MULTIPLICITY FLUCTUATIONS

- BASICS
- MEAN MULTIPLICITY VS HIGHER MOMENTS: ACCEPTANCE

BASICS

EVENT MULTIPLICITY (OR JUST MULTIPLICITY) ≡
≡ NUMBER OF PARTICLES IN AN EVENT
MULTIPLICITY IS TYPICALLY DENOTED BY N

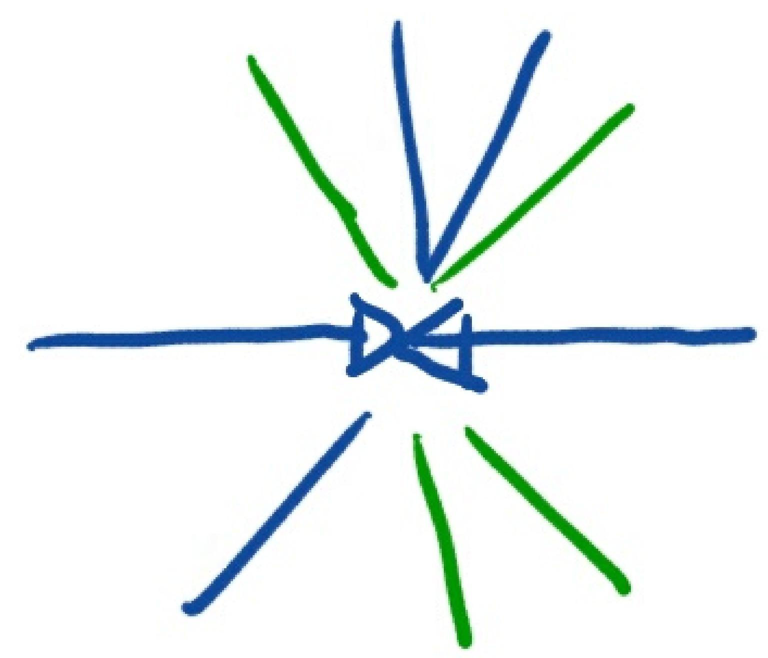
MULTIPLICITY OF PARTICELS OF DIFFERENT TYPES CAN BE CONSIDERED, E.G.:

- CHARGED PARTICLE MULTIPLICITY
- NEGATIVELY CHARGED PARTICLE MULTIPLICITY
- K^+ MULTIPLICITY

MULTIPLICITY MAY REFER TO PARTICLE NUMBER IN THE FULL ACCEPTANCE (4π MULTIPLICITY) OR IN A LIMITED ACCEPTANCE
(IN EXPERIMENTS ACCEPTANCE REFERS ONLY TO ACCEPTANCE IN MOMENTUM SPACE)

