

Dynamical coupling of hydrodynamics and transport for heavy ion collisions

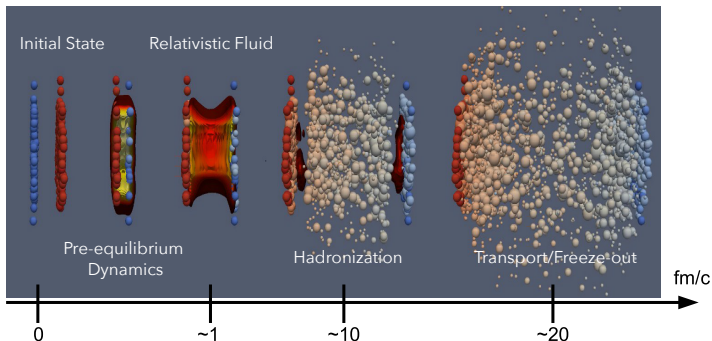
Dmytro Oliinychenko

Frankfurt Institute of Advanced Studies
oliiny@fias.uni-frankfurt.de
in collaboration with Hannah Petersen
[arXiv:1609.01087](https://arxiv.org/abs/1609.01087)



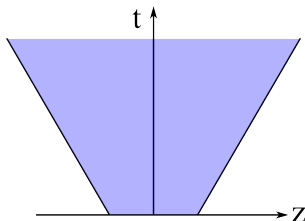
September 13, 2016

Heavy ion collision in the view of hybrid models



- **Hydrodynamics**: local thermal equilibrium,
 $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^\mu = 0$, EoS, boundary conditions
Applicability: $\lambda \simeq (n\sigma)^{-1} \ll L \implies$ **high density**, $Kn = \frac{\lambda}{L} < 1$
- **Transport**: Monte-Carlo simulation of particle collisions
Applicability: negligible multi-particle collisions \implies **low density**,
 $Kn > 1$
- **Hybrid**: hydro at high density + transport at low density

Conventional hybrid models



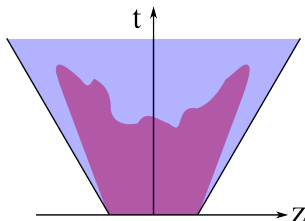
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *a posteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, Huovinen, HP [PRC **91**, 2, 024906 (2015)]

Conventional hybrid models



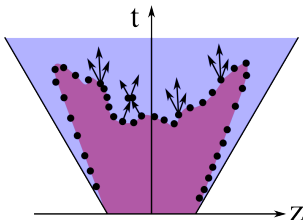
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *a posteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, Huovinen, HP [PRC **91**, 2, 024906 (2015)]

Conventional hybrid models



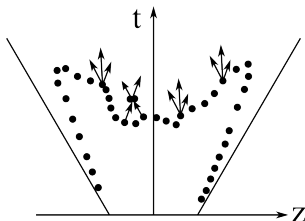
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *a posteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, Huovinen, HP [PRC **91**, 2, 024906 (2015)]

Conventional hybrid models



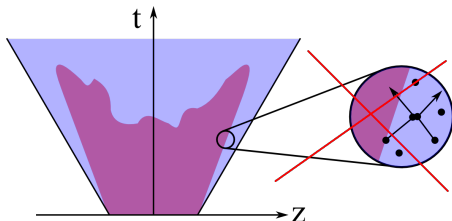
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *a posteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, Huovinen, HP [PRC **91**, 2, 024906 (2015)]

Conventional hybrid models



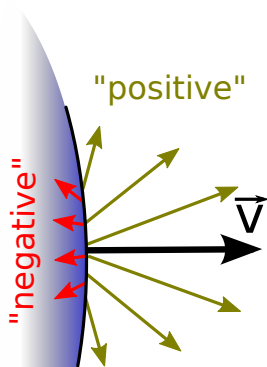
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *a posteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, Huovinen, HP [PRC **91**, 2, 024906 (2015)]

Particlization and negative contributions



$d\sigma_\mu$ - normal 4-vector
 $u_\mu = (\gamma, \gamma \vec{v})$ - 4-velocity
 T - temperature
 μ - chemical potential

- Particlization
 - ▶ know ϵ , p , u_μ on the surface
 - ▶ from EoS - T , μ
 - ▶ want particles

