Cosmic Microwave Background

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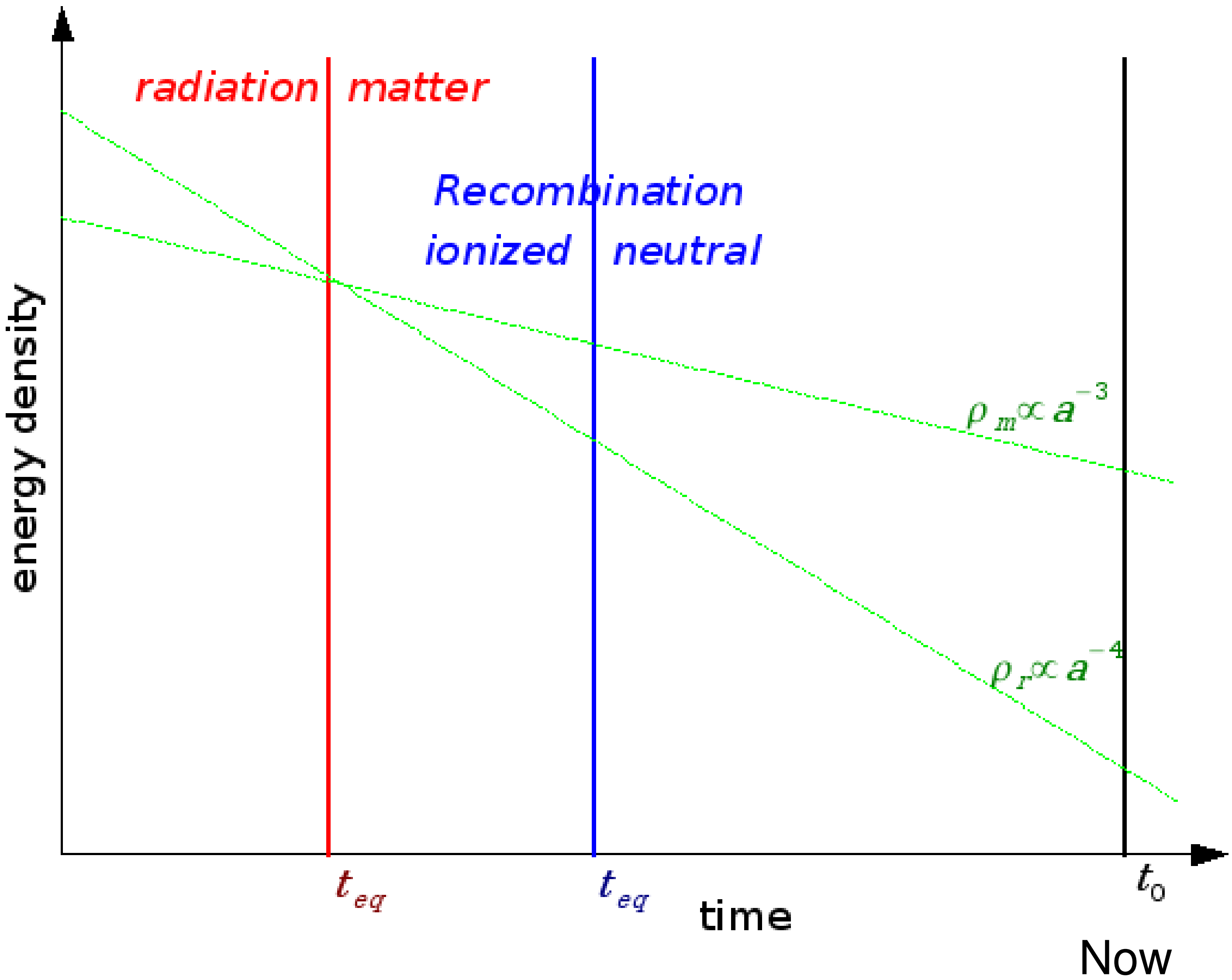
Thermal history of the Universe



Physics of recombination: $H + \gamma \leftrightarrow e^- + p, \quad B_H = 13.6 \text{ eV}$

In thermal equilibrium, for $T \ll m_p$, number densities are

$$n_X = g_X \left(\frac{m_X T}{2\pi}\right)^{3/2} e^{(m_X - \mu_X)/T}$$

Using $n_e = n_p$, $n_B = n_p + n_H$, Saha's equation for ionized fraction $x_e = n_e/n_B$ reads

$$\frac{1 - x_e}{x_e^2} \approx 3.84 \eta_B \left(\frac{T}{m_e}\right)^{3/2} e^{B_H/T}$$

$T_{rec} = 0.32$ eV

where η_B is the photon-to-baryon ratio

$$\eta_B = \frac{n_B}{n_\gamma} = 2.7 \times 10^{-8} \left(\frac{\Omega_b h^2}{T_{2.7}^3}\right)$$

Thompson scattering maintains photon temperature close to matter temperature.
Decoupling occurs when $\Gamma_T(T_{dec}) = H(T_{dec})$.

$T_{dec} = 0.26$ eV

Last scattering – visibility function

The visibility function g(z) is the probability that a CMB photon was last scattered in the redshift interval [z, z+dz]

$$g(z) = e^{-\tau}\frac{d\tau}{dz}, \quad \tau(z) = \int_0^z dz\,\frac{\sigma_T n_e(z)}{(1+z)H(z)}$$

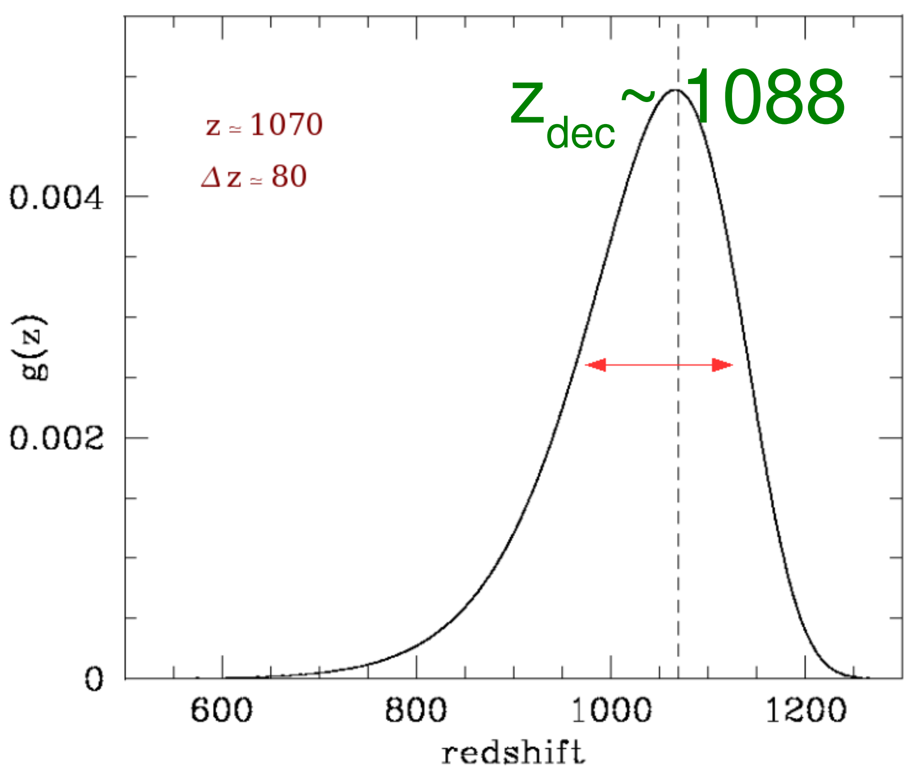
Below z_{dec}, the photon distribution f_γ evolves according to the Liouville equation

$$\frac{df_\gamma}{d\eta} = 0$$

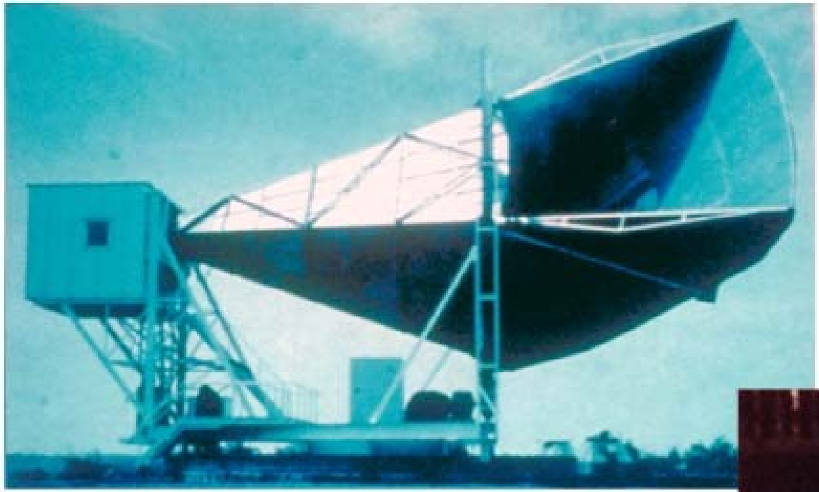
$$T_\gamma(0) = T_\gamma(z_{dec})/(1+z_{dec})$$

$\sim 2.5\ 10^{-4}$ eV

~ 2.7 K

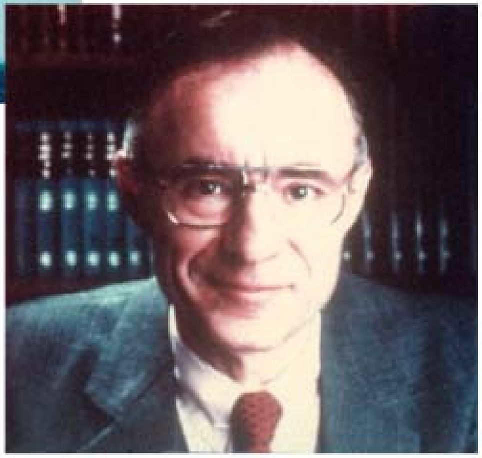


Discovery of Cosmic Background (1965)