LET US DEFINE EVENT QUANTITY AS EVENT MULTIPLICITY

$A = N$



$N=3$ BLUE MULTIPLICITY

$N=4$ GREEN MULTIPLICITY

$N=7$ TOTAL MULTIPLICITY

MULTIPLICITY IS DISCRETE MATERIAL EVENT QUANTITY

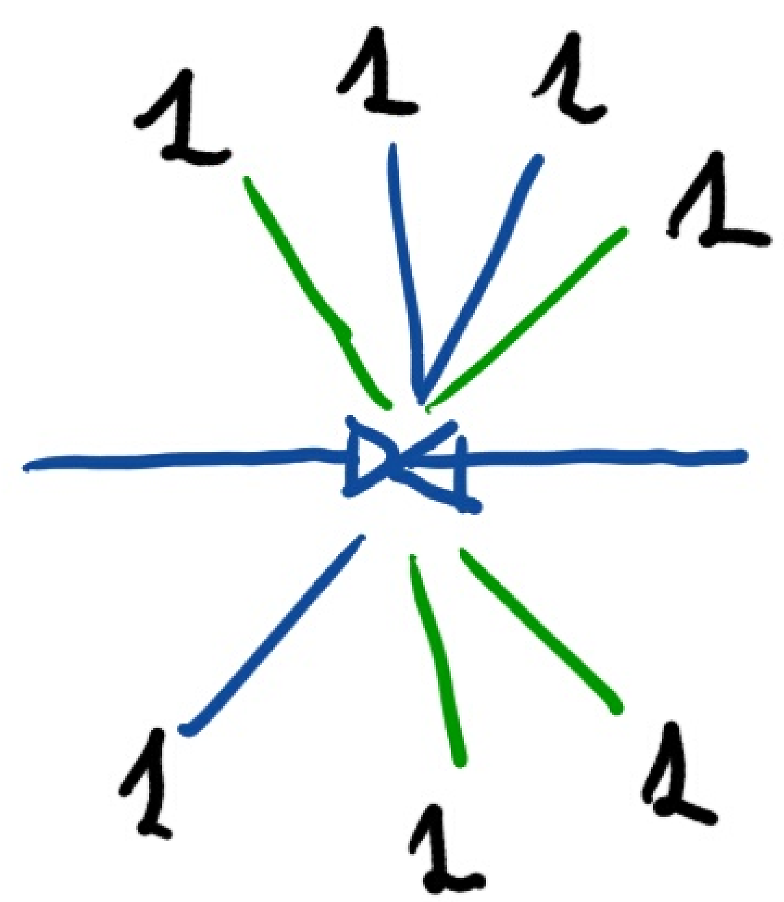
MULTIPLICITY FLUCTUATIONS ARE CHARACTERIZED BY PROBABILITY DISTRIBUTION:

$P(N)$ AND/OR ITS MOMENTS: $\langle N \rangle$, $\mathrm{Var}[N]$, ...

MULTIPLICITY CAN BE EXRESSED AS A SUM RUNNING OVER ALL PARTICELS :

$$N = \sum_{i=1}^{N} 1 = \sum_{i=1}^{N} \delta_{tt_i}$$

WHERE FOR ALL PARTICLES $t_i = t$ AND THUS THE KRONECKER $\delta_{tt_i} = 1$ FOR ALL i



