Aspects of Gauge-Strings Duality

Carlos Nunez

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Outline

1. I will comment on recent work in the area of non-Abelian T-duality. The focus will be on the general ideas and outcomes.
2. The knowledge of field theory results at strong coupling allows us to say things about a geometry (ranges of coordinates, smoothing-out of singularities, etc).
3. What I will discuss today will be in the context of an $N = 2$ SCFT in four dimensions and a Matrix Model with sixteen supercharges.
4. This talk is a schematic selection of topics taken from two papers with: Yolanda Lozano (2016) and Yolanda Lozano and Salomón Zacarías in March 2017. Previous works with G. Itsios, Y. Bea, A. Sierra, C. Whitting, J. Montero, N. Macpherson, D. Zoakos J. Edelstein, K. Sfetsos, D. Thompson, E. Caceres, V. Rodgers, L. Pando Zayas.

Let us now move into the the contents of the seminar.
I will start with an introduction to what non-Abelian T-duality actually is, trying to put things in context.
This audience is very used to work with proposed or conjectured dualities:
AdS/CFT, Seiberg-Witten, Seiberg duality, etc. Many tests for these proposals and Physics predictions have been produced.
Other (quite restrictive) dualities can be proven, using a very simple method. One finds and interpolating Action. Integrating out fields alternatively, one finds two different sides of the duality.
For example, the Particle-Vortex duality (Peskin 1977, Dasgupta and Halperin 1977). In spite of being very simple, it has very nice applications in condensed matter.

$$Z_{MASTER} = \int D\phi_1 D\phi_2\, e^{-S[\phi_1,\phi_2]}$$

$$\begin{tikzcd}
 & Z_{MASTER} \arrow[dl, "\int D\phi_1"'] \arrow[dr, "\int D\phi_2"] & \\
Z_{particle} = \int D\phi_2\, e^{-S_2[\phi_2]} \arrow[rr, leftrightarrow, "\text{dual to}"'] & & Z_{vortex} = \int D\phi_1\, e^{-S_1[\phi_1]}
\end{tikzcd}$$

Other dualities can be proven following a prescriptive method. This method was introduced by Buscher, then streamlined by Rocek and Verlinde. Given an Action, four-steps are followed:

- Detect a global symmetry. Gauge it.
- Impose the gauge field has no dynamics using a Lagrange multiplier.
- Integrate out the original degrees of freedom and gauge field. The Action in terms of the Lagrange multiplier is the 'Dual Action'

There are many relevant examples, interesting in Physics
The procedure in Statistical Mechanics is exemplified with the Ising Model self-duality (Kramers-Wannier, 1940).
In 2-d QFT is exemplified by Bosonisation (Coleman, Mandelstam, 1975).
In String Theory is exemplified by T-duality (Buscher, 1985, Rocek and Verlinde, 1990).
To fix the idea, let us see this briefly in the case of Bosonisation. This is taken from Burgess-Quevedo, 1995.

Suppose that you consider the Lagrangian (in two dimensions),

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi, \rightarrow Z[j] = \int D\bar{\psi} D\psi e^{-\int d^2x (L + j_\mu \bar{\psi}\gamma^\mu \psi)}$$

In the following and for clarity, allow me to ignore the source terms
One notices a vectorial global symmetry

$$\psi \rightarrow e^{i\alpha}\psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{-i\alpha}.$$

We will *gauge* this global symmetry introducing A_μ. But, imposing that $F_{\mu\nu} = 0$. The generating functional will read,

$$Z = \int DA_\mu D\Lambda D\psi D\bar{\psi} \exp\left[-\int d^2x \bar{\psi}\gamma^\mu (i\partial_\mu + A_\mu)\psi + \Lambda \epsilon_{\mu\nu} F^{\mu\nu}\right]$$

If we integrate the Lagrange multiplier Λ, we are back to the fermionic Action above.

If, on the other hand, we integrate the fermions and then the gauge field, we will then get a partition function, 2 d is important!

$$Z = \int D\Lambda \exp\left[-\int d^2x(\partial_\mu\Lambda)^2\right],$$

Which is the characteristic bosonisation prescription. Had we kept the two source terms, the more meaningful prescription arises,

$$i\bar{\psi}\gamma^\mu\partial_\mu\psi \to (\partial_\mu\Lambda)^2$$
$$i\bar{\psi}\gamma_\mu\psi \to \frac{\epsilon_{\mu\nu}\partial_\nu\Lambda}{\sqrt{\pi}}$$
$$i\bar{\psi}\gamma_\mu\gamma_5\psi \to \frac{\partial_\mu\Lambda}{\sqrt{\pi}}.$$

Exactly the same procedure can be followed in the presence of interactions. Coleman-Mandelstam's (1975) Thirring-Sine Gordon duality is derived.

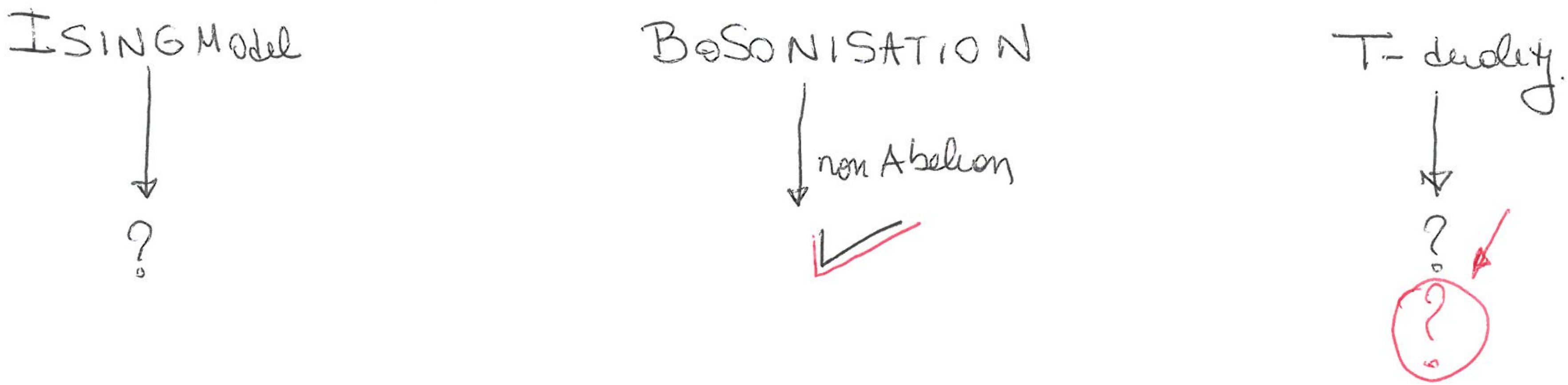
One can apply this procedure to the situation in which the fermions transform under a non-Abelian global symmetry, like $O(N)$

In that case, after some gymnastics with the Polyakov-Wiegmann identity, one obtains Witten's 1982 WZW bosonization.

