MEASURING FLUCTUATIONS: CORRECTIONS: IDENTITY METHOD

■ IMPERFECTNESS OF MEASUREMENTS

■ ■ THE SPECIAL CASE: INCOMPLETE PARTICLE IDENTIFICATION

■ ■ ■ THE IDENTITY METHOD

IMPERFECTNESS OF MEASUREMENTS

MEASUREMENTS ARE NEVER PERECT.
THEIR RESULTS DIFFER FROM A TRUE (REQUESTED, WANTED) RESULT DUE TO:

- STATISTICAL FLUCTUATIONS AND
- SYSTEMATIC BIASES.

CLEARLY, IN ORDER TO QUANTIFY, AND POSSIBLY CORRECT FOR THE DIFFERECES, ONE HAS TO DEFINE THE TRUE RESULT.
THIS IS NOT TRIVIAL BY ITSELF AND IT WILL BE DISCUSSED LATER.

STATISTICAL FLUCTUATIONS

STATISTICAL FLUCTUATIONS ARE DIFFERENCES BETWEEN THE TRUE RESULT DEFINED FOR AN INFINITE EVENT SAMPLE (UNIVERSE) DUE TO FINITE STATISTICS OF MEASURED EVENTS.

STATISTICAL FLUCTUATIONS DECREASE WITH THE EVENT STATISTICS M AS:

$$1/\sqrt{M}$$

THE ONLY WAY TO DECREASE ("CORRECT FOR") STATISTICAL FLUCTUATIONS IS TO INCREASE EVENT STATISTICS.

THERE ARE WELL DEFINED METHODS TO QUANTIFY A MAGNITUDE OF STATISTICAL FLUCTUATIONS (CALCULATE STATISTICAL UNCERTAINTIES), E.G., THE BOOTSTRAP METHOD (TO BE DISCUSSED LATER).

SYSTEMATIC BIASES

SYSTEMATIC BIASES ARE DIFFERENCES BETWEEN THE TRUE RESULT DEFINED FOR A GIVEN EVENT SAMPLE AND THE MEASURED ONE FOR THIS SAMPLE.

SYSTEMATIC BIASES ARE INDEPENDENT OF EVENT STATISTICS, AND THUS ARE TYPICALLY QUANTIFIED USING SIMULATED EVENTS WITH STATISTICS MANY TIMES LARGER THAN THE DATA STATISTICS.

HIGH STATISTICS OF SIMULATED EVENTS IS NEEDED TO REDUCE STATISTICAL FLUCTUATIONS OF DIFFERENCES BETWEEN PURE MODEL RESULTS (MC TRUE) AND PURE MODEL EVENTS PROCESSED BY DETECTOR SIMULATION, RECONSTRUCTION AND ANALYSIS CHAINS (MC MEASURED).

MEASURED RESULTS CAN/SHOULD BE CORRECTED FOR SYSTEMATIC BIASES. THE CORRESPONDING CORRECTIONS ARE NEVER PERFECT. POSSIBLE BIASES DUE TO IMPERFECTNESS OF THE CORRECTIONS ARE QUANTIFIED BY SYSTEMATIC UNCERTAINTIES.

SYSTEMATIC UNCERTAINTIES ARE MORE DIFFICULT TO CALCULATE THAN STATISTICAL ONES.
THE CORRESPONDING METHODS ARE NOT VERY WELL ESTABLISHED.

FOR MORE ON SYSTEMATIC UNCERTAINTIES SEE:

BARLOW, HEP-EX/020726
SYSTEMATIC ERRORS: FACTS AND FICTIONS

SYSTEMATIC BIASES:
MULTIPLICITY DISTRIBUTION $M(N)$

N_T – TRUE EVENT MULTIPLICITY
N_M – MEASURED EVENT MULTIPLICITY

$M_T(N_T)$ – TRUE MULTIPLICITY DISTRIBUTION
$M_M(N_M)$ – MEASURED MULTIPLICITY DISTRIBUTION

$$M_T = \sum_{N_T} M_T(N_T)$$

$$M_M = \sum_{N_M} M_M(N_M)$$

$$P(M_T) = M_T(N_T) / M_T$$

$$P(M_M) = M_M(N_M) / M_M$$

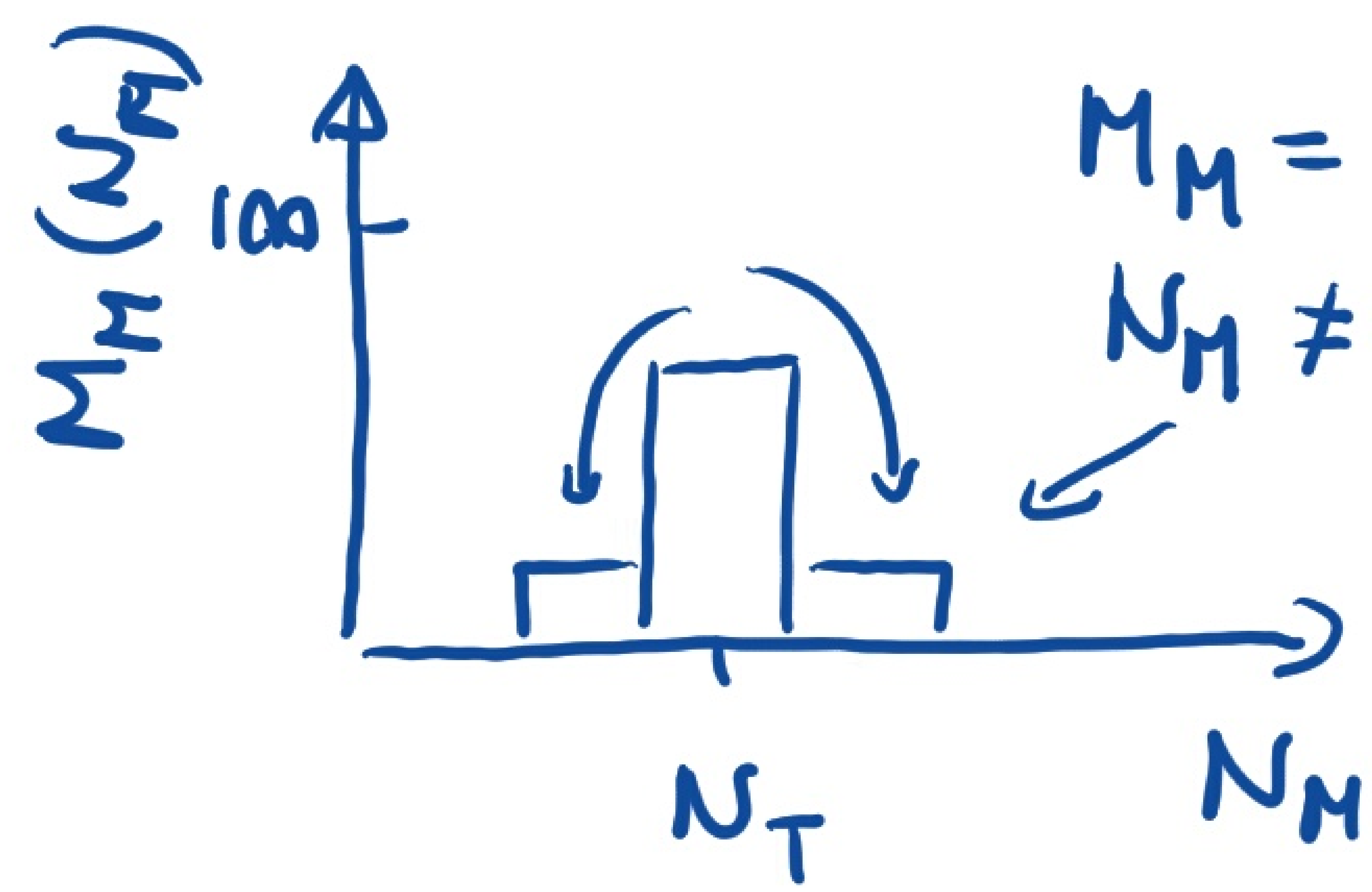
TWO CASES OF BIASES

- ALL TRUE EVENTS AND ONLY TRUE EVENTS ARE MEASURED $M_M = M_T$ (EVENT DETECTION EFF. = 1) BUT MEASURED EVENT MULTIPLICITIES ARE BIASED $N_M \neq N_T$ E.G. DUE TO:

  - TRUE TRACK DETECTION EFFICIENCY < 1, $N \searrow$
  - MEASURED TRACKS INCLUDE TRACKS FROM SECONDARY INTERACTIONS, WEAK DECAYS $N \nearrow$
  - INCOMPLETE PARTICLE IDENTIFICATION $N \nearrow \searrow$

  TO BE DISCUSSED NEXT

$M_T(N_T) = 100$



$M_M = M_T$
$N_M \neq N_T$

EVENT MIGRATION

USUALLY EVENT MIGRATION IS SIGNIFICANT:

