Degenerate scalar-tensor theories
a geometrical approach

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Based on works with Javier Chagoya, Marco Crisostomi

+ Kazuya Koyama, David Langlois, Karim Noui

Question

What is the **most general** *consistent scalar-tensor theory?*

- **General Relativity** is the only consistent 4d theory describing a massless, interacting spin two field (2 tensor dofs).
- Is there any analogue result for **covariant scalar-tensor** set-ups propagating massless spin two + spin zero fields (2+1 dofs)?

Plan

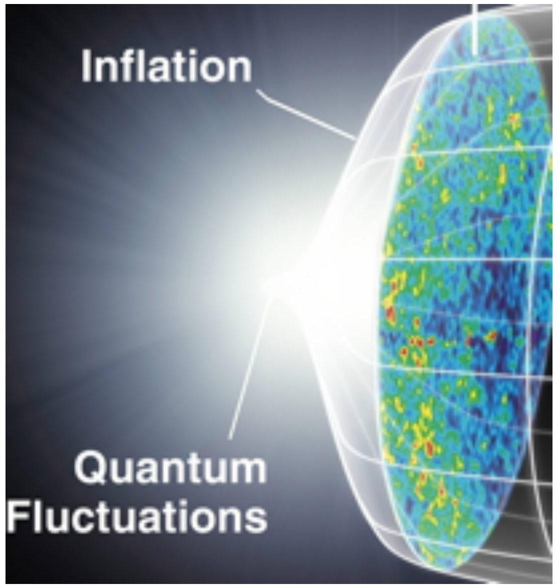
1. **Motivations** for considering this question
2. **What do we know** about its answer

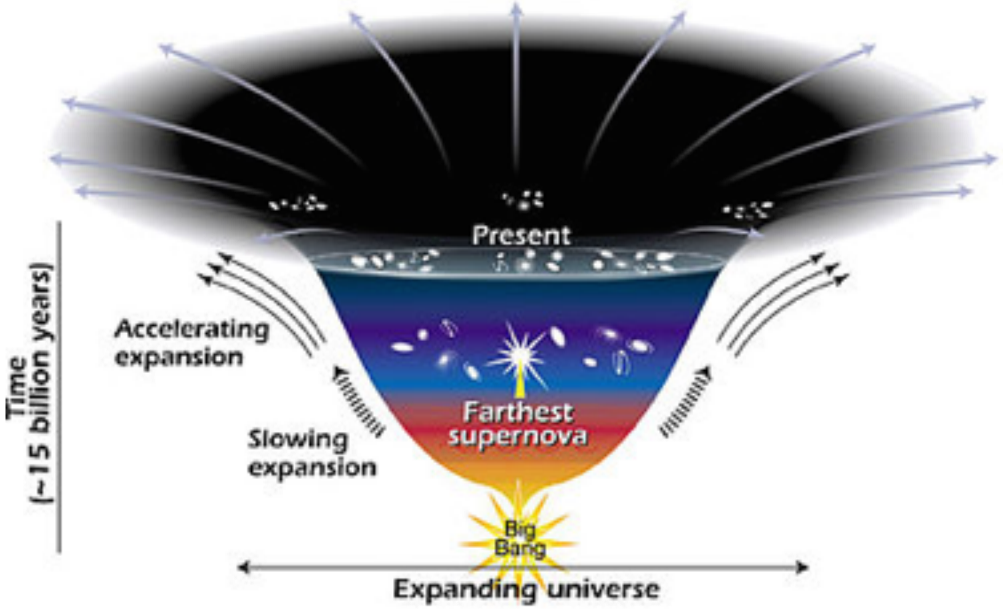
little, but a geometrical approach can be useful (maybe)

Scalar-Tensor Theories

Why considering them?

- Naturally arise from **string theory**
- Important applications to **cosmology**





Scalar-Tensor Theories

Why considering them?

- Naturally arise from **string theory**
- Important applications to **cosmology**

`Prototype` $$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi}_{\texttt{Canonical kinetic term}} - \underbrace{V(\phi)}_{\texttt{self-interactions}} \right]$$

- **Much more** can be done...

Scalar-Tensor Theories

What about derivative scalar self-interactions ?

- interactions involving **single derivative** of scalars. Call $X = \partial_\mu \pi \partial^\mu \pi$
  - Prototype: **DBI action** $S = \int d^4x \sqrt{-g} \sqrt{1+X}$
  - More generally: any function $S = \int d^4x \sqrt{-g}\, P(X)$ is fine
    EOMs are second order

- interactions including **second derivatives** of scalars

$$\boxed{\texttt{The Galileons}} \left\{ \begin{array}{l} \text{- EOMs contain at most second derivatives} \\ \\ \text{- EOMs invariant under symmetry } \pi \to \pi + b_\mu x^\mu + c \end{array} \right.$$

[Nicolis, Rattazzi, Trincherini]