- "Cooper-Frye formula"

$$d^3 N(p) = f(p) \frac{d^3 p}{(2\pi\hbar)^3} \frac{p^\mu}{p^0} d\sigma_\mu$$

$$\frac{p^\mu}{p^0} \cdot d\sigma_\mu - \text{analog of } n \cdot V$$

$$\text{e.g. ideal hydro } f(p) = \left(e^{\frac{p^\mu u_\mu - \mu}{T}} \pm 1 \right)^{-1}$$

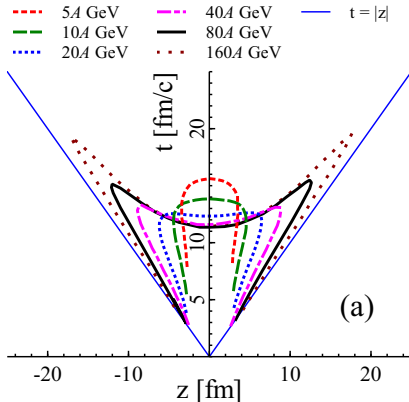
- Negative contribution

- ▶ $p^\mu d\sigma_\mu > 0$: positive contribution, particles fly out
- ▶ $p^\mu d\sigma_\mu < 0$: negative contribution, particles fly in

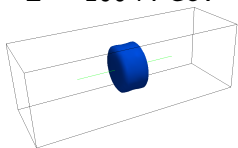
Negative contributions with coarse-grained UrQMD

DO, Huovinen, HP, PRC 91 (2015)

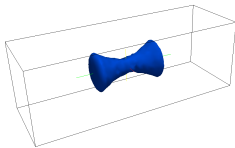
Au+Au central collisions
typical particulization
hypersurface
 $\epsilon_{LRF} = 0.3 \text{ GeV}/\text{fm}^3$



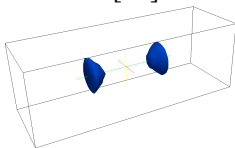
$E = 160 \text{ A GeV}$



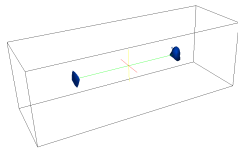
$t = 4 \text{ fm}/c$



$t = 11 \text{ fm}/c$

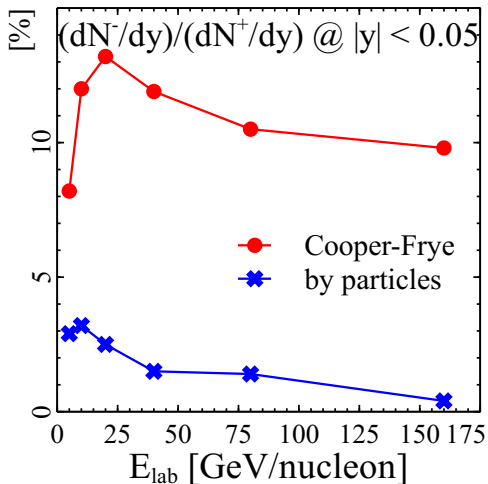


$t = 13 \text{ fm}/c$



$t = 18 \text{ fm}/c$

Negative contributions: energy dependence

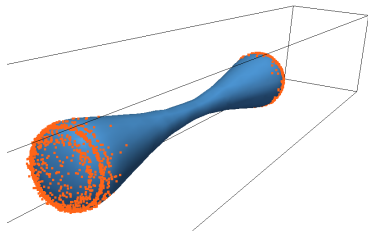


Lower collision energy - slower expansion - larger negative contributions
Non-equilibrium calculation gives much smaller values

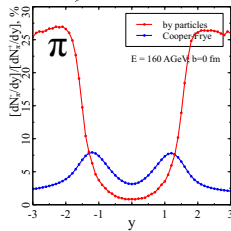
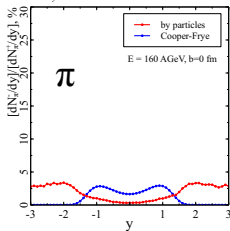
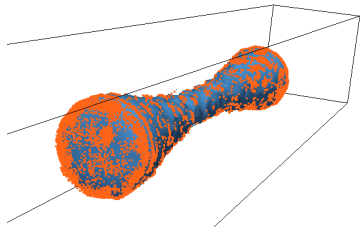
Negative contributions: surface lumpiness