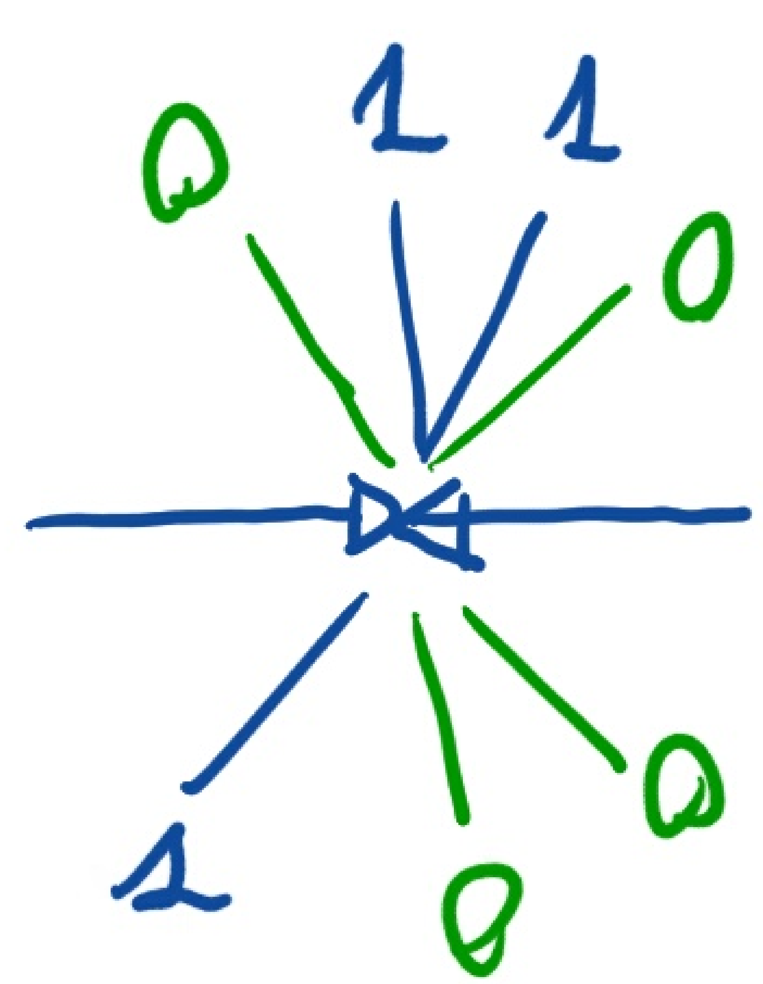
$$N = \sum_{i=1}^{7} 1 = 7$$

THEN MULTIPLICITY OF PARTICLES OF A GIVEN TYPE t (E.G. NEGATIVELY CHARGED PARTICLES) CAN BE EXPRESSED AS:

$$N_t = \sum_{i=1}^{N} \delta_{t t_i}$$

WHERE $\delta_{tt_i} = 1$ FOR t-PARTICLES $(t_i = t)$
AND $\delta_{tt_i} = 0$ FOR OTHER PARTICLES $(t_i \neq t)$

$t = B$(LUE)



$$N_B = \sum_{i=1}^{N} \delta_{Bt_i} = 3$$

(FROM THIS PRESENTATION OF MULTIPLICITY THE IDENTITY METHOD ORIGINATES)

MEAN MULTIPLICITY VS HIGHER MOMENTS: ACCEPTANCE

EVENT SAMPLE : $\{N_1, N_2 \dots N_M\}$

SAMPLE MEAN MULTIPLICITY:

$$\langle N \rangle = \frac{1}{M} \sum_{i=1}^{M} N_i$$

BUT $N_i = \sum_{j=1}^{N_i} \delta_{t t_j}$ AND THUS $\left(\mathcal{N} \equiv \sum_{i=1}^{M} N_i \right)$

$$\langle N \rangle = \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{N_i} \delta_{t t_j} = \frac{1}{M} \sum_{j=1}^{\mathcal{N}} \delta_{t t_j} =$$

$$= \frac{(\text{TOTAL NUMBER OF PARTICLES IN EVENT SAMPLE}, \mathcal{N})}{(\text{NUMBER OF EVENTS}, M)}$$

FOR EVENT SAMPLE OF M EVENTS WITH $\mathcal{N}$ PARTICLES MEAN MULTIPLICITY IS GIVEN BY:

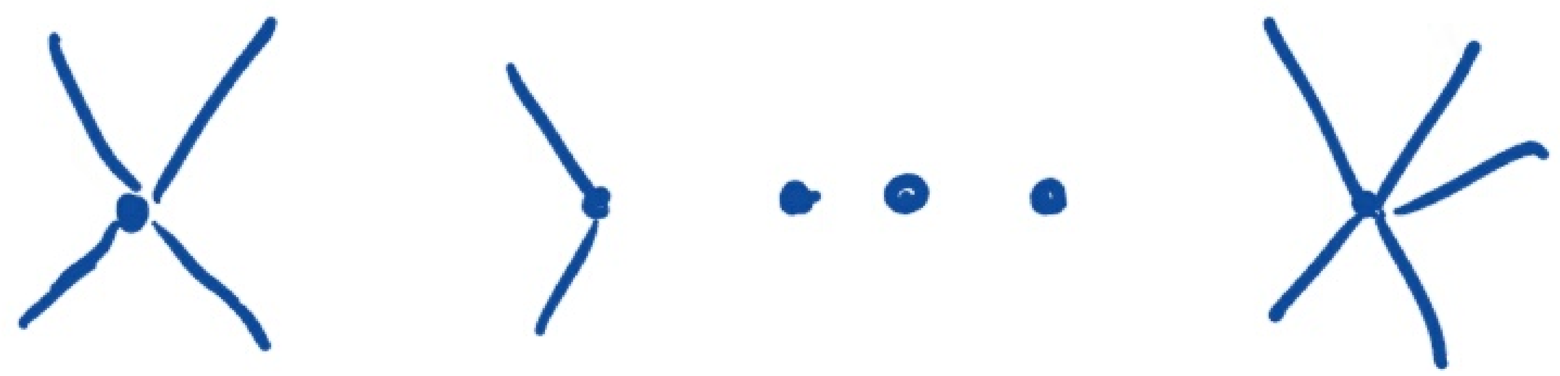
$$\langle N \rangle = \frac{\mathcal{N}}{M}$$

