INTRODUCTION TO EVENT-BY-EVENT FLUCTUATIONS IN HIGH ENERGY COLLISIONS

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PRELIMINARY AGENDA:

- BASIC IDEAS, NOTATION
- WHY FLUCTUATIONS?
- MULTIPLICITY FLUCTUATIONS IN ELEMENTARY INTERACTIONS
- WOUNDED NUCLEON AND STATISTICAL MODELS
- EXTENSIVE, INTENSIVE AND STRONGLY INTENSIVE QUANTITIES
- DATA/MODEL COMPARISON p+p/A+A
- IMPERFECT MASS MEASUREMENTS – IDENTITY METHOD
- FLUCTUATIONS AND ONSET OF DECONFINEMENT
- FLUCTUATIONS AND CRITICAL POINT

LITARATURE:

- COULD NOT FIND A TEXTBOOK
- REFERENCES TO ORIGINAL PAPERS, REVIEWS ON SLIDES

LOGISTICS:

- LECTURES FROM APRIL 14 TO JULY 14?
- THURSDAY

START: 14:00

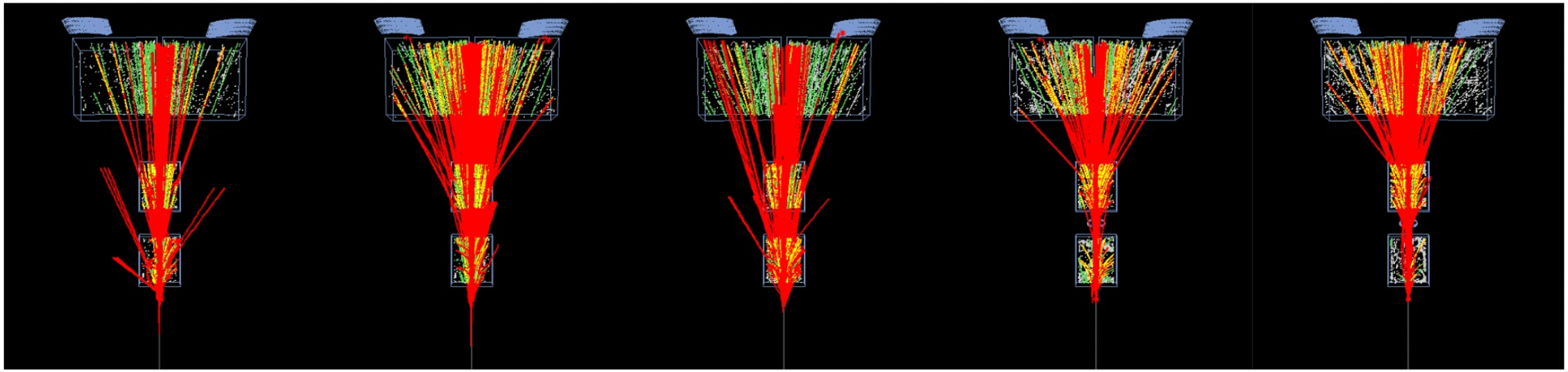
STOP: ~15:09 (< 15:30)

NO LECTURES ON
MAY 5 AND 25 (HOLIDAYS IN DE) AND
JUNE 2 AND 9 (MG MEETINGS)

SLIDES IN INDICO (indico.cern.ch/event/520685)
VIDYO PARTICIPATION POSSIBLE (VIDYO ROOM: NA61-Lectures)
QUESTIONS/DISCUSSION DURING LECTURES VERY WELCOME

BASIC IDEAS AND NOTATION

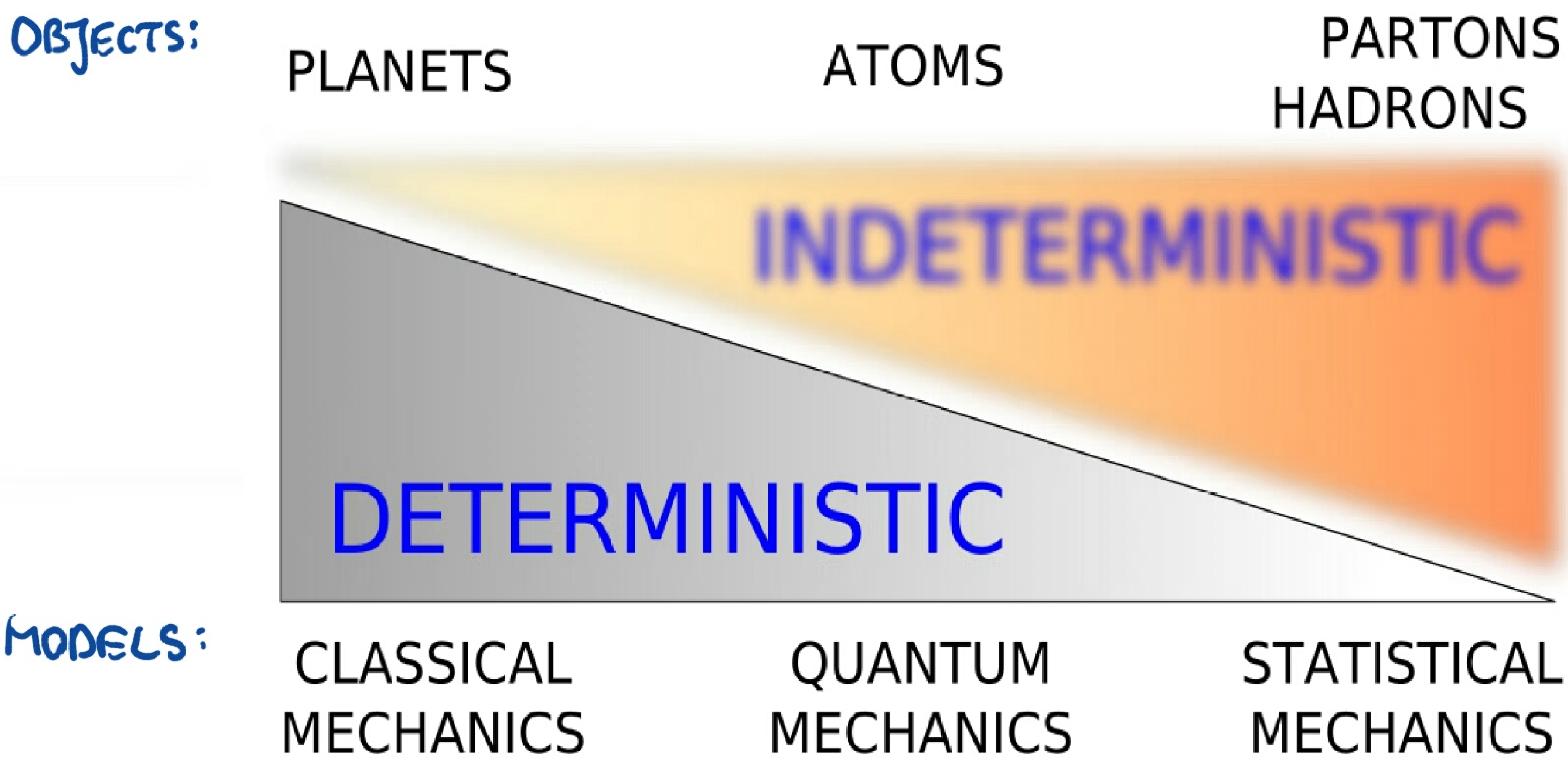
DIFFERENT COLLISIONS (EVENTS)