EXAMPLE: FOR A SINGLE BIN DISTRIBUTION AT N_T AND A SINGLE TRACK DETECTION EFFICIENCY ε

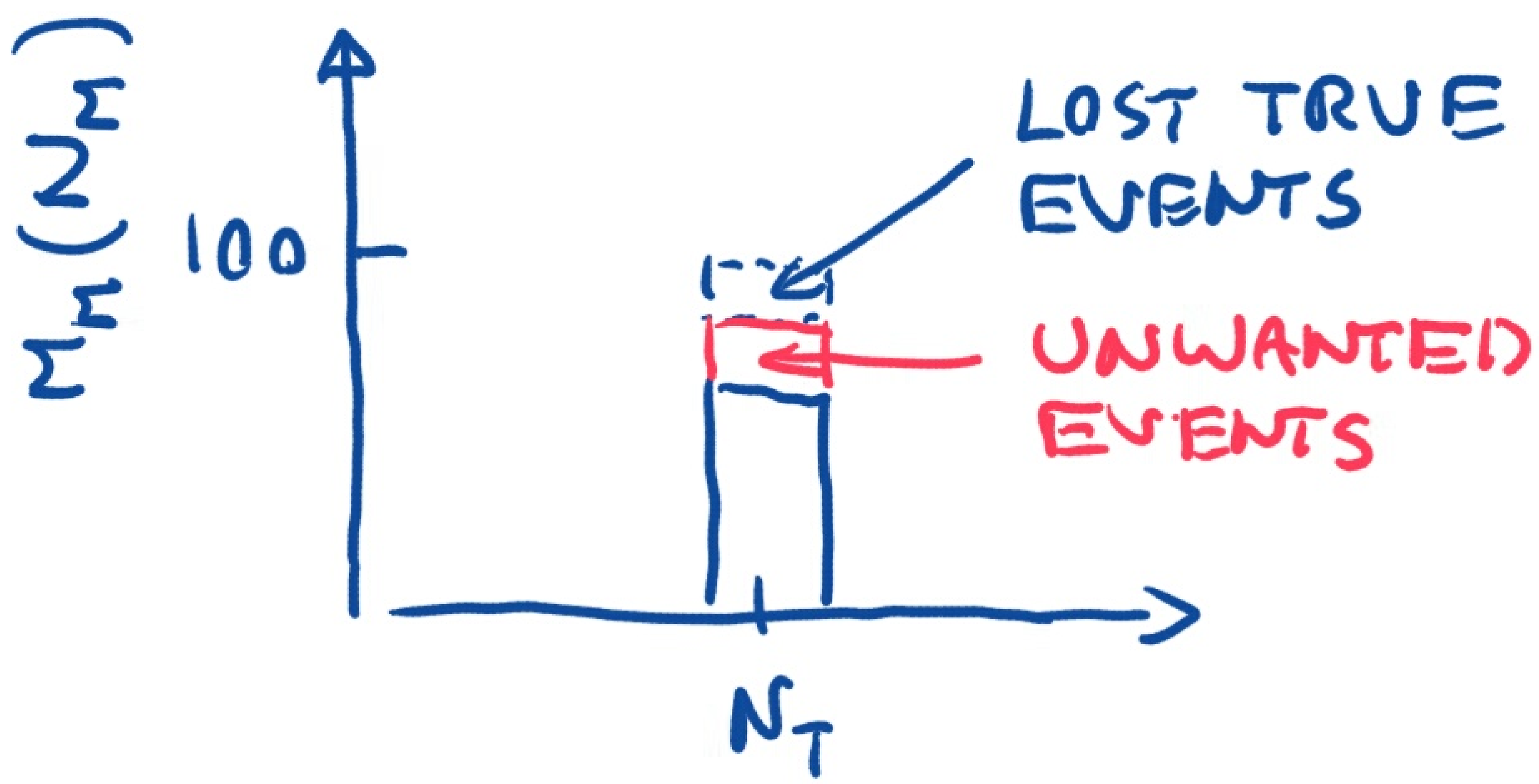
$\Rightarrow$ (PROBABILITY OF $N_M \neq N_T$) $= P_{MIGRATION} = 1 - \varepsilon^{N_T}$

FOR $\varepsilon = 0.9$ AND $N_T = 10$ $\quad P_{MIGRATION} \approx 0.65$

— " — $N_T = 100$ $\quad P_{MIGRATION} \approx 1$

FOR A REALISTIC DISTRIBUTION WITH MANY NON-EMPTY BINS THE MIGRATION EFFECT IS REDUCED DUE TO MIGRATION OF EVENTS FROM OTHER BINS TO A BIN OF INTREST.

- MULTIPLICITY OF ALL EVENTS IS CORRECTLY MEASURED, $N_M = N_T$
  BUT SOME EVENTS ARE LOST AND SOME UNWANTED EVENTS ARE MEASURED, E.G., DUE TO
  - TRIGGER INEFFICIENCY $M \searrow$
  - OFF-TARGET, OFF-TIME INTERACTIONS $M \nearrow$



LOST/UNWANTED EVENTS

IN A REAL EXPERIMENT BIASES DUE TO BOTH EVENT MIGRATION AND LOST/UNWANTED EVENTS ARE PRESENT.

UNFOLDING: THE STANDART CORRECTION METHOD

(DECONVOLUTION, UNSMEARING)

THE MEASURED $P(N_M)$ CAN BE RELATED TO THE TRUE ONE $P(N_T)$ BY INTRODUCING A RESPONSE MATRIX, $R[N_M, N_T]$

$$\overrightarrow{M_M(N_M)} = \sum_{N_T} R[N_M, N_T] \, \overrightarrow{M_T[N_T]}$$

THIS INVOLVES BIASES DUE TO:

- LOST EVENTS
- EVENT MIGRATION (OFF-DIAGONAL ELEMENTS)

GIVEN R (E.G. CALCULATED USING MC) ONE CAN INVERT IT (R^{-1}) AND CALCULATE THE TRUE DISTRIBUTION AS

$$\overrightarrow{M_T(N_T)} = \sum_{N_M} R^{-1}[N_M, N_T] \, \overrightarrow{M_M(N_M)}$$

THE RESPONSE MATRIX IS USUALLY CALCULATED USING A MONTE CARLO SIMULATION.

FOR 1D DATA, A 2D HISTOGAM IS FILLED WITH MC EVENTS:

$$\begin{array}{cc} N_M \uparrow & \begin{bmatrix} \\ R[N_M, N_T] \\ \\ \end{bmatrix} \\ & \longrightarrow N_T \end{array}$$

EACH N_T COLUMN SHOULD BE NORMALIZE TO ITS MEASUREMENT EFFICIENCY.

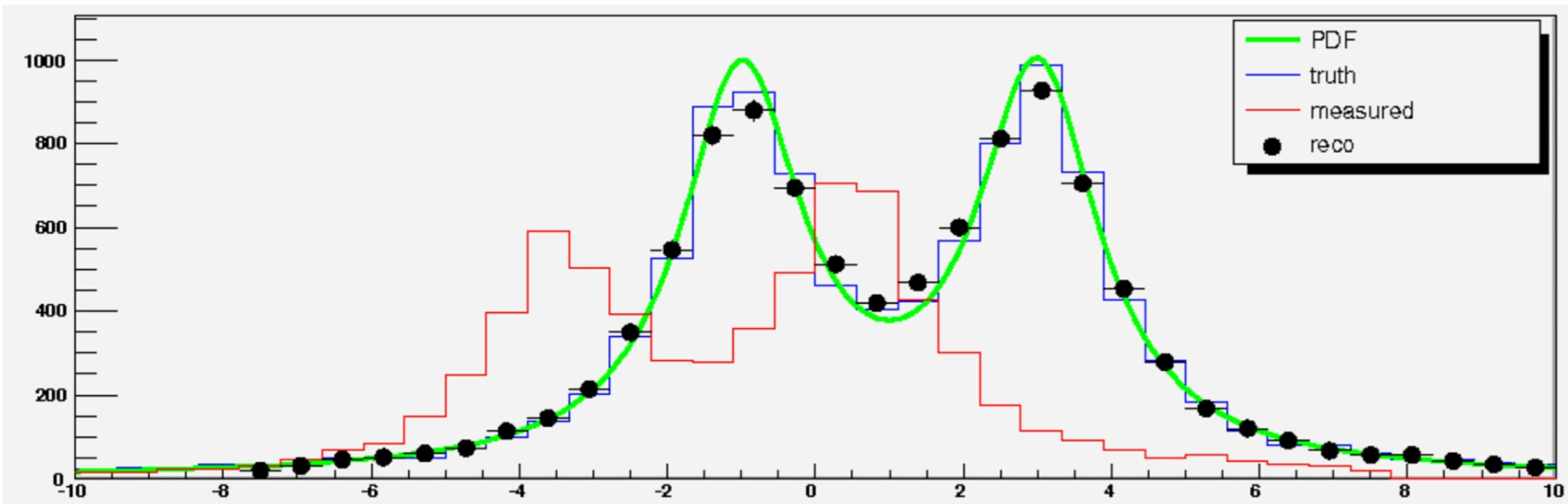
AN EVENT IS EITHER MEASURED WITH A VALUE N_M, OR ACCOUNTED IN INEFFICIENCY.

BEFORE UNFOLDING THE $M_M[N_M]$ DISTRIBUTION SHOULD BE CORRECTED FOR A CONTAMINATION OF UNWANTED EVENTS (DATA OR MC BASED CORRECTION)

THEN:

$$\begin{bmatrix} M_T(0) \\ M_T(1) \\ \vdots \\ \vdots \\ M_T(N_T^{MAX}) \end{bmatrix} = \begin{bmatrix} \\ R^{-1}[N_M, N_T] \\ \\ \end{bmatrix} \cdot \begin{bmatrix} M_M(0) \\ M_M(1) \\ \vdots \\ \vdots \\ M_M(N_M^{MAX}) \end{bmatrix}$$

EXAMPLE :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+ WITH INFINITE STATISTICS (DATA AND MONTE CARLO) IT IS POSSIBLE TO RECONSTRUCT THE TRUE DISTRIBUTION CORRECTLY.