Scalar-Tensor Theories

The Galileons

$$\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2}(\partial\pi)^2 \\
\mathcal{L}_3 &= (\partial\pi)^2\,\Box\pi \\
\mathcal{L}_4 &= (\partial\pi)^2\left[(\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right] \\
\mathcal{L}_5 &= (\partial\pi)^2\left[(\Box\pi)^3 + 2\,(\partial_\mu\partial_\nu\pi)^3 - 3\Box\pi\,(\partial_\mu\partial_\nu\pi)^2\right]
\end{aligned}$$

Screening effects

$\Updownarrow$

Acceleration of the universe $\Leftrightarrow$

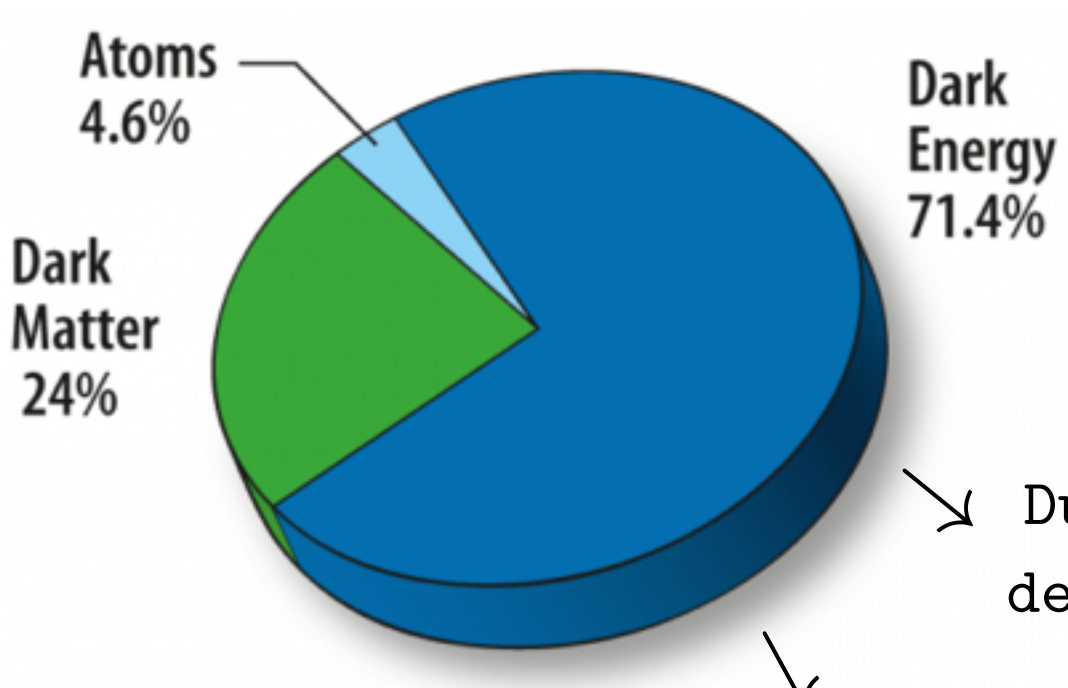
Non-renormalization theorems

Scalar-Tensor Theories

`The Galileons`

$\boxed{\text{Acceleration of the universe}}$ **self-acceleration**

There exist branches of `cosmological solutions` that are `asymptotically de Sitter`, with no need of positive cosmological constant.



`Due to massless scalar with derivative self-interactions`

`Regime where second derivatives` $\partial^2\pi$ `are important`

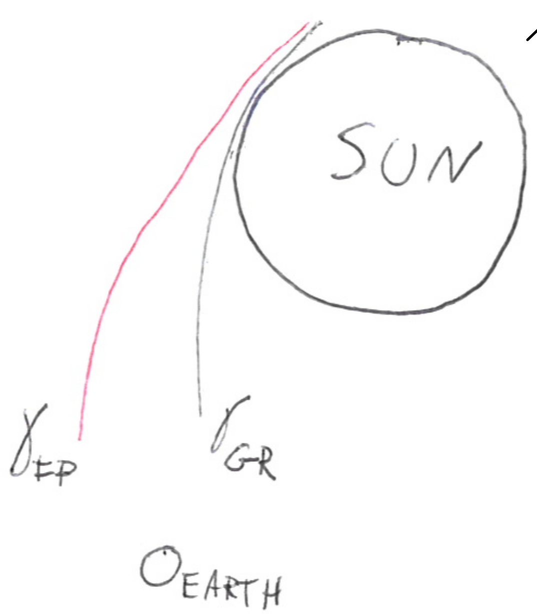
Scalar-Tensor Theories

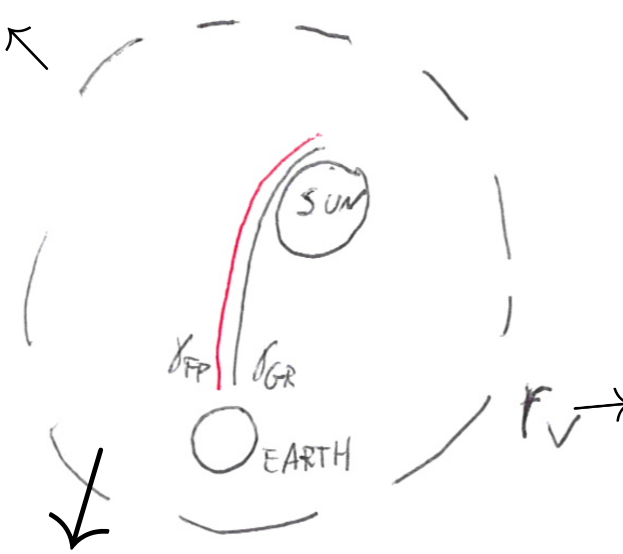
The Galileons

$\boxed{\text{Screening effects}}$ `why don't we see it?` **Vainshtein mechanism:**