In exactly the same way, Buscher presented the T-duality rules.

Then this should be complemented by the transformations for RR fields and fermions (Bergshoeff, Hull, Ortin; Benichou, Policastro, Troost; Hassan....)

It is natural to wonder if T-duality can be made 'non-Abelian' (it is also natural to ask the same about the Ising model!)



A non-Abelian extension of T-duality exist. It follows the procedure above. de la Ossa Quevedo
Rocek+Verlinde, Rocek+Greene, Alvarez, Alvarez Gaumé, Borlaf, Lozano ..... Sfetsos
Sfetsos (1995) Thompson (2010)

Allow me NOT to pass you through the whole formalism

But the idea is that given a background with—for example– an $SU(2)$-isometry,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \sum_{i=1}^{3} f_i(x)(\omega_i - A_i(x))^2, \quad \Phi(x).$$

$\omega_i \equiv$ left invariant forms of $SU(2)$

$$C_{p,RR} \sim B_2 \sim a_{\mu\nu}dx^\mu \wedge dx^\nu + b_{\mu i}dx^\mu \wedge \omega^i + c_{ij}\omega_i \wedge \omega_j.$$

Non-Abelian T-duality can be applied and a new background (solution of Type II eqs. of motion) is generated.

In terms of the new coordinates or Lagrange multipliers.

$\omega_i(\theta, \varphi, \psi) \rightarrow z_a(\eta, \chi, \xi) \equiv z_a(\eta, \Omega_2(\chi, \xi))$.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \sum_{i=1}^{3} g_{ab}dz_a dz_b, \quad \tilde{\Phi}, \quad \tilde{B}_2, \quad \tilde{C}_p.$$

Precise formulas have been developed for g_{ab} and the RR, NS fields on interesting classes of backgrounds. The non-Abelian T-duality acts in those cases as a solution generating technique. This was proven in a wide variety of SUSY preserving backgrounds, where new (unclassified) solutions were generated.

Various things are unclear about this non-Abelian version of T-duality.

- What is the range of the dual coordinates (Lagrange multipliers)?
- Can we dualise again? $(\eta, \chi, \xi) \rightarrow (\theta, \varphi, \psi)$?
- What happens if we perform this duality on a world-sheet with genus $g > 0$?
- How does the initial world-sheet CFT relates to the final one?
- When is the generated background singular? Can these singularities be resolved?

These issues are not well understood. There were various papers in the 1990's trying to address them, but the results are not conclusive.

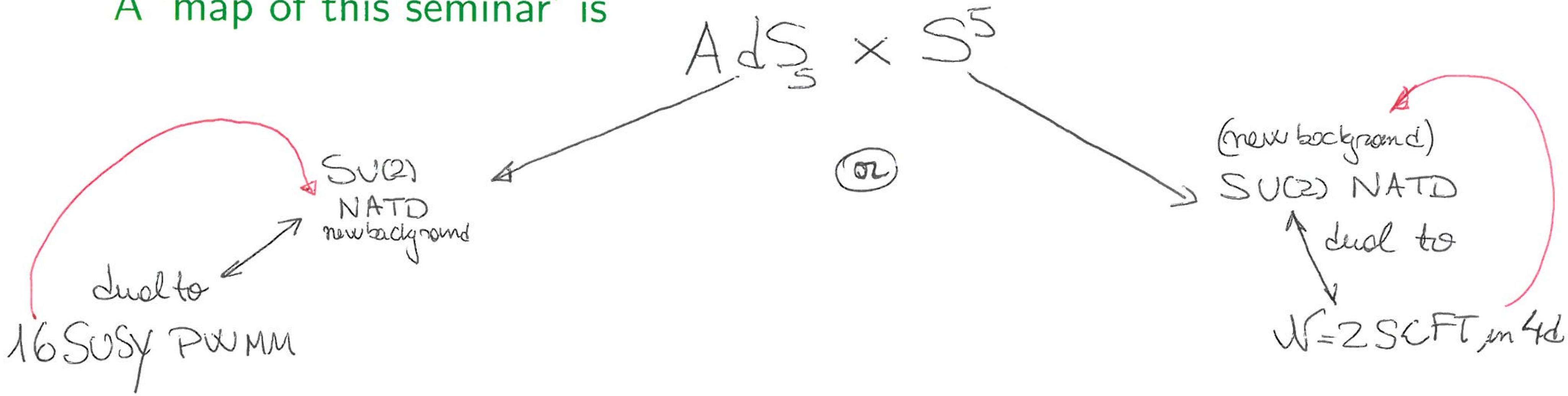
The point of the papers on which this talk is based is to contribute to the understanding of the problems above.