Microwave Receiver

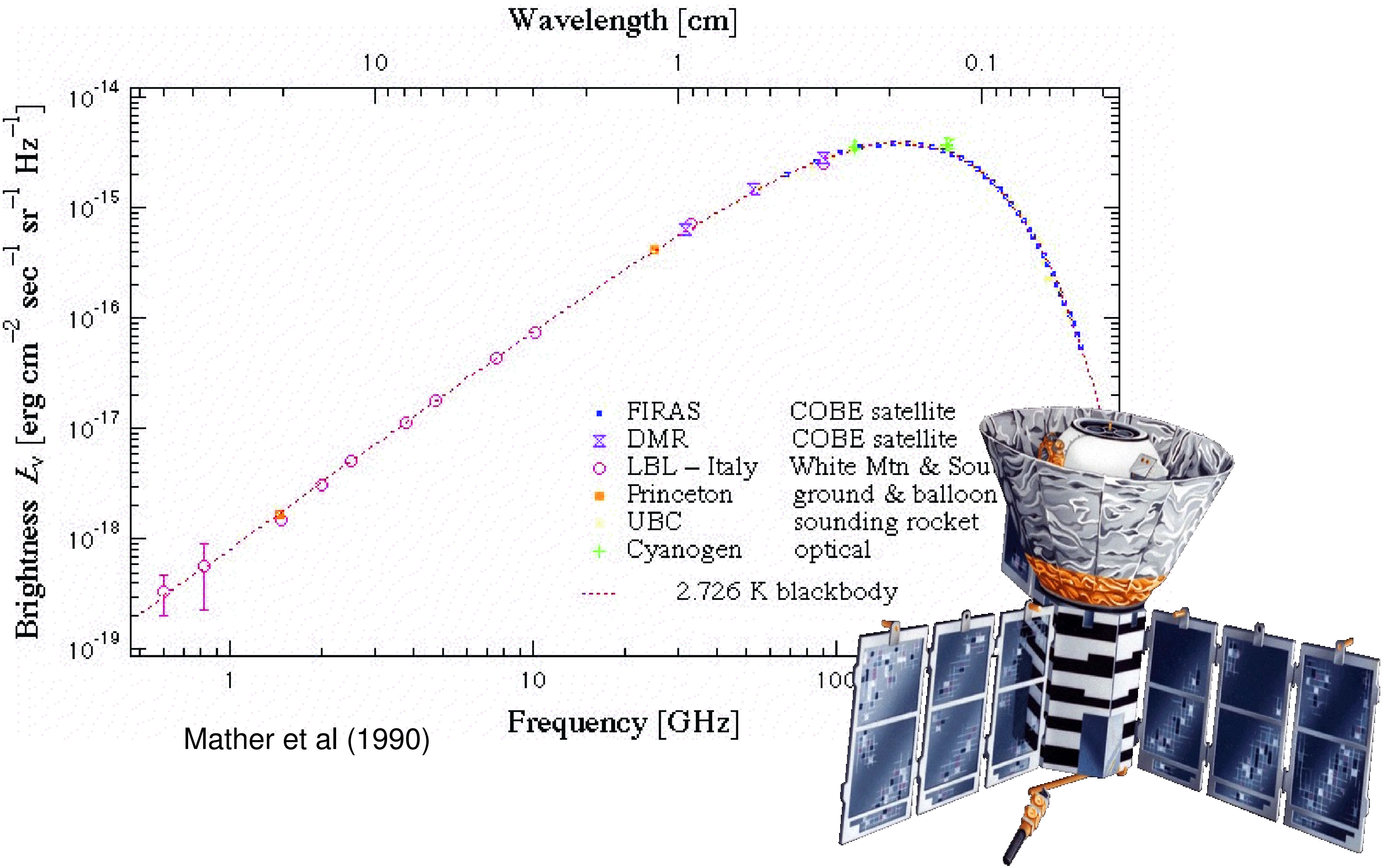
Robert Wilson



Arno Penzias

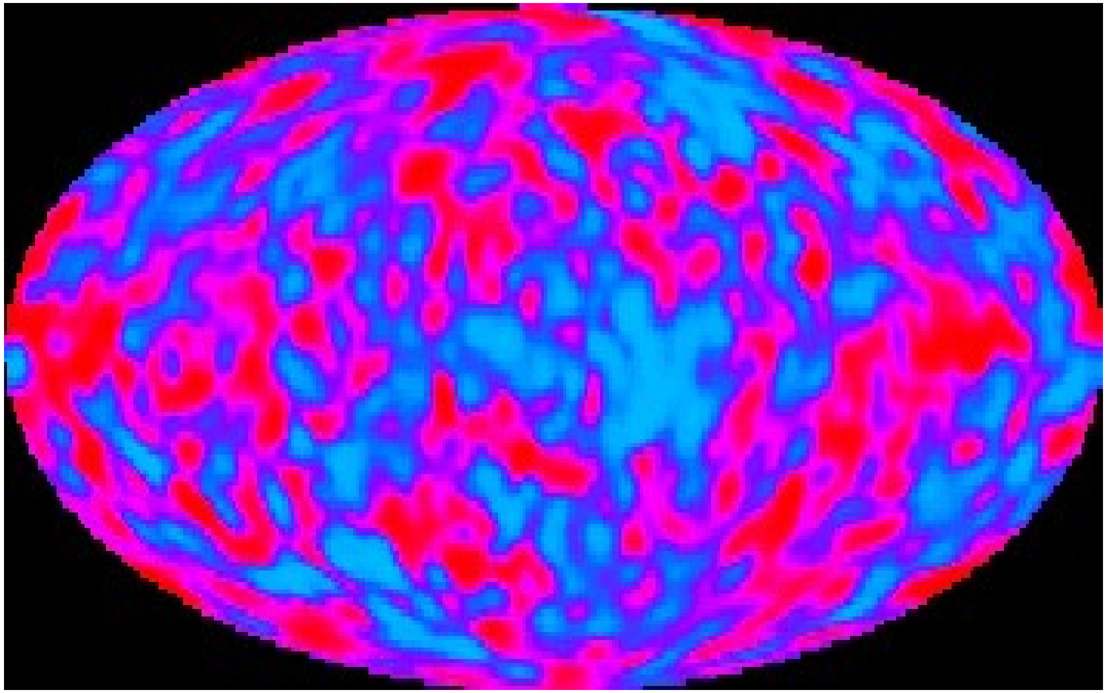
MAP990045

COBE/FIRAS (1990) : Nearly perfect blackbody at T_0=2.73 K



Mather et al (1990)

COBE/DMR (1992) : temperature anisotropies $\Delta T/T_0 \sim 10^{-5}$



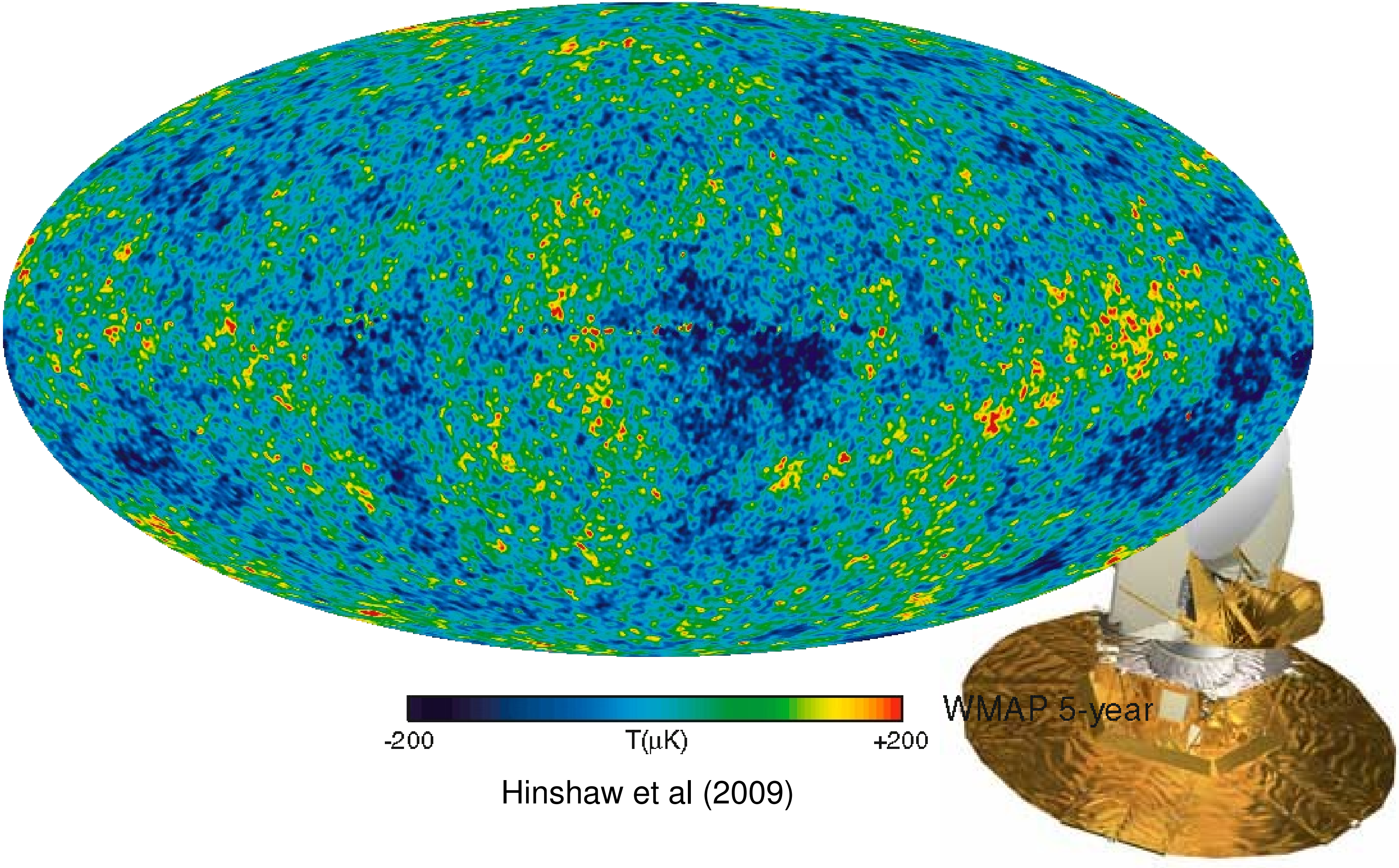
Smoot et al. (1992)

Sachs-Wolfe Effect (1967): $\Delta T/T_0 = \Psi/3$

Cold spots = larger photon overdensity
= stronger gravity

WMAP (2000 – present):