$E = 160 \text{ AGeV}, b = 0$

Smooth surface



Lumpy surface

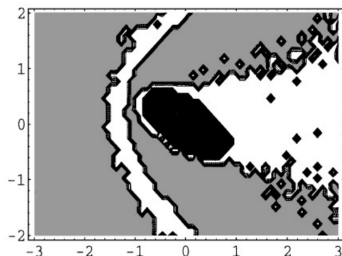


Lumpier surface - larger negative contributions

Hybrid model in a perfect world

- Coupled hydrodynamics and kinetic equations
- Transition surface found dynamically (so-called dynamical decomposition)
- Some works in this direction

K. Bugaev, *Phys Rev Lett.* 2003; L. Czernai, *Acta Phys. Hung.*, 2005



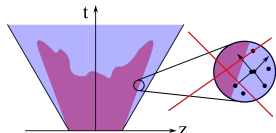
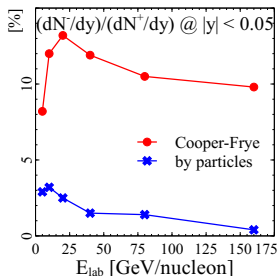
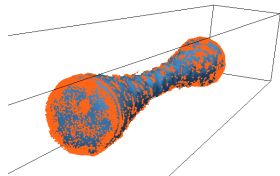
Example from non-relativistic hydrodynamics [Tiwari, *J. Comp. Phys.* 144, 710726 (1998)]

2D flow of gas around solid ellipse

White domains - Boltzmann equation, grey domains - Euler equation

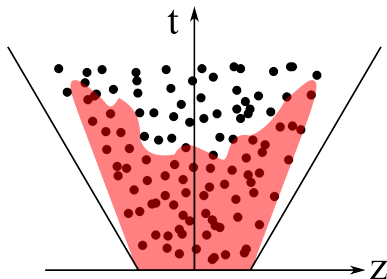
Summary of introduction

- Hybrid approaches adopt approximations:
 - aposteriori determination of particlization surface
 - particles decouple from hydrodynamics once and cannot get back into it
- These approximations become inadequate for
 - low collision energies
 - large fluctuations (event-by-event/fluctuating hydrodynamics)
- In non-relativistic hydrodynamics there are dynamic decomposition approaches, which go beyond these approximations



Alternative way: forced thermalization in transport

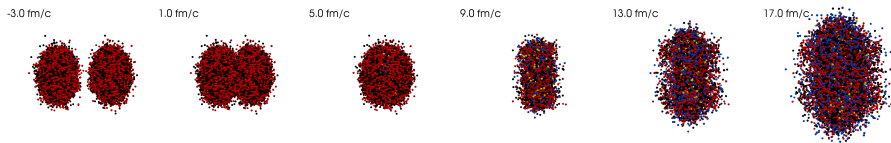
- Pure transport
- Force instant local thermalization, where density is high
- Effectively accounts for multiparticle collisions
- Conceptually similar to hybrid model, where
 - ▶ "Hydro" region defined dynamically
 - ▶ "Hydro" and transport are coupled



SMASH transport model

Weil et al., arXiv:1606.06642

- hadronic cascade, $2 \leftrightarrow 2$ and $2 \leftrightarrow 1$ reactions
 - ▶ most of established hadrons from PDG up to mass 2 GeV
- simulates AA collision as a sequence of elementary reactions
- timesteps: $\dots \rightarrow \text{collide/decay} \rightarrow \text{propagate} \rightarrow \dots$
- testparticles ansatz: $N \rightarrow N \cdot N_{\text{test}}, \sigma \rightarrow \sigma / N_{\text{test}}$
- model in active development, strings are being tested
 - ▶ currently reliable at low energies



central Au+Au collision, $E_{\text{kin}} = 2 \text{ GeV}$, $N_{\text{test}} = 100$
color coding: neutrons, protons, Δ , π

