Kerr-Newman Black Holes in Supergravity

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Corrections to Black Hole Entropy

- Precision studies of BHs in string theory usually focus on BPS black holes (and their near BPS relatives).

- Conventional wisdom: far from BPS there are large and complicated corrections.

- This talk: ***explicitly compute corrections*** to black hole entropy far from the BPS limit.

- Two types: ***quantum corrections*** and ***higher derivative corrections***.

- Generally the corrections are found to be fairly complicated, as expected.

Environmental Dependence

- Black holes are often solutions to many different theories.
- For example, ***Kerr-Newman black holes*** are usually considered ***solutions to the Einstein-Maxwell theory***

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- A simple variation: augment the theory by adding a field that appears only quadratically in the action (such as a fermion ψ.)
- The ***solution is "the same"*** because it is consistent to assume that the additional field vanishes $\psi = 0$.
- Environmental dependence: ***corrections depend on such additional fields*** (for example, these fields run in quantum loops).
- This talk: ***Kerr-Newman black holes are simpler in an environment with $\mathcal{N} \geq 2$ supersymmetry***.

This Talk

- ***Embedding*** of Kerr-Newman black holes into theories with $\mathcal{N} \geq 2$ SUSY.
- ***Quantum corrections*** to black hole entropy.
- ***Higher derivative corrections*** to black hole entropy: Weyl^2.
- ***Discussion***: structure of black hole entropy.

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Black Hole Solutions

- Starting point: consider ***any solution to Einstein-Maxwell theory***

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)$$

- The general asymptotically flat stationary black holes: Kerr-Newman (quantum numbers: M, J, Q).

- Special cases:
  - Schwarzchild: M general but J=Q=0
  - Kerr: M and J general but Q=0
  - Reissner-Nordström: M and Q general but J=0
  - BPS M=Q and J=0

- We want to consider these as solutions to $\mathcal{N} \geq 2$ SUGRA.

N=2 SUGRA

- The simplest case: $\mathcal{N}=2$ ***minimal*** SUGRA has bosonic part identical to Einstein-Maxwell theory.
- The solution remains a solution after two gravitini multiplets are added because they can be consistently set to zero.
- ***Coupling to n_V vector multiplets is a more significant challenge***:

$$\mathcal{L} = \frac{1}{2\kappa^2}R - g_{\alpha\bar{\beta}}\nabla^\mu z^\alpha \nabla_\mu z^{\bar{\beta}} + \frac{1}{2}\mathrm{Im}\left[\mathcal{N}_{IJ}F^{+I}_{\mu\nu}F^{+\mu\nu J}\right]$$

- Comments:
  - Complex scalar fields in vector multiplets: z^α, $\alpha = 1, \ldots, n_V$.
  - Vector fields A^I_μ include the graviphoton so $I = 0, \ldots, n_V$ (one more value).
  - Kähler metric $g_{\alpha\bar{\beta}}$ and vector couplings $\mathcal{N}_{IJ}$ depend on scalars and are related by special geometry.

Adding Scalars to Kerr-Newman

- Kerr-Newman does not have scalars so to maintain the "same" solution we take the $\mathcal{N} = 2$ ***scalars constant***.

- An obstacle: ***generally the vector fields source the scalars*** so they cannot be constant.

- The sources on the scalars all cancel if we specify the $\mathcal{N} = 2$ vectors in terms of the Einstein-Maxwell vector as:

$$F^{+I}_{\mu\nu} = X^I F^+_{\mu\nu} \, .$$

  with X^I projective coordinates for the scalars.

- Interpretation 1: specify moduli, then pick the vectors (Q^I, P_I) so that the BPS attractor equations are satisfied for these moduli.

- Interpretation 2: the ***Einstein-Maxwell field is the graviphoton***.

More General Embedding

- We consider ***all theories with $\mathcal{N} \geq 2$ SUSY***.
- It is convenient to summarize the matter content in terms of $\mathcal{N} = 2$ fields: one SUGRA multiplet, $\mathcal{N} - 2$ (massive) gravitini, n_V vector multiplets, n_H hyper multiplets.
- This ***decomposition is useful for both BPS and non-BPS***.
- Our embedding takes the geometry unchanged, matter fields “minimal”, and guarantees that all equations of motion of $\mathcal{N} \geq 2$ SUGRA are satisfied.
- We want to compute quantum corrections of Kerr-Newman as a solution in $\mathcal{N} \geq 2$ SUGRA.