non-linearities associated with self-interactions **hide the effect of scalar** nearby spherically symmetric sources

Evade stringent bounds on deviations from GR





Vainshtein radius

Regime where second derivatives $\partial^2\pi$ are important

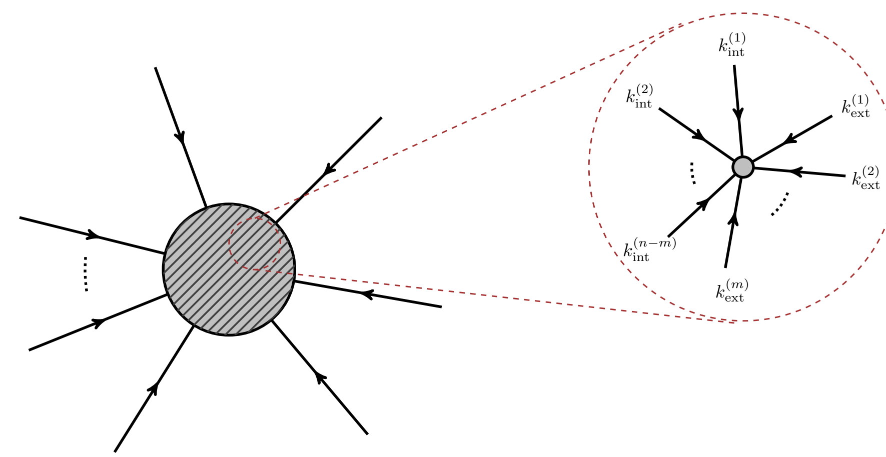
Scalar-Tensor Theories

The Galileons

Non-renormalization theorems [Nicolis, Rattazzi; Goon, Hinterbichler, Trodden]

`Galileon interactions don't get renormalized in perturbation theory`

Thanks to structure of interactions + Galilean symmetry



Safe radial interval
where $\partial^2\pi$ is large and theory
`technically natural`

$\Downarrow$

Scalar-Tensor Theories

The Galileons

$$
\begin{aligned}
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\end{aligned}
$$

Screening effects

Acceleration of the universe

Non-renormalization theorems

Good for Dark Energy!

Scalar-Tensor Theories

Horndenski (1974)

Question

What's the **most general scalar-tensor action** leading to 2nd order equations of motion?

Scalar-Tensor Theories

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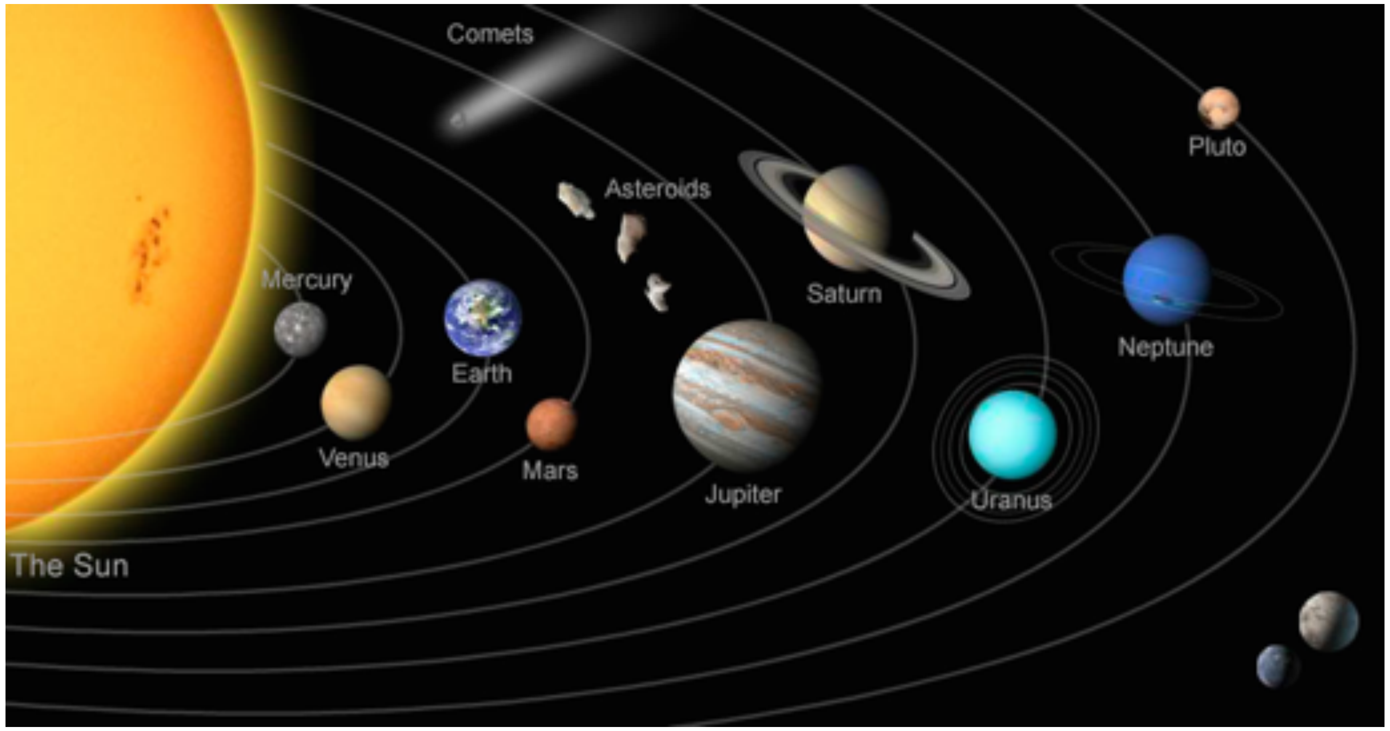
What's the **most general scalar-tensor action** leading to 2nd order equations of motion?

