The background of the slide is a complex, fractal-like network of purple and yellow lines, representing the cosmic web of dark matter filaments and nodes. The lines are interconnected, forming a dense, web-like structure that fills the entire frame.

Managing astrophysical uncertainties in direct dark matter detection

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TH BSM Forum – CERN. July 14, 2016

Outline

Introduction

Astrophysical uncertainties in DM detection

Indirect detection – The density distribution

Direct detection – the velocity distribution

A halo-independent bound

FF, A. Ibarra & S. Wild, JCAP 09052 (2015); J. Phys. Conf. Ser. 718 (2016)



Outline

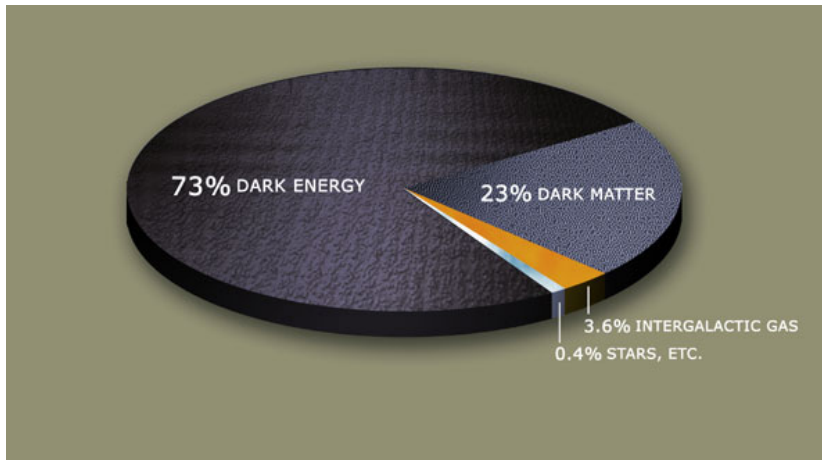
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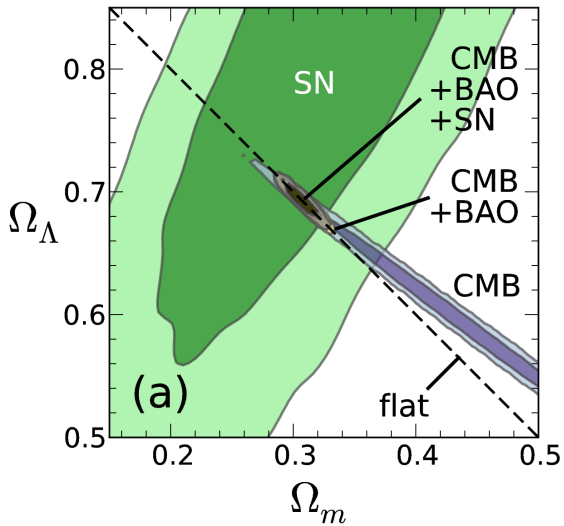
Indirect detection – The density distribution

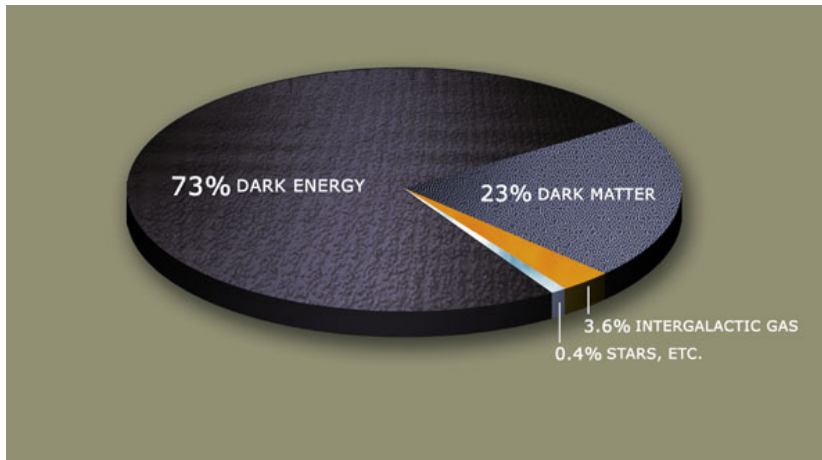
Direct detection – the velocity distribution

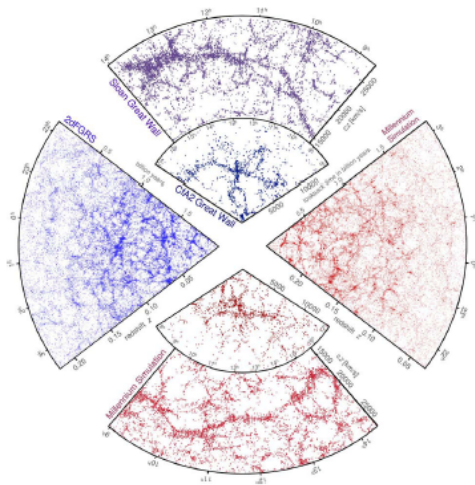
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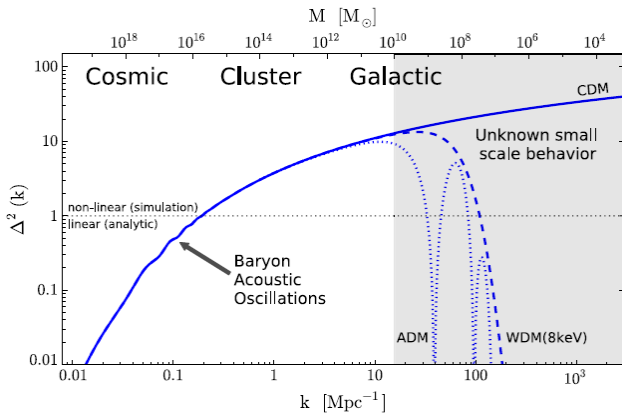








Frenk & White 12

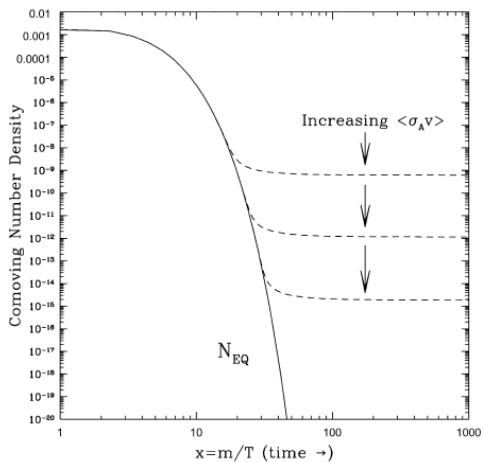


Kuhlen, Vogelsberger & Angulo 12

Cold Dark Matter

- ▶ Exotic, i.e. non-baryonic
- ▶ Non-relativistic, i.e. cold
- ▶ Required to explain cosmological expansion; dynamics of galaxies, clusters, . . . ; the existence of structure itself.