– HOWEVER, FOR A LIMITED NUMBER OF EVENTS STATISTICAL FLUCTUATIONS APPEAR AND THEY ARE WASHED OUT BY THE EVENT MIGRATION. CONSEQUENTLY, R^{-1} CAN NOT DISTINGUISH BETWEEN WIDELY FLUCTUATING AND SMOOTH $P(N_T)$ (METHODS HELPING TO SUPPRESS NON-PHYSICAL SOLUTIONS EXIST)

– MOREOVER, UNFOLDING OF MULTI-DIMENSIONAL DATA IS DIFFICULT TO IMPLEMENTED, USUALLY ONLY 1D AND 2D CASES ARE CODED.

+ READY TO USE SOFTWARE: <u>RooUnfold</u>

THE SPECIAL CASE: INCOMPLETE PARTICLE IDENTIFICATION

START WITH A SIMPLE REQUEST:
MEASURE MULTIPLICITY DISTRIBUTION OF K^+ MESONS

$\Rightarrow$ NEED TO MEASURE K^+ MULTIPLICITY EVENT-BY-EVENT

$\Rightarrow$ NEED TO IDENTIFIED K^+ MESONS AMONG ALL POSITIVELY CHARGED PARTICLES (p, K^+, π^+, e^+) BY MEASURING PARTICLE MASS

PARTICLE MASS MEASURMENT

$$p = m \; \beta / \sqrt{1-\beta^2}$$

$\Rightarrow$ THE METHOD:

MEASURE p $\leftarrow$ CHARGE PARTICLE TRAJECTORY IN A MAGNETIC FIELD

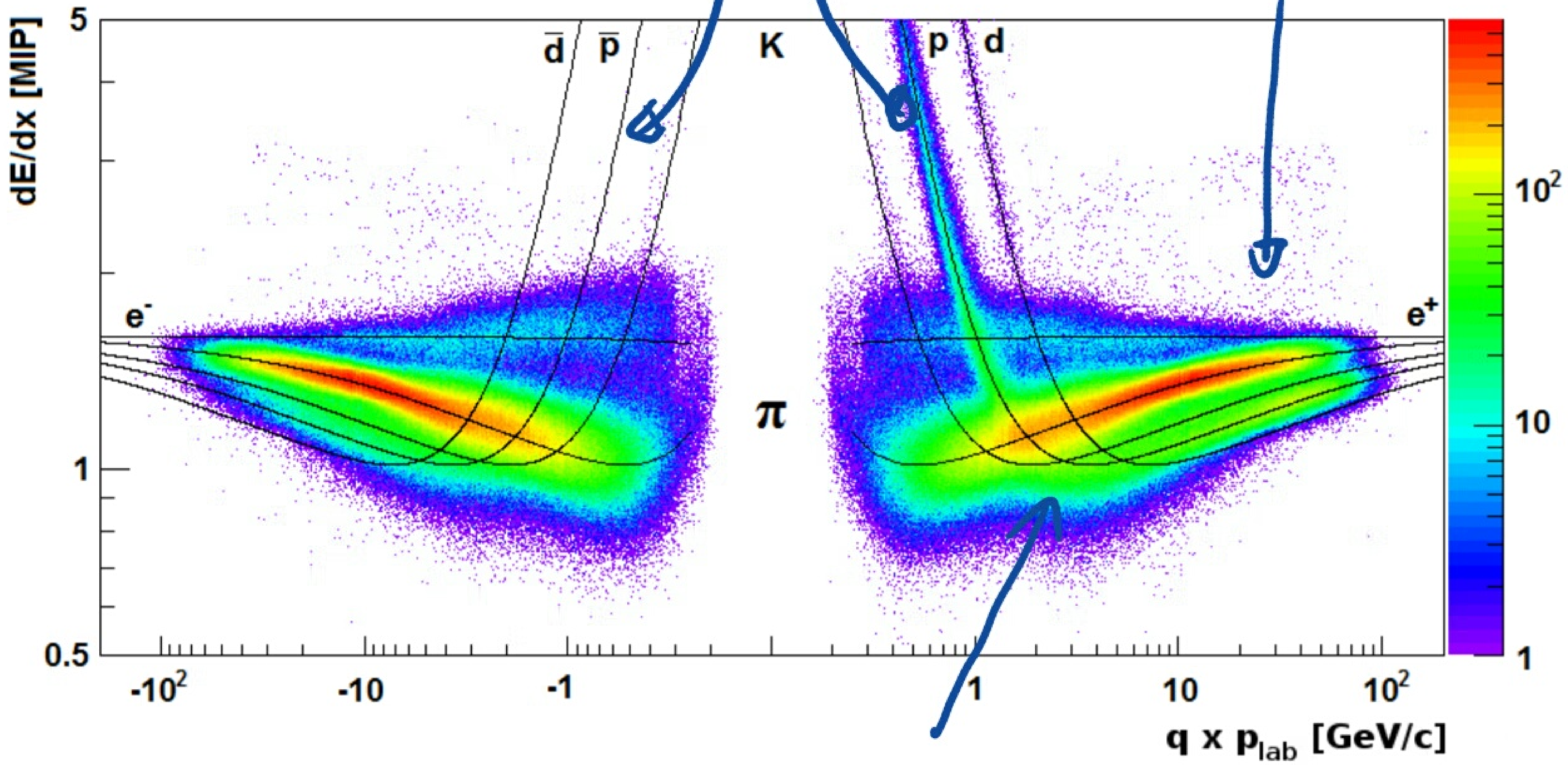
MEASURE β $\leftarrow$ SPECIFIC ENERGY LOSS $\left(dE/dx\,(\beta) \right)$, TIME-OF-FLIGHT $\left(\beta = L/t \right)$