THERE ARE TWO METHODS TO CALCULATE $\mathcal{N}$

- EVENT-BY-EVENT METHOD:
  MEASURE N_i IN EACH EVENT AND ADD THEM:
  $\mathcal{N} = \sum N_i$

- SINGLE PARTICLE METHOD:
  MEASURE MULTIPLICTIES IN A LIMITED ACCEPTANCE N_i^A $(<(\ll) N_i)$, ADD THEM
  $\mathcal{N}_i^A = \sum_i N_i^A$ AND CORRECT TO THE REQUIRED ACCEPTANCE USING SYMMETRIES OF SINGLE PARTICLE SPECTRA IN THE EVENT SAMPLE

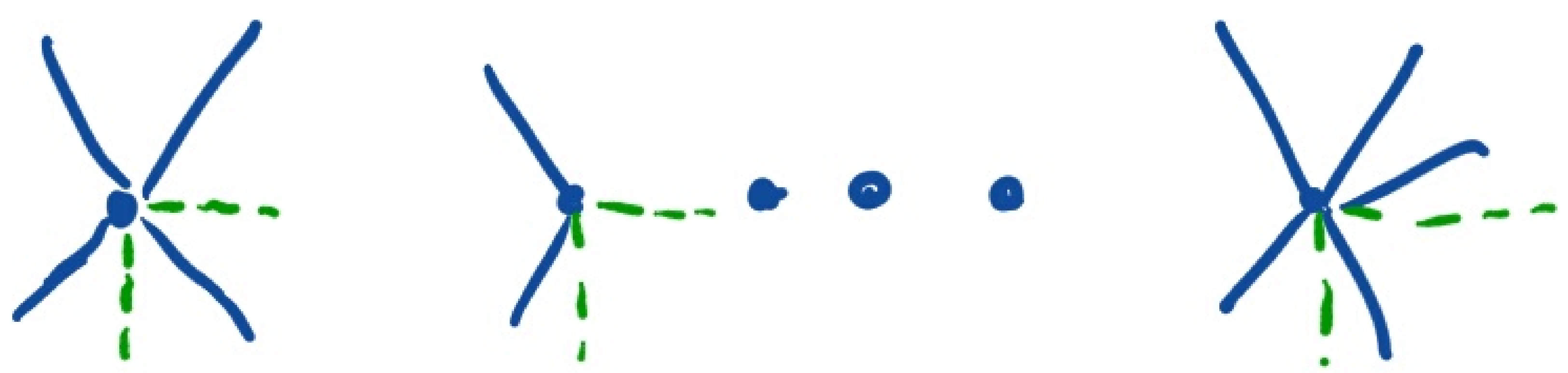
E-BY-E METHOD:



$N_1 = 4$ $\quad N_2 = 2$ $\quad N_M = 5$

$$\langle N \rangle = \frac{1}{M} \sum_{i=1}^{M} N_i$$

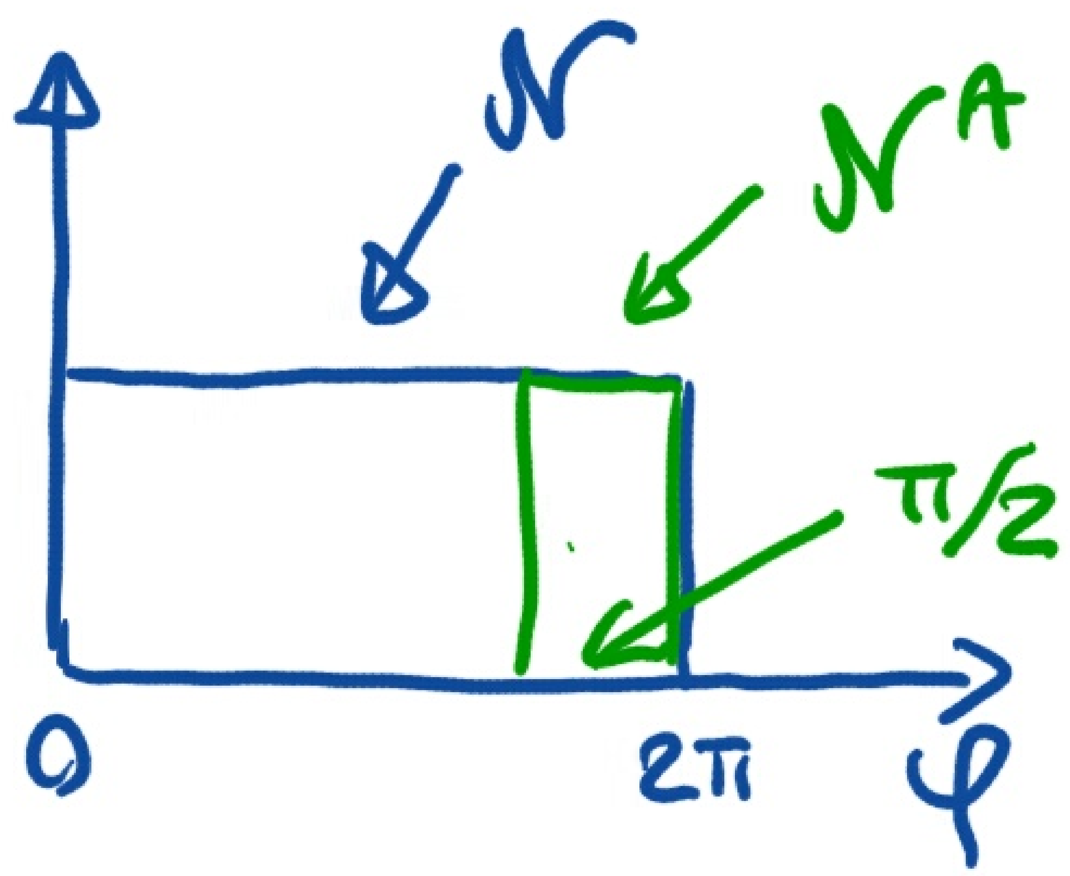
SINGLE PARTICLE METHOD:



$N_1^A = 1$ $\quad N_2^A = 0$ $\quad N_M^A = 1$

$$\mathcal{N}^A = \sum_{i}^{M} N_i^A$$

$$\mathcal{N} = \mathcal{N}^A \cdot (2\pi) / (\pi/2) = 4 \cdot \mathcal{N}^A$$



$$\langle N \rangle = \frac{1}{M} \cdot \mathcal{N}^A \cdot \left(\text{MODEL INDEPENDENT CORRECTION} \right)$$

MEAN MULTIPLICITY CAN BE MEASURED WITHOUT MEASURING MULTIPLICITIES EVENT-BY-EVENT

THIS EXPLAINS WHY RESULTS ON MEAN MULTIPLICTIES (AND THEIR MOMENTUM SPECTRA) ARE RICH AND OF HIGH QUALITY IN COMPARISON TO HIGHER MOMENTS

ACTUALLY MEAN (FIRST MOMENT) OF ANY EVENT QUANTITY WHICH CAN BE EXPRESSED AS A SUM OF SINGLE PARTICLE QUANITIES HAS THE SAME PROPERTIES AS MEAN MULTIPLICITY.

SECOND AND HIGHER MOMENTS REQUIRE MEASUREMENTS OF MULTIPLICITIES EVENT-BY-EVENT

THE SINGLE PARTICLE METHOD DOES NOT WORK FOR THEM.

