Numerical relativity: (Almost) all you need to know

Seth Hopper

Vitor Cardoso

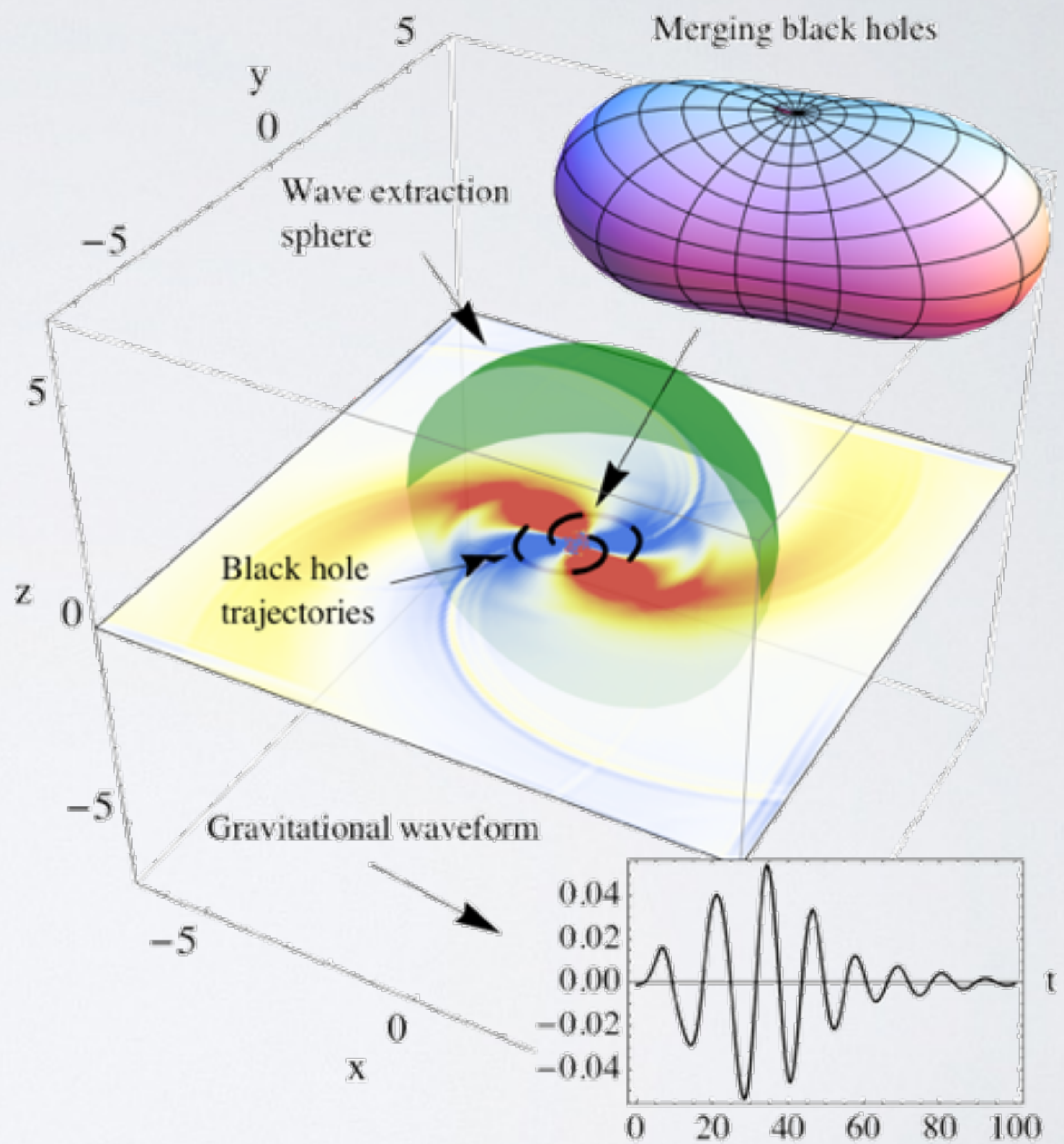


Fig: Ian Hinder

centra

TÉCNICO LISBOA

CERN - November 16, 2016

Binary black holes come in a few different varieties

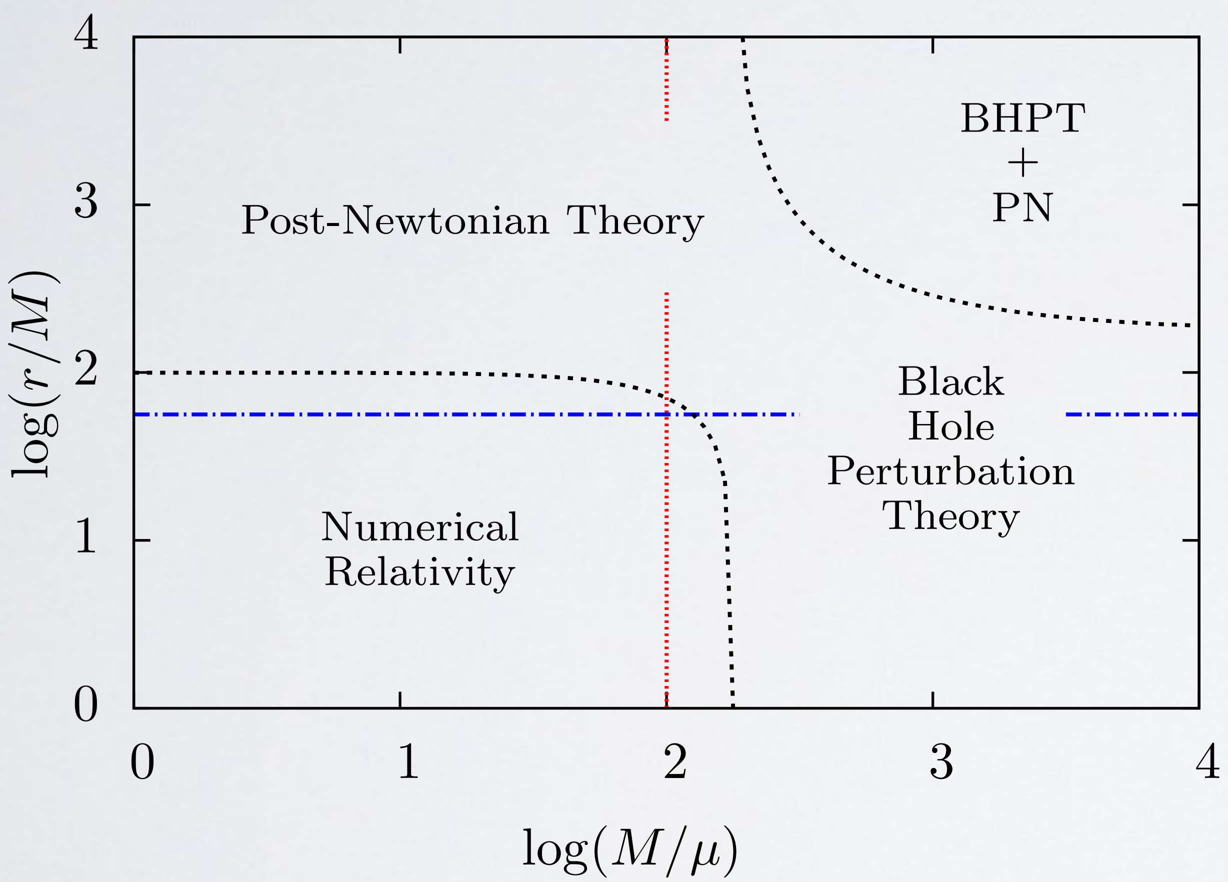


Fig: Forseth et al.