CMB temperature anisotropies resolved down to ~ 13 arcmins

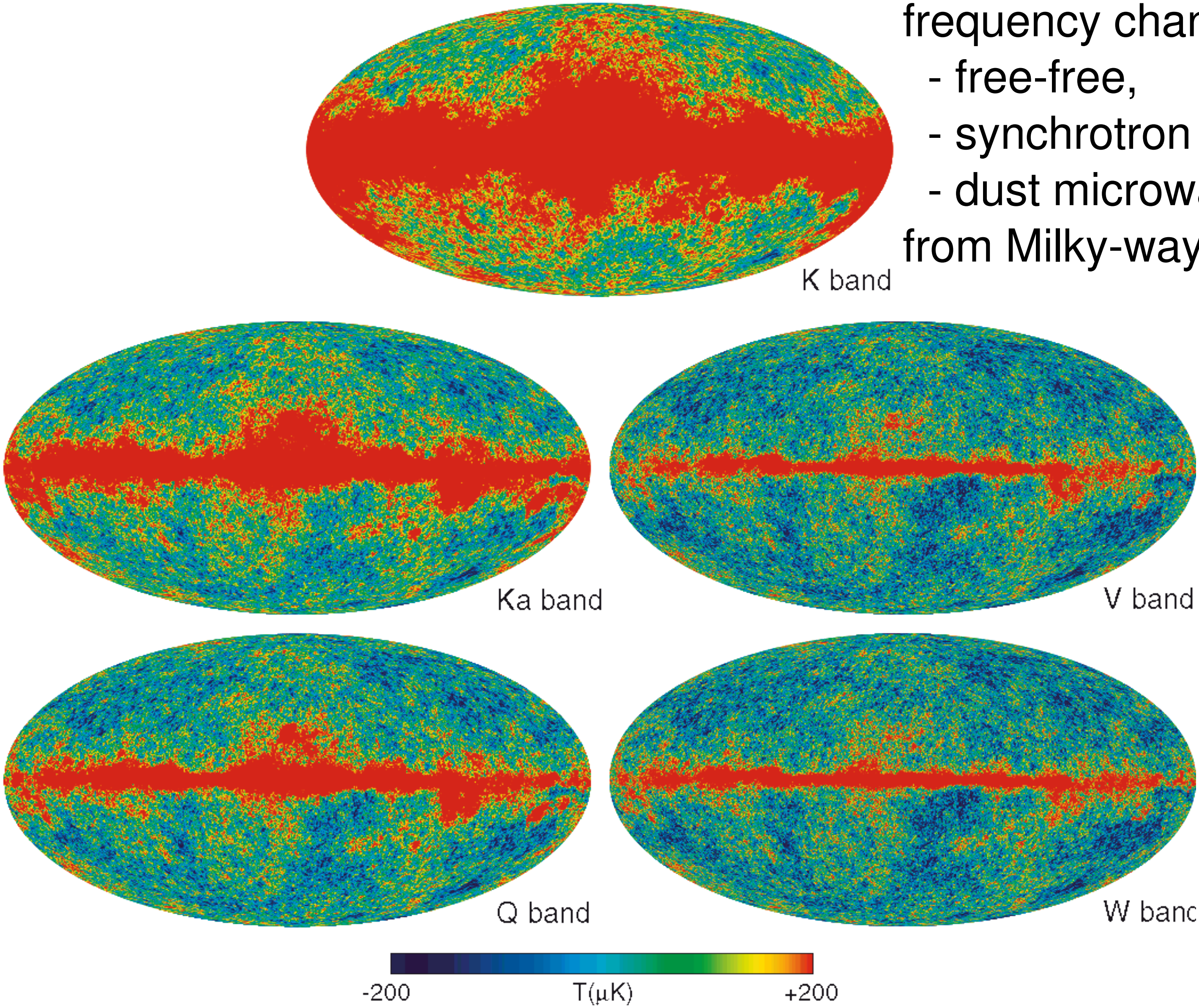


Hinshaw et al (2009)

Foreground contamination

Measure CMB anisotropies in several frequency channels to remove
- free-free,
- synchrotron
- dust microwave emission

from Milky-way + extragalactic sources



Hinshaw et al (2009)

Why are CMB anisotropies so useful ?

Baryon-to-photon ratio η_{B}

- Sound speed and inertia of photon-baryon fluid

Matter-to-radiation ratio $\Omega_{\mathrm{m}} / \Omega_{\gamma}$

- Dark matter abundance
- Neutrinos + other relativistic components

Angular diameter distance to Last Scattering Surface

- Position of acoustic peaks

Time dependence of gravitational potential

- Integrated Sachs-Wolfe effect, Dark Energy

Optical depth

- cosmic reionization

Primordial power spectrum (scalar+tensor)

- Slow-roll inflation
- gravity waves (CMB polarization)

Primordial non-Gaussianity

- constraints on inflationary models

CMB temperature power spectrum

CMB temperature anisotropies are expanded in spherical harmonics

$$\frac{\Delta T}{T_0}(\hat{\mathbf{n}}) = \sum_{lm} \Theta_l^m Y_l^m(\hat{\mathbf{n}})$$

The CMB temperature power spectrum is the ensemble average

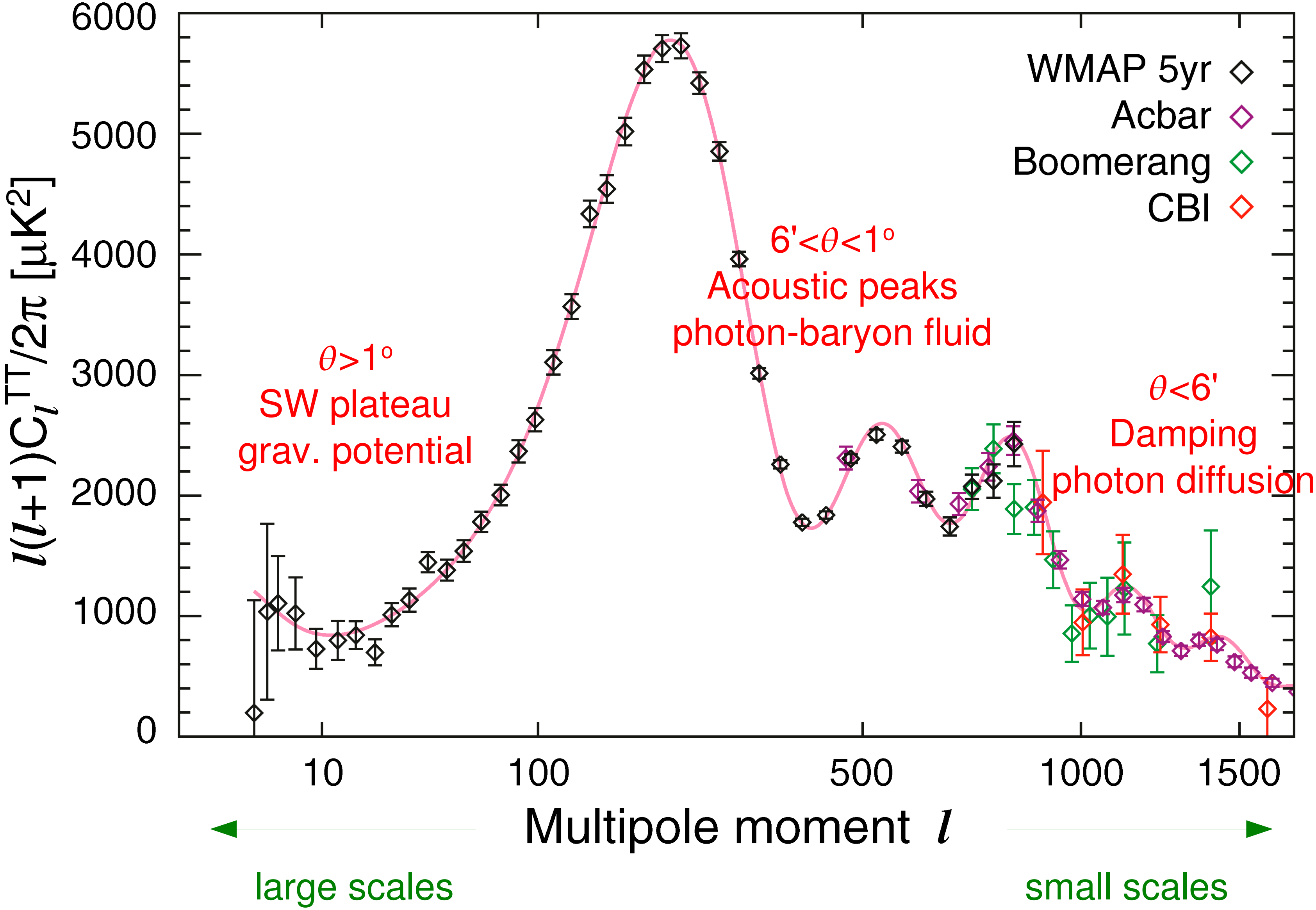
$$\langle \Theta_l^m \Theta_{l'}^{m'} \rangle = \delta_{ll'} \delta_{mm'} C_l^{TT}$$

For Gaussian fluctuations, Θ_l^m are Gaussian distributed and C_l^{TT} contain the full statistical information of the CMB temperature distribution

Fundamental limitation set by cosmic variance:

$$\Delta C_l = \sqrt{\frac{2}{2l+1}} C_l$$

Measured C_l^{TT}



Nolta et al (2009)

The physics of CMB temperature anisotropies

Solve a coupled set of relativistic Boltzmann equations

$$\frac{df_X}{d\eta} = C[f], \quad X = "\gamma", "B", "c", "\nu" \cdots$$

and the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

to first order in perturbations around a FRW Universe with metric

$$ds^2 = a^2 \left[-d\eta^2 + \gamma_{ij}(\kappa) dx^i dx^j \right], \qquad \kappa = 0, \pm 1$$

and stress-energy tensor

$$T^{\mu}_{\nu} = \mathrm{diag}(-\bar{\rho}, \bar{p}, \bar{p}, \bar{p})$$

Cosmological perturbation theory

A general perturbation to the FRW metric may be represented as

$$ds^2 = a^2 \left\{ -\left(1+2\Psi\right) d\eta^2 + 2B_i dx^i d\eta + \left[\left(1+2\Phi\right)\gamma_{ij} + 2H_{ij}\right] dx^i dx^j \right\}, \quad H^i_i = 0$$

while for the stress-energy tensor one may choose

$$T^0_0 = -\rho - \delta\rho$$

$$T^0_i = \left(\rho + p\right)\left(V_i - B_i\right)$$

$$T^i_0 = -\left(\rho + p\right) V^i$$

$$T^i_j = \left(p + \delta p\right)\delta^i_j + p\Pi^i_j, \quad \Pi^i_i = 0$$

i) Go to Fourier space. Translation invariance of the linearized equations of motion implies that different Fourier modes do not interact.
ii) Decompose the perturbations into scalar (S), vector (V) and tensor (T) components, each of which evolves independently at first order.
iii) Choose a gauge (coordinates) according to the problem considered (i.e. evolution of CMB, inflationary, neutrinos fluctuations etc.)

