The muon magnetic moment in new physics

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Motivation

The latest $(g-2)_\mu$ experimental result at BNL:

$$a_\mu^{\text{E821}} = (11659208.9 \pm 6.3) \times 10^{(-10)}$$ [Bennett et al. '06]

$$\Delta a_\mu^{(\text{E821}-\text{SM})} = \begin{cases} (28.7 \pm 8.0) \times 10^{-10} & \text{[Davier et al.]} \\ (26.1 \pm 8.0) \times 10^{-10} & \text{[Hagiwara et al.]} \end{cases}$$

$3 \sim 4\sigma \Rightarrow$ New physics

New experiment at Fermilab(E969): ~ 0.14 ppm
Current accuracy of a_μ^{SM}: 0.42 ppm [Davier et al. '10]

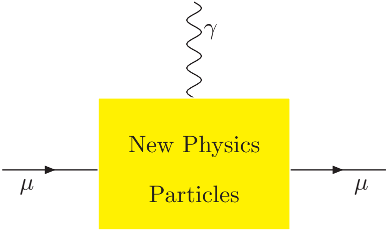
need to improve the accuracy of all aspects of the theory prediction:
higher order loop corrections required.

Outline

Why 2HDM and MSSM?

- Question of the possibility of enlarged scalar sector
- Extension in EWSB sector
- 2HDM: the simplest extension to the SM, compatible with current experimental results
- MSSM: still best motivated, the discoverd Higgs boson with $M_h = 125$ GeV in agreement with SUSY prediction
- explains the anomalous magnetic moment of muon
- suggest solutions for other physical problems, e.g. Dark Matter

Overview of new physics contributions



$a_\mu^{\mathsf{NP}} = C_{\mathsf{NP}} \frac{m_\mu^2}{M_{\mathsf{NP}}^2}$,

C_{NP}: model dependent

$\propto \frac{1}{M_{\mathsf{NP}}^2}$ interpreted as decoupling behavior

Overview of new physics contributions

$$a_\mu^{\mathsf{NP}} = C_{\mathsf{NP}} \frac{m_\mu^2}{M_{\mathsf{NP}}^2},\ C_{\mathsf{NP}}\text{: model dependent}$$

2HDM	**MSSM**	**Radiative m_μ generation**
•2 Higgs doublets	•Supersymmetry	•$v_d \to 0,\ \tan\beta \to \infty$
•$h, H, A, H^\pm$	•Sparticles:$\tilde{\chi}^{0/\pm}$, $\tilde{\mu}$, $\tilde{\nu}_\mu$	•$m_\mu = \delta m_\mu(\tilde{\chi}^{0/\pm}, \tilde{\mu}, \tilde{\nu}_\mu)$
•α^2 correction	•α^1 correction	•α^0 correction
•$M_{\mathsf{NP}} < 100$ GeV	•$M_{\mathsf{NP}} \sim 5 \times 10^2$ GeV	•$M_{\mathsf{NP}} \sim 10^3$ GeV

2HDM

2HDM

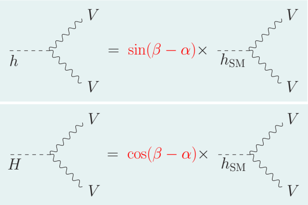
Two Higgs doublets with same hypercharge:

$$\phi_1,\ \phi_2,\ v^2 = v_1^2 + v_2^2,\ v = 246\ \text{GeV}$$

$$\Phi_v = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + S_1 + iG^0) \end{pmatrix},\ \Phi_\perp = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2 + iA) \end{pmatrix}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos(\beta-\alpha) & -\sin(\beta-\alpha) \\ \sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix},\ \tan\beta \equiv \frac{v_2}{v_1}$$

h and H CP-even mass eigenstates



$\beta - \alpha = \frac{\pi}{2},\ h = h_{SM}$

$\Rightarrow$ alignment limit

[Craig, Galloway, Thomas '13][Haber '13]

$$\begin{aligned} V(\phi_1,\phi_2) = {} & m_{11}^2 \phi_1^\dagger\phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 \\ & - m_{12}^2 (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) \\ & + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2^2)^2 \\ & + \lambda_3 \phi_1^\dagger\phi_1\phi_2^\dagger\phi_2 + \lambda_4 \phi_1^\dagger\phi_2\phi_2^\dagger\phi_1 \\ & + \frac{\lambda_5}{2}\{(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\} \end{aligned}$$

CP conserving: real m_{12}^2 and λ_5

$$\boxed{m_{11}^2,\ m_{22}^2,\ m_{12}^2,\ \lambda_1\ \cdots\ \lambda_5}$$

$$\downarrow$$

$$\boxed{M_h,\ M_H,\ M_A,\ M_{H^\pm}}\ \boxed{\beta,\ \alpha}\ \boxed{v}\ \boxed{\lambda_1}$$

Small deviation from the SM-limit:

$\beta - \alpha = \frac{\pi}{2} - \eta.$

allowed from LHC, $\sin(\beta-\alpha) \sim 0.7$

2HDM: Yukawa interaction in the Aligned 2HDM

[Pich, Tuzón '09]

$$\mathcal{L}_Y = \sqrt{2} H^+ (\bar{u}[V_{\mathsf{CKM}} y_d^A P_{\mathsf{R}} + y_u^A V_{\mathsf{CKM}} P_{\mathsf{L}}] d + \bar{\nu} y_l^A P_{\mathsf{R}} l) - \sum_{\mathcal{S},f} \mathcal{S} \bar{f} y_f^{\mathcal{S}} P_{\mathsf{R}} f + h.c$$

Type	I	II	X	Y
u	Φ_2	Φ_2	Φ_2	Φ_2
d	Φ_2	Φ_1	Φ_2	Φ_1
l	Φ_2	Φ_1	Φ_1	Φ_2
ζ_u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ζ_d	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
ζ_l	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$

Type II: MSSM-like
Type X: lepton-specific Type Y: flipped

$$y_f^{\mathcal{S}} = \frac{Y_f^{\mathcal{S}}}{v} M_f,\ \mathcal{S} \in h, H, A$$

$M_f = 3 \times 3$ mass matrix,

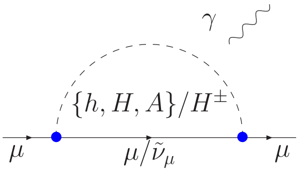
$f = u, d, l$

$$Y_f^h = \sin(\beta-\alpha) + \cos(\beta-\alpha)\zeta_f = 1 + \eta\zeta_f,$$

$$Y_f^H = \cos(\beta-\alpha) - \sin(\beta-\alpha)\zeta_f = -\zeta_f + \eta, \qquad (\beta-\alpha = \frac{\pi}{2} - \eta)$$

$$Y_f^A = -\Theta_f^A \zeta_f, \Theta_{d,l}^A = 1, \Theta_u^A = -1$$

2HDM: One-loop contribution



- : $y_\mu \propto m_\mu \zeta_l$

$a_\mu^{\text{2HDM,1}} \propto \alpha \frac{m_\mu^2}{M_s^2} m_\mu^2 \zeta_l^2$:

$\frac{m_\mu^2}{M_S^2}$ from loop calculation,

m_μ^2 from Yukawa coupling.

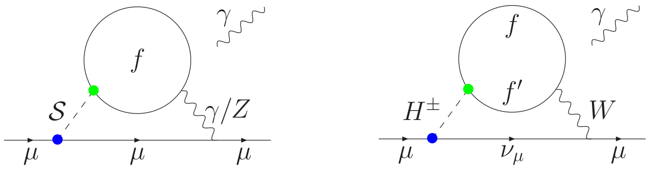
$$a_\mu^{\text{2HDM,1}} \simeq \left(\frac{\zeta_l}{100}\right)^2 \times 10^{-10} \left\{ \frac{3.3+0.5\ln(\hat{x}_H)}{\hat{x}_H^2} - \frac{3.1+0.5\ln(\hat{x}_A)}{\hat{x}_A^2} - \frac{0.04}{\hat{x}_{H^\pm}^2} \right\},\ \hat{x}_{\mathcal{S}} \equiv \frac{M_{\mathcal{S}}}{100\text{ GeV}}$$

$a_\mu^{\text{2HDM,1}} \approx 0.13 \times 10^{-10}$ for $M_H = M_A = M_{H^\pm} = 100$ GeV

$a_\mu^{\text{2HDM,1}} \approx 0.03 \times 10^{-10}$ for $M_H = M_A = M_{H^\pm} = 200$ GeV

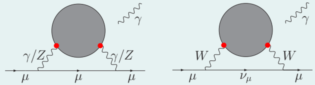
2-loop Feynman diagrams without Yukawa couplings or with only one Yukawa coupling provide terms$\propto m_\mu^2 \Rightarrow$ $a_\mu^{\text{2HDM,2}} > a_\mu^{\text{2HDM,1}}$

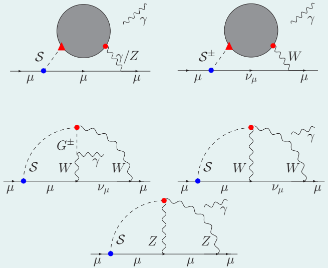
2HDM: Fermionic contribution



- Neutral and charged Higgs contributions with fermion inner loops
- $y_\mu y_f \longrightarrow \zeta_l \zeta_f$
- $a_\mu^{\text{2HDM, F}} \propto \alpha^2 \left(\frac{m_\mu^2}{M_S^2}\right) m_f^2 \zeta_l \zeta_f$
- Constraint, $\zeta_u \simeq 1 \Rightarrow \zeta_l^2$ term with τ-loop is dominant.
- $50 \text{ GeV} < M_A < 100 \text{ GeV}$, $\zeta_l = 100$, $a_\mu^{\text{2HDM, F}} = (15 \cdots 30) \times 10^{-10}$

2HDM: Bosonic contribution





- Yukawa interaction, $y_\mu \propto m_\mu \zeta_l$
- Higgs-gauge coupling

$\propto \sin(\beta - \alpha)$ for h

$\propto \cos(\beta - \alpha)$ for H

▲ including Triple Higgs coupling($\tan\beta$)

Diagrams with (● × ●):

independent of ζ_l

H-terms are suppressed by

$\cos(\beta - \alpha)^2 = \eta^2$

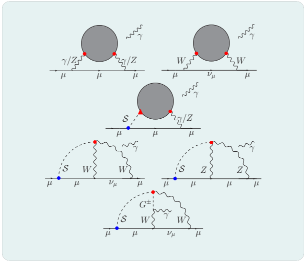
Diagrams with (● × ▲):

$\propto \zeta_l$, enhanced by $\tan\beta$.

2HDM: Bosonic contribution

$$a_\mu^{\text{B}} = a_\mu^{\text{EW add.}} + a_\mu^{\text{non-Yuk}} + a_\mu^{\text{Yuk}}$$

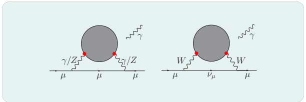
$a_\mu^{\text{EW add.}}$



$\mathcal{S} = h$

$a_\mu^{\text{EW add.}} = 2.3 \times 10^{-11} \eta \zeta_l$

$a_\mu^{\text{non-Yuk}}$



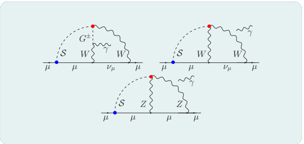
$\mathcal{S} = H, A,$

$\mathcal{S}^\pm = H^\pm$

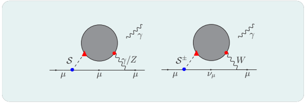
no dependence on $\tan\beta$

M_A-dependency

a_μ^{Yuk}



$\mathcal{S} = H$

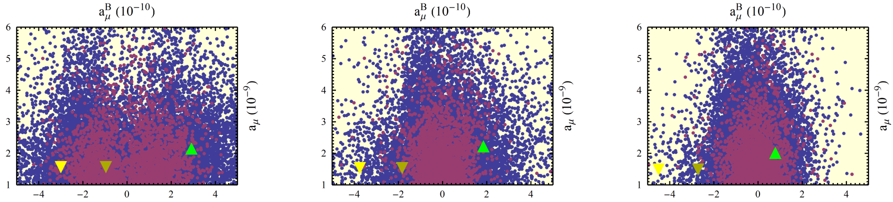


$\mathcal{S} = h, H, A,\ \mathcal{S}^\pm = H^\pm$

triple Higgs couplings $\Rightarrow$

dependence on $\tan\beta$

2HDM: Numerical analysis



$\eta = -0.1$ $\eta = 0$ $\eta = 0.1$

125 GeV$<M_H<500$ GeV, $M_A<500$ GeV, 80 GeV$<M_{H^\pm}<500$ GeV

$1<\tan\beta<100$, $|\eta|<0.1$, $0<\lambda_1<4\pi$, $|\zeta_u|<1.2$, $|\zeta_d|<50$,$|\zeta_l|<100$

blue/red points before/after applytin constraints:

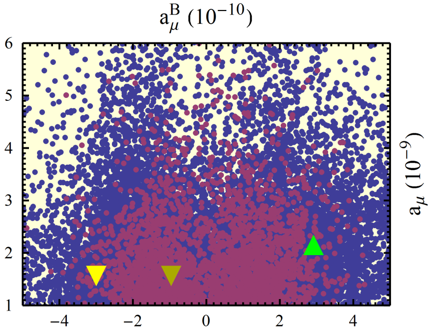
Vacuum stability, Global minimum

Perturbativity, EW and experimental constraints

$a_\mu^{\mathsf{B}} = (2 \cdots 4) \times 10^{-10}$: reduces the uncertainty

[Cherchiglia, Kneschke, Stöckinger, S-K '16]

2HDM: Numerical analysis



$\eta = -0.1$

Benchmark points:

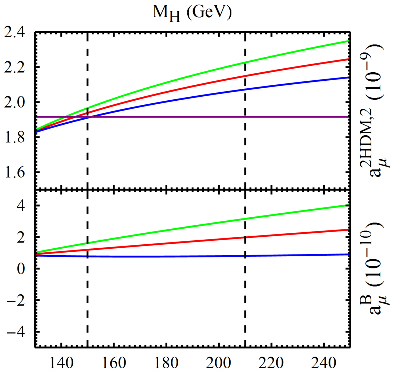
$M_A = 50$ GeV, $M_H = M_{H\pm} = 200$ GeV,

$\zeta_l = -100$, $\zeta_u = \zeta_d = 0.01$.

(compatible with $\tan\beta = 100$ for Type X)

- ▼ $\tan\beta = 2$, $\lambda_1 = 4\pi$
- ▼ $\tan\beta = 2$, $\lambda_1 = 2\pi$
- ▲ $\tan\beta = 100$

2HDM: Numerical analysis



red: $\eta = 0$
blue: $\eta = 0.1$
green: $\eta = -0.1$

$M_A = 50$ GeV, $M_{H^\pm} = 200$ GeV,
$\zeta_l = -100$, $\zeta_u = \zeta_d = 0.01$
$\tan\beta = 100$, $\lambda_1 = 4\pi$

$$a_\mu^{\mathrm{B}}|_{\eta=0} \approx -6.3 \times 10^{-7} M_H^2 \zeta_l \tan\beta \times \mathcal{F}(M_H, M_{H^\pm})$$

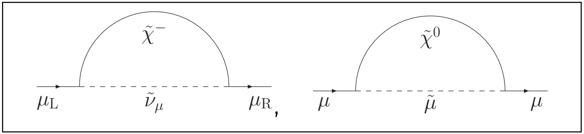
$$\mathcal{F}(M_H, M_{H^\pm}) \propto \frac{1}{M_{H^\pm}^2}$$

$\propto M_H^2$ from m_{12}^2
From EW constraint:
Small splitting between M_H and $M_{H^\pm}$: all values allowed for M_A
Large splitting between M_H and $M_{H^\pm}$: M_A almost degenerate with $M_{H^\pm}$

MSSM

MSSM

$$\mathcal{L}_{\mathsf{int}} = \tilde{\nu}^\dagger \overline{\chi^-}(c_{\mathsf{L}}^* P_{\mathsf{L}} + c_{\mathsf{R}} P_{\mathsf{R}})\mu + \tilde{\mu}^\dagger \overline{\chi^0}(n_{\mathsf{L}}^* P_{\mathsf{L}} - n_{\mathsf{R}} P_{\mathsf{R}})\mu + \mathsf{h.c}$$



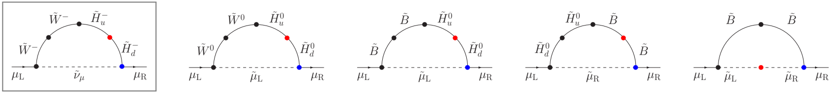
$\times$ external photon

One-loop contribution

[Fayet '80]...[Moroi '96]

Parameter dependence: $\mu, M_1, M_2, M_E, M_L, \tan\beta$
One-loop correction dominant: $\mathcal{O}(\alpha)$
enhanced by $\tan\beta$, dependent on sign(μ)

MSSM: One-loop corrections



● : $y_\mu = \frac{m_\mu}{v_d}$

$y_\mu v_u = \frac{m_\mu}{v_d} v_u = m_\mu \tan\beta$

● : μ

One-loop contribution

$$a_\mu^{\text{SUSY,1L}} \propto \alpha \frac{m_\mu^2}{M_{\text{SUSY}}^2} \tan\beta \,\text{sign}(\mu)$$

$$\Downarrow$$

$$a_\mu^{\text{SUSY,1L}} \approx 13 \times 10^{-10} \tan\beta \,\text{sign}(\mu) \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}}\right)^2$$

$a_\mu^{\text{SUSY,1L}} \approx 26 \times 10^{-10}$, for $\tan\beta = 50$ and $M_{\text{SUSY}} = 500$ GeV.

MSSM: Two-loop corrections

SUSY two-loop corrections to SM 1L diagrams: $\sim 10^{-10}$

[Chen, Geng '01],[Arhib, Baek '02],[Heinemeyer, Stöckinger, Weiglein '03, '04]

Photonic corrections to SUSY 1L diagrams:

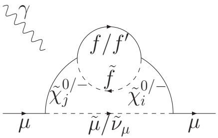
[v. Weitershausen, Schäfer, Stöckinger, S-K '10]

$\mu, M_1, M_2, M_E, M_L, \tan\beta$

$a_\mu^{\text{2L},(\gamma)} \approx \frac{4\alpha}{\pi} \log \frac{m_\mu}{M_{\text{SUSY}}} a_\mu^{\text{SUSY, 1}}$,

$-(7\cdots9)\%$ corrections

for $100 < M_{\text{SUSY}} < 1000$ GeV



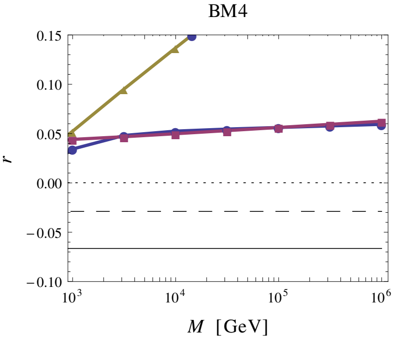
fermion/sfermion two-loop corrections

[Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K, '13]

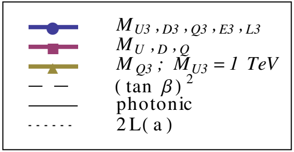
$M_{Q_i}, M_{U_i}, M_{D_i}, M_{L_i}, M_{E_i}$, $i\in\{1,2,3\}$

non-decoupling behavior: term $\propto \ln(\frac{m_{\tilde{f}}^2}{m_{\tilde{\nu}_\mu}^2})$, when large splitting between $m_{\tilde{f}}$ and $m_{\tilde{\nu}_\mu}$.

MSSM: Non-decoupling behavior of $f\tilde{f}$-loop corrections



- $M_2, m_{\tilde{\mu}_L} >> M_1, m_{\tilde{\mu}_R}$
- $M_1 = 140$ GeV
- $m_{\tilde{\mu}_R} = 200$ GeV
- $M_2 = m_{\tilde{\mu}_L} = 2000$ GeV
- $\mu = -160,\ \tan\beta = 40$
- $\mathcal{O}(10 \cdots 30\%)$



Radiative muon mass generation

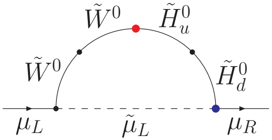
Radiative muon mass generation

$$v_d \rightarrow 0, \qquad \tan\beta \equiv \frac{v_u}{v_d} \rightarrow \infty, \qquad m_\mu^{\text{tree}} = y_\mu v_d \Rightarrow 0$$

- m_μ generated via coupling to v_u

[Dobrescu, Fox '10][Altmannshofer, Straub '10]

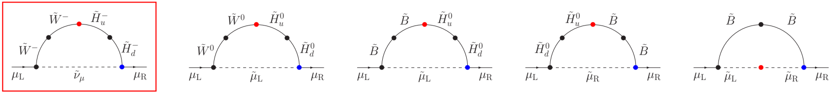
- $m_\mu \equiv \cancel{y_\mu v_d} + y_\mu v_u \Delta_\mu^{\text{red}}$
- y_μ obtained from one-loop self energy.
- $a_\mu^{\text{SUSY}} = \frac{y_\mu v_u}{m_\mu} a_\mu^{\text{red}}$
- $a_\mu^{\text{SUSY}} \propto y_\mu$ and $m_\mu \propto y_\mu$



$$\Rightarrow \quad a_\mu^{\text{SUSY}} = \frac{a_\mu^{\text{red}}}{\Delta_\mu^{\text{red}}}$$

[1504.05500][Bach, Park, Stöckinger,S-K]

Radiative muon mass generation



$$
\begin{aligned}
a_\mu^{\text{red}} &= a_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu}) + a_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\mu}_L) + a_\mu^{\text{red}}(\tilde{B}\tilde{H}\tilde{\mu}_L) + a_\mu^{\text{red}}(\tilde{B}\tilde{H}\tilde{\mu}_R) + a_\mu^{\text{red}}(\tilde{B}\tilde{\mu}_L\tilde{\mu}_R) \\
\Delta_\mu^{\text{red}} &= \Delta_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu}) + \Delta_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\mu}_L) + \Delta_\mu^{\text{red}}(\tilde{B}\tilde{H}\tilde{\mu}_L) + \Delta_\mu^{\text{red}}(\tilde{B}\tilde{H}\tilde{\mu}_R) + \Delta_\mu^{\text{red}}(\tilde{B}\tilde{\mu}_L\tilde{\mu}_R)
\end{aligned}
$$

- a_μ^{MSSM} sign depends on the mass ratios.
- $\text{sgn}(\mu)$ and $\tan\beta$ dependence disappears.
- α^0 order correction
- $a_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu})$ and $\Delta_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu})$ have opposite signs.
- For the equal mass case, $a_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu})$ and $\Delta_\mu^{\text{red}}(\tilde{W}\tilde{H}\tilde{\nu})$ dominate $\Longrightarrow$ negative a_μ^{MSSM}

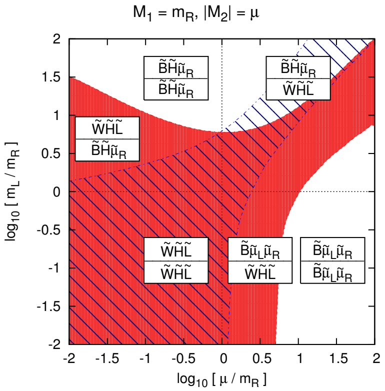
$$a_\mu^{\text{MSSM}} = \frac{a_\mu^{\text{red}}}{\Delta_\mu^{\text{red}}}$$

equal mass case

$$\approx \frac{g_1^2+5g_2^2}{3(g_1^2-3g_2^2)}\frac{m_\mu^2}{M_{\text{SUSY}}^2}$$

$$\approx -72 \times 10^{-10}\left(\frac{1\text{TeV}}{M_{\text{SUSY}}}\right)^2$$

Radiative muon mass generation



white: positive for $M_2 > 0$, $M_2 < 0$

red: negative for $M_2 > 0$

blue: negative for $M_2 < 0$

μ_R-dominance: top middle
$\tilde{B}\tilde{H}\tilde{\mu}_R$ dominant

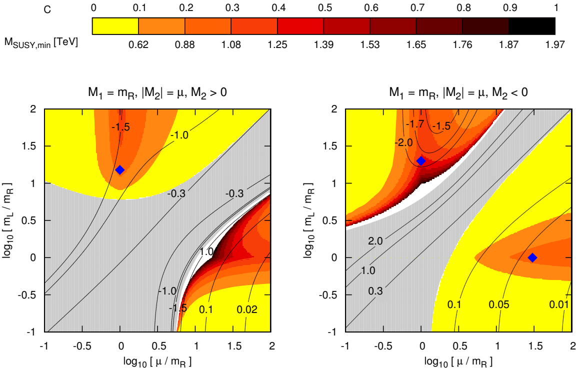
At the center, the equal mass case,

$$a_\mu^{\text{MSSM}} \approx -72 \times 10^{-10} \left(\frac{1\ \text{TeV}}{M_{\text{SUSY}}}\right)^2$$

Large μ-limit: right end
$\tilde{B}\tilde{\mu}_L\tilde{\mu}_R$ dominant

What can be the C-value/a_μ^{MSSM} for the given parameter ratio space?

Radiative muon mass generation



TeV			
μ	1	1.3	30
M_1	1	1.3	1
M_2	1	-1.3	-30
m_L	15	26	1
m_R	1	1.3	1
$\left(\frac{a_\mu}{10^{-10}}\right)$	26.4	29.0	28.0

$$a_\mu \approx -72 \times 10^{-10} \left(\frac{1\text{Tev}}{M_{\text{SUSY}}}\right)^2$$

$$|\mu| \gg |M_1| = m_L = m_R \equiv M_{\text{SUSY}}$$

$$m_L \gg |\mu| = |M_1| = m_R \equiv M_{\text{SUSY}}$$

$$a_\mu \approx 37 \times 10^{-10} \left(\frac{1\text{TeV}}{M_{\text{SUSY}}}\right)^2$$

GM2Calc

https://gm2calc.hepforge.org/

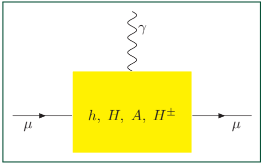
[Athron et. al. '15]

$$a_\mu^{\text{SUSY}} = \left(a_\mu^{1\text{L}} + a_\mu^{2\text{L(a)}} + a_\mu^{2\text{L, photonic}} + a_\mu^{2\text{L, } f\tilde{f}} \right)_{\tan\beta\text{-resummed}}$$

- A stand alone program to evaluate $(g-2)_\mu$ in MSSM.
- includes all known loop corrections, particularily $\tilde{f}f$ 2-loop.
- allows $\tan\beta \to \infty$.
- computing in on-shell scheme: no error caused by $m_{\tilde{f}}$ like in $\overline{\text{DR}}$ mass.
- in standard SLHA input

Summary

2HDM



$a_\mu^{\text{2HDM}} \propto \alpha^2 m_\mu^2 \frac{m_f^2}{M_S^2} \zeta_l^2$

τ-loop, enhanced by ζ_l^2

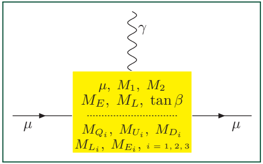
$a_\mu^{\text{2HDM, B}} \approx (2 \cdots 4) \times 10^{-10}$

$a_\mu \approx (15 \cdots 30) \times 10^{-10}$

$50\ \text{GeV} < M_A < 100\ \text{GeV}$

$\zeta_l = 100$

MSSM



$a_\mu^{\text{SUSY, 1}} \propto \alpha^1 \frac{m_\mu^2}{M_{\text{SUSY}}^2} t_\beta \text{sign}(\mu)$

$a_\mu^{2L,\gamma} \approx -(7 \cdots 9)\% \times a_\mu^{\text{SUSY},1}$

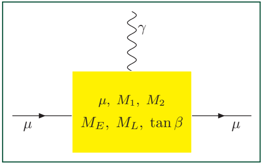
$a_\mu^{2L,\tilde{f}f} \approx (10 \cdots 30)\% \times a_\mu^{\text{SUSY},1}$

$a_\mu \approx 26 \times 10^{-10}$

$M_{\text{SUSY}} \approx 500$ GeV

$t_\beta = 50$

Radiative m_μ generation



$a_\mu^{\text{Rad.}} \propto \alpha^0 \frac{m_\mu^2}{M_{\text{SUSY}}^2}$

$|\mu| \gg |M_1| = m_L = m_R \equiv M_{\text{SUSY}}$

$m_L \gg |\mu| = |M_1| = m_R \equiv M_{\text{SUSY}}$

$a_\mu \approx 37 \times 10^{-10} \left(\frac{1\ \text{TeV}}{M_{\text{SUSY}}}\right)^2$

$M_{\text{SUSY}} \approx 1000$ GeV