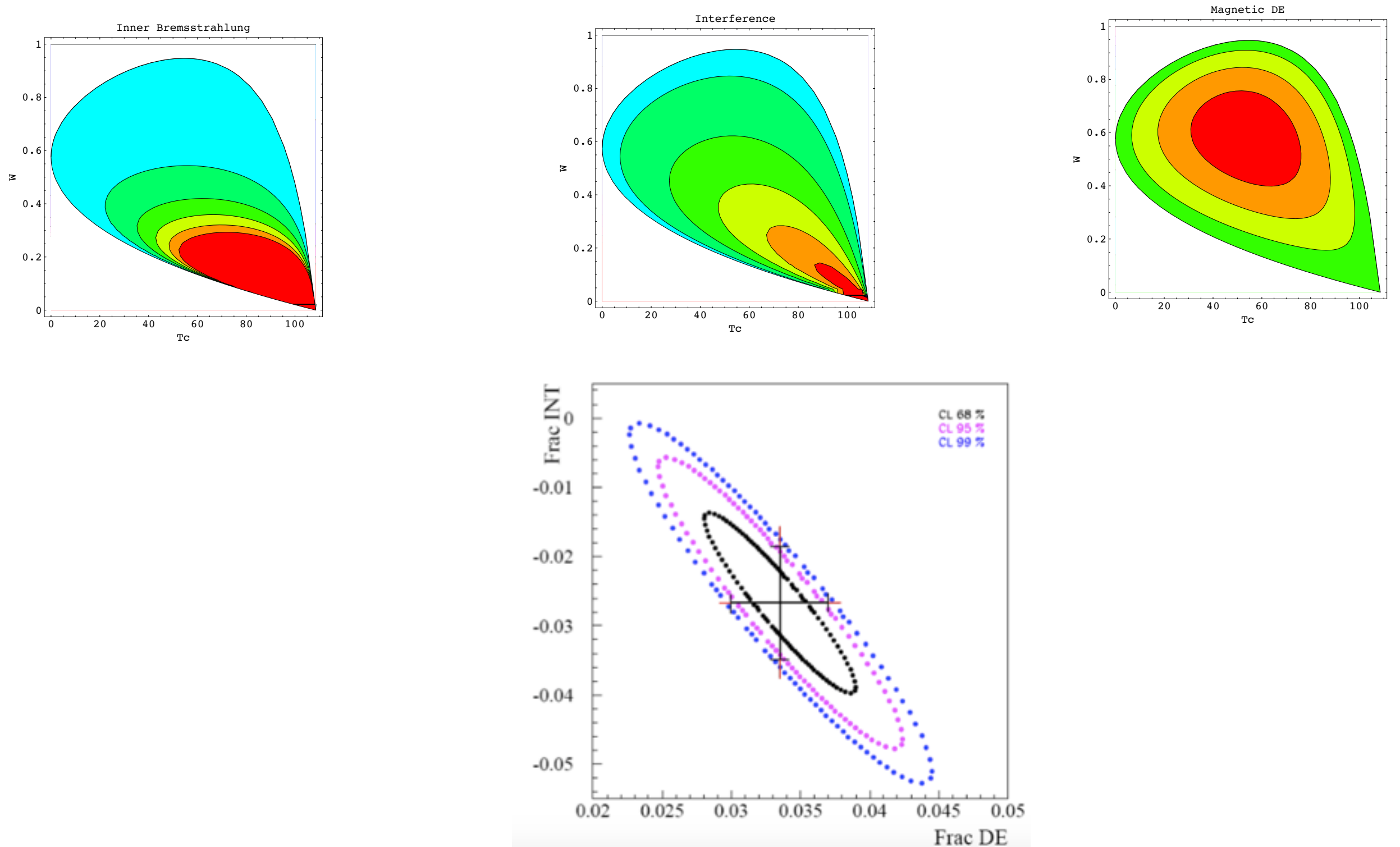
$E1$ and $M1$ are measured with Dalitz plot

$$\frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K^2} 2 \operatorname{Re} \left(\frac{E1}{eA} \right) W^2 \right. \\ \left. + \frac{m_{\pi^+}^4}{m_K^2} \left(\left| \frac{E1}{eA} \right|^2 + \left| \frac{M1}{eA} \right|^2 \right) W^4 \right]$$

