MEASURING FLUCTUATIONS: UNCERTAINTIES

- STATISTICAL UNCERTAINTIES
- SYSTEMATIC UNCERTAINTIES

STATISTICAL UNCERTAINTIES

STATISTICAL UNCERTAINTIES (ERRORS) QUANTIFY DIFFERENCES (STATISTICAL FLUCTUATIONS BETWEEN A TRUE RESULT FOR A FINITE EVENT SAMPLE AND ONE FOR THE INFINITE SAMPLE OF EVENTS.

STATISTICAL FLUCTUATIONS OF THE MEASURED RESULTS (UNAVOIDABLY FOR A FINITE SAMPLE OF EVENTS) ARE CAUSED BY:

- INDETERMINISTIC NATURE OF COLLISIONS AT HIGH ENERGIES
  (E.G. EVENT MULTIPLICITY VARIES RANDOMLY EVENT-BY-EVENT)
- STATISTICAL FLUCTUATIONS IN A MEASUREMENT PROCESS
  (E.G. MEASURED dE/dx VARIES RANDOMLY AROUD ITS EXPECTED VALUE)

THE ASSUMPTION OF INDEPENDENT EVENTS:

STATISTICAL FLUCTUATIONS (DUE TO PROPERTIES OF STUDIED PHYSICS AND MEASURMENTS) TAKE PLACE INDEPENDENDENTLY IN INDIVIDUAL EVENTS.
THEY DO NOT CORRELATE EVENT QUANTITIES MEASURED IN DIFFERENT EVENTS.

E.G. FOR MEASURED EVENT MULTIPLICTIES, N_i $(i=1,\ldots M)$, ONE GETS:

$$P(N_1, N_2 \ldots N_M) = P(N_1)\cdot P(N_2) \ldots P(N_M)$$

WHERE M IS NUMBER OF EVENTS IN A SAMPLE

DISTRIBUTION OF EVENT QUANTITY

THEN A NUMBER OF EVENTS WITH MULTIPLICITY N, M(N) IS DISTRIBUTED APPROXMATELY ACCORDING TO

THE BINOMIAL DISTRIBUTION:

$$P(M(N)) = \binom{M}{M(N)} q^{M(N)} \cdot (1-q)^{M-M(N)},$$

WHERE $q = M(N)/M \approx \langle M(N) \rangle / M$

THIS DISTRIBUTION OF M(N) IS EXPECTED WHEN REPEATING THE EXPERIMENT (MEASUREMENT OF M EVENTS) MANY TIMES.

THE VARIANCE OF THE M(N) DISTRIBUTION IS

$$\mathrm{Var}[M(N)] = M \cdot q(1-q) = M(N)\left(1 - M(N)/M\right)$$

FOR $q \ll 1$: BINOMIAL → POISSON

WITH $\mathrm{Var}[M(N)] \approx M(N)$

AND, IN ADDITION, FOR $M(N) \gg 1$: POISSON → GAUSS

WITH $\sigma \approx \sqrt{\mathrm{Var}[M(N)]} \approx \sqrt{M(N)}$

POPULAR APPROXIMATION OF STATISTICAL ERRORS OF DISTRIBUTIONS OF EVENT QUANTITIES (VALID UNDER ABOVE ASSUMPTIONS)

A RELATIVE STATISTICAL ERROR OF $M(N)$,

$\sigma(M(N))/M(N)$ IS:

$$\sigma(M(N))/M(N) = \sqrt{M(N)}/M(N) = 1/\sqrt{M(N)} \sim 1/\sqrt{M}$$

SIMILARLY:

$$\sigma(P(N)) \approx P(N) \cdot 1/\sqrt{M}$$

$$\Downarrow$$

STATISTICAL UNCERTANTIES OF NORMALIZED DISTRIBUTIONS OF EVENT QUANTITIES DECREASE WITH THE NUMBER OF EVENTS M AS:

$$1/\sqrt{M}$$

MEASURES OF E-BY-E FLUCTUATIONS

DISTRIBUTIONS OF EVENT QUANTITIES ARE FREQUENTLY CHARACTRERIZED BY THEIR MOMENTS AND FUNCTIONS OF THEM – MEASURES OF E-BY-E FLUCTUATIONS.