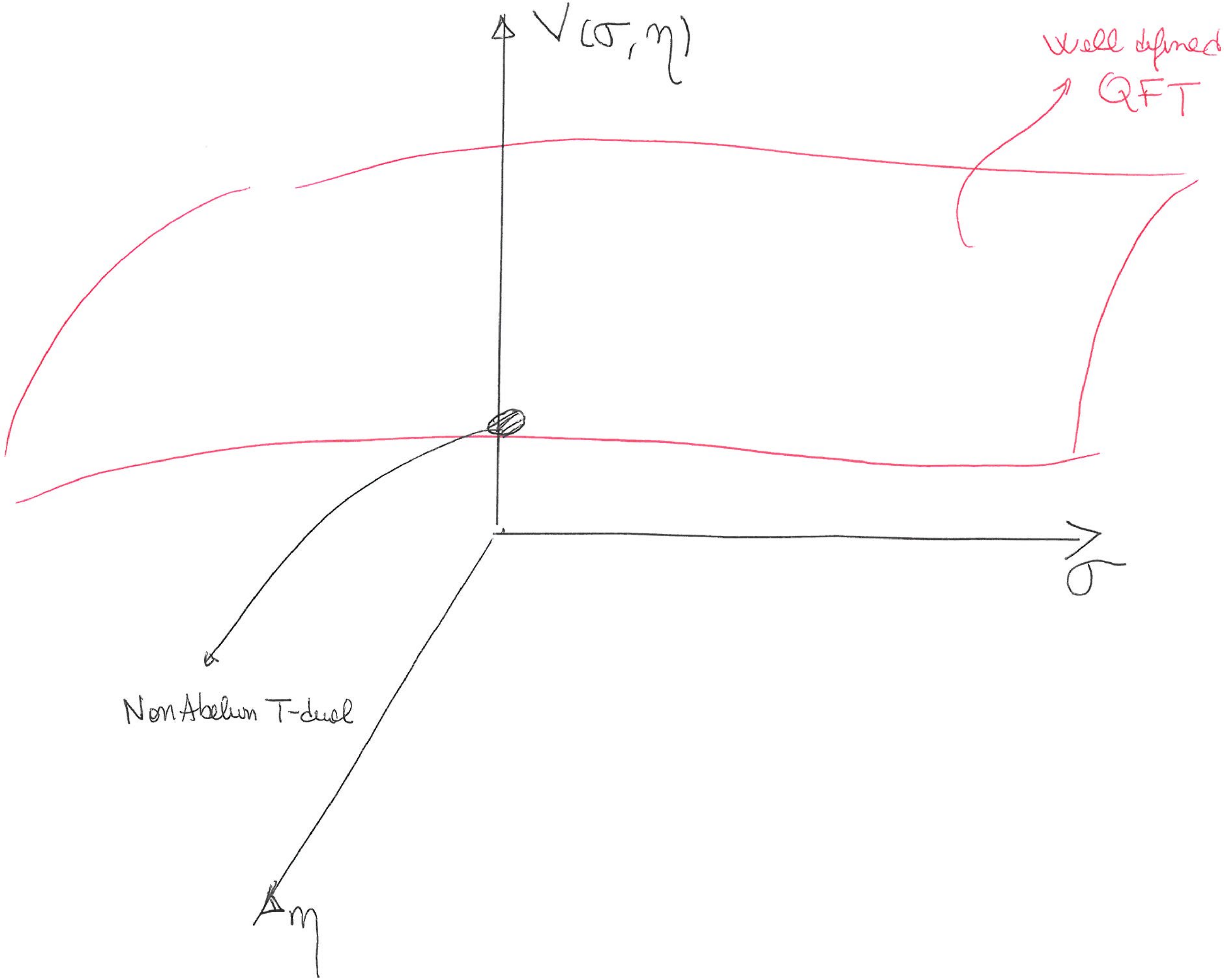
To do so, we will focus on a very simple example dual to a CFT and, after non-Abelian T-duality, use the QFT dual to 'inform' the background.

The example chosen for this talk is that of $AdS_5 \times S^5$. Studying the non-Abelian T dual background, we will find it can be put in correspondence with either an $\mathcal{N} = 2$ SCFT or a Matrix Model with 16 SUSYs. This will depend on "which $SU(2)$ we dualise".

We have a powerful formalism to study the field theory and learn things about the background.

A 'map of this seminar' is





V(σ, η)
Well defined QFT
σ
Non Abelian T-dual
η

Let us then consider $AdS_5 \times S^5$ in $g_s = 1$ units, written as

$$ds^2 = 4L^2 \left(AdS_5 + S^5\right)$$

$$ds^2_{\text{AdS5}} = \left[\frac{d\sigma^2}{(\sigma^2-1)} - \sigma^2 dt^2 + (\sigma^2-1) d\Omega_3^2\right], \qquad (\sigma = \cosh r).$$

Change variables → usual global AdS

$$d\Omega_5 = \left[\frac{d\sigma^2}{(1-\sigma^2)} + \sigma^2 d\beta^2 + (1-\sigma^2) d\tilde{\Omega}_3^2\right], \qquad (\sigma = \sin\alpha).$$

Change variables → usual $S^5 = S^2 \times S^3$

$$d\Omega_3^2 = \sum_{i=1}^{3} \frac{\omega_i^2}{4}, \qquad d\tilde{\Omega}_3^2 = \sum_{i=1}^{3} \frac{\tilde{\omega}_i^2}{4}.$$

$$F_5 = \frac{4}{L}(\mathrm{Vol}_{AdS_5} - \mathrm{Vol}_{S^5}), \quad N_{D3} = \frac{\int F_5}{2\kappa_{10}^2 T_{D3}} \to \frac{L^4}{\alpha'^2} = 4\pi N_{D3}.$$

$$\sum_{i=1}^{3} \omega_i^2 = (d\psi + \cos\theta d\varphi)^2 + d\theta^2 + \sin^2\theta d\varphi^2.$$

We will calculate the non-Abelian T-dual on this background, using the SU(2) inside $SO(2,4)$ or $SO(6)$, realised in ω_i's .

This two different non-Abelian T-dualities will generate two different new solutions of Type IIA with the following characteristics.

- Both solutions are singular. Close to the singularity, the space asymptotes to NS5 branes.
- Both solutions will preserve SUSY.
- Both solutions are supported by NS flux. There is also charge of D_p and D_{p+2} branes (for $p = 4$ or $p = 0$).

For the purposes of this presentation, let me set $L = \alpha' = g_s = 1$. You will only miss the nice quantisations conditions that are properly written in our papers, namely $\frac{L^4}{\alpha'^2} \sim N_p$. Also, allow me to quote only the NS-fields

The generated backgrounds read,

Non-abelian T-duality on $SU(2)$ inside $SO(6)$, $(\tilde{\theta}, \tilde{\varphi}, \tilde{\psi}) \to (\eta, \chi, \xi)$.

$$ds^2_{IIA,st} = 4AdS_5 + 4\left[\frac{d\sigma^2 + d\eta^2}{(1-\sigma^2)} + \sigma^2 d\beta^2\right] + \frac{4\eta^2(1-\sigma^2)}{(4\eta^2 + 1 - \sigma^2)} d\Omega_2(\chi, \xi).$$

$$B_2 = \frac{8\eta^3}{(4\eta^2 + 1 - \sigma^2)} d\Omega_2(\chi, \xi), \quad e^{-2\Phi} = (1-\sigma^2)(4\eta^2 + 1 - \sigma^2),$$

Non-abelian T-duality on $SU(2)$ inside $SO(2,4)$, $(\theta, \varphi, \psi) \to (\eta, \chi, \xi)$.

$$ds^2_{IIA,st} = 4d\Omega_5 + 4\left[\frac{d\sigma^2 + d\eta^2}{(\sigma^2 - 1)} - \sigma^2 dt^2\right] + \frac{4\eta^2(\sigma^2 - 1)}{(4\eta^2 + \sigma^2 - 1)} d\Omega_2(\chi, \xi).$$

$$B_2 = \frac{8\eta^3}{(4\eta^2 + \sigma^2 - 1)} d\Omega_2(\chi, \xi), \quad e^{-2\Phi} = (\sigma^2 - 1)(4\eta^2 + \sigma^2 - 1),$$

Both non-Abelian T-dual backgrounds are complemented by F_2 and $F_4 = B_2 \wedge F_2$. They are both singular at $\sigma = 1$.
Which is understandable, since the size of the (θ, φ, ψ)-cycle before the duality vanishes at $\sigma = 1$.

