Introduction to Accelerator Physics

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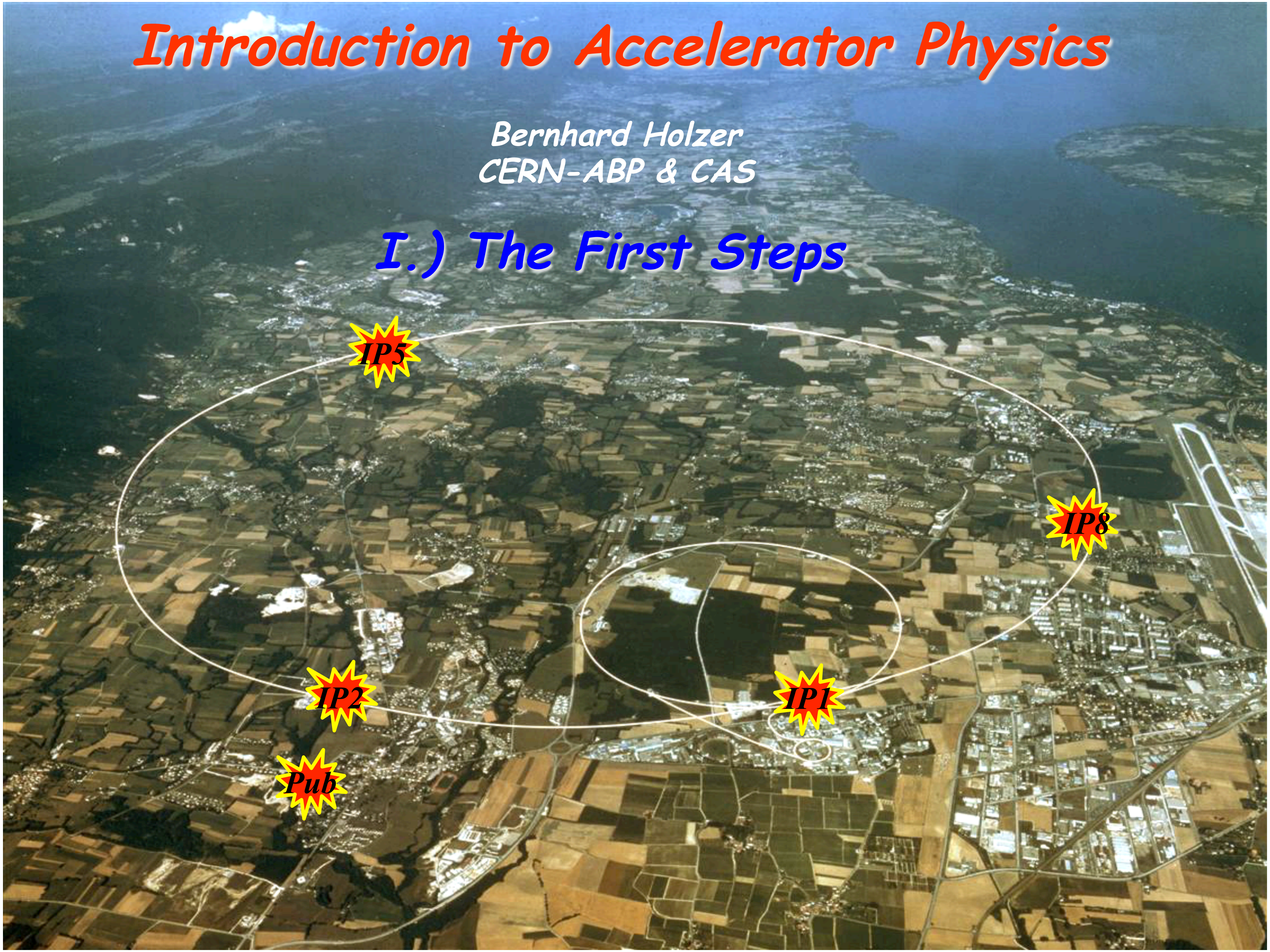
I.) The First Steps



Outline:

what you need to build a Storage Ring

- ***Albert & James-Clark***
- ***Liquid Helium & Beer***
- ***Beethoven***
- ***Big Ben & Big Bang***
- ***Aircraft Carriers in the tunnel***
- ***Cats (but no dogs)***

the beta function is usually obtained via the matrix element „m12", which is in Twiss form for the undistorted case

$$m_{12} = \beta_0 \sin 2\pi Q$$

$$(1) \quad m^*_{12} = \beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds$$

and including the error:

$$\underbrace{m^*_{12}}_{} = b_{11} a_{12} + b_{12} a_{22} - \cancel{b_{12} a_{12}} \Delta k ds$$

$$m_{12} = \beta_0 \sin 2\pi Q$$

As M is still a matrix for one complete turn we still can express the element m_{12} in twiss form:*

$$(2) \quad m^*_{12} = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

$$-a_{12} b_{12} \Delta k ds = \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

$$dQ = \frac{\Delta k \beta_1 ds}{4\pi}$$

Equalising (1) and (2) and assuming a small error

What we will NOT do .....

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi Q \underbrace{\cos 2\pi dQ}_{\approx 1} + \cos 2\pi Q \underbrace{\sin 2\pi dQ}_{\approx 2\pi dQ}$$

$$-a_{12} b_{12} \Delta k ds = \frac{\beta_0 \Delta k \beta_1 ds}{2} \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

$$\beta_0 \sin 2\pi Q - \cancel{a_{12} b_{12}} \Delta k ds = \beta_0 \sin 2\pi Q + \beta_0 \cancel{2\pi dQ \cos} 2\pi Q + d\beta_0 \sin 2\pi Q + d\beta_0 2\pi dQ \cos 2\pi Q$$

$$d\beta_0 = \frac{-1}{2 \sin 2\pi Q} \{2 a_{12} b_{12} + \beta_0 \beta_1 \cos 2\pi Q\} \Delta k ds$$

replace by ...

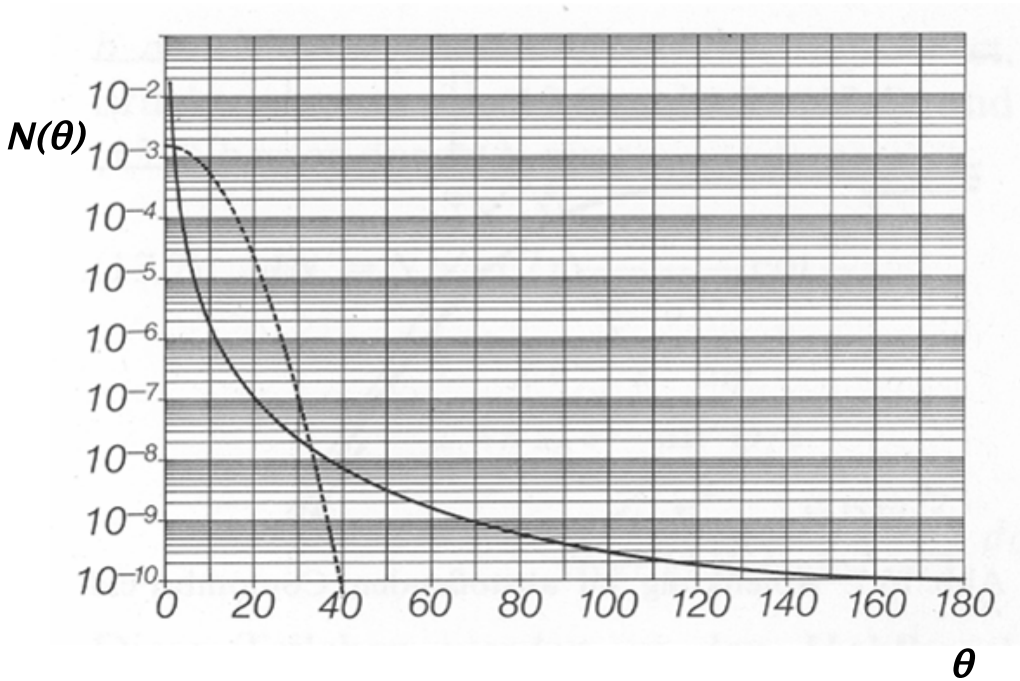
"after some TLC transformations" ... or ... *" after some beer"*

A Bit of History

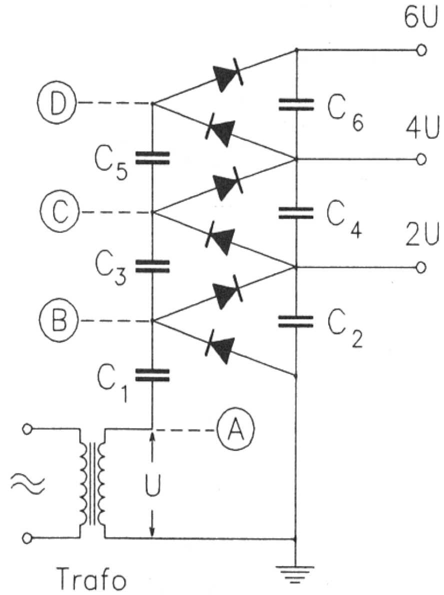
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$