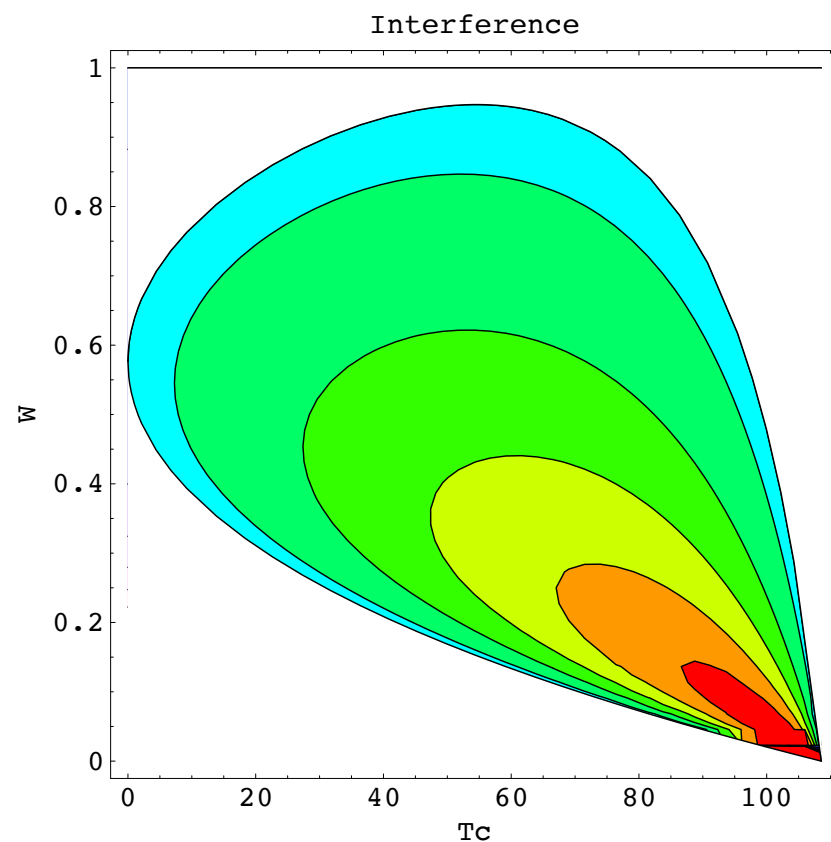
$$W^2 = (q \cdot p_K)(q \cdot p_+) / (m_\pi^2 m_K^2)$$

$$A = A(K^+ \rightarrow \pi^+ \pi^0)$$

Dalitz plot NA48/2



NA48/2 CP violation



Dalitz plot analysis crucial

$$\text{SM} \leq \mathcal{O}(10^{-5})$$

Paver et al.

$$\text{NP} \leq \mathcal{O}(10^{-4})$$

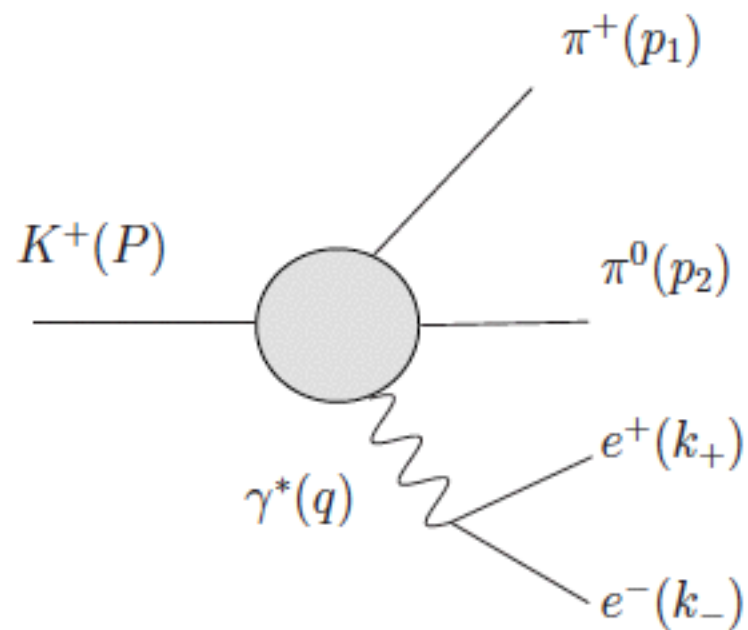
Colangelo et al.

$$\text{NA48/2} \quad < 1.5 \cdot 10^{-3} \quad \text{at} \quad 90\% \quad \text{CL}$$

BUT NOT in the interesting interf. kin. region (statistics)

$$K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$$

Sehgal et al; Savage, Wise et al



- $\mathcal{M}_{LD} = \frac{e}{q^2} \bar{e} \gamma^\mu (1 - \gamma^5) e H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

- Interference $E \quad M$ novel compared to $K_L \rightarrow \pi^+ \pi^- \gamma$
- $E \quad M$ known from $K_L \rightarrow \pi^+ \pi^- \gamma$ (IB and DE)