Thermal relics



The WIMP miracle

For $10 \text{ GeV} \lesssim m_\chi \lesssim 10 \text{ TeV}$ weak-scale interactions produce observed abundance from thermal decoupling:

$$\Omega_\chi h^2 = 0.1 \frac{3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle}$$

The same interactions make it potentially detectable:

- ▶ $\chi\chi \rightarrow \gamma\gamma, \pi^0, e^\pm, \dots$
- ▶ $\chi N \rightarrow \chi N$

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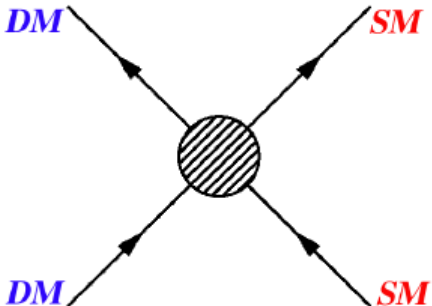
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thermal freeze-out (early Univ.)
indirect detection (now)



direct detection 



production at colliders

Outline

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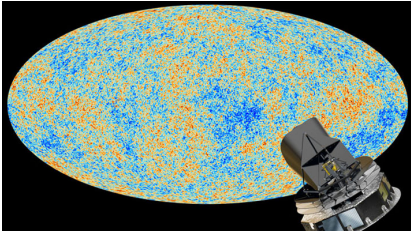
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Indirect detection of DM



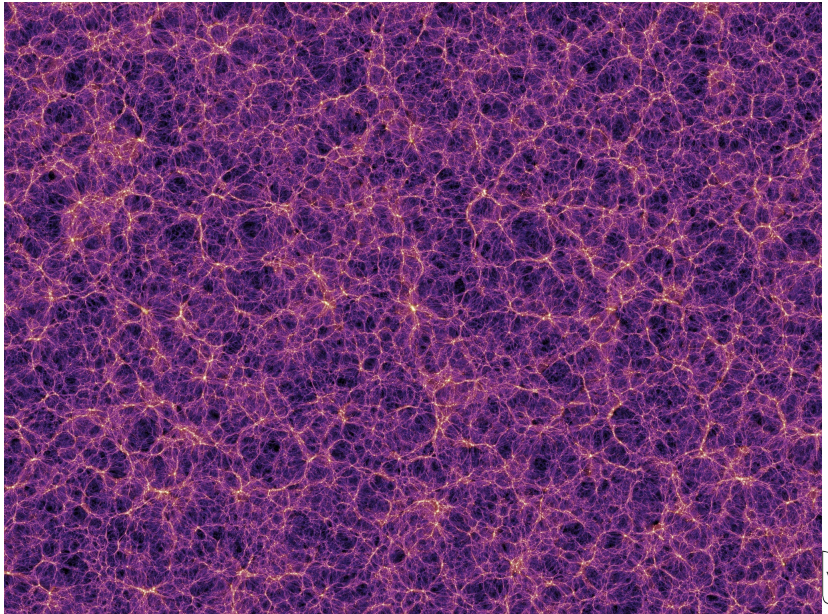
$$\Phi = \underbrace{\frac{N_i}{8\pi m_\chi^2} \langle \sigma v \rangle}_{\text{\# collisions/volume giving SM particles}} \times \underbrace{\int_{\text{line of sight}} ds \rho^2}_{\text{Amount of DM}^2}$$

- ▶ Sensitive to the annihilation cross-section, which sets also the relic density.
- ▶ Requires knowledge of the dark matter density profile (and velocity distribution in some cases) at the local, galactic, ... level.

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Simulations



N-body simulations (DM only)

- ▶ Follow about 10^{14} particles, each weighting $10^3 M_{\odot}/10^9 M_{\odot}/10^{12} M_{\odot}$, using $> 10^6$ CPU-hours.
- ▶ DM halos follow a *universal* profile:

$$\rho_{NFW} = \frac{\rho_0 a^3}{r(a+r)^2}$$

$$\rho_{Einasto} = \rho_0 \exp\left(-\frac{2}{\gamma} \left[\left(\frac{r}{a}\right)^{\gamma} - 1\right]\right)$$

- ▶ Substructure down to Earth mass clumps

$$\frac{dN}{dM} \propto M^{-2}$$

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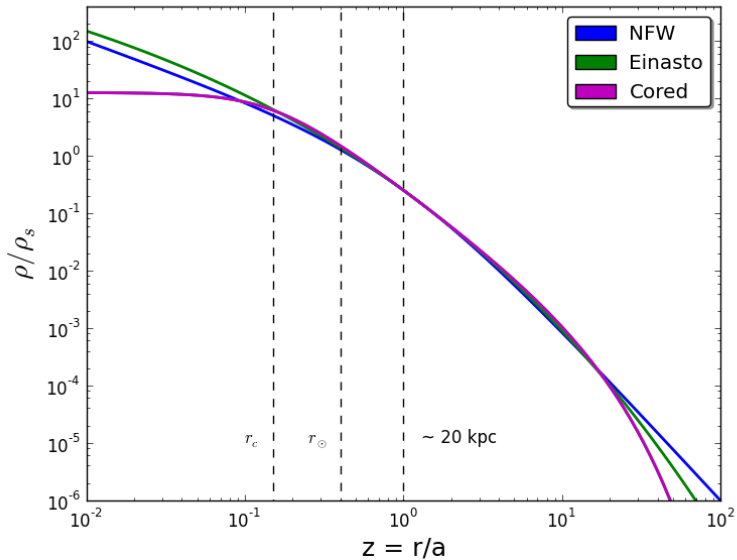
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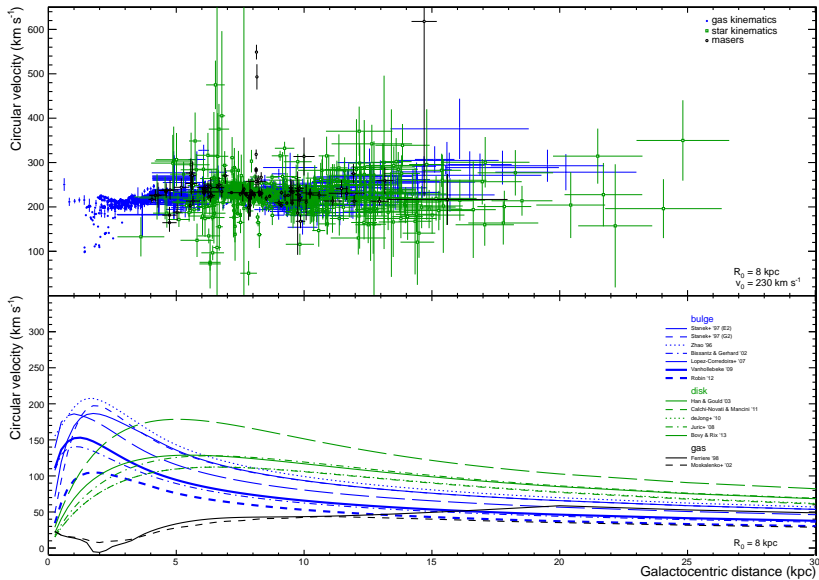