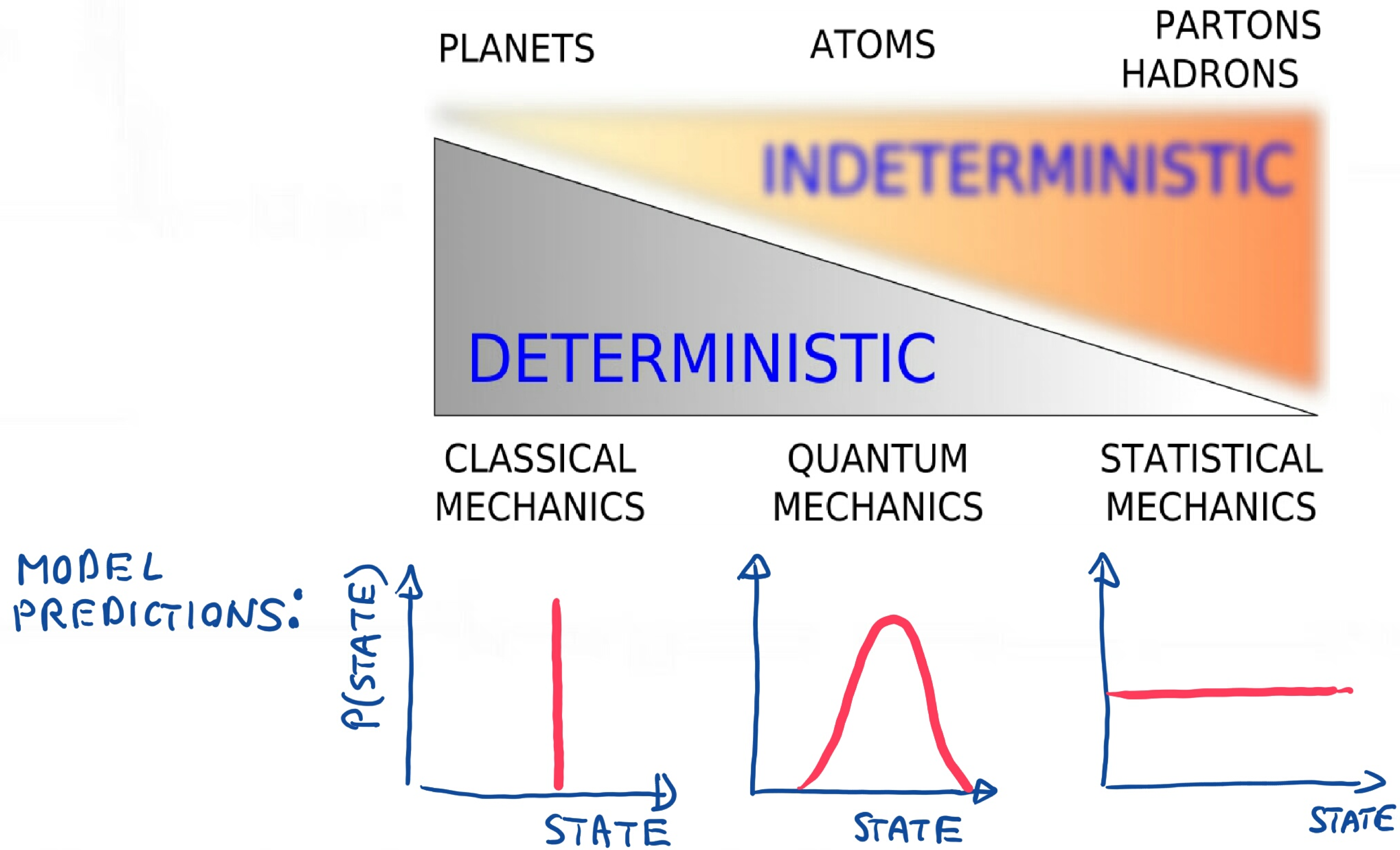
DIFFER !

