Future Higgs Studies: A Theorist's Outlook

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Prospects for Charged Higgs Discovery at Colliders

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With the discovery of the Higgs boson on
4 July 2012, the Standard Model is triumphant.

But, theorists are never satisfied!

Instead, we ask: is that all there is?

But, be careful what you ask for… you may be responsible for a twitter storm!

Back to the Higgs boson…

Why do theorists expect more than just the Higgs boson of the Standard Model?

Some fundamental microscopic phenomena must necessarily lie outside of the purview of the Standard Model (SM).

- Neutrinos are not massless.
- Dark matter is not accounted for.
- No explanation for the baryon asymmetry of the universe.
- The solution to the strong CP puzzle lies outside of the SM.
- Gauge coupling unification fails (is this some hint?)
- No explanation for the inflationary period of the very early universe.
- The gravitational interaction is omitted.

Outline

- Implications of a new fundamental scale beyond the SM
- Three big questions for the LHC program
- What if additional Higgs scalars exist?
- A SM-like Higgs boson cries out for the alignment limit
- The alignment limit with or without decoupling
- Achieving alignment without decoupling: case studies
  - a one-doublet one-singlet scalar sector
  - the two Higgs doublet model (2HDM)
  - accidental alignment in the MSSM
  - models with doublet and triplet scalars
- Conclusions

A new fundamental high energy scale

New high energy scales must exist where new degrees of freedom and/or more fundamental physics reside. Let Λ denote the energy scale at which the SM breaks down.

Predictions made by the SM depend on a number of parameters that must be taken as input to the theory. These parameters are sensitive to ultraviolet (UV) physics, and since the physics at very high energies is not known, one cannot predict their values.

However, one can determine the sensitivity of these parameters to the UV scale Λ.

In the 1930s, it was already appreciated that a critical difference exists between bosons and fermions. Fermion masses are logarithmically sensitive to UV physics. Ultimately, this is due to the chiral symmetry of massless fermions, which implies that

$$\delta m_F \sim m_f \ln(\Lambda^2/m_F^2)\,.$$

No such symmetry exists for scalar bosons* (in the absence of supersymmetry), and consequently we expect quadratic sensitivity of the scalar boson squared-mass to UV physics

$$\delta m_B^2 \sim \Lambda^2\,.$$

*In the case of the photon, gauge invariance (assuming no spontaneous symmetry breaking) implies that $\delta m_\gamma^2 = 0$.

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On the Self-Energy and the Electromagnetic Field of the Electron

V. F. WEISSKOPF
University of Rochester, Rochester, New York
(Received April 12, 1939)

In 1939, Weisskopf announces in the abstract to this paper that "the self-energy of charged particles obeying Bose statistics is found to be quadratically divergent"….

…. and concludes that in theories of elementary bosons, new phenomena must enter at an energy scale of order m/e (e is the relevant coupling)—the first application of naturalness.

which is about 10^{-58} times smaller than the classical electron radius. The "critical length" of the positron theory is thus infinitely smaller than usually assumed.

The situation is, however, entirely different for a particle with Bose statistics. Even the Coulombian part of the self-energy diverges to a first approximation as $W_{st} \sim e^2h/(mca^2)$ and requires a much larger critical length that is $a = (hc/e^2)^{-\frac{1}{2}} \cdot h/(mc)$, to keep it of the order of magnitude of mc^2. This may indicate that a theory of particles obeying Bose statistics must involve new features at this critical length, or at energies corresponding to this length; whereas a theory of particles obeying the exclusion principle is probably consistent down to much smaller lengths or up to much higher energies.

The tyranny of naturalness

In the SM the Higgs scalar potential,

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \tfrac{1}{2}\lambda(\Phi^\dagger\Phi)^2\,,$$

where $\mu^2 = \frac{1}{2}\lambda v^2$ in terms of the vacuum expectation value v of the Higgs field. The parameter μ^2 is quadratically sensitive to Λ. Hence, to obtain $v = 246$ GeV in a theory where $v \ll \Lambda$ requires a significant fine-tuning of the ultraviolet parameters of the fundamental theory.

Indeed, the one-loop contribution to the squared-mass parameter μ^2 would be expected to be of order $(g^2/16\pi^2)\Lambda^2$. Setting this quantity to be of order of v^2 (to avoid an *unnatural* fine-tuning of the tree-level parameter and the loop contribution) yields

$$\Lambda \simeq 4\pi v/g \sim \mathcal{O}(1\text{ TeV})$$

A *natural* theory of electroweak symmetry breaking (EWSB) would seem to require new physics at the TeV scale to govern the EWSB dynamics.

Origin of the electroweak scale?

- Naturalness is restored by supersymmetry which ties the bosons to the more well-behaved fermions

- The Higgs boson is an approximate Goldstone boson, the only other known mechanism for keeping an elementary scalar light. Example: neutral naturalness

- The Higgs boson is a composite scalar, with an inverse length of order the TeV-scale

- The TeV-scale is chosen by some vacuum selection mechanism

- It's just fine-tuned. Get over it!

What next at the LHC?

- Experimentalists—Of course, keep searching for new physics beyond the Standard Model (BSM)

- Theorists—Find new ways BSM physics (which might provide natural relief) can be hiding at the TeV-scale

But, if no signals for BSM physics emerge soon, what then?
My answer: look to the Higgs sector, of course!

After all, we have only recently discovered a most remarkable particle that seems to be like nothing that has ever been seen before—an elementary scalar boson. Shouldn't we probe this state as thoroughly as possible and explore its properties?

The three really big questions

1. Are there additional Higgs bosons to be discovered? (To paraphrase I.I. Rabi, "Who ordered that?") If fermionic matter is non-minimal why shouldn't scalar matter also be non-minimal?

2. If we measure the Higgs properties with sufficient precision, will deviations from SM-like Higgs behavior be revealed?

3. The operator $H^\dagger H$ is the unique relevant operator of the SM that is a Lorentz invariant gauge group singlet. As such, does it provide a "Higgs portal" to BSM physics that is neutral with respect to the SM gauge group?

Do more Higgs bosons mean more fine-tuning?

There are many examples in which natural explanations of the EWSB scale (e.g., the MSSM with TeV-scale SUSY-breaking) employ BSM physics with extended Higgs sectors.

If you give up on naturalness (e.g., vacuum selection), it has been argued that it may be difficult in some cases to accommodate more than one Higgs doublet at the electroweak scale.

However, it is possible to construct "partially natural" extended Higgs sectors in which the electroweak vacuum expectation value is fine-tuned (as in the SM), but additional scalar masses are related to the electroweak scale by a symmetry.

The partially natural two-Higgs doublet model

By imposing two discrete symmetries,

$$\mathbb{Z}_2^m : \quad \Phi_1 \Longleftrightarrow \Phi_2 .$$

$$\mathbb{Z}_2^i : \quad \Phi_1 \Longleftrightarrow -\Phi_1 , \quad \Phi_2 \Longleftrightarrow \Phi_2 ,$$

the 2HDM scalar potential is given by

$$\begin{aligned} V &= m^2 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right) + \tfrac{1}{2}\lambda \left[(\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 \right] + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\ &\quad + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right\} , \end{aligned}$$

which requires one fine tuning of the parameter m^2 (as in the SM). However, these discrete symmetries cannot be successfully implemented in the Higgs-fermion Yukawa interactions.

But, if one adds vector-like fermion top partners, then one can extend the discrete symmetries such that top quarks transform into their top partners.

To construct a successful model,[†] one will need to introduce a bare mass M for the top partners, which will softly break one of the two discrete symmetries. We assume that this soft-breaking is generated at a cutoff scale Λ. This re-introduces some fine-tuning (which grows with M), although it is not quadratically sensitive to Λ. The end result is that the top partners should not be too heavy (good for LHC discovery!).

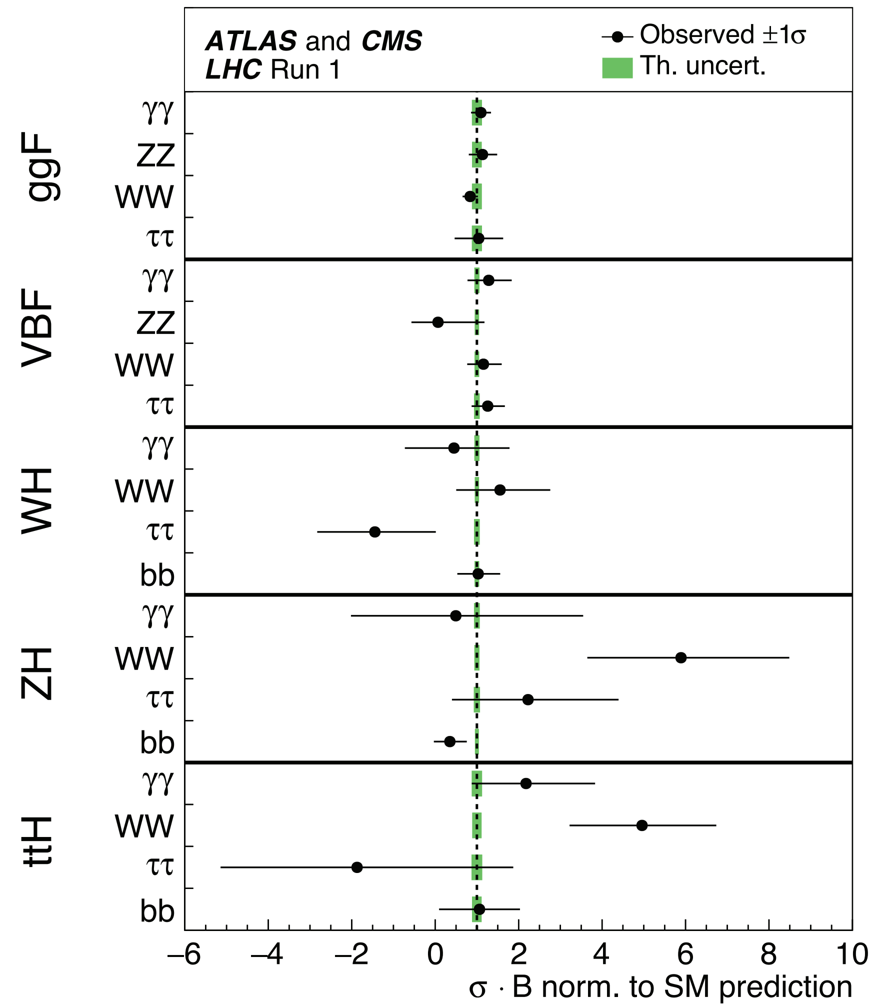
[†] For details, see P. Draper, H.E. Haber and J. Ruderman, JHEP 06 (2016) 124 [arXiv:1605.03237].

Theoretical implications of a SM-like Higgs boson

We already know that the observed Higgs boson is SM-like.

Thus any model of BSM physics, including models of extended Higgs sectors must incorporate this observation.

In models of extended Higgs sectors, a SM-like Higgs boson can be achieved in a particular limit of the model called the alignment limit.



The alignment limit—approaching the SM Higgs boson

Consider an extended Higgs sector with n hypercharge-one Higgs doublets Φ_i and m additional singlet Higgs fields ϕ_i.

After minimizing the scalar potential, we assume that only the neutral Higgs fields acquire vacuum expectation values (in order to preserve U(1)$_{\rm EM}$),

$$\langle\Phi_i^0\rangle = v_i/\sqrt{2}\,, \qquad\qquad \langle\phi_j^0\rangle = x_j\,.$$

Note that $v^2 \equiv \sum_i |v_i|^2 = 4m_W^2/g^2 = (246~\text{GeV})^2$.

We define new linear combinations of the hypercharge-one doublet Higgs fields (the so-called *Higgs basis*). In particular,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \frac{1}{v}\sum_i v_i^*\Phi_i\,, \qquad\qquad \langle H_1^0\rangle = v/\sqrt{2}\,,$$

and $H_2, H_3, \ldots, H_n$ are the other linear combinations of doublet scalar fields such that $\langle H_i^0\rangle = 0$ (for $I = 2, 3, \ldots, n$).

That is H_1^0 is aligned in field space with the direction of the Higgs vacuum expectation value (vev). Thus, if $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is a mass-eigenstate, then the tree-level couplings of this scalar to itself, to gauge bosons and to fermions are precisely those of the SM Higgs boson. This is the exact alignment limit.

In general, $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ is not a mass-eigenstate due to mixing with other neutral scalars. In this case, the observed Higgs boson is SM-like if either

- the elements of the scalar squared-mass matrix that govern the mixing of $\sqrt{2}\,\mathrm{Re}(H_1^0) - v$ with other neutral scalars are suppressed,

and/or

- the diagonal squared masses of the other scalar fields are all large compared to the mass of the observed Higgs boson (the so-called *decoupling limit*).