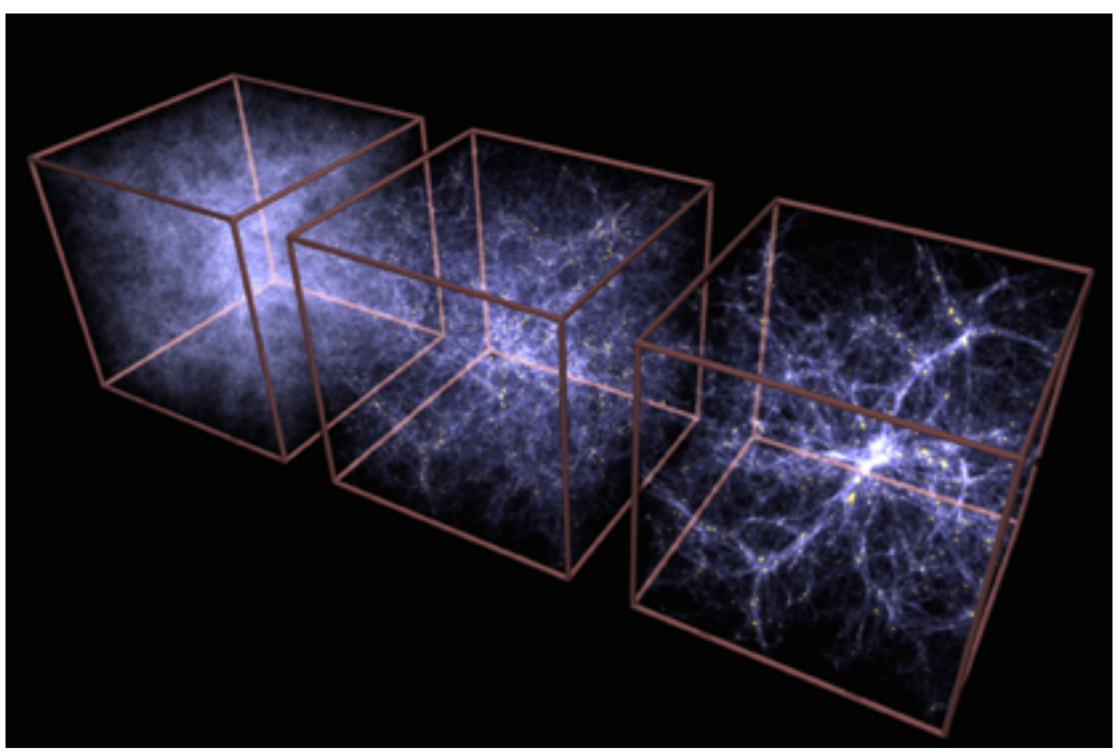
$$
\begin{aligned}
L_2^{\mathrm{H}} &\equiv G_2(\phi, X)\ , \qquad\qquad L_3^{\mathrm{H}} \equiv G_3(\phi, X)\,\Box\phi\ , \\
L_4^{\mathrm{H}} &\equiv G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})\ , \\
L_5^{\mathrm{H}} &\equiv G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})\,,
\end{aligned}
$$

where the G_i are **arbitrary functions** of ϕ, X

- contains up to three powers of $\partial^2\phi$. Flat space $\rightarrow$ Galileons
- **lot of activity** for applications to cosmology, gravity/BHs etc etc