THEY MAY INVOLVE QUANTITIES WHICH ARE CORRELATED WITH RESPECT OF STATISTICAL FLUCTUATIONS.
THUS THE ERROR PROPAGATION FROM $P(M(N))$ TO MEASURES OF FLUCTUATIONS MAY BE DIFFICULT OR EVEN ALMOST IMPOSSIBLE

THUS IT IS POPULAR TO CALCULATE STATISTICAL UNCERTAINTIES OF FLUCTUATION MEASURES USING EITHER THE SUBSAMPLE OR BOOTSTRAR METHODS.

SUBSAMPLE METHOD

DIVIDE THE TOTAL SAMPLE OF M EVENTS INTO K SUBSAMPLES EACH WITH $M_S = M/K$ EVENTS

EVENTS ARE INDEPENDENT WITH RESPECT OF STATISTICAL FLUCTUATIONS AND THUS SUBSAMPLES ARE INDEPENDENT.

FOR EACH SUBSAMPLE CALCULATE A MEASURE OF FLUCTUATIONS, E.G. $\langle N_\pi^3 \rangle$ USING THE IDENTITY METHOD.
THE VALUE OF $\langle N_\pi^3 \rangle$ FOR k $(k=1 \ldots K)$ SUBSAMPLE IS DENOTED:

$$\langle N_\pi^3 \rangle_k$$

CALCULATE MEAN OF $\langle N_\pi^3 \rangle_k$ DISTRIBUTION:

$$\langle N_\pi^3 \rangle = \frac{1}{K} \sum_{k=1}^{K} \langle N_\pi^3 \rangle_k$$

AND ITS VARIANCE:

$$\mathrm{Var}\left[\langle N_\pi^3 \rangle_k\right] = \frac{1}{K-1} \sum_{k=1}^{K} \left(\langle N_\pi^3 \rangle_k - \langle N_\pi^3 \rangle \right)^2$$

THEN ACCORDING TO THE CENTRAL LIMIT THEOREM, FOR SUFFICIENTLY LARGE K:

$\langle N_\pi^3 \rangle$ IS GAUSS DISTRIBUTED WITH

$$\sigma(\langle N_\pi^3 \rangle) = \sqrt{\mathrm{Var}\left[\langle N_\pi^3 \rangle_k\right]} \Big/ \sqrt{K}$$

NOTE, FOR THE FIXED M_S, K IS PROPORTIONAL TO M AND THUS:

$$\sigma(\langle N_{\pi}^{3} \rangle) \sim 1/\sqrt{M}$$

NOTE, WHEN STUDING THE DEPENDENCE OF RESULTS ON THE NUMBER OF SUBSAMPLES K ONE SHOULD RANDOMLY ALLOCATE EVENTS TO SUBSAMPLES FOR EACH K.

OTHERWISE THE K DEPENDENCE WILL BE BIASED BY A STRONG CORRELATION OF SUBSAMPLES WITH A LARGE NUMBER OF THE SAME EVENTS

NOTE, OF COURSE, $\langle N_{\pi}^{3} \rangle$ CAN BE REPLACED BY ANY OTHER FLUCTUATION MEASURE.

BOOTSTRAP METHOD

IN SOME CASES THE SUBSAMPLE METHOD CAN NOT BE USED, THIS IS WHEN THE MINIMUM EVENT STATISTICS REQUIRED FOR THE ANALYSIS IS LARGER THAN $M_S = M/K$

THEN THE BOOTSTRAP METHOD IS USUALLY USED.

B. EFRON, BOOTSTRAP METHODS: ANOTHER LOOK AT
THE JACKKNIFE
THE ANALYSIS OF STATISTICS 7(1) 1.(1979)

BOOTSTRAP = GENERATING NEW EVENT SAMPLES USING THE MEASURED EVENT SAMPLE AS AN APPROXIMATION OF THE TRUE ONE BY RANDOM RESAMPLING WITH REPLACEMENT.