Although the alignment limit is most naturally achieved in the decoupling regime, it is possible to have a SM-like Higgs boson without decoupling. In the latter case, the masses of the additional scalar states could lie below ~ 500 GeV and be accessible to LHC searches.

Extending the SM Higgs sector with a singlet scalar

The simplest example of an extended Higgs sector adds a real scalar field S. The most general renormalizable scalar potential (subject to a $\mathbb{Z}_2$ symmetry to eliminate linear and cubic terms) is

$$\mathcal{V} = -m^2\Phi^\dagger\Phi - \mu^2 S^2 + \tfrac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \tfrac{1}{2}\lambda_2 S^4 + \lambda_3(\Phi^\dagger\Phi)S^2\,.$$

After minimizing the scalar potential, $\langle\Phi^0\rangle = v/\sqrt{2}$ and $\langle S\rangle = x/\sqrt{2}$. The squared-mass matrix of the neutral Higgs bosons is

$$\mathcal{M}^2 = \begin{pmatrix} \lambda_1 v^2 & \lambda_3 vx \\ \lambda_3 vx & \lambda_2 x^2 \end{pmatrix}\,.$$

The corresponding mass eigenstates are h and H with $m_h \leq m_H$. An approximate alignment limit can be realized in two different ways.

- $x \gg v$. This is the *decoupling limit*, where h is SM-like and $m_H \gg m_h$.
- $|\lambda_3|x \ll v$. Then h is SM-like if $\lambda_1 v^2 < \lambda_2 x^2$. Otherwise, H is SM-like.

The Higgs mass eigenstates are explicitly defined via

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\;\Phi^0 - v \\ \sqrt{2}\,S - x \end{pmatrix},$$

where

$$\lambda_1 v^2 = m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha\,,$$

$$\lambda_2 x^2 = m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha\,,$$

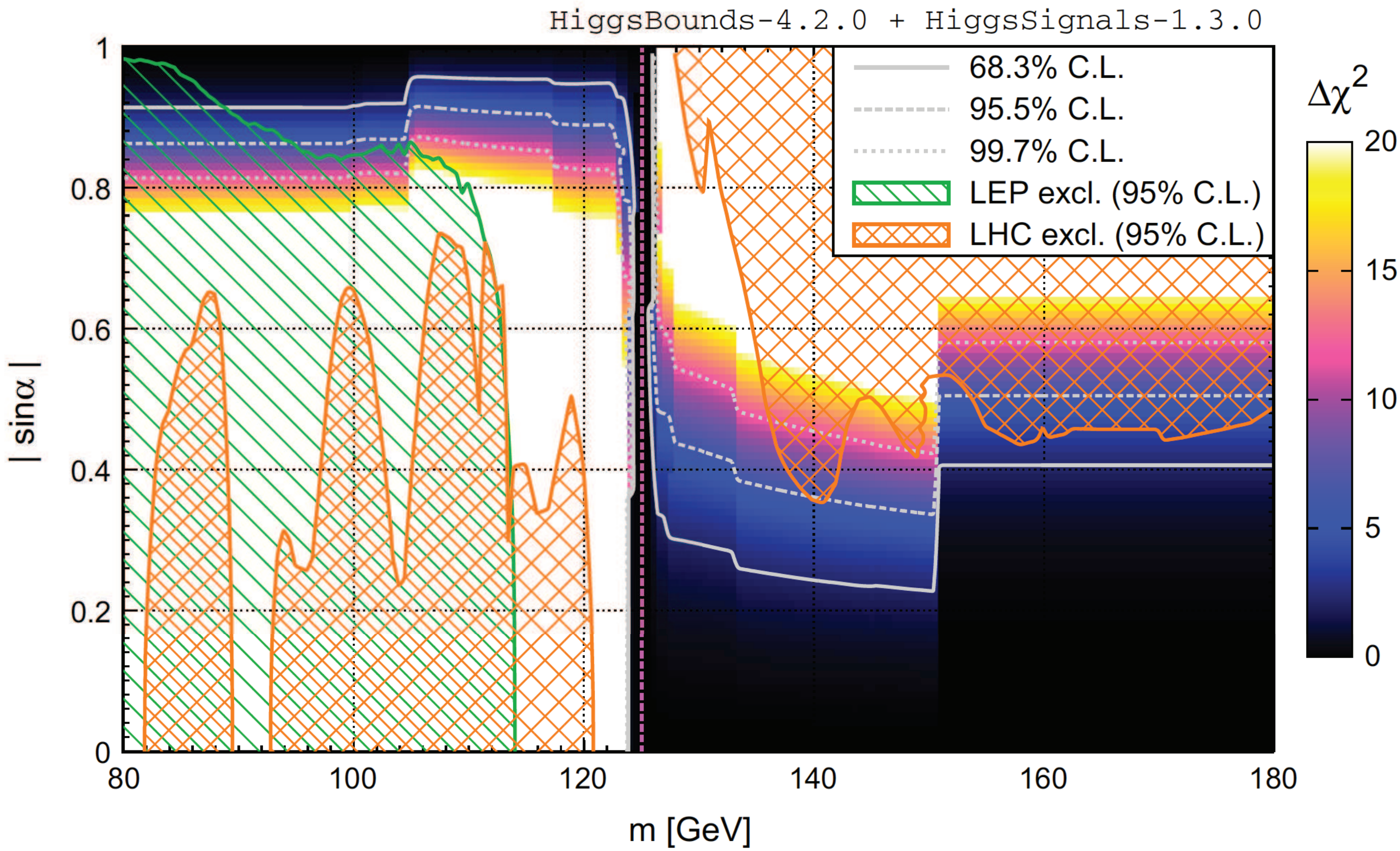
$$\lambda_3 x v = (m_H^2 - m_h^2)\sin\alpha\cos\alpha\,.$$

The SM-like Higgs must be approximately $\sqrt{2}\,\mathrm{Re}\;\Phi^0 - v$.
If h is SM-like, then $m_h^2 \simeq \lambda_1 v^2$ and

$$|\sin\alpha| = \frac{|\lambda_3| v x}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - \lambda_1 v^2)}} \simeq \frac{|\lambda_3| v x}{m_H^2 - m_h^2} \ll 1\,,$$

If H is SM-like, then $m_H^2 \simeq \lambda_1 v^2$ and

$$|\cos\alpha| = \frac{|\lambda_3| v x}{\sqrt{(m_H^2 - m_h^2)(\lambda_1 v^2 - m_h^2)}} \simeq \frac{|\lambda_3| v x}{m_H^2 - m_h^2} \ll 1\,.$$



Taken from T. Robens and T. Stefaniak, Eur. Phys. J. **C75**, 104 (2015).

Theoretical structure of the 2HDM

Consider the most general renormalizable 2HDM potential,

$$\begin{aligned}\mathcal{V} = {} & m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ & + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\} .\end{aligned}$$

After minimizing the scalar potential, assume that $\langle\Phi_i^0\rangle = v_i$ (for $i = 1, 2$).

Define the Higgs basis fields,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} \equiv \frac{v_1^*\Phi_1 + v_2^*\Phi_2}{v}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \equiv \frac{-v_2\Phi_1 + v_1\Phi_2}{v},$$

such that $\langle H_1^0\rangle = v/\sqrt{2}$ and $\langle H_2^0\rangle = 0$. The Higgs basis is uniquely defined up to an overall rephasing, $H_2 \to e^{i\chi}H_2$.

In the Higgs basis, the scalar potential is given by:

$$
\begin{aligned}
\mathcal{V} = {} & Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\
& + \tfrac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \left\{ \tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\} ,
\end{aligned}
$$

where Y_1, Y_2 and $Z_1, \ldots, Z_4$ are real and uniquely defined, whereas Y_3, Z_5, Z_6 and Z_7 are complex and transform under the rephasing of H_2,

$$
[Y_3, Z_6, Z_7] \to e^{-i\chi} [Y_3, Z_6, Z_7] \quad \text{and} \quad Z_5 \to e^{-2i\chi} Z_5 \,.
$$

Physical observables must be independent of χ.

After minimizing the scalar potential, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$.

Remark: Generically, the Z_i are $\mathcal{O}(1)$ parameters.

Type I and II Higgs-quark Yukawa couplings in the 2HDM

In the Φ_1–Φ_2 basis, the 2HDM Higgs-quark Yukawa Lagrangian is:

$$-\mathscr{L}_{\rm Y} = \overline{U}_L \Phi_i^{0\,*} h_i^U U_R - \overline{D}_L K^\dagger \Phi_i^- h_i^U U_R + \overline{U}_L K \Phi_i^+ h_i^{D\,\dagger} D_R + \overline{D}_L \Phi_i^0 h_i^{D\,\dagger} D_R + {\rm h.c.}\,,$$

where K is the CKM mixing matrix, and there is an implicit sum over i. The $h^{U,D}$ are 3×3 Yukawa coupling matrices.

In order to naturally eliminate tree-level Higgs-mediated FCNC, we shall impose a discrete symmetry to restrict the structure of $\mathscr{L}_{\rm Y}$.

Under the discrete symmetry, $\Phi_1 \to +\Phi_1$ and $\Phi_2 \to -\Phi_2$, which restricts the form of the scalar potential by setting $m_{12}^2 = \lambda_6 = \lambda_7 = 0$.Two different choices for how the discrete symmetry acts on the fermions then yield:

- Type-I Yukawa couplings: $h_1^U = h_1^D = 0$,

- Type-II Yukawa couplings: $h_1^U = h_2^D = 0$.

If the discrete symmetry is unbroken, then the scalar potential and vacuum are automatically CP-conserving (and all scalar potential parameters and the Higgs vevs can be chosen real).

Actually, it is sufficient for the discrete symmetry to be broken softly by taking $m_{12}^2 \neq 0$. In this case, an additional source of CP-violation will be present if $\mathrm{Im}(\lambda_5^*[m_{12}^2]^2) \neq 0$. Nevertheless, Higgs-mediated FCNC effects remain suppressed.

Note that the parameter

$$\tan\beta \equiv \frac{v_2}{v_1}\,,$$

is now meaningful since it refers to vacuum expectation values with respect to the basis of scalar fields where the discrete symmetry has been imposed.

The alignment limit in the CP-conserving 2HDM

In the case of a CP-conserving scalar potential, one can choose χ such that $\mathrm{Im}Z_5 = \mathrm{Im}Z_6 = \mathrm{Im}Z_7 = 0$, corresponding to a *real Higgs basis*. We identify the CP-odd Higgs boson as $A = \sqrt{2}\,\mathrm{Im}\,H_2^0$, with $m_A^2 = Y_2 + \frac{1}{2}(Z_3 + Z_4 - Z_5)v^2$. After eliminating Y_2 in favor of m_A^2, the CP-even Higgs squared-mass matrix with respect to the Higgs basis states, $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\,\sqrt{2}\,\mathrm{Re}\,H_2^0\}$ is given by,

$$\mathcal{M}_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}\,.$$

The CP-even Higgs bosons are h and H with $m_h \leq m_H$. The couplings of $\sqrt{2}\,\mathrm{Re}\;H_1^0 - v$ coincide with those of the SM Higgs boson. Thus, the alignment limit corresponds to two limiting cases:

1. $m_A^2 \gg (Z_1 - Z_5)v^2$. This is the *decoupling limit*, where h is SM-like and $m_A \sim m_H \sim m_{H^\pm} \gg m_h$.

2. $|Z_6| \ll 1$. h is SM-like if $m_A^2 + (Z_5 - Z_1)v^2 > 0$. Otherwise, H is SM-like.

In particular, the CP-even mass eigenstates are:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_{\beta-\alpha} & -s_{\beta-\alpha} \\ s_{\beta-\alpha} & c_{\beta-\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{2}\,\mathrm{Re}\,H_1^0 - v \\ \sqrt{2}\,\mathrm{Re}\,H_2^0 \end{pmatrix},$$

where $c_{\beta-\alpha} \equiv \cos(\beta-\alpha)$ and $s_{\beta-\alpha} \equiv \sin(\beta-\alpha)$ are defined in terms of the mixing angle α that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the original basis of scalar fields, $\{\sqrt{2}\,\mathrm{Re}\,\Phi_1^0 - v_1\,,\ \sqrt{2}\,\mathrm{Re}\,\Phi_2^0 - v_2\}$, and $\tan\beta \equiv v_2/v_1$.