Let us look at the isometries of both backgrounds.
When the non-Abelian T-duality acts on an $SU(2)$ inside $SO(6)$, we find

$$SO(2,4) \times SO(3) \times U(1), \quad 16 \text{ SUSY}.$$

When the non-Abelian T-duality acts on an $SU(2)$ inside $SO(2,4)$, we find

$$SO(6) \times SO(3) \times R_t, \quad 16 \text{ SUSY}.$$

This very large isometry group implies strong restrictions on the backgrounds. There exist classifications and our backgrounds are new solutions within those classifications.
We can calculate the charges associated to each of these backgrounds. This give valuable information about the dual QFT.

In calculating the Page charges, we find, for the case in which we dualise on the S^5 (we have magnetic F_2, F_4),

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{\eta,\chi,\xi} H_3,$$

$$Q_{D_6} = \frac{1}{2\kappa_{10}^2 T_{D_6}} \int_{\sigma,\beta} F_2, \quad Q_{D_4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\sigma,\beta,\chi,\xi} F_4 - B_2 \wedge F_2.$$

In the example in which the duality goes on AdS_5 (we have magnetic F_6, F_8),

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int_{\eta,\chi,\xi} H_3,$$

$$Q_{D_2} = \frac{1}{2\kappa_{10}^2 T_{D_2}} \int_{\eta,\Omega_5} F_6, \quad Q_{D_0} = \frac{1}{2\kappa_{10}^2 T_{D0}} \int_{\eta,\Omega_5,\Omega_2} F_8 - B_2 \wedge F_6,$$

Where $2\kappa_{10}^2 T_p = (2\pi)^{7-p}$ in the conventions I am using here.

This calculation is one place where some of the unknown things about non-Abelian T-duality, shows itself.

What is the range of the coordinate η?

A very nice observation was made by Lozano+Macpherson in 2014:

Bound $0 < \frac{1}{4\pi^2} \oint_{\chi,\xi} B_2 = \frac{\eta}{\pi} < 1$.

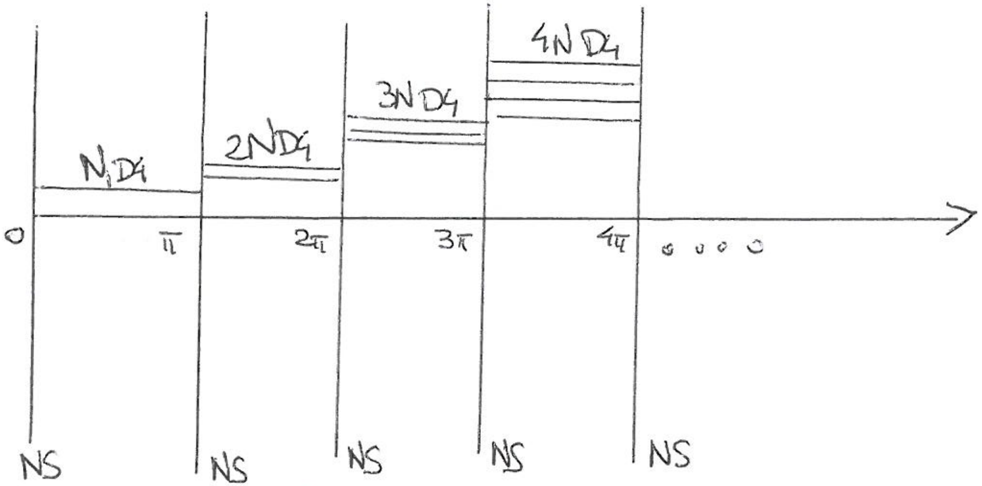
This has some very interesting consequences

- The coordinate η, naturally divides itself in intervals of size π.
- Each time we cross $\eta = k\pi$, a large gauge transformation that restores the integral above to $[0, 1]$ should be performed.
- This large gauge transformation of B_2 generates charge of NS5 branes. Also, it changes the Page charges due to the $B_2 \wedge F_p$ terms.
- The η-coordinate turns out to be a sort of "theory space" coordinate. All the QFT information will be encoded by a function of η.

Let us briefly describe the dual QFTs. I will be very sketchy in the arguments that lead to these proposals

The dual field theories can be 'induced' by a combination metric, fluxes, SUSY study. This gives clues about the branes involved, their relative positions in space, etc.

In the case in which we perform the duality on the S^5, we find a system of D4, NS5 branes that correspond to a Hanany-Witten set-up,



That suggests an $\mathcal{N} = 2$ four dimensional quiver

In the case in which we dualised along the S^3 in AdS_5
we found a system containing D0 and D2 branes (Myers blown up D0's). The system is a Matrix Model, the Plane wave Matrix Model—a deformation of BFSS. The vacuum of this Matrix Model is in this case,

$$J^i = \begin{bmatrix} \overbrace{J^i_{j=0}}^{1\text{ time}} & & & & & & \\ & \overbrace{J^i_{j=\frac{1}{2}}}^{2\text{ times}} & & & & & \\ & & \overbrace{J^i_{j=1}}^{3\text{ times}} & & & & \\ & & & \overbrace{J^i_{j=\frac{3}{2}}}^{4\text{ times}} & & & \\ & & & & \ddots & & \\ & & & & & \overbrace{J^i_{j=2k+1}}^{k\text{ times}} & \\ & & & & & & \ddots \end{bmatrix}$$

Different calculations support these claims. For example, the SUSY probe branes in the non-abelian T-dual backgrounds coincide with the structure we are suggesting.

Perhaps more definitive, is the fact that these quivers and matrix models have very precise string duals.

Indeed, due to the large isometry groups involved, both the QFTs and their respective gravity duals are well understood and classified.