CALCULATE m

EXAMPLE: MASS MEASURMENT VIA dE/dx

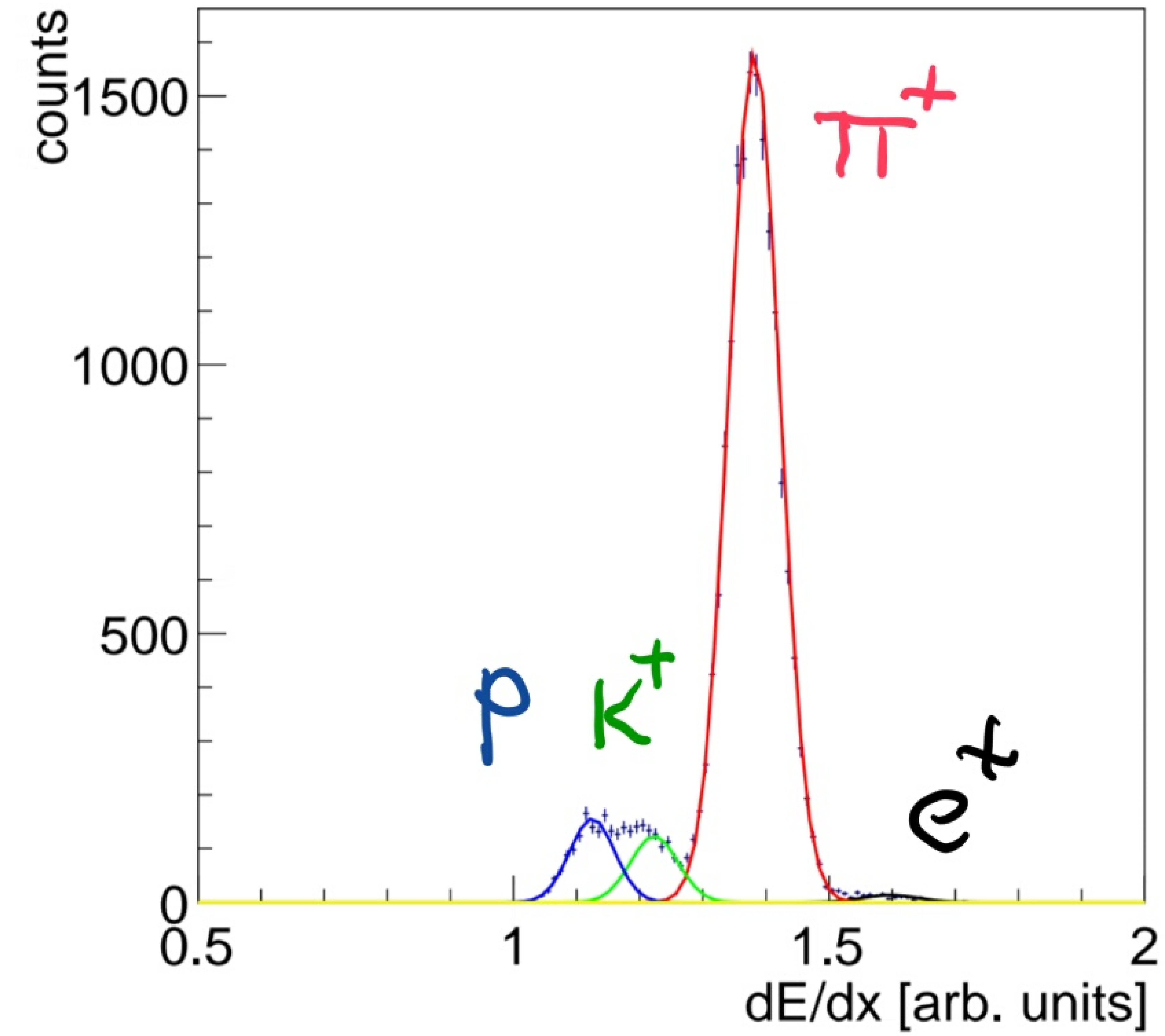
$1/\beta^2$ REGION

RELATIVISTIC RISE REGION



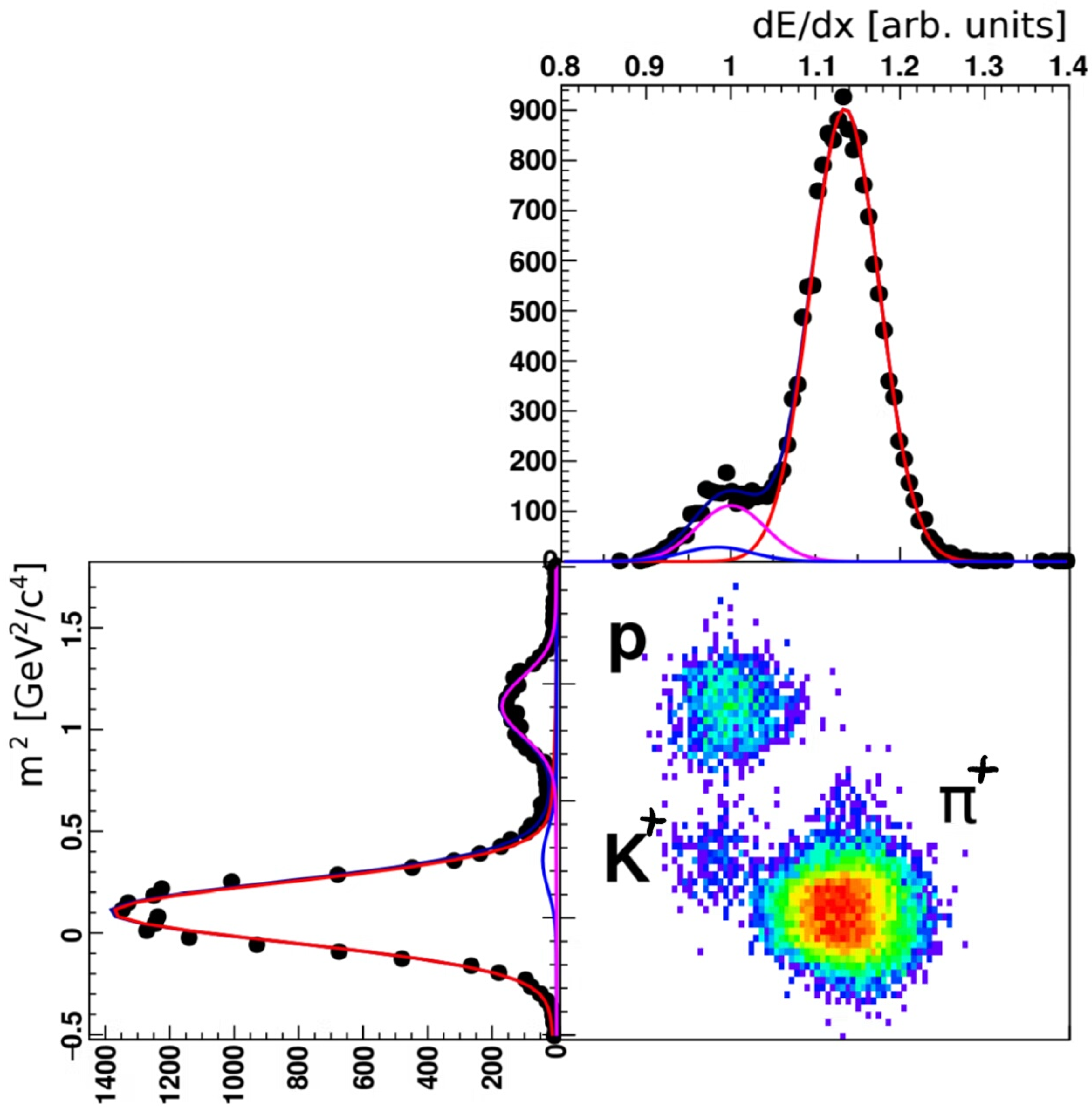
CROSS-OVER REGION

Δp IN RR REGION



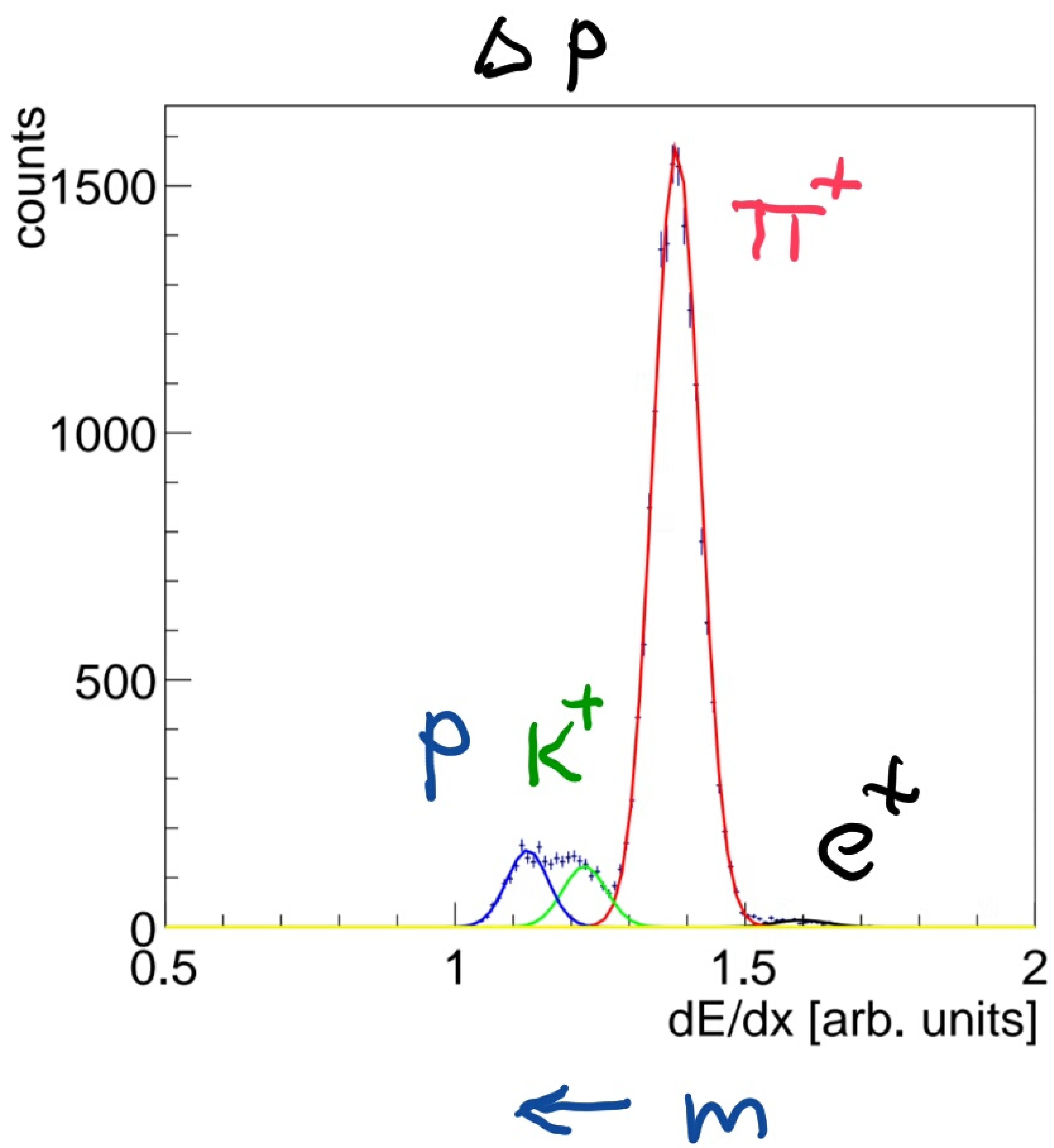
$\leftarrow m$

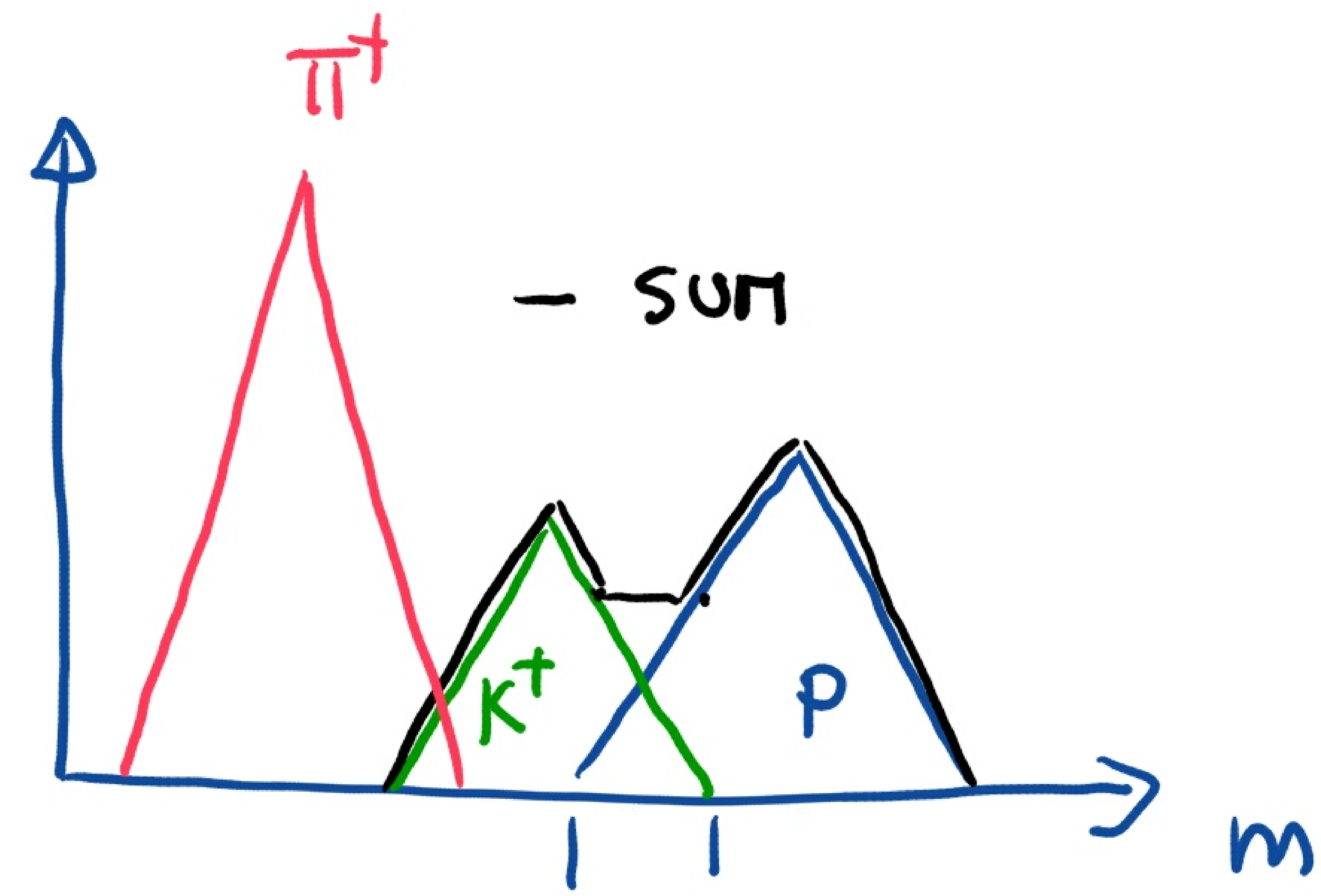
USUALLY A RESOLUTION OF MASS MEASURMENTS IS NOT SUFFICIENT TO UNIQUELY IDENTFY ALL PARTICLES