Since the SM-like Higgs must be approximately $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, it follows that

- h is SM-like if $|c_{\beta-\alpha}| \ll 1$,
- H is SM-like if $|s_{\beta-\alpha}| \ll 1$.

Alignment *without* decoupling is required to have a SM-like H.

Remark: Although the tree-level couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson, the one-loop couplings can differ due to the exchange of non-minimal Higgs states (if not too heavy). For example, the $H^\pm$ loop contributes to the decays of the SM-like Higgs boson to $\gamma\gamma$ and γZ.

The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

$$
\begin{aligned}
Z_1 v^2 &= m_h^2 s_{\beta-\alpha}^2 + m_H^2 c_{\beta-\alpha}^2 \,, \\
Z_6 v^2 &= (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha} \,, \\
Z_5 v^2 &= m_H^2 s_{\beta-\alpha}^2 + m_h^2 c_{\beta-\alpha}^2 - m_A^2 \,.
\end{aligned}
$$

If h is SM-like, then $m_h^2 \simeq Z_1 v^2$ and

$$
|c_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,,
$$

If H is SM-like, then $m_H^2 \simeq Z_1 v^2$ and

$$
|s_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1 \,.
$$

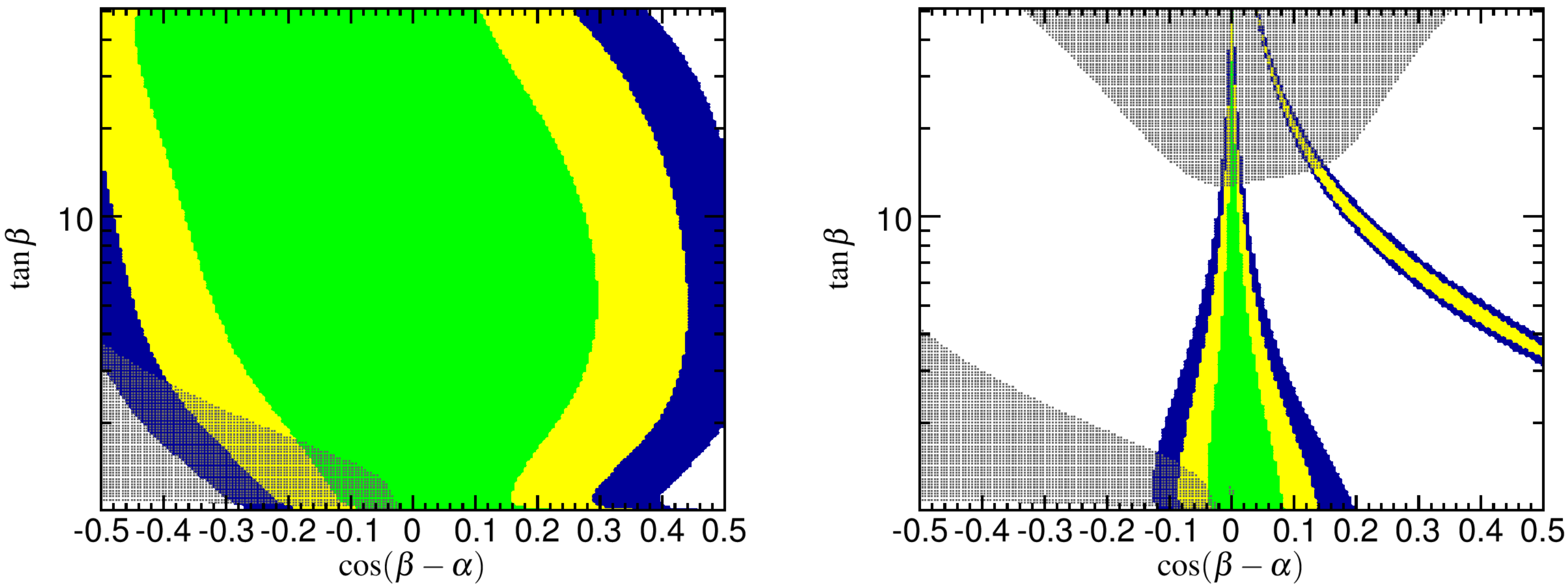
Higgs interaction	2HDM coupling	approach to alignment limit
hVV	$s_{\beta-\alpha}$	$1-\frac{1}{2}c^2_{\beta-\alpha}$
hhh	*	$1+2(Z_6/Z_1)c_{\beta-\alpha}$
hH^+H^-	*	$\frac{1}{3}\left[(Z_3/Z_1)+(Z_7/Z_1)c_{\beta-\alpha}\right]$
Hhh	*	$-Z_6/Z_1+\left[1-\frac{2}{3}(Z_{345}/Z_1)\right]c_{\beta-\alpha}$
$hhhh$	*	$1+3(Z_6/Z_1)c_{\beta-\alpha}$
$h\overline{D}D$	$s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^D_R$	$\mathbb{1}+c_{\beta-\alpha}\rho^D_R$
$h\overline{U}U$	$s_{\beta-\alpha}\mathbb{1}+c_{\beta-\alpha}\rho^U_R$	$\mathbb{1}+c_{\beta-\alpha}\rho^U_R$

Type I and II 2HDM couplings of the SM-like Higgs boson h normalized to those of the SM Higgs boson, in the alignment limit. The hH^+H^- and Hhh couplings given above are normalized to the SM hhh coupling (where $Z_{345}\equiv Z_3+Z_4+Z_5$). The scalar Higgs potential is taken to be CP-conserving. For the fermion couplings, D is a column vector of three down-type fermion fields (either down-type quarks or charged leptons) and U is a column vector of three up-type quark fields. In the third column, the first non-trivial correction to alignment is exhibited. Finally, complete expressions for the entries marked with a * can be found in H.E. Haber and D. O'Neil, Phys. Rev. D **74**, 015018 (2006) [Erratum: ibid. D **74** (2006) 059905].

$$\text{Type I}: \qquad \rho^D_R=\rho^U_R=\mathbb{1}\cot\beta\,,$$

$$\text{Type II}: \qquad \rho^D_R=-\mathbb{1}\tan\beta\,, \qquad \rho^U_R=\mathbb{1}\cot\beta\,.$$

Constraints on Type-I and II 2HDMs from Higgs data



Direct constraints from LHC Higgs searches for Type-I (left) and Type-II (right) 2HDM with $m_H = 300$ GeV with $m_h = 125$ GeV, $Z_4 = Z_5 = -2$ and $Z_7 = 0$. Colors indicate compatibility with the observed Higgs signal at $1\,\sigma$ (green), $2\,\sigma$ (yellow) and $3\,\sigma$ (blue). Exclusion bounds at 95% C.L. from the non-observation of the additional Higgs states overlaid in gray. From H.E. Haber and O. Stål, Eur. Phys. J. C **75**, 491 (2015) [Erratum: ibid., **76**, 312 (2016)].

The MSSM Higgs Sector at tree-level

The MSSM Higgs sector is a CP-conserving 2HDM. The dimension-four terms of the scalar potential constrained by supersymmetry. At tree level,

$$\lambda_1 = \lambda_2 = -\lambda_3 - \lambda_4 - \lambda_5 = \tfrac{1}{4}(g^2 + g'^{\,2})\,, \quad \lambda_4 = -\tfrac{1}{2}g^2\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0\,.$$

The corresponding real Higgs basis parameters of interest are:

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2\,, \qquad Z_5 v^2 = m_Z^2 s_{2\beta}^2\,, \qquad Z_6 v^2 = -m_Z^2 s_{2\beta} c_{2\beta}\,.$$

in a convention where $\tan\beta \geq 0$. It follows that,

$$\cos^2(\beta - \alpha) = \frac{m_Z^4\, s_{2\beta}^2 c_{2\beta}^2}{(m_H^2 - m_h^2)(m_H^2 - m_Z^2 c_{2\beta}^2)}\,.$$

The decoupling limit is achieved when $m_H \gg m_h$ as expected. Alignment without decoupling is (naively) possible at tree-level when $Z_6 = 0$, which yields $\sin 4\beta \simeq 0$. However, this limit is not phenomenologically viable. In any case, radiative corrections are required to obtain the observed Higgs mass.

Tree-level MSSM Higgs couplings to quarks and squarks

The MSSM employs the Type–II Higgs–fermion Yukawa couplings. Employing the more common MSSM notation, $H_D^i \equiv \epsilon_{ij}\Phi_1^{j\,*}$ and $H_U^i = \Phi_2^i$ (where $i,j=1,2$ are weak SU(2) indices), the tree-level Yukawa couplings are:

$$-\mathscr{L}_{\rm Yuk} = \epsilon_{ij}\big[h_b \bar{b}_R H_D^i Q_L^j + h_t \bar{t}_R Q_L^i H_U^j\big] + {\rm h.c.}\,,$$

which yields

$$m_b = h_b v c_\beta/\sqrt{2}\,, \qquad\qquad m_t = h_t v s_\beta/\sqrt{2}\,.$$

The leading terms in the coupling of the Higgs bosons to third generation squarks are proportional to the Higgs–top quark Yukawa coupling, h_t,

$$\mathscr{L}_{\rm int} \ni h_t\big[\mu^*(H_D^\dagger \widetilde{Q})\widetilde{U} + A_t\epsilon_{ij}H_U^i\widetilde{Q}^j\widetilde{U} + {\rm h.c.}\big] - h_t^2\big[H_U^\dagger H_U(\widetilde{Q}^\dagger\widetilde{Q} + \widetilde{U}^*\widetilde{U}) - |\widetilde{Q}^\dagger H_U|^2\big]\,,$$

where $\widetilde{Q} = \begin{pmatrix}\widetilde{t}_L\\ \widetilde{b}_L\end{pmatrix}$ and $\widetilde{U} \equiv \widetilde{t}_R^*$.

In terms of the Higgs basis fields H_1 and H_2,

$$\begin{aligned}
\mathscr{L}_{\rm int} \ni h_t\epsilon_{ij}\big[(\sin\beta X_t H_1^i+\cos\beta Y_t H_2^i)\widetilde{Q}^j\widetilde{U}+{\rm h.c.}\big] \\
-h_t^2\bigg\{\bigg[s^2_\beta|H_1|^2+c^2_\beta|H_2|^2+\sin\beta\cos\beta(H_1^\dagger H_2+{\rm h.c.})\bigg](\widetilde{Q}^\dagger\widetilde{Q}+\widetilde{U}^*\widetilde{U}) \\
-s^2_\beta|\widetilde{Q}^\dagger H_1|^2-c^2_\beta|\widetilde{Q}^\dagger H_2|^2-\sin\beta\cos\beta\big[(\widetilde{Q}^\dagger H_1)(H_2^\dagger\widetilde{Q})+{\rm h.c.}\big]\bigg\}\,,
\end{aligned}$$

where

$$X_t\equiv A_t-\mu^*\cot\beta\,,\qquad\qquad Y_t\equiv A_t+\mu^*\tan\beta\,.$$

Assuming CP-conservation for simplicity, we shall henceforth take μ, A_t real.

The radiatively corrected MSSM Higgs Sector

To illustrate the leading one-loop effects, we work in the limit where m_h, m_A, m_H, $m_{H^\pm} \ll M_S$, where M_S is the scale of SUSY-breaking. In this case, we can formally integrate out the squarks and generate a low-energy effective 2HDM Lagrangian (which is no longer of the tree-level MSSM form).

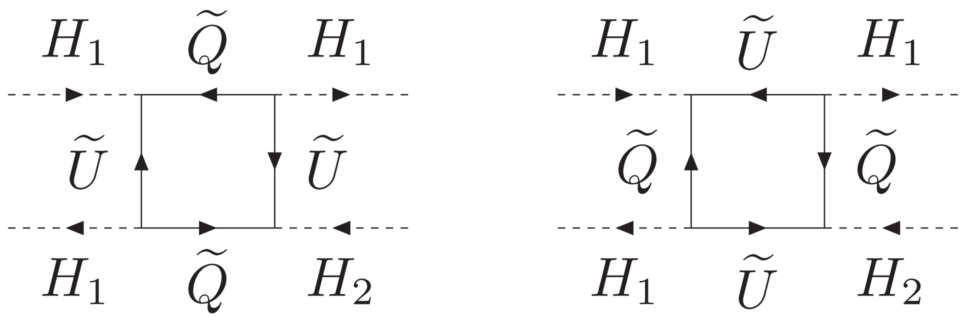
The dominant one-loop corrected expressions for Z_1 and Z_6 are given by[‡]

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

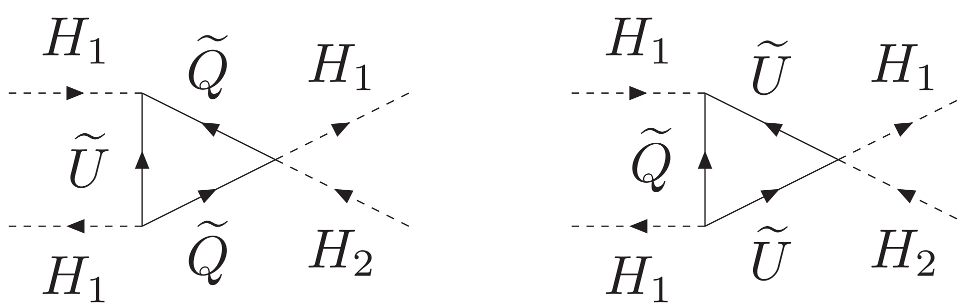
$$Z_6 v^2 = -s_{2\beta}\left\{m_Z^2 c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\},$$

where $M_S^2 \equiv m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu\cot\beta$ and $Y_t = A_t + \mu\tan\beta$.