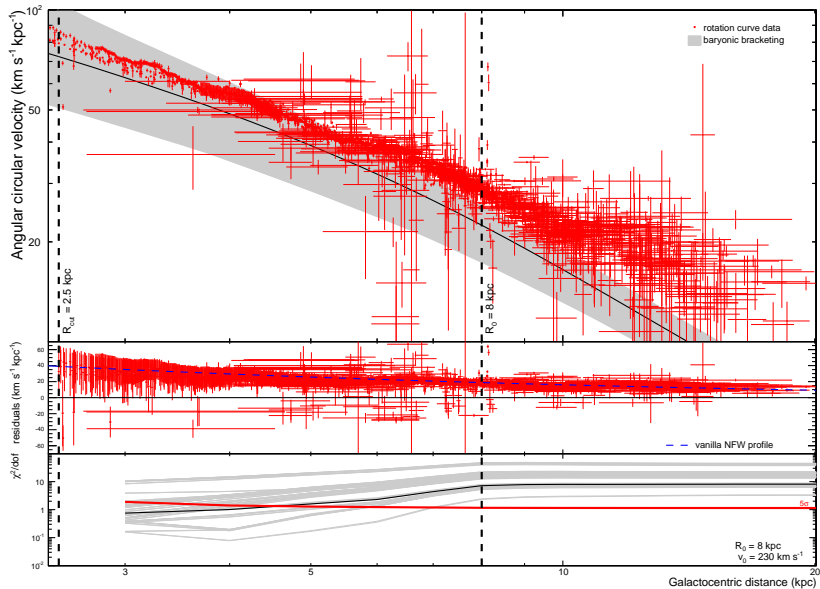
Stadel et al. 09

Einasto provides a slightly better fit to DM only simulations,
 $c \approx 6 - 18$, $\alpha \approx 0.08 - 0.32$, $\Delta M(r_{\odot}) \approx 4$.

Observations



Observations



Baryonic effects

Several physical processes can modify the structure of DM haloes:

- ▶ Smooth and slow accretion: *adiabatic contraction*, $rM(r) = \text{const.}$
- ▶ Dynamical friction: *expansion*. Satellite/clumpy accretion, galactic bars, . . .
- ▶ Gas outflows: *expansion*. Strong mass outflows cause rapid perturbations to potential, particles gain energy on average.

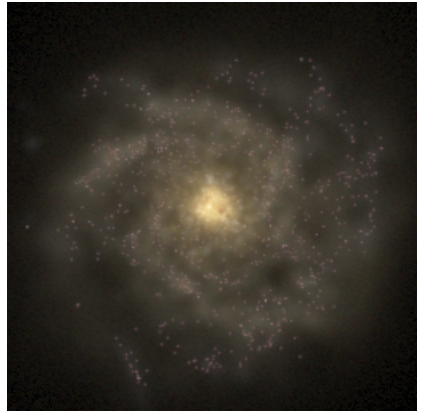
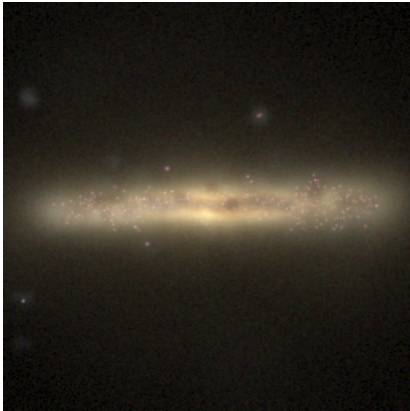
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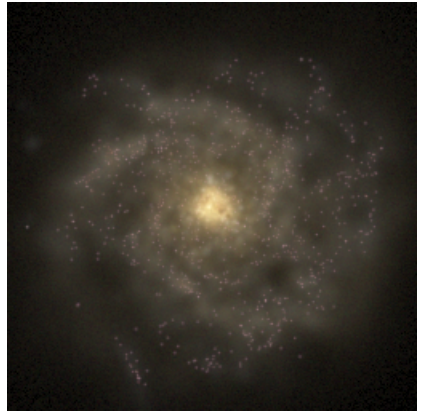
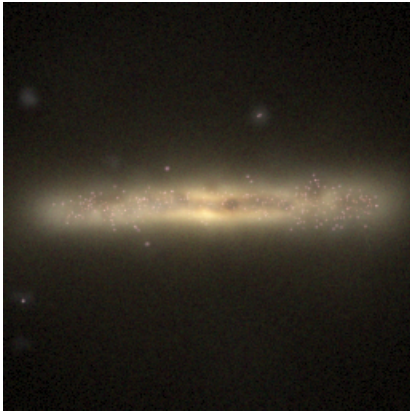
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Calore et al. 16

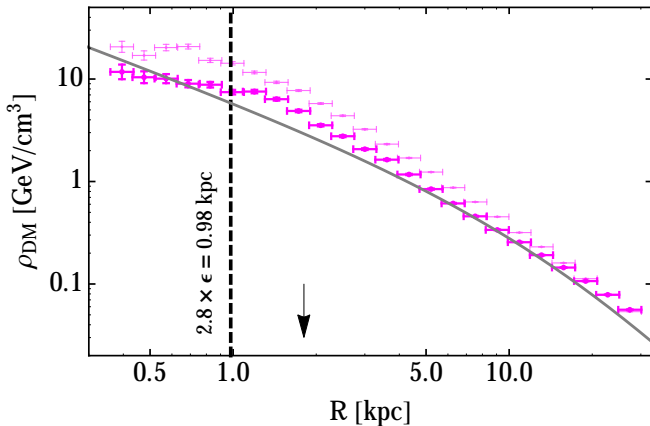
Fit to MW stellar mass, rotation curve and galaxy shape instead of total virial mass.