PROBABILITY AND STATISTICS
IN PARTICLE PHYSICS
FRODESEN, SKJEGGESTAD, TOFTE
ISBN 82-00-01906-3 1979

DIFFERENCES CANNOT BE ELIMINATED NEITHER BY FIXING INITIAL PARAMETERS OF AN EXPERIMENT (E.G. COLLISION ENERGY, BEAM, TARGET COMPOSITION) NOR BY SELECTING EVENTS BASED ON THEIR PROPERTIES

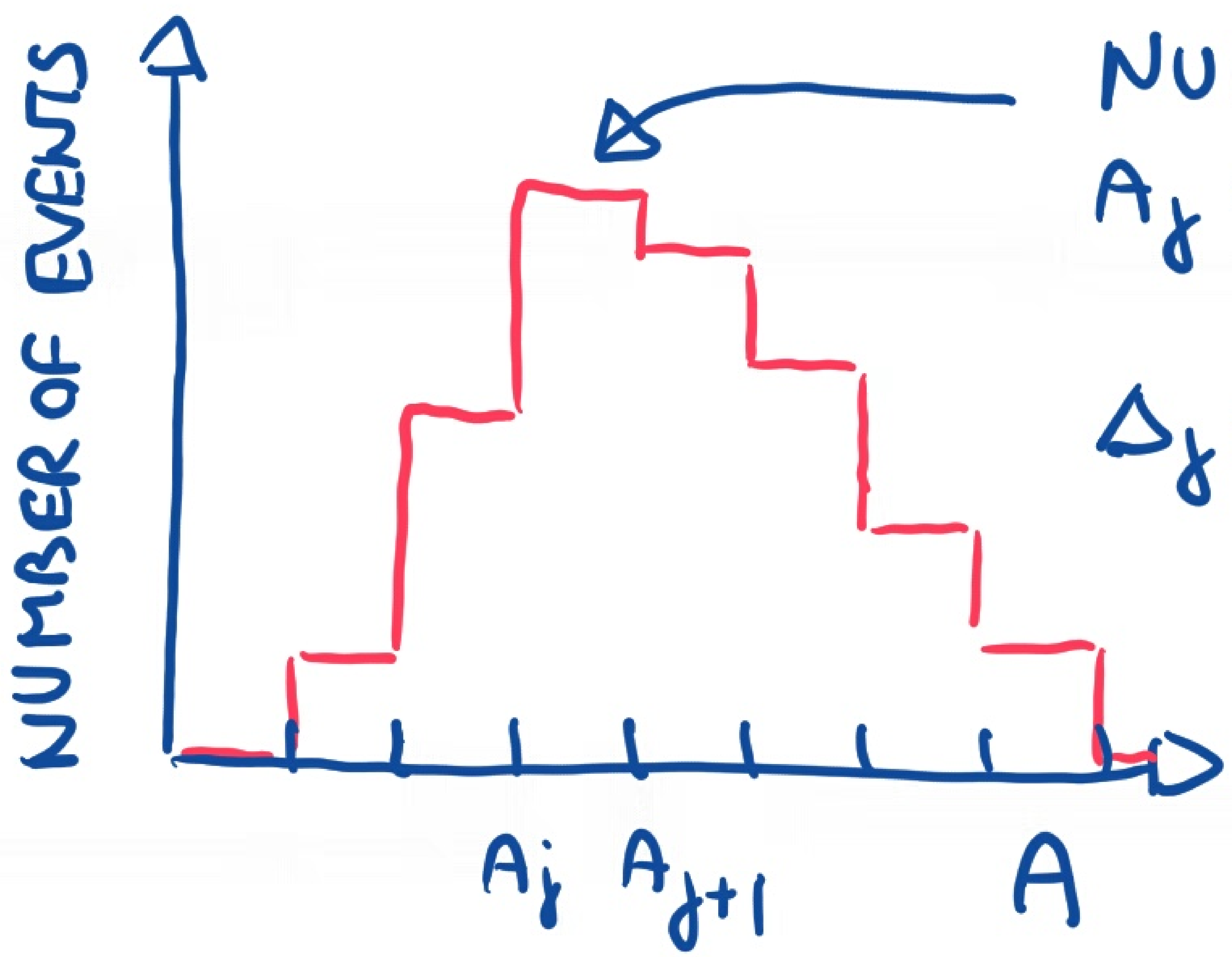


REMAINING DIFFERENCES ARE DUE TO INDETERMINISTIC NATURE OF NATURE


PLANETS
ATOMS
PARTONS
HADRONS
INDETERMINISTIC
DETERMINISTIC
CLASSICAL MECHANICS
QUANTUM MECHANICS
STATISTICAL MECHANICS
MODEL PREDICTIONS:
P(STATE)
STATE
STATE
STATE

HOW DO WE MEASURE EVENT-BY-EVENT FLUCTUATIONS?

- DEFINE A QUANTITY (EVENT QUANTITY, "A") WHICH CAN BE MEASURED FOR EACH EVENT, E.G. NUMBER OF PARTICLES (EVENT MULTIPLICITY) N $(A = N)$ OR SUM OF PARTICLE TRANSVERES MOMENTA p_T
- MEASURE "A" FOR ALL EVENTS IN AN EVENT SAMPLE $\{A_1, A_2 \dots A_M\}$ AND PLOT ITS DISTRIBUTION