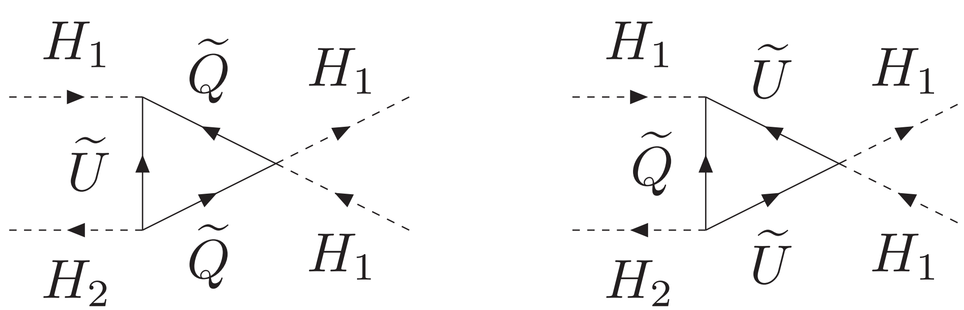
[‡]CP-violating phases that could appear in the MSSM parameters such as μ and A_t are neglected. The above expression for Z_6 was first written down in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **91**, 035003 (2015).



$\propto s_\beta^3 c_\beta X_t^3 Y_t$



$\propto s_\beta^3 c_\beta X_t^2$



$\propto s_\beta^3 c_\beta X_t Y_t$

Example: One-loop threshold corrections to Z_6

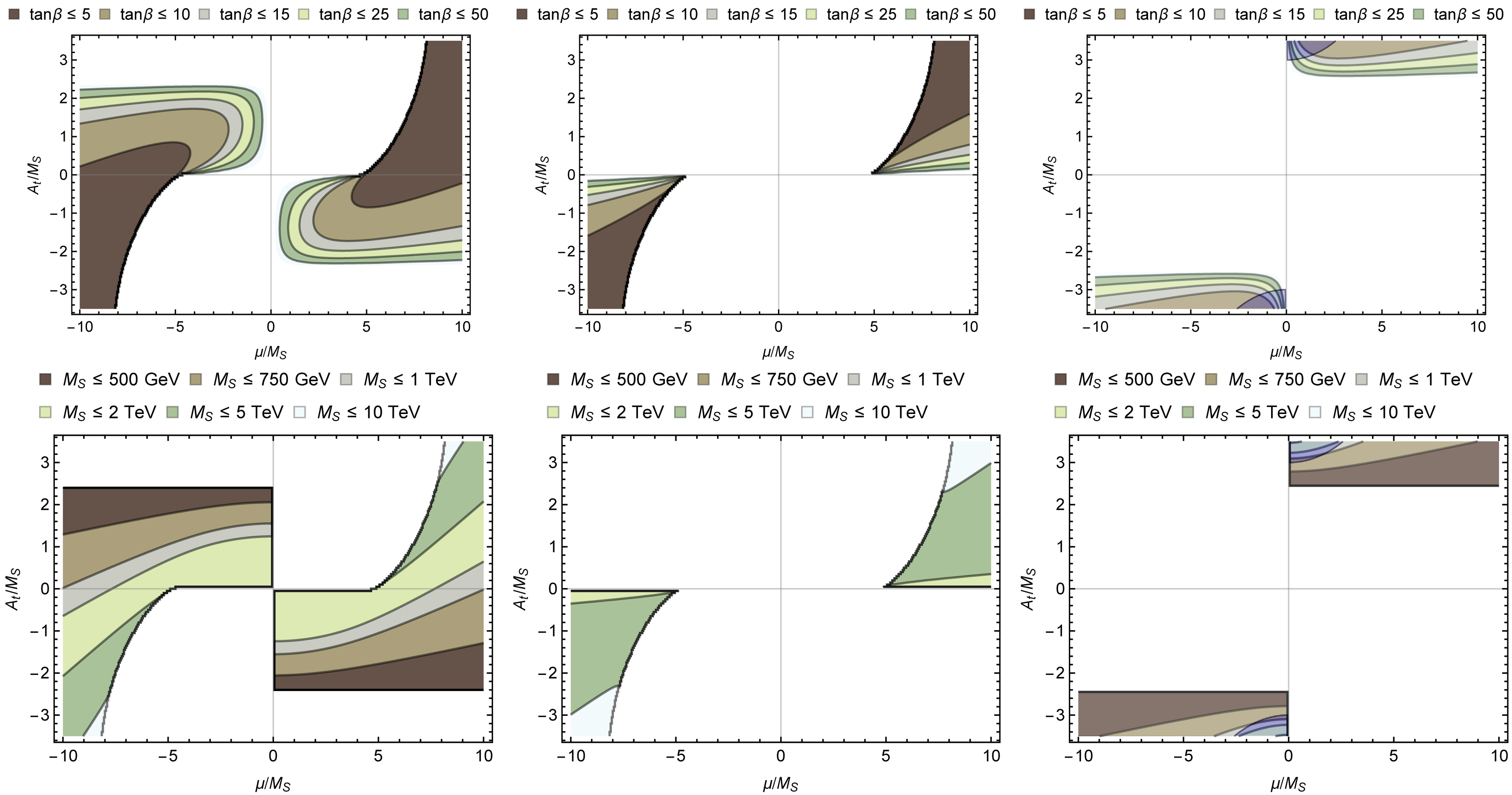
Note that $m_h^2 \simeq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for M_S and X_t. Exact alignment (i.e., $Z_6 = 0$) can now be achieved due to an accidental cancellation between tree-level and loop contributions,

$$m_Z^2 c_{2\beta} = \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12 M_S^4}\right].$$

That is, $Z_6 \simeq 0$ for particular choices of $\tan\beta$. The alignment condition is then achieved by (numerically) solving a 7th order polynomial equation for positive real solutions of $t_\beta \equiv \tan\beta$ (where $\widehat{A}_t \equiv A_t/M_S$ and $\widehat{\mu} \equiv \mu/M_S$),

$$m_Z^2 t_\beta^4(1-t_\beta^2) - Z_1 v^2 t_\beta^4(1+t_\beta^2) + \frac{3m_t^4 \widehat{\mu}(\widehat{A}_t t_\beta - \widehat{\mu})(1+t_\beta^2)^2}{4\pi^2 v^2}\left[\tfrac{1}{6}(\widehat{A}_t t_\beta - \widehat{\mu})^2 - t_\beta^2\right] = 0\,.$$

REMARK: Typically, we identify h as the SM-like Higgs boson. However, in the alignment limit there exist parameter regimes, corresponding to the case of $m_A^2 + (Z_5 - Z_1)v^2 < 0$ (where the radiatively corrected Z_1 and Z_5 are employed), in which H is the SM-like Higgs boson. In either case, $Z_1 v^2$ is the (approximate) squared mass of the SM-like Higgs boson.



Top panels: Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane, in the one-loop approximation. Z_1 is adjusted to give the correct Higgs mass. Taking the three top panels together, one can immediately discern the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

Bottom panels: Contours of the top squark mass parameter M_S, which depends on the values of μ/M_S and A_t/M_S, needed to obtain the correct Higgs squared-mass in the alignment limit, $Z_1 v^2 = 125$ GeV. The three figures correspond to the three $\tan\beta$ solutions of exact alignment previously exhibited.

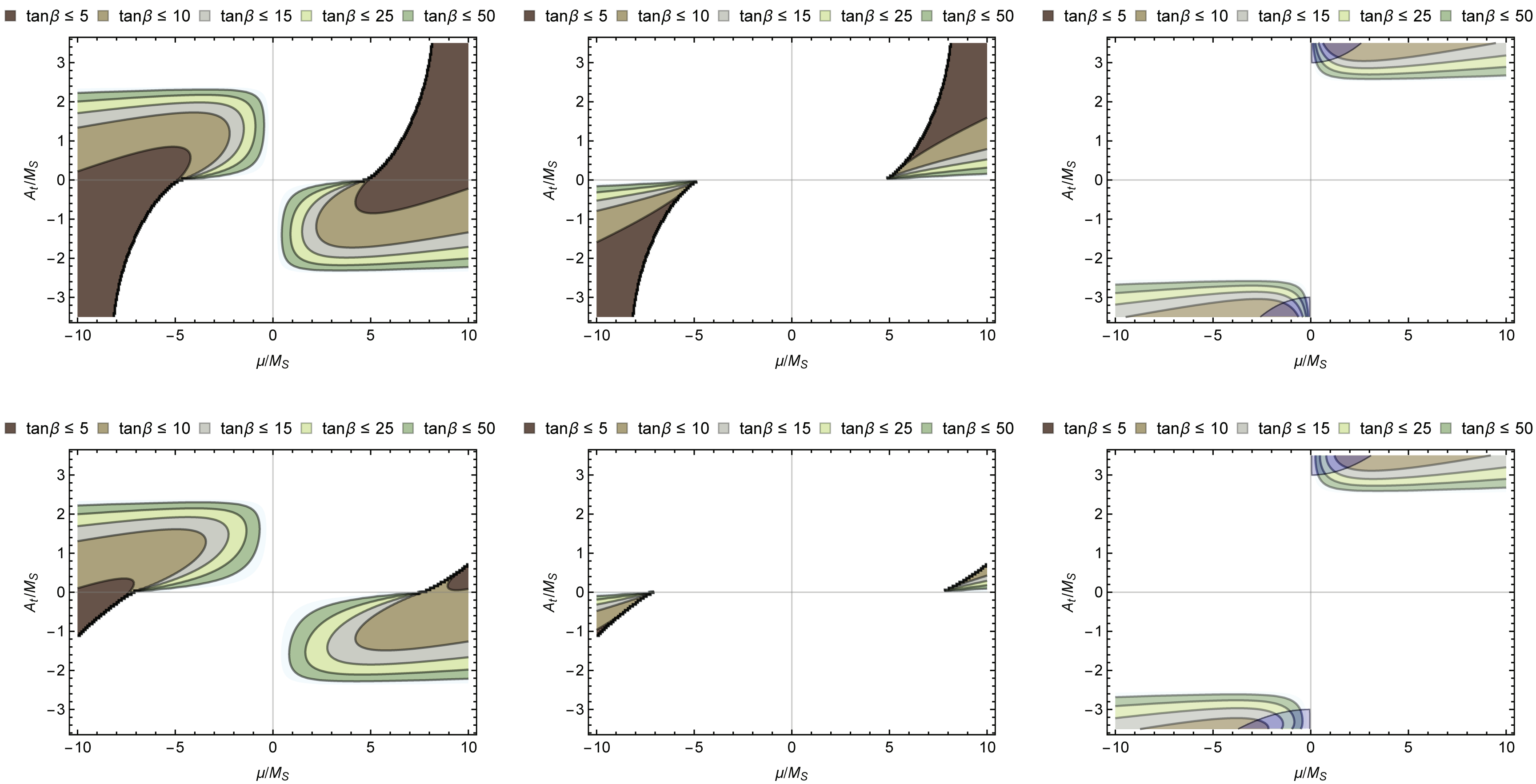
Leading two-loop corrections of $\mathcal{O}(\alpha_s h_t^2)$

Leading two-loop corrections of $\mathcal{O}(\alpha_s h_t^2)$ can be obtained from the leading one-loop corrected results by replacing h_t with $h_t(\lambda)$, where $\lambda \equiv \left[m_t(m_t)M_S\right]^{1/2}$ in the one-loop leading log pieces and $\lambda \equiv M_S$ in the leading threshold corrections. Imposing $Z_6 = 0$ now leads to a 11th order polynomial equation in t_β that can be solved numerically.[§]

In the region of interest in the $(\mu/M_S, A_t/M_S)$ plane, we find that the previous one-loop real $\tan\beta$ solutions are still present (appropriately perturbed at the two-loop level). In addition, another real $\tan\beta$ solution emerges with $|X_t/M_S| \gtrsim 3$, and is therefore discarded.