BOOTSTRAP: STEP-BY-STEP

- CREATE K BOOTSTRAP SAMPLES (RESAMPLES) BY DRAWING RANDOMLY WITH REPLACEMENT EVENTS FROM THE MEASURED EVENT SAMPLE.
  THE SIZE OF EACH RESAMPLE IS THE SAME AS THE SIZE OF THE MEASURED SAMPLE, M.
  THIS IS THE MAIN ADVANTAGE OF THE BOOTSTRAP OVER SUBSAMPLE METHOD

NOTE, THAT AN EVENT CAN APPEAR MANY TIMES IN ONE BOOTSTRAP SAMPLE, BUT IT ALSO MAY NOT BE INCLUDED IN ANY RESAMPLE.
ALSO THE SAME EVENT CAN BE INCLUDED IN SEVERAL RESAMPLES.
THUS RESULTS CALCULATED FOR RESAMPLES ARE CORRELATED.

THIS IS THE MAIN DISADVENTAGE OF THE BOOSTRAP IN COMARISON TO THE SUBSAMPLE METHOD

- FOR EACH RESAMPLE CALCULATE A MEASURE OF E-BY-E FLUCTUATIONS, E.G. $\langle N_\pi^3 \rangle_k$ USING THE IDENTITY METHOD.

- CALCULATE MEAN OF $N_\pi \; {}_k$ DISTRIBUTION:

$$\langle N_\pi^3 \rangle_B = \frac{1}{K} \sum_{k=1}^{K} \langle N_\pi^3 \rangle_k$$

- AND ITS VARIANCE:

$$Var\left[\langle N_\pi^3 \rangle_k\right] = \frac{1}{K-1} \sum_{k=1}^{K} \left(\langle N_\pi^3 \rangle_k - \langle N_\pi^3 \rangle_B\right)^2$$

- PROVIDING $\langle N_\pi^3 \rangle_B \simeq \langle N_\pi^3 \rangle$ (MEAN FROM THE MEASURED SAMPLE AND THE DISTRIBUTION OF $\langle N_\pi^3 \rangle_K$ IS APPROXIMATELY GAUSSIAN

CALCULATE STATISTICAL ERROR OF $\langle N_\pi^3 \rangle$ AS

$$\sigma(\langle N_\pi^3 \rangle) = \sqrt{\mathrm{Var}[\langle N_\pi^3 \rangle]}$$

NOTE, THAT THE FACTOR $1/\sqrt{K}$ IS NOT PRESENT HERE AS THE BOOTSTRAP SAMPLES HAVE THE SAME EVENT STATISTICS AS THE MEASURED SAMPLE

NOTE, OF COURSE, $\langle N_\pi^3 \rangle$ CAN BE REPLACED BY ANY OTHER FLUCTUATION MEASURE.