Given the isometries we mentioned before, we can propose that backgrounds dual to QFTs with those global symmetries, must be

$$ds^2 \sim f_1 AdS_5 + f_2 d\Omega_2 + f_3 d\beta^2 + f_4 d\eta^2 + f_5 d\sigma^2; \quad f_i(\sigma,\eta)$$

for $SO(2,4) \times SO(3) \times U(1)$.

$$ds^2 \sim -g_1 dt^2 + g_2 d\Omega_5 + g_3 d\Omega_2 + g_4 d\eta^2 + g_5 d\sigma^2; \quad g_i(\sigma,\eta)$$

for $SO(6) \times SO(3) \times R_t$.

And a similar proposal for the fluxes. Imposing the preservation of 16 SUSYs, Lin, Lunin and Maldacena found an explicit expression for the $f_i(\sigma,\eta), g_i(\sigma,\eta)$.

. There is a generic background geometry dual to an 4d-$\mathcal{N}=2$ SCFT and Matrix Models with the isometries above. Lin, Lunin, Maldacena (2004), Lin, Maldacena (2005), Gaiotto, Maldacena (2009).

After some simplifications are made, the configuration is described in terms of a function $V(\sigma,\eta)$. Defining $\dot{V}=\sigma\partial_\sigma V$, $V'=\partial_\eta V$ the equation solved by $V(\sigma,\eta)$ is,

$$\underline{\ddot{V}+\sigma^2 V''=0.}$$

This differential equations, valid for both the SCFT in 4d or the Matrix Model in (0+1)-d, needs to be supplemented by boundary conditions. This is what distinguishes between different dynamics.

$\underline{\mathcal{N}=2\ \text{SCFT}}$

Boundary conditions.

$\underline{\text{PWMM}}$

Boundary conditions

For the 4d SCFTs, the metric reads, Gaiotto-Maldacena (2009)

$$ds^2_{IIA,st} = \Big(\frac{2\dot{V}-\ddot{V}}{V''}\Big)^{1/2}\Big[4AdS_5 + \frac{2V''\dot{V}}{\Delta}d\Omega_2^2(\chi,\xi) + \frac{2V''}{\dot{V}}(d\sigma^2+d\eta^2) + \frac{4V''\sigma^2}{2\dot{V}-\ddot{V}}d\beta^2\Big],$$
$$\Delta = (2\dot{V}-\ddot{V})V'' + (\dot{V}')^2.$$

For the case of the Plane Wave Matrix Model, Lin and Maldacena (2005) determined that,

$$ds^2_{IIA,st} = \Big(\frac{-2\dot{V}+\ddot{V}}{-V''}\Big)^{1/2}\Big[4d\Omega_5 + \frac{2V''\dot{V}}{\Delta}d\Omega_2^2(\chi,\xi) + \frac{-2V''}{\dot{V}}(d\sigma^2+d\eta^2) - \frac{4V''\sigma^2}{-2\dot{V}+\ddot{V}}dt^2\Big],$$
$$\Delta = (-2\dot{V}+\ddot{V})V'' - (\dot{V}')^2.$$

And analog expressions for the rest of the RR and NS fields.

AS you imagine, we can find the expression for $V(\sigma, \eta)$, that satisfies the Laplace equation and gives the two non-Abelian T-dual backgrounds.

Indeed, for the case in which we dualised an $SU(2)$ insider S^5, one finds,

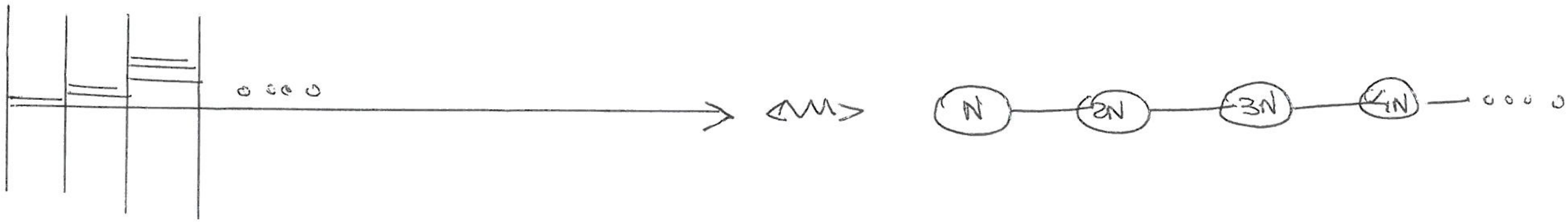
$$V_{CFT}(\sigma, \eta) = \eta(\log \sigma - \frac{\sigma^2}{2}) + \frac{1}{3}\eta^3.$$

While, for the case in which we dualised on AdS_5, the function $V(\sigma, \eta)$ is,

$$V_{MM}(\sigma, \eta) = \eta(-\log \sigma + \frac{\sigma^2}{2}) - \frac{1}{3}\eta^3.$$

Both non-Abelian T-dual backgrounds are new solutions that fit in the classifications. Nevertheless, the backgrounds are singular, boundary conditions are not satisfied. What is wrong with the Physics dual these non-Abelian T-dual backgrounds?

We can have an intuition for what goes wrong. Consider the Hanany-Witten set-up or quiver associated with the $\mathcal{N} = 2$ SCFT dual to the non-Abelian dual on S^5.



For a Hanany-Witten set-up like the one above, some relations between linking numbers have to be satisfied

Defining linking numbers for NS5 and D6 branes as

$$\hat{L}_{NS5} = Net_{D4} - Right_{D6}, \quad L_{D6} = Net_{D4} + Left_{NS5}.$$

The linking numbers satisfy a relation— for P NS5-branes and D6 branes that appear with multiplicity d_i,

$$\sum_{i=1}^{P} \hat{L}_{NS,i} + \sum_{i=1}^{P-1} d_i L_{D6,i} = 0.$$

One learns from here that if we decide to limit or bound the size of the η-coordinate, we will need to add D6 branes so to satisfy the relation above.