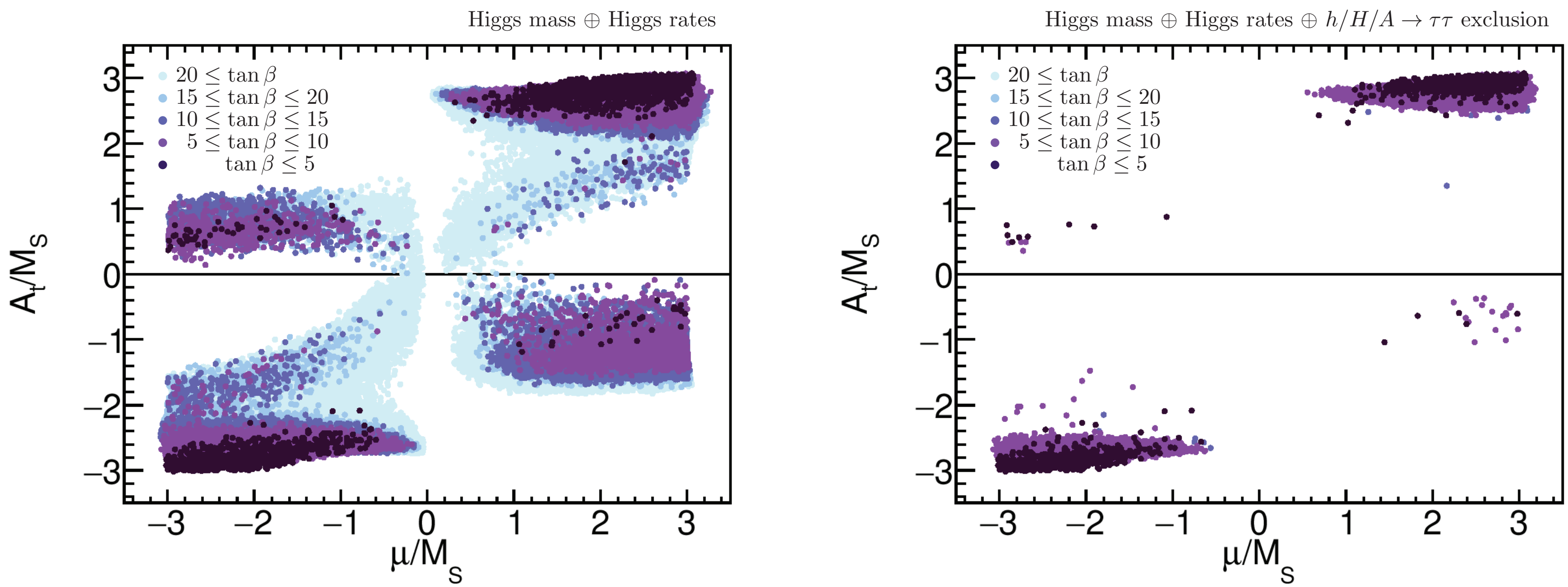
[§] P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638 [hep-ph], and in preparation.

Comparing the one-loop results for $\tan\beta$ solutions at exact alignment (top panels) to the corresponding two-loop improved results (bottom panels).



Contours of $\tan\beta$ corresponding to exact alignment, $Z_6 = 0$, in the $(\mu/M_S, A_t/M_S)$ plane. Z_1 is adjusted to give the correct Higgs mass. Top panels: Approximate one-loop result. Bottom panels: Two-loop improved result. Taking the top (bottom) three panels together, one can immediately discern the regions of zero, one, two and three values of $\tan\beta$ in which exact alignment is realized. In the overlaid blue regions we have (unstable) values of $|X_t/M_S| \geq 3$.

How well do the approximate two-loop results for the exact alignment limit[¶] match a comprehensive scan over the MSSM parameter space? In a recent paper,[‖] an 8-parameter pMSSM scan was performed to determine allowed parameter regimes which contain a light CP-odd Higgs boson A. Typically, h is SM-like, although one cannot yet rule out the possibility of a SM-like H.

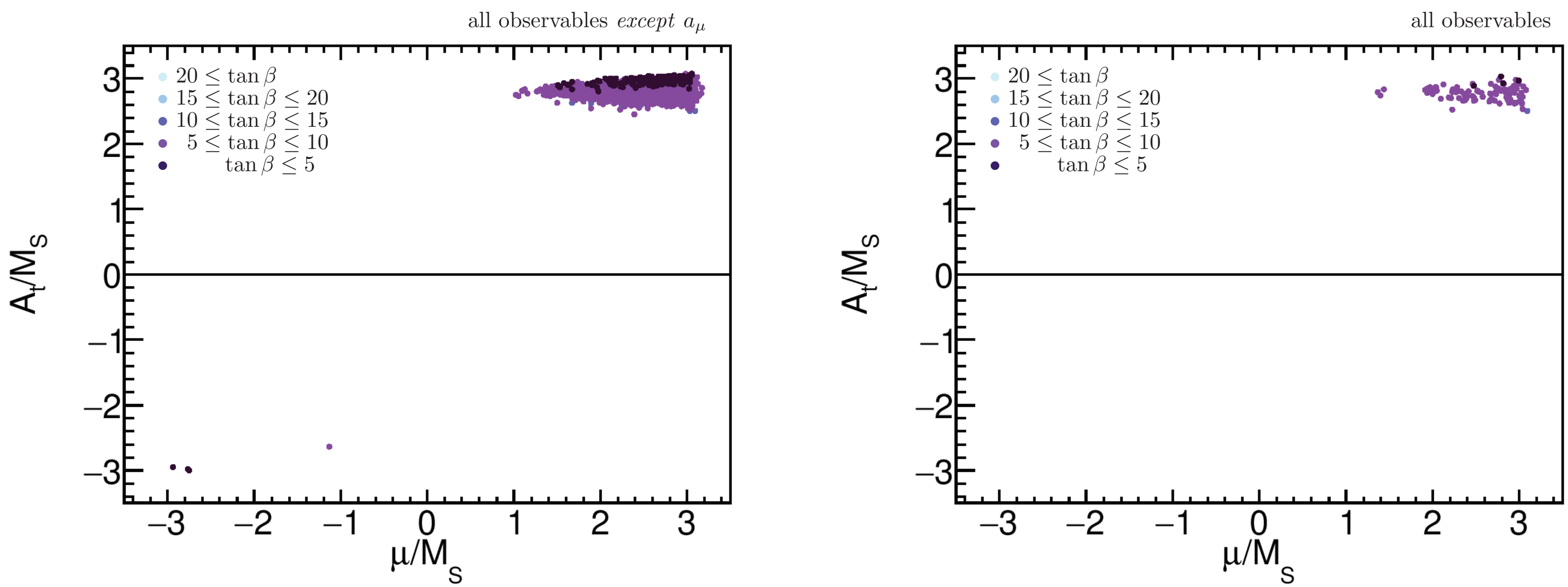


Preferred points of the pMSSM-8 scan with low $m_A \le 350$ GeV for different selections of observables. The points are within the (approximate) 95% CL region, based on the following observables. Left panel: only Higgs mass and signal rates; Right panel: Higgs mass, signal rates and $h/H/A \to \tau^+\tau^-$ exclusion likelihood.

[¶]Of course, the precision Higgs data only requires that the condition of alignment is approximately satisfied.

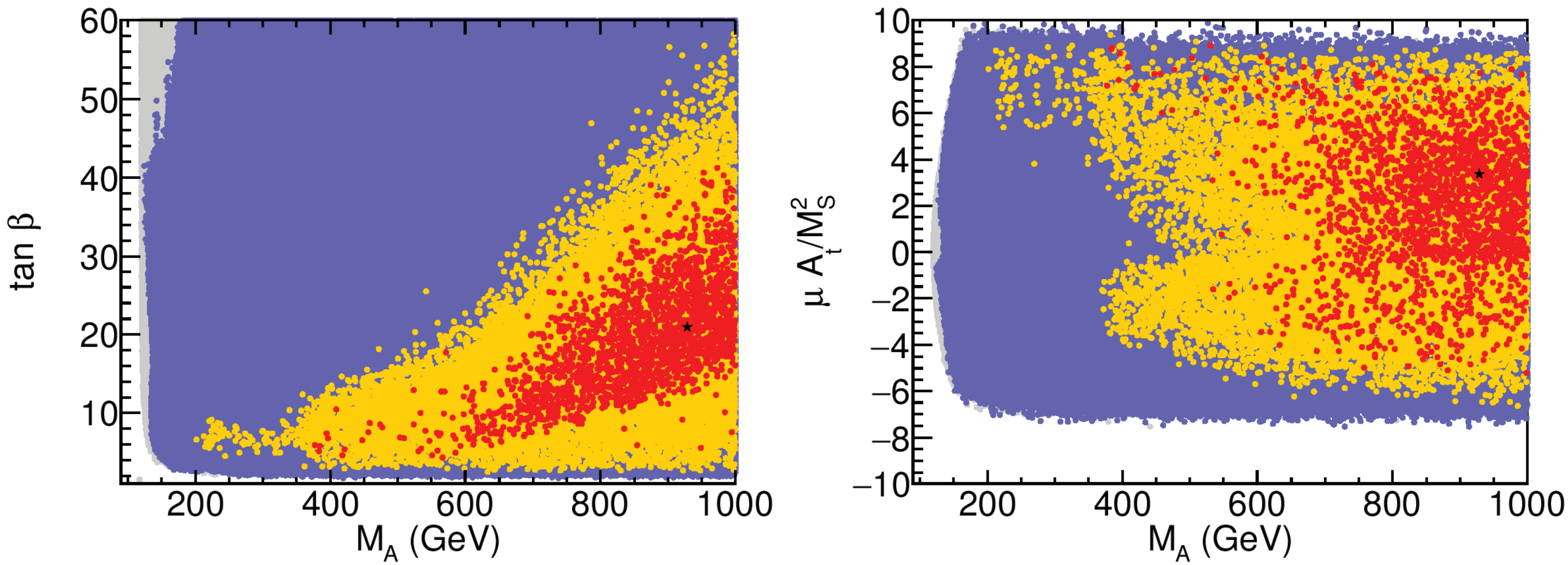
[‖]P. Bechtle, H.E. Haber, S. Heinemeyer, O. Stål, T. Stefaniak, G. Weiglein and L. Zeune, arXiv:1608.00638.

Including additional constraints from SUSY particle searches and the impact of SUSY radiative corrections on SM observables, the allowed parameter regions of the pMSSM-8 scan shrinks further. For example, results from the SuperIso program show that the negative μ region is mostly disfavored by BR($B \to X_s\gamma$), whereas negative A_t is disfavored by BR($B_s \to \mu^+\mu^-$).



Preferred points of the pMSSM-8 scan with low $m_A \leq 350$ GeV for all observables except a_μ (left panel), and for all observables (right panel).

Bottom line: m_A values as low as 200 GeV are still allowed in the MSSM.



Preferred parameter regions in the $(M_A,\ \tan\beta)$ plane (left) and $(M_A,\ \mu A_t/M_S^2)$ plane (right), where $M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$ and h is the SM-like Higgs boson, in a pMSSM-8 scan. Points that do not pass the direct constraints from Higgs searches from `HiggsBounds` and from LHC SUSY particle searches from `CheckMATE` are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta\chi_h^2 < 2.3$, yellow for $\Delta\chi_h^2 < 5.99$ and blue otherwise. The best fit point is indicated by a black star.

Beyond singlets and doublets

If one considers a scalar sector with triplet Higgs fields, then one must include addition Higgs multiplets in such a way that $\rho \simeq 1$.

Georgi and Machacek constructed an amusing model in which $\rho = 1$ at tree-level due to a well chosen scalar potential that respects custodial symmetry. The model contains a complex $Y = 1$ doublet, a complex $Y = 2$ triplet and a real $Y = 0$ singlet. Without going into details, there is a doublet vev, v_ϕ, and a common triplet vev, v_χ, with $v^2 \equiv v_\phi^2 + 8v_\chi^2 = (246~\text{GeV})^2$.