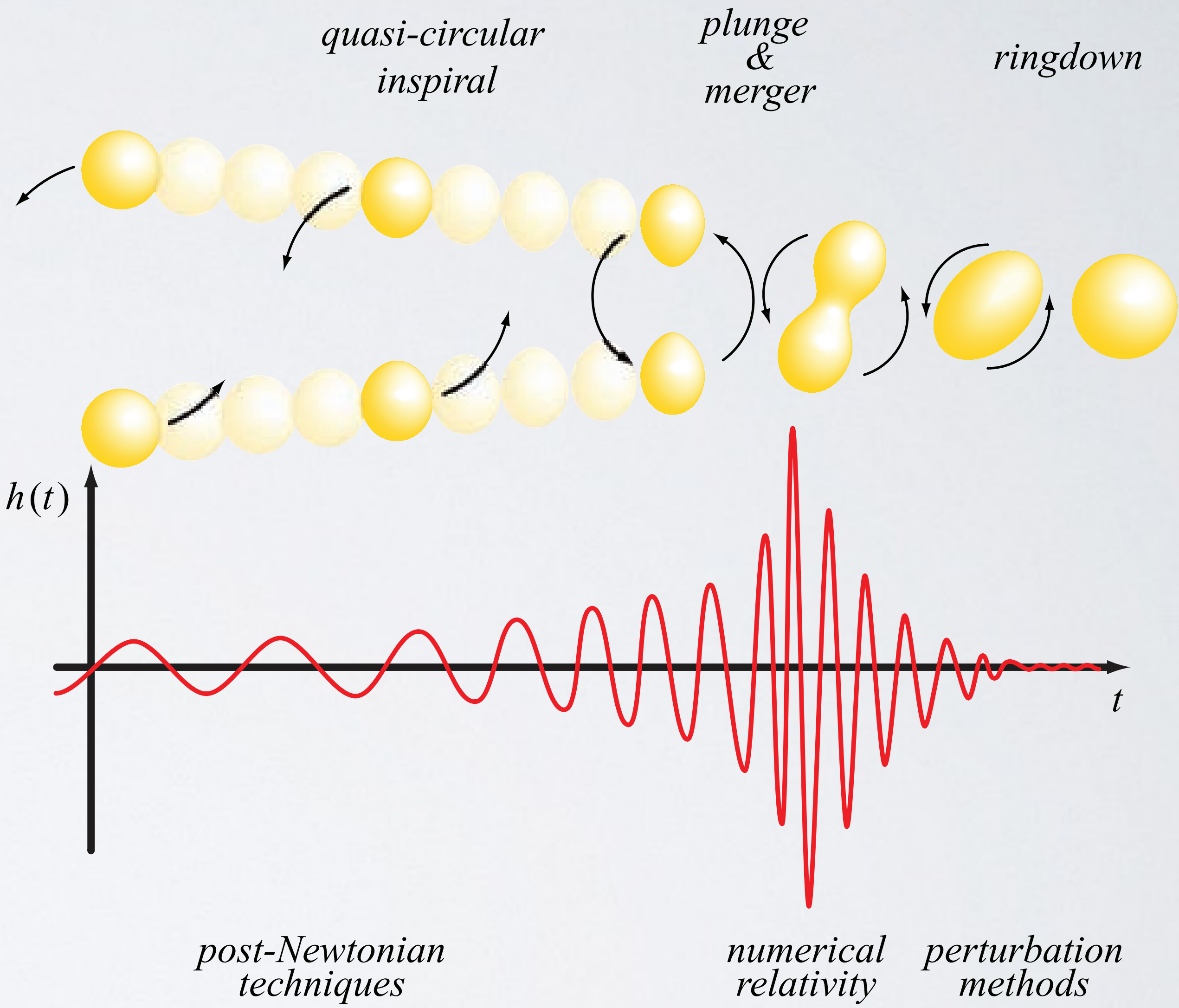


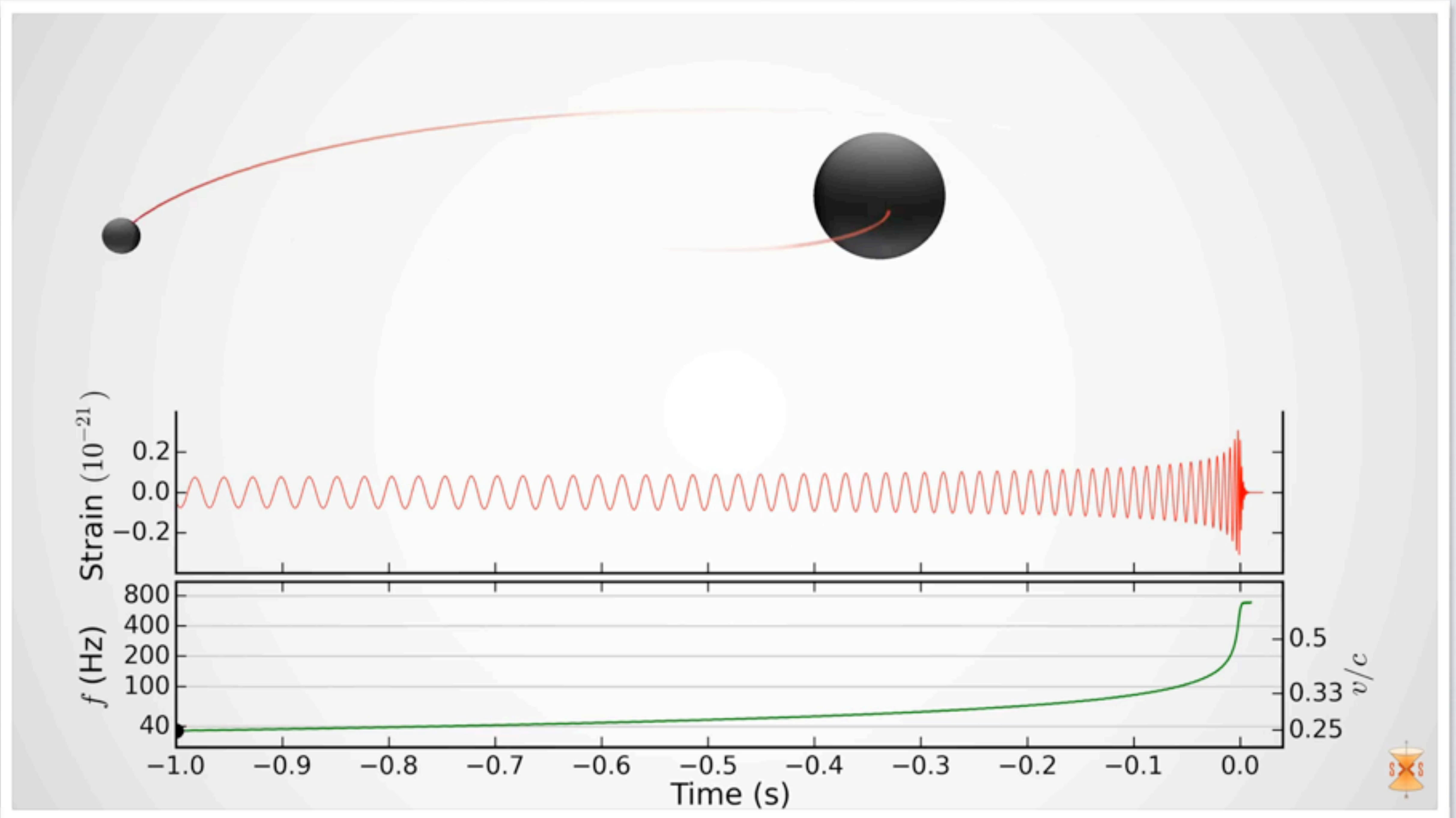
Fig: Baumgarte & Shapiro

This is a video simulating the second detection, GW151226

$M_1 = 14.2\ M_\odot$ $M_2 = 7.5\ M_\odot$

$M_{Final} = 20.8\ M_\odot$ $M_{GWs} = 1\ M_\odot$

- Simulation: Simulating eXtreme Spacetime project

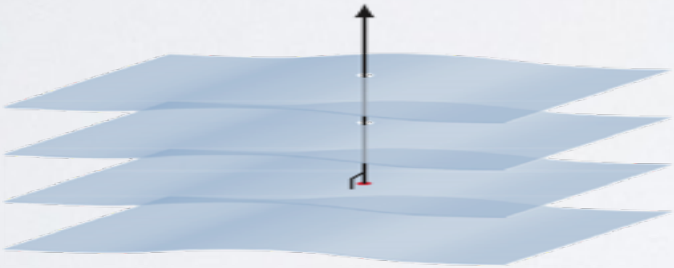


Outline

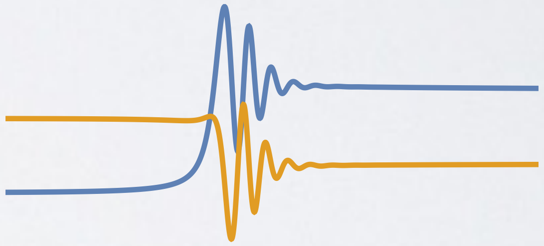
History and formalism

Initial data

Evolution of field equations

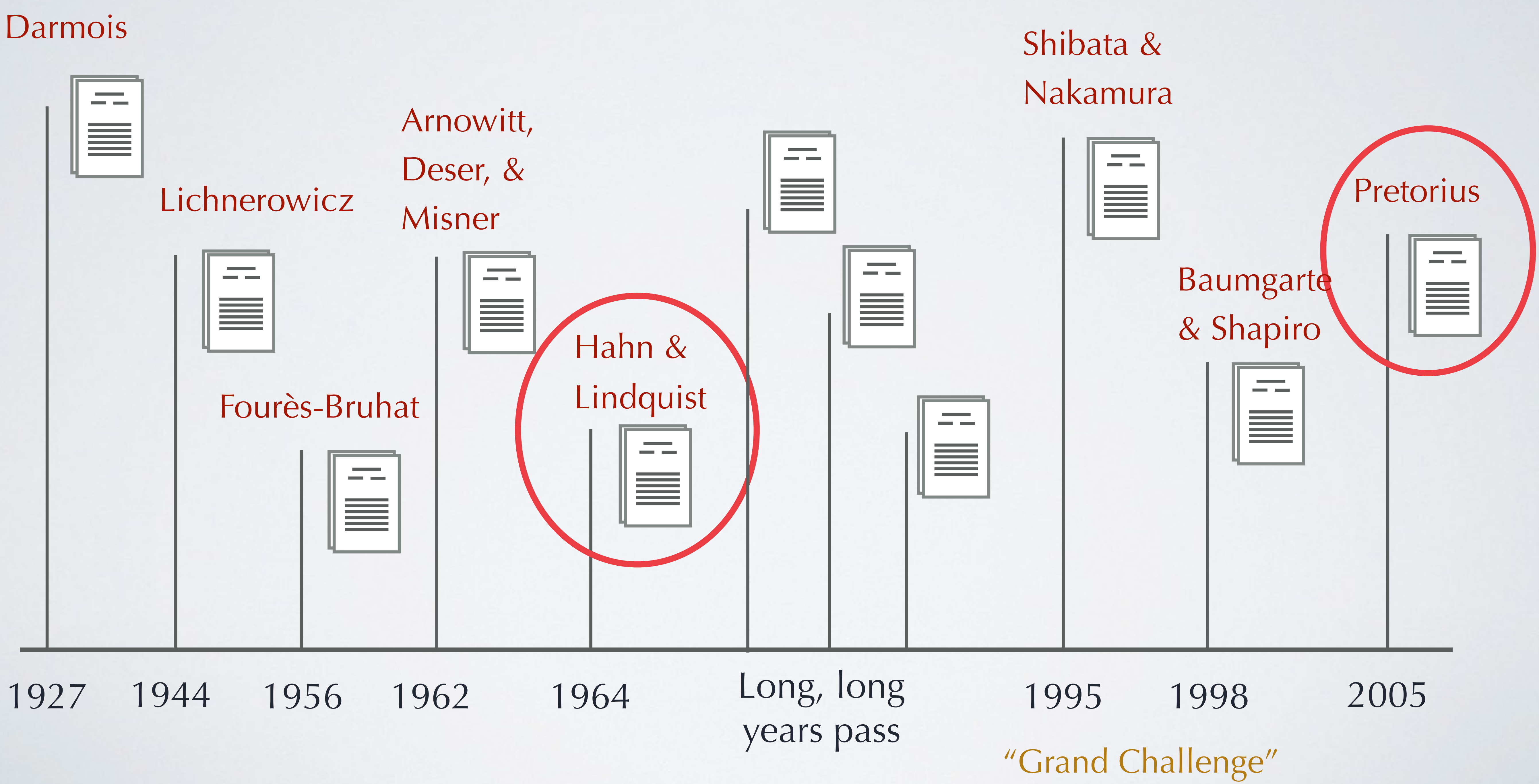


Wave extraction

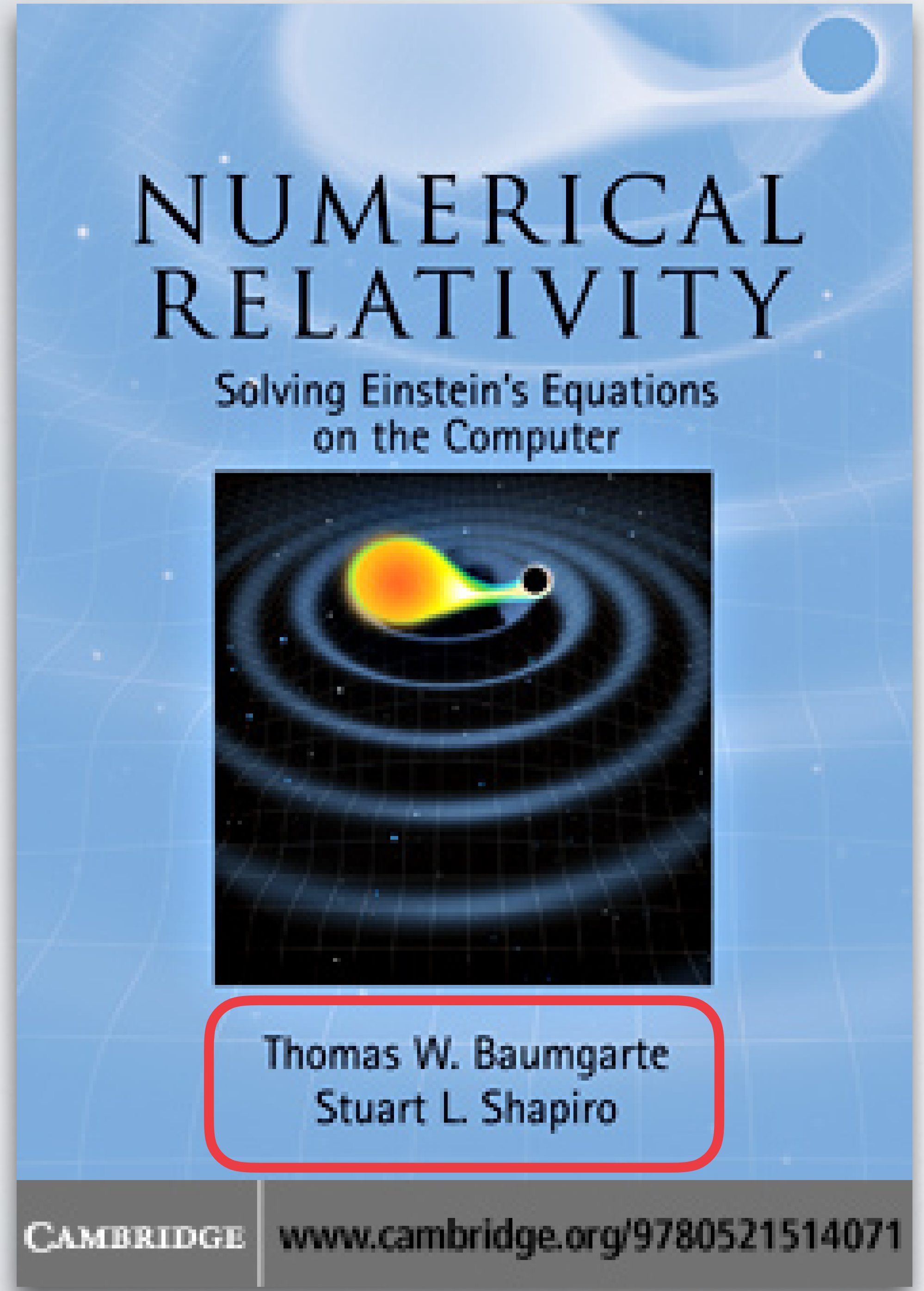


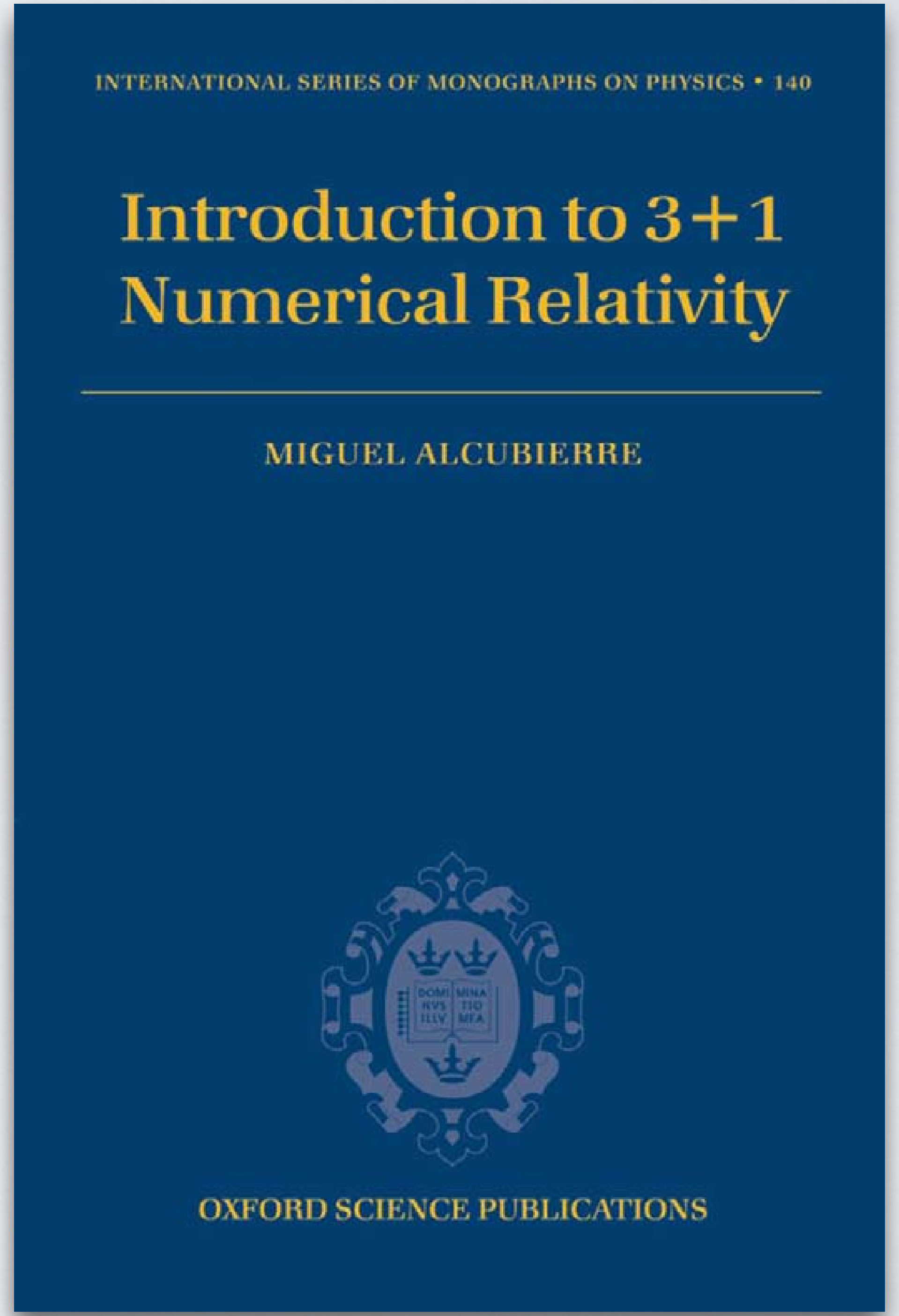
The tools we use

There is a long history of successes and failures in numerical relativity



Here are two good resources





Hahn & Lindquist did the first work in 1964

ANNALS OF PHYSICS: **29**, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

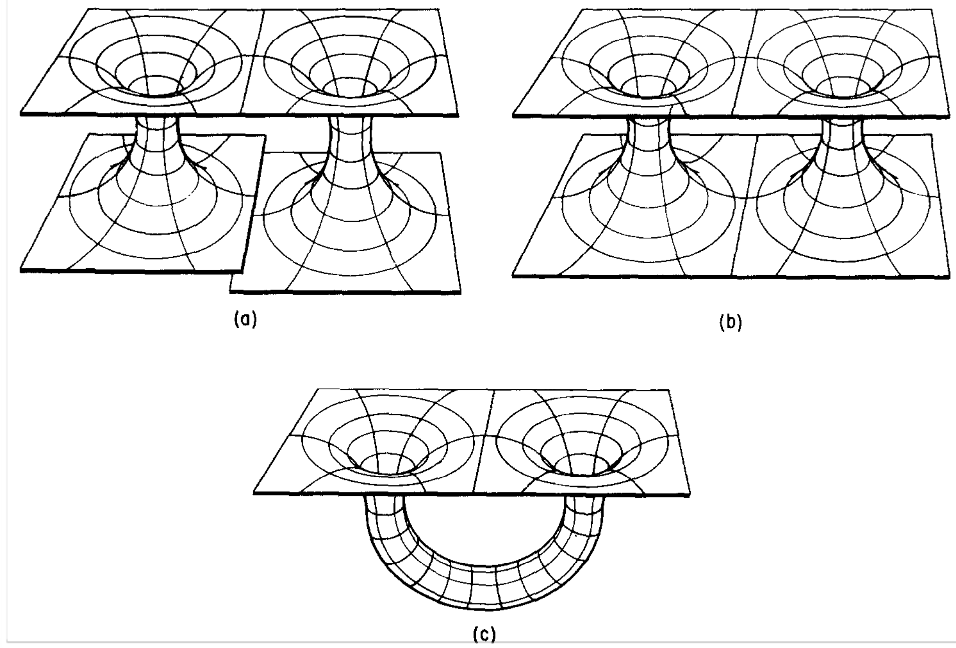
SUSAN G. HAHN

International Business Machines Corporation, New York, New York

AND

RICHARD W. LINDQUIST

Adelphi University, Garden City, New York



In summary, the numerical solution of the Einstein field equations presents no insurmountable difficulties. Much still remains to be done, however, in the investigation both of stable difference schemes (a proof of stability being one of the outstanding unsolved problems) and of coordinate conditions that are well suited to numerical work. The practical impossibility of carrying the numerical solution sufficiently far into the future limits the conclusions which can be drawn about the dynamical behavior of the wormhole system. Nevertheless, one sees evidence for a gravitational collapse of each mouth, analogous to that of the Schwarzschild metric, together with an interaction between the two of them. These two effects can only be properly disentangled through measurements in the asymptotic region; with the limited data at our disposal, such an analysis has not been possible.