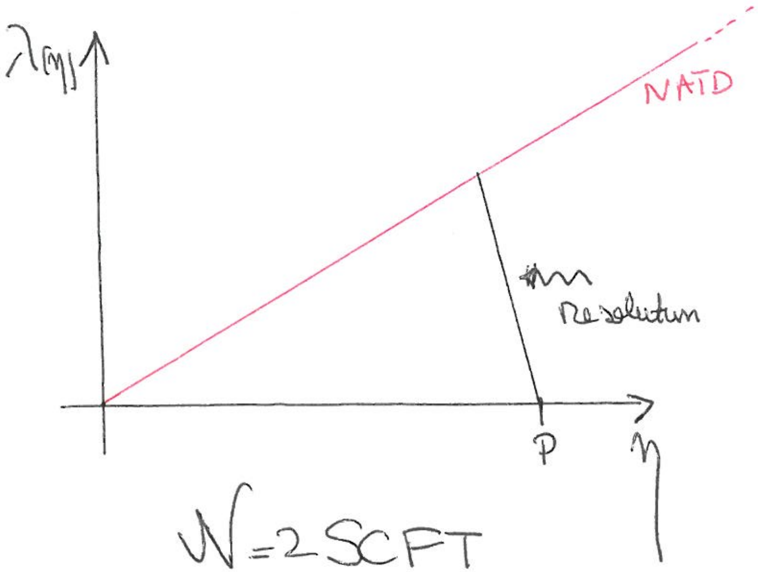
Something very similar happens in the matrix model case.
Indeed, there there is an ever-growing dimension of the matrix that represents the vacuum. This changes the correct D0 brane asymptotics into something else.
In some way, we need to 'stop' this growth in the representation-sizes. Same with the ever growing ranks of the quiver in the CFT case.
The way to do amend this problem is to find a new set of solutions that in some form 'reduce' to the non-Abelian T-dual ones, or such that they capture the same information.
Technically, the ways of finding solutions have been discussed in various papers, but I would like to emphasise the works of
Reid-Edwards and Stefanski (2010), Aharony Berkooz and Berdichevsky (2012), for the case of SCFTs
Bak, Siwach, Yee (2005), Shieh, van Anders, Van Raamsdonk (2007); Donos, Simon (2010)

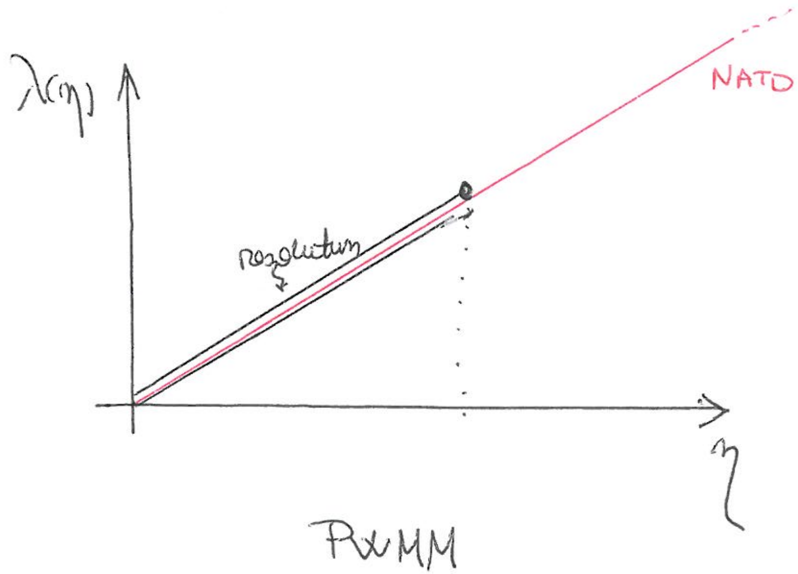
In other words, our procedure is to let the QFT inform the backgrounds what to do, how to resolve their singularities, how to bound its coordinates, if needed.

As I mentioned before, a lot of information is contained in the η-coordinate. For example, one can define a 'charge density'

$$\lambda(\eta) = \sigma \partial_\sigma V(\sigma, \eta)|_{\sigma=0} = \eta.$$

We wish to find solutions that solve the problems of the non-Abelian T-duals, by 'bounding' $\lambda(\eta)$





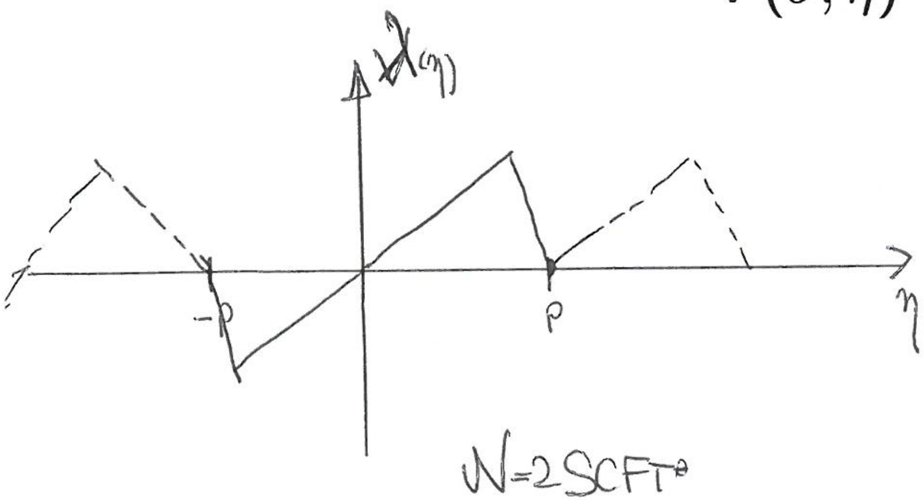
For the case in which the non-Abelian duality acts on S^3 inside S^5, the solution for the $V(\sigma, \eta)$ that does what we ask above is

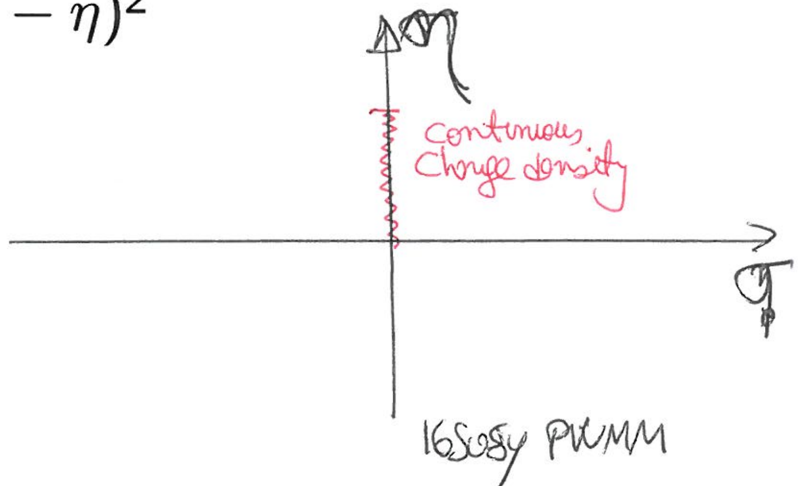
$$\dot{V}(\sigma, \eta) = \sum_{m=1}^{\infty} A_m \sigma K_1\left(\frac{m\pi\sigma}{N_5}\right) \sin\left(\frac{m\pi\eta}{N_5}\right).$$

Where A_m are the Fourier coefficients of $\lambda(\eta)$ defined in $[0, N_5]$ and odd-periodically extended.