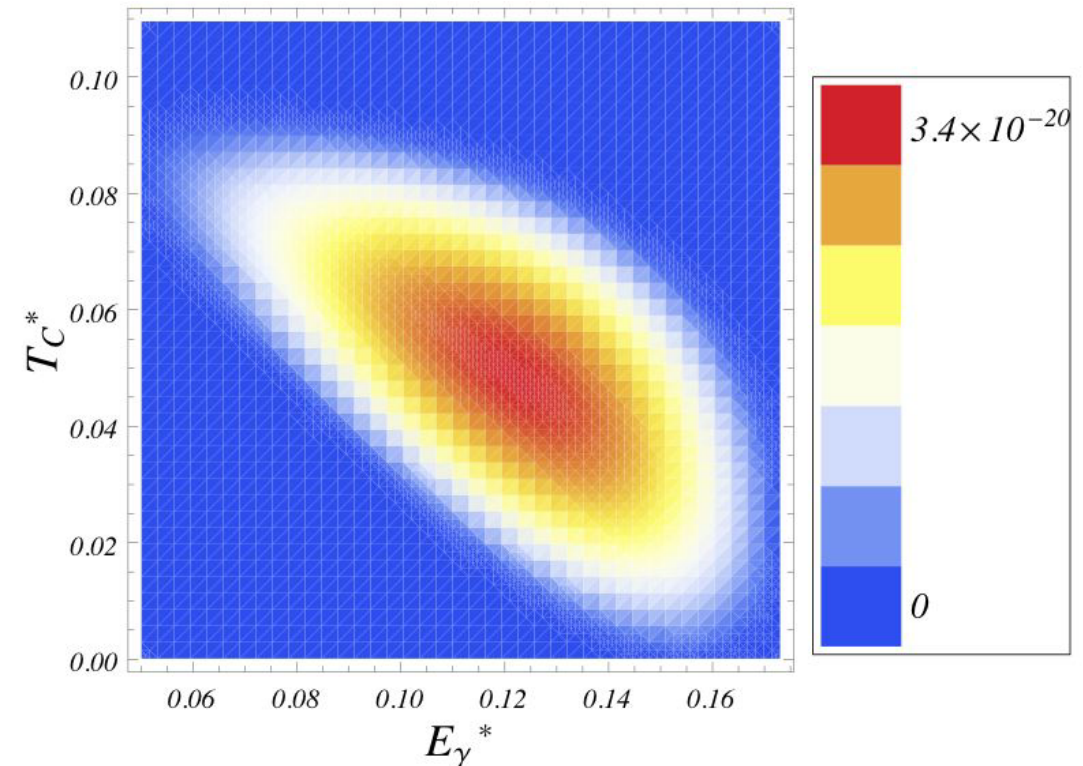
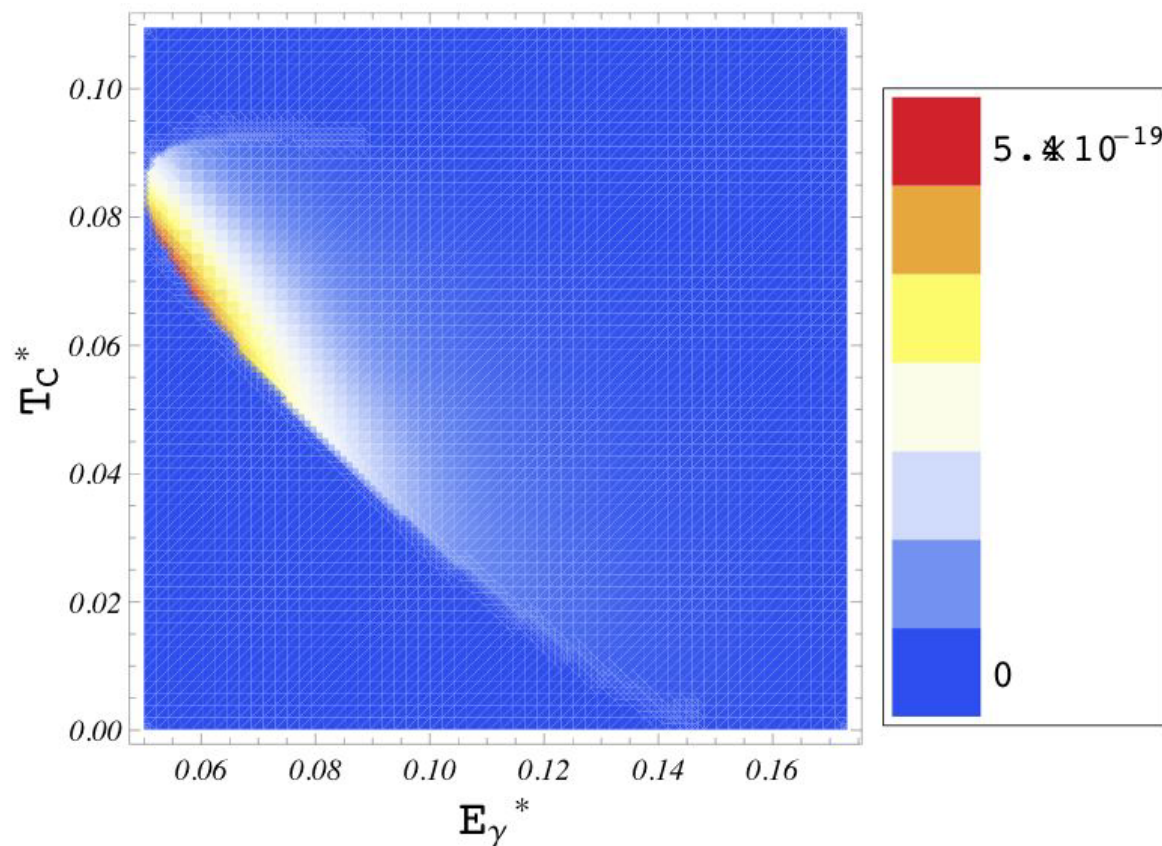
$$K^+ \rightarrow \pi^+ \pi^0 \gamma^* \rightarrow \pi^+ \pi^0 e^+ e^-$$