The Maxwell equations serve as a guide

Maxwell's equations

$$D_i E^i - 4\pi\rho = 0 \qquad D_i B^i = 0$$

2 constraint equations

$$\partial_t E_i = \epsilon_{ijk} D^j B^k - 4\pi j_i \qquad \partial_t B_i = -\epsilon_{ijk} D^j E^k$$

6 evolution equations

Einstein's equations

$$G_{ab} = 8\pi T_{ab},$$

$$G^{a0} = 8\pi T^{a0}$$

4 second order constraint equations

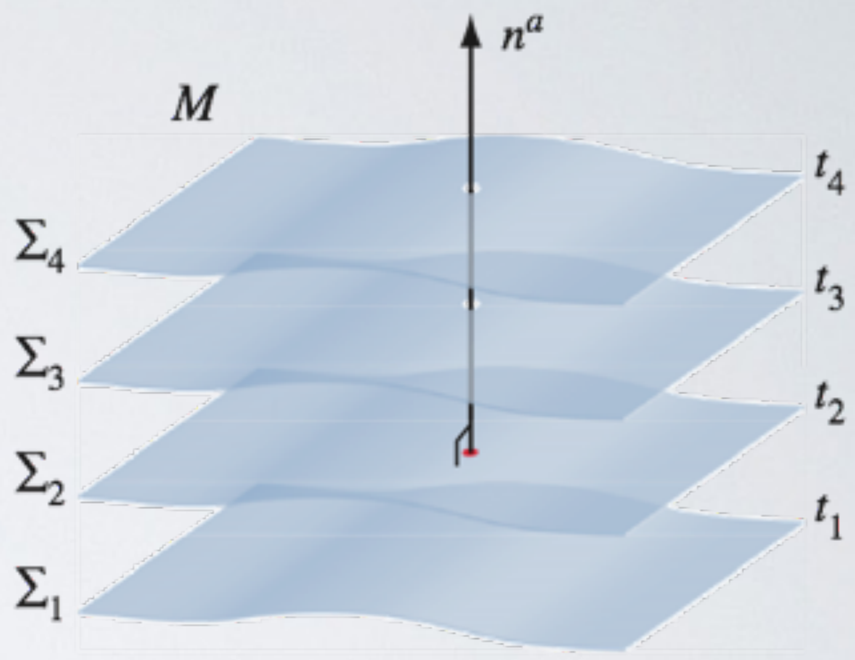
$$G^{ij} = 8\pi T^{ij}$$

6 second order evolution equations

The Maxwell equations can be written in a 3+1 form

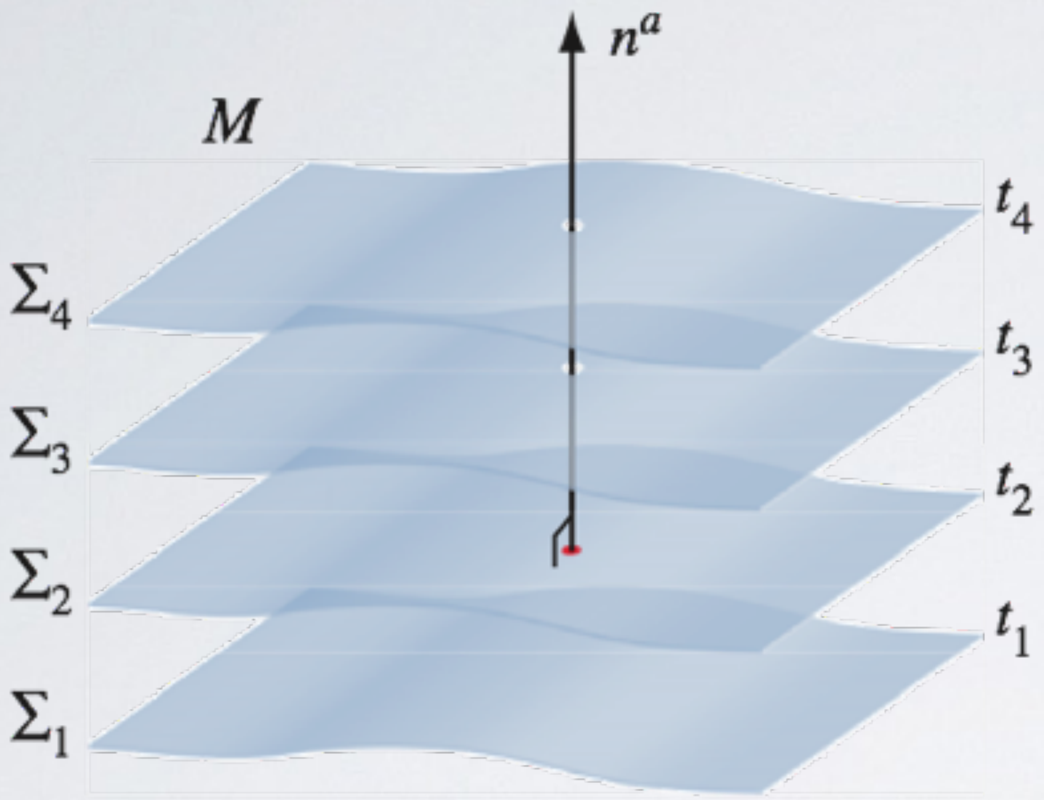
	Maxwell	Einstein
Primary field	A_i	?
Secondary field	E_i	
Gauge variables	Φ	
Matter sources	ρ_e and j_i	
Evolution equations	$\partial_t A_i = -E_i - D_i \Phi$ $\partial_t E_i = -D^j D_j A_i + D_i D^j A_j - 4\pi j_i$	
Constraint equations	$D_i E^i = 4\pi \rho_e$	

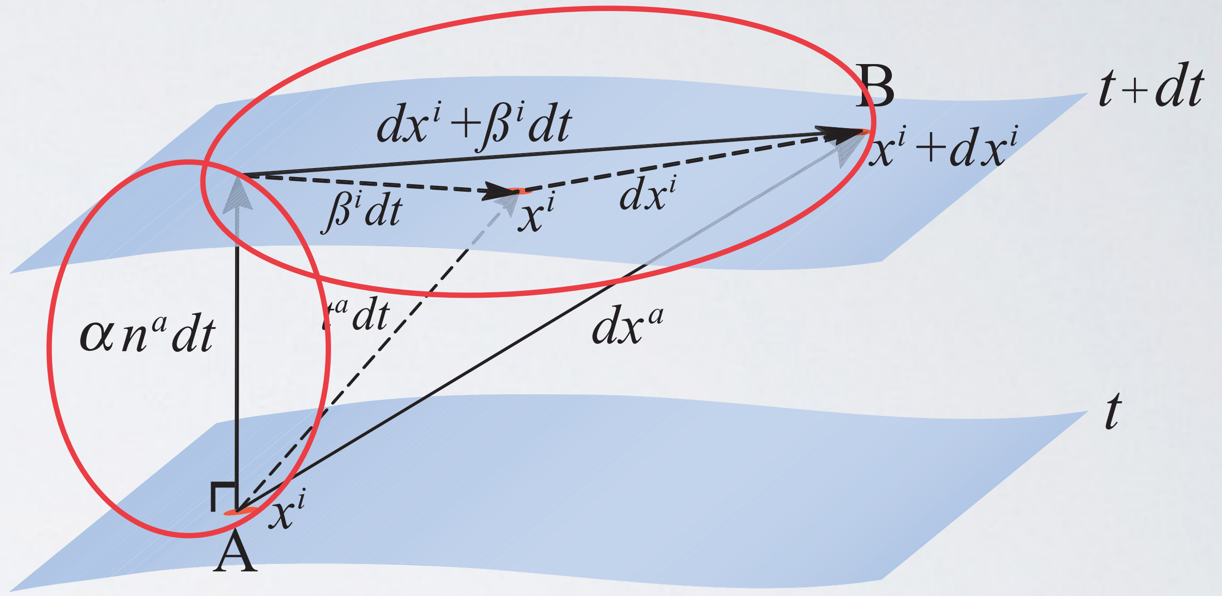
We foliate spacetime into spacelike slices



- Define surfaces of constant t: $\Omega_a = \nabla_a t \qquad n_a = -\alpha\Omega_a$
- 3 spacelike basis vectors: $n_a e^a_{(i)} = 0$
- Lie drag them to get them on next slice: $\mathcal{L}_t e^a_{(i)} = 0$
- Fourth basis vector defined by t: $t^a = e^a_{(0)} = (1, 0, 0, 0)$
- Time vector has normal part and "shift": $t^a = \alpha n^a + \beta^a$
- Define spatial metric tensor: $\gamma_{ij} = g_{ij} \qquad \gamma_{ab} = g_{ab} + n_a n_b \qquad n^a \gamma_{ab} = 0$
- Define spatial projection operator: $\gamma^a_{\ b} = g^a_{\ b} + n^a n_b$
- Define normal projection operator: $-n^a n_b$
- Decompose tensors into timelike and spacelike parts

The lapse and the shift can be defined with great freedom





$$g_{ab} = \begin{pmatrix} -\alpha^2 + \beta_l \beta^l & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

$$t^a = \alpha n^a + \beta^a$$

The vector t^a connects points with the same spatial coordinates on neighboring time slices.

Lapse: α Measures proper time along normal vector

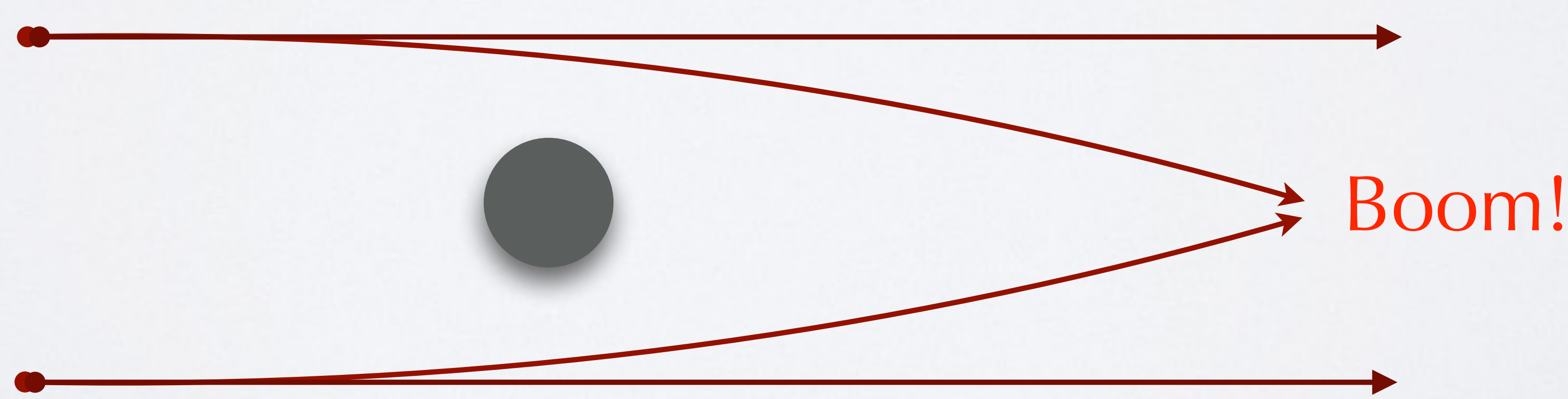
Shift: β^i How coordinates "move" on the slice as time evolves

I am forced to ask:

Why are you making things so complicated?