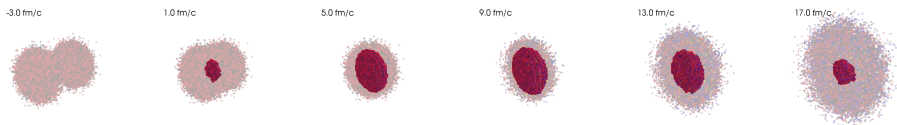
Forced grand-canonical thermalization

Every time interval Δt_{th} :

- 1 Span a lattice and compute $T^{\mu\nu}$, j_B^μ , j_S^μ on it
- 2 Find rest frame ϵ , n_b , n_S , T , μ from EoS in every cell
- 3 Remove particles from $\epsilon > \epsilon_c$ region
- 4 Sample new particles in $\epsilon > \epsilon_c$ region
 - ▶ Thermal distribution function
 - ▶ Conserving charges, energy and momentum globally
- 5 Run usual transport model until next thermalization

Ways to interpret the procedure:

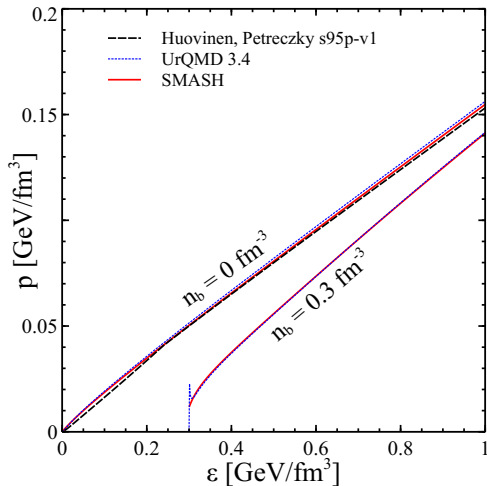
- changing local distribution function to the thermal one
- effective treatment of multi-particle collisions
- "Zero-time hydro" = fluidization + immediate particlization



central Au+Au collision, $E_{kin} = 2$ GeV, $N_{test} = 100$, purple region - thermalization region, $\epsilon_c = 0.3$ GeV/fm³:

SMASH ideal hadron gas EoS

More hadron sorts - smaller pressure at given energy density



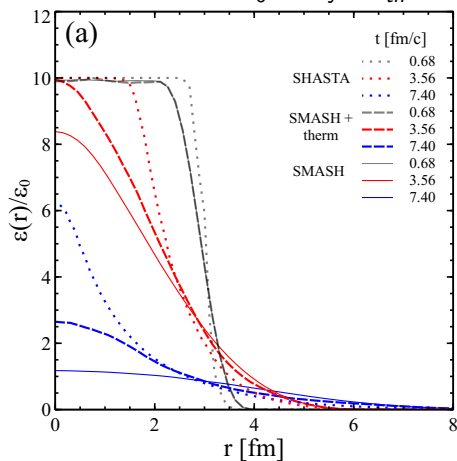
P. Huovinen, P. Petreczky, Nucl.Phys. A837 (2010) 26-53

UrQMD \equiv Hadron Gas EoS from UrQMD tables by J. Steinheimer

Forced thermalization: expanding sphere setup

Evolution of $R = 3$ fm sphere, initial $\epsilon = 10\epsilon_0$, $n_B = 0$.

Thermalization at $\epsilon > 2\epsilon_0$ every $\Delta t_{th} = 1$ fm.

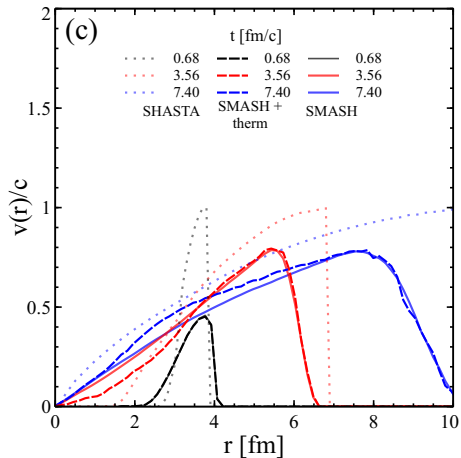


SMASH + thermalization is between transport and hydro

Forced thermalization: expanding sphere setup

Evolution of $R = 3$ fm sphere, initial $\epsilon = 10\epsilon_0$, $n_B = 0$.