Cappiello, Cata, G.D. and Gao,

- the asymm. , $\frac{\Re(E_B M^*)}{|E_B|^2 + |M|^2}$, not as lucky $E_B \gg M$:
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6} \sim 50 B(K^+)_{M}$
- Short distance info without having simultaneously K^+ and K^- , asymm. in phase space, (P-violation) interesting! No ϵ -contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\text{MeV}$ IB only 10 times larger than DE

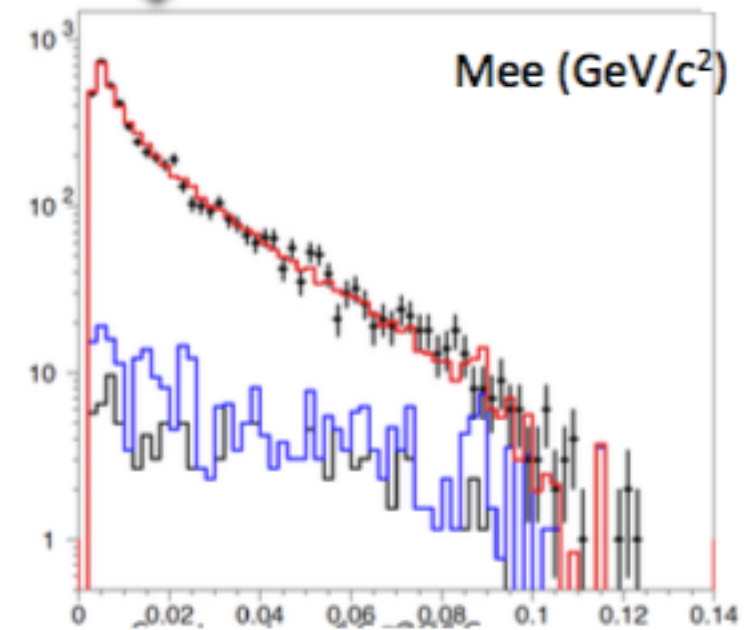
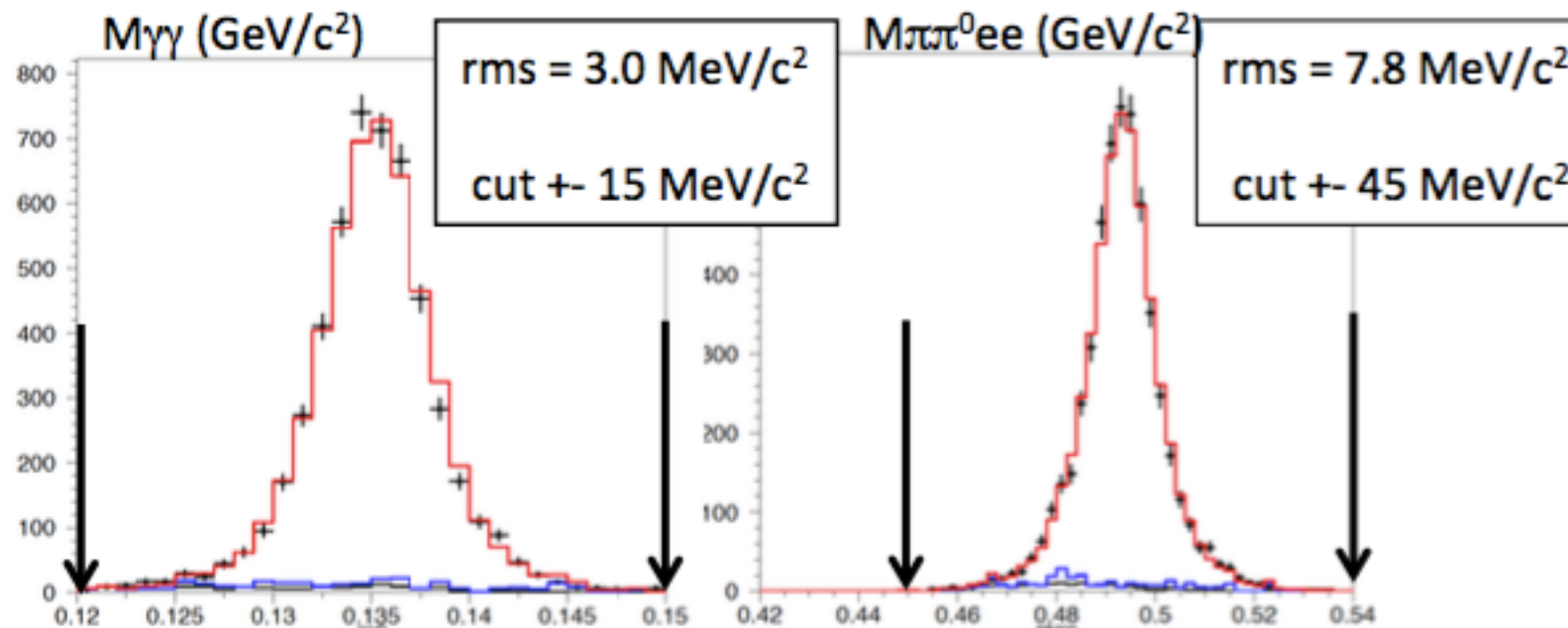
Starting from CP conserving IB, DE