Rutherford Scattering, 1906

Using radioactive particle sources:
α-particles of some MeV energy

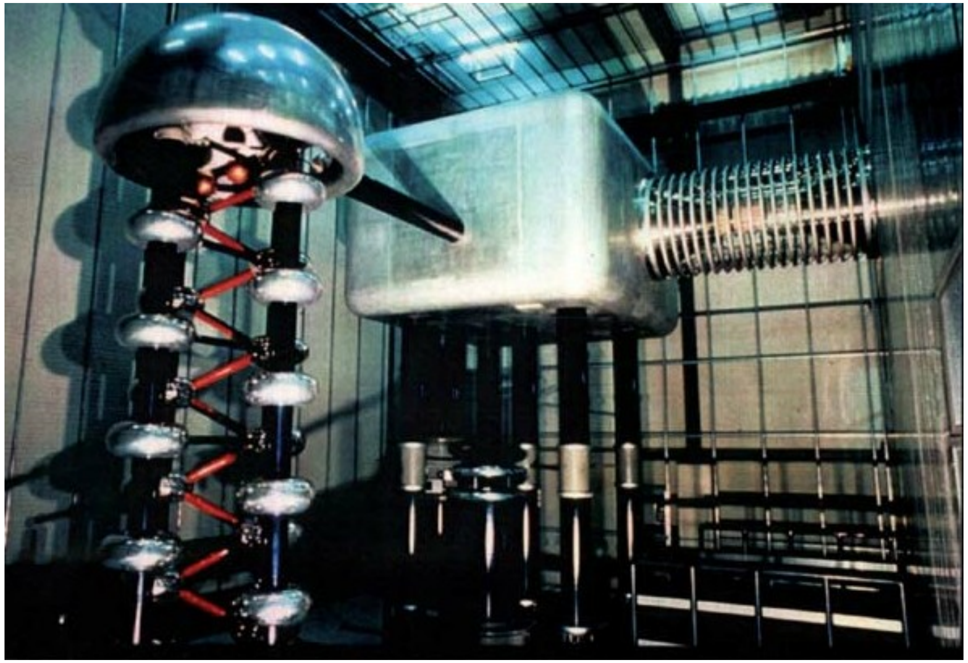


1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: *Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam*

1932: *First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV*



Particle source: ***Hydrogen discharge tube on 400 kV level***
Accelerator: ***evacuated glas tube***
Target: ***Li-Foil on earth potential***

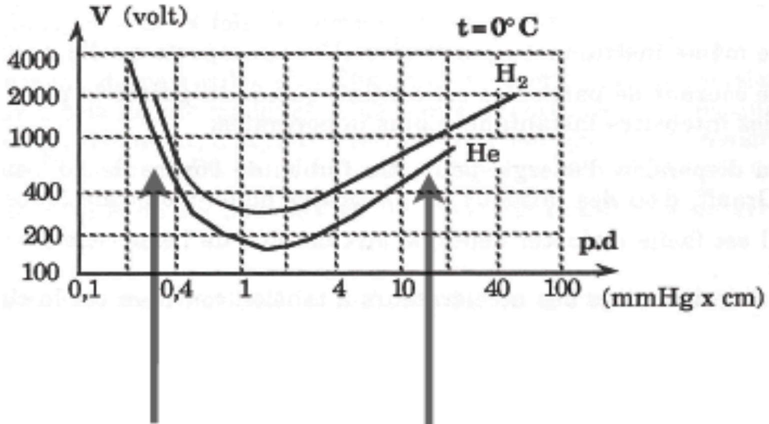
Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

robust, simple, on-knob machines
largely used in history as pre-accelerators for proton and ion beams
recently replaced by modern structures (RFQ)

Main limitation

Main limitation:
electric discharge due to too high Voltage.
Maximum limit: 1 MV

Limit set by Paschen law:
the breaking Voltage between two parallel electrodes depends only on the pressure of the gas between the electrodes and their distance

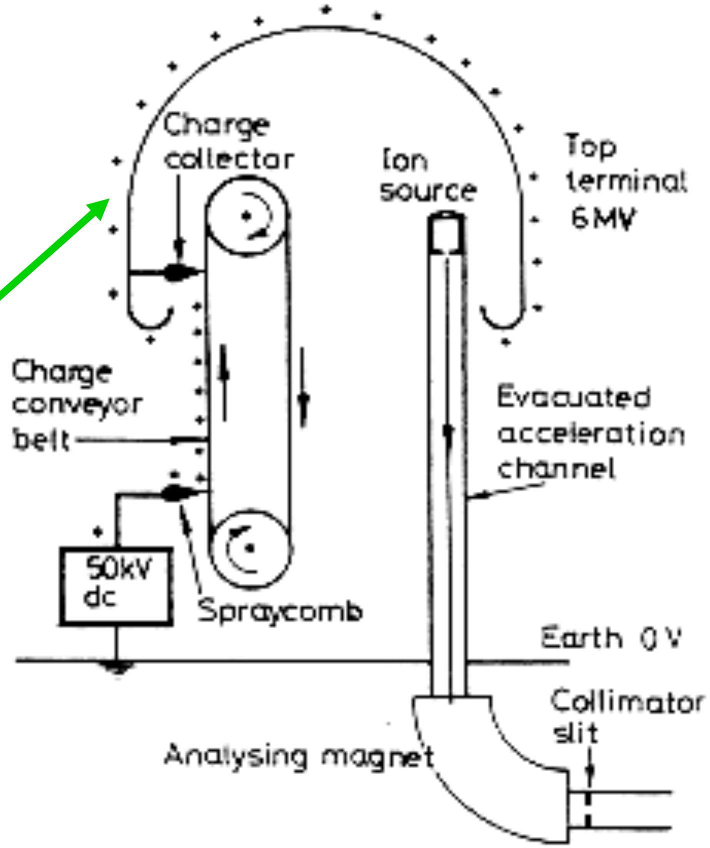


Low pressure: gas not too dense, long mean average path of electrons

High pressure: dense gas, large Voltage needed for gas ionisation

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

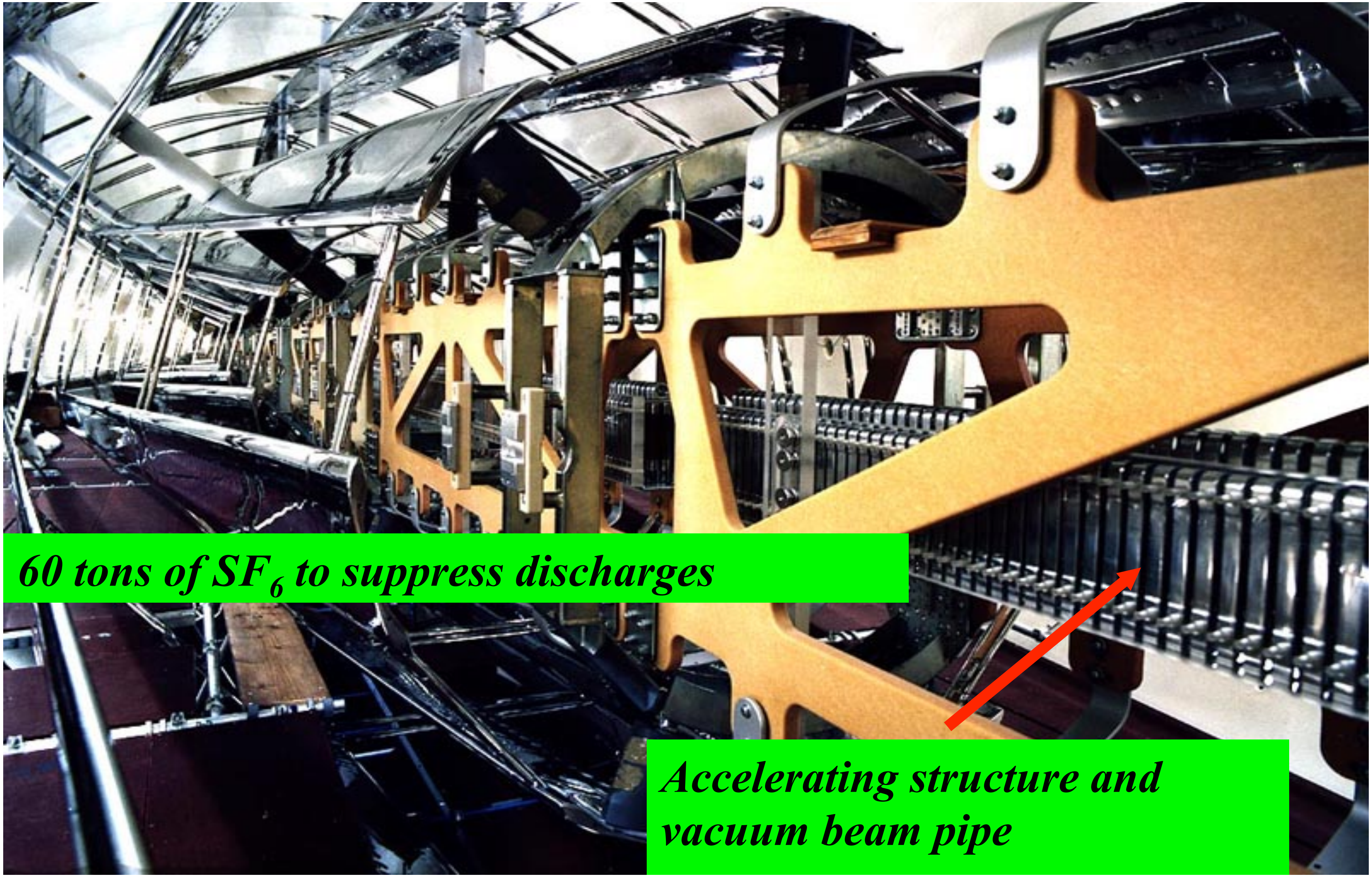
The „Tandem principle": Apply the accelerating voltage twice ...

... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

... and how it looks inside

"Vivitron" Strassbourg

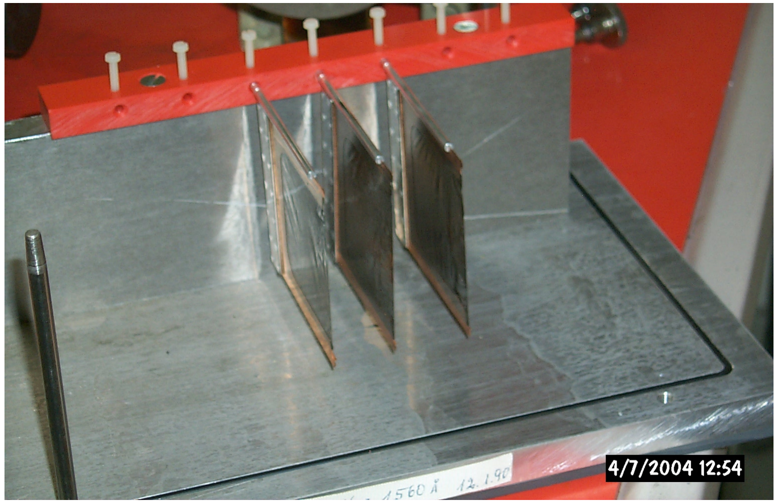


The Principle of the "Steam Engine": Mechanical Transport of Charge via a rotating chain or belt

stripping foils: 1500 Å

$H^- \rightarrow p$

$C^- \rightarrow C^{6+}$



Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

Antwort: JA !!

Gretchen Frage (J.W. Goethe, Faust)

Do they actually drop ?

Yes, they do !!

$$l_{VdG} = 30m$$

$$v \approx 10\%\, c \approx 3*10^7 m/s$$

$$\Delta t = 1\mu s$$

Free Fall in Vacuum:

$$s = \frac{1}{2} g t^2$$

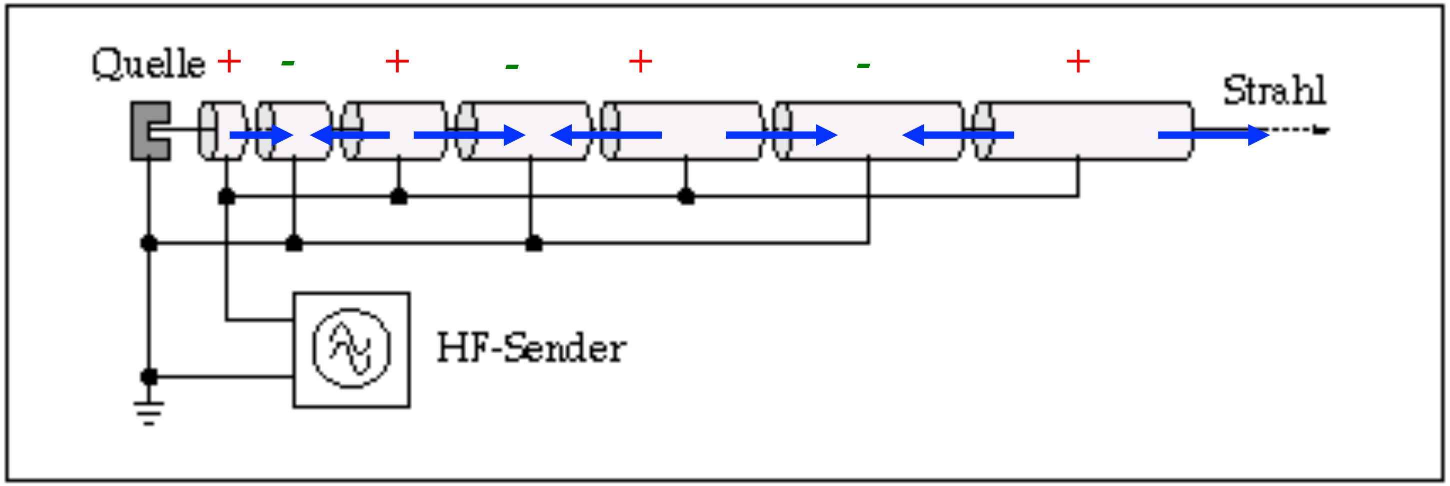
$$= \frac{1}{2} 10 \frac{m}{s^2} * (1\mu s)^2$$

$$= 5*10^{-12} m = 5pm$$

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

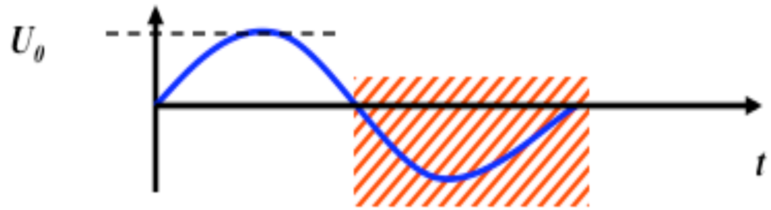
$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped“ to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$\rightarrow l_i = \frac{1}{f_{rf}} * \sqrt{\frac{i * q * U_0 * \sin\psi_s}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: E_{kin} *= 20 MeV per Nucleon,* $\beta = 0.04 \ldots 0.6$*, Particles: Protons/Ions*

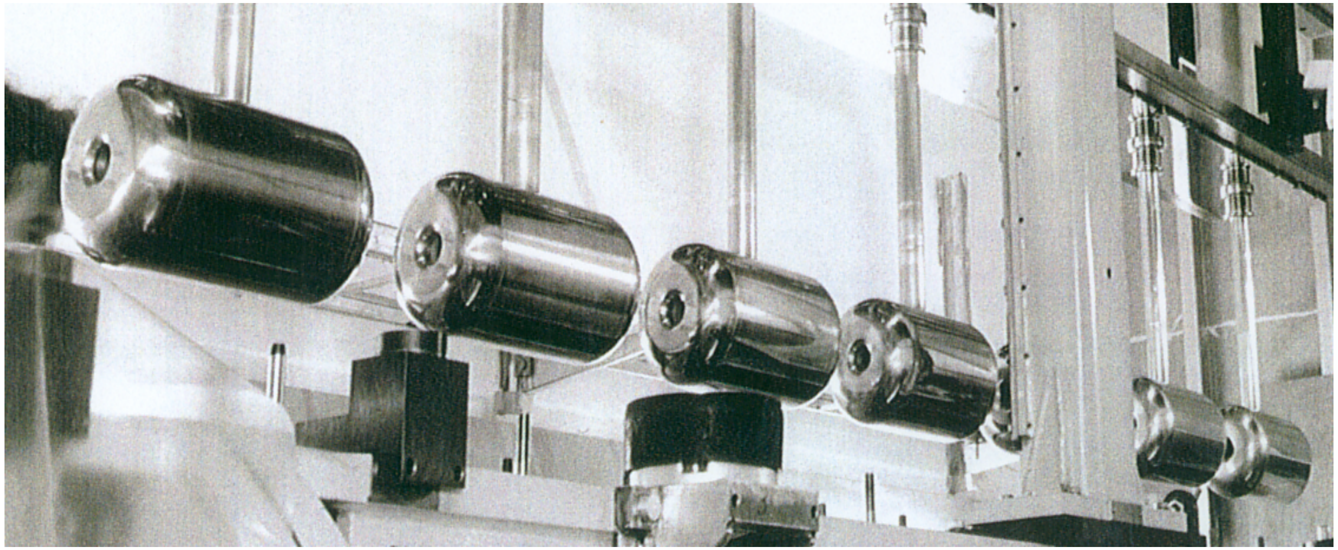
Example: DESY ***Accelerating structure of the Proton Linac***