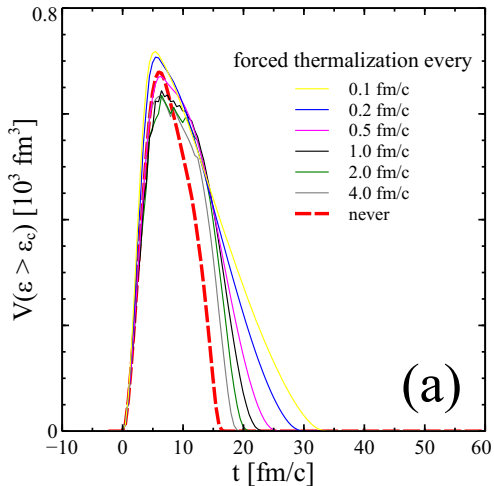
Thermalization at $\epsilon > 2\epsilon_0$ every $\Delta t_{th} = 1$ fm.



SMASH + thermalization is between transport and hydro

Effects of forced thermalization

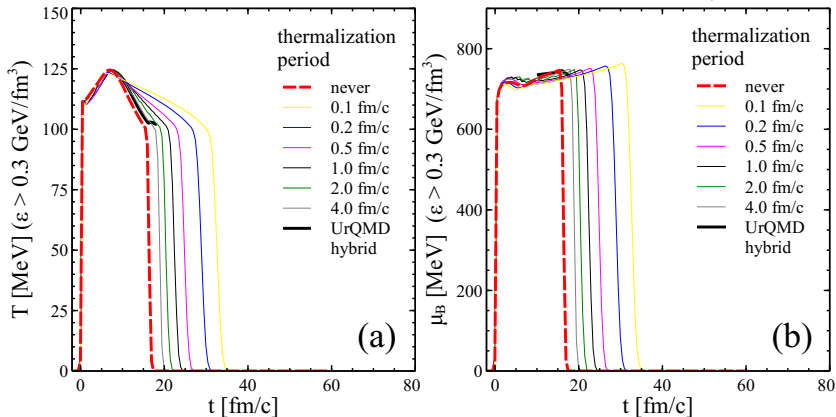
Au+Au central collision, $\sqrt{s} = 3$ GeV, $\epsilon_c = 2\epsilon_0$



High-density region exists longer

$\langle T \rangle$ and $\langle \mu_B \rangle$ in the thermalization region

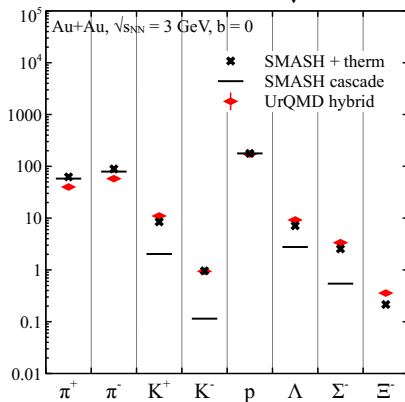
Au+Au central collision, $\sqrt{s} = 3$ GeV, $\langle A \rangle = \frac{\sum \epsilon_{ijk} A_{ijk}}{\sum \epsilon_{ijk}}$



Similar to UrQMD hybrid

Effects of forced thermalization: multiplicities

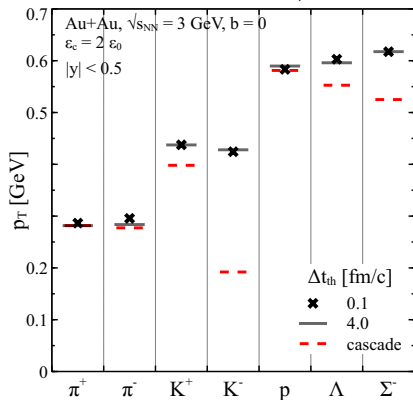
Au+Au central collision, $\sqrt{s} = 3$ GeV