Geodesic slicing:

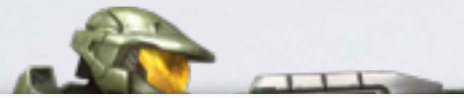
$$\alpha = 1, \qquad \beta^i = 0.$$



We now have Einstein's equations in a 3+1 form

	Maxwell	Einstein: ADM Equations
Primary field	A_i	γ_{ij}
Secondary field	E_i	K_{ij}
Gauge variables	Φ	α and β^i
Matter sources	ρ_e and j_i	ρ, S^i and S_{ij}
Evolution equations	$\partial_t A_i = -E_i - D_i \Phi$ $\partial_t E_i = -D^j D_j A_i + D_i D^j A_j - 4\pi j_i$	$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$ $\partial_t K_{ij} = \alpha(R_{ij} - 2K_{ik}K^k{}_j + K K_{ij})$ $- D_i D_j \alpha - 8\pi\alpha(S_{ij} - \frac{1}{2}\gamma_{ij}(S - \rho))$ $+ \beta^k \partial_k K_{ij} + K_{ik}\partial_j \beta^k + K_{kj}\partial_i \beta^k$
Constraint equations	$D_i E^i = 4\pi \rho_e$	$R + K^2 - K_{ij}K^{ij} = 16\pi\rho$ $D_j(K^{ij} - \gamma^{ij}K) = 8\pi S^i$

Curvature can be intrinsic or extrinsic



Intrinsic curvature:

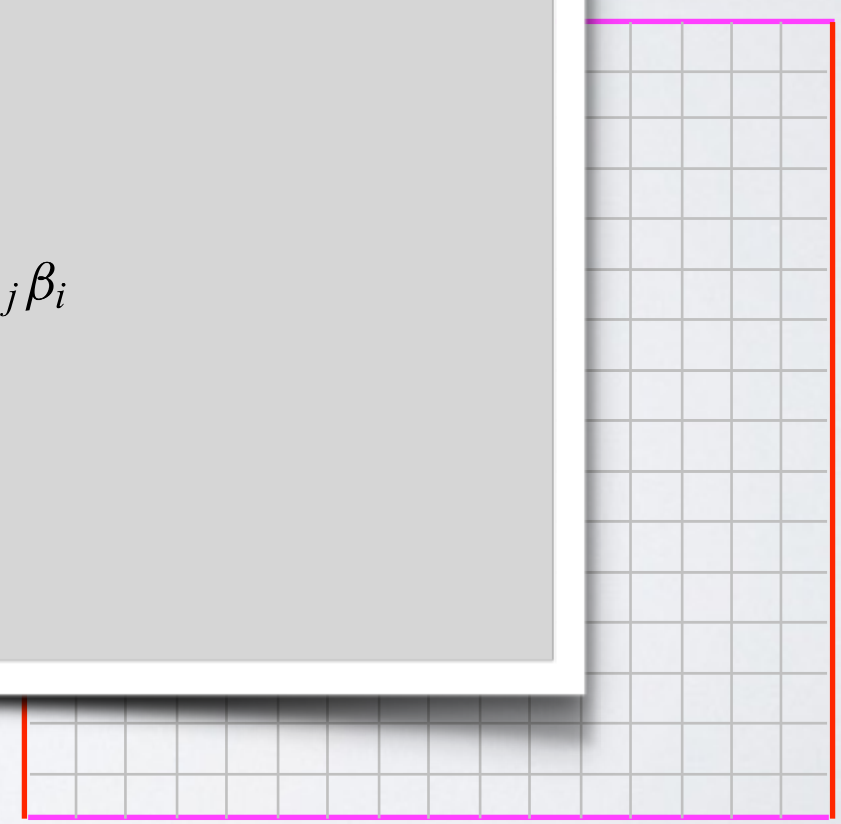
$$\gamma_{ij}$$

Extrinsic curvature:

$$K_{ij}$$

Relation:

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$



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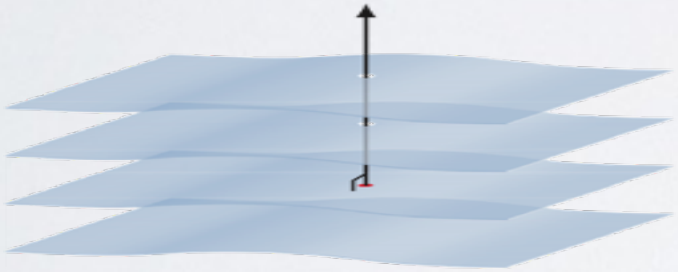
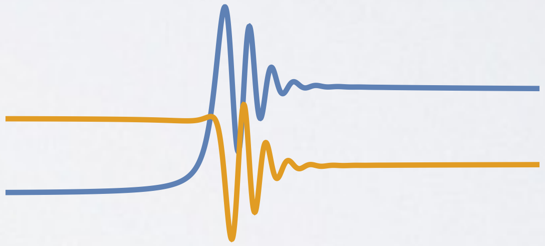


Wave extraction



The tools we use

Initial data

12 initial degrees of freedom γ_{ij} spatial metric K_{ij} extrinsic curvature

- 4 constraint equations $R + K^2 - K_{ij}K^{ij} = 16\pi\rho$ $D_j(K^{ij} - \gamma^{ij}K) = 8\pi S^i$

- 4 coordinate conditions (lapse and shift) α β^i

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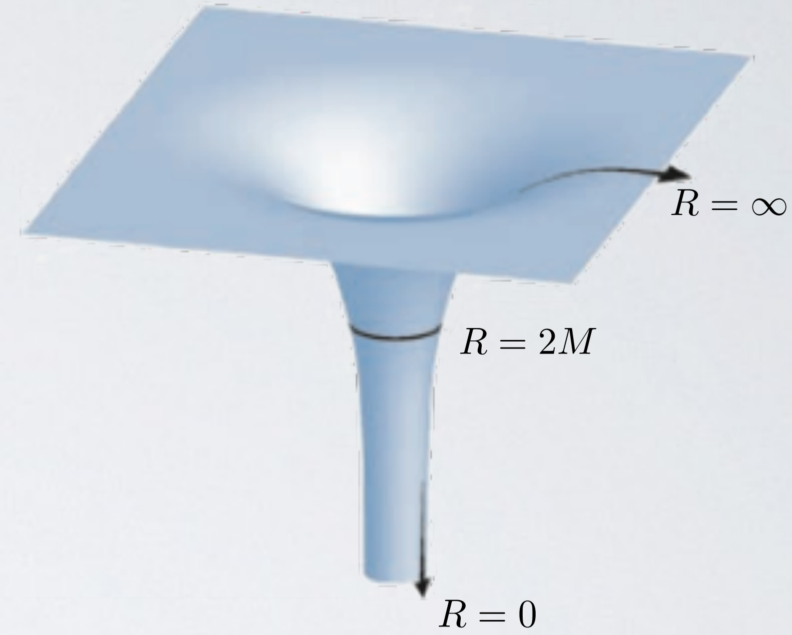
4 remaining degrees of freedom ⟷ 2 dynamical GW polarizations

How do you specify those degrees of freedom?

Schwarzschild in isotropic coordinates is good for initial data and avoiding singularities

Schwarzschild coordinates:

$$dl^2 = \left(1 - \frac{2M}{R}\right)^{-1} dR^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

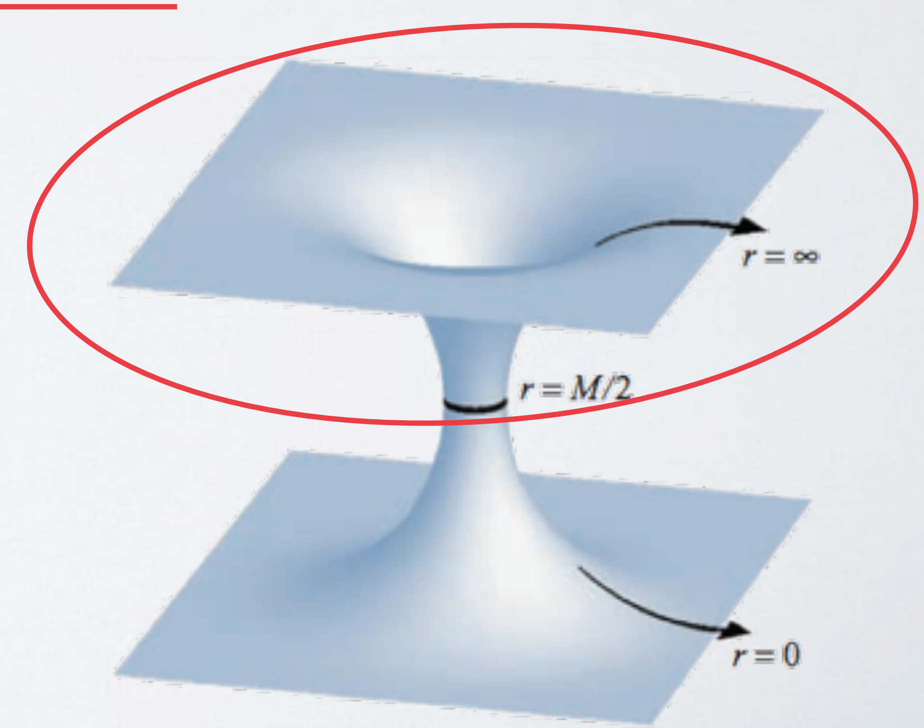


Isotropic coordinates:

$$dl^2 = \underbrace{\left(1 + \frac{M}{2r}\right)^4}_{\text{Conformal factor}} \underbrace{\left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)}_{\text{Flat space}}$$

Conformal factor

Flat space



- r = 0 is a coord. singularity, corresponding to infinity in the "other universe"
- Outside the horizon this is standard Schwarzschild

This generalizes to multiple black holes

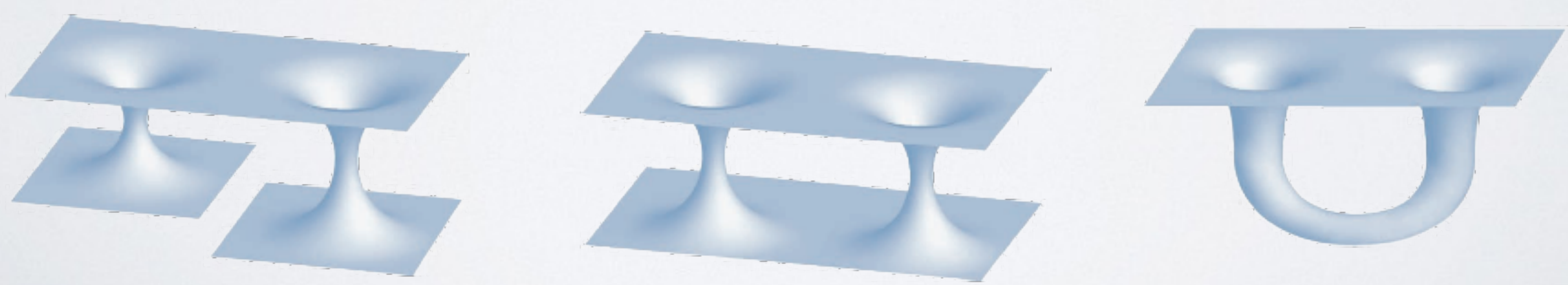
Assume:

- Conformal flatness: $\gamma_{ij} = \psi^4 \eta_{ij}$
- Maximal slicing: $K = 0.$
- Time reversal invariance: $t \to -t.$

Then:

- Momentum constraint $D_j(K^{ij} - \gamma^{ij} K) = 8\pi S^i$ satisfied trivially
- Hamiltonian constraint: $\bar{D}^2 \psi = 0$

This equation is linear! $\psi = 1 + \sum_\alpha \frac{\mathcal{M}_\alpha}{2r_\alpha}$ $\psi = 1 + \frac{\mathcal{M}_1}{2r_1} + \frac{\mathcal{M}_2}{2r_2}$



Misner, 1960

Bowen - York initial data for binary black holes is (relatively) simple

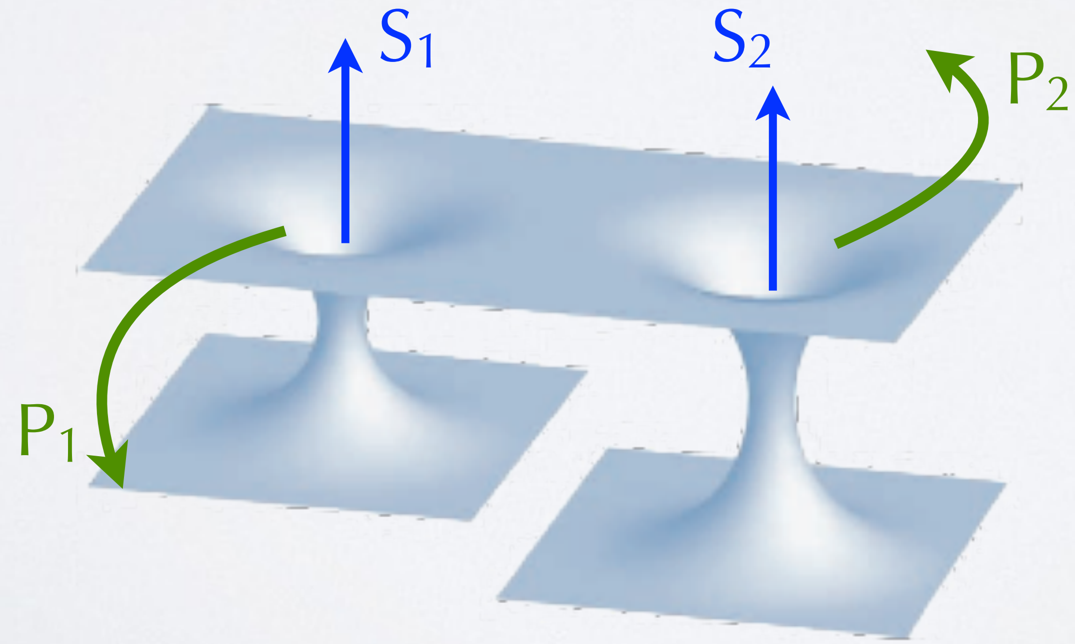
Assume:

- Conformal flatness: $\gamma_{ij} = \psi^4 \eta_{ij}$
- Maximal slicing: $K = 0.$

Then:

- Momentum constraint with analytic solutions: $\bar{D}_j \bar{A}^{ij} = 0$
- Use linearity for binary black holes: $\bar{A}^{ij} = \bar{A}^{ij}_{\mathbf{C}_1\mathbf{P}_1} + \bar{A}^{ij}_{\mathbf{C}_1\mathbf{S}_1} + \bar{A}^{ij}_{\mathbf{C}_2\mathbf{P}_2} + \bar{A}^{ij}_{\mathbf{C}_2\mathbf{S}_2}$

One equation to solve numerically: $\bar{D}^2\psi = -\frac{1}{8}\psi^{-7}\bar{A}_{ij}\bar{A}^{ij}$



The initial data is mathematically correct, but astrophysically wrong

Does this assumption make sense? $\gamma_{ij} = \psi^4 \eta_{ij}$

No!