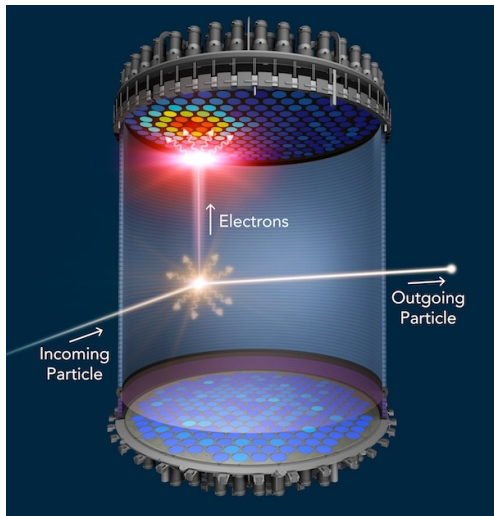
Calore et al. 16

Fit to MW stellar mass, rotation curve and galaxy shape instead of total virial mass.



The ten selected galaxies exhibit consistent DM profiles inside the solar circle. Baryonic effects make the halo shallower than required to explain the Fermi GeV excess.

Direct detection



Expected recoil events at a DD experiment:

$$R = \mathcal{E} \cdot \sum_i \int_0^\infty dE_R \epsilon(E_R) \frac{\xi_i \rho_{\text{loc}}}{m_{A_i} m_{\text{DM}}} \times$$
$$\int_{v \geq v_{\text{min},i}^{(\text{DD})}(E_R)} d^3 v v f(\vec{v} + \vec{v}_{\text{obs}}(t)) \frac{d\sigma_i}{dE_R}.$$

Probes the DM scattering cross section with nuclei:

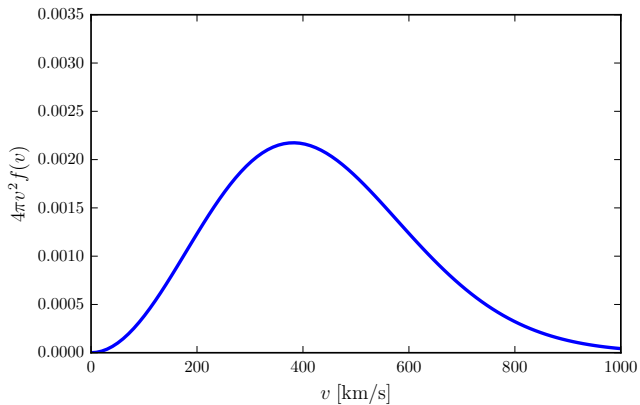
$$\frac{d\sigma_i}{dE_R} = \frac{m_{A_i}}{2\mu_{A_i}^2 v^2} (\sigma_{\text{SI}} F_{i,\text{SI}}^2(E_R) + \sigma_{\text{SD}} F_{i,\text{SD}}^2(E_R)),$$

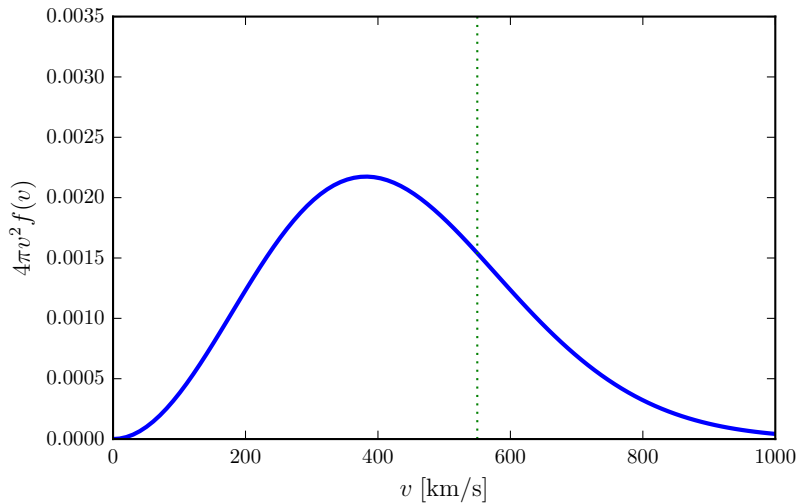
Astrophysical inputs required:

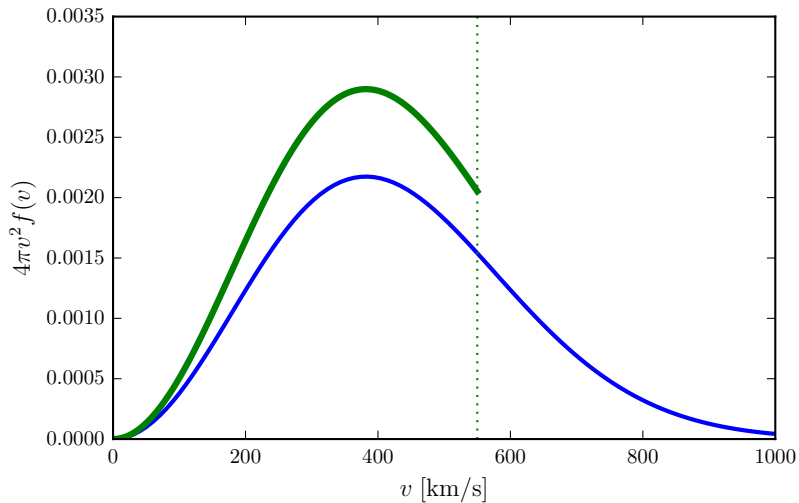
- ▶ Local DM density ρ_{loc}
- ▶ Local velocity distribution $f(\vec{v} + \vec{v}_{\text{obs}}(t))$.
- ▶ Minimum DM velocity to trigger a recoil event

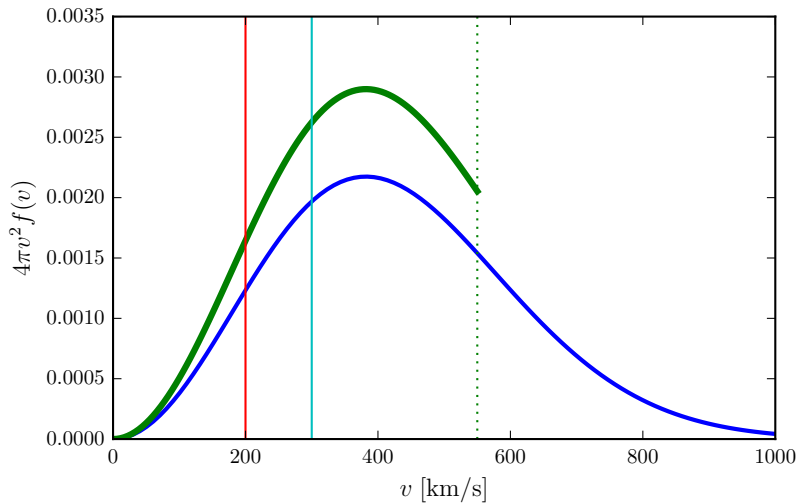
$$v_{\text{min}} = \sqrt{\frac{m_T E_R}{2\mu_T^2}}$$