EXAMPLE: BOOTSTRAP TEST FOR $\omega[N_K]$ FROM IDENTITY METHOD

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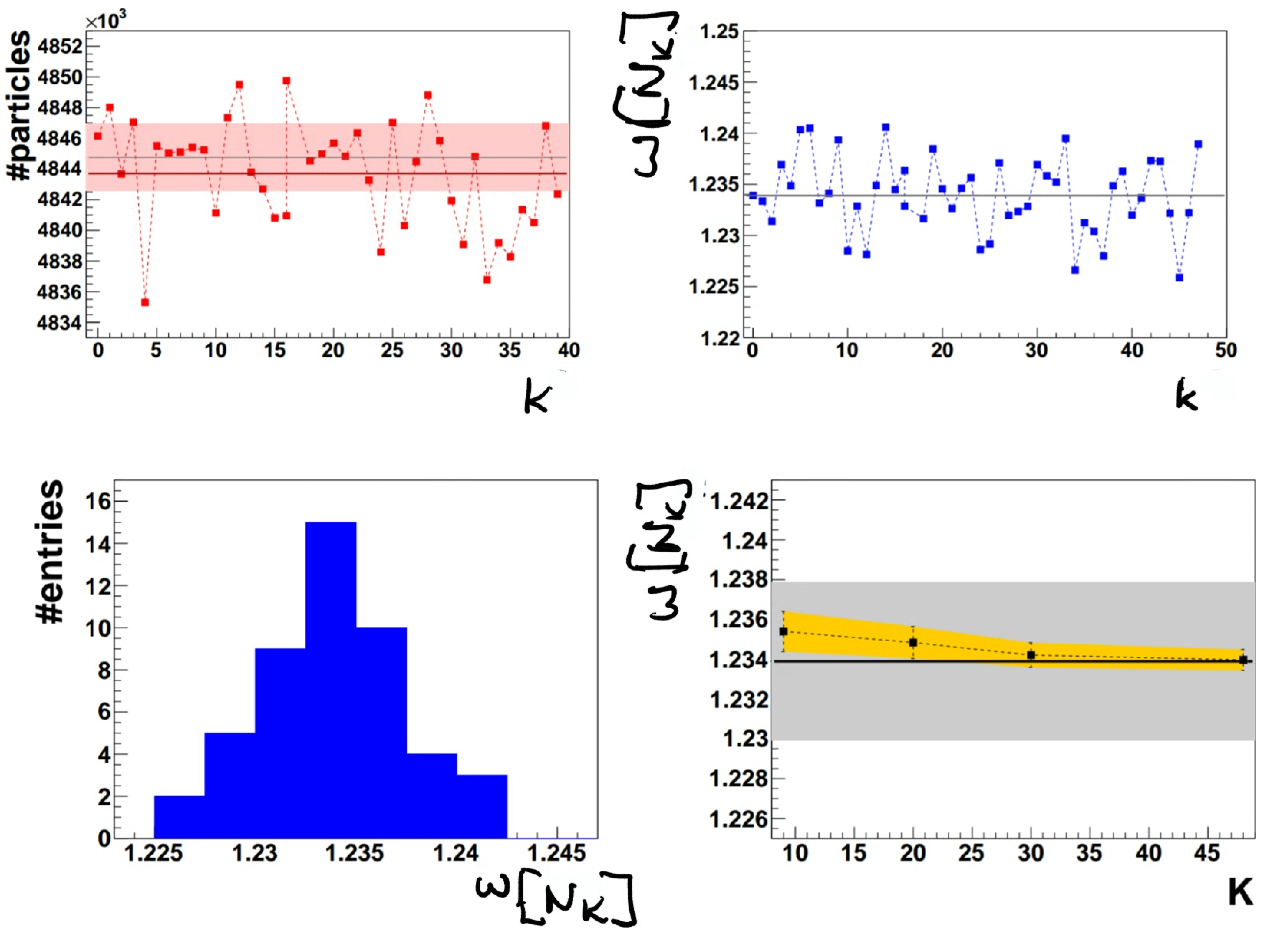

SYSTEMEMATIC UNCERTAINTIES

R. BARLOW, SYSTEMATIC ERRORS: FACTS AND FICTION
HEP-EX/020726

SYSTEMATIC BIASES/EFFECTS ARE DIFFERENCES BETWEEN THE TRUE RESULT FOR A GIVEN SAMPLE OF EVENTS AND THE MEASURED ONE FOR THIS SAMPLE. SYSTEMATIC BIASES ARE INDEPENDENT OF EVENT STATISTICS.

SYSTEMATIC BIASES MAY BE CAUSED BY MANY EFFCTS, E.G. IMPERFECT EVENT SELECTION PROCEDURE (ON-LINE AND OFF-LINE), INEFFICIENCY OF EVENT AND TRACK RECONSTRUCTION, IMPERFECT PARTICLE IDENTIFICATION...

SYSTEMATIC EFFECTS SHOULD BE IDENTIFIED AND CORRECTED FOR.

THE UNCERTAINTY IN THE ESTIMATION OF THE CORRESPONDING CORRECTIONS IS CALLED A SYSTEMATIC ERROR/UNCERTAINTY.