But we have bigger problems …

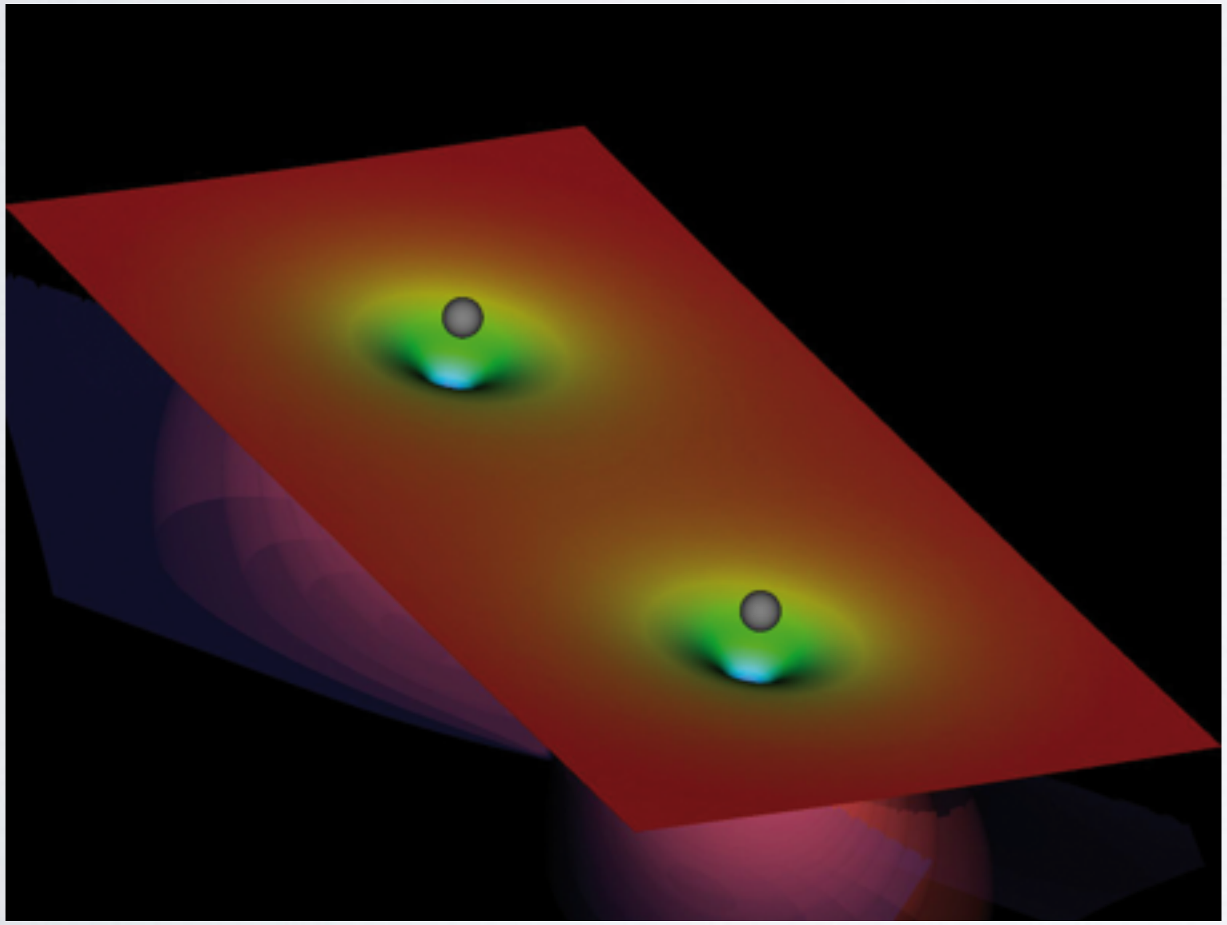


Fig: `inside.hlrs.de/`

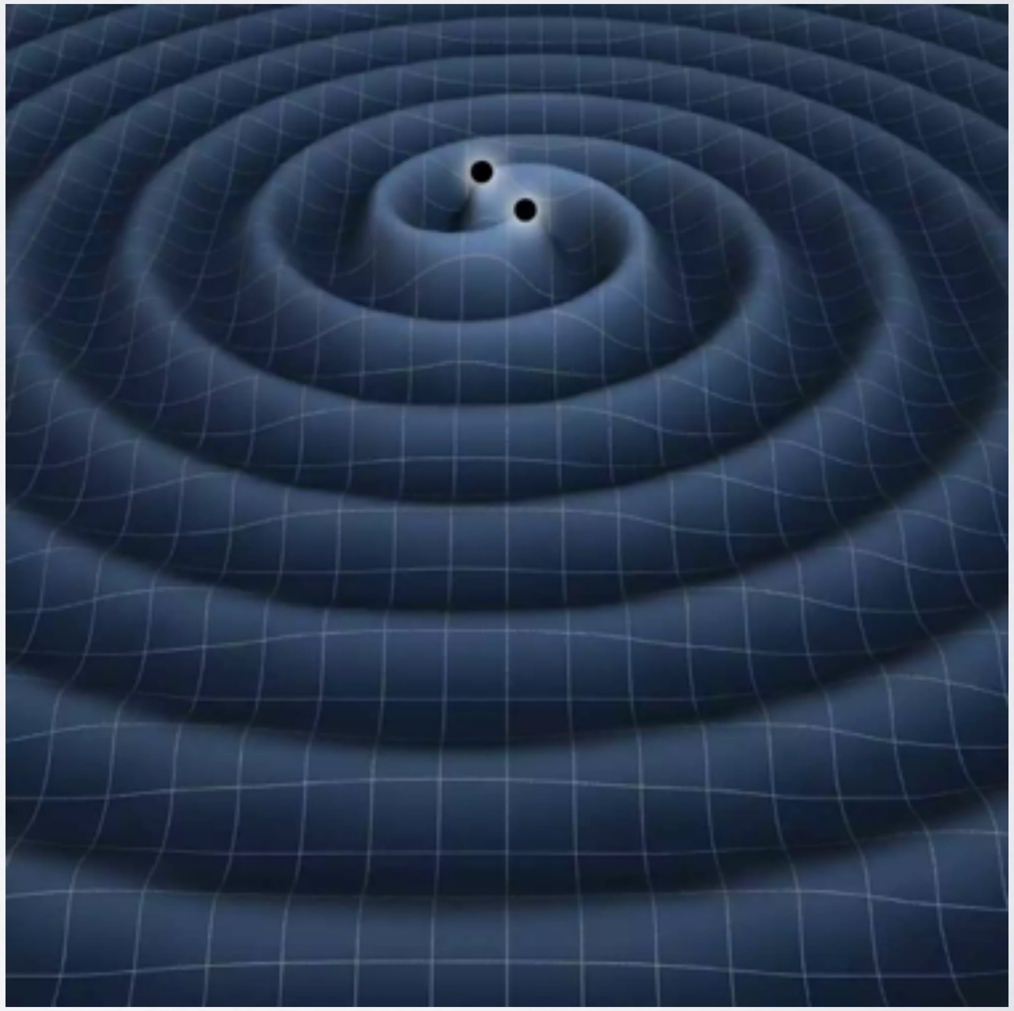


Fig: `spaceplace.nasa.gov/`

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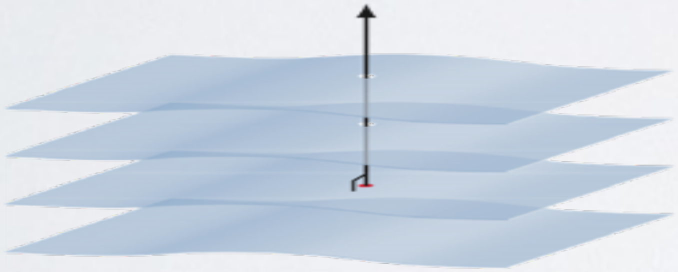
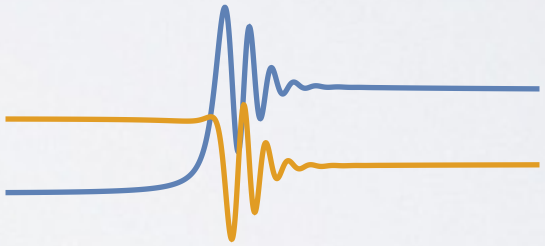
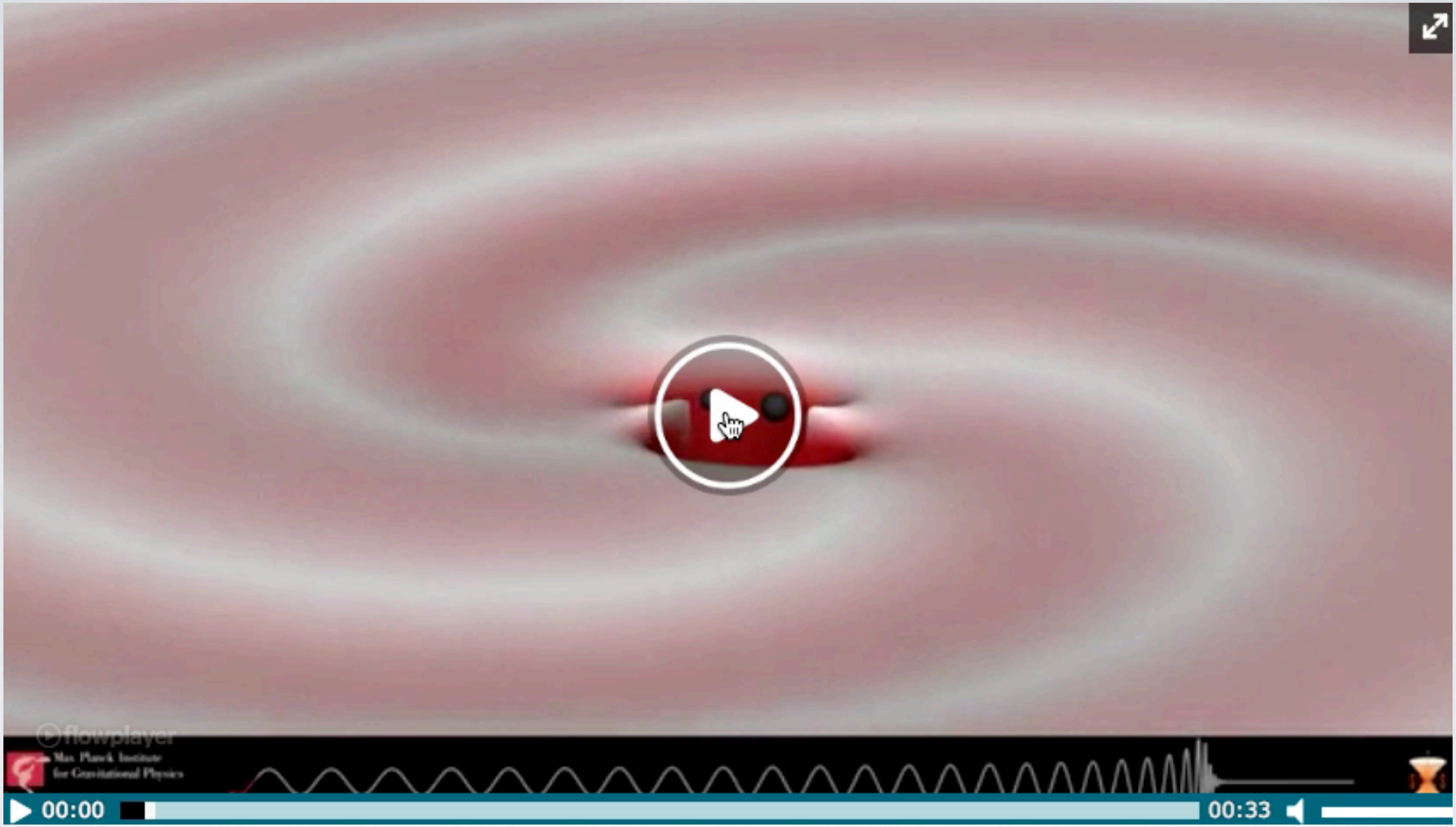


Wave extraction



The tools we use

Here is a simulation of the first detection, GW150914



- Simulation: Simulating eXtreme Spacetime project
- Movie: Roland Haas, Max Planck Institute
- Real time: ~ 1 s

We're gonna need a stronger set of equations

Hyperbolic equation in first order form:

$$\partial_t \mathbf{u} + \mathbf{A}^i \cdot \partial_i \mathbf{u} = \mathbf{S}$$

Well-posedness:

$$\|\mathbf{u}(t, x^i)\| \leq k e^{\alpha t} \|\mathbf{u}(0, x^i)\|$$

Strongly hyperbolic: The matrix $\mathbf{A}^i n_i$ has real eigenvalues and a complete set of eigenvectors for all unit vectors n^i

Weakly hyperbolic: The matrix $\mathbf{A}^i n_i$ has real eigenvalues but an incomplete set of eigenvectors

Key point 1: Strongly hyperbolic systems are well-posed

Key point 2: The ADM equations are only weakly hyperbolic

We can identify the troublesome terms

Maxwell:

$$-\partial_t^2 A_i + D^j D_j A_i - D_i D^j A_j = D_i \partial_t \Phi - 4\pi j_i$$

d'Alembertian

Strongly hyperbolic without this

Einstein:

$$R_{ij} = \frac{1}{2}\gamma^{kl}\left(\partial_l \partial_i \gamma_{kj} + \partial_j \partial_k \gamma_{il} - \partial_j \partial_i \gamma_{kl} - \partial_l \partial_k \gamma_{ij}\right)$$

$$+ \gamma^{kl}\left(\Gamma^m_{il}\Gamma_{mkj} - \Gamma^m_{ij}\Gamma_{mkl}\right)$$

Laplacian

"Generalized Coulomb gauge" provides insight for gravity gauge choices

Standard E&M:

Maxwell's equations: $$-\partial_t^2 A_i + D^j D_j A_i - D_i D^j A_j = D_i \partial_t \Phi - 4\pi j_i$$

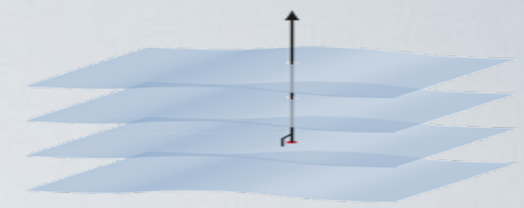
Coulomb gauge: $$D^i A_i = 0 \longrightarrow -\partial_t^2 A_i + D^j D_j A_i = D_i \partial_t \Phi - 4\pi j_i$$

Alternatively:

Define gauge source function: $$H(t, x^i) \equiv D^i A_i$$

Generalized Coulomb gauge: $$-\partial_t^2 A_i + D_j D^j A_i - H(t, x^i) = D_i \partial_t \Phi - 4\pi j_i$$

Generalized harmonic coordinates



Einstein equations:

$$^{(4)}R_{ab} = 8\pi(T_{ab} - (1/2)g_{ab}T)$$

Define gauge source function:

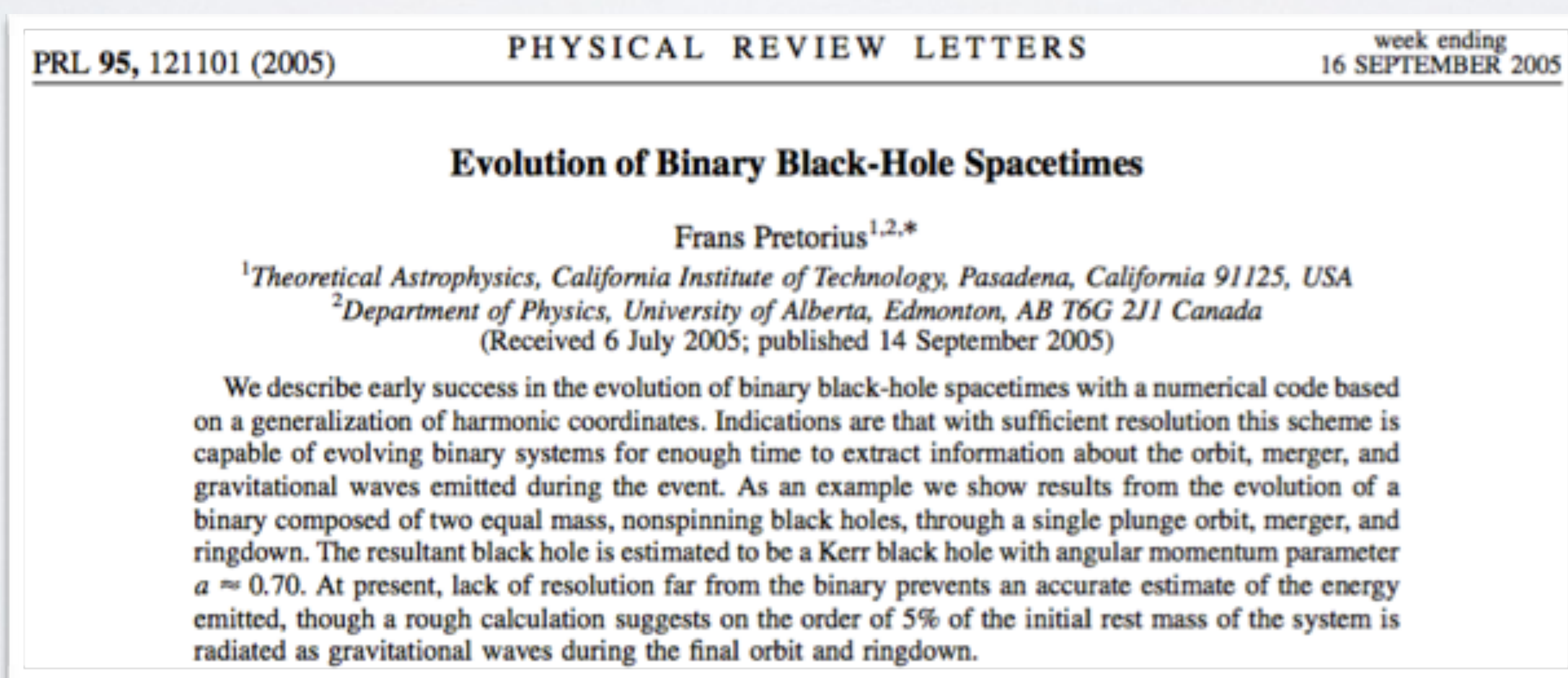
$$H^a(t, x^i) \equiv g^{bc\,(4)}\Gamma^a_{bc}$$

Field equations in generalized harmonic coordinates:

$$g^{cd}\partial_d\partial_c g_{ab} + 2\partial_{(a}g^{cd}\partial_c g_{b)d} + 2H_{(a,b)} - 2H_d\,{}^{(4)}\Gamma^d_{ab} + 2\,{}^{(4)}\Gamma^c_{bd}\,{}^{(4)}\Gamma^d_{ac} = -8\pi\,(2T_{ab} - g_{ab}T)$$

New constraint equation:

$$\mathcal{C}^a \equiv H^a - g^{bc\,(4)}\Gamma^a_{bc} = 0$$



PRL 95, 121101 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Evolution of Binary Black-Hole Spacetimes

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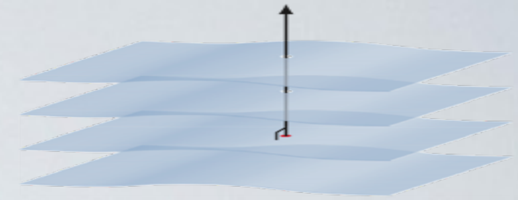
(Received 6 July 2005; published 14 September 2005)

We describe early success in the evolution of binary black-hole spacetimes with a numerical code based on a generalization of harmonic coordinates. Indications are that with sufficient resolution this scheme is capable of evolving binary systems for enough time to extract information about the orbit, merger, and gravitational waves emitted during the event. As an example we show results from the evolution of a binary composed of two equal mass, nonspinning black holes, through a single plunge orbit, merger, and ringdown. The resultant black hole is estimated to be a Kerr black hole with angular momentum parameter $a \approx 0.70$. At present, lack of resolution far from the binary prevents an accurate estimate of the energy emitted, though a rough calculation suggests on the order of 5% of the initial rest mass of the system is radiated as gravitational waves during the final orbit and ringdown.

The BSSN formalism is due to Shibata, Nakamura, Baumgarte and Shapiro



Primary and secondary fields

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

$$K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

Evolution equations

$$\partial_t\phi = -\frac{1}{6}\alpha K + \beta^i\partial_i\phi + \frac{1}{6}\partial_i\beta^i$$

$$\partial_t\bar{\gamma}_{ij} = -2\alpha\tilde{A}_{ij} + \beta^k\partial_k\bar{\gamma}_{ij} + \bar{\gamma}_{ik}\partial_j\beta^k + \bar{\gamma}_{kj}\partial_i\beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k\beta^k$$

$$\begin{aligned}\partial_t\tilde{A}_{ij} = {} & e^{-4\phi}\left(-(D_iD_j\alpha)^{TF} + \alpha(R_{ij}^{TF} - 8\pi S_{ij}^{TF})\right) + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l{}_j) \\ & + \beta^k\partial_k\tilde{A}_{ij} + \tilde{A}_{ik}\partial_j\beta^k + \tilde{A}_{kj}\partial_i\beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k\beta^k.\end{aligned}$$

$$\partial_t K = -\gamma^{ij}D_jD_i\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i\partial_i K$$

Constraint equations

$$0 = \mathcal{M}^i = \bar{D}_j(e^{6\phi}\tilde{A}^{ji}) - \frac{2}{3}e^{6\phi}\bar{D}^i K - 8\pi e^{6\phi}S^i$$

$$0 = \mathcal{H} = \bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\tilde{A}_{ij}\tilde{A}^{ij} - \frac{e^{5\phi}}{12}K^2 + 2\pi e^{5\phi}\rho$$

The computational details get complicated fast

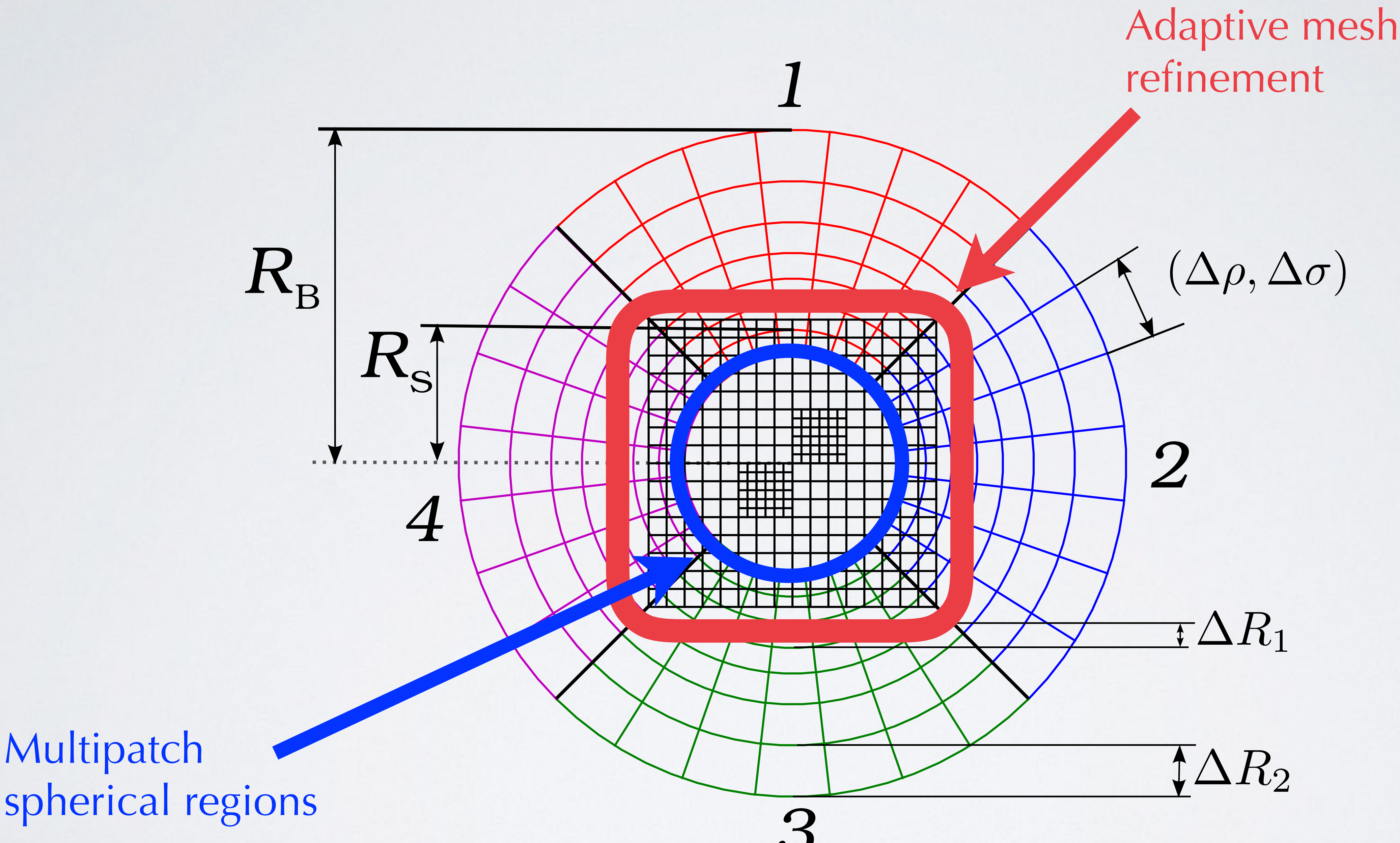
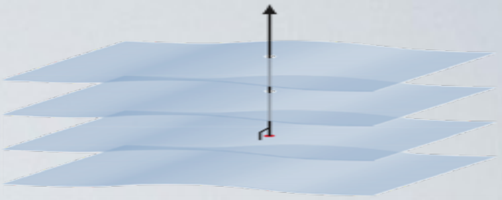


Fig: Reisswig et al.

Event horizons cannot be used in simulations



Black hole:

Region of spacetime that null geodesics cannot escape

Event horizon:

2+1 dim hypersurface defined by future directed null geodesics

Apparent horizon:

2 dim surface defined on slice by future directed null geodesics

Must be inside the event horizon

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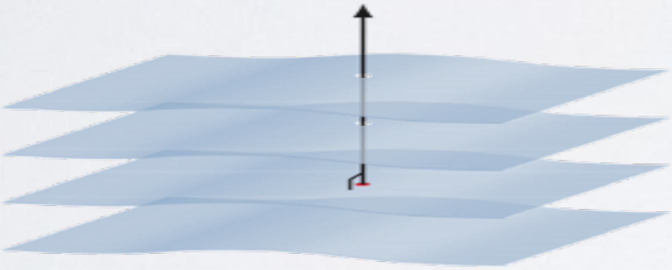
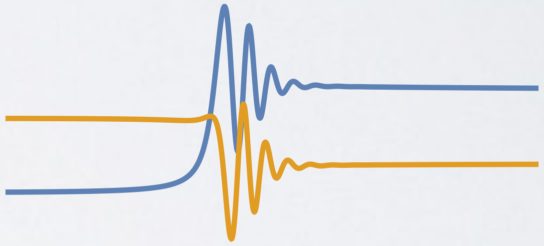


Wave extraction



The tools we use

Wave extraction requires a known, perturbed background

Wave zone: $g_{ab} = \eta_{ab} + h^{\mathrm{rad}}_{ab}$

Strong gravity, no approx.