NUMBER OF EVENTS WITH $A_j < A < A_{j+1}$

$\Delta_j = A_{j+1} - A_j$

– THEN PROBABILITY DENSITY DISTRIBUTION OF A IS ESTIMATED AS:

$$h_j(A) = \frac{M(A_j < A < A_{j+1})}{\Delta_j \cdot M}$$

WHERE: Δ_j – BIN SIZE

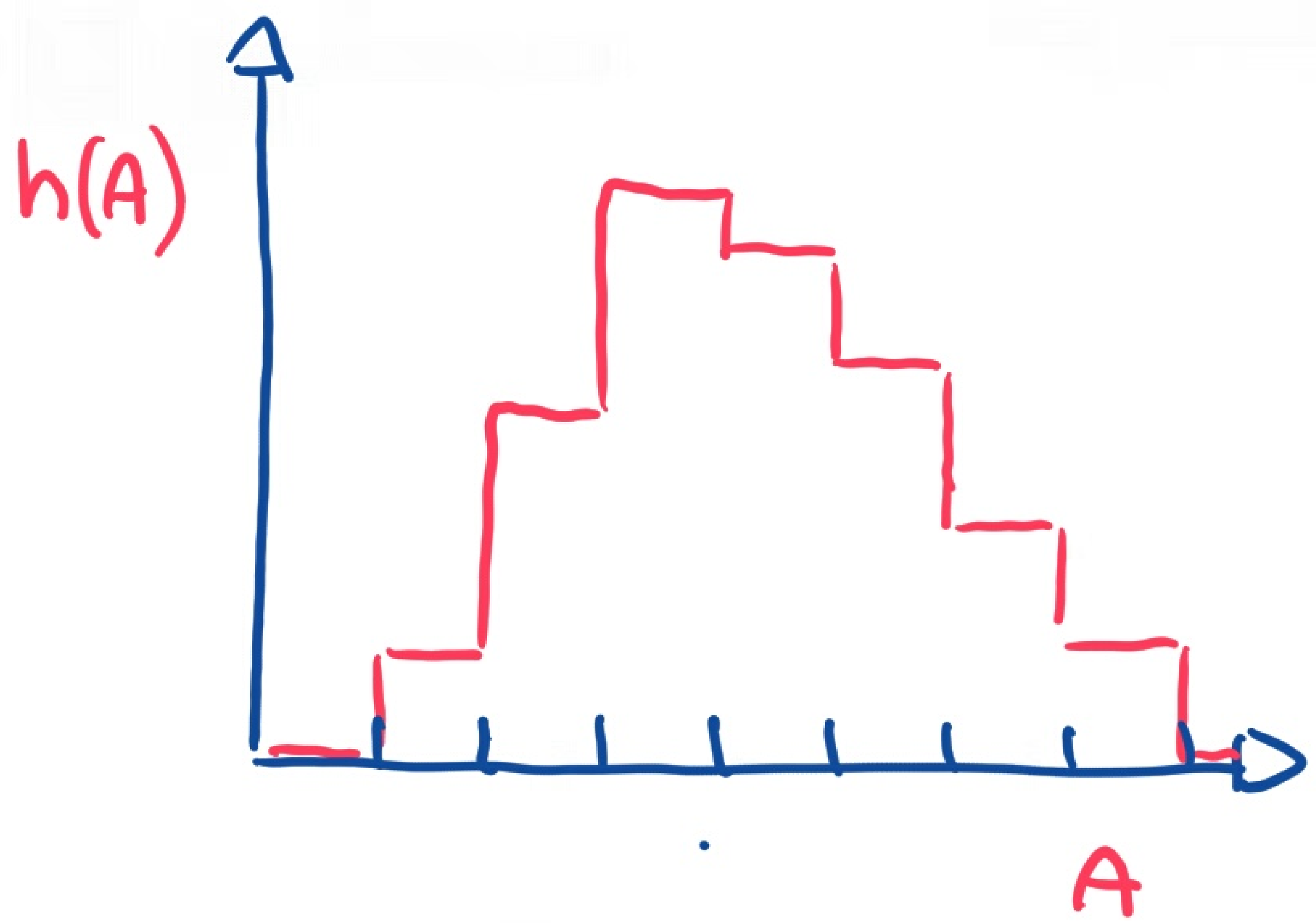
$M(A_j < A < A_{j+1})$ – # EVENTS IN BIN $A_j < A < A_{j+1}$

M – TOTAL NUMBER OF EVENTS

BY CONSTRUCTION:

$$\sum_j h_j(A)\,\Delta_j = 1 \qquad \left(\int h(A)\,dA = 1 \right)$$

- MEASUREMENTS OF E-BY-E FLUCTUATIONS OF A
  MEAN MEASUREMENT OF $h(A)$



AND/OR A CHRACTERIZATION OF THE EVENT SAMPLE BY A LIMITTED NUMBER OF QUANTITIES (FLUCTUATION MEASURES, QUANTITIES) E.G. SAMPLE MOMENTS

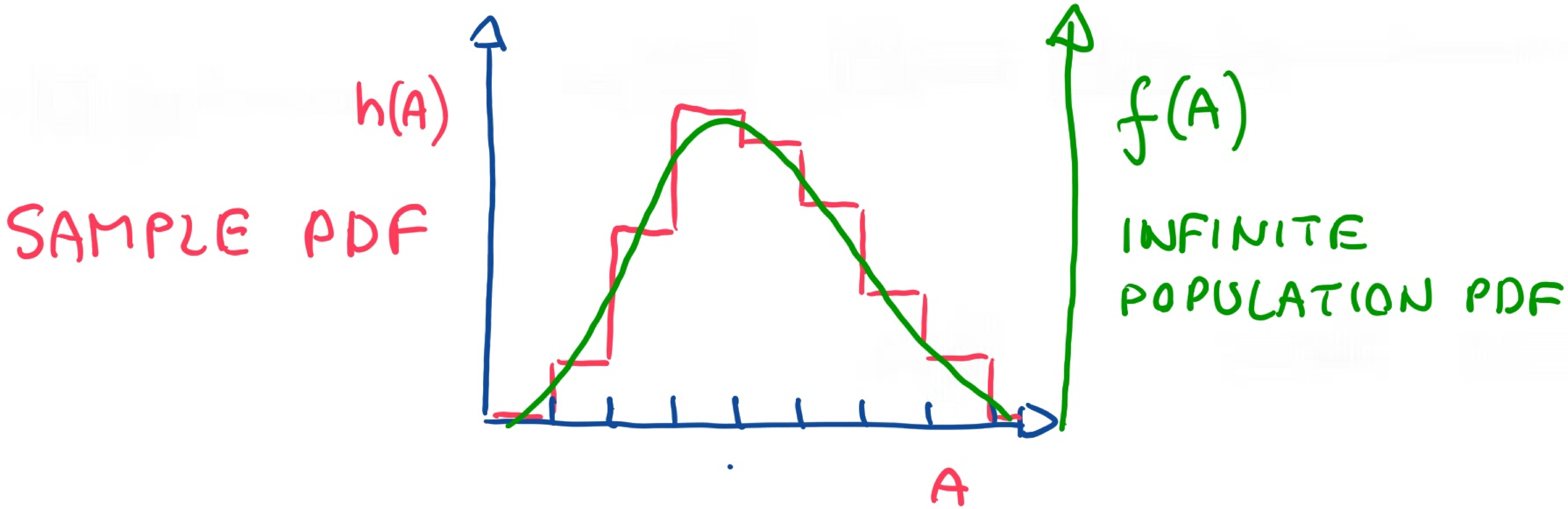
POSTULATE:

FOR $\Delta_j \to 0$ AND
$M \to \infty$

$$h(A) = \frac{M(A_j < A < A_{j+1})}{\Delta_j \cdot M} \to f(A)$$

$f(A)$ - (INFINITE) POPULATION PROBABILITY DENSITY FUNCTION OF THE RANDOM VARIABLE A

OF COURSE $\int f(A)\,dA = 1$



$h(A) \longrightarrow f(A)$

FOR

$M \longrightarrow \infty$

$\Delta \longrightarrow 0$

MOMENTS OF A DISTRIBUTED ACCORDING TO $f(A)$

EXPECTED VALUE OF ANY FUNCTION $g(A)$:

$$E[g(A)] = \int g(A) \cdot f(A)\, dA$$

$$E[c] = c \qquad c - \text{CONSTANT}$$

$$E[c \cdot g(A)] = c \cdot E[g(A)]$$

$$E[c_1 g_1(A) + c_2 g_2(A)] = c_1 E[g_1(A)] + c_2 E[g_2(A)]$$

(E - LINEAR OPERATOR)

RAW (ALGEBRAIC) MOMENTS ($g(A) = A^k$):

$$\mu'_k \equiv E[A^k] = \int A^k \cdot f(A)\, dA$$

MEAN A

CENTRAL MOMENTS ($g(A) = (A - \mu'_1)^k$):

$$\mu_k \equiv E[(A-\mu'_1)^k] = \int (A-\mu'_1)^k \cdot f(A)\, dA$$

RAW MOMENTS, EXAMPLES

MEAN – FIRST RAW MOMENT

$$E[A] = \int A \cdot f(A)\, dA \qquad (= \mu_1' = \mu)$$

SECOND RAW MOMENT;

$$E[A^2] = \int A^2 f(A)\, dA \qquad (= \mu_2')$$

THIRD RAW MOMENT:

$$E[A^3] = \int A^3 f(A)\, dA \qquad (= \mu_3')$$

...

CENTRAL MOMENTS, EXAMPLES

FIRST CENTRAL MOMENT:

$$E[(A-\mu)^1] = E[A] - \mu = 0 \qquad (= \mu_1)$$

SECOND CENTRAL MOMENT - VARIANCE

$$E[(A-\mu)^2] = E[A^2] - \mu^2 \qquad (= \mu_2 = \sigma^2)$$

THIRD MOMENT

$$E[(A-\mu)^3] = E[A^3] - 3\mu E[A^2] + 2\mu^3 \quad (= \mu_3)$$

...

BACK TO EVENT-BY-EVENT MEASUREMENTS:

$A_1, A_2 \ldots \quad A_M$

BASIC ASSUMPTION:

- $A_1, A_2, \ldots A_M$ ARE INDEPENDENTLY "DRAWN" FROM $f(A)$ (INFINITE POPULATION PDF)

CALCULATE ARTHMETIC MEAN OF $\{A_1, \ldots A_M\}$

$$\bar{A} = \frac{1}{M} \sum_{i=1}^{M} A_i$$

EXPECTED VALUE OF $\bar{A}$ IS

$$E[\bar{A}] \overset{E\ \text{LINEAR}}{=} \frac{1}{M} \sum_i E[A_i] \overset{f(A)\ \text{FOR ALL}\ A_i}{=} \frac{1}{\cancel{M}} \cdot \cancel{M} \cdot \mu = \mu$$

AND ITS VARIANCE

$$E[(\bar{A} - \mu)^2] = \frac{\sigma^2}{M}$$

ARTHMETIC MEAN OF EVENT QUANTITY "A" CALCULATED FOR EVENT SAMPLE M ESTIMATES EXPECTED VALUE OF "A"

$$\bar{A} = \frac{1}{M} \sum_{i}^{M} A_i \approx E[A] = \int A\, f(A)\, dA$$

WITH PRECISION (DISPERSION = VARIANCE$^{1/2}$)

$\sigma / \sqrt{M}$ $\quad (\bar{A} \to E[A]$ FOR $M \to \infty)$

FREQUENTLY BOTH $E[A]$ (POPULATION MEAN) AND $\bar{A}$ (SAMPLE MEAN) ARE CALLED MEAN AND DENOTED AS $\langle A \rangle$

EVENT SAMPLE VARIANCE ESTIMATES SECOND CENTRAL MOMENT (POPULATION VARIANCE)

$$s^2 = \frac{1}{M-1} \sum_{i}^{M} (A_i - \bar{A})^2 \approx E\left[(A-\mu)^2\right] = \sigma^2$$

($s^2 \to \sigma^2$ FOR $M \to \infty$)