$$E_{total} = 988\, MeV$$

$$m_0 c^2 = 938\, MeV$$

$$p = 310\, MeV / c$$

$$E_{kin} = 50\, MeV$$



Beam energies

1.) reminder of some relativistic formula

rest energy $E_0 = m_0 c^2$

total energy $E = \gamma * E_0 = \gamma * m_0 c^2$

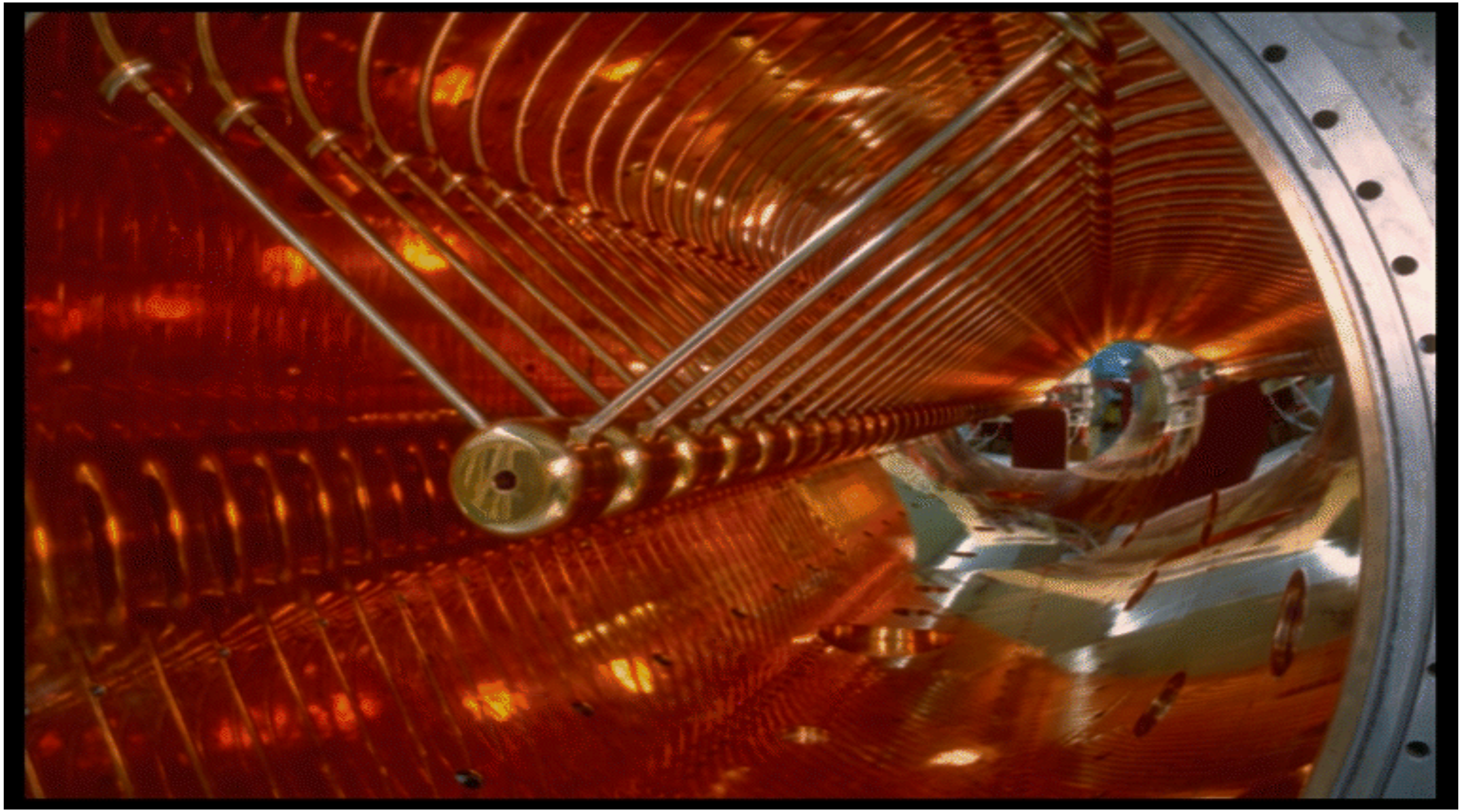
momentum $E^2 = c^2 p^2 + m_0^2 c^4$

kinetic energy $E_{kin} = E_{total} - m_0 c^2$

GSI: ***Unilac, typical Energie E_{kin}= 20 MeV per Nukleon, β = 0.04 … 0.6, Protons/Ions, f_{rf} = 110 MHz***

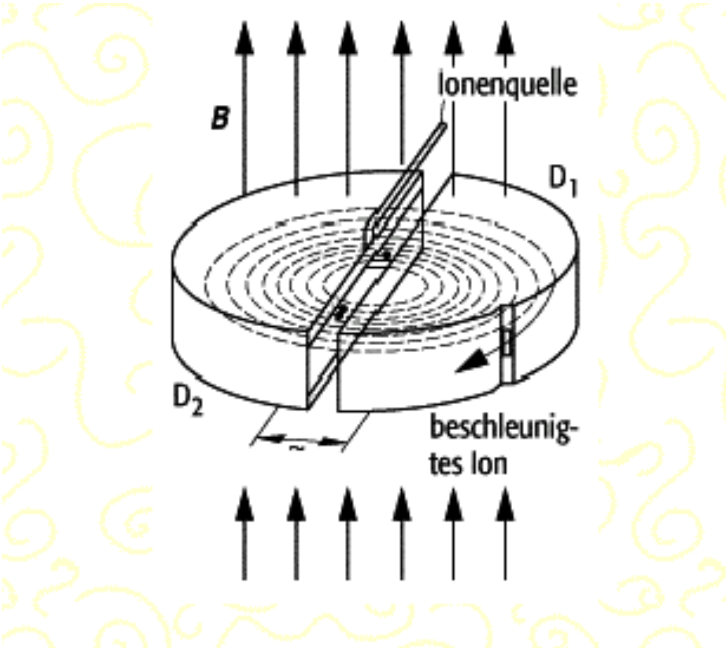
Energy Gain per „Gap“:

$$W = q\,U_0 \sin \omega_{RF} t$$



Application: ***until today THE standard proton / ion pre-accelerator***
CERN Linac 4 is being built at the moment

4.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: Bend a Linac on a Spiral
Application of a constant magnetic field
keep B = const, RF = const

➔ ***Lorentzforce***

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p/q$$

increasing radius for
increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$\omega_z = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency
is independent of the momentum

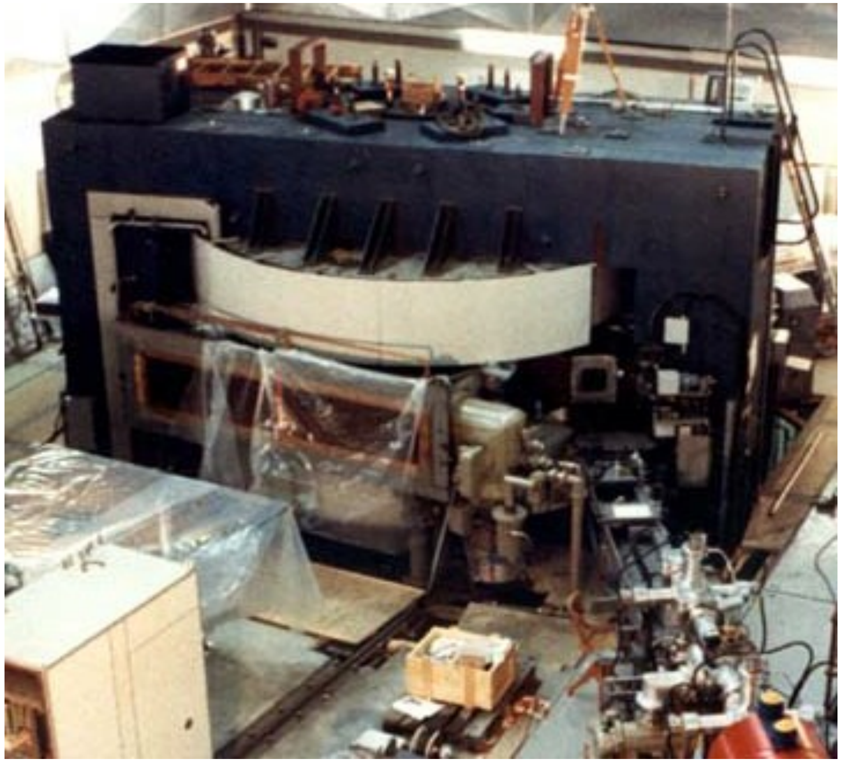
Cyclotron:

! **ω is constant** *for a given q & B*

!! *B*R = p/q*

large momentum → huge magnet

!!!! **ω ~ 1/m ≠ const** *works properly only for non relativistic particles*



PSI Zurich

Application:

Work horses for medium energy protons

Proton / Ion Acceleration up to ≈ 60 MeV (proton energy)

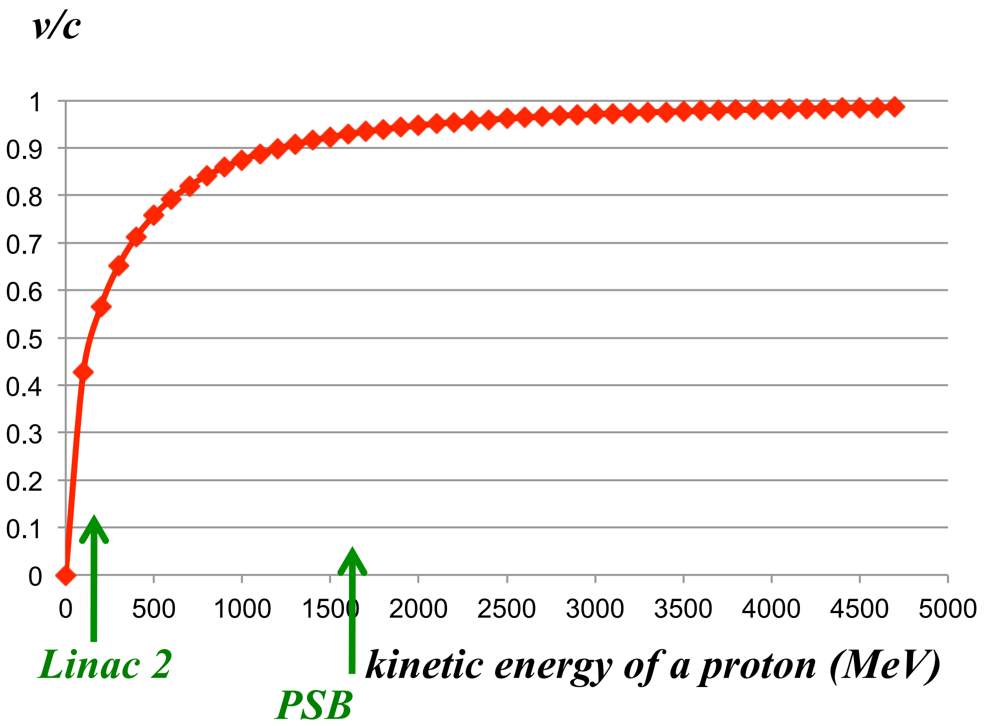
nuclear physics

radio isotope production, proton / ion therapy

Beam Energy

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$



CERN Accelerators

	kin. Energy	γ
Linac 2	60 MeV	1.06
PS	26 GeV	27
SPS	450 GeV	480
LHC	7 TeV	7460

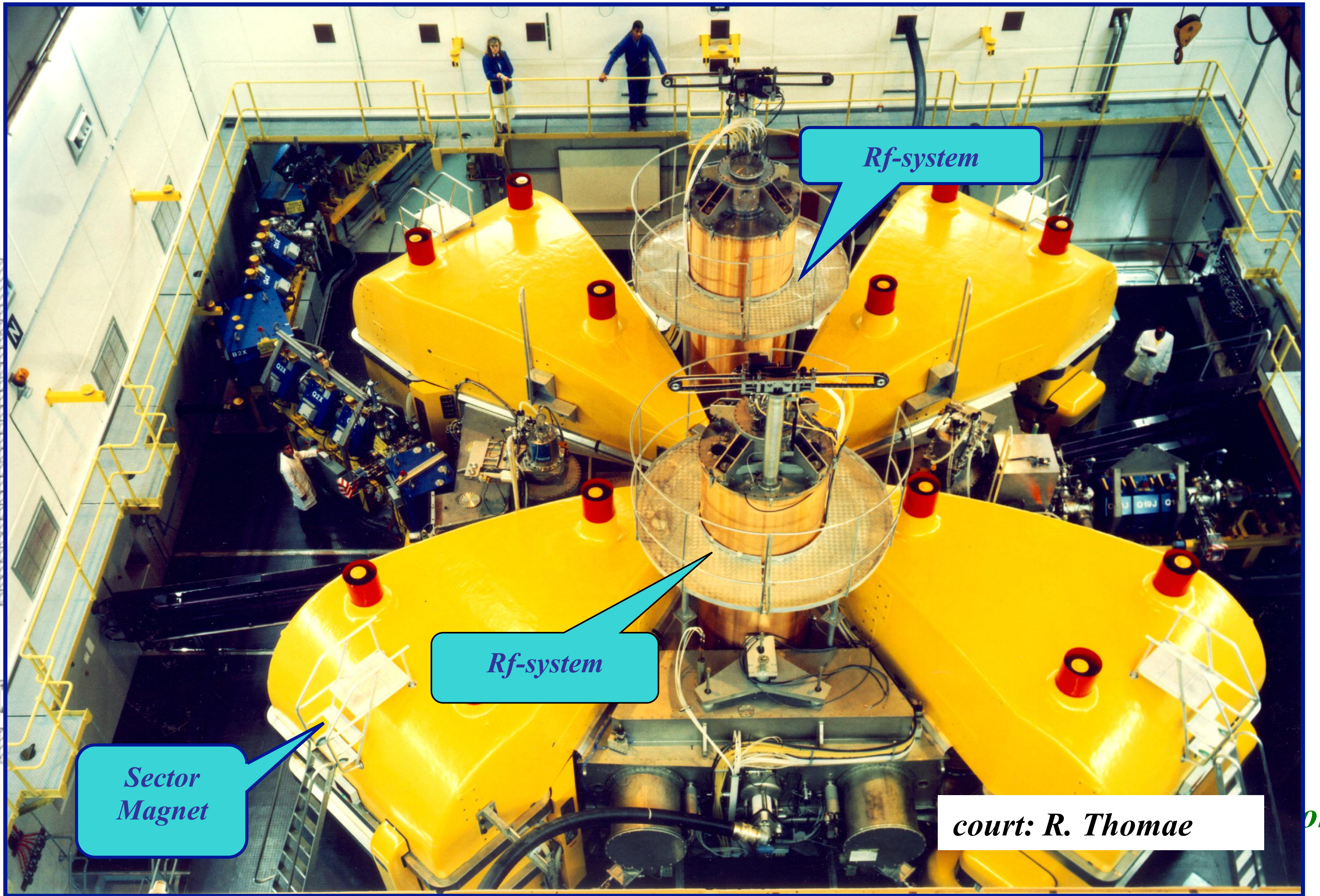
remember: proton mass = 938 MeV

Cyclotron:

modern trends: Problem: m ≠ const.

➔ non relativistic machine

$$\omega_z = \frac{e * B_z}{\gamma * m_0}$$



ons

5.) The Betatron: Wideroe 1928/ Kerst 1940

...apply the transformer principle to an electron beam: no RF system needed,
changing magnetic B field

Idea: a time varying magnetic field induces a voltage that will accelerate the particles

Farady induction law

$$\oint \vec{E} d\vec{s} = -\int_A \dot{B} df = -\dot{\Phi}$$

circular orbit

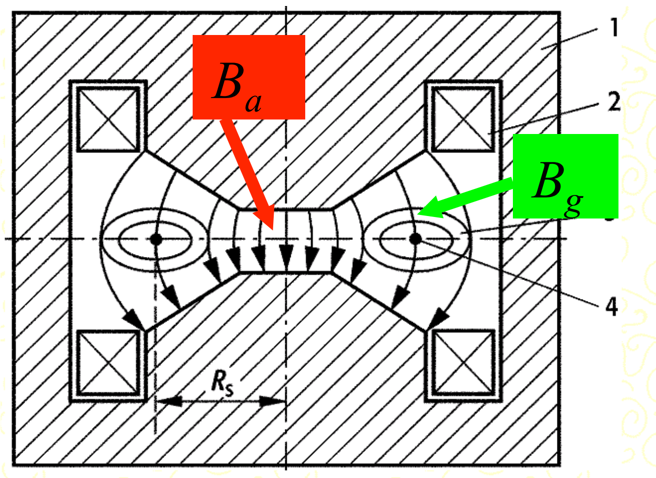
$$\frac{mv^2}{r} = e * v * B$$

$$\rightarrow \quad p = e * B * r$$

magnetic flux through this orbit area

$$\Phi = \int B df = \pi r^2 * B_a$$

schematic design



induced electric field

$$\oint \vec{E} ds = \vec{E} * 2\pi r = -\dot{\Phi} \quad \Rightarrow \quad \vec{E} = \frac{-\pi r^2 * \dot{B}_a}{2\pi r} = -\frac{1}{2} \dot{B}_a r$$

force acting on the particle: $\dot{p} = -\left|\vec{E}\right| e = \frac{1}{2} \dot{B}_a r * e$