$\Leftarrow$ **ok**



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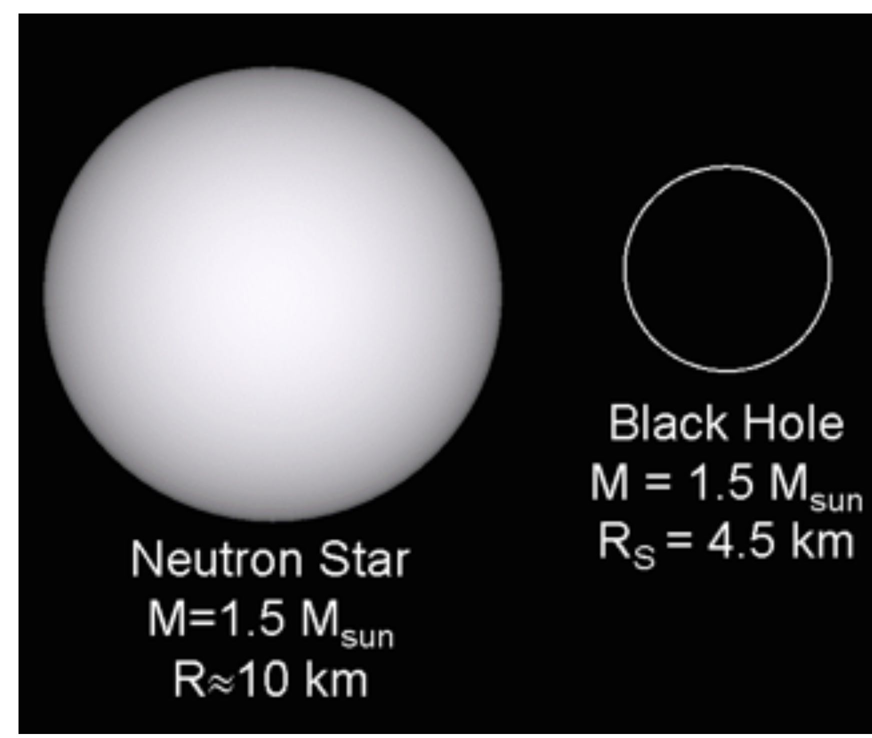

Scalar-Tensor Theories

Horndenski (1974)

Question

What's the **most general scalar-tensor action** leading to 2nd order equations of motion?

$\Downarrow$

`Is this really necessary?`

Scalar-Tensor Theories

Horndenski (1974)

Question

What's the **most general scalar-tensor action** leading to 2nd order equations of motion?

Ostrogradsky theorem

[see e.g. Woodard]

Any non-degenerate theory with **EOMs of order higher than two** has **Hamiltonian unbounded from below**

Ostrogradsky ghost

Scalar-Tensor Theories

Horndenski (1974)

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What's the **most general scalar-tensor action** leading to 2nd order equations of motion?

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[see e.g. Woodard]

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Scalar-Tensor Theories

~~Ostrogradsky theorem~~

Let's consider **degenerate scalar-tensor theories**

Relations defining conjugate momenta can't be fully inverted:
velocities can't be expressed in terms of fields and their conjugate momenta

Constraint conditions exist!

In this way new consistent covariant scalar-tensor theories can be found, that propagate

only up to three degrees of freedom

no Ostrogradsky ghost mode

Scalar-Tensor Theories

~~**Ostrogradsky theorem**~~

`Let's consider` **degenerate scalar-tensor theories**

Relations defining conjugate momenta can't be fully inverted:
velocities can't be expressed in terms of fields and their conjugate momenta

Beyond Horndenski [Gleyzes et al]

$$L_4^{\rm bH} \equiv F_4(\phi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\, \epsilon^{\mu'\nu'\rho'\sigma} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \ ,$$
$$L_5^{\rm bH} \equiv F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'} \ ,$$