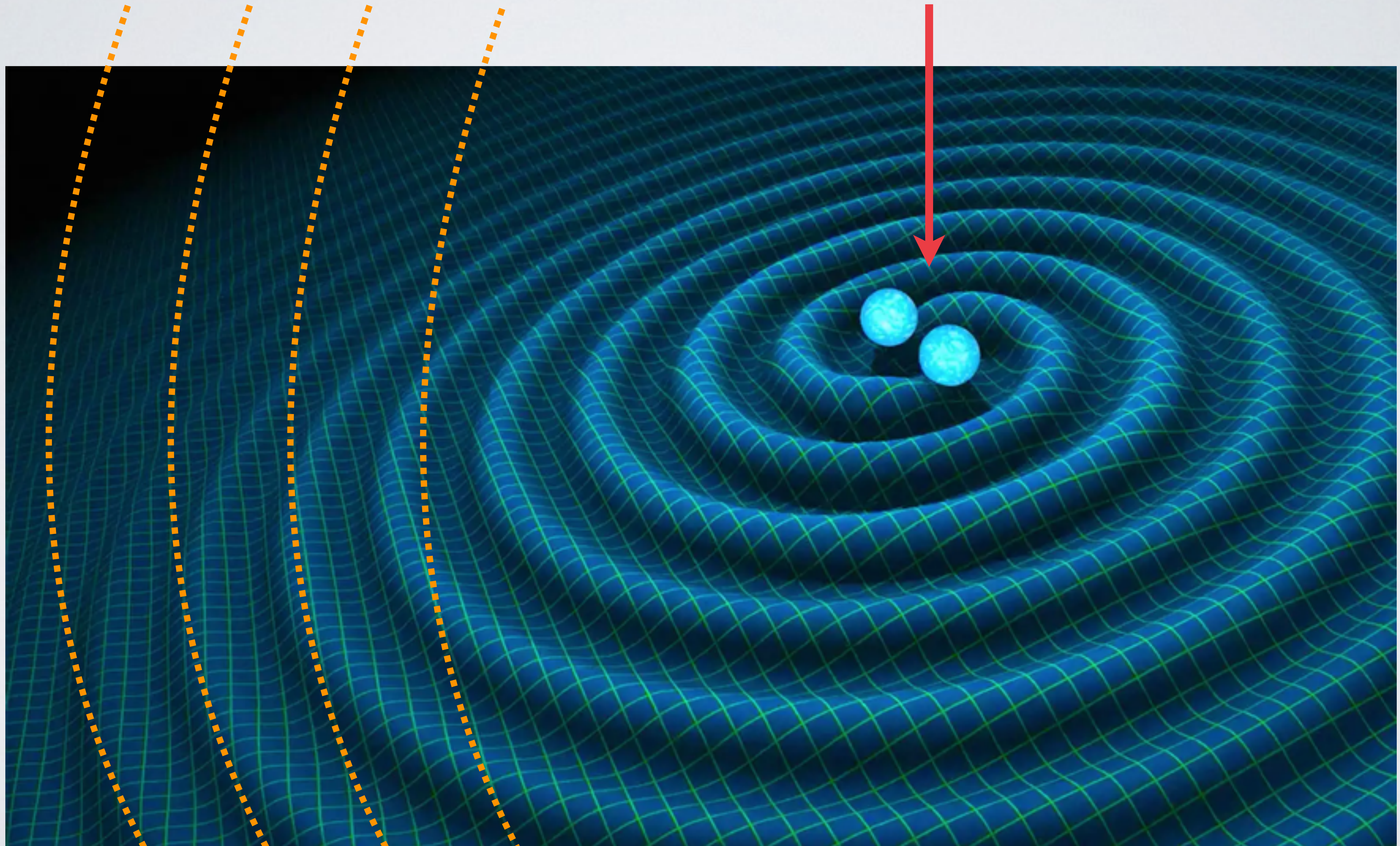
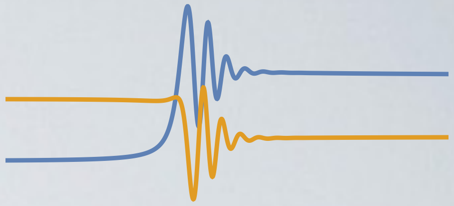
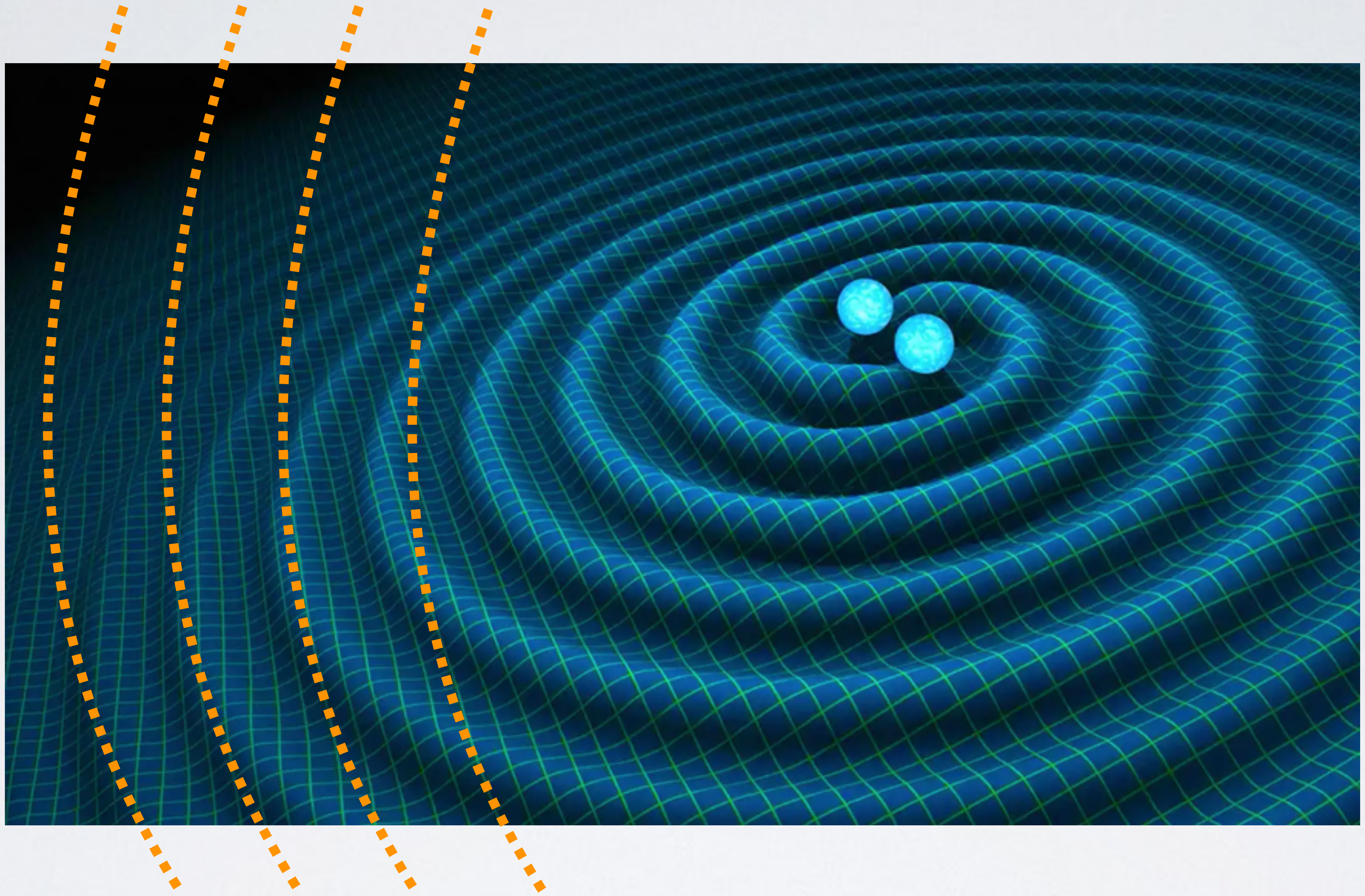


Fig: `jpl.nasa.gov/`

After wave extraction, we build a correspondence





$$h_{ab}^{\mathrm{rad}} \longleftrightarrow \begin{array}{c} S_1, S_2 \\ J \\ M_1/M_2 \end{array}$$

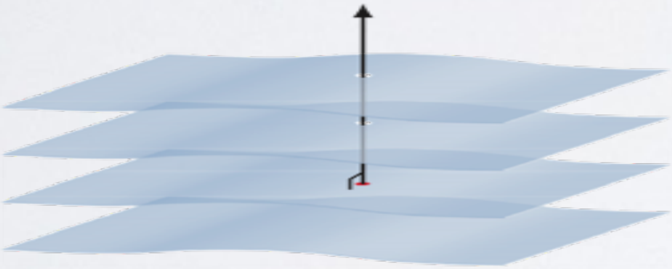
Do it 249,999 more times!

Outline

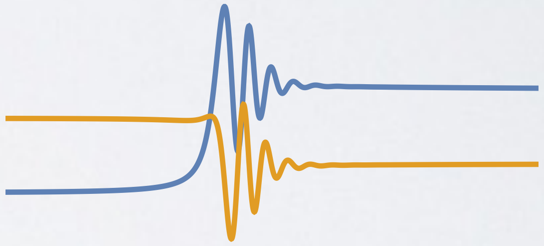
History and formalism

Initial data

Evolution of field equations



Wave extraction



The tools we use

The Einstein Toolkit is free, open source, and actively developed

- 113 Members
- 69 Groups

einstein toolkit

Consortium Members

We are building a consortium of users and developers for the Einstein Toolkit. Users of the Einstein Toolkit are encouraged to register on this page.

Current users include:

- **Albert Einstein Institute**
  - Ian Hinder
- **Aristotle University of Thessaloniki**
  - Nick Stergioulas
- **Aveiro University**
  - Juan Carlos Degollado
  - Carlos Herdeiro
- **Belmont University**
  - Scott Hawley
- **California Institute of Technology**
  - Christian D. Ott
  - Peter Kalmus
  - Philipp Mösta
  - David Radice
  - Christian Reisswig
  - Béla Szilágyi
- **California State University East Bay**
  - Ernest Schleicher
- **Università di Catania**
  - Eloisa Bentivegna
- **Chinese Academy of Sciences**
  - Mew Bing Wan
- **Christian-Albrechts-Universität zu Kiel**
  - Stefan Rühe
- **Eastern New Mexico University**
  - William L. Andersen
- **Masarykova Univerzita (Masaryk University)**
  - Radek Sevcik
- **McNeese State University**
  - Megan Miller
- **Monash University**
  - Hayley Macpherson
- **NASA Goddard Space Flight Center**
  - John Baker
  - Bernard Kelly
  - Jennifer Seiler
- **National Center for Supercomputing Applications**
  - Gabrielle Allen
  - Roland Haas
  - Edward Seidel
- **Nicolaus Copernicus Astronomical Center (NCAC)**
  - Antonios Manousakis
  - Bhupendra Prakash Mishra
  - Varadarajan Parthasarathy
- **North Carolina State University**
  - Cody Simmons
  - David Brown
- **Northwestern University**
  - Carl Rodriguez
- **Osaka University**
  - Luca Baiotti
  - Antonio Figura
  - Michele Grasso
- **Universitat de les Illes Balears**
  - Sascha Husa
- **Universidad de Los Andes**
  - Willians Barreto
- **Universidad Michoacana**
  - Francisco Guzmán
- **Universidad Nacional de Ingeniería (National University of Engineering)**
  - Renato Tovar Landeo
- **Universidad Nacional Autónoma de México**
  - Jose Manuel Torres
- **Universidade Federal do Espírito Santo**
  - Carlos Lobo
- **Universität Bremen**
  - Oleg Korobkin
- **University of California**
  - David Rideout
- **University of Cambridge**
  - Pau Figueras
  - Helvi Witek
- **University College Dublin**
  - Barry Wardell
- **Università degli Studi di Firenze (University**

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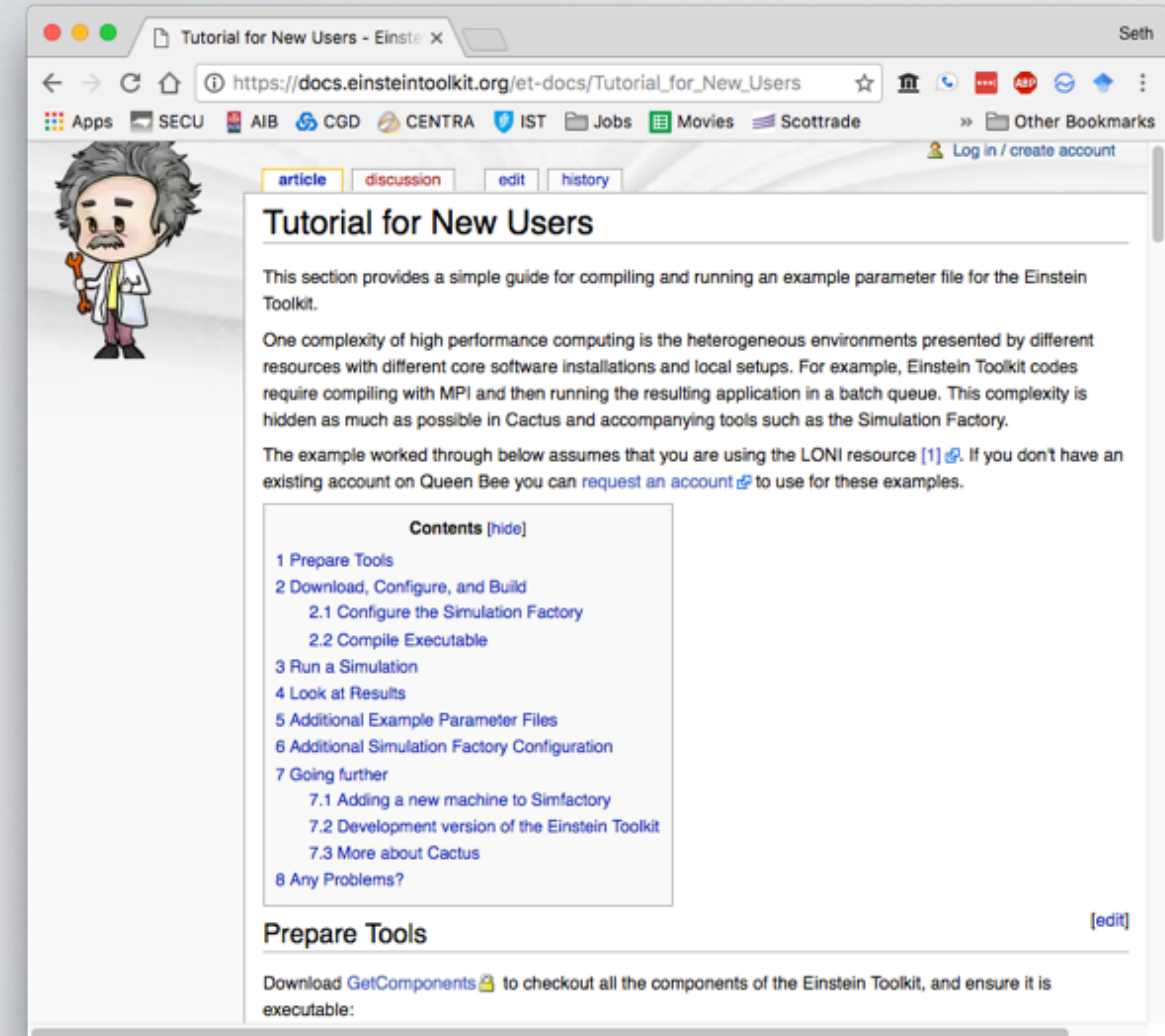
There are tutorials for new users



- You need a cluster
- Binary black holes
- Relativistic magnetohydrodynamics
- ~5000 cpu hrs / run