Quantum Corrections: Generalities

- The entropy of a large black hole allows the expansion:

$$S = \frac{A}{4G} + \frac{1}{2} D_0 \log A + \ldots \ .$$

- ***Taking all parameters with dimension length large***: area $A \sim (2MG)^2$ by dimensional analysis ***up to a function of dimensionless ratios*** $J/M^2, Q/M$ that is nontrivial.

- In the same limit, the logarithmic correction is $\log A \sim \log 2MG$ up to the function D_0 of dimensionless ratios that is interesting.

- The area A and the coefficient D_0 can both be ***computed from the low energy theory***: only massless fields contribute.

- They each offer an ***infrared window into the ultraviolet theory***.

Quantum Fluctuations: Strategy

- All contributions from quadratic fluctuations around the classical geometry take the schematic form

$$e^{-W} = \int \mathcal{D}\phi \, e^{-\phi\Lambda\phi} = \frac{1}{\sqrt{\det\Lambda}} \, .$$

- The quantum corrections are encoded in the heat kernel

$$D(s) = \mathrm{Tr} \, e^{-s\Lambda} = \sum_i e^{-s\lambda_i} \, .$$

- The effective action becomes

$$W = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} D(s) = -\frac{1}{2}\int_{\epsilon^2}^{\infty} \frac{ds}{s} \int d^D x K(s) \, .$$

- The leading corrections are encoded in the ***the s-independent term in $D(s)$ denoted D_0***, *a.k.a.* the 2nd Seeley-deWitt coefficient, *a.k.a.* the trace anomaly.

Interactions

- ***In principle***: computations are straightforward applications of techniques from the 70's.

- But: our embedding into SUGRA gives ***nonminimal couplings***.

- For example, for fermions in $\mathcal{N} = 2$ hypermultiplets the background enter through Pauli couplings

$$\mathcal{L}_{\rm hyper} = -2\bar{\zeta}_A \gamma^\mu D_\mu \zeta^A - \frac{1}{2}\left(\bar{\zeta}^A F_{\mu\nu} \Gamma^{\mu\nu} \zeta^B \epsilon_{AB} + {\rm h.c.} \right) \ .$$

- Bosons in $\mathcal{N} = 2$ vector multiplets (some effort to show)

$$\mathcal{L}_{\rm vector} = -g_{\alpha\bar{\beta}} \left(\nabla_\mu z^\alpha \nabla^\mu \bar{z}^{\bar{\beta}} + \frac{1}{2} f^\alpha_{\mu\nu} f^{\mu\nu\bar{\beta}} - \frac{1}{2} (F^-_{\mu\nu} f^{\alpha\mu\nu} \bar{z}^{\bar{\beta}} + {\rm h.c.}) \right)$$

- Such nonminimal couplings force us to compute some new heat kernels.

Heat Kernel Technology

- Perturbative expansion of the (equal point) heat kernel density:

$$K(x,x;s) = \sum_{n=0}^{\infty} s^{n-2} a_{2n}(x)$$

- We need a_4 (the D_0 coefficient is the spacetime integral over a_4)

- Schematic for Λ:

$$\Lambda^n_m = -(\mathcal{D}^\mu \mathcal{D}_\mu) I^n_m - (2\omega^\mu D_\mu)^n_m - P^n_m$$

- General result:

$$(4\pi)^2 a_4(x) = \mathrm{Tr}\left[\frac{1}{2}E^2 + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{180}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu})I\right] .$$

Notation:

$$E = P - \omega^\mu \omega_\mu - (D^\mu \omega_\mu) \ , \quad \mathcal{D}_\mu = D_\mu + \omega_\mu \ , \quad \Omega_{\mu\nu} \equiv [\mathcal{D}_\mu, \mathcal{D}_\nu] \, .$$