COMBINING DIFFERENT TECHNICS HELPS, BUT STILL NOT PERFECT, WORKS IN A LIMITED ACCEPTANCE, EXPENSIVE

THE QUESTION IS HOW TO COUNT IDENTIFIED PARTICLES (E.G. K^+ MESONS) IN THE CASE YOU CANNOT UNIQUELY IDENTIFY THEM BECAUSE THEIR MASS SPECTRA OVERLAP.



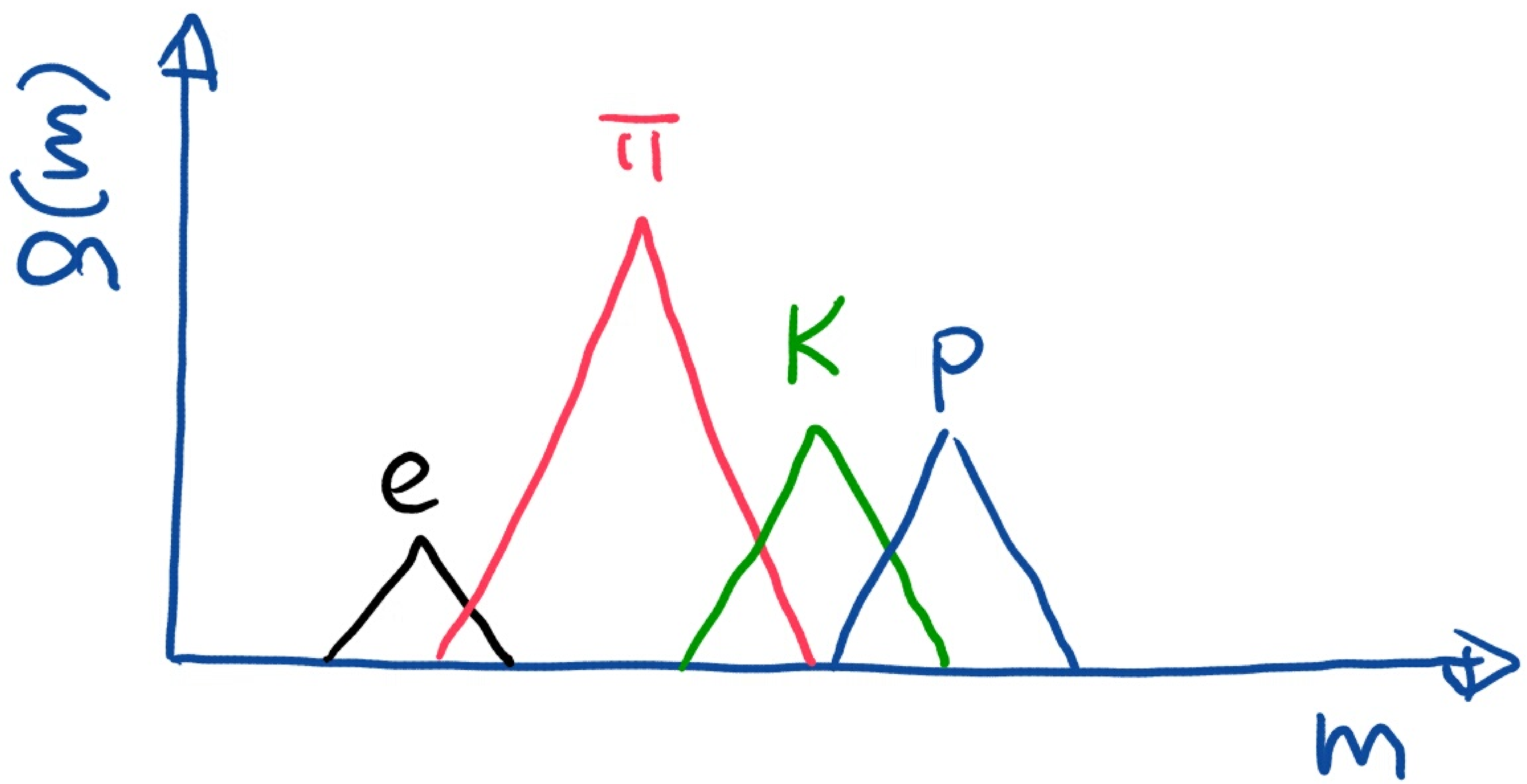


INCOMPLETE PARTICLE IDENTIFICATION

→ THE IDENTITY METHOD

THE IDENTITY METHOD

ASSUME MASS DENSITY FUNCTIONS
$\rho_e(m)$, $\rho_\pi(m)$, $\rho_k(m)$, $\rho_p(m)$, ARE GIVEN:



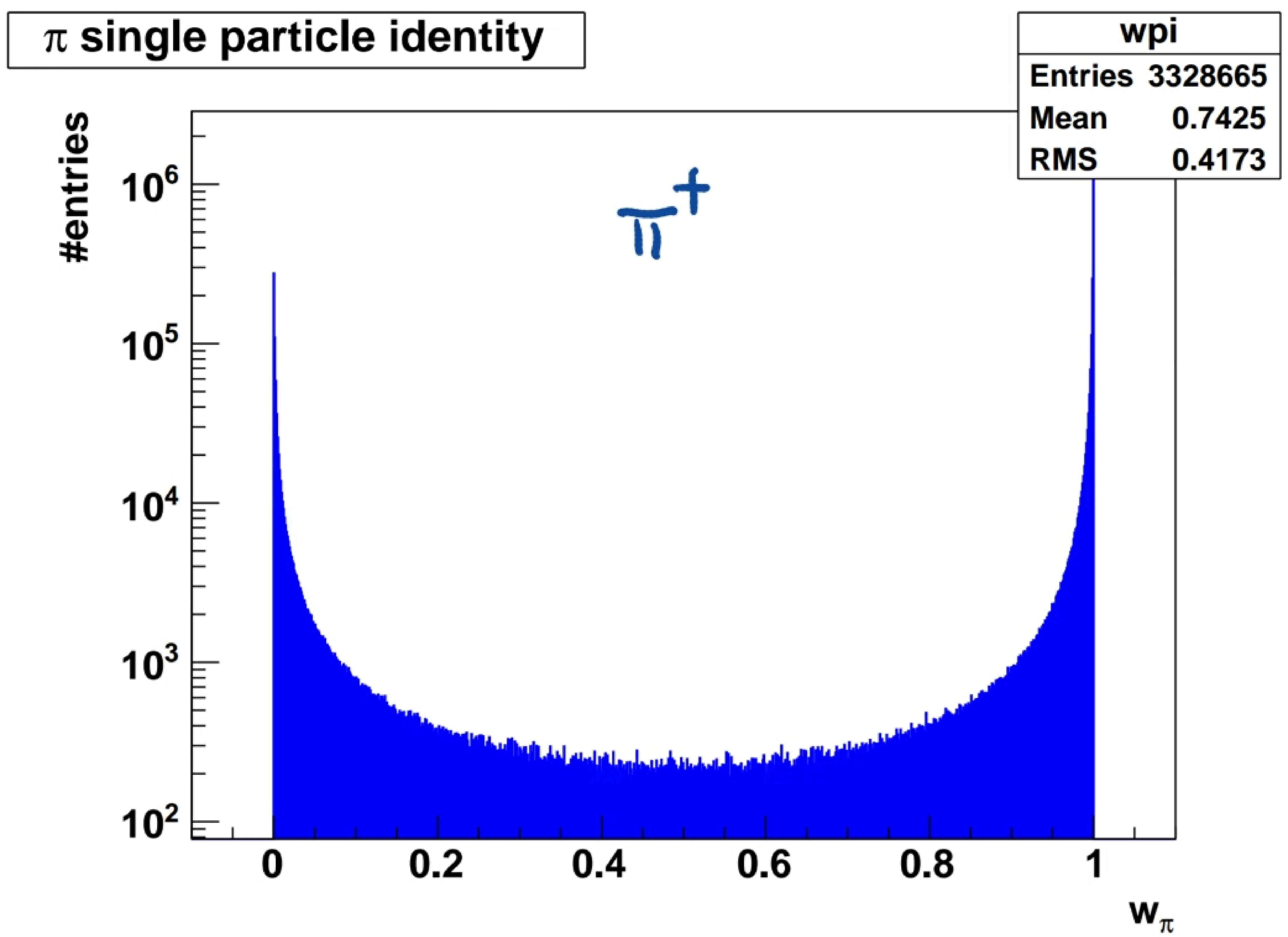
$$\int \rho_j(m)\,dm = \langle N_j \rangle$$

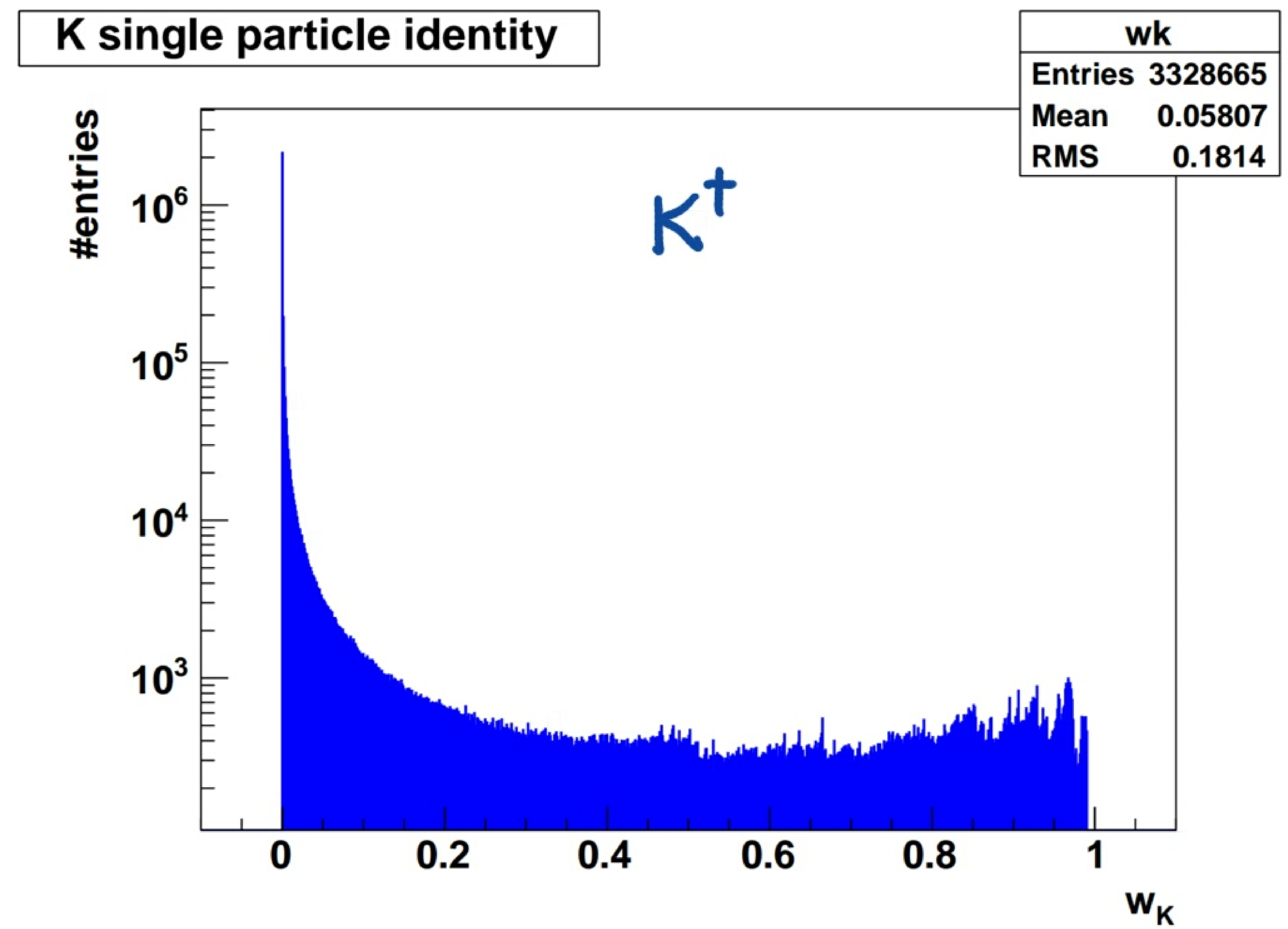
$$j = e, \pi, k, p$$

THEN ONE DEFINES A PARTICLE IDENTITY AS:

$$w_j(m) \equiv \rho_j(m) / \left(\sum_i \rho_i(m) \right), \qquad 0 \leq w_j(m) \leq 1$$

PARTICLE IDENTITY DISTRIBUTION USING MASS MEASUREMENT VIA dE/dx IN RELATIVISTIC MASS REGION: EXAMPLES





THERE ARE SIGNIFICANT REGIONS OF ALMOST UNIQUE IDENTIFICATION OF π^+ and p
(MAXIMA OF w_π, w_p at ≈ 1)

K^+ IDENTIFICATION IS POOR

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THEN AN "EVENT MULTIPLICITY" OF IDENTIFIED PARTICLES IN THE CASE OF INCOMPLETE IDENTIFICATION CAN BE INTRODUCED:

$$W_j = \sum_{i=1}^{N_M} w_j(m_i) \quad \leftarrow \text{IDENTITY EVENT MULTIPLICITY}$$

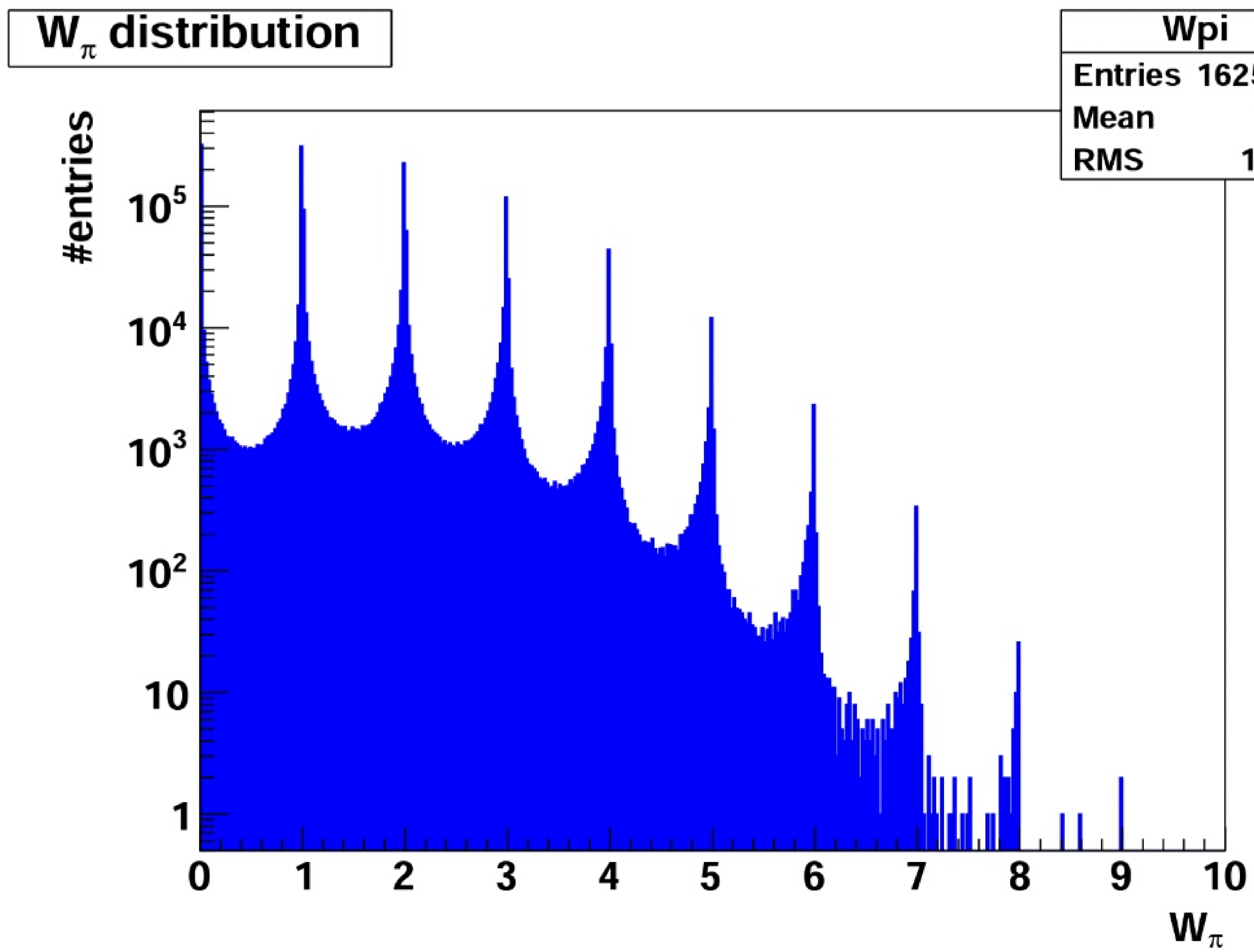
HERE THE SUM RUNS OVER N_M PARTICLES MEASURED IN AN EVENT.