The increasing momentum of the particle has to be accompanied by a rising magnetic guide field:

$$\dot{p} = e * \dot{B}_g r$$

$$\mathbf{B_g = \frac{1}{2} B_a}$$

robust, compact machines,
Energy ≤ 300...500 MeV,
limit: Synchrotron radiation

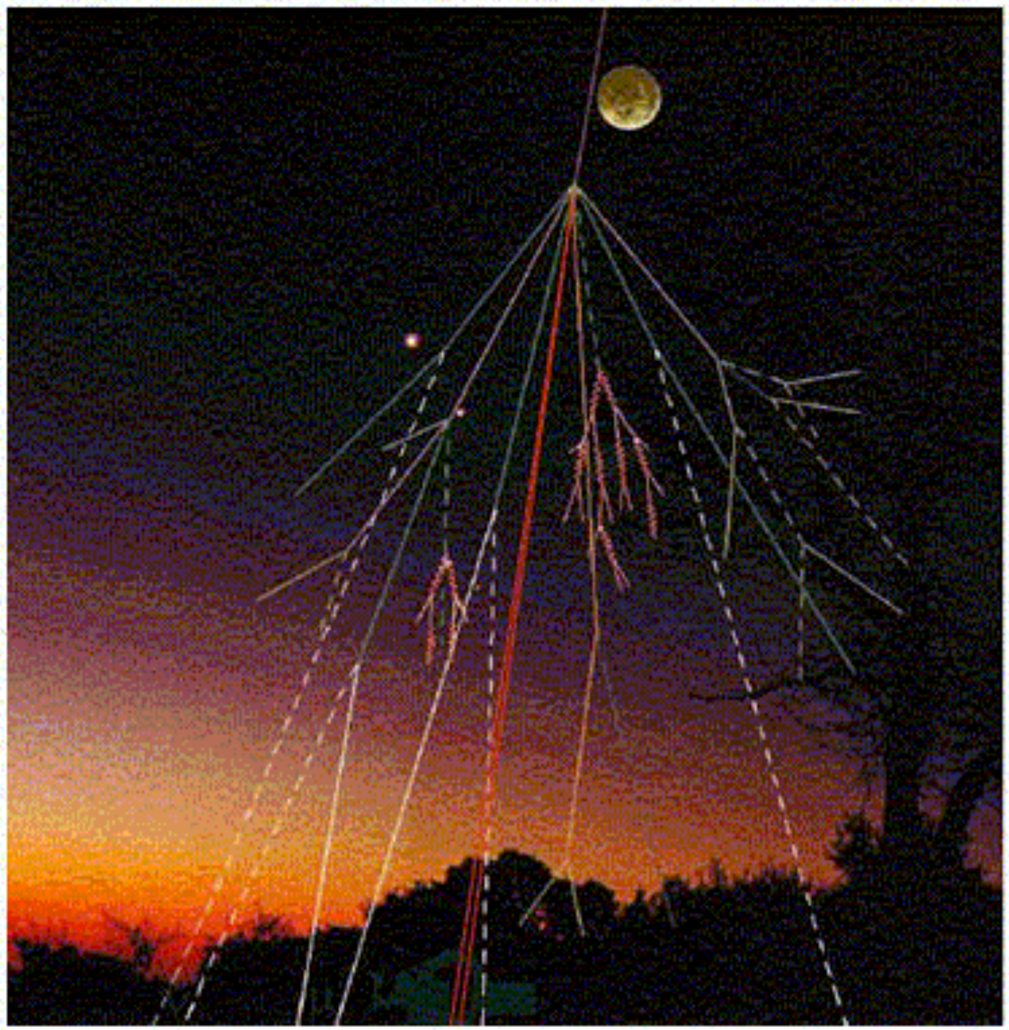
6.) Synchrotrons / Storage Rings / Colliders:

~~Wideroe 1943, McMillan, Veksler 1944,~~
Courant, Livingston, Snyder 1952

Idea: define a circular orbit of the particles,
keep the beam there during acceleration,
put magnets at this orbit to guide and focus

LEP, CERN

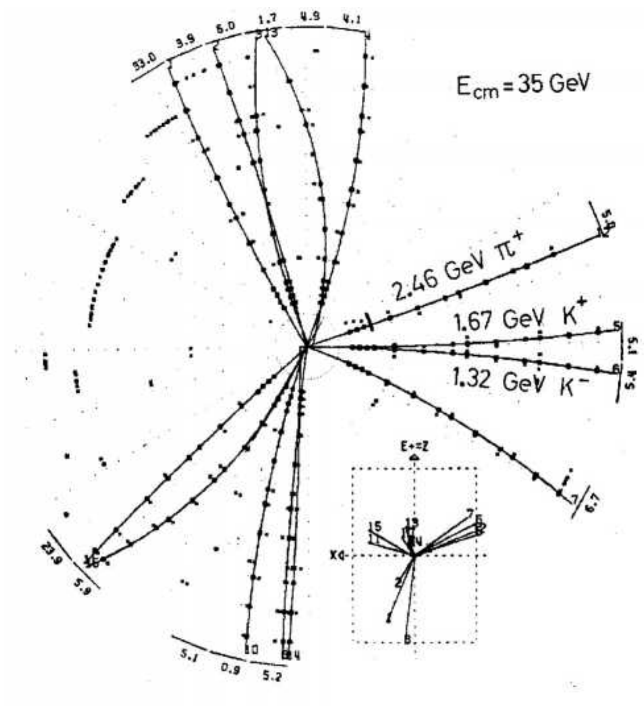
Synchrotrons as Discovery Machines



Creation of μ showers in the earth atmosphere

$$E_{IP} = mc^2$$

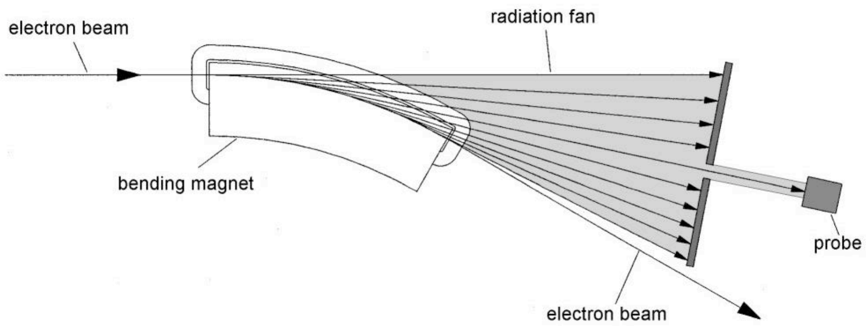
$$E_{IP} = E_1 + E_2$$

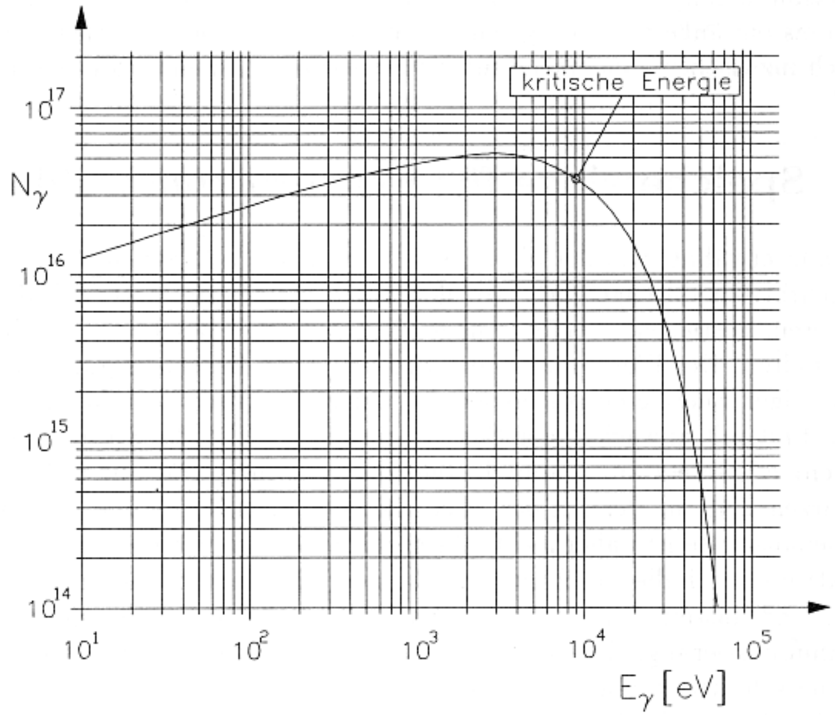


Three Jet event, gluon discovery at PETRA

7.) Electron Storage Rings

Production of Synchrotron Light





$$P_s = \frac{e^2 c}{6\pi\varepsilon_0} * \frac{1}{(m_0 c^2)^4} \frac{E^4}{R^4}$$