Examples

- Lagrangian for minimally coupled scalar with mass m:

$$\mathcal{L} = -\frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \frac{1}{2}m^2\phi^2\,.$$

  Heat kernel coefficient

$$(4\pi)^2 a_4^{\text{minimal scalar}}(x) = \frac{1}{2}m^4 + \frac{1}{180}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}\right)\ .$$

- Gravitino in the $\mathcal{N}=2$ SUGRA multiplet:

$$\mathcal{L}_{\text{gravitini}} = -\frac{1}{2\kappa^2}\bar{\Psi}_{A\mu}\gamma^{\mu\nu\rho}D_\nu\Psi_{A\rho} + \frac{1}{4\kappa^2}\bar{\Psi}_{A\mu}\left(F^{\mu\nu} + \gamma_5\widetilde{F}^{\mu\nu}\right)\epsilon_{AB}\Psi_{B\nu}$$

  Heat kernel coefficient:

$$\begin{aligned}(4\pi)^2 a_4^{\text{gravitino}}(x) = -\frac{1}{360}\Big(&212R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 32R_{\mu\nu}R^{\mu\nu}\\ &-360R_{\mu\nu}(F^{\mu\rho}F^{\nu}_{\ \ \rho} - \widetilde{F}^{\mu\rho}\widetilde{F}^{\nu}_{\ \ \rho}) + 180R_{\mu\nu\rho\sigma}(F^{\mu\nu}F^{\rho\sigma} - \widetilde{F}^{\mu\nu}\widetilde{F}^{\rho\sigma})\\ &+45(F^{\mu\rho}F_{\nu\rho} - \widetilde{F}^{\mu\rho}\widetilde{F}_{\nu\rho})(F_{\mu\sigma}F^{\nu\sigma} - \widetilde{F}_{\mu\sigma}\widetilde{F}^{\nu\sigma})\Big)\ .\end{aligned}$$

Simplifications

- General form of 2nd Seeley-deWitt coefficient:

$$a_4(x) = \alpha_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \alpha_2 R_{\mu\nu} R^{\mu\nu} + \alpha_3 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \ldots$$

- After simplifications using Einstein equation, Bianchi identities, ...

$$a_4(x) = \frac{c}{16\pi^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2} E_4 ,$$

  Euler density

$$E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 .$$

- Note: ***terms dependent on field strength all cancel***.

- Final results can be expressed in terms of c, a only.

- For each $\mathcal{N} = 2$ multiplet: record c, a for bosons and fermions independently.

Integrals

- Form of quantum corrections to the entropy:

$$\delta S = \frac{1}{2} D_0(\frac{Q}{M}, \frac{J}{M^2}) \log A_H$$

- The a-term gives a ***universal*** (independent of BH parameters) contribution to D_0 because

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{-g}\, E_4 = 2\ .$$

- The c-term gives a complicated contribution to D_0:

$$\int d^4x \sqrt{-g}\, W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} = 64\pi^2 + \frac{\pi\beta Q^4}{b^5 r_H^4 (b^2 + r_H^2)} \Big[4b^5 r_H + 2b^3 r_H^3 + 3(b^2 - r_H^2)(b^2 + r_H^2)^2 \tan^{-1}\left(\frac{b}{r_H}\right) + 3b r_H^5 \Big]\ .$$

$b = J/M$, $r_H = M + \sqrt{M^2 - b^2}$, $\beta = 1/T$.

Results: Logarithmic Corrections

- Contributions from bosons in $\mathcal{N} \geq 2$ theory:

$$c^{\text{boson}} = \frac{1}{60}(137 + 12(\mathcal{N} - 2) - 3n_V + 2n_H)$$
$$a^{\text{boson}} = \frac{1}{90}(106 + 31(\mathcal{N} - 2) + n_V + n_H)$$

- Contributions from fermions in $\mathcal{N} \geq 2$ theory:

$$c^{\text{fermion}} = \frac{1}{60}(-137 - 12(\mathcal{N} - 2) + 3n_V - 2n_H)$$
$$a^{\text{fermion}} = \frac{1}{360}(-589 + 41(\mathcal{N} - 2) + 11n_V - 19n_H)$$

- The ***c coefficent vanishes in $\mathcal{N} \geq 2$ theory!***

- A ***huge simplification***: Weyl^2 terms are complicated in general backgrounds.

