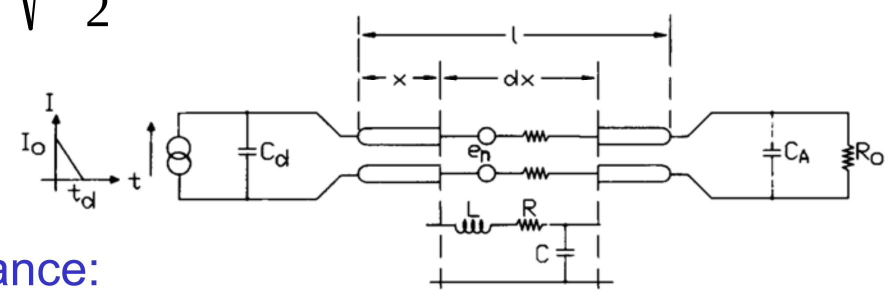
R.L. Chase, C. de La Taille, S, Rescia and N. Seguin, Nucl . Instr . and Meth. A 330 (1993) 228

- Skin effect:
  - Tendency of AC current to distribute itself within a conductor so that the current density near the surface of it is greater than at its core
  - f↑ → R↑
  - Define penetration or “skin” depth δ: $\delta = \sqrt{\frac{2\rho}{\omega\mu}}$
- Signal attenuation due to skin effect
  - Resistance per unit length R_S: $R_S = \frac{1}{W}\sqrt{\frac{\omega\mu\rho}{2}}$
  - It can be shown that:

$$Z_S = \frac{\sqrt{\mu\rho}}{W}\sqrt{j\omega}$$



  - Transmission line characteristic impedance:

$$Z_0 = \sqrt{\frac{Z_S + j\omega L}{j\omega C}} \approx \sqrt{\frac{L}{C}}$$

Skin effect

- Signal attenuation due to skin effect (continued)
  - Propagation constant:

$$\gamma = \sqrt{(Z_S + j\omega L) j\omega C} \approx \frac{\sqrt{\mu\rho}}{2Z_0 W}\sqrt{j\omega} + \sqrt{LC}\, j\omega$$

  - Transfer function of a length of line:

$$\frac{V(x+l,\omega)}{V(x,\omega)} = e^{-\gamma l} = e^{-\frac{\sqrt{\mu\rho l}}{2Z_0 W}\sqrt{j\omega}} e^{-\sqrt{LC} l j\omega}$$

  - Inverse Fourier transform:

$$h_1(t) = \frac{\tau_0}{2\sqrt{\pi}} t^{-\frac{3}{2}} e^{-\frac{\tau_0}{4t}} U(t) \qquad \text{with} \quad \tau_0 = \frac{\sqrt{\mu\rho l}}{2Z_0 W}$$

  - And the step response of the transmission line:

$$u_1(t) = erfc\left[\frac{1}{2}\sqrt{\frac{\tau_0}{t}}\right]$$

Introduction of long time constants → strong attenuation

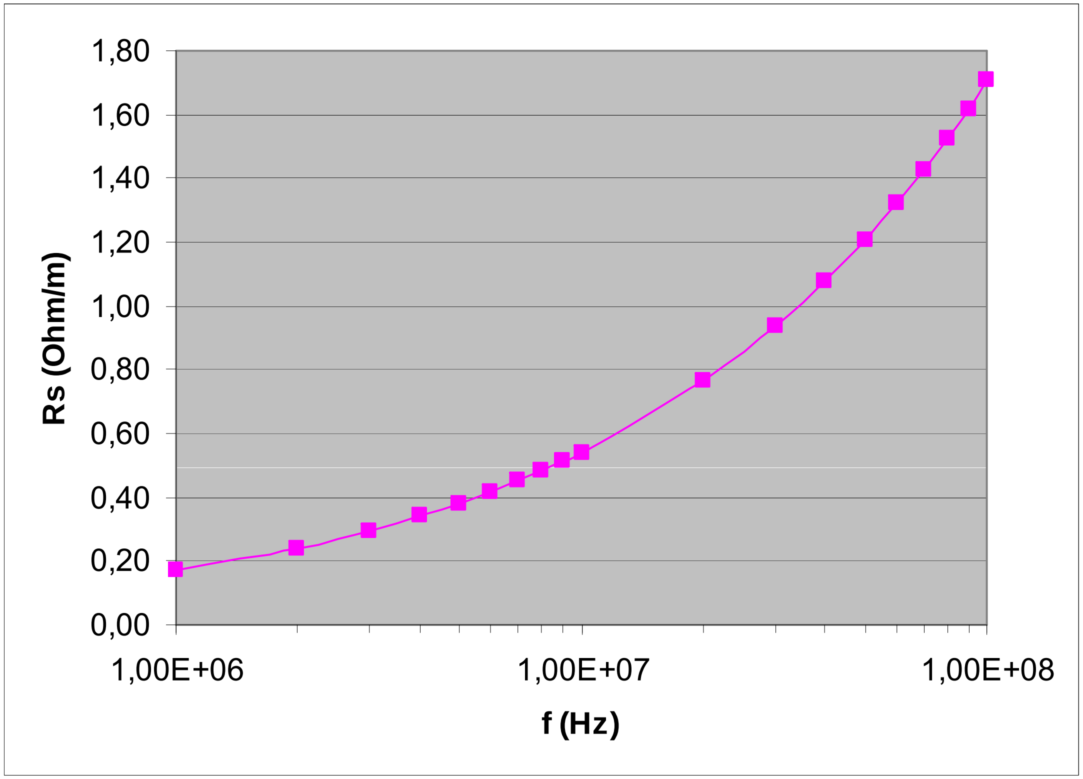
Skin effect: noise contribution

- Resistance per unit length R_S: $$R_S = \frac{1}{W}\sqrt{\frac{\omega\mu\rho}{2}}$$
- Noise generator per unit length: $$e_{n,l}^{\ 2}(f) = 4KTR_S(f)$$
- Propagation constant (rearranged): $$\gamma = \frac{R_S(\omega)}{2R_0} + j\frac{\omega}{v_p}$$
- Impedance seen at position x towards the detector: $$Z(x,\omega) = R_0 \frac{\frac{1}{j\omega C_d} + R_0 tgh(\gamma x)}{R_0 + \frac{1}{j\omega C_d} tgh(\gamma x)}$$
- The noise current per unit length through the terminating resistor R_0:

$$i_{n,l}^{\ 2}(x) = e_{n,l}^{\ 2} \frac{1}{\left|Z(x,\omega) + R_0\right|^2} \left|e^{-\gamma(l-x)}\right|^2 \longrightarrow i_n^{\ 2}(\omega) = \int_0^l i_{n,l}^{\ 2}(x)\,dx$$

Skin effect: noise contribution

- Rs values:



Skin effect noise measurements

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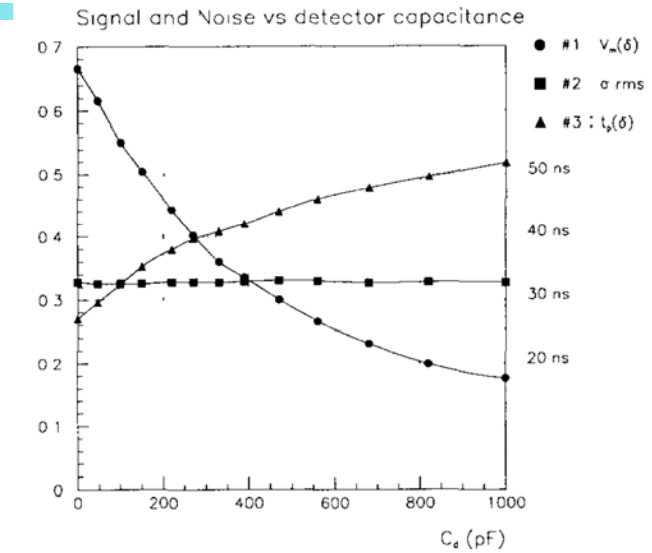

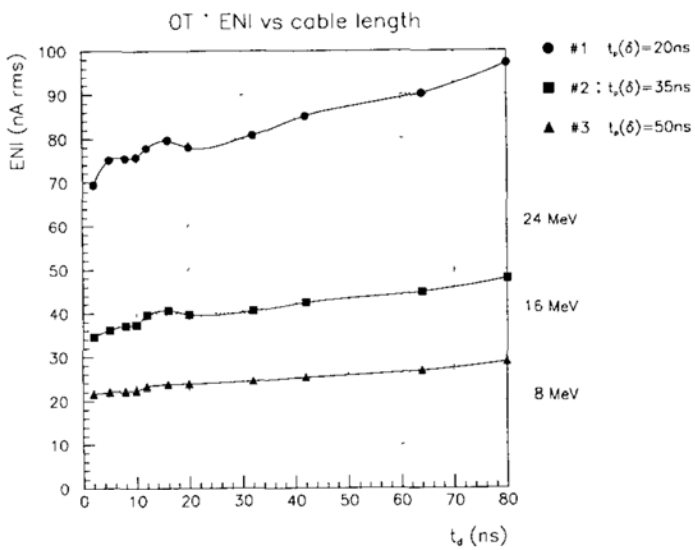

- Plots from noise measurements for ATLAS calorimeter (include amp+CRRC^2)
- Signal and noise vs C_d
  - In ECAL case $C_d < 20$ pF
  - Observed correspondent $t_p \sim 10$ ns
- ENI vs cable length:
  - Cable length = 60 ns (= 12 m)
  - $t_p \sim 10$ ns
  - Using ENI formula, extrapolate: ENI ~ 344 nA rms

$$ENI = e_n C_t \frac{B_S}{t_p^{\frac{3}{2}}} \oplus i_n \frac{B_S}{t_p^{\frac{1}{2}}}$$

Plans

- It seems not clear that the approximated results are correct
  - Review calculations
  - Calculate exact theoretical values for:
    - different C_d
    - cable lengths
    - t_p