MEAN - THE FIRST MOMENT:

$$\langle N^1 \rangle = \frac{1}{M} \sum_{i=1}^{M} \left(\sum_{=1}^{N_i} \delta_{t t_j} \right)^1 = \frac{1}{M} \sum_{j=1}^{\mathcal{N}} \delta_{t t_j}^1$$

$$(a+b)^1 = a^1 + b^1$$

THE SECOND MOMENT:

$$\langle N^2 \rangle = \frac{1}{M} \sum_{i=1}^{M} \left(\sum_{=1}^{N_i} \delta_{t t_j} \right)^2 \neq \frac{1}{M} \sum_{j=1}^{\mathcal{N}} \delta_{t t_j}^2$$

$$(a+b)^2 \neq a^2 + b^2$$

THE ABOVE EXPLAINS WHY TRADITIONALLY ONE LABELS RESULTS AS EVENT-BY-EVENT RESULTS ONLY IF THEY INCLUDE THE SECOND AND HIGER MOMENTS.

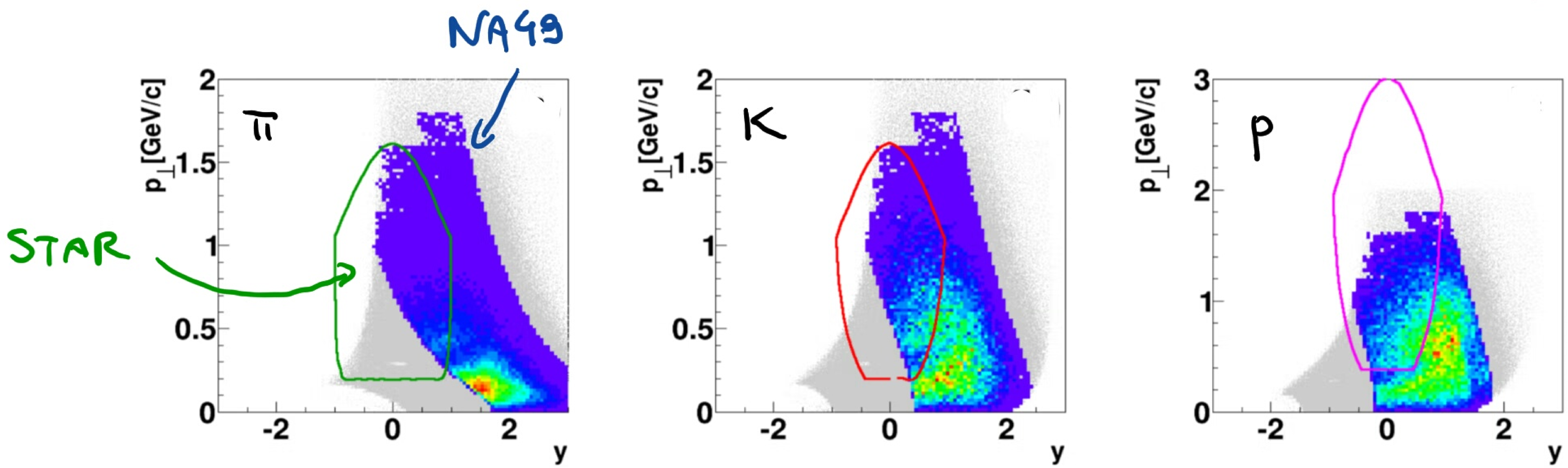
RESULTS ON THE FIRST MOMENT ARE CALLED RESULTS ON MEAN MULTIPLICITIES OR SINGLE PARTICLE SPECTRA (MEAN MULTIPLICITY IN MOMENTUM BINS)

- RESULTS ON SECOND AND HIGHER MOMENTS CANNOT BE CORRECTED IN A MODEL INDEPENDENT WAY FOR A LIMITED EXPERIMENTAL ACCEPTANCE

- THE RESULTS DEPEND ON ACCEPTANCE.

$\Downarrow$

DIFFICULT SITUATION FOR EXPERIMENTS WITH INCOMPLETE ACCEPTANCE (ALL PRESENT EXPERIMENTS) !



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