FREQUENTLY BOTH σ^2 (POPULATION VARIANCE) AND s^2 (SAMPLE VARIANCE) ARE CALLED VARIANCE AND DENOTED $\mathrm{Var}[A]$

MULTI-DIMENSIONAL E-BY-E ANALYSIS

ONE CAN DEFINE AND MEASURE MORE THAN ONE EVENT QUANTITY, E.G. TWO EVENT QUANTITIES A ANB B

IN THIS CASE:
EVENT SAMPLE IS GIVEN BY
$\{(A_1, B_1), (A_2, B_2) \ldots (A_M, B_M)\}$

AND POPULATION DISTRIBUTION BY
$f(A, B)$

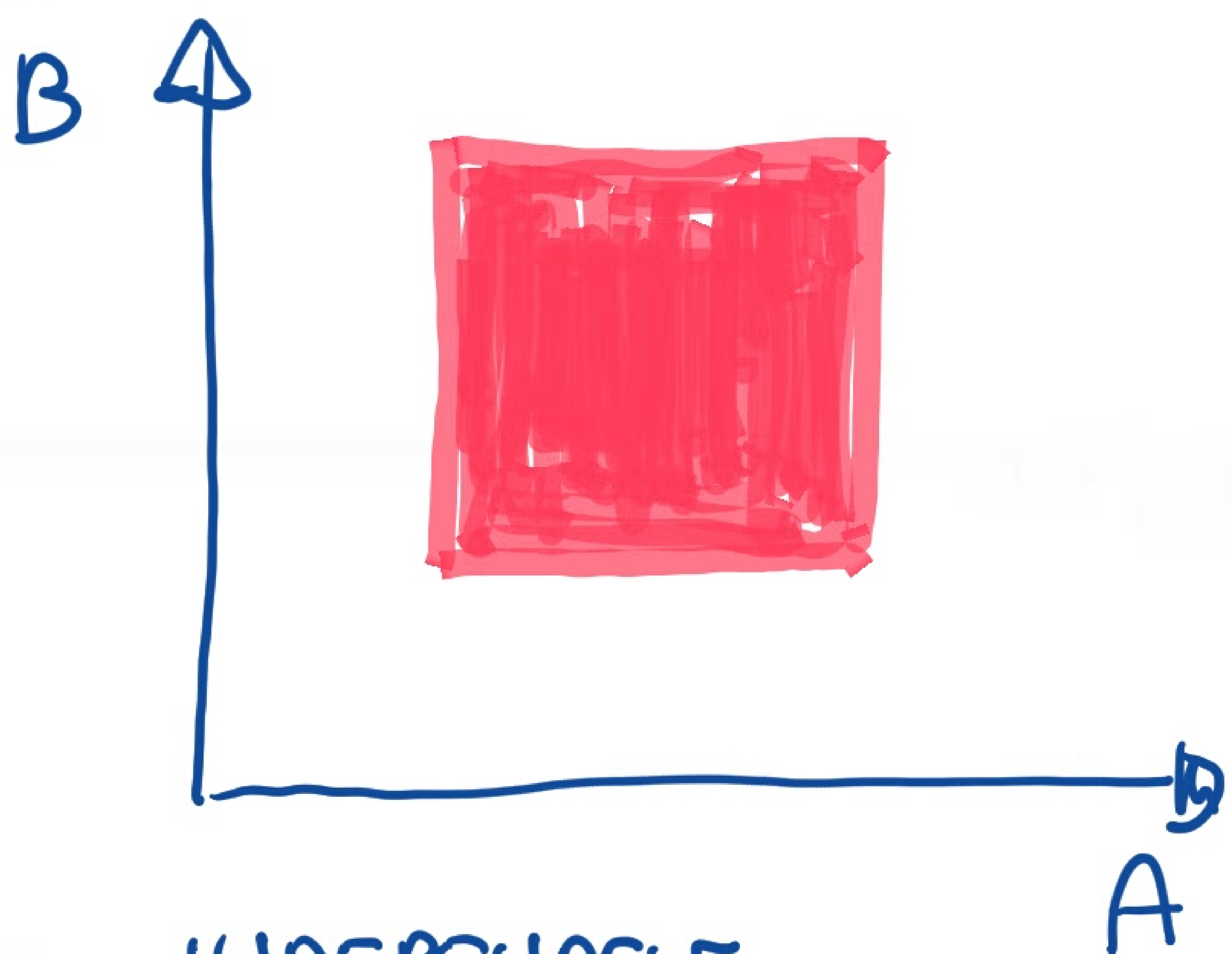
IN ADDITION TO "PURE" MOMENTS:
$\langle A^k \rangle, \langle B^k \rangle$
"MIXED" ONES APPEAR
$\langle A^k B^l \rangle$