The physical scalars make up custodial SU(2) multiplets: a 5-plet of states ($H_5^{\pm\pm}$, $H_5^{\pm}$ and H_5^0) with common mass m_5, a triplet ($H_3^{\pm}$, H_3^0) with common mass m_3, and custodial singlets that mix with squared-mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} Z_{11} v_\phi^2 & v_\phi v_\chi (Z_{12} - 2\sqrt{3}\, m_3^2/v^2) \\ v_\phi v_\chi (Z_{12} - 2\sqrt{3}\, m_3^2/v^2) & \frac{3}{2} m_3^2 - \frac{1}{2} m_5^2 + v_\chi^2 (Z_{22} - 12 m_3^2/v^2) \end{pmatrix},$$

where the Z_{ij} depend on dimensionless quartic couplings.

The custodial singlet CP-even Higgs bosons are h and H with $m_h \leq m_H$. An approximate alignment limit can be realized in two different ways.

1. In the *decoupling limit*, h is SM-like and $m_H \simeq m_3 \simeq m_5 \gg m_h$.**

2. $v_\chi \ll v$. Then h is SM-like if $Z_{11}v^2 < \frac{3}{2}m_3^2 - \frac{1}{2}m_5^2$. Otherwise, H is SM-like.

Remark: Implications of a modified unitarity sum rule

In the Georgi-Machacek model, the existence of doubly-charged Higgs bosons implies that

$$\sum_i g^2_{h_i W^+W^-} = g^2 m_W^2 + \sum_k |g_{\phi_k^{++}W^-W^-}|^2 \,,$$

where the sum is taken over all CP-even Higgs bosons of the model. The presence on the last term on the right hand side above means that individual h_iVV couplings can exceed the corresponding coupling of the SM.

For details, see K. Hartling, K. Kumar, and H.E. Logan, Phys. Rev. **D90, 015007 (2014).

It is convenient to write $c_H \equiv \cos\theta_H = v_\phi/(v_\phi^2 + 8v_\chi^2)^{1/2}$, and $s_H \equiv \sin\theta_H$. Then, the following couplings are noteworthy:

$$H_1^0 W^+W^- : \quad g c_H m_W \,, \qquad\qquad H_1^{\prime\,0} W^+W^- : \quad \sqrt{8/3}\, g m_W s_H \,,$$
$$H_5^0 W^+W^- : \quad \sqrt{1/3}\, g m_W s_H \,, \qquad H_5^{++} W^-W^- : \quad \sqrt{2}\, g m_W s_H \,,$$

where H_1^0 and $H_1^{\prime\,0}$ are the custodial singlet interaction eigenstates. Note that $H_1^{\prime\,0}$ and H_5^0, H_5^{++} have no coupling to fermions, whereas

$$H_1^0 f\bar{f} : \quad \frac{g m_q}{2 m_W c_H} \,.$$

In the absence of H_1^0–$H_1^{\prime\,0}$ mixing, $c_H = 1$ corresponds to the alignment limit. But consider the strange case of $s_H = \sqrt{3/8}$. In this case, the $H_1^{\prime\,0}$ coupling to W^+W^- matches that of the SM. Nevertheless, this does not saturate the HWW sum rule! Moreover, it is possible that the $H_1^{\prime\,0}W^+W^-$ coupling is *larger* than gm_W, without violating the HWW sum rule. Including H_1^0–$H_1^{\prime\,0}$ mixing allows for even more baroque possibilities not possible in a multi-doublet extension of the SM.

Conclusions

Pursuing Higgs physics into the future by theorists and experimentalists is likely to lead to profound insights into the fundamental theory of particles and their interactions.

Backup Slides

The alignment limit in the general 2HDM

The neutral Higgs mass-eigenstates, denoted by $\{h_1, h_2, h_3\}$, are linear combinations of $\{\sqrt{2}\,\mathrm{Re}\,H_1^0 - v\,,\ \sqrt{2}\,\mathrm{Re}\,H_2^0\,,\ \sqrt{2}\,\mathrm{Im}\,H_2^0\}$, and are determined by diagonalizing the 3×3 real symmetric squared-mass matrix,

$$\mathcal{M}^2 = v^2 \begin{pmatrix} Z_1 & \mathrm{Re}(Z_6) & -\mathrm{Im}(Z_6) \\ \mathrm{Re}(Z_6) & \frac{1}{2}Z_{345} + Y_2/v^2 & -\frac{1}{2}\mathrm{Im}(Z_5) \\ -\mathrm{Im}(Z_6) & -\frac{1}{2}\mathrm{Im}(Z_5) & \frac{1}{2}Z_{345} - \mathrm{Re}(Z_5) + Y_2/v^2 \end{pmatrix},$$

where $Z_{345} \equiv Z_3 + Z_4 + \mathrm{Re}(Z_5)$. The diagonalizing matrix is a 3×3 real orthogonal matrix that depends on three angles: θ_{12}, θ_{13} and θ_{23}, such that θ_{12} and θ_{13} are invariant whereas $\theta_{23} \to \theta_{23} - \chi$ under the rephasing of H_2.*

The couplings of $\sqrt{2}\,\mathrm{Re}\,H_1^0 - v$ coincide with those of the SM Higgs boson.

Thus, the alignment limit corresponds to two limiting cases:

1. $Y_2 \gg v^2$, corresponding to the decoupling limit.
2. $|Z_6| \ll 1$, corresponding to alignment with or without decoupling.

We identify the SM-like Higgs boson, $h_1 \simeq \sqrt{2}\,\mathrm{Re}\,H_1^0 - v$, with $m_h^2 \simeq Z_1 v^2$.

*See H.E. Haber and D. O'Neil, Phys. Rev. **D74**, 015018 (2006) [Erratum: ibid., **D74**, 059905 (2006)].

The alignment limit of the general 2HDM in equations

To obtain the conditions in which h_1 is the SM-like Higgs boson, noting that:

$$\frac{g_{h_1VV}}{g_{h_{\rm SM}VV}} = c_{12}c_{13}\,, \qquad \text{where } V = W \text{ or } Z\,,$$

where $h_{\rm SM}$ is the SM Higgs boson, we demand that

$$s_{12}\,,\; s_{13} \ll 1\,.$$

Here, $s_{12} \equiv \sin\theta_{12}$, $c_{12} \equiv \cos\theta_{12}$, etc. We denote the masses of the neutral Higgs mass eigenstates by m_1, m_2 and m_3. It follows that:

$$Z_1 v^2 = m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2\,,$$

$$\mathrm{Re}(Z_6\, e^{-i\theta_{23}})\, v^2 = c_{13}s_{12}c_{12}(m_2^2 - m_1^2)\,,$$

$$\mathrm{Im}(Z_6\, e^{-i\theta_{23}})\, v^2 = s_{13}c_{13}(c_{12}^2 m_1^2 + s_{12}^2 m_2^2 - m_3^2)\,,$$

$$\mathrm{Re}(Z_5\, e^{-2i\theta_{23}})\, v^2 = m_1^2(s_{12}^2 - c_{12}^2 s_{13}^2) + m_2^2(c_{12}^2 - s_{12}^2 s_{13}^2) - m_3^2 c_{13}^2\,,$$

$$\mathrm{Im}(Z_5\, e^{-2i\theta_{23}})\, v^2 = 2s_{12}c_{12}s_{13}(m_2^2 - m_1^2)\,.$$

Assuming no mass degeneracies in the neutral scalar sector, it then follows that in the alignment limit,

$$\boxed{\begin{aligned} s_{12} \equiv \sin\theta_{12} &\simeq \frac{\mathrm{Re}(Z_6 e^{-i\theta_{23}})v^2}{m_2^2 - m_1^2} \ll 1\,, \\ s_{13} \equiv \sin\theta_{13} &\simeq -\frac{\mathrm{Im}(Z_6 e^{-i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,, \end{aligned}}$$

One additional small quantity characterizes the alignment limit,

$$\mathrm{Im}(Z_5 e^{-2i\theta_{23}}) \simeq \frac{(m_2^2 - m_1^2)s_{12}s_{13}}{v^2} \simeq -\frac{2\,\mathrm{Im}(Z_6^2 e^{-2i\theta_{23}})v^2}{m_3^2 - m_1^2} \ll 1\,.$$

Finally, the following mass relations in the alignment limit are noteworthy,

$$m_1^2 \simeq Z_1 v^2\,,$$

$$m_2^2 - m_3^2 \simeq \mathrm{Re}(Z_5 e^{-2i\theta_{23}})v^2\,.$$

A symmetry origin for alignment without decoupling

For simplicity, we examine the CP-conserving 2HDM, for which one can rephase the Higgs basis field H_2 such that Z_5, Z_6 and Z_7 are real. Given a scalar potential in the Φ_1–Φ_2 basis, one can derive

$$Z_6 = -\tfrac{1}{2}\big[\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta}\big] s_{2\beta} + \lambda_6 c_\beta c_{3\beta} + \lambda_7 s_\beta s_{.}eps3\beta\,,$$

$$Z_7 = -\tfrac{1}{2}\big[\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta}\big] s_{2\beta} + \lambda_6 s_\beta s_{3\beta} + \lambda_7 c_\beta c_{3\beta}\,.$$

If the alignment condition $Z_6 = 0$ holds independently of $\tan\beta$, then it follows that[†]

$$\lambda_1 = \lambda_2 = \lambda_{345}\,, \qquad \lambda_6 = \lambda_7 = 0\,.$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. The above *natural alignment condition* can be achieved by imposing a particular Higgs flavor or generalized CP symmetry.

Note that the natural alignment condition also sets $Z_7 = 0$. Indeed, if the natural alignment condition holds in one basis, then it holds in any basis.

[†]See P.S. Bhupal Dev and A. Pilaftsis, JHEP **1412**, 024 (2014) 024 [Erratum: ibid. **1511**, 147 (2015)].

The natural alignment condition can be relaxed. It is sufficient to impose a discrete $\mathbb{Z}_2$ symmetry where the Higgs basis field H_1 is unchanged but $H_2 \to -H_2$. It then follows that

$$Y_3 = Z_6 = Z_7 = 0\,.$$

Note that the minimum condition $Y_3 = -\frac{1}{2}Z_6 v^2$ requires that $Y_3 = 0$ if $Z_6 = 0$, so this $\mathbb{Z}_2$ symmetry *cannot* be softly broken.

No conditions are imposed on $Z_1, \ldots, Z_5$. The natural alignment condition is a special case where $Z_1 = Z_2 = Z_{345}$.

Having imposed the above $\mathbb{Z}_2$ symmetry in the bosonic sector of the theory, we can extend it to the Yukawa interactions. If we demand that all fermions are even under the $\mathbb{Z}_2$ symmetry, then the H_1 couplings to fermions are those of the SM Higgs boson and the Yukawa couplings of H_2 to the fermions are absent. This is the inert doublet model (IDM).

Further details on the IDM

By imposing the discrete $\mathbb{Z}_2$ symmetry, the scalar potential is CP-conserving. The SM Higgs state is $h = \sqrt{2}\,\mathrm{Re}\,H_1^0 - v$. The inert doublet is

$$H_2 = \begin{pmatrix} H^+ \\ (H + iA)/\sqrt{2} \end{pmatrix} ,$$

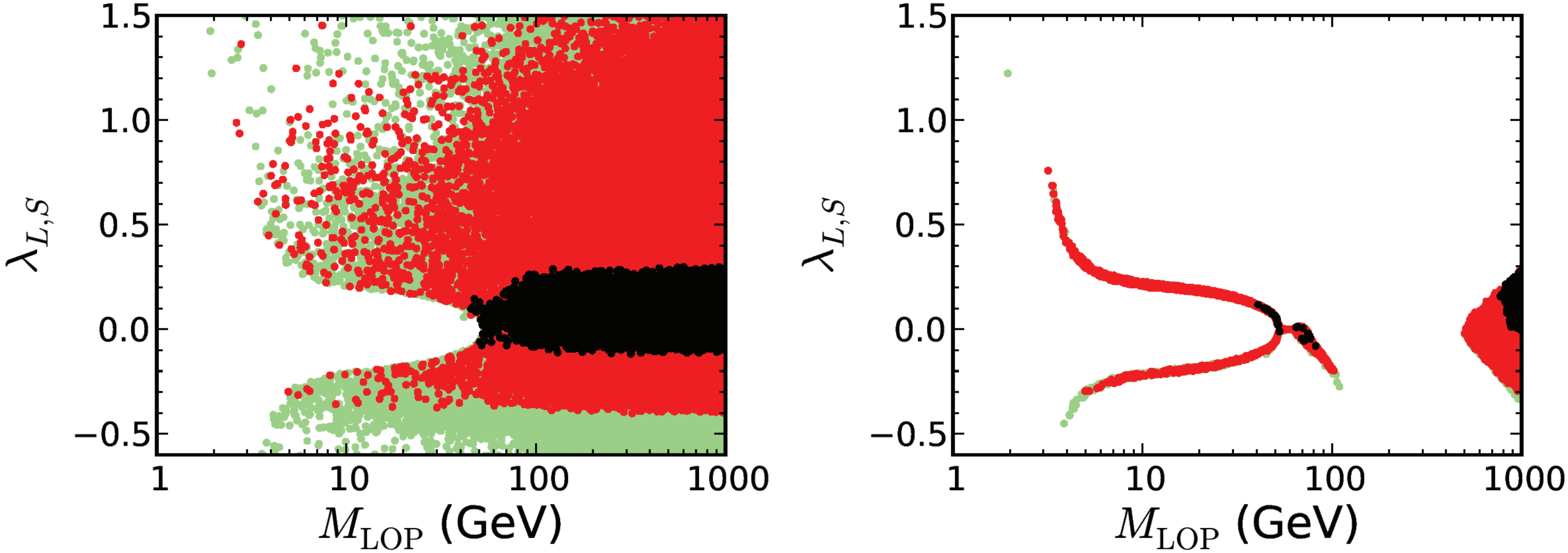
where the mass eigenstates consist of two neutral scalars, H, A and a charged Higgs pair. The physical Higgs masses are

$$m_h^2 = Z_1 v^2 \,, \qquad m_{H^\pm}^2 = Y_2 + \tfrac{1}{2} Z_3 v^2 \,,$$
$$m_{H,A}^2 = m_{H^\pm}^2 + \tfrac{1}{2}(Z_4 \pm |Z_5|) v^2 \,.$$

H and A have opposite CP-quantum numbers, but there is no interaction that can determine separate CP quantum number for these states. The lighter of these two states will henceforth be denoted as H_L.