Photon propagation

The distribution function for photons of energy E and propagation direction **n** is a perturbed Black-Body spectrum,

$$f_\gamma(\mathbf{x}, \hat{\mathbf{n}}, E, \eta) = f_P\left(\frac{aE}{1+\Theta}\right), \quad f_P(\epsilon) \equiv 2\left[\exp(\epsilon/T_0) - 1\right]^{-1}$$

In the presence of gravity and collisions, the evolution of $\Theta = \Delta T/T_0$ in the Newtonian gauge is governed by

$$\frac{d\Theta}{d\eta} = n^\alpha \partial_\alpha \Phi + \partial_\eta \Psi + \left(\frac{d\Theta}{d\eta}\right)_{\mathrm{coll}}$$

$$\left(\frac{d\Theta}{d\eta}\right)_{\mathrm{coll}} = \dot{\tau}_c \left(-\Theta + \frac{1}{4}\delta_\gamma + \mathbf{V}_b \cdot \hat{\mathbf{n}}\right)$$

photons scattered out of the beam

photons scattered into the beam

e^- are not at rest w.r.t. cosmic frame

$$\dot{\tau}_c \equiv x_e n_B \sigma_T a \quad \text{(Differential Thomson optical depth)}$$

Multiplying by exp(-τ) and integrating over the conformal time,

$$\Theta(\hat{\mathbf{n}}, \eta_0) = \int_0^{\eta_0} d\chi\, g(\chi) \left(\frac{1}{4}\delta_\gamma + \Psi + \mathbf{V} \cdot \hat{\mathbf{n}} \right) + \int_0^{\eta_0} d\chi\, e^{-\tau(\chi)} \left[\partial_\eta \Psi - \partial_\eta \Phi \right]$$

$$\tau(\chi) \equiv \int_\chi^{\eta_0} d\eta\, \dot{\tau}_c, \quad g(\chi) = e^{-\tau} \dot{\tau}_c$$

In the instantaneous recombination limit,

$$\Theta(\hat{\mathbf{n}}, \eta_0) \approx \left[\frac{1}{4}\delta_\gamma + \Psi + \mathbf{V} \cdot \hat{\mathbf{n}} \right] (\mathbf{x}_{dec}, \eta_{dec}) + \int_{\eta_{dec}}^{\eta_0} d\eta\, \left[\partial_\eta \Psi - \partial_\eta \Phi \right]$$

acoustic peaks
+ SW effect

Doppler

Integrated Sachs-Wolfe (ISW)

Photon-baryon fluid equations

In the Newtonian Gauge, the equations for the scalar modes of a relativistic fluid of photons and baryons are

$$\dot{\delta}_\gamma = -\frac{4}{3}kV_\gamma - 4\dot{\Phi}$$

$$\dot{\delta}_B = -kV_B - 3\dot{\Phi}$$

CONTINUITY

$$\dot{V}_\gamma = k\left(\frac{1}{4}\delta_\gamma + \Psi - \frac{1}{6}\Pi_\gamma\right) - \dot{\tau}_c\left(V_\gamma - V_B\right)$$

$$\dot{V}_B = -\frac{\dot{a}}{a}V_B + k\Psi + \dot{\tau}_c\frac{V_\gamma - V_b}{R}$$

EULER

$$\left(k^2 - 3\kappa\right)\Phi = 4\pi G a^2 \sum_{X=\gamma,B}\left[\rho_X\delta_X + 3\frac{\dot{a}}{a}\left(\rho_X + p_X\right)\frac{V_X}{h}\right]$$

$$k^2\left(\Psi + \Phi\right) = -8\pi G a^2 p_\gamma \Pi_\gamma$$

EINSTEIN (POISSON)

$$\dot{\tau}_c \equiv x_e n_B \sigma_T a \quad \text{(Differential Thomson optical depth)}$$

$R=3\rho_B/4\rho_\gamma \sim \eta_B$ is the baryon-photon momentum density ratio, and Π_γ is the photon anisotropic stress perturbation

Tight coupling approximation

When the scattering is rapid compared to the travel time across a wavelength,

$$k/\dot{\tau}_c \ll 1 \rightarrow V_b \approx V_\gamma, \quad \Pi_\gamma \approx 0$$

The baryon velocity can be eliminated and the eqs. for photons combine into

$$\frac{d}{d\eta}\left[(1+R)\,\dot{\delta}_\gamma\right] + \frac{k^2}{3}\delta_\gamma = -\frac{4}{3}k^2\,(1+R)\,\Psi + 4\frac{d}{d\eta}\left[(1+R)\,\dot{\Psi}\right]$$

Below the sound horizon $r_s(\eta) = \int_0^\eta d\eta'\, c_s(\eta') = \int_0^\eta d\eta' \, \frac{1}{\sqrt{3(1+R)}}$

photon pressure resits gravitational compression and sets up acoustic oscillations. Neglecting the time variation of R, the effective temperature evolves as

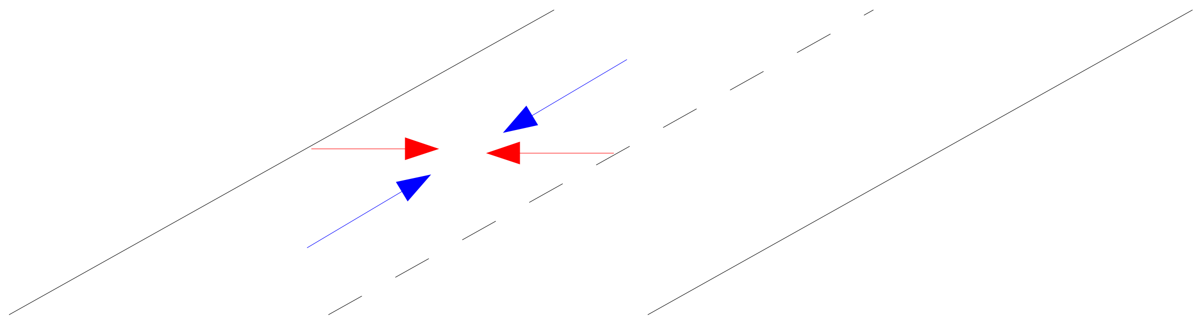
$$\left[\frac{1}{4}\delta_\gamma + \Psi\right](\eta) = A\cos\left(kr_s\right) + B\sin\left(kr_s\right) - R\Psi$$

The initial conditions (ICs) determine A and B: B=0 (adiabatic)
A=0 (isocurvature)

Peebles & Yu (1970), Hu & Sugiyama (1995)

Photon diffusion

Photons have a finite mean free path -> the coupling baryon-photon is not perfect at small scales where photon diffusion erases temperature differences and causes anisotropic stress (viscosity).



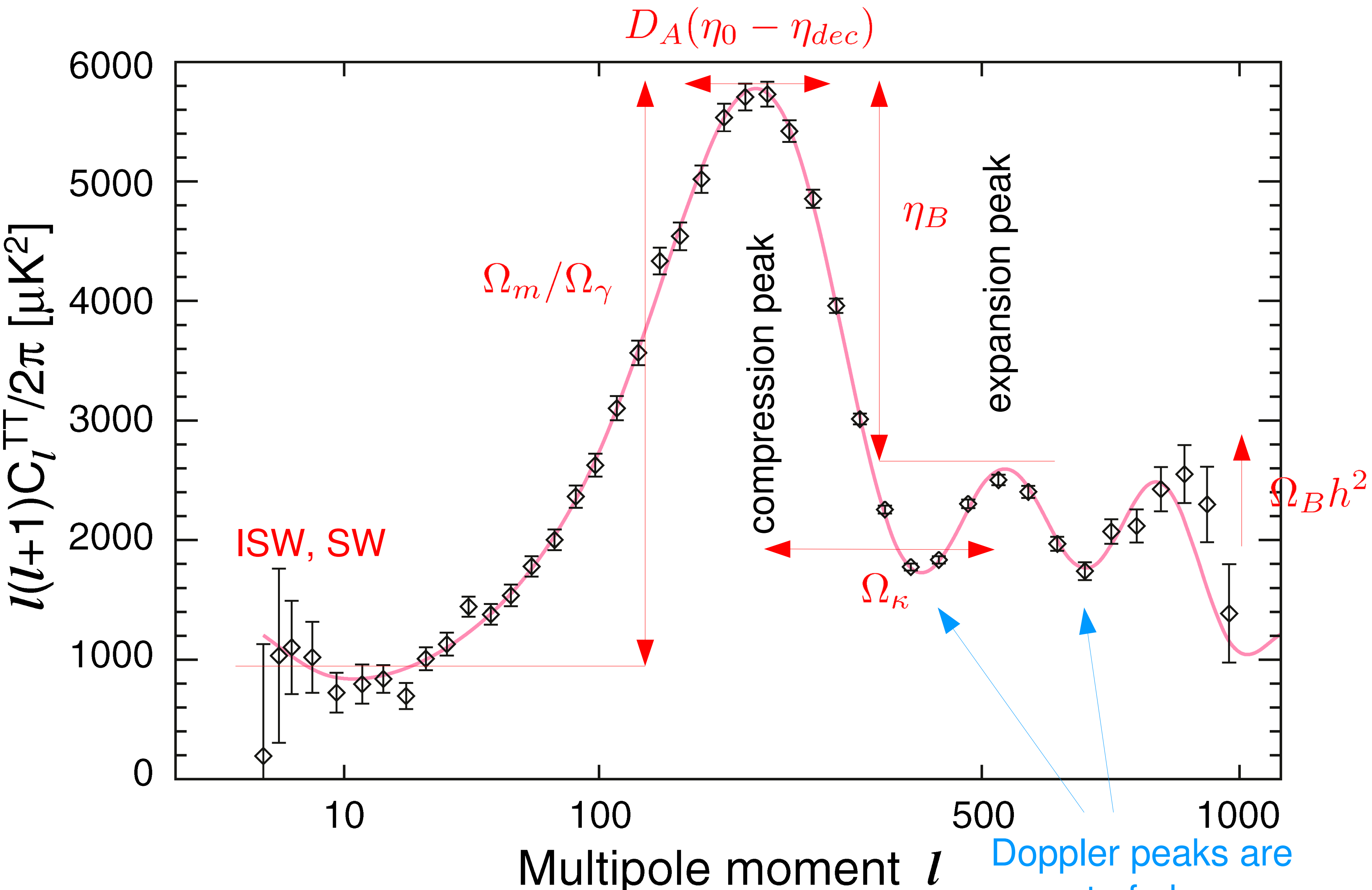
$V_\gamma > 0$ $\qquad$ $V_\gamma < 0$ $\qquad$ $V_\gamma > 0$

$$\Pi_\gamma \approx \frac{8}{5}\left(\frac{k}{\dot{\tau}_c}\right) f_2^{-1} V_\gamma, \quad f_2 \sim \mathcal{O}(1)$$