A AND B ARE UNCORRELATED (INDEPENDENT) WHEN

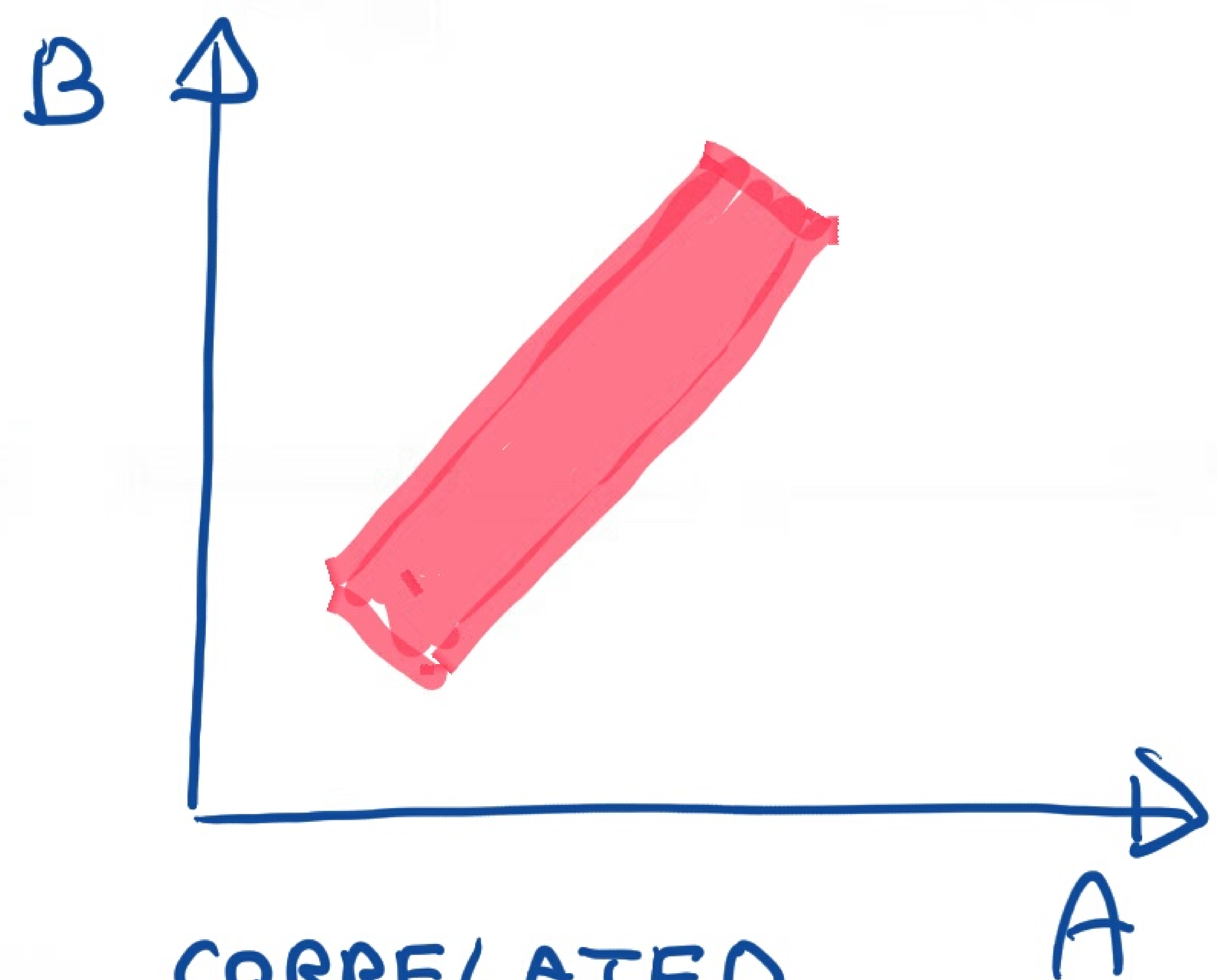
$$f(A,B) = f_A(A) \cdot f_B(B) \Rightarrow E[A^k B^l] = E[A^k] E[B^l]$$