Thermalization leads to strangeness enhancement - similarly to UrQMD hybrid

Effects of forced thermalization: transverse momenta

Au+Au central collision, $\sqrt{s} = 3$ GeV



Thermalization leads to pressure isotropization and transverse push

Summary

- Hybrid approaches adopt approximations:
 - ▶ a posteriori determination of particlization surface
 - ▶ particles decouple from hydrodynamics once and cannot get back into it
- These approximations become inadequate for
 - ▶ low collision energies
 - ▶ large fluctuations (event-by-event/fluctuating hydrodynamics)
- Suggestion: pure transport, forced thermalization in regions with high energy density
 - ▶ one can plug in arbitrary EoS
 - ▶ backflow is automatically taken into account
 - ▶ transition hypersurface is determined dynamically
- Tested on SMASH, observed
 - ▶ longer lifetime of high-density region
 - ▶ more energy transferred to midrapidity
 - ▶ strangeness enhancement

Outlook: further testing, plug in EoS with 1st order phase transition

"Jets + hydro" scenario with feedback to hydro using this idea?

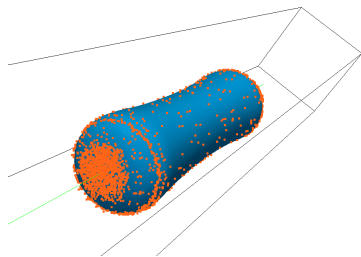
Definitions for negative contributions

- Hypersurface of constant Landau rest frame energy density
 - ▶ A) T and μ from Hadron Gas EoS, Cooper-Frye formula
 - ▶ B) Many UrQMD events, count particles crossing hypersurface
- Will coincide if particle distribution from UrQMD is exactly equilibrated

A) Cooper-Frye

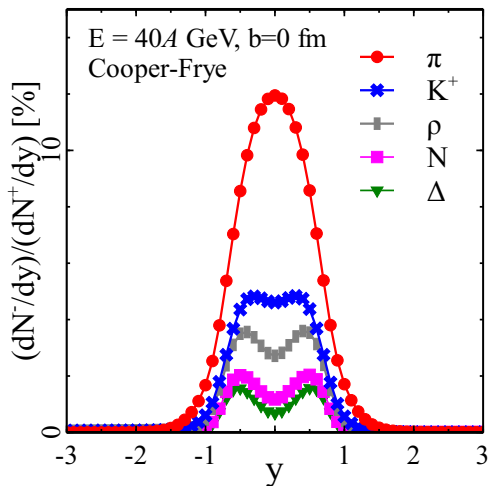
$$p^0 \frac{d^3 N^+}{dp^3} = \frac{p^\mu d\sigma_\mu}{\exp(p^\nu u_\nu / T) \pm 1} \theta(p^\nu d\sigma_\nu)$$
$$p^0 \frac{d^3 N^-}{dp^3} = \frac{p^\mu d\sigma_\mu}{\exp(p^\nu u_\nu / T) \pm 1} \theta(-p^\nu d\sigma_\nu)$$

B) "by particles"



Negative contributions: particle mass dependence

$E = 40$ AGeV, $b = 0$, $\epsilon_0 = 0.3$ GeV/fm³, dN/dy distributions



Smaller mass - larger negative contribution

The energy-momentum tensor $T^{\mu\nu}$ is constructed as

$$T^{\mu\nu}(\vec{r}) = \frac{1}{N_{ev}} \sum_{events} \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\vec{r} - \vec{r}_i, p_i) \quad (1)$$

Smearing kernel $K(r)$ should be such that $K(r)d^3r$ is Lorentz-scalar

$$\Delta x^i = \Lambda_j^i \Delta x'^j \quad (2)$$

$$\Lambda_j^i = \delta_j^i + (u^i u_j)/(1 + \gamma) \quad (3)$$

$$(\Delta x^i)^2 = \Lambda_j^i \Delta x'^j \Lambda_k^i \Delta x'^k \quad (4)$$

$$\Lambda_j^i \Lambda_k^i = \delta_{jk} + u_j u_k \quad (5)$$

$$(\Delta \vec{x})^2 = (\Delta \vec{x}')^2 + (\Delta \vec{x}' \cdot \vec{u})^2 \quad (6)$$