Free streaming transfers power to higher multipoles so, in principle, one should solve the full Boltzmann hierarchy

Acoustic oscillations are frozen in the CMB at decoupling

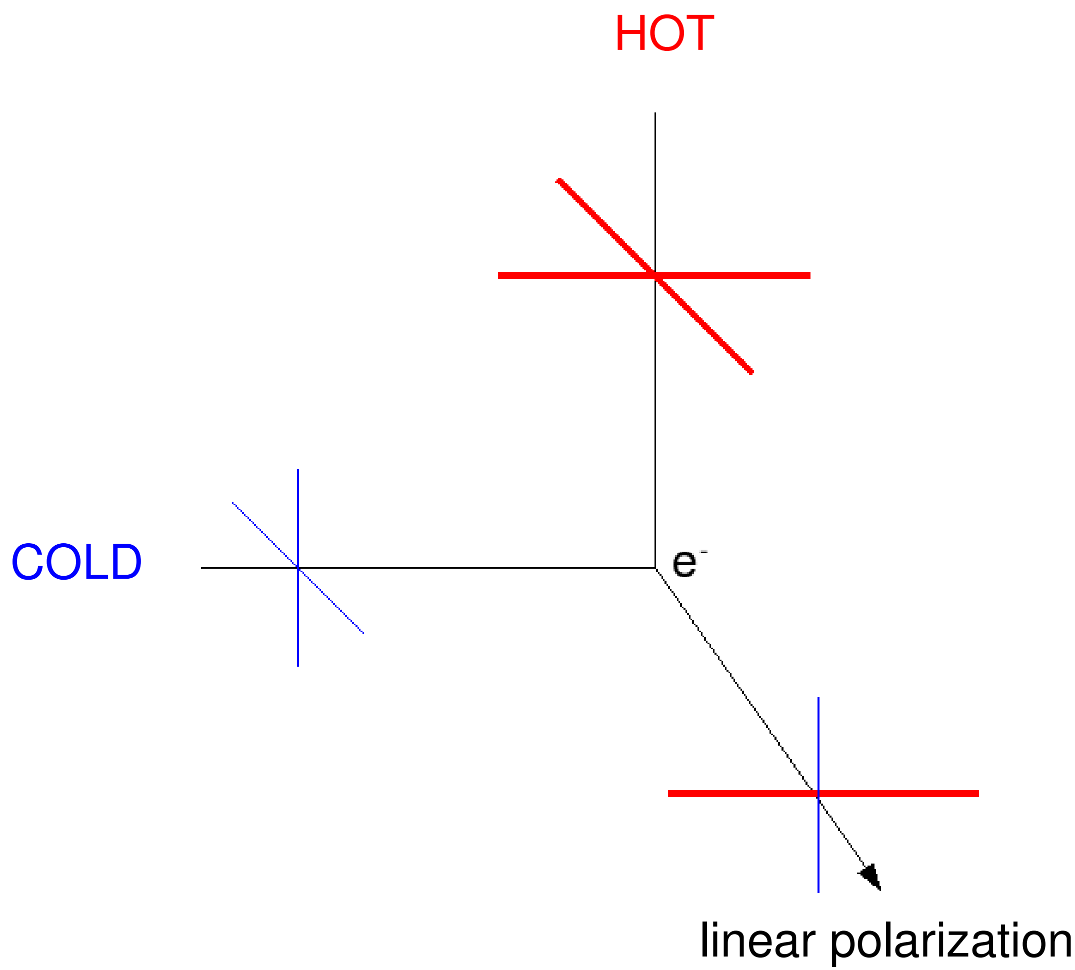
First peak at $l_{peak} \approx 200$ + cosine series -> inflation !



Doppler peaks are out of phase

CMB polarization

Thomson scattering of radiation with a temperature quadrupole anisotropy generates linear polarization at recombination



Intensity matrix: $\begin{pmatrix} \delta_\gamma^{11} & \delta_\gamma^{12} \\ \delta_\gamma^{21} & \delta_\gamma^{22} \end{pmatrix}$

Stokes parameters:

$$\frac{\Delta T}{T} = \frac{1}{4}\left(\delta_\gamma^{11} + \delta_\gamma^{22}\right)$$

$$Q = \frac{1}{4}\left(\delta_\gamma^{11} - \delta_\gamma^{22}\right)$$

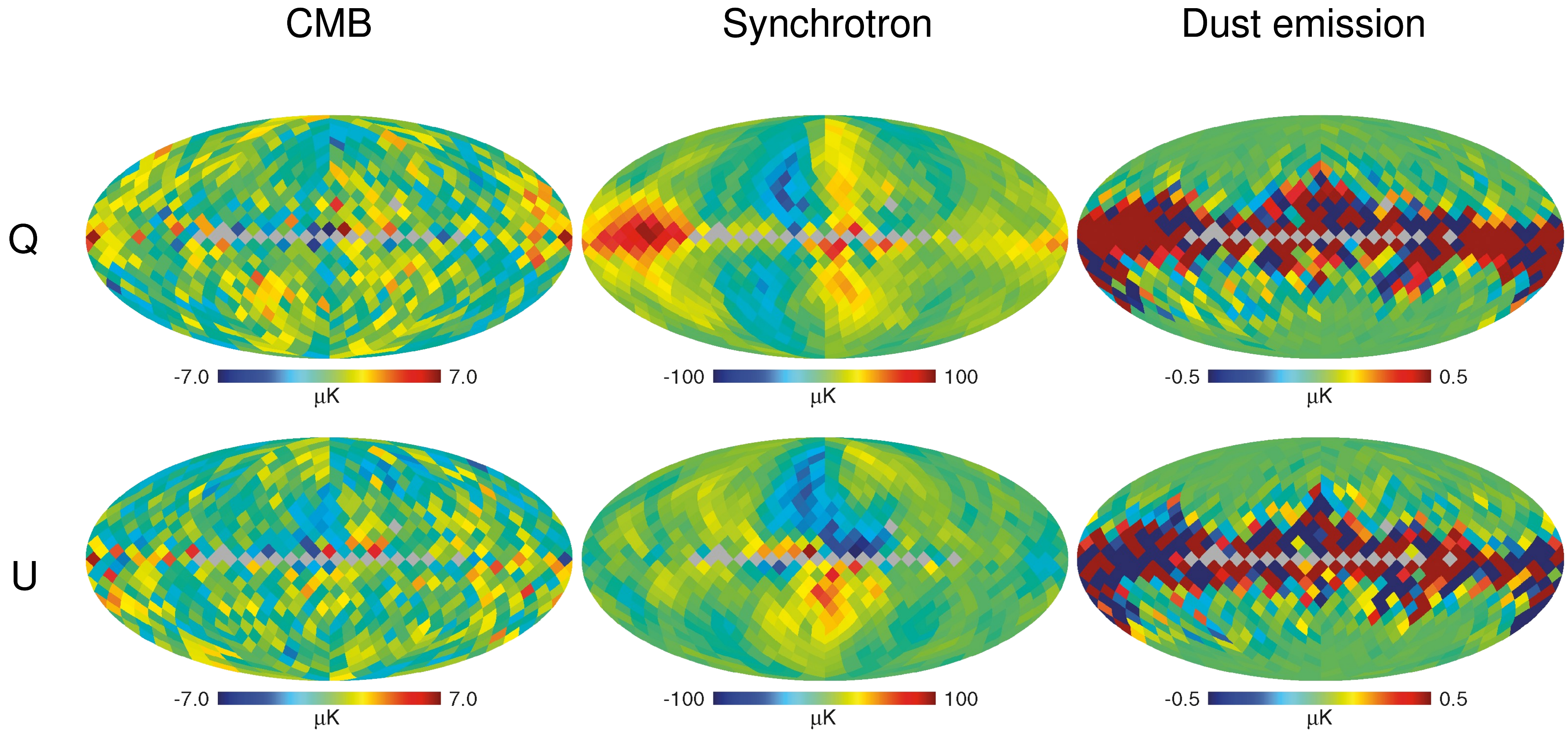
$$U = \frac{1}{2}\delta_\gamma^{12}$$

Theory: CMB anisotropies polarized at 5-10% level

Bond & Efstathiou (1984)

WMAP: CMB anisotropies indeed are polarized !

constrain optical depth to reionization $\tau_{rei} = 0.090 \pm 0.019$



Dunkley et al (2009)

E-mode and B-mode decomposition

The Stokes parameters Q,U transform as a spin-2 field under rotation

$$(Q \pm iU)(\hat{\mathbf{n}}) \to e^{\mp 2i\psi}(Q \pm iU)(\hat{\mathbf{n}})$$

hence may be expanded in terms of tensor (spin-2) spherical harmonics

$$(Q \pm iU)(\hat{\mathbf{n}}) = \sum_{lm} {}_{\pm 2}a_l^m \, {}_{\pm 2}Y_l^m(\hat{\mathbf{n}})$$