SYSTEMATIC BIASES WHICH WERE NEGLECTED OR OVERLOOKED ARE CALLED SYSTEMATIC MISTAKES

EXAMPLE: ENERGY OF PROJECTILE SPECTATORS MEASURED BY A PSD CALORIMETER

THE PSD RESPONSE DEPENDS ON TEMPERATURE OF PHOTODETECTORS (MPPC)

CASE A:

IF THE TEMPERATURE DEPENDECE IS KNOWN (MEASURED DURING CALIBRATION RUNS) AND THE MPPC TEMPERATURE WAS MEASURED DURING THE PHYSICS DATA TAKING, THE DATA CAN BE CORRECTED FOR THE BIAS.

THE SYSTEMATIC EFFECT IS KNOWN EXACTLY AND THERE IS NO SYSTEMATIC ERROR.

CASE B

IF THE BIAS IS IGNORED THEN THIS IS A MISTAKE.
HOPEFULLY CONSISTENCY CHECKS WILL BE DONE, E.G.
THE BEAM ENERGY WILL SHOW TIME (TEMPERATURE)
DEPENDENCE, AND THE EFFCT WILL BE PROPERLY
IDENTIFIED AND CORRECTED FOR.

FREQUENTLY ONE MAKES A SHORT CUT AND FROM
FAILED CONSISTENCY TESTS GUESSES SYSTEMATIC
ERRORS OF THE FINAL RESULTS.
THIS SHOULD BE AVOIDED.
IF DONE THE OBTAINED ERRORS ARE CALLED
APOSTERIORI SYSTEMATIC ERRORS

TYPICALLY MORE CHECKS ARE DONE THE LARGER
APOSTERIORI SYSTEMATIC ERROR IS !

CASE C

IF THE EFFECT IS KNOWN TO EXIST BUT THE MPPC TEMPERATURE WAS NOT MEASURED AND ONE CAN ESTIMATE IT ONLY WITHIN A FEW DEGREES, THAT IS A SYSTEMATIC UNCERTAINTY ON A SYSTEMATIC EFFECT AND A SYSTEMATIC ERROR IN THE ACCEPTED SENSE, THE SO-CALLED A PRIORI SYSTEMATIC ERROR

THE RECOMMENDED PROCEDURE

1. IDENTIFY AND CORRECT FOR SYSTEMATIC EFFECTS

2. ESTIMATE UNCERTAINTIES (STATISTICAL AND/OR SYSTEMATIC) OF "INPUT PARAMETERS" TO THE CORRCTIONS

3. PROPAGATE THEM TO THE FINAL RESULT ≡ CALCULATE
   A PRIORI SYSTEMATIC ERRORS

4. PERFORM CROSS CHECKS / CONSISTENKY TESTS
   IF FAILED GO BACK TO 1.
   AFTER SEVERAL LOOPS, WHEN YOUR SUPERVISOR'S PATIENCE IS EXHAUSTED GO TO 5.

5. GUESS A POSTERIORI SYSTEMATIC ERRORS
   BASED ON FAILED CROSS CHECKS.

ROGER BARLOW 0207026:

5. CONCLUSIONS: ADVICE FOR PRACTITIONERS

The following should be printed in large letters and hung on the wall of every practising particle physicist.

I Thou shalt never say 'systematic error' when thou meanest 'systematic effect' or 'systematic mistake'.

II Thou shalt not add uncertainties on uncertainties in quadrature. If they are larger than chickenfeed thou shalt generate more Monte Carlo until they shrink to become so.

III Thou shalt know at all times whether what thou performest is a check for a mistake or an evaluation of an uncertainty.

IV Thou shalt not incorporate successful check results into thy total systematic error and make thereby a shield behind which to hide thy dodgy result.

V Thou shalt not incorporate failed check results unless thou art truly at thy wits' end.

VI Thou shalt say what thou doest, and thou shalt be able to justify it out of thine own mouth; not the mouth of thy supervisor, nor thy colleague who did the analysis last time, nor thy local statistics guru, nor thy mate down the pub.

Do these, and thou shalt flourish, and thine analysis likewise.