The Standard Halo Model

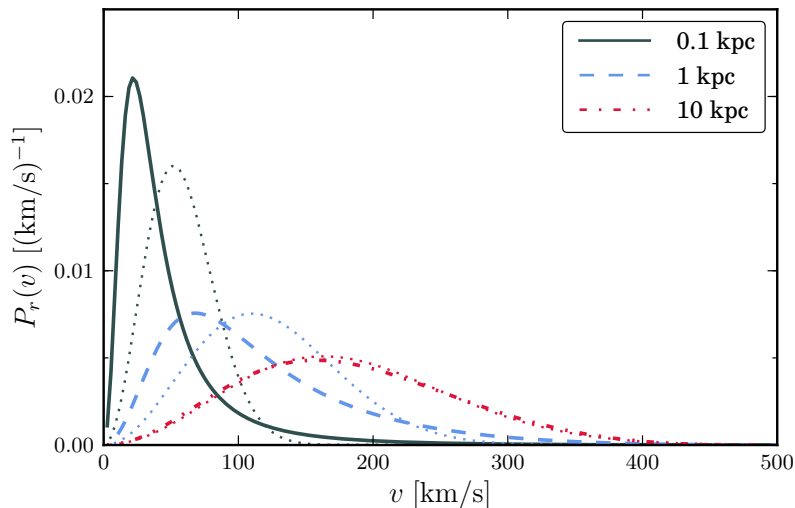




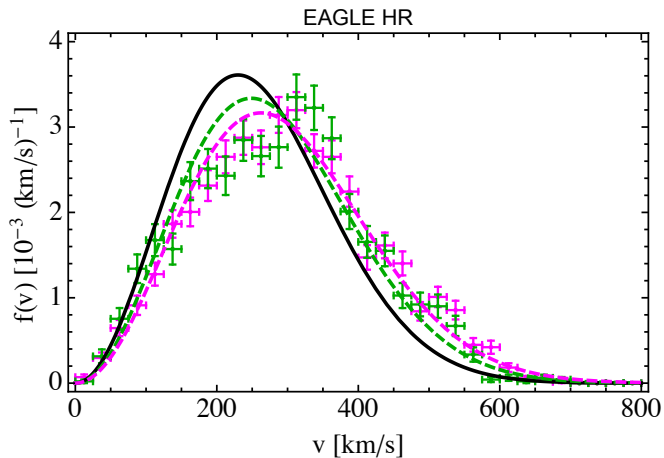




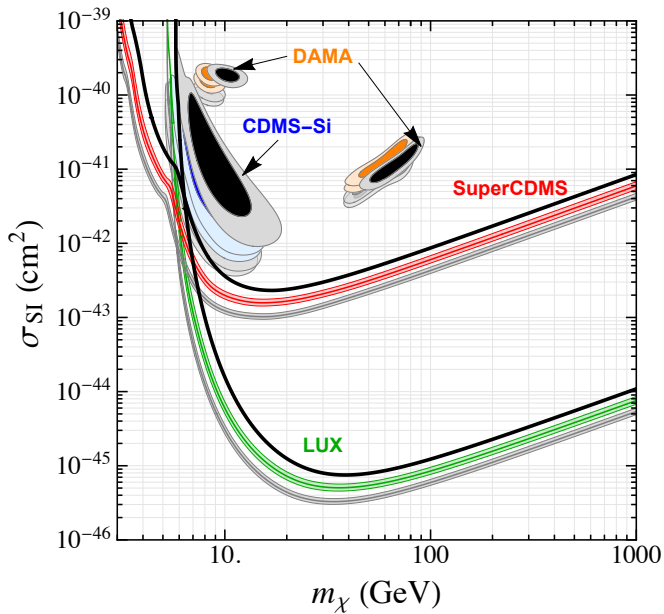
For an NFW profile



The velocity distribution from simulations



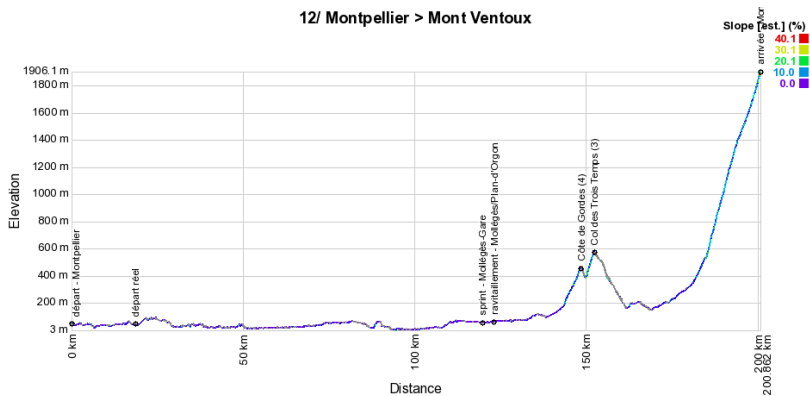
Bozorgnia et al. 1601.04707







12/ Montpellier > Mont Ventoux



created by GPSTracker.com

Halo independent methods

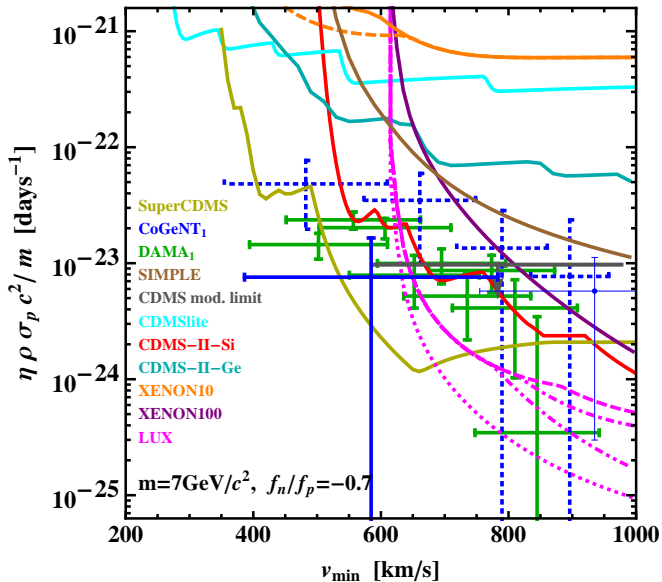
$$\eta(v_{\min}, t) \equiv \frac{\rho\sigma_{\text{ref}}}{m_{\chi}} \int_{v_{\min}}^{\infty} dv \frac{f(v, t)}{v}$$

$$\frac{dR}{dE_R} = \frac{N_T M_T}{2\mu^2} \eta(v_{\min}, t)$$

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Inputs:

- ▶ Neither direct detection experiments nor neutrino telescopes have detected dark matter.