It is convenient to introduce the spin-0 (rotationally invariant) E- and B-modes

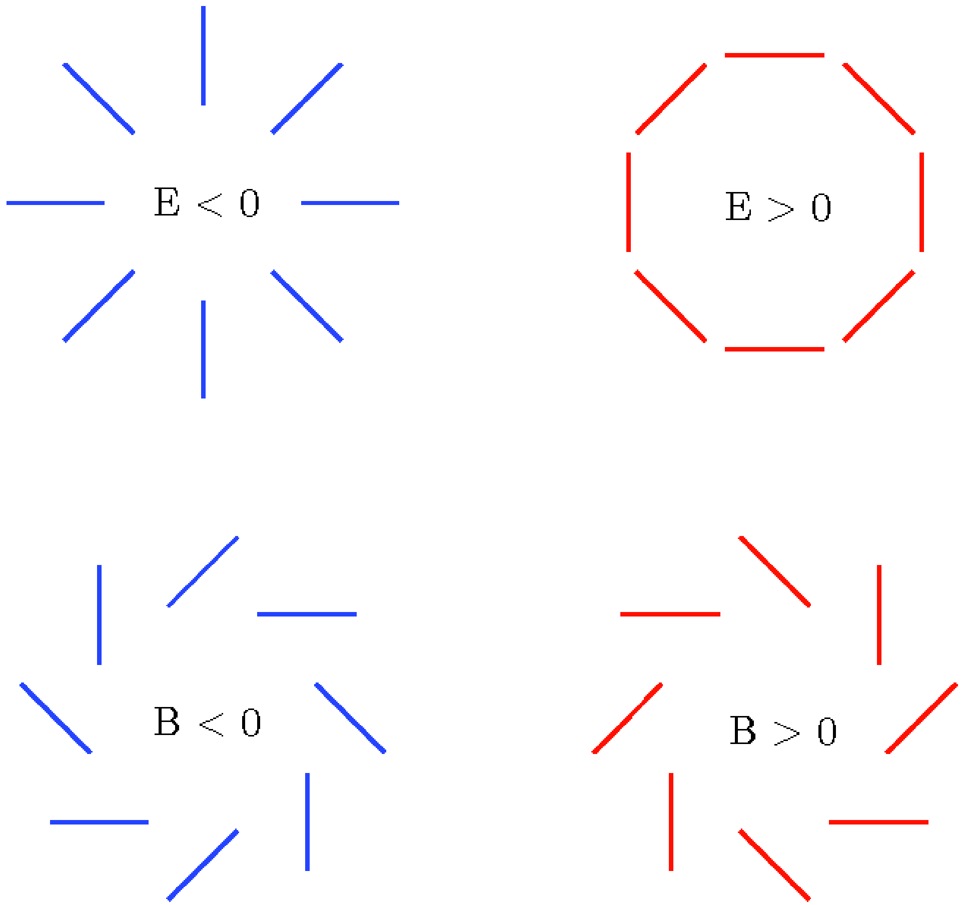
$$E(\hat{\mathbf{n}}) = \sum_{lm} E_l^m Y_l^m(\hat{\mathbf{n}}) \qquad \text{(even parity)}$$
$$B(\hat{\mathbf{n}}) = \sum_{lm} B_l^m Y_l^m(\hat{\mathbf{n}}) \qquad \text{(odd parity)}$$

where

$$E_l^m = -\frac{1}{2}\left({}_{+2}a_l^m + {}_{-2}a_l^m\right)$$
$$B_l^m = -\frac{1}{2i}\left({}_{+2}a_l^m - {}_{-2}a_l^m\right)$$

Seljak & Zaldarriaga (1997), Kamionkowski et al (1997)

Polarization pattern = E-modes (curl-free) + B-modes (divergence-free)



- scalar (density) perturbations create only E-modes
- vector (vorticity) perturbations create mainly B-modes
- tensor (gravity waves) perturbations create both E- and B-modes

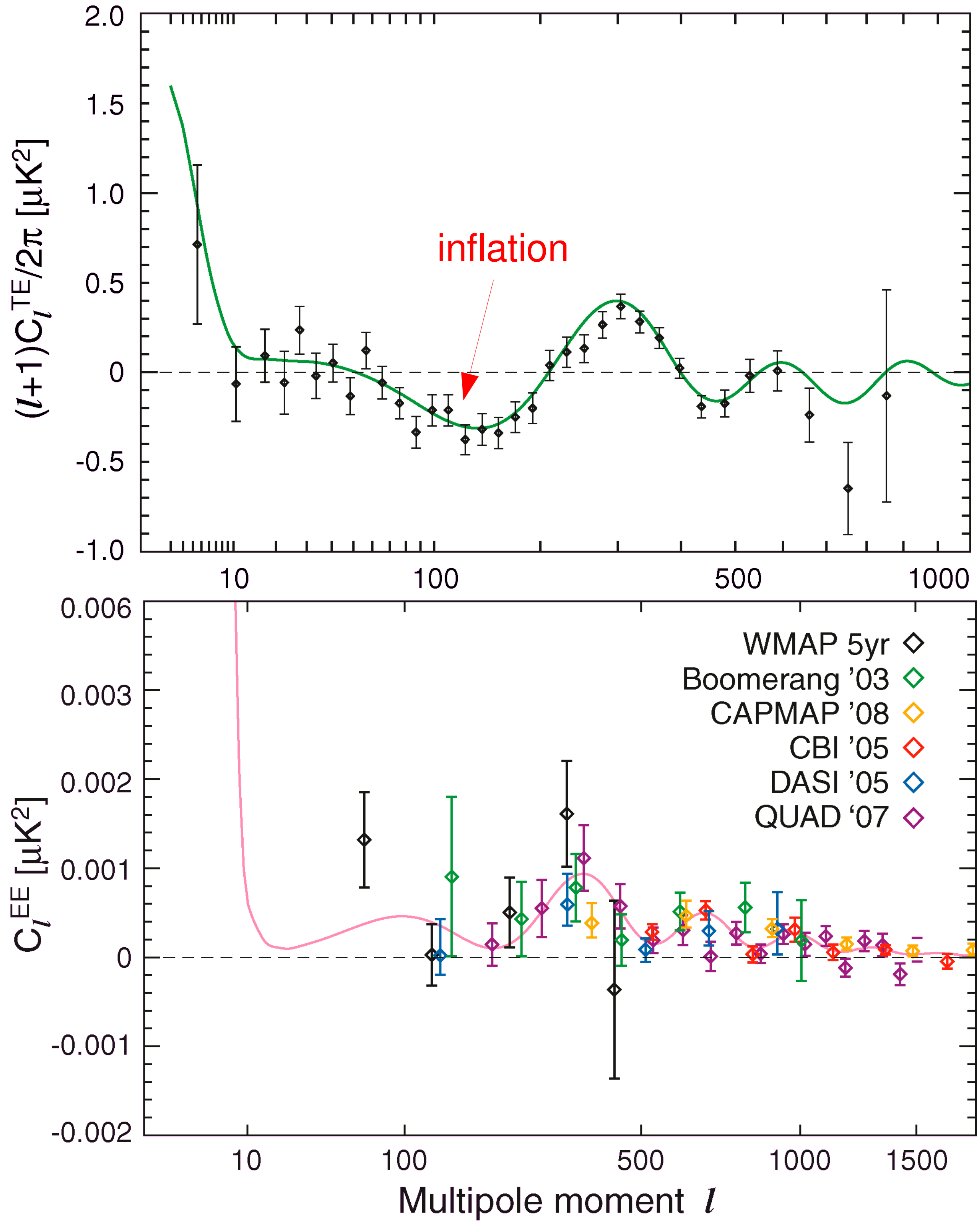
B-mode = smoking gun of inflation

In a parity-conserving Universe, there are four observable angular power spectra commonly named TT, TE, EE and BB

WMAP 5-year C_l^{TE} , C_l^{EE}

E-mode detected at high level of significance !

C_l^{BB} consistent with zero



How do we compute CMB power spectra ?

Use the line-of-sight technique, i.e. write down an integral solution to the Boltzmann equation. This is the method implemented in CMBFast.

$$C_{Xl}^{(S)} = (4\pi)^2 \int dk\, k^2 P_s(k) |g_{Xl}^{(S)}(k)|^2$$

SCALAR

$$C_{Xl}^{(T)} = (4\pi)^2 \int dk\, k^2 P_t(k) |g_{Xl}^{(T)}(k)|^2$$

TENSOR

The transfer functions are

$$g_{Xl}^{(S,T)}(k) = \int_0^{\eta_0} d\eta\, S^{(S,T)}(k,\eta)$$

$$X = T, E, B$$

Seljak & Zaldarriaga (1996)

Primordial perturbations

A convenient phenomenological parametrization of the power spectrum of primordial scalar (density) and tensor (gravity waves) perturbations is

$$P_s(k) = A_s(k_0)\left(\frac{k}{k_0}\right)^{n_s(k_0)-1+\frac{1}{2}\alpha_s(k_0)\ln(k/k_0)}$$