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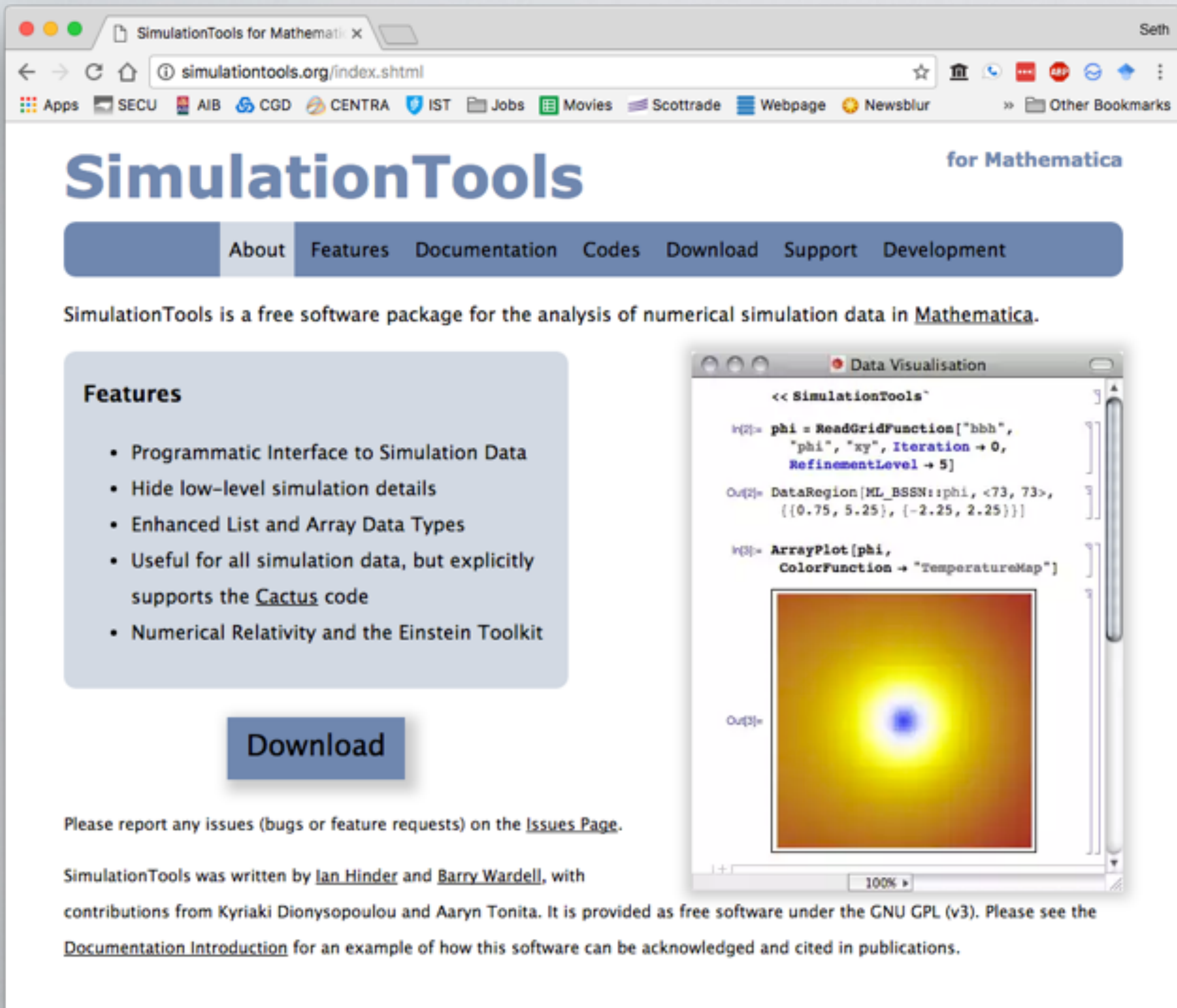
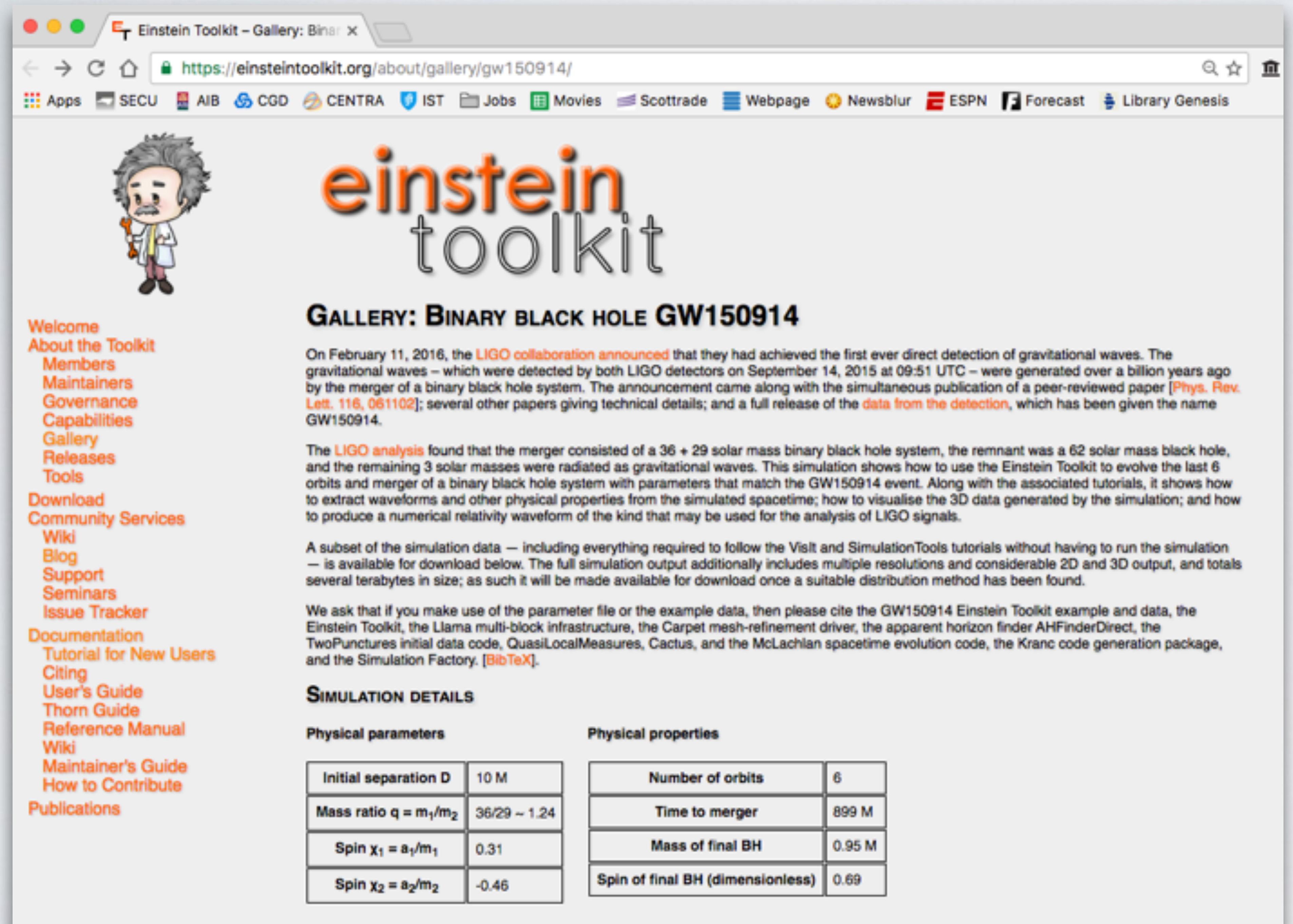
SimulationTools is a free and open source package for analyzing simulation data in Mathematica



- The simulation is only half the effort
  - ➡ Post processing!
- Ian Hinder & Barry Wardell

simulationtools.org

You can simulate the GW150914 event yourself!


Einstein Toolkit – Gallery: Binar

https://einsteintoolkit.org/about/gallery/gw150914/

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How to Contribute
Publications

Gallery: Binary black hole GW150914

On February 11, 2016, the LIGO collaboration announced that they had achieved the first ever direct detection of gravitational waves. The gravitational waves – which were detected by both LIGO detectors on September 14, 2015 at 09:51 UTC – were generated over a billion years ago by the merger of a binary black hole system. The announcement came along with the simultaneous publication of a peer-reviewed paper [Phys. Rev. Lett. 116, 061102]; several other papers giving technical details; and a full release of the data from the detection, which has been given the name GW150914.

The LIGO analysis found that the merger consisted of a 36 + 29 solar mass binary black hole system, the remnant was a 62 solar mass black hole, and the remaining 3 solar masses were radiated as gravitational waves. This simulation shows how to use the Einstein Toolkit to evolve the last 6 orbits and merger of a binary black hole system with parameters that match the GW150914 event. Along with the associated tutorials, it shows how to extract waveforms and other physical properties from the simulated spacetime; how to visualise the 3D data generated by the simulation; and how to produce a numerical relativity waveform of the kind that may be used for the analysis of LIGO signals.

A subset of the simulation data — including everything required to follow the VisIt and SimulationTools tutorials without having to run the simulation — is available for download below. The full simulation output additionally includes multiple resolutions and considerable 2D and 3D output, and totals several terabytes in size; as such it will be made available for download once a suitable distribution method has been found.

We ask that if you make use of the parameter file or the example data, then please cite the GW150914 Einstein Toolkit example and data, the Einstein Toolkit, the Llama multi-block infrastructure, the Carpet mesh-refinement driver, the apparent horizon finder AHFinderDirect, the TwoPunctures initial data code, QuasiLocalMeasures, Cactus, and the McLachlan spacetime evolution code, the Kranc code generation package, and the Simulation Factory. [BibTeX].

Simulation details

Physical parameters

Initial separation D	10 M
Mass ratio $q = m_1/m_2$	36/29 ~ 1.24
Spin $\chi_1 = a_1/m_1$	0.31
Spin $\chi_2 = a_2/m_2$	-0.46

Physical properties

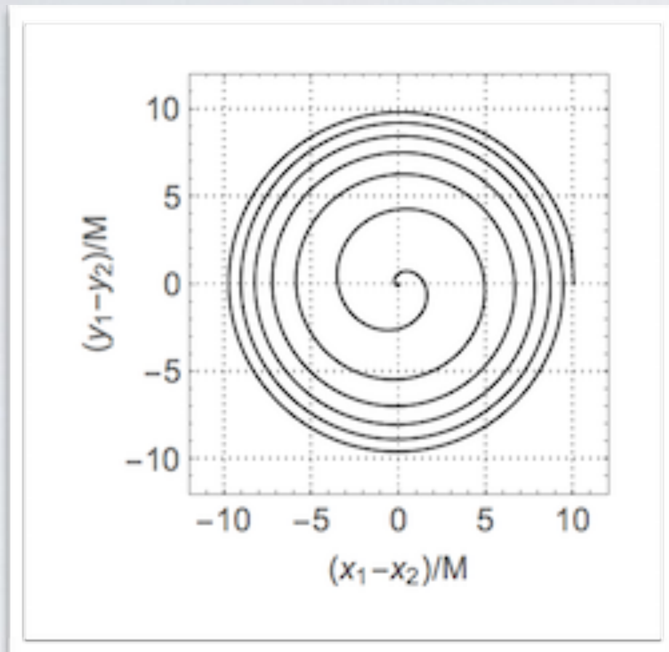
Number of orbits	6
Time to merger	899 M
Mass of final BH	0.95 M
Spin of final BH (dimensionless)	0.69

- Barry Wardell & Ian Hinder
- Parameter files for simulation
- 16,108 cpu hrs
- Post processing scripts

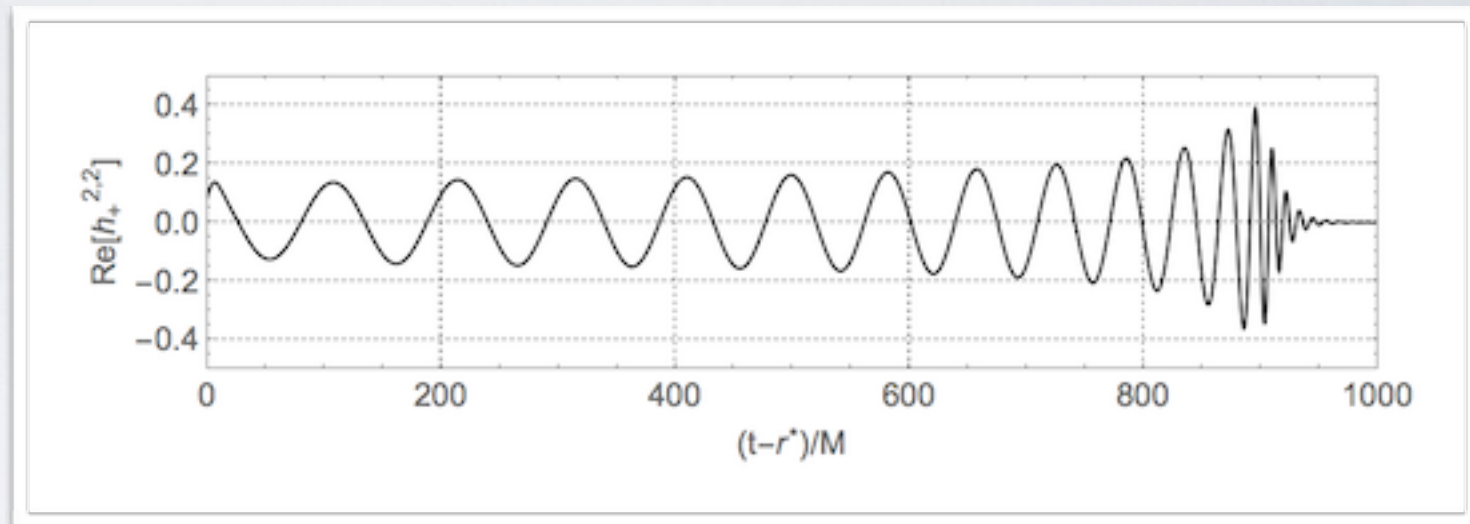
einsteintoolkit.org/about/gallery/gw150914/

You can simulate the GW150914 event yourself!

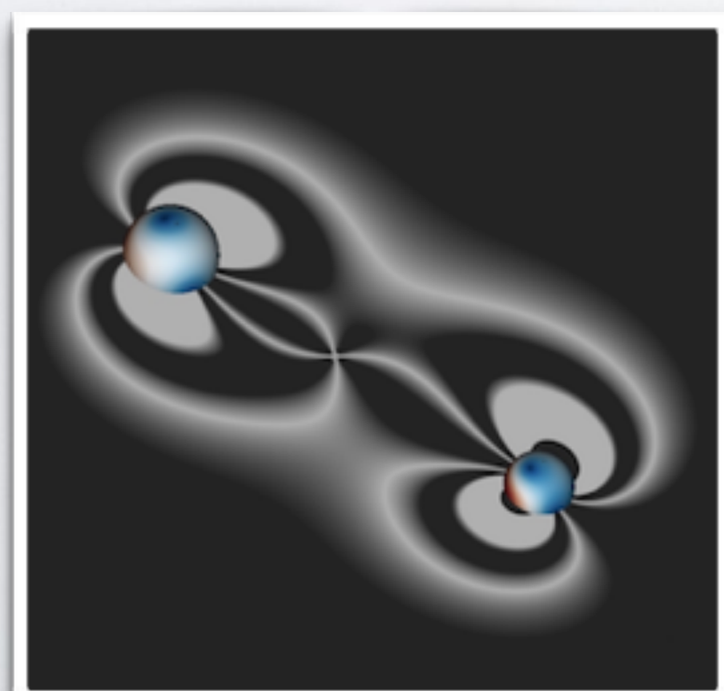
Trajectories



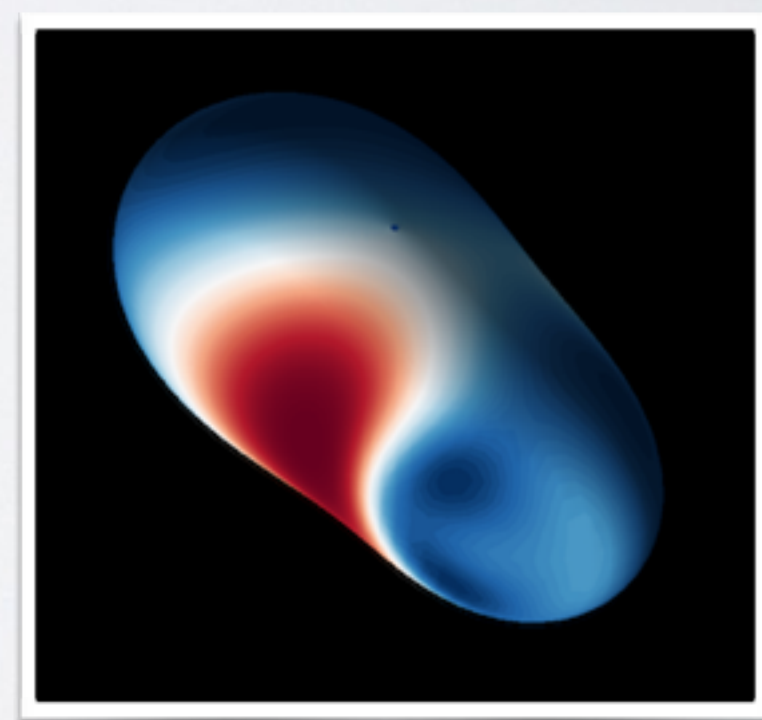
Waveform