Scalar-Tensor Theories

~~**Ostrogradsky theorem**~~

`Let's consider` **degenerate scalar-tensor theories**

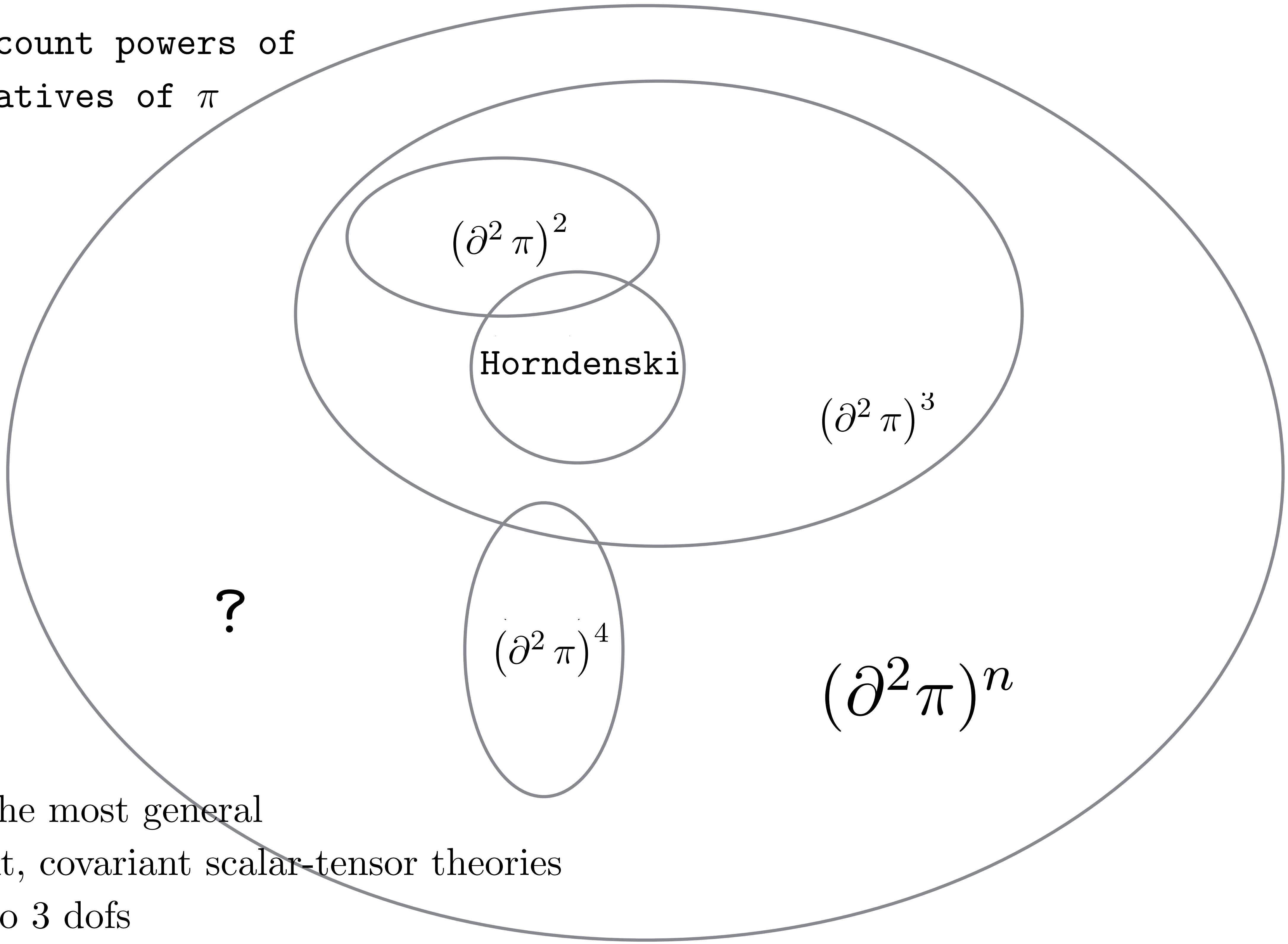
Relations defining conjugate momenta can't be fully inverted:
velocities can't be expressed in terms of fields and their conjugate momenta

Extended Scalar Tensor [Langlois, Noui; Crisostomi, Koyama, GT; Ben Achour et al]

- Use Hamiltonian approach to systematically find **all consistent theories**, classifying them in terms of **number of second derivatives of scalar**
- New theories found, with interesting consequences for dark energy (growth of structure) + screening mechanisms (neutron stars etc)

Most general scalar-tensor theory?

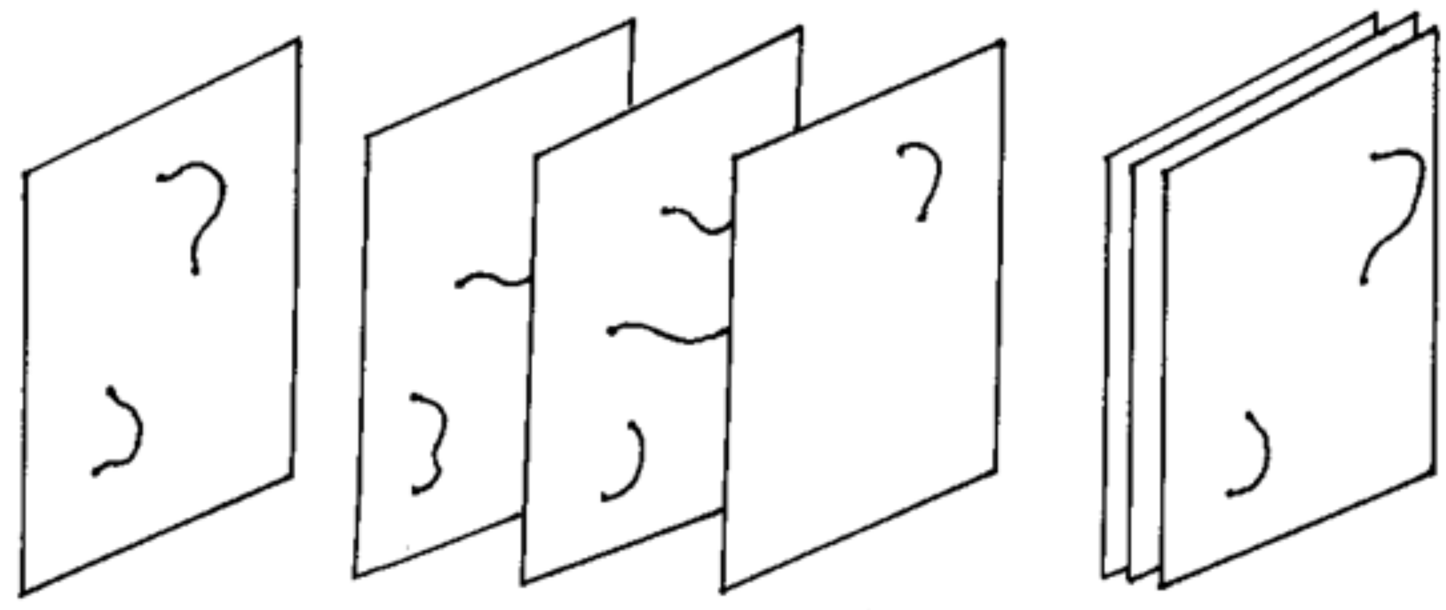
`Criterion: count powers of second derivatives of` π