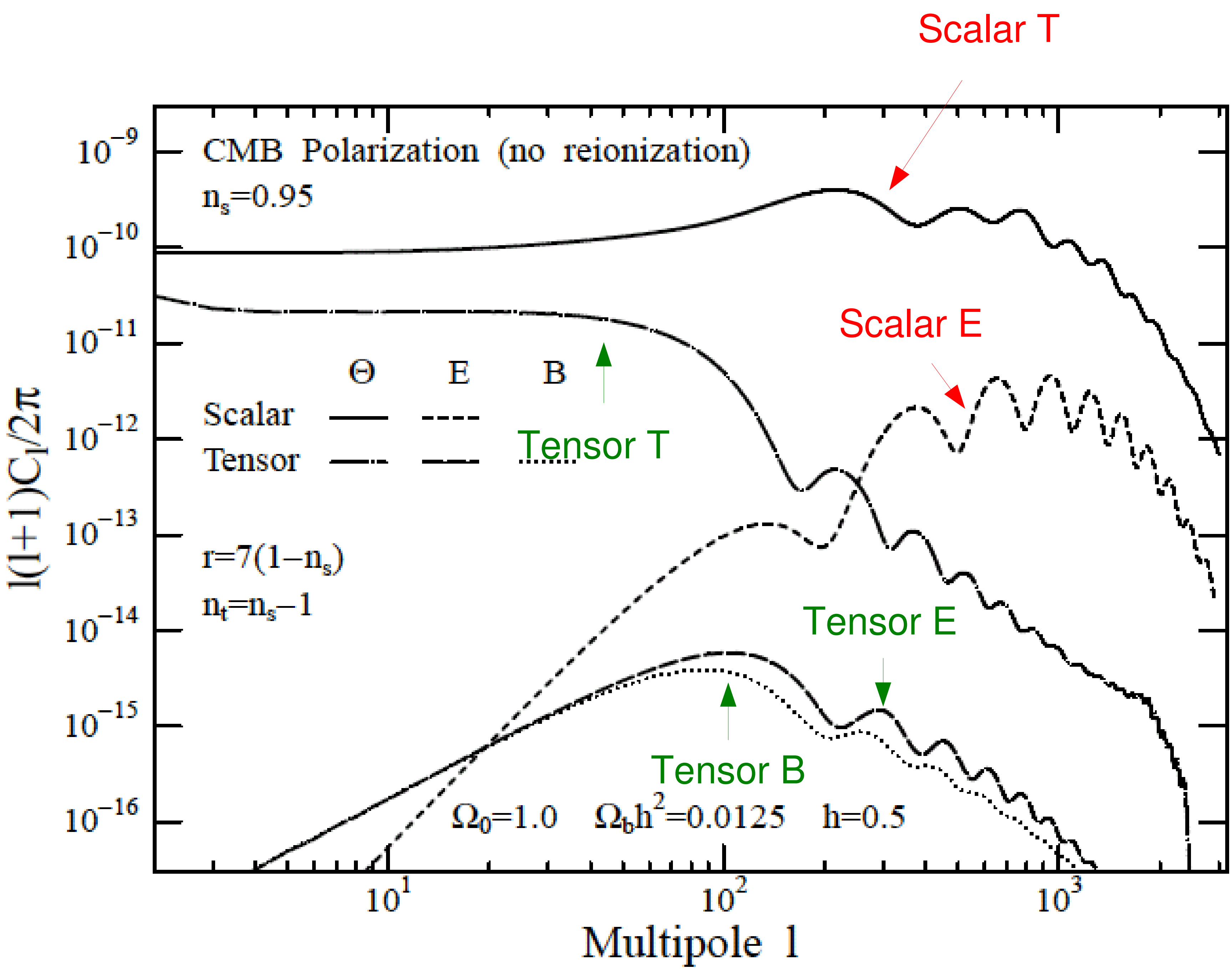
$$P_t(k) = A_t(k_0)\left(\frac{k}{k_0}\right)^{n_t(k_0)}$$

$$n_s - 1 \equiv \frac{d\ln P_s}{d\ln k}, \quad \alpha_s \equiv \frac{dn_s}{d\ln k}, \quad n_t \equiv \frac{d\ln P_t}{d\ln k}$$

CMB polarization measurements are sensitive to the tensor-to-scalar ratio

$$r \equiv \frac{P_t}{P_s}$$

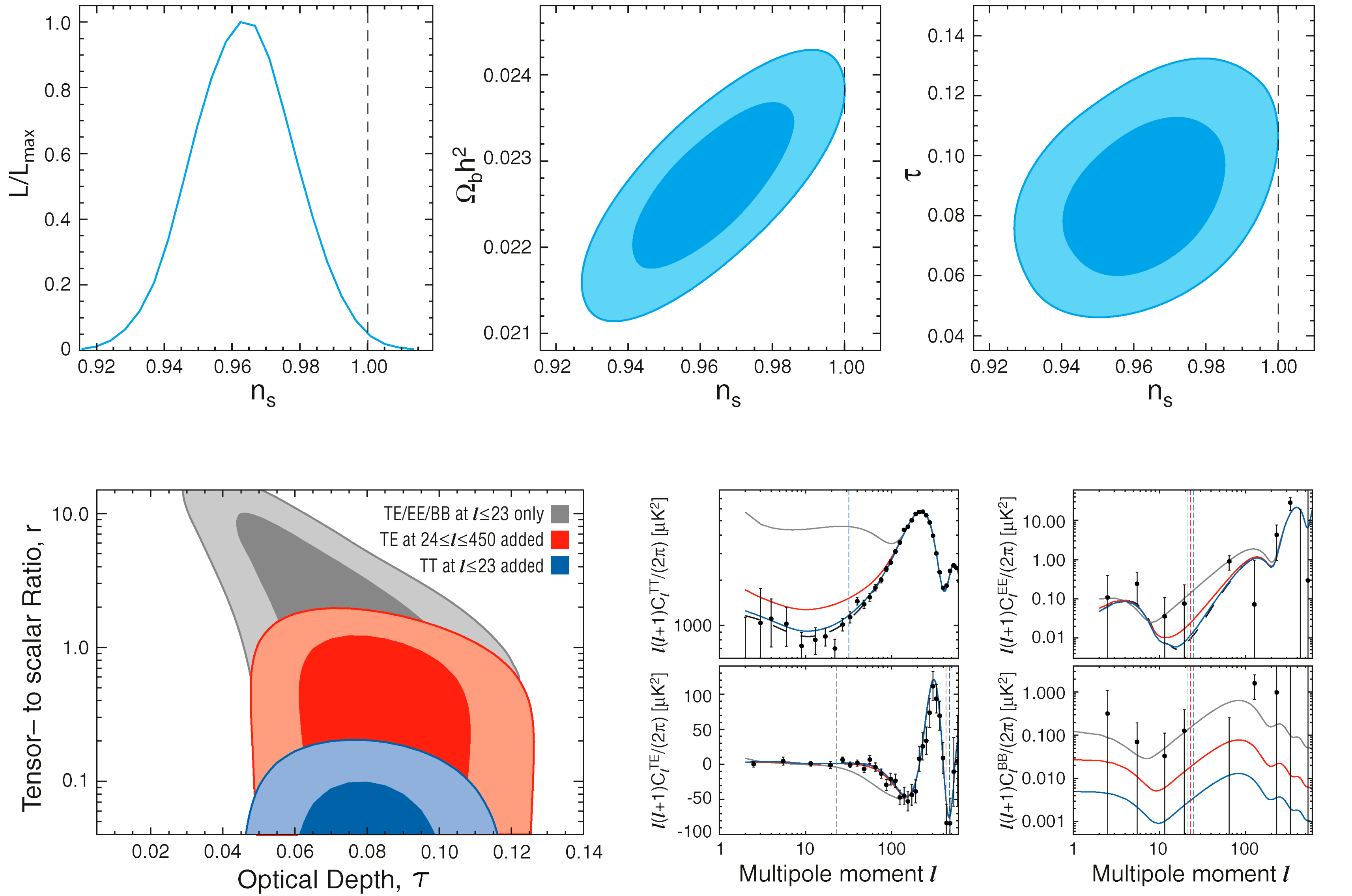
n_s, α_s, n_t, A_s and r can be related to the model-dependent shape of the inflaton potential



Scalar T
Scalar E
Tensor T
Tensor E
Tensor B
CMB Polarization (no reionization)
n_s=0.95
Θ E B
Scalar
Tensor
r=7(1−n_s)
n_t=n_s−1
Ω_0=1.0 $\Omega_b h^2$=0.0125 h=0.5
l(l+1)C_l/2π
Multipole l
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{1}
10^{2}
10^{3}

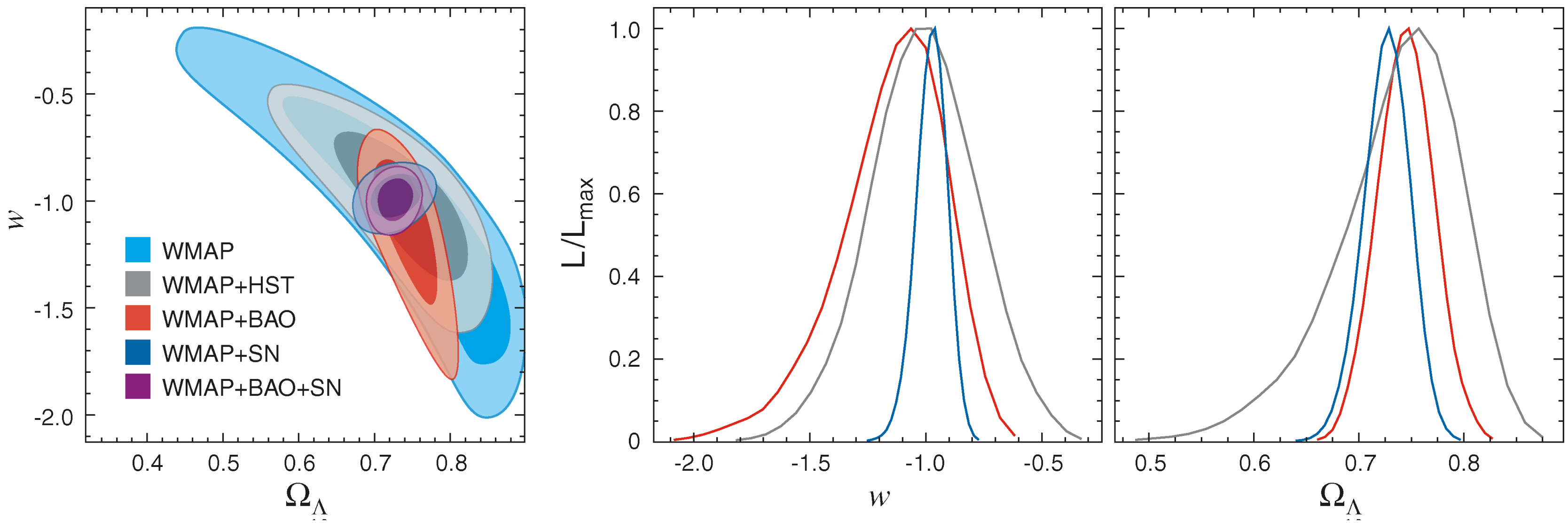
WMAP 5-year constraints on cosmological parameters

Scale-invariant (HZ) spectrum excluded at 3σ (r=0) !