- It is ***a surprise***: SUSY of the background $\Rightarrow$ $AdS_2 \times S^2$ $\Rightarrow$ $\text{Weyl}^2 = 0$

Summary: Quantum Corrections

- Logarithmic corrections to black hole entropy in $\mathcal{N} \geq 2$ SUGRA are determined by the coefficient of the Euler invariant.

- This coefficient ***depends only on the theory*** (not on parameters of the black hole) so the ***logarithmic corrections are universal:***

$$\delta S = \frac{1}{12}\left(23 - 11(\mathcal{N} - 2) - n_V + n_H\right) \log A_H \ .$$

- These corrections can be reproduced from microscopic theory in some BPS cases.

- The IR theory is a window into the UV theory: apparently the ***deformation (far!) off extremality is independent of coupling!***

- A minor caveat: fermionic zero modes (due to enhanced SUSY) gives a jump at extremality (in most ensembles).

Higher Derivative Corrections

- String theory corrections can give a Weyl^2 term ***directly in the action***.
- Quantum result: the coefficient of this term does not receive quantum corrections, it is not renormalized in $\mathcal{N} \geq 2$ SUGRA.
- ***A term directly in the action is complicated***: the Einstein equation is modified by the Bach tensor

$$B_{\mu\nu} = 4R_{\mu\rho}R_{\nu}^{\rho} - g_{\mu\nu}R_{\rho\sigma}R^{\rho\sigma} - \frac{4}{3}R_{\mu\nu}R + \frac{1}{3}g_{\mu\nu}R^2 - 2R_{\mu\nu}R^{\mu\nu} + 4D^{\rho}D_{\mu}R_{\nu\rho} + \frac{1}{3}g_{\mu\nu}R^2 - \frac{4}{3}D_{\mu}D_{\nu}R \,.$$

- A perturbative approach: evaluate $B_{\mu\nu}$ on the Kerr-Newman background, use result to correct geometry, evaluate $B_{\mu\nu}$ on the corrected geometry,....
- This procedure is simple conceptually but the results are complicated and not illuminating.

SUSY Theory and BPS Black Holes

- We are interested in corrections with $\mathcal{N} = 2$ SUSY, eg. a Weyl^2 term ***along with its N=2 partners.***
- Details of the action: off-shell formalism from reduction of superconformal supersymmetry, a lot of auxiliary fields.
- To find ***BPS solutions*** to the resulting action:
  - Constrain solution using BPS conditions (independent of action in off-shell theory)
  - Use action for the final details.
- This ***program has proven extremely successful***:
  - Corrections to BPS BH entropy were computed in detail.
  - They were found to agree with microscopic considerations.
- Summary of agreements: OSV conjecture ***relating BPS black hole entropy to topological string amplitudes***.

SUSY Theory and Kerr-Newman

- We have analyzed the full ***equations of motion*** of $\mathcal{N}=2$ SUGRA with Weyl^2 (***including SUSY partners***).

- The equations of motion are extremely elaborate due to all the terms required by SUSY.

- In the simplest case of pure $\mathcal{N}=2$ SUGRA (no vector multiplets): ***the full equations motion are satisfied by Kerr-Newman***.

- Cartoon: there is an elaborate cancellation between gravitational terms (Weyl^2), their matter partners (F^4), and cross-terms (RF^2).

- The simplifications are for a ***theory with*** $\mathcal{N}=2$ ***SUSY*** but ***solutions that preserve no SUSY***.

Wald Entropy

- The geometry is the same, but the Wald entropy is changed.

- ***Corrections to Wald entropy simplify greatly***. Schematically:

$$\partial_{\mathrm{Riem}}(\mathrm{Weyl}^2 + \mathrm{SUSY\ partners}) = \partial_{\mathrm{Riem}}\left((\mathrm{Riem}^2 - 2\mathrm{Ric}^2 + \frac{1}{3}R^2) + \frac{1}{4}\mathrm{Ric}\ F^2\right)$$
$$= \partial_{\mathrm{Riem}}\left(\mathrm{Riem}^2 - 4\mathrm{Ric}^2 + R^2\right) = \partial_{\mathrm{Riem}} E_4$$

- The correction to the Wald entropy due to higher order derivatives is a ***constant, independent of black hole parameters***:

$$\Delta S = 256\pi c_2$$

- The ***value of the constant is related to the prepotential*** according to the ***OSV conjecture for BPS black holes***.