For the case of the matrix model, one can find the potential by a 'method of images'

$$V(\sigma, \eta) = \int_{-\infty}^{\infty} dz \frac{\lambda(z)}{\sqrt{\sigma^2 + (z-\eta)^2}}.$$





Calculating explicitly one finds the full expressions for these potentials. From here, one finds new backgrounds.
In the case of the $\mathcal{N} = 2$ SCFT, the characteristics of the new solution are

- It is fully encoded in the function $\dot{V}(\sigma, \eta)$. A solution to a Laplace problem in the $[\sigma, \eta]$ plane, with charge density $\lambda(\eta)$.
- The background is smooth, except at the position of the D6 branes. They realise the global $SU(P)$ flavor symmetry.
- The coordinate η is now bounded—like the quiver, whose length is finite.
- The geometry that before was singular at $\sigma = 1$ is now smooth there. The CFT gave a completion to the originally singular metric

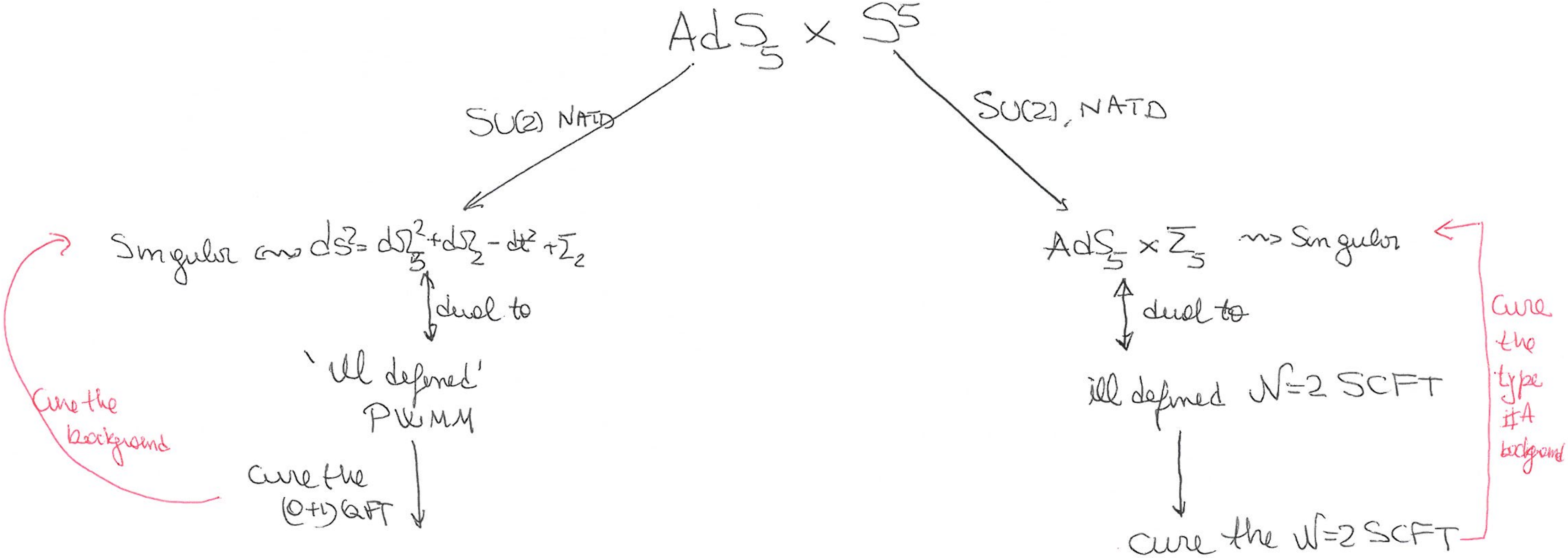
For the case of the matrix model, the new background's characteristics are:

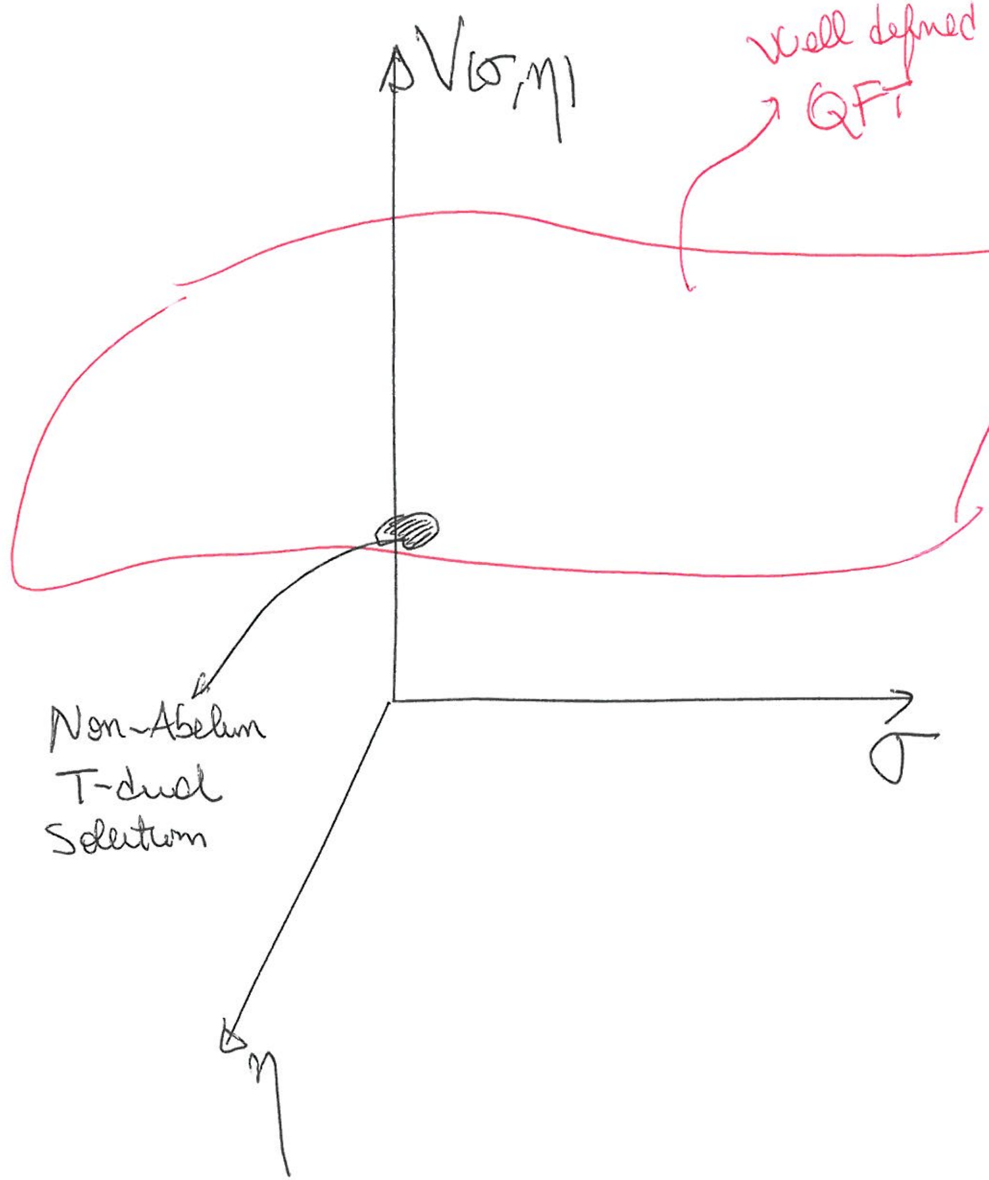
- It is fully encoded in the function $\dot{V}(\sigma,\eta)$. A solution to a Laplace problem in the $[\sigma,\eta]$ plane, with charge density $\lambda(\eta)$–bounded.
- The background is smooth, except at the position $\sigma = 0$, where a continuos charge distribution $\lambda(z)$ was placed.
- The coordinate η is not bounded, but the number of $SU(2)$ representations in the vacuum is finite, as read from $\lambda(\eta)$.
- The geometry that before was singular at $\sigma = 1$ is now smooth there. The QFT gave a completion to the originally singular metric

In both cases, one can see very explicitly that there is an scaling of the coordinates of the new 'completed' backgrounds, that gives as a result the non-Abelian T-dual ones,

In other words, we are finding either a 4d SCFT or a (0+1)d matrix model that 'complete' the ones that correspond with the non-Abelian T-dual backgrounds.

In a drawing, the idea could be expressed like this.





"Somewhat analogous"

Klebanov-Witten

↓

Klebanov-Tseytlin

↓

Singularity

↓

Klebanov-Strassler (Baryonic branch)

SCFT

well defined QFT

confines

χ-Symmetry breaks

cure Singularity

Finally, let me end with something that I believe is quite nice

For the moment, we succeeded to make this work, only in the case of the matrix model.

Suppose that we consider a particular solution of eleven dimensional supergravity. The solution is called 'superstar' —Leblond, Myers and Page, (2001).

It can be understood as the relevant deformation (by giant gravitons) of the M5 branes 6d CFT.

$$ds^2 = \Delta^{1/3}\left(-fH^{-1}dt^2 + f^{-1}dr^2 + r^2 d\Omega_5^2 + L^2\frac{d\theta^2}{4}\right)$$
$$+\frac{\Delta^{-2/3}L^2}{4}\left(\sin^2\theta\, d\Omega_2^2 + H\cos^2\theta(dx_{11} + \frac{2}{HL}dt)^2\right),$$
$$\Delta = 1 + \frac{Q}{r^4}\sin^2\theta, \quad H = 1 + \frac{Q}{r^4}, \quad f = 1 + \frac{H}{L^2}r^2.$$

One can rescale the coordinates

$$r = L\tilde{r}, \qquad t = Lt, \qquad Q = \tilde{q}L^4, \quad x_{11} = \frac{4}{L^2}\tilde{x}_{11}.$$

$$\tilde{r} = \frac{r_5}{L}, \quad \theta = \frac{2}{L}r_2. \quad q = \tilde{q}L^2.$$

After taking $L \to \infty$, keeping leading terms and changing coordinates as $r_5^2 = 4(\sigma^2 - 1)$, $r_2 = 2\eta$ and setting $q = 4$, the metric reads

$$\begin{aligned} ds_{11}^2 &= (\sigma^2-1)^{1/3}(4\eta^2+(\sigma^2-1)^2)^{1/3} \times \\ &\left(-4\sigma^2 dt^2 + \frac{4}{\sigma^2-1}(d\sigma^2+d\eta^2) + 4d\Omega_5 + \frac{4\eta^2(\sigma^2-1)}{4\eta^2+(\sigma^2-1)^2}d\Omega_2\right) \\ &+ \frac{1}{(\sigma^2-1)^{2/3}(4\eta^2+(\sigma^2-1)^2)^{2/3}}\left(dx_{11} - 2(\sigma^2-1)^2 dt\right)^2. \end{aligned}$$

This is the eleven dimensional lift of the non-Abelian T-dual on AdS_5

In this way, we have found another 'completion' of the metric dual to the (0+1)-d QFT. This completion requires to 'grow dimensions' in the QFT. To finally reach a 6d CFT.

There are various interesting comments and speculations one can make from here, but let me close it here.

Summary, Conclusions and Final Comments

The usual T-duality and its non-Abelian version can be seen as "solution generating techniques".

In the case of T-duality, the stringy character of the transformation is well understood. This is not the case in the non-abelian generalisation.

Using CFT information, we obtained information about the manifold generated by non-Abelian T-duality acting on $AdS_5 \times S^5$.

Other examples follow a similar logic, the non-Abelian T-duality, 'focuses' on a patch of a more generic manifold.

On the other hand, non-Abelian T-duality can be applied to a large variety of examples and it has proven very useful to:

- Find new backgrounds, avoiding known classifications.
- Generate new backgrounds with 'dynamic' $SU(2)$-structure and other G-structures.
- Use these new backgrounds to 'define' new QFTs at strong coupling, by the calculation of the QFT observables.

The "QFT perspective" promoted in this talk may help to clarify problems and issues present in the points above.