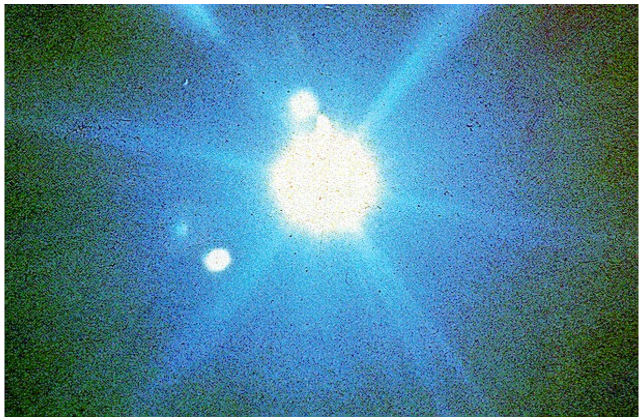
Radiation Power

$$\Delta E = \frac{e^2}{3\varepsilon_0 (m_0 c^2)^4} \frac{E^4}{R}$$

Energy Loss per turn

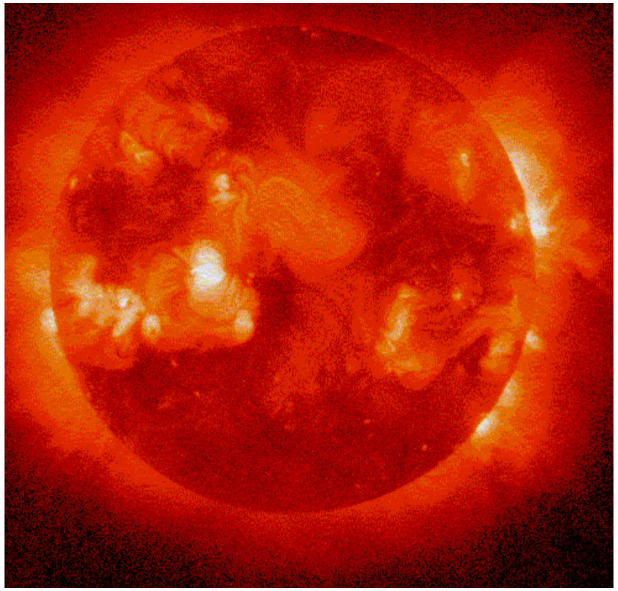
$$\omega_c = \frac{3c\gamma^3}{2R}$$

„typical Frequency" of emitted light

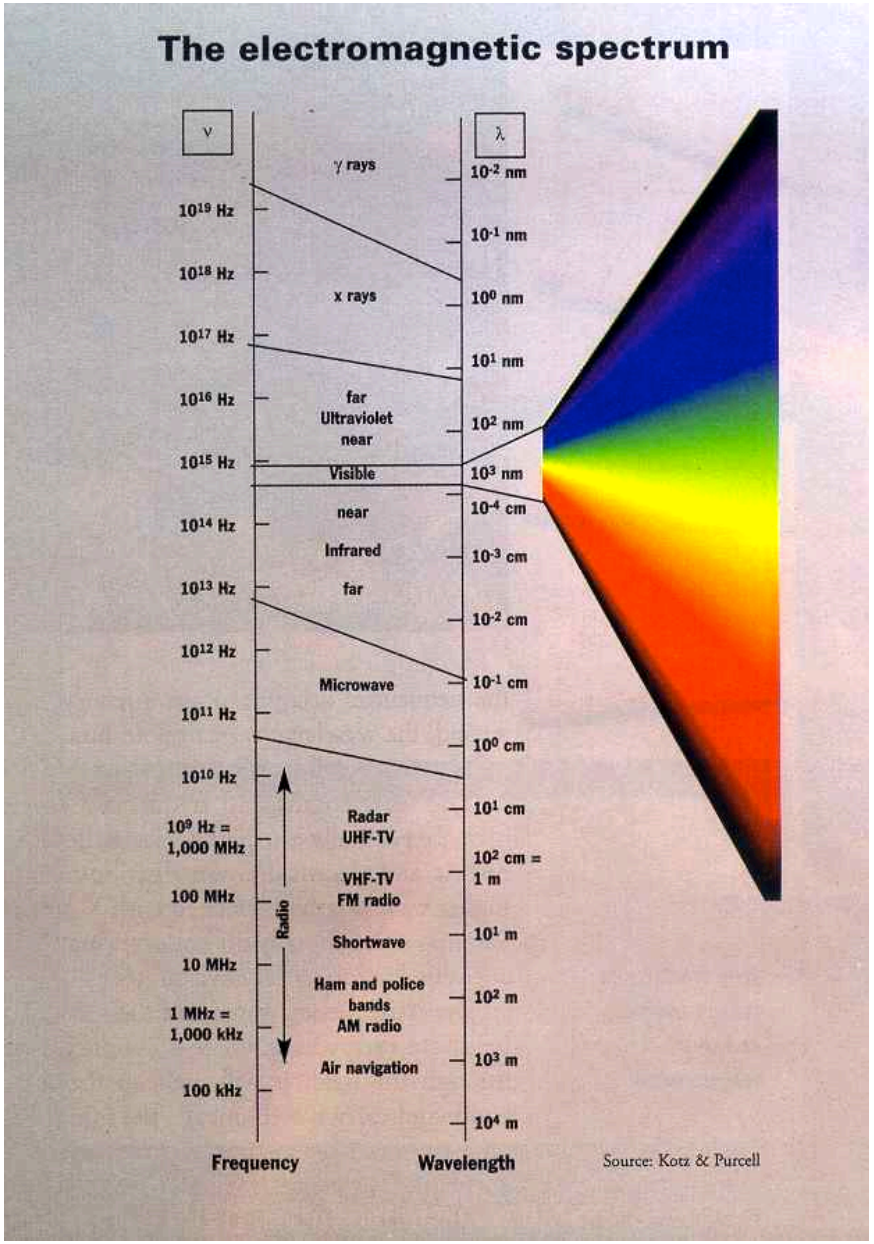


Application of Synchrotron Light
Analysis at Atoms & Molecules

The electromagnetic Spectrum:



having a closer look at the sun ...



Light:

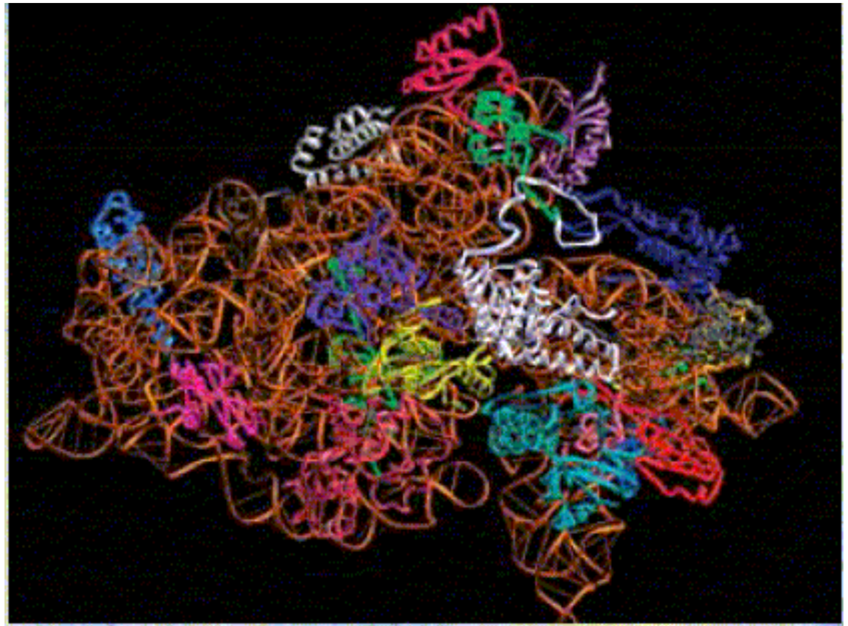
$\lambda \approx 400$ nm … 800 nm

1 Oktave

Analysis of Cell structures

Structure of a Ribosom

Ribosomen are responsible for the protein production in living cells.
The structure of these Ribosom molecules can be analysed using brilliant synchrotron light from electron storage rings
(Quelle: Max-Planck-Arbeitsgruppen für Strukturelle Molekularbiologie)