The lightest $\mathbb{Z}_2$–odd particle (LOP) is stable. If $Z_4 < |Z_5|$ (in which case H_L is lighter than $H^\pm$), then the LOP is a neutral scalar.

The LOP is a candidate for dark matter. Including the exclusion limits from the current dark matter direct detection experiments, a cosmologically relevant LOP is ruled out by Goudelis, Herrmann and Stål for all LOP masses below 500 GeV except for a narrow window around $\frac{1}{2}m_h$.



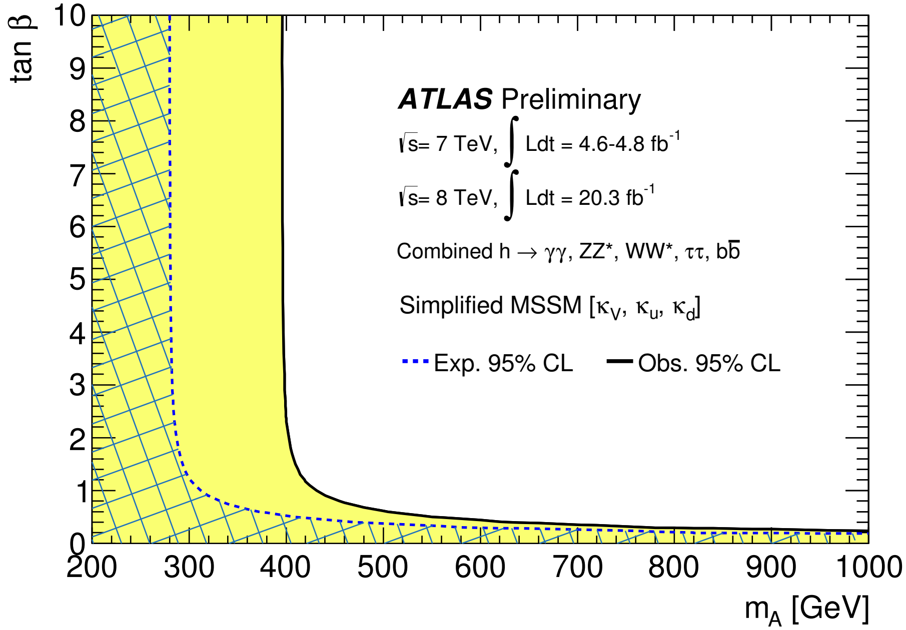
The viable IDM parameter space projected on the $(M_{\rm LOP}\,,\,\lambda_{L,S})$ plane imposing only the upper limit (left) and the upper and lower limits (right) of the WMAP range, $0.1018 \leq M_{\rm LOP}h^2 \leq 0.1234$. The green points correspond to all valid points in the scan, while the red and black regions show the points which remain valid when the model satisfies stability and perturbativity up to a scale $\Lambda = 10^4$ GeV and the GUT scale $\Lambda = 10^{16}$ GeV, respectively. Above, $\lambda_{L,S} \equiv \frac{1}{2}(Z_3 + Z_4 \mp |Z_5|)$; when multiplied by v the latter corresponds to the hH_LH_L coupling. Taken from A. Goudelis, B. Herrmann and O. Stål, JHEP **1309** (2013) 106.

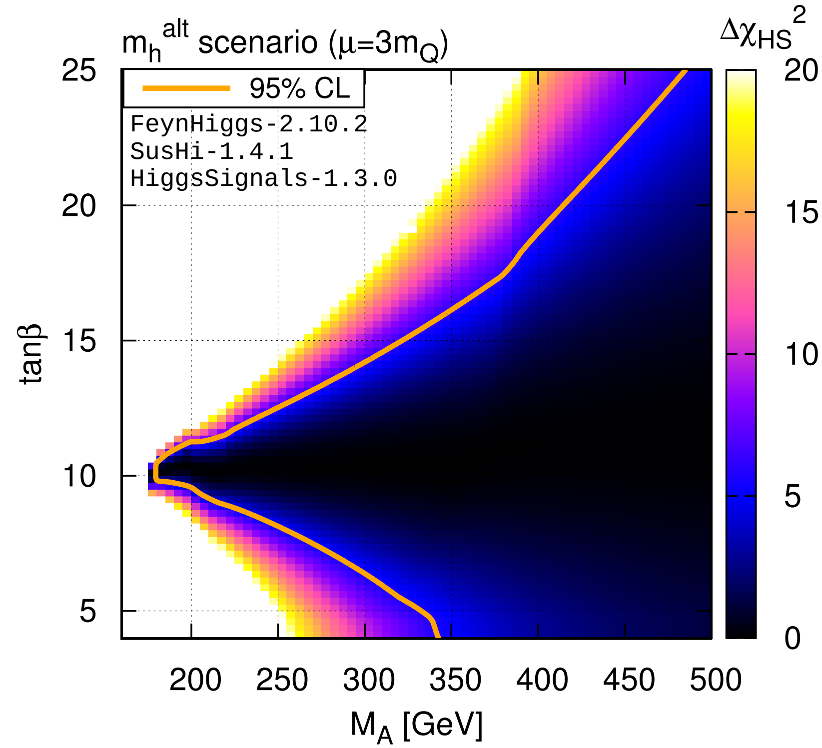
The MSSM Higgs sector in light of precision Higgs data

The observed Higgs boson at 125 GeV is SM-like (to within roughly an accuracy of 20%). The common wisdom is that this observation implies that additional Higgs states of the MSSM Higgs sector must be rather heavy (corresponding to the decoupling limit).

Indeed, ATLAS has claimed to rule out $m_A \lesssim 400$ GeV based on Run 1 precision Higgs data. But, one needs to be careful about the underlying assumptions...

For example, in the so called MSSM $m_h^{\rm alt}$ benchmark scenario introduced in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **91**, 035003 (2015), the Run 1 precision Higgs data places virtually no bound on m_A if $\tan\beta \sim 10$. This is a consequence of the alignment limit without decoupling, which is achieved in the $m_h^{\rm alt}$ benchmark scenario when $\tan\beta \simeq 10$.

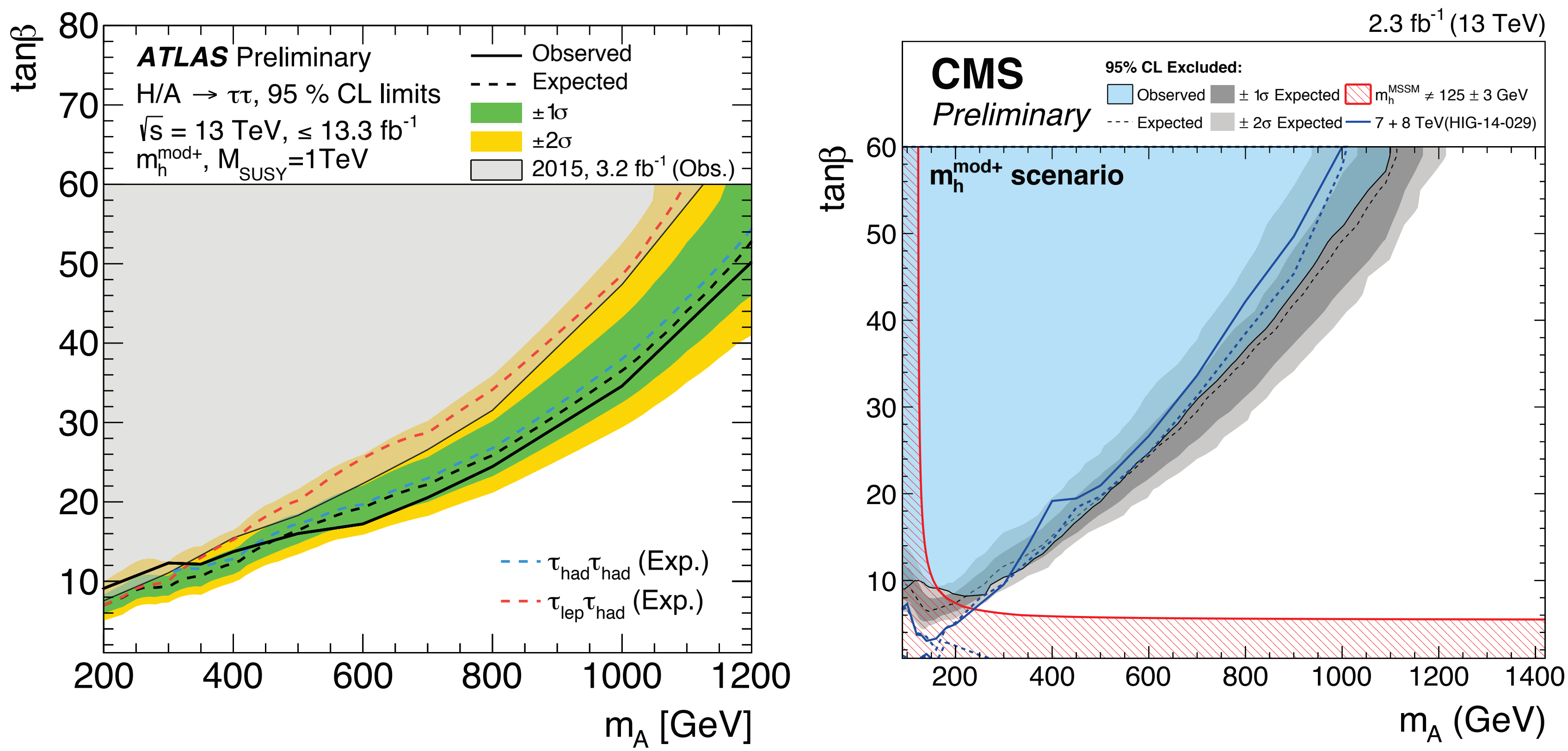




Left panel: Regions of the $(m_A, \tan\beta)$ plane excluded in a simplified MSSM model via fits to the measured rates of the production and decays of the SM-like Higgs boson h. Taken from ATLAS-CONF-2014-010.

Right panel: Likelihood distribution, $\Delta\chi^2_{\rm HS}$ obtained from testing the signal rates of‘h against a combination of Higgs rate measurements from the Tevatron and LHC experiments, obtained with `HiggsSignals`, in the alignment benchmark scenario of Carena et al. (op. cit.). From P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, EPJC **75**, 421 (2015).

Direct searches for the additional Higgs states also suggest that these states must be heavy, although the sensitivity of these searches are limited if $\tan\beta \lesssim 10$.



The observed and expected 95% CL limits on $\tan\beta$ as a function of m_A in the MSSM $m_h^{\text{mod+}}$ benchmark scenario. Left panel: ATLAS results taken from ATLAS-CONF-2016-085. Right panel: CMS results taken from CMS-PAS-HIG-16-006.