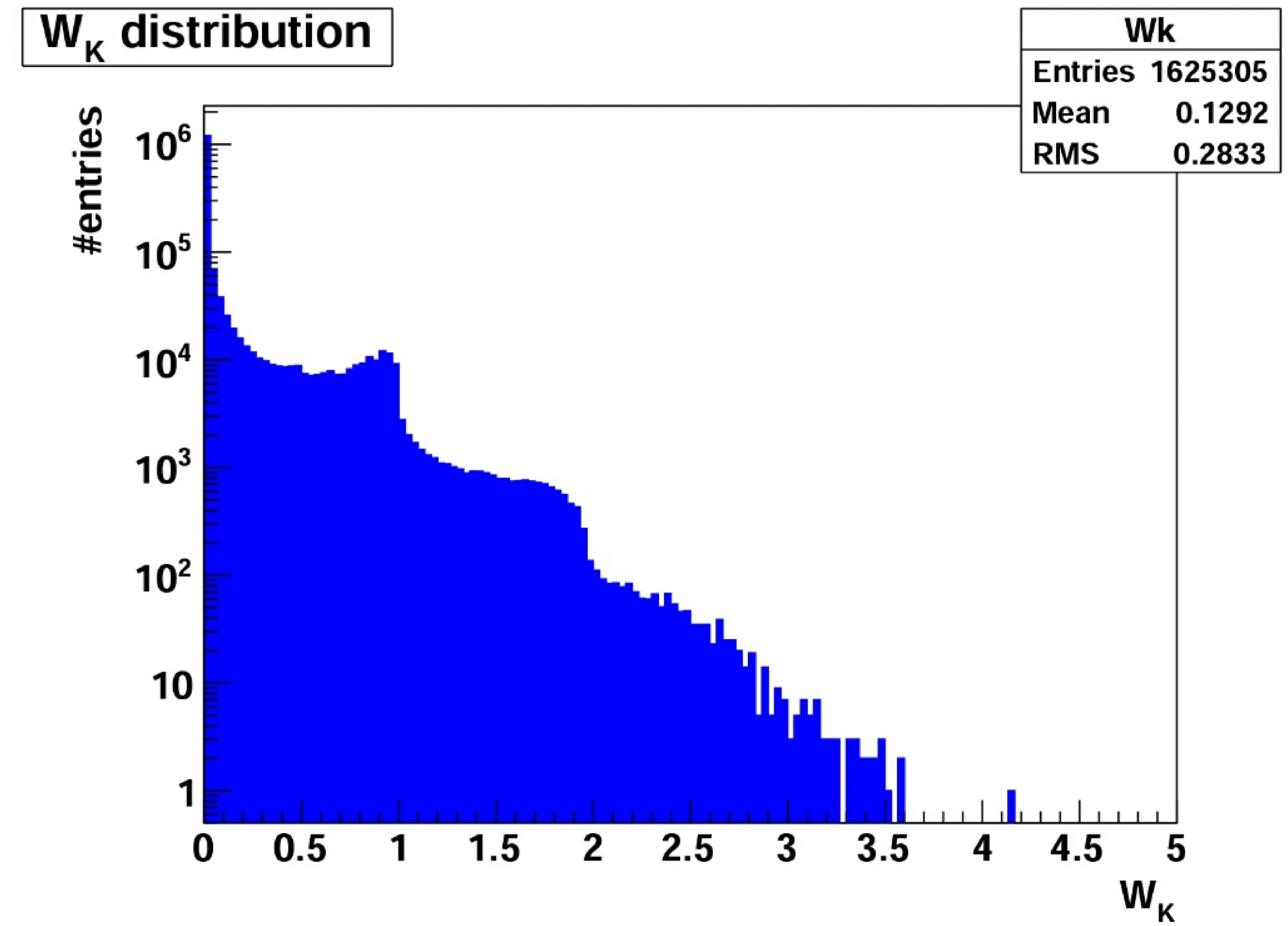
NOTE, THAT UNLIKE TRUE EVENT MULTIPLICITY, THE MEASURED IDENTITY MULTIPLICITY IS A REAL NON-NEGATIVE NUMBER.

THE INTRODUCTION OF THE IDENTITY MULTIPLICITY IS THE FIRST ELEMENT OF THE IDENTITY METHOD.