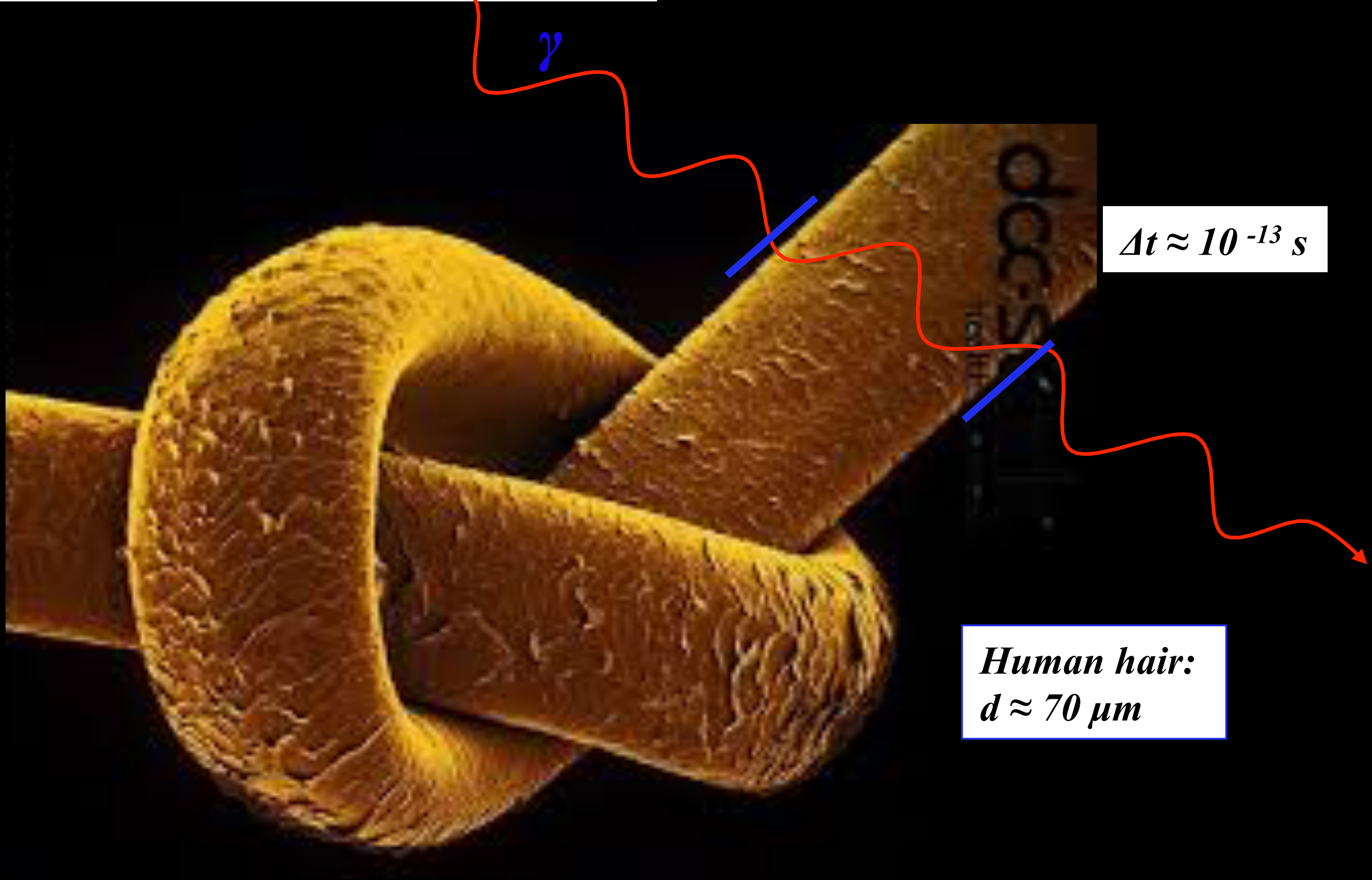
Structure of the ribosome, the "protein factory" in living cells

Angiographie

x-ray method applicable for the imaging of coronar heart arteria

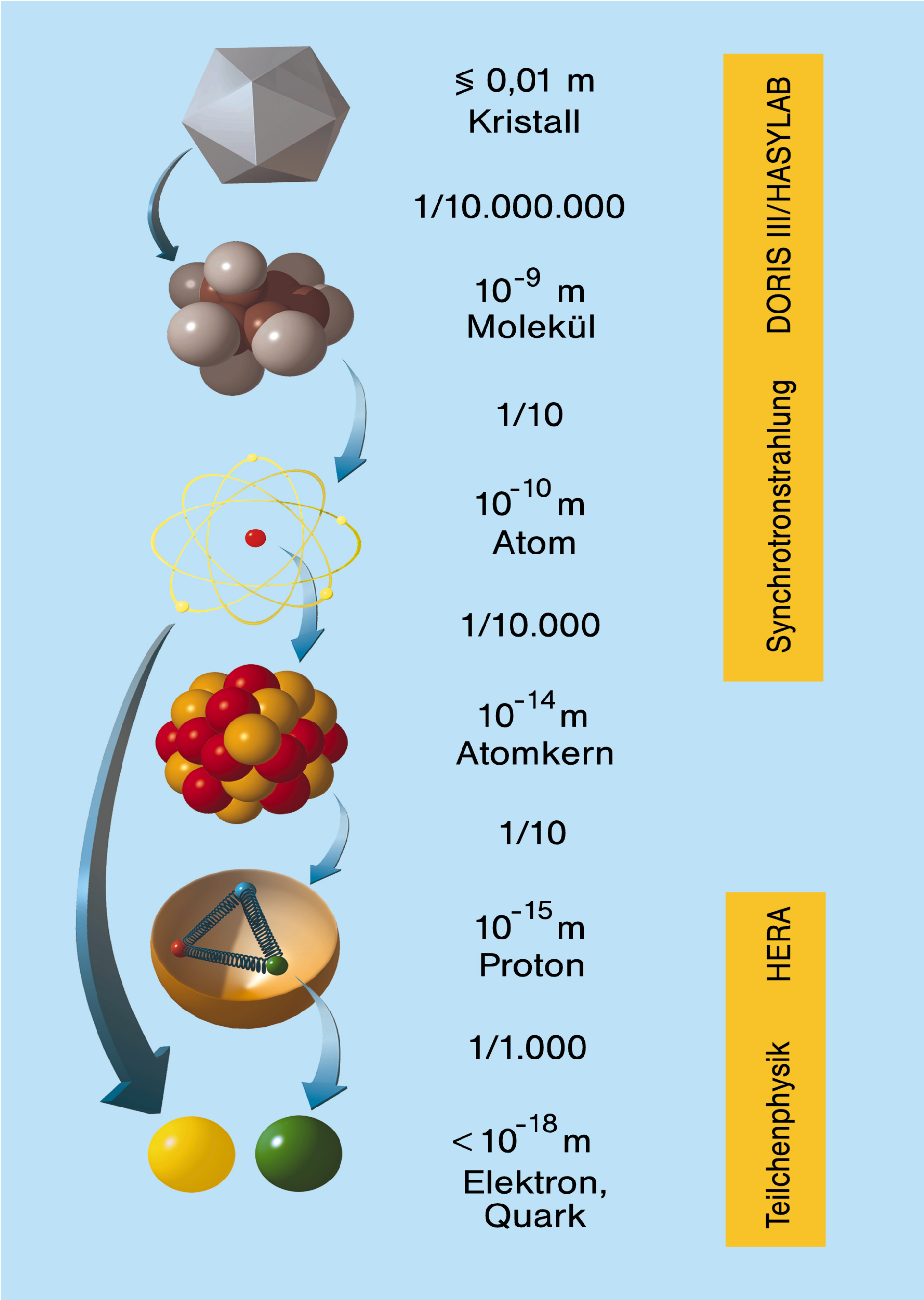
Beam Dynamics in FELs and ERLs:

Smallest time scale in FELs and Linacs:



FEL pulses allow observation of processes in the femto-sec scale

Structure of Matter



9.) Storage Rings for Structure Analysis

synchrotron light: nm

electron scattering: Å ... 10^{-18} m

de Broglie:

$$\lambda = \frac{h}{p} = \frac{ch}{E} \qquad E \approx pc$$