Adding a Higgs singlet to the 2HDM

Consider a Higgs sector that consists of two hypercharge-one complex doublet and a complex neutral singlet S. We can define the doublet fields of the Higgs basis, H_1 and H_2 as before. The relevant scalar potential is more complicated than that of the 2HDM. Here we focus on the terms that are relevant for the scalar squared-mass matrices.

$$
\begin{aligned}
\mathcal{V} \ni \ldots &+ \tfrac{1}{2}Z_1(H_1^\dagger H_1)^2 + \ldots + \left[\tfrac{1}{2}Z_5(H_1^\dagger H_2)^2 + Z_6(H_1^\dagger H_1)H_1^\dagger H_2 + \text{h.c.}\right] + \ldots \\
&+ S^\dagger S\left[Z_{s1}H_1^\dagger H_1 + \ldots + (Z_{s3}H_1^\dagger H_2 + \text{h.c.}) + Z_{s4}S^\dagger S\right] \\
&+ \Big\{Z_{s5}H_1^\dagger H_1 S^2 + \ldots + Z_{s7}H_1^\dagger H_2 S^2 + Z_{s8}H_2^\dagger H_1 S^2 + Z_{s9}S^\dagger S\, S^2 + Z_{s10}S^4 + \text{h.c.}\Big\} \\
&+ \left[C_1 H_1^\dagger H_1 S + \ldots + C_3 H_1^\dagger H_2 S + C_4 H_2^\dagger H_1 S + C_5(S^\dagger S)S + C_6 S^3 + \text{h.c.}\right].
\end{aligned}
$$

For simplicity, we shall assume that the scalar potential is CP-invariant. We then write the squared-mass matrix of the CP-even Higgs bosons with respect to the basis $\{\sqrt{2}\,\text{Re}\,H_1^0 - v\,,\ \sqrt{2}\,\text{Re}\,H_2^0\,,\ \sqrt{2}\,(\text{Re}\,S - v_s)\}$.

The squared-mass matrix for the CP-even scalars is a real symmetric matrix,

$$\mathcal{M}_S^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 & \sqrt{2}\, v\big[C_1 + (Z_{s1} + 2Z_{s5})v_s\big] \\ & \overline{M}_A^2 + Z_5 v^2 & \dfrac{v}{\sqrt{2}}\Big[C_3 + C_4 + 2(Z_{s3} + Z_{s7} + Z_{s8})v_s\Big] \\ & & -C_1 \dfrac{v^2}{2v_s} + 3(C_5 + C_6)v_s + 4(Z_{s4} + 2Z_{s9} + 2Z_{s10})v_s^2 \end{pmatrix},$$

where $\overline{M}_A^2$ is the 11 element of the CP-odd squared-mass matrix with respect to the basis $\{\sqrt{2}\,\mathrm{Im}\ H_2^0,\ \sqrt{2}\,\mathrm{Im}\ S\}$.

Exact alignment occurs when $(\mathcal{M}_S^2)_{12} = (\mathcal{M}_S^2)_{13} = 0$. That is,

$$Z_6 = 0\,, \qquad\qquad C_1 + (Z_{s1} + 2Z_{s5})v_s = 0\,.$$

The decoupling limit corresponds to $\overline{M}_A \gg v$ and $v_s \gg v$ and yields approximate alignment.

Approximate alignment can also be achieved with a combination of a subset of the above conditions. For example, $C_1 + (Z_{s1} + 2Z_{s5})v_s \simeq 0$ and $\overline{M}_A \gg v$ [with $Z_6 \sim \mathcal{O}(1)$] yields approximate alignment.

The alignment limit of the Higgs sector of the NMSSM

In the NMSSM, including the leading one-loop radiative corrections,

$$Z_1 v^2 = (m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta}^2 + \tfrac{1}{2}\lambda^2 v^2 + \frac{3v^2 s_\beta^4 h_t^4}{8\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right],$$

$$Z_6 v^2 = -s_{2\beta}\left\{(m_Z^2 - \tfrac{1}{2}\lambda^2 v^2)c_{2\beta} - \frac{3v^2 s_\beta^2 h_t^4}{16\pi^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t(X_t+Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4}\right]\right\}.$$

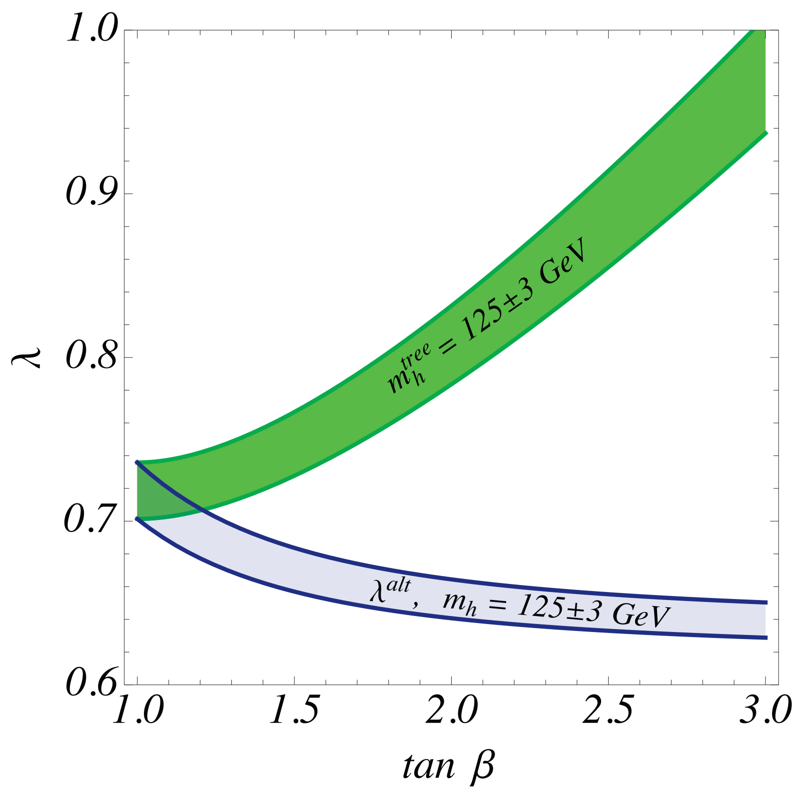
The exact alignment limit requires that $Z_6 = 0$ and $C_1 + (Z_{s1} + 2Z_{s5})v_s = 0$. In the NMSSM, the latter condition yields

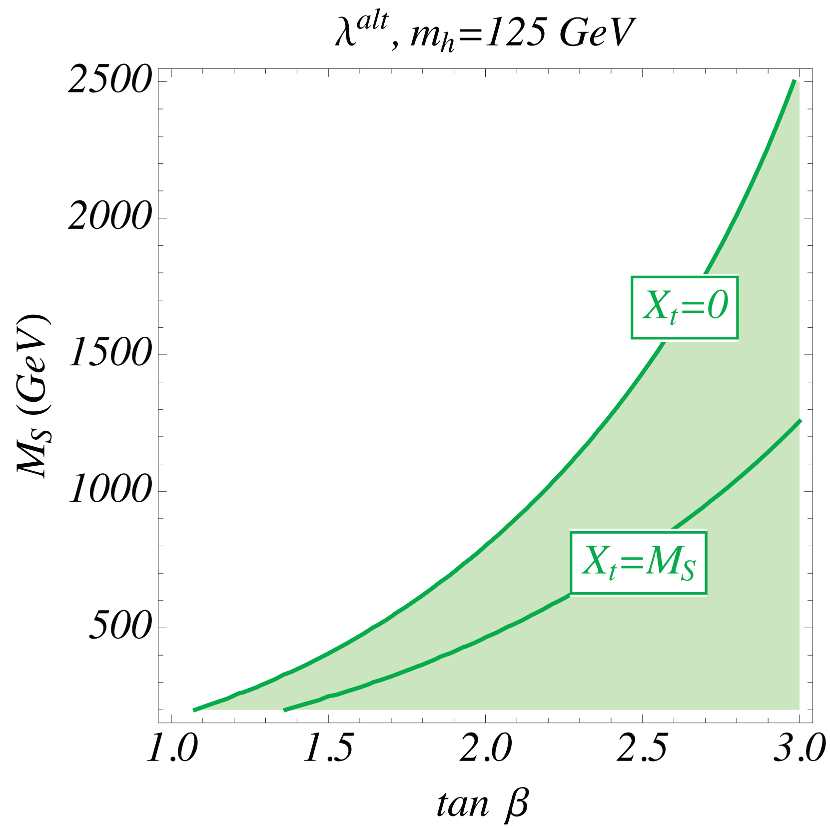
$$\frac{\overline{M}_A^2 s_{2\beta}^2}{4\mu^2} + \frac{\kappa s_{2\beta}}{2\lambda} = 1\,,$$

where $\overline{M}_A^2 \equiv 2\mu(A_\lambda + \kappa v_s)/s_{2\beta}$ and $\mu \equiv \lambda v_s$. Note that κ governs the self-coupling of the singlet scalar field.

In contrast to the MSSM, in the NMSSM one can set $Z_6 = 0$ and obtain $m_h = 125$ GeV, with only small contributions from the one-loop radiative corrections. This leads to a preferred choice of NMSSM parameters,[‡]

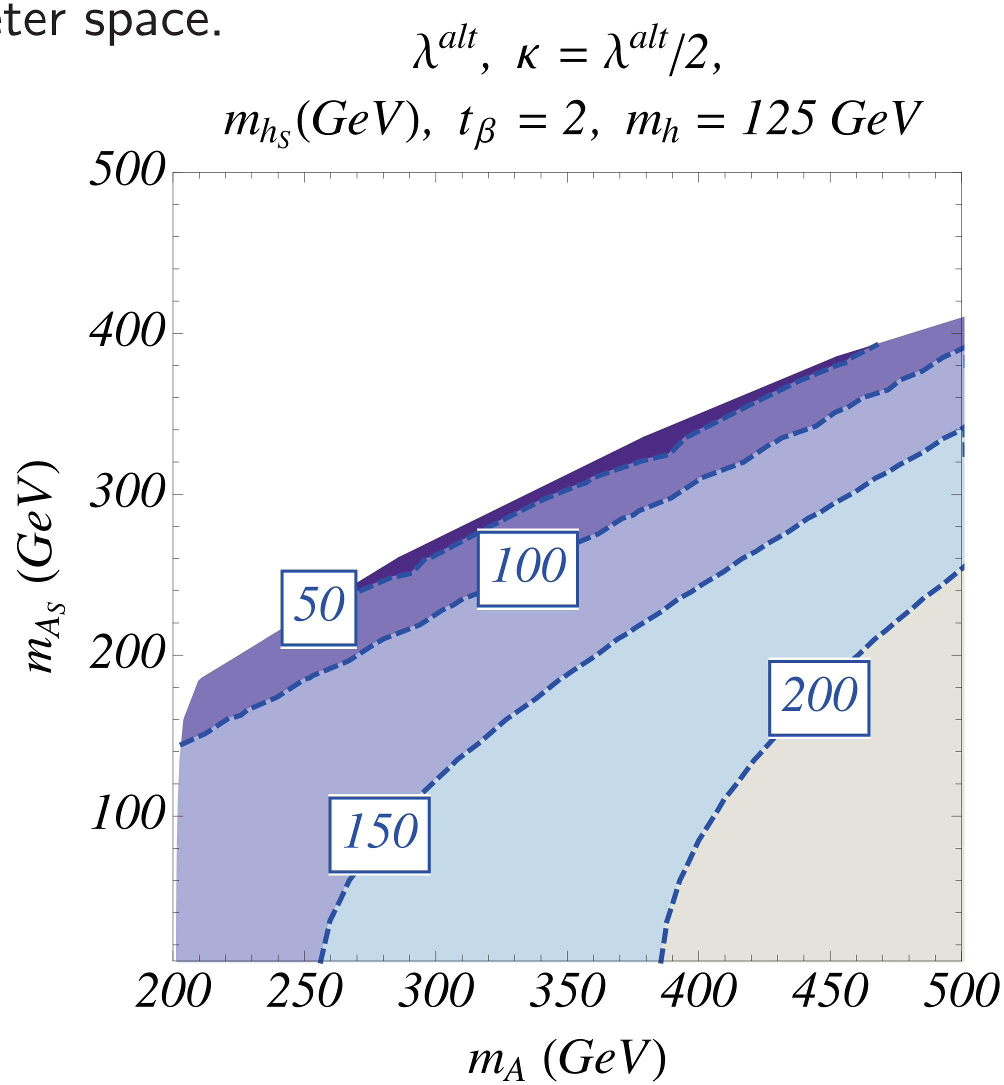
$$\lambda \sim 0.65\,, \qquad\qquad \tan\beta \sim 2\,.$$





[‡]See M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D **93**, 035013 (2016).

The second alignment limit condition leads to further correlations of the NMSSM parameter space.



Near the alignment limit, we have $m_A \simeq m_H \simeq \overline{M}_A$.