We don't know the most general
form of consistent, covariant scalar-tensor theories
propagating up to 3 dofs

Question

Do some of these theories admit a geometrical interpretation, which might reveal some symmetries (recall Galilean symmetry)



Recall lessons from Galileons: let's think to DE applications

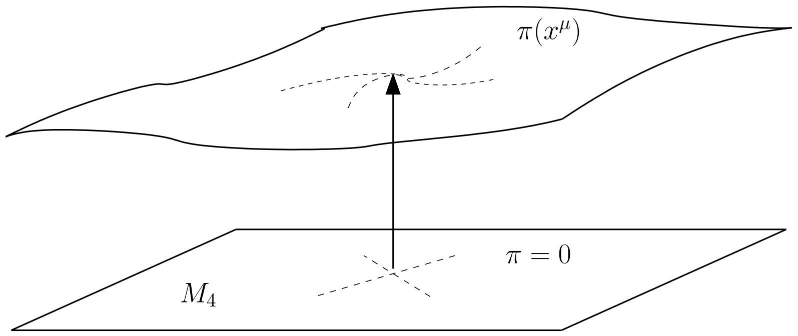
Galileons from branes

[de Rham, Tolley]

- Probe brane in Minkowski space: **brane position** π

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

DBI action: $$S_\lambda = -\lambda \int \mathrm{d}^4x\sqrt{-g} = -\lambda \int \mathrm{d}^4x\sqrt{1+(\partial\pi)^2}$$



- Action is invariant under a symmetry

$$\delta_v\pi(x) = v_\mu x^\mu + \pi(x)v^\mu\partial_\mu\pi(x)$$

inherited from 5d Poincaré symmetry

Galileons from branes

- **Generalization**

  Consider higher dimensional Lovelock invariants + associated Gibbons-Hawking terms

$$S_K = -M_5^3 \int \mathrm{d}^4x\sqrt{-g}K \quad ; \quad S_R = \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}R \quad ;$$

$$S_{GB} = -\beta\frac{M_5^3}{m^2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{K}_{\mathrm{GB}} \quad ; \quad \mathcal{K}_{\mathrm{GB}} = -\frac{2}{3}K^3_{\mu\nu} + KK^2_{\mu\nu} - \frac{1}{3}K^3 - 2G_{\mu\nu}K^{\mu\nu}$$

  - EOMs for π remain second order
  - Symmetry is still preserved $\delta_v\pi(x) = v_\mu x^\mu + \pi(x)v^\mu\partial_\mu\pi(x)$

Galileons from branes

- **Generalization**

  Consider higher dimensional Lovelock invariants + associated Gibbons-Hawking terms

$$S_\lambda = -\lambda \int \mathrm{d}^4x\sqrt{-g} = -\lambda \int \mathrm{d}^4x\sqrt{1+(\partial\pi)^2}$$

$$S_K = -M_5^3 \int \mathrm{d}^4x\sqrt{-g}K = M_5^3 \int \mathrm{d}^4x\ \left([\Pi]-\gamma^2[\phi]\right)$$

$$S_R = \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}R = \frac{M_4^2}{2}\int \mathrm{d}^4x\,\gamma\left(\left([\Pi]^2-[\Pi^2]\right)+2\gamma^2\left([\phi^2]-[\Pi][\phi]\right)\right)$$

$$\begin{aligned} S_{GB} &= -\beta\frac{M_5^3}{m^2}\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{K}_{\mathrm{GB}} \\ &= \beta\frac{M_5^3}{m^2}\int \mathrm{d}^4x\,\gamma^2\Big(\frac{2}{3}\left([\Pi]^3+2[\Pi^3]-3[\Pi][\Pi^2]\right)+4\gamma^2([\Pi][\phi^2]-[\phi^3]) \\ &\qquad\qquad -2\gamma^2([\Pi]^2-[\Pi^2])[\phi]\Big)\,, \end{aligned}$$

Galileons from branes

- **Generalization**

  Consider higher dimensional Lovelock invariants + associated Gibbons-Hawking terms