Assumptions:

- ▶ The dark matter density and velocity distribution at the position of the Sun and the Earth are identical and constant over 10 – 100 million years.

Output:

- ▶ An upper bound on the DM-nucleon scattering cross section that is independent of the velocity distribution.

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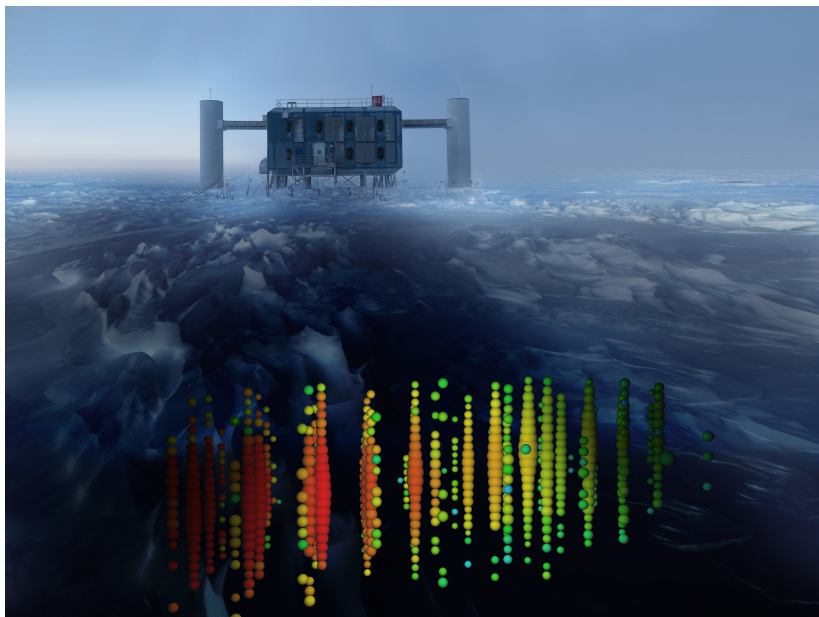
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Neutrino telescopes



The neutrino flux is determined by the capture rate:

$$\begin{aligned}
 C = & \sum_i \int_0^{R_\odot} 4\pi r^2 dr \eta_i(r) \frac{\rho_{\text{loc}}}{m_{\text{DM}}} \times \\
 & \int_{v \leq v_{\text{max},i}^{\text{(Sun)}}(r)} d^3v \frac{f(\vec{v})}{v} \left(v^2 + [v_{\text{esc}}(r)]^2 \right) \times \\
 & \int_{m_{\text{DM}} v^2/2}^{2\mu_{A_i}^2 (v^2 + [v_{\text{esc}}(r)]^2) / m_{A_i}} dE_R \frac{d\sigma_i}{dE_R},
 \end{aligned}$$

DM as a superposition of streams

We can view the DM velocity distribution in the solar system as a superposition of hypothetical streams with fixed velocity \vec{v}_0 with respect to the solar frame:

$$f(\vec{v}) = \int_{|\vec{v}_0| \leq v_{\max}} d^3 v_0 \delta^{(3)}(\vec{v} - \vec{v}_0) f(\vec{v}_0),$$

We can express the expected scattering events R in a direct detection experiment and the capture rate in the Sun C as:

$$R = \int_{|\vec{v}_0| \leq v_{\max}} d^3 v_0 f(\vec{v}_0) R_{\vec{v}_0},$$
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By construction,

$$R_{\vec{v}_0} \leq R_{\max}$$

This gives a limit on $\sigma_{\tilde{\chi}N}$.

For a given $v_0 \equiv |\vec{v}_0|$ find the weakest limit amongst all angles between \vec{v}_0 and the fixed Earth velocity \vec{v}_E . By construction,

$$R_{\vec{v}_0}(\sigma) \geq R_{\max} \quad \text{for} \quad \sigma \geq \sigma_{\max}^{\text{DD}}(v_0).$$

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Similarly, for every stream there is an upper limit on the scattering cross section which is allowed by a neutrino telescope:

$$C_{\vec{v}_0} \leq C_{\max}.$$

The most conservative upper limit, $\sigma_{\max}^{\text{NT}}(v_0)$, for a given stream speed v_0 satisfies