Komatsu et al (2009)

Dark Energy / Cosmological Constant



Komatsu et al (2009)

CMB data alone cannot constrain the Dark Energy equation of state.
Combine with large-scale structure data to break degeneracies.

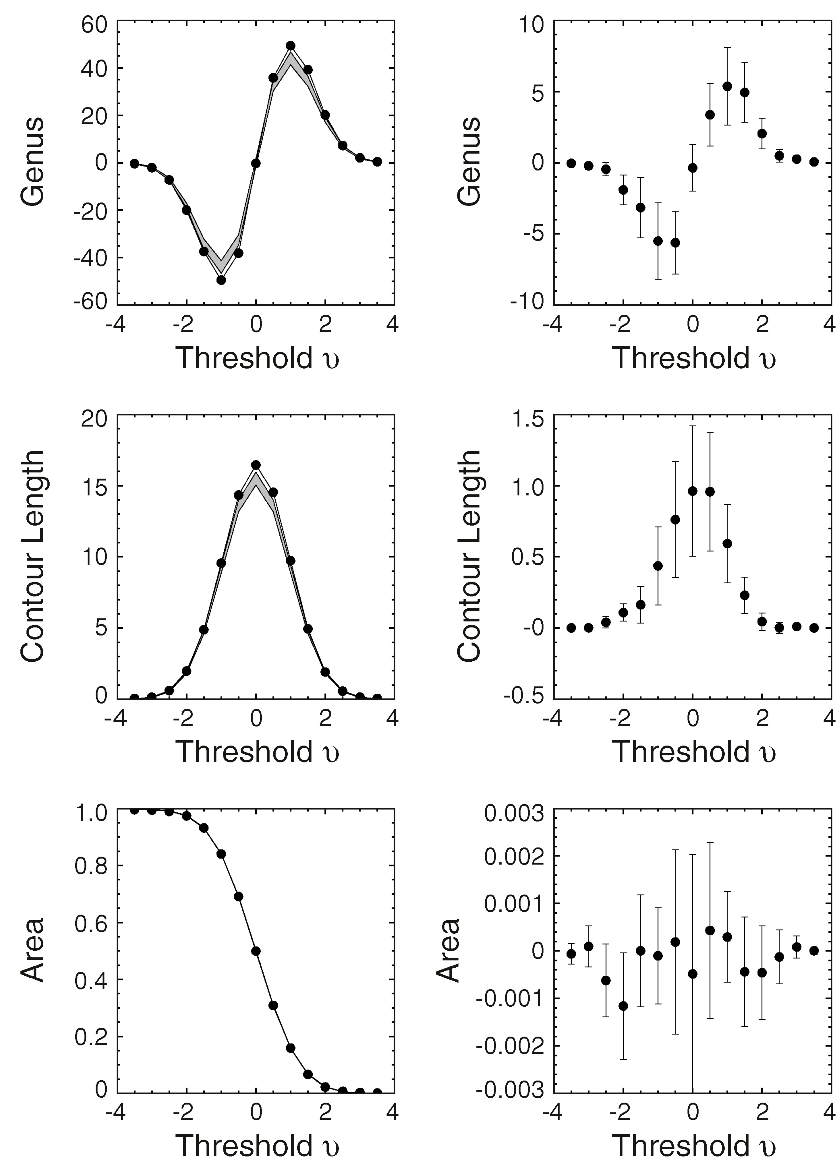
Primordial non-Gaussianity

WMAP 5-year measured Minkowski functionals and three-point function of CMB temperature fluctuations.

$$-9 < f_{\rm NL}^{\rm local} < 111 \quad (95\% \text{ C.L.})$$

$$-151 < f_{\rm NL}^{\rm equil.} < 253 \quad (95\% \text{ C.L.})$$

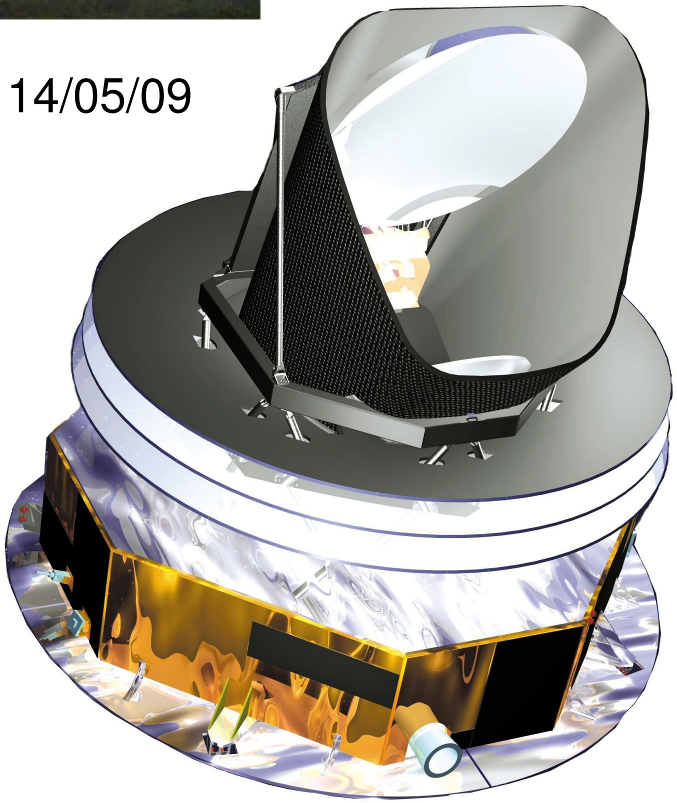
Minkowski functionals



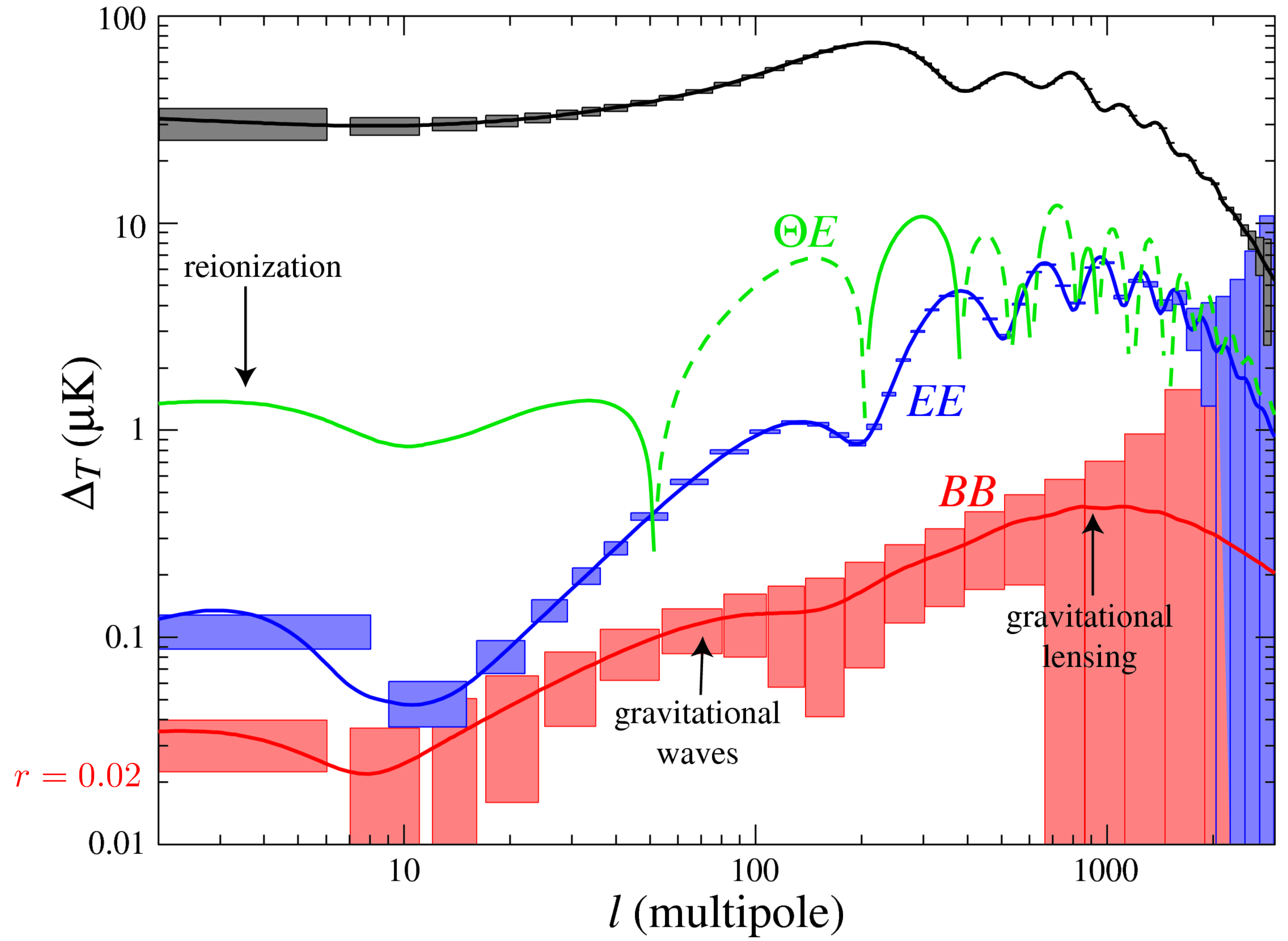
Planck satellite

lauchned on 14/05/09

- highly sensitive bolometer detectors cooled down to 0.1K
- high angular resolution (l ~ 1500)
- map of the whole sky in 9 frequency channels -> CMB anisotropies, SZ, extra-galactic sources etc.



- CMB secondary anisotropies (SZ, lensing etc.)
- tighten constraints on cosmological parameters
- study ionization history of the Universe
- probe the dynamic of the inflationary era
- test fundamental physics, e.g. brane-world or pre-Big Bang cosmologies



100
10
1
0.1
0.01
$r = 0.02$
Δ_T (μK)
10
100
1000
l (multipole)
reionization
ΘE
EE
BB
gravitational waves
gravitational lensing

Summary

A flat, nearly scale-invariant ΛCDM model fits the CMB temperature and polarization data very well.
The measurements are consistent with single-field slow roll inflation .

Future CMB experiments (Planck, CMBPol):

improve B-mode/non-Gaussianity measurement (detection/upper limit)

$\downarrow$

constrain inflation !