> `Take non-relativistic limit` $\partial\pi \ll 1$ `(but still` $\partial^2\pi$ `large)`

$$S_2 = S_\lambda^{NR} = -\frac{\lambda}{2}\int \mathrm{d}^4x\,(\partial\pi)^2$$

$$S_3 = S_K^{NR} = \frac{M_5^3}{2}\int \mathrm{d}^4x\,(\partial\pi)^2\,\Box\pi$$

$$S_4 = S_R^{NR} = \frac{M_4^2}{4}\int \mathrm{d}^4x\,(\partial\pi)^2\,\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right)$$

$$S_5 = S_{GB}^{NR} = \beta\frac{M_5^3}{3m^2}\int \mathrm{d}^4x\,(\partial\pi)^2\left((\Box\pi)^3 + 2(\partial_\mu\partial_\nu\pi)^3 - 3\Box\pi(\partial_\mu\partial_\nu\pi)^2\right)$$

- Symmetry becomes Galilean symmetry

$$\delta_v\pi(x) = v_\mu x^\mu + \pi(x)v^\mu\partial_\mu\pi(x) \quad\Rightarrow\quad \delta\pi = v_\mu\,x^\mu$$

What about degenerate scalar-tensor theories?

Can we find an analogous geometric construction for EST ?

- `Build probe brane actions made of powers of extrinsic curvature`

$$
\begin{aligned}
S_K &= \int \sqrt{-g}\, K_\mu{}^\mu \\
S_{K^2} &= \int \sqrt{-g}\, K_\mu{}^\nu\, K_\nu{}^\mu \\
&\ldots
\end{aligned}
$$

- These actions have symmetries associated with bulk isometries

$$\delta_v \pi(x) = v_\mu x^\mu + \pi(x) v^\mu \partial_\mu \pi(x).$$

...but when coupled with gravity, their EOMs have higher order derivatives

What about degenerate scalar-tensor theories?

Can we find an analogous geometric construction for EST ?

- Take 'ultrarelativistic' limit $\partial\pi \gg 1$

$$
\begin{aligned}
\mathcal{L}_1 &= \Lambda^2 \sqrt{|X|} \\
\mathcal{L}_2 &= \Lambda \left([\Pi] - \frac{1}{X}[\Phi] \right) \\
\mathcal{L}_3 &= \frac{1}{\sqrt{|X|}} \left([\Pi]^2 - [\Pi^2] + \frac{2}{X}\left([\Phi^2] - [\Phi][\Pi]\right) \right) \\
\mathcal{L}_4 &= \frac{1}{\Lambda X} \left([\Pi]^3 + 2[\Pi^3] - 3[\Pi^2][\Pi] + \frac{3}{X}\left(2[\Pi][\Phi^2] - 2[\Phi^3] - [\Phi][\Pi]^2 + [\Phi][\Pi^2]\right) \right)
\end{aligned}
$$

`symmetry left`

$$\boxed{\delta\pi = \pi\, w^\mu \partial_\mu \pi}$$

`First time that such symmetry is noticed for these theories`

What about degenerate scalar-tensor theories?

Can we find an analogous geometric construction for EST ?

- Take ‘ultrarelativistic’ limit $\partial\pi \gg 1$

$$
\begin{aligned}
\mathcal{L}_1 &= \Lambda^2 \sqrt{|X|} \\
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\mathcal{L}_3 &= \frac{1}{\sqrt{|X|}} \left([\Pi]^2 - [\Pi^2] + \frac{2}{X} \left([\Phi^2] - [\Phi][\Pi] \right) \right) \\
\mathcal{L}_4 &= \frac{1}{\Lambda\, X} \left([\Pi]^3 + 2[\Pi^3] - 3[\Pi^2][\Pi] + \frac{3}{X} \left(2[\Pi][\Phi^2] - 2[\Phi^3] - [\Phi][\Pi]^2 + [\Phi][\Pi^2] \right) \right)
\end{aligned}
$$

- `when coupled with gravity, one gets` $\boxed{\texttt{degenerate scalar tensor theories}}$

- **Good news for model building**: we can model models of dark energy/inflation using EST with underlying symmetry preserving structure of action

What about degenerate scalar-tensor theories?

Can we find an analogous geometric construction for EST ?

yes

`Take 'ultra-relativistic' limit of of actions built with combinations of` $K_\mu{}^\nu$

Does the same work with induced action made with other combinations of curvature fluctuations? $R_{\mu\nu}R^{\mu\nu}$ **etc?**

? `work in progress....` ?

Conclusions

- The structure of **scalar-tensor** theories is richer than expected: degenerate systems
  - We still don't know the **most general** theory for massless spin 2 + spin 0 fields
  - Cosmologists are very interested to this question, for building new models of **dark energy** and **inflation**
- I presented
  - **most recent findings** about what we know of these theories
  - A geometrical approach which makes apparent previously unnoticed **underlying symmetry**

TO DO NEXT

- Continue the exploration of **geometrical approach** by studying **ultrarelativistic** limit of **brane induced actions** made with curvature invariants