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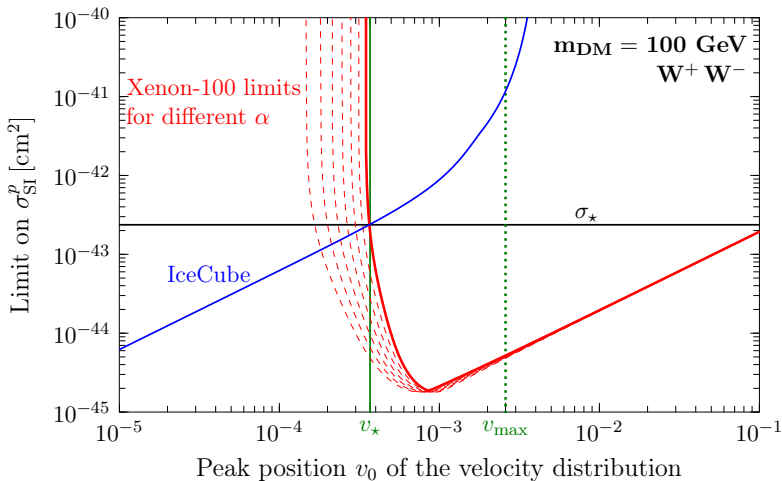
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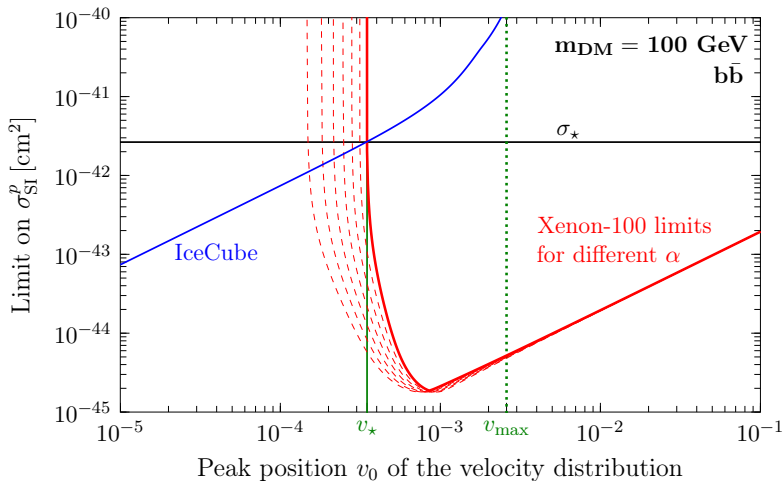
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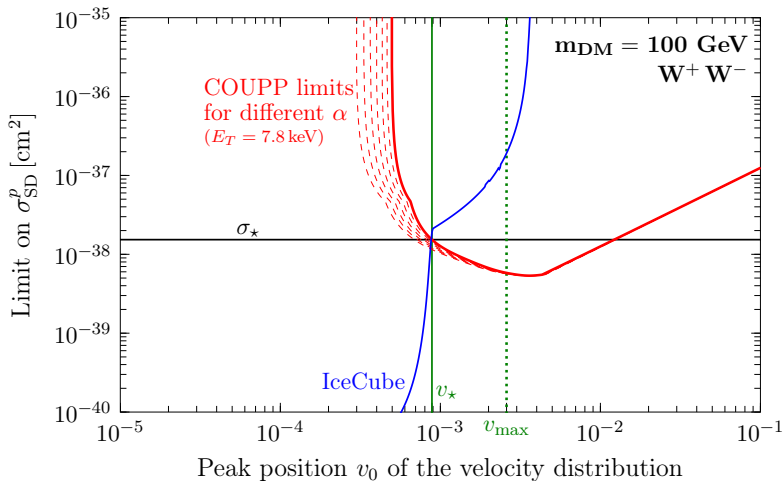
Upper limit on σ_{SI}^p , for $f(\vec{v}) = \delta^3(\vec{v} - \vec{v}_0)$



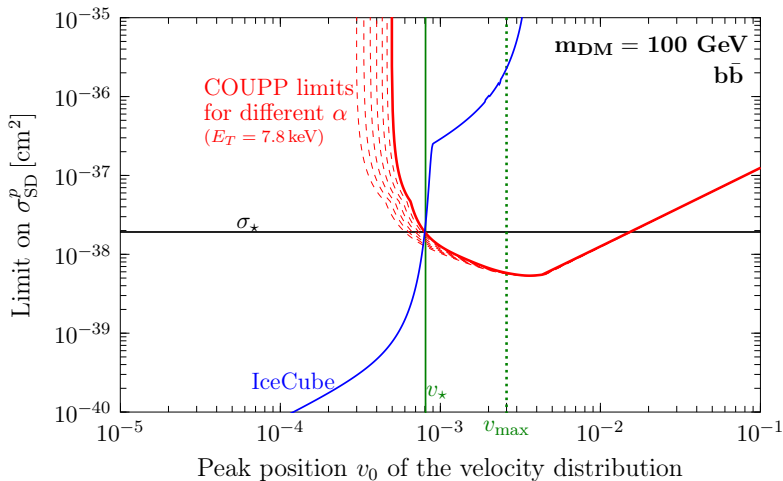
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Upper limit on σ_{SD}^p , for $f(\vec{v}) = \delta^3(\vec{v} - \vec{v}_0)$



Knowing $\sigma_{\max}^{\text{DD/NT}}(v_0)$ we can calculate limits for a general $f(\vec{v})$.
Since

$$R_{\vec{v}_0}(\sigma) = \frac{\sigma}{\sigma_{\max}^{\text{DD}}(v_0)} R_{\vec{v}_0}[\sigma_{\max}^{\text{DD}}(v_0)] \geq \frac{\sigma}{\sigma_{\max}^{\text{DD}}(v_0)} R_{\max},$$

we have

$$R(\sigma) \geq \int_{|\vec{v}_0| \leq v_{\max}} d^3 v_0 f(\vec{v}_0) \frac{\sigma}{\sigma_{\max}^{\text{DD}}(v_0)} R_{\max}.$$

Requiring $R(\sigma) \leq R_{\max}$, we deduce

$$\sigma \leq \left[\int_{|\vec{v}_0| \leq v_{\max}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\max}^{\text{DD/NT}}(v_0)} \right]^{-1}.$$

A halo-independent upper limit on $\sigma_{\tilde{\chi}N}$

DD experiments are insensitive to slowly moving WIMPs. But, these can be efficiently captured in the Sun.

They probe the WIMP population in a complementary way: for every stream speed v_0 there is a finite upper bound.

We can use this fact to our advantage. Consider the largest value allowed between 0 and v_{\max} :

$$\sigma_* \equiv \max \left\{ \sigma_{\max}^{\text{DD}}(\tilde{V}), \sigma_{\max}^{\text{DD}}(v_{\max}) \right\},$$

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Also

$$\begin{aligned}\sigma &\leq \left[\int_{0 \leq v_0 \leq v_{\max}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\max}^{\text{DD}}(v_0)} \right]^{-1} \\ &\leq \left[\int_{\tilde{v} \leq v_0 \leq v_{\max}} d^3 v_0 \frac{f(\vec{v}_0)}{\sigma_{\max}^{\text{DD}}(v_0)} \right]^{-1}.\end{aligned}$$