GAZDZICKI, GREBIESZKOW, MACKOWIAK, MROWCZYNSKI
PRC83 (2011) 054907

EXAMPLES OF 1D IDENTITY MULTIPLICITY DISTRIBUTIONS





NOTE ASSYMETRIC SMEARING OF THE TRUE $M_I(N)$

STILL $\underline{\langle N_j \rangle = \langle W_j \rangle}$

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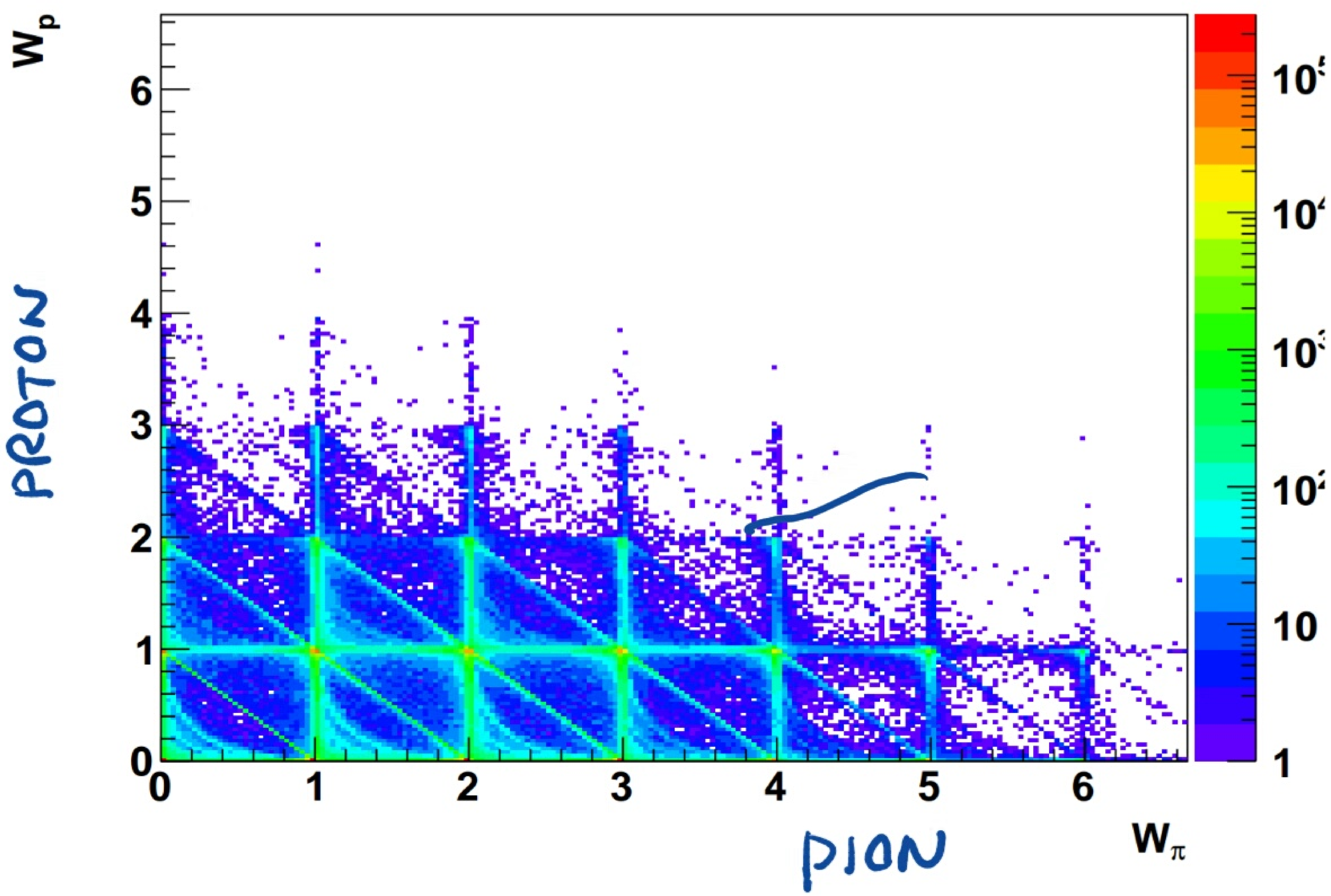
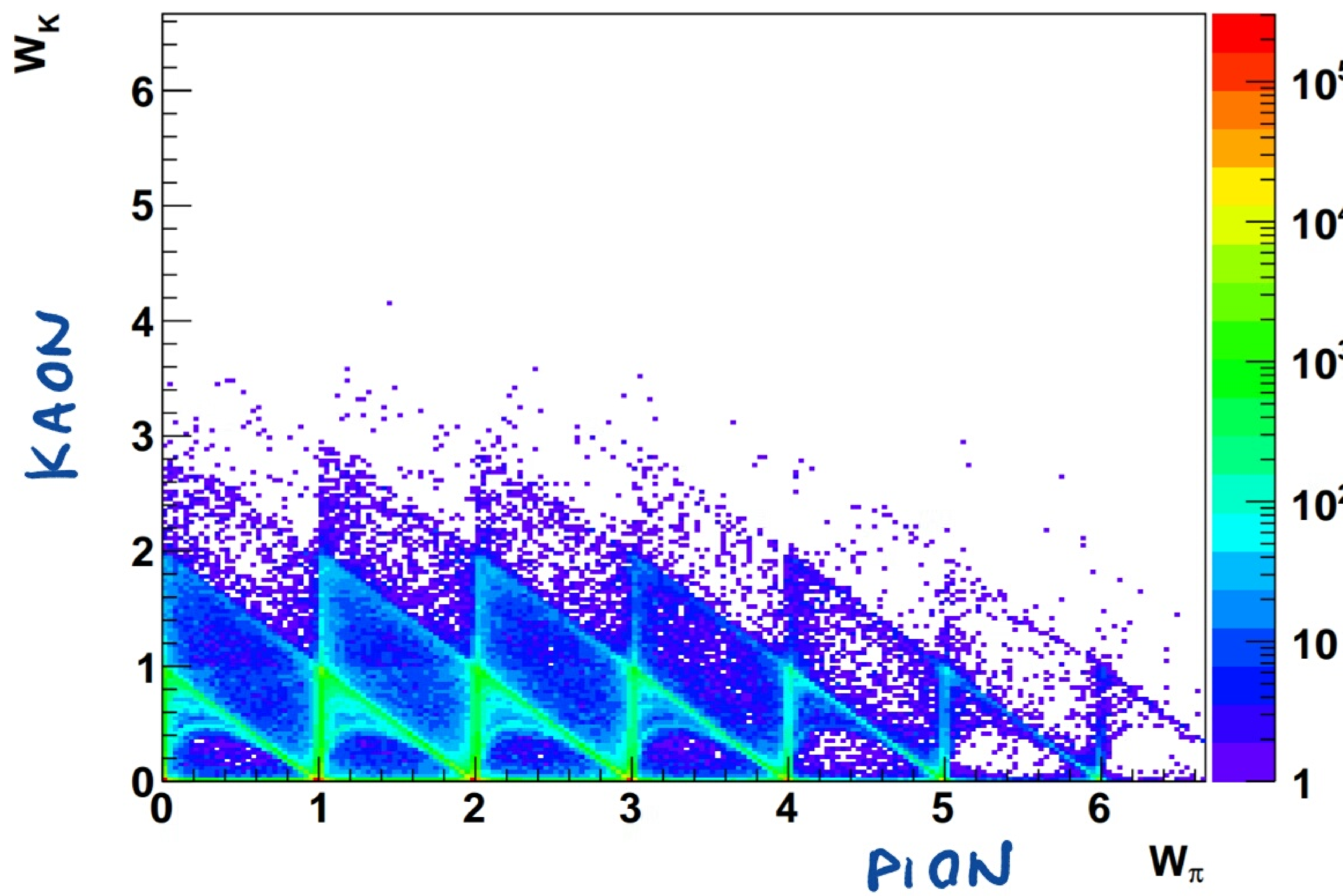
EXAMPLES OF 2D IDENTITY MULTIPLICITY DISTRIBUTIONS





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FOR AN EVENT SAMPLE THE BINNED IDENTITY MULTIPLICITY DISTRIBUTION CAN BE OBTAINED:

$M_M(\bar{\omega}_e^B, \bar{\omega}_\pi^B, \bar{\omega}_K^B, \bar{\omega}_p^B)$, WHERE $\bar{\omega}_j^B$ LABELS BINS IN ω_j

THEN, IN THEORY THE UNFOLDING CAN BE USED TO GET THE TRUE MULTIPLICITY DISTRIBUTION:

$$M_T(\vec{N}) = R^{-1}[\vec{\omega}, \vec{N}] \times M_M(\vec{\omega}^B)$$

WHERE

$$\vec{\omega}^B = (\bar{\omega}_e^B, \bar{\omega}_\pi^B, \bar{\omega}_K^B, \bar{\omega}_p^B)$$

$$\vec{N} = (N_e, N_\pi, N_K, N_p)$$

... BUT THE DATA ARE 4D AND THE RESPONSE MATRIX 8D.

I AM NOT AWARE OF ANY NUMERICAL PROCEDURE WHICH CAN BE USED IN THIS CASE.

⇓

A NEW METHOD IS NEEDED:

⇓

THE MOMENT UNFOLDING – THE SECOND ELEMENT OF THE IDENTITY METHOD.

THE MOMENT UNFOLDING : THE SECOND MOMENTS

GORENSTEIN, PR C84 (2011) 024902

- DEFINE VECTORS OF TRUE AND MEASURED MOMENTS

$$\overrightarrow{\langle N_j N_k \rangle} \equiv \begin{bmatrix} \langle N_e^2 \rangle \\ \langle N_\pi^2 \rangle \\ \langle N_K^2 \rangle \\ \langle N_p^2 \rangle \\ \langle N_e N_\pi \rangle \\ \langle N_e N_K \rangle \\ \langle N_e N_p \rangle \\ \langle N_\pi N_K \rangle \\ \langle N_\pi N_p \rangle \\ \langle N_K N_p \rangle \end{bmatrix} \quad \text{AND} \quad \overrightarrow{\langle W_j W_k \rangle} \equiv \begin{bmatrix} \langle W_e^2 \rangle \\ \vdots \\ \langle W_K W_p \rangle \end{bmatrix}$$

- CALCULATE THE MOMENT RESPONSE MATRIX

$$R^{-1}\left[\overrightarrow{\langle W_j W_k \rangle}, \overrightarrow{\langle N_j N_k \rangle}\right]$$

THE MOMENT RESPONSE MATRIX IS CALCULATED USING THE MASS DENSITY FUNCTIONS EITHER
BY A NUMERICAL INTEGRATION
(IMPLEMENTED IN TIDENTITY METHOD, RUSTAMOV)
BY A MONTE CARLO SIMULATION
(UP TO NOW NOT TRIED)

- INVERSE THE MOMENT RESPONSE MATRIX

CALCULATE TRUE MOMENTS:

- $$\overrightarrow{\langle N_j N_k \rangle} = R^{-1}\left[\overrightarrow{\langle W_j W_k \rangle}, \overrightarrow{\langle N_j N_k \rangle}\right] \overrightarrow{\langle W_j W_k \rangle}$$

... OTHER MOMENTS ALSO CAN BE UNFOLDED

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THE FIRST MOMENTS:

$$\begin{bmatrix} \langle N_e \rangle \\ \langle N_\pi \rangle \\ \langle N_K \rangle \\ \langle N_p \rangle \end{bmatrix} = \begin{bmatrix} R^{-1}\left[\overline{\langle W \rangle}, \overline{\langle N \rangle}\right] \end{bmatrix} \times \begin{bmatrix} \langle W_e \rangle \\ \langle W_\pi \rangle \\ \langle W_K \rangle \\ \langle W_p \rangle \end{bmatrix}$$

HERE INSTEAD OF MASS DENSITY FUNCTIONS, MASS PDFs ARE USED TO CALCULATE IDENTITY

AND HIGHER MOMENTS

...

$\Leftarrow$ $g_j(m)$ AND LOWER ORDER MOMENTS ARE NEEDED

FOR ALL MOMENTS THE DATA IS 1D (AND THE RESPONSE MATRIX 2D) THUS THERE ARE NO TECHNICAL PROBLEMS TO IMPLEMENT THE MOMENT UNFOLDING.

IN GENERAL, IN ADDITION TO THE BIAS DUE TO INCOMPLETE PARTICLE IDENTIFICATION, THERE ARE OTHER BIASES.

IT SEEMS TO BE POSSIBLE TO GENERALIZE THE IDENTITY METHOD TO INCLUDE CORRECTIONS FOR OTHER BIASES

HOMEWORK !

REFERENCES TO THE IDENTITY METHOD

THE FIRST IDEA:

GAZDZICKI, GREBIESZKOW, MACKOWIAK, MROWCZYNSKI
PRC83 (2011) 054907

2ND MOMENTS:

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OTHER MOMENTS:

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