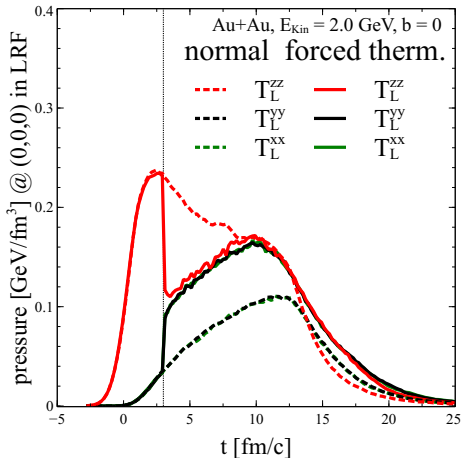
$$K(\vec{r}) = \gamma (2\pi\sigma^2)^{-3/2} \exp\left(-\frac{\vec{r}^2 + (\vec{r} \cdot \vec{u})^2}{2\sigma^2}\right) \quad (7)$$

Normalization using $\int (\prod_{i=1}^n dx_i) e^{-x_i A^{ij} x_j} = \pi^{n/2} (\det A)^{-1/2}$

Effects of forced isotropization: pressure

Forced isotropization at $t > 3$ fm/c, where $\epsilon > 0.3$ GeV/fm³.

Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$

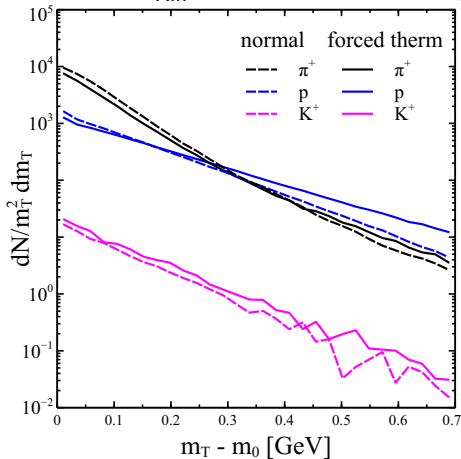


Pressures rapidly equalize

Effects of forced isotropization: m_T spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm³.

Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$

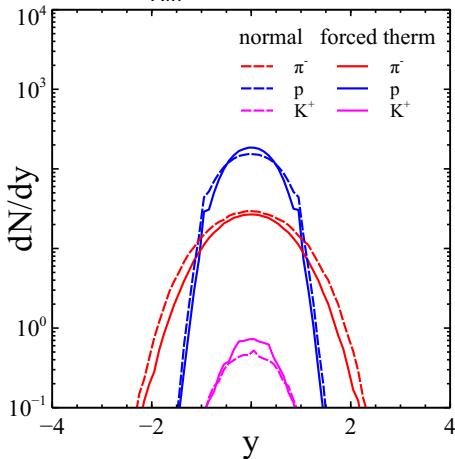


Particles move from low p_T to high p_T : "transverse push"

Effects of forced isotropization: y spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm³.

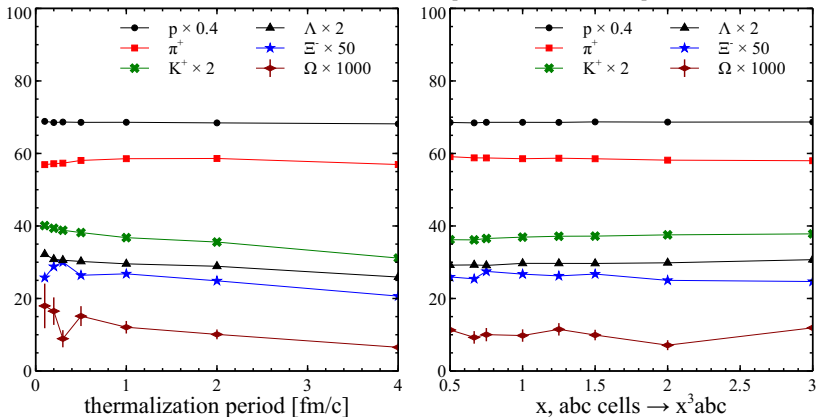
Au+Au, CM frame, $E_{Kin} = 2 A$ GeV, $b = 0$, $N_{test} = 100$



Nucleons move to midrapidity, less pions, more kaons

Effects of forced thermalization: multiplicities

Au+Au central collision, $\sqrt{s} = 3$ GeV, $[1.0 \times 1.0 \times 0.5]$ lattice spacing



Thermalization period and lattice spacing are not important for multiplicities