IN THIS CASE FOR AN EVENT SAMPLE ONE GETS

$$\langle A^k \cdot B^l \rangle \cong \langle A^k \rangle \langle B^l \rangle$$



INDEPENDENT

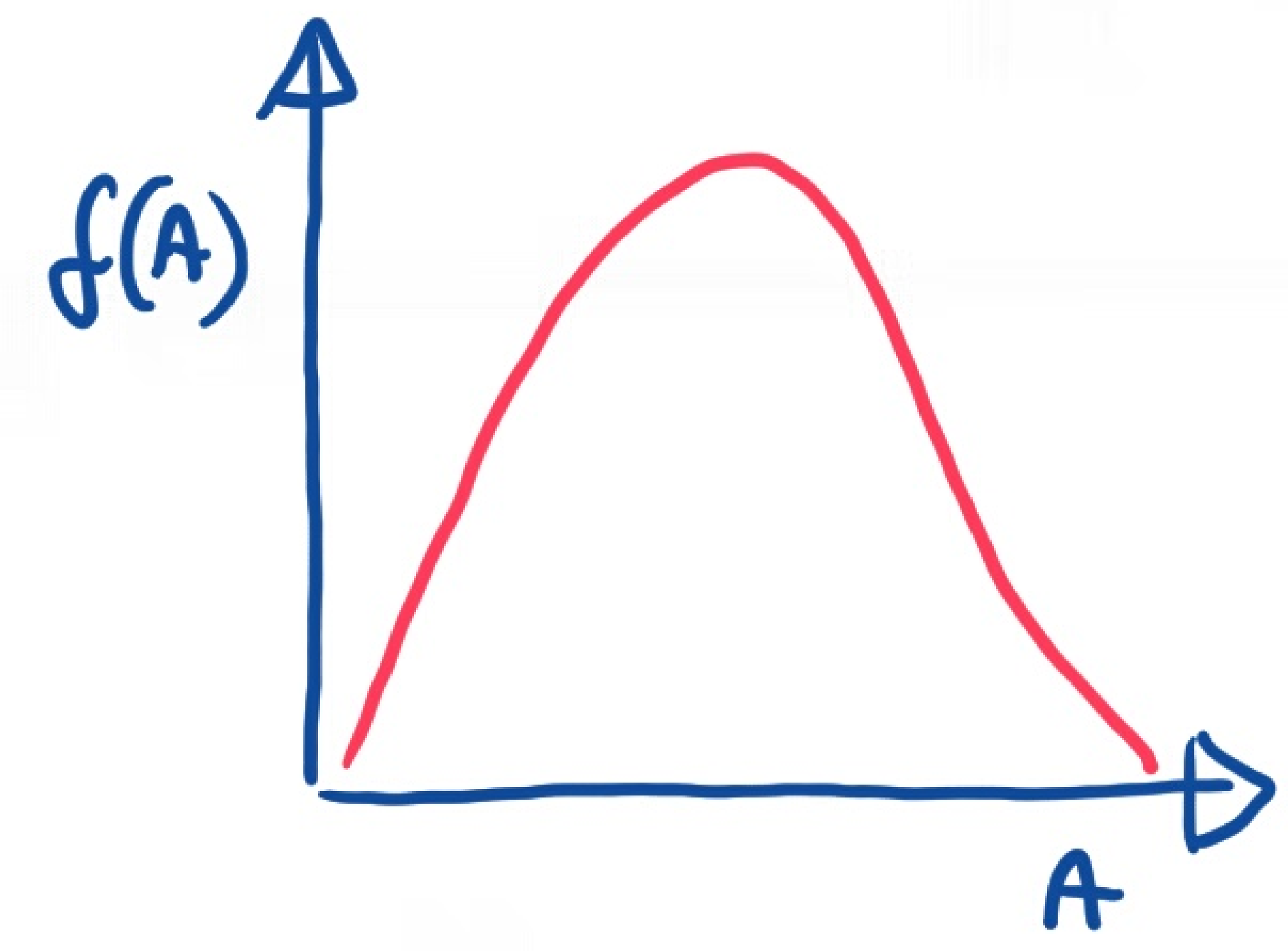


CORRELATED

CONTINOUS AND DISCRETE EVENT QUANTITIES

$f(A)$ IS A CONTINOUS FUNCTION OF A

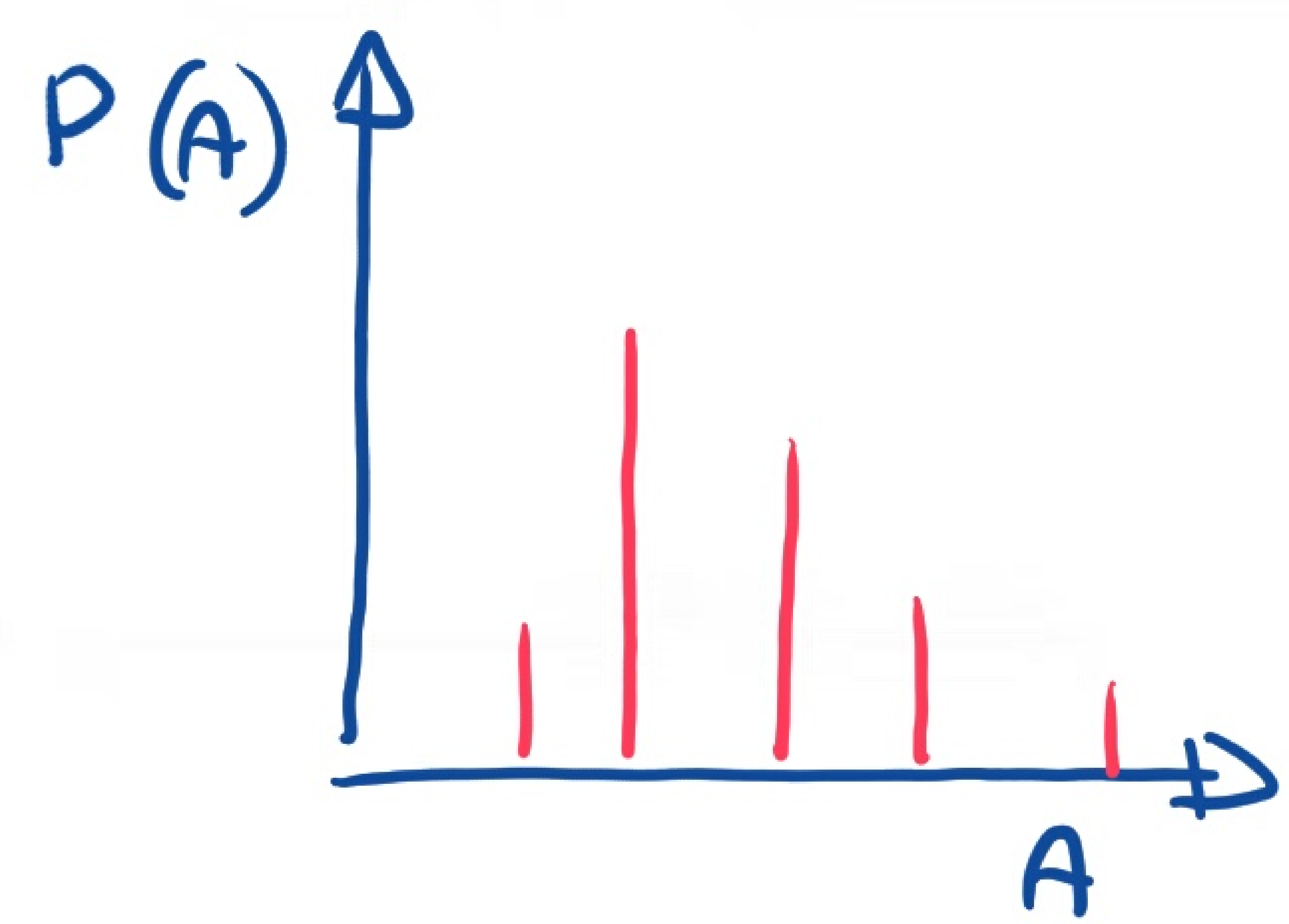
$\Rightarrow$ A – CONTINOUS EVENT QUANTITY



$$f(A) = f_c(A) \cdot \sum_j \delta(A - D_j)$$

$\Rightarrow$ A – DISCRETE EVENT QUANTITY

ONLY VALUES OF $A = D_j$ HAVE NON-ZERO PROBABILITIES $(f_c(D_j))$



$$P(A) = f(A) \Big/ \underbrace{\sum f(A)}_{\text{NON-ZERO VALUES}} \Rightarrow \sum P(A) = 1$$

EXAMPLES:

CONTINUOUS EVENT QUANTITIES:

$A = P_T$, $A = E$ $\longrightarrow f(A)$

(MOTIONAL EVENT QUANTITIES)

DISCRETE EVENT QUANTITIES:

$A = Q$, $A = N$, $A = B$ $\longrightarrow P(A)$

(MATERIAL EVENT QUANTITIES)