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Defining

$$\delta_f \equiv \int_{\tilde{v} \leq v_0 \leq v_{\max}} d^3 v_0 f(\vec{v}_0) ,$$

we obtain

$$\sigma \leq \frac{\sigma_*}{\delta_f} . \quad (1)$$

An analogous calculation for the neutrino telescope limit gives

$$\sigma \leq \frac{\sigma_*}{1 - \delta_f} , \quad (2)$$

Together, they imply:

$$\sigma \leq 2\sigma_* \quad (3)$$

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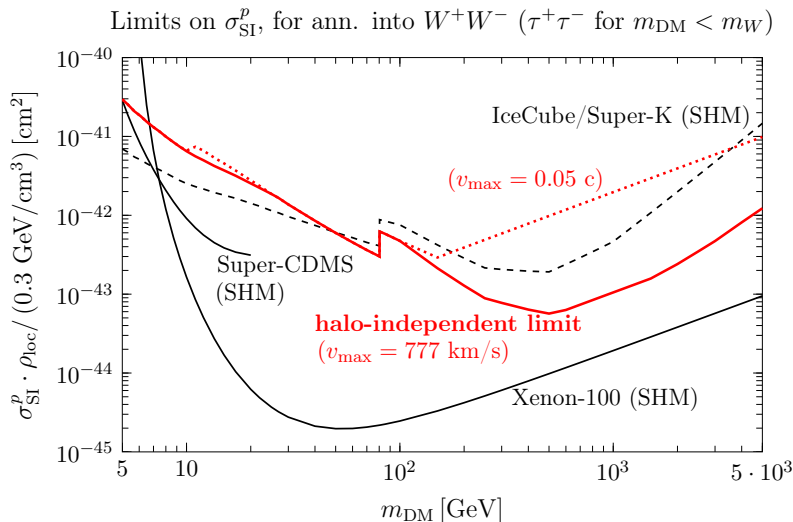
An analogous calculation for the neutrino telescope limit gives

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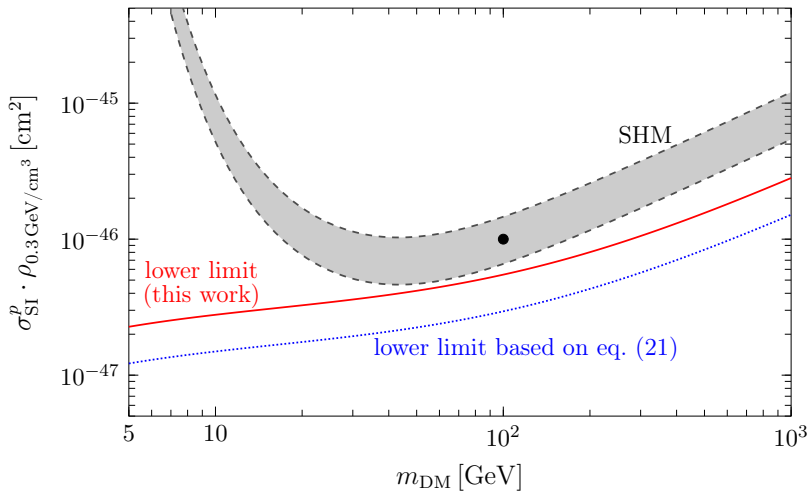
Together, they imply:

$$\sigma \leq 2\sigma_* \quad (3)$$

Comparison with model dependent limits



In case of a positive signal



Conclusions

- ▶ Knowledge of the density profile and velocity distribution is a crucial ingredient for dark matter searches.
- ▶ There is very little information about the inner region of the MW. Hydrodynamic simulations suggest a \sim kpc core that seems at odds with a DM interpretation of the Fermi GeV excess.
- ▶ Null results from direct searches and neutrino telescopes imply a robust upper bound on the DM-nucleon cross section that is independent of the velocity distribution.

$f(v)$ also matters

If there are new light particles mediating long-range forces between the dark matter,

$$\sigma \rightarrow \sigma \times \frac{\pi\alpha}{v}$$

the indirect detection fluxes will depend on $f(v)$. Instead of:

$$\Phi = \underbrace{\frac{N_i}{8\pi m_\chi^2} \langle \sigma v \rangle}_{\text{\# collisions giving SM particles}} \times \underbrace{\int_{\text{line of sight}} ds \rho^2}_{\text{Amount of DM}^2}$$

We should be doing:

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Obtaining the phase-space distribution

Assume that dark matter satisfies the collisionless Boltzmann equation,

$$\frac{df}{dt} = 0$$

Very hard to solve! Only a few exact solutions known, found finding integrals of motion (*singular isothermal sphere*, Hernquist, Jaffe, ...).

Use Eddington's formula:

$$f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \int_0^{\mathcal{E}} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{d^2\rho}{d\Psi^2}. \quad (4)$$

Caveats: we are assuming $\beta = 0$.

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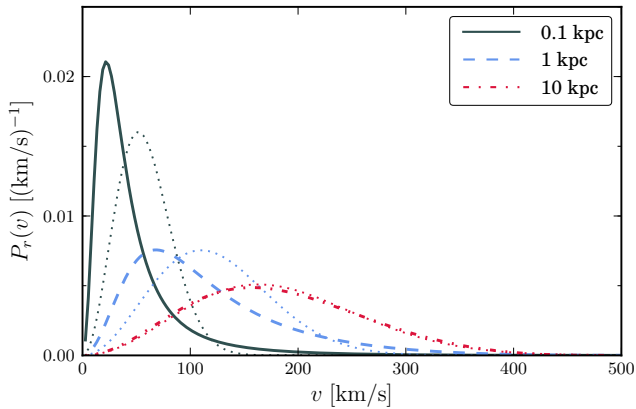
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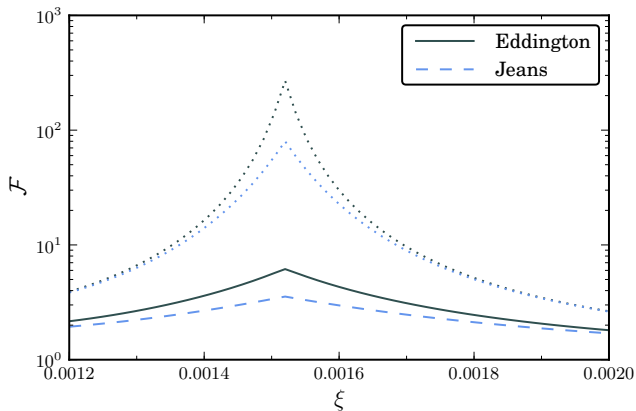
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Velocity distribution: Eddington

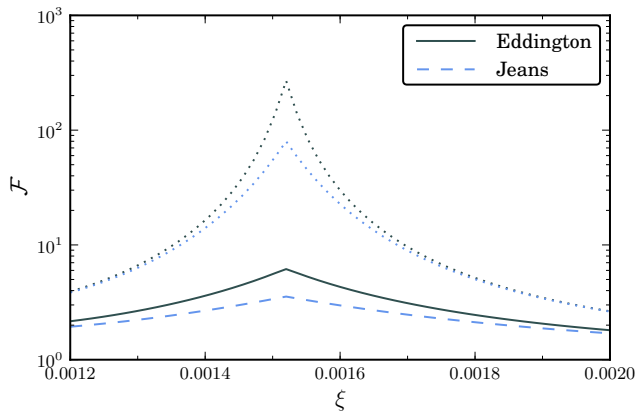


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