Entropy Phenomenology

- ***Entropy of Kerr Newman BHs***:

$$S = 2\pi \left((M^2 - \frac{1}{2}Q^2) + \sqrt{M^2(M^2 - Q^2) - J^2} \right)$$

- The form of the formula is ***reminiscent of the Cardy formula for entropy of 2D CFTs***:

$$S = 2\pi \left[\sqrt{\frac{c_L h_L}{6}} + \sqrt{\frac{c_R h_R}{6}} \right]$$

- The resemblance may be an important clue.

- Dependence on angular momentum J accounted for by the model: identify it with the $SU(2)_R$ quantum number in a $(0, 4)$ CFT (as for 5D BPS BHs).

Matching Condition

- The levels assigned by the phenomenology satisfy a ***matching condition***

$$\frac{c_L h_L}{6} - \frac{c_R h_R}{6} = \frac{1}{4} Q^4 = \text{integer} = \text{independent of M}$$

- This would be expected from ***modular invariance of a CFT*** (at least in the short string sector $c_L = c_R = 6$).

- Complaint: $Q^2 = n^2 \alpha$ with $\alpha \simeq 1/137$ in QED is not an integer.

- Response: embedding into $\mathcal{N} = 2$ SUGRA: $Q^2 \to 4QP$.
  Dirac's quantization rule gives $QP = \frac{1}{2} \times \text{integer}$.

- So: $\mathcal{N} = 2$ ***of theory*** gives simplifications for generic black holes.

Derivative Corrections to Entropy

- Higher derivative corrections (Euler or $\mathrm{Weyl}^2 + \mathrm{SUSY}$) ***add a constant to the entropy***.

- The value is ***independent of mass*** so same as BPS case so accounted for by BPS CFT (the OSV conjecture).

- The quantum corrections are also the same as the BPS case so accounted for by BPS CFT (the OSV conjecture).

- So ***all corrections pertain to L sector*** (as for BPS black holes).

The Inner Horizon Revisited

- For any black hole geometry there is an ***algorithm to split the entropy into "L" and "R" contributions***:

$$S_+ = S_L + S_R = \frac{1}{2}(S_+ + S_-) + \frac{1}{2}(S_+ - S_-)$$

where $S_\pm = \frac{A_\pm}{4G_N}$ with $\pm$ referring to outer and ***inner horizon***.

- Higher derivative corrections are the same for outer and inner horizon.

- This algorithm agrees that ***all corrections are captured by*** S_L.

- Slogan for matching condition (leading order)

$$\frac{1}{(8\pi G_4)} A_+ A_- = \text{integer}$$

- Generalization to higher derivatives using $S_{L,R}$ as simple as using $S_\pm$ and apparently the correct one.

Nonextreme BH Entropy (Scenario)

- Extremal Black hole entropy is accounted for by ***chiral string field theory*** (multiple strings).

- These theories are complicated but ***we have much experience with them*** at the level of indices and/or localization.

- A scenario for nonextreme black hole entropy: ***couple L and R chiral string field theory*** minimally.

- The L theory same as BPS theory, the R theory contains oscillator excitations but no new corrections.

- ***This gives the correct result*** for the black hole entropy.

Summary

- We evaluated corrections to Kerr-Newman black holes motivated by string theory: quantum corrections and higher derivative corrections.

- Perspective: $\mathcal{N} \geq 2$ ***SUSY of the theory simplifies results greatly even when BHs preserve no SUSY.***

- Quantum corrections: independent of mass (so the same as for BPS black holes)

- Higher derivative corrections $(\mathrm{Weyl}^2 + \mathrm{SUSY})$ also independent of mass (so the same as for BPS black holes)

- Significance: evidence that ***black hole entropy far from extremality is accounted for by weakly coupled strings.***