q_c (MeV)	B [10^{-8}]	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



NA48/2: $K^{+/-} \rightarrow \pi^{+/-} \pi^0 e^+ e^-$

Bloch-Devaux



New Result!

$$BR = (4.22 \pm 0.06_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.13_{\text{ext}}) 10^{-6}$$

dominated by external error on BR($\pi^0 D$)

In perfect agreement with

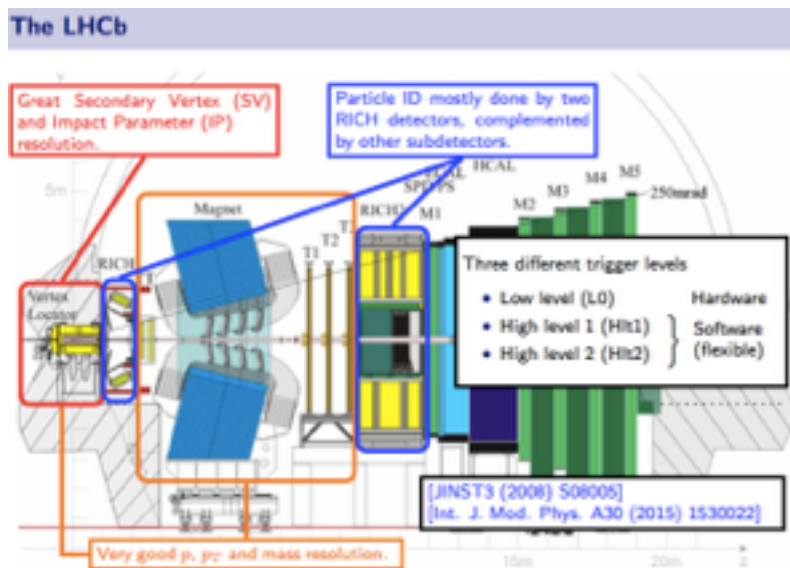
Theory : ChPT calculations EPJ C72 (2012)

IB + DE + INT

BR (IB) = $4.19 \cdot 10^{-6}$ no Rad Cor, No Isospin breaking Cor	Total $4.29 \cdot 10^{-6}$
BR (IB) = $4.10 \cdot 10^{-6}$ no Rad Cor, with Isospin breaking Cor**	Total $4.19 \cdot 10^{-6}$

$K_S \rightarrow \mu \bar{\mu}$ LHCb

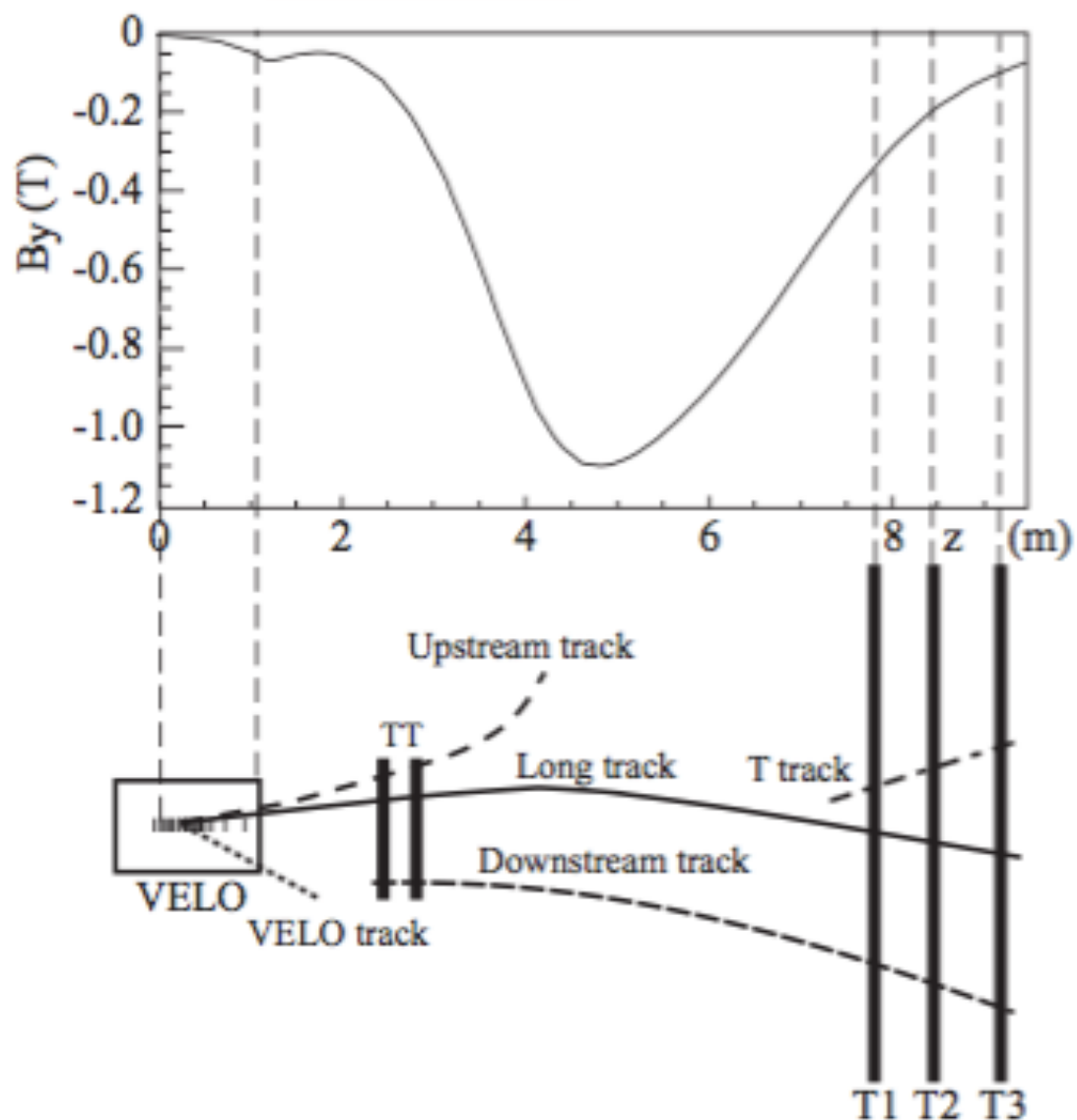
CERN SPS



After 40 years improvement by 3 orders of magnitudes from LHCb

$$B(K_S \rightarrow \mu^+ \mu^-) < 3.1 \times 10^{-7} \text{ at } 90\% \text{ CL}$$

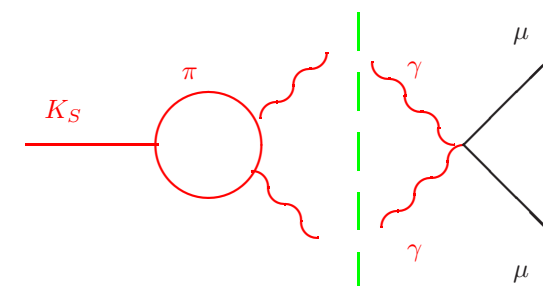
$$B(K_S \rightarrow \mu^+ \mu^-) < 6.9(5.8) \times 10^{-9} \text{ at } 95(90)\% \text{ CL}$$



[Int. J. Mod. Phys. A30 (2015) 1530022]

SM

$$\sim 5 \times 10^{-12}$$



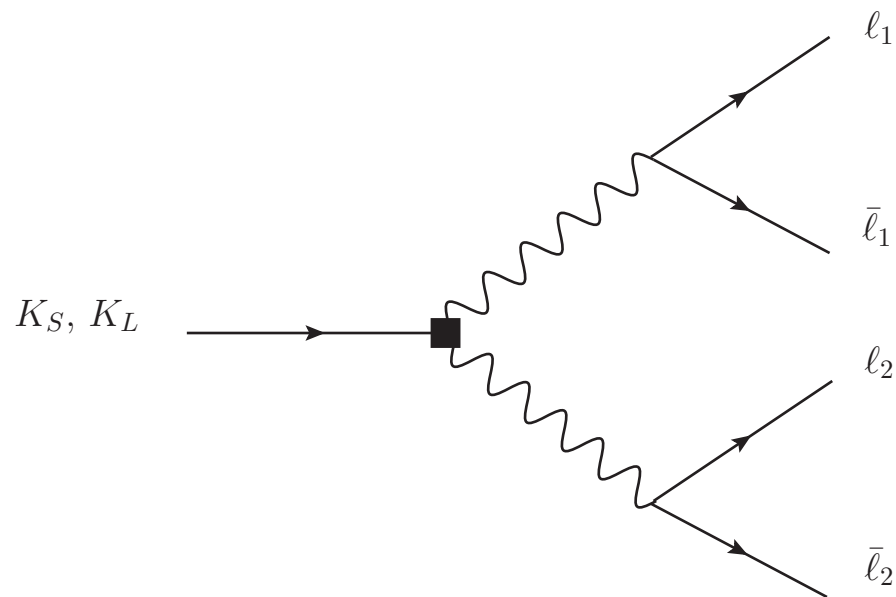
SD $1.5 \cdot 10^{-12}$

NP $1.5 \cdot 10^{-11}$
Allowed

NP Limits from CPviol in $K_L \rightarrow \mu\mu$

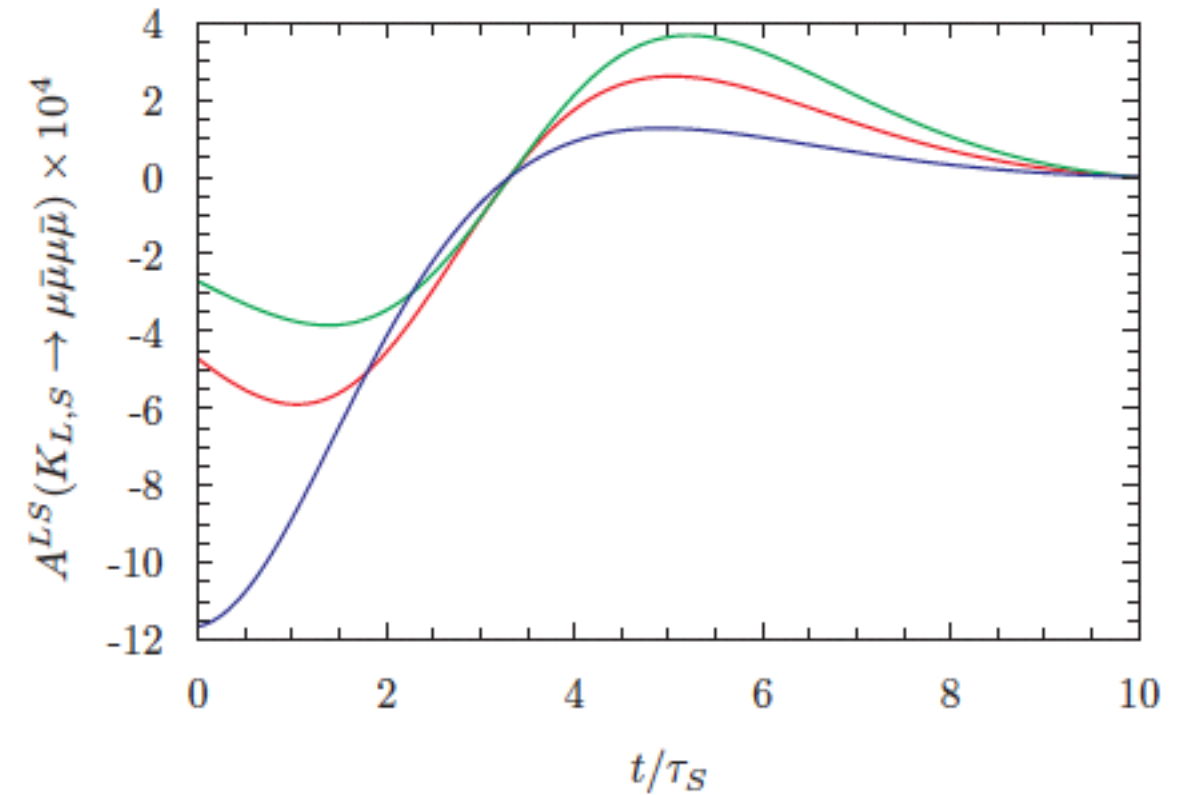
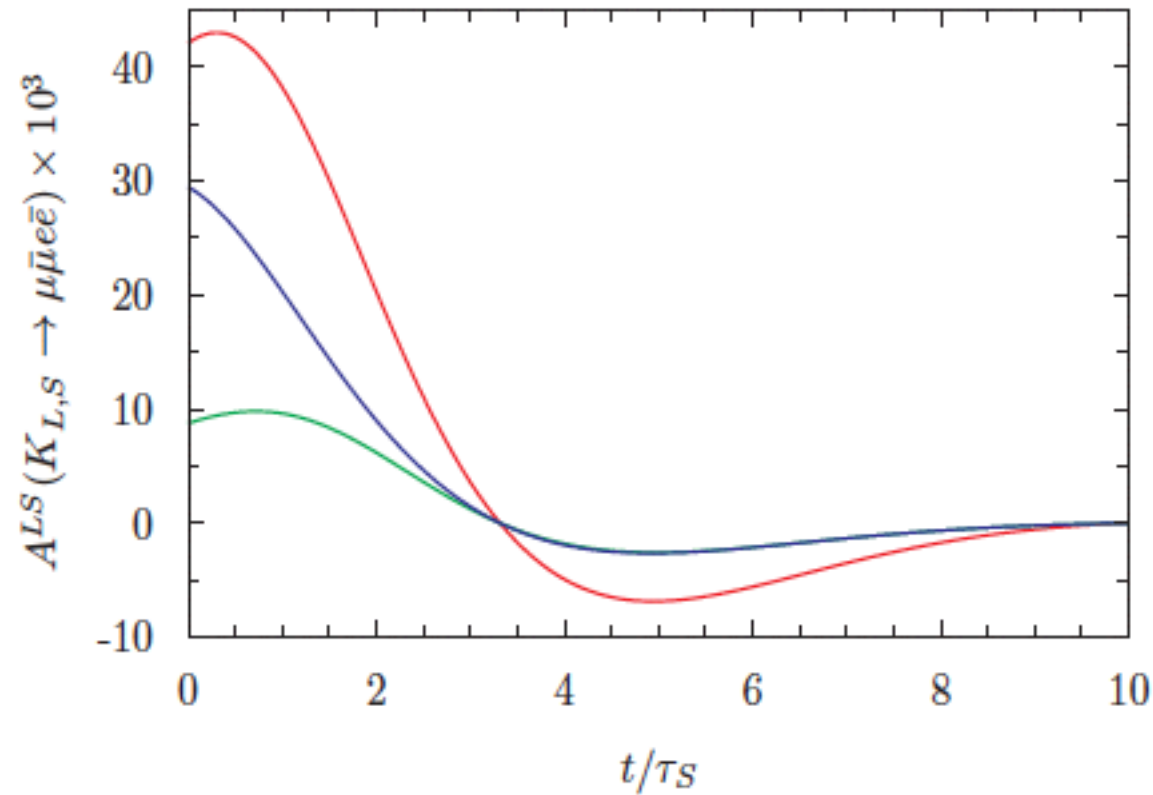
Other interesting channels

$K_S \rightarrow \mu\mu\mu\mu$	—	SM LD	$\sim 2 \times 10^{-14}$
$K_S \rightarrow e e \mu \mu$	—		$\sim 10^{-11}$
$K_S \rightarrow e e e e$	—		$\sim 10^{-10}$



GD, Greynat, Vulvert

Time interference effects



Interferences between K_L and $K_S \rightarrow \ell_1\bar{\ell}_1\ell_2\bar{\ell}_2$. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. As explained in the text we assume the sign $K_L \rightarrow \gamma\gamma$. For 4μ 's 10^{14} K_S needed, $e\bar{e}\mu\bar{\mu}$ 10^{12}

Conclusion

- LFUV interesting
- BBG interesting and positive news from our calculations
- $K^+ \rightarrow \pi^+ \pi^0 l^+ l^-$ hard fight but good perspectives, more CP violating observables
- $K_S \rightarrow \pi^+ \pi^- \mu^+ \mu^-$ Br $\sim 10^{-14}$



	$K_L \rightarrow \mu^\pm e^\mp$	$K^+ \rightarrow \pi^+ \mu^\pm e^\mp$	$K_L \rightarrow \pi^0 \mu^\pm e^\mp$	$K^+ \rightarrow \pi^+ \mu^\pm e^\mp$ (NA62 projection)
$(C_{7V}^{\mu e} ^2 + C_{7A}^{\mu e} ^2)^{1/2}$	$< 1.3 \times 10^{-6}$	$< 2.2 \times 10^{-5}$		$< 5.1 \times 10^{-6}$
$(y_{7V}^{\mu e} ^2 + y_{7A}^{\mu e} ^2)^{1/2}$			< 0.040	
$(C_9^{B,\mu e} ^2 + C_{10}^{B,\mu e} ^2)^{1/2}$	< 0.71	< 12	< 35	< 2.7

Kaon physics

Tests of CPV already among most stringent (ϵ_K, ϵ')

Near future improvements mostly due to theory (Lattice)

More progress foreseen in rare decays

$$\Rightarrow K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}$$

\Rightarrow rare K decays at HL-LHCb?

d'Ambrosio, PoS(FPCP2015)018

	PDG	Prospects	
$K_S \rightarrow \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL (LD)	$(5.0 \pm 1.5) \cdot 10^{-12}$	NP $< 10^{-11}$
$K_L \rightarrow \mu\mu$	$(6.84 \pm 0.11) \times 10^{-9}$	difficult : SD \ll LD	
$K_S \rightarrow \mu\mu\mu\mu$	—	SM LD $\sim 2 \times 10^{-14}$	} NP?
$K_S \rightarrow ee\mu\mu$	—	$\sim 10^{-11}$	
$K_S \rightarrow eeee$	—	$\sim 10^{-10}$	
$K_S \rightarrow \pi^+ \pi^- \mu^+ \mu^-$	—	SM LD $\sim 10^{-14}$	