

Extractions of the proton and deuteron charge radii from scattering experiments

Hadronic Contributions to New Physics Searches 25-30 Sept. 2016, Tenerife, Spain

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Until 2010, conventional wisdom had the proton's rms radius $r_p=0.875(6)$ fm



This came both from electron scattering and hydrogen Lamb shift measurements

In 2010, muonic hydrogen Lamb shift measurements found rp = 0.84184(67) fm



Either some extractions of the radius from data are wrong or there is new physics

If the muon, for example, couples to a yet unknown particle, possibly dark matter, and the electron doesn't, this could explain the discrepancy

Everyone hopes for new physics, but it has been hard to find



r_p History

Pohl, doi:10.1146/annurev-nucl-102212-170627





Can we really determine the proton's radius by electron scattering experiments?

If so, what have we been missing?

The answer, perhaps, lies not in the realm of new physics, but in the mundane mechanics of modeling real and imperfect experimental data



8.4 meV

How to Measure r_p with μH

Pohl, doi:10.1038/nature09250





Muonic Hydrogen





Hydrogen Lamb Shift

Pohl, doi:10.1146/annurev-nucl-102212-170627





 $G_E(Q^2)=1-[r_p{}^2/6]Q^2+cQ^4+\dots$ r_p is gotten from the slope of $G_E(Q^2)$ at Q^2 =0

This definition of r_p is consistent with that measured in Lamb-Shift atomic transitions

Determining the proton's rms radius from ep elastic scattering is as easy and as hard as determining the slope of $G_E(Q^2)$ at $Q^2 = 0$

The problem is nobody can measure at Q² =0, so everyone must extrapolate.



Can't we just fit G_E to a power series in Q^2 and extract the linear term?

Yes, but...

One must match the power series to the correct range of Q² in order to extrapolate well to Q²=0

No...

A global fit that includes regions of G_E where the linear term is small can skew the fit at the origin



Fit Bias





Fit Bias with Statistical Data



Generated: $f(x) = e^{-x} [0, 1]$ Fit: $f(x) = C_0 [1 + C_1 Q^2 + C_2 Q^4]$ [0.02,0.1] 15 trials of 10⁹ events [0.02,0.1] 50 trials of 10⁷ events [0.00,0.1] 50 trials of 10⁷ events [0.02,0.2-4] 50 trials of 10⁷ events No bias in slope



^x_{max} Hadronic Contributions to New Physics Searches

0.3

0.2

0.1

 χ^2

0.4



From Monte Carlo Studies we conclude that fits of the form

$$C_0G_E(Q^2) = C_0[1 - C_1Q^2 + C_2Q^2]$$

applied over a range where G_E drops by 10% from unity at Q²=0 yield an unbiased c₁ (*i.e.* an unbiased radius)

Let's apply this to data going back to early 1960s and to what happens



REVIEWS OF MODERN PHYSICS

VOLUME 35, NUMBER 2

APRIL 1963

Electric and Magnetic Form Factors of the Nucleon*

L. N. HAND, D. G. MILLER, AND RICHARD WILSON Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts







Systematic errors, added linearly, overestimate the statistical fluctuations



ELECTROMAGNETIC FORM FACTORS OF THE PROTON AT LOW FOUR-MOMENTUM TRANSFER (II)

Nuclear Physics B93 (1975) 461-478 © North-Holland Publishing Company

F. BORKOWSKI, G.G. SIMON, V.H. WALTHER and R.D. WENDLING* Institut für Kernphysik der Universität Mainz, D-65 Mainz, Germany





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ABSOLUTE ELECTRON-PROTON CROSS SECTIONS AT LOW MOMENTUM TRANSFER MEASURED WITH A HIGH PRESSURE GAS TARGET SYSTEM

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PRL 105, 242001 (2010)

PHYSICAL REVIEW LETTERS

week ending 10 DECEMBER 2010

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High-Precision Determination of the Electric and Magnetic Form Factors of the Proton

J. C. Bernauer,^{1,*} P. Achenbach,¹ C. Ayerbe Gayoso,¹ R. Böhm,¹ D. Bosnar,² L. Debenjak,³ M. O. Distler,^{1,†} L. Doria,¹ A. Esser,¹ H. Fonvieille,⁴ J. M. Friedrich,⁵ J. Friedrich,¹ M. Gómez Rodríguez de la Paz,¹ M. Makek,² H. Merkel,¹ D. G. Middleton,¹ U. Müller,¹ L. Nungesser,¹ J. Pochodzalla,¹ M. Potokar,³ S. Sánchez Majos,¹ B. S. Schlimme,¹ S. Širca,^{6,3} Th. Walcher,¹ and M. Weinriefer¹



 $c_0=0.9992(3)$ $r_p=0.850(19)$ fm

Low Q² Mainz 2010 data





 $r_p \, (fm)$



rp from Higinbotham

- Stepwise regression
- F tests
- Akaike information criterion
- Multivariate error estimates.
- Systematical determination of predictive variables for a give of electron scattering data

FIG. 1. The 68% (inner) and 95% (outer) confidence ellipsoids associated with the covariance matrix from the three-parameter fit of the Mainz80 and Saskatoon74 data. The plane representing the muonic Lamb shift result of rp = 0.84 fm is shown at its corresponding a1 value of -0.1176 fm² and is clearly not ruled out by this fit.

Higinbotham et al.

DOI: 10.1103/PhysRevC.93.055207





Global Fitting Models



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High-order polynomials are a bad idea



• Over-fitting





- Quick divergence outside the fit range
- Oscillations at the edges (Runge Phenomena)

If you're going to extrapolate, use low-order polynomials or Padé approximates



PHYSICAL REVIEW C 93, 065207 (2016)

Consistency of electron scattering data with a small proton radius

Keith Griffioen, Carl Carlson, and Sarah Maddox

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We determine the charge radius of the proton by analyzing the published low momentum transfer electronproton scattering data from Mainz. We note that polynomial expansions of the form factor converge for momentum transfers squared below $4m_{\pi}^2$, where m_{π} is the pion mass. Expansions with enough terms to fit the data, but few enough not to overfit, yield proton radii smaller than the CODATA or Mainz values and in accord with the muonic atom results. We also comment on analyses using a wider range of data, and overall obtain a proton radius $R_E = 0.840(16)$ fm.

DOI: 10.1103/PhysRevC.93.065207

Extract G_E from Mainz 2000 Cross sections Fit!



$$G_E(Q^2) = G_D(Q^2) \left(\frac{\sigma}{\sigma_D}\right)^{1/2} \left[1 + \tau \mu_p^2 \frac{G_M^2 / (\mu_p G_E)^2 - 1}{\varepsilon + \tau \mu_p^2}\right]^{-1/2}$$

$$\mu_p G_E/G_M = 1 - Q^2/Q_0^2$$
Fit to world data
$$Q_0^2 = 8.02 \pm 0.05 \text{ with } \chi^2/\text{dof} = 2.3$$

 $\mathcal{E} =$





A continued Fraction with 4 parameters fits the data well. Extracted **r**_p=0.8389(4) Extracting G_E by adjusting Q_0 in G_E/G_M yields best value of 8 GeV²





Orange: 1-Q²/(8.02 GeV²)







0.8

1











With errors on G_E are inflated by 15% the full 1422 points obey perfect statistics







Polynomial Terms





Arrington

World G_E data corrected *ad hoc* for 2-photon effects

DOI: 10.1103/PhysRevC.71.015202

Our fit extrapolates well



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• Muonic Hydrogen Lamb Shift: $r_p = 0.84087(39) \text{ fm}$

CREMA Collaboration

• Electron Scattering

 $r_p = 0.840(16) \text{ fm}$

Griffioen, Carlson, Maddox Reanalysis of the Mainz 2010 data

• Hydrogen Lamb Shift:

r_p = 0.8297(91) fm

L. Maisenbacher, A. Beyer, et al.

Max-Planck-Institut für Quantenoptik, Garching, Germany



Laser spectroscopy of Science **muonic deuterium 669** 12 AUGUST 2016 • VOL 353 ISSUE 6300 Randolf Pohl,^{1,2*} François Nez,³ Luis M. P. Fernandes,⁴ Fernando D. Amaro,⁴ F = 5/2 $2S_{1/2}^{F=3/2} \rightarrow 2P_{3/2}^{F=5/2}$ 10 CODATA $\mu p + iso$ | this value #1 $2P_{3/2}$ F = 1/2signal [arb. units] F = 3/2F=3/2 $2P_{1/2}$ F=1/2 ν 800 900 700 v - 50.0 THz (GHz) В $2S_{1/2}^{F=1/2} \rightarrow 2P_{3/2}^{F=3/2}$ $\mu p + iso_{\text{this value } \#2}$ CODATA signal [arb. units] $2S_{1/2}^{F=1/2} \to 2P_{3/2}^{F=1/2}$ F = 3/2this value #3 2S_{1/2} \mathbf{v}_2 v_3 F = 1/2100 200

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v - 52.0 THz (GHz)



$$\Delta E_{\rm LS}^{\rm theo} = 228.7766(10) \,{
m meV} + \Delta E_{\rm LS}^{\rm TPE}$$

-6.1103(3) $r_{\rm d}^2 \,{
m meV}/{
m fm}^2$

$$\Delta E_{\rm LS}^{\rm TPE} (\text{theo}) = 1.7096(200) \,\text{meV}$$
$$\Delta E_{\rm LS}^{\rm exp} = 202.8785(31)_{\rm stat} (14)_{\rm syst} \,\text{meV}$$
$$\mu + \int_{\rm d} + \int_{\rm d} \int_{\rm d}$$



Isotope Shift

$$\delta^{(2)}(\mathrm{H},\mathrm{D}) \equiv r_{\mathrm{d}}^2 - r_{\mathrm{p}}^2 = 3.82007(65) \mathrm{fm}^2$$

$r_{\rm p}(\mu d + iso) = 0.8356(20) \,{\rm fm}$ $r_{\rm p}(\mu p) = 0.84087(39) \,{\rm fm}$



$r_{\rm d}({\rm D \ spectroscopy}) = 2.1415(45) \ {\rm fm}$ $r_{\rm d}({\rm CODATA}) = 2.1424(21)~{\rm fm}$ $r_{\rm d}(\mu \rm p + iso) = 2.12771(22) ~\rm fm$ $r_{\rm d}(\mu {\rm d}) = 2.12562(13)_{\rm exp}(77)_{\rm th}$

rd

Smaller than CODATA



 $\Delta E_{\rm HFS}^{\rm pol}(\exp) = 0.2178(74)\,\rm meV$ (theo) = 0.2226(49) meV

 $\Delta E_{\rm LS}^{\rm TPE}(\exp) = 1.7638(68)\,{\rm meV}$ ${}_{S}^{PE}(\text{theo}) = 1.7096(200) \,\text{meV}$



μ Hyperfine Splitting

PHYSICAL REVIEW A 83, 042509 (2011)

Proton-structure corrections to hyperfine splitting in muonic hydrogen

Carl E. Carlson,^{1,2} Vahagn Nazaryan,³ and Keith Griffioen² ¹Helmholtz Institut Mainz, Johannes Gutenberg-Universität, D-55099 Mainz, Germany ²Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA ³Center for Advanced Medical Instrumentation, Department of Physics, Hampton University, Hampton, VA 2 (Received 7 February 2011; published 19 April 2011)

$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol} = \frac{E_{2\gamma}^{box}}{E_F} - \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2}$$
$$\Delta_Z = \frac{8\alpha m_r}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(\frac{G_E(Q^2)G_M(Q^2)}{1 + \kappa_p} - 1 \right)$$
$$\equiv -2\alpha m_r r_Z,$$

$$\Delta_{1} = \int_{0}^{\infty} \frac{dQ^{2}}{Q^{2}} \left\{ \beta_{1}(\tau_{\ell}) F_{2}^{2}(Q^{2}) + 4m_{p} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^{2}} \frac{Q^{4}\beta_{1}(\tau) - 4m_{\ell}^{2}\nu^{2}\beta_{1}(\tau_{\ell})}{Q^{4} - 4m_{\ell}^{2}\nu^{2}} g_{1}(\nu, Q^{2}) \right\},$$

$$\Delta_{2} = -12m_{p}^{2} \int_{0}^{\infty} \frac{dQ^{2}}{Q^{2}} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^{2}} \frac{Q^{4}[\beta_{2}(\tau) - \beta_{2}(\tau_{\ell})]}{Q^{4} - 4m_{\ell}^{2}\nu^{2}} g_{2}(\nu, Q^{2})$$

$$\Delta_{\rm pol} = \frac{\alpha m_{\ell}}{2(1+\kappa_p)\pi m_p} (\Delta_1 + \Delta_2)$$



$$\beta_1(\tau) = -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)},$$

The hyperfine splittings in μ H depend on proton form factors GE and GM proton spin structure functions g1 and g2



PHYSICAL REVIEW A 89, 022504 (2014)

Nuclear-structure contribution to the Lamb shift in muonic deuterium

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Mikhail Gorchtein^{*} and Marc Vanderhaeghen Institut für Kernphysik, Johannes Gutenberg-Universität, Mainz, Germany and PRISMA Cluster of Excellence, Johannes Gutenberg-Universität, Mainz, Germany (Received 3 December 2013; published 4 February 2014)

We consider the two-photon exchange contribution to the 2P - 2S Lamb shift in muonic deuterium in the framework of forward dispersion relations. The dispersion integrals are evaluated using experimental data on elastic deuteron form factors and inelastic electron-deuteron scattering, both in the quasielastic and hadronic range. The subtraction constant that is required to ensure convergence of the dispersion relation for the forward Compton amplitude $T_1(v, Q^2)$ is related to the deuteron magnetic polarizability $\beta(Q^2)$. Based on phenomenological information, we obtain for the Lamb shift $\Delta E_{2P-2S} = 2.01 \pm 0.74$ meV. The main source of the uncertainty of the dispersion analysis is due to lack of quasielastic data at low energies and forward angles. We show that a targeted measurement of the deuteron electrodesintegration in the kinematics of upcoming experiments A1 and MESA at Mainz can help in quenching this uncertainty significantly.

DOI: 10.1103/PhysRevA.89.022504

PACS number(s): 31.30.jr, 13.40.Gp, 14.20.Dh, 36.10.Ee



| | TABLE I. | TPE | corrections | to | the | $2S_{1/2}$ | energy | level | in | muoni | С |
|----------------------------|----------|-----|-------------|----|-----|------------|--------|-------|----|-------|---|
| deuterium in units of meV. | | | | | | | | | | | |

| $\begin{array}{ccc} \Delta E^{\rm PWBA} & -1.616(739) \\ \Delta E^{\rm FSI} & -0.391(44) \\ \Delta E^{\perp} & -0.322(3) \\ \Delta E^{\rm hadr} & -0.028(2) \\ \Delta E^{\beta} & 0.740(40) \end{array}$ | $\Delta ar{E}^{el}$ | -0.417(2) |
|--|---|--|
| ΔE^{Th} 0.023(1) | ΔE^{PWBA} ΔE^{FSI} ΔE^{\perp} ΔE^{hadr} ΔE^{β} ΔE^{Th} | -1.616(739) -0.391(44) -0.322(3) -0.028(2) 0.740(40) 0.023(1) 2.011(740) |



Precise quasi-elastic data at low Q² is needed for TPE. Mainz A1 has data being analyzed





Volume 47B, number 4

PHYSICS LETTERS

26 November 1973

Measured D/p Ratio

ELASTIC ELECTRON DEUTERON SCATTERING

R.W. BERARD, F.R. BUSKIRK, E.B. DALLY, J.N. DYER, X.K. MARUYAMA, R.L. TOPPING and T.J. TRAVERSO Naval Postgraduate School, Monterey, California, USA

Received 3 October 1973

Measurements of the ratio of the deuteron to proton electric form factors were made for low q. The rms radius of the deuteron structure factor was found to be 1.9635 \pm 0.0045 fm, yielding an rms charge radius of 2.095 \pm 0.006 fm.

Form factor drops much faster than for the proton



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Measured D/p Ratio

 $r_{\rm c} = 2.116 \pm 0.006 \, {\rm fm}$



Nuclear Physics A364 (1981) 285-296 © North-Holland Publishing Company

ELASTIC ELECTRIC AND MAGNETIC e-d SCATTERING AT LOW MOMENTUM TRANSFER

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Received 5 December 1980

Abstract: Up to $q^2 = 4 \text{ fm}^{-2}$ differential cross sections for elastic electron-deuteron scattering have been measured for a wide range of scattering angles. Using a combination of high pressure gas target and liquid target systems we have obtained high-accuracy data for the ratio e-p to e-d scattering and a small normalization error for backward scattering data. For extreme low momentum transfer the elastic structure deuteron rms radius has been extracted: $r_d = 1.9660 \pm 0.0068$ fm. This quantity, which is a characteristic parameter of the nucleon-nucleon potential was compared with the results of different models. The analysis of the isoscalar form factor indicates no significant contribution of a non-resonant three-pion state. The high-accuracy data of the electric neutron form factor could be extended to higher q^2 . All nucleon form factors are described with a consistent ansatz. The magnetic scattering data indicate the important role of isobar configuration in nucleon-nucleon interactions.

Form factor drops much faster than for the proton

Slope at origin is underestimated assuming r_p=0.84 fm



Schlimme et al., DOI: 10.1051/epjconf/2016113 04017

A1 Mainz, Elastic/Quasi-Elastic eD scattering

Currently being analyzed

Deuteron form factor measurements at low momentum transfers

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Abstract. A precise measurement of the elastic electron-deuteron scattering cross section at four-momentum transfers of $0.24 \text{ fm}^{-1} \le Q \le 2.7 \text{ fm}^{-1}$ has been performed at the Mainz Microtron. In this paper we describe the utilized experimental setup and the necessary analysis procedure to precisely determine the deuteron charge form factor from these data. Finally, the deuteron charge radius r_d can be extracted from an extrapolation of that form factor to $Q^2 = 0$.



- Low Q² electron scattering data are consistent with the muonic Lamb shift value r_p=0.84 fm.
- Several experiments including PRAD at JLab and MUSE at PSI will weigh in on this issue int he coming years.
- More precision low-Q² G_E, G_M, F₁, F₂, g₁ and g₂ data are needed for improving the nuclear physics contributions to atomic transitions. Mainz deuteron data will contribute to this. Some JLab data on g₁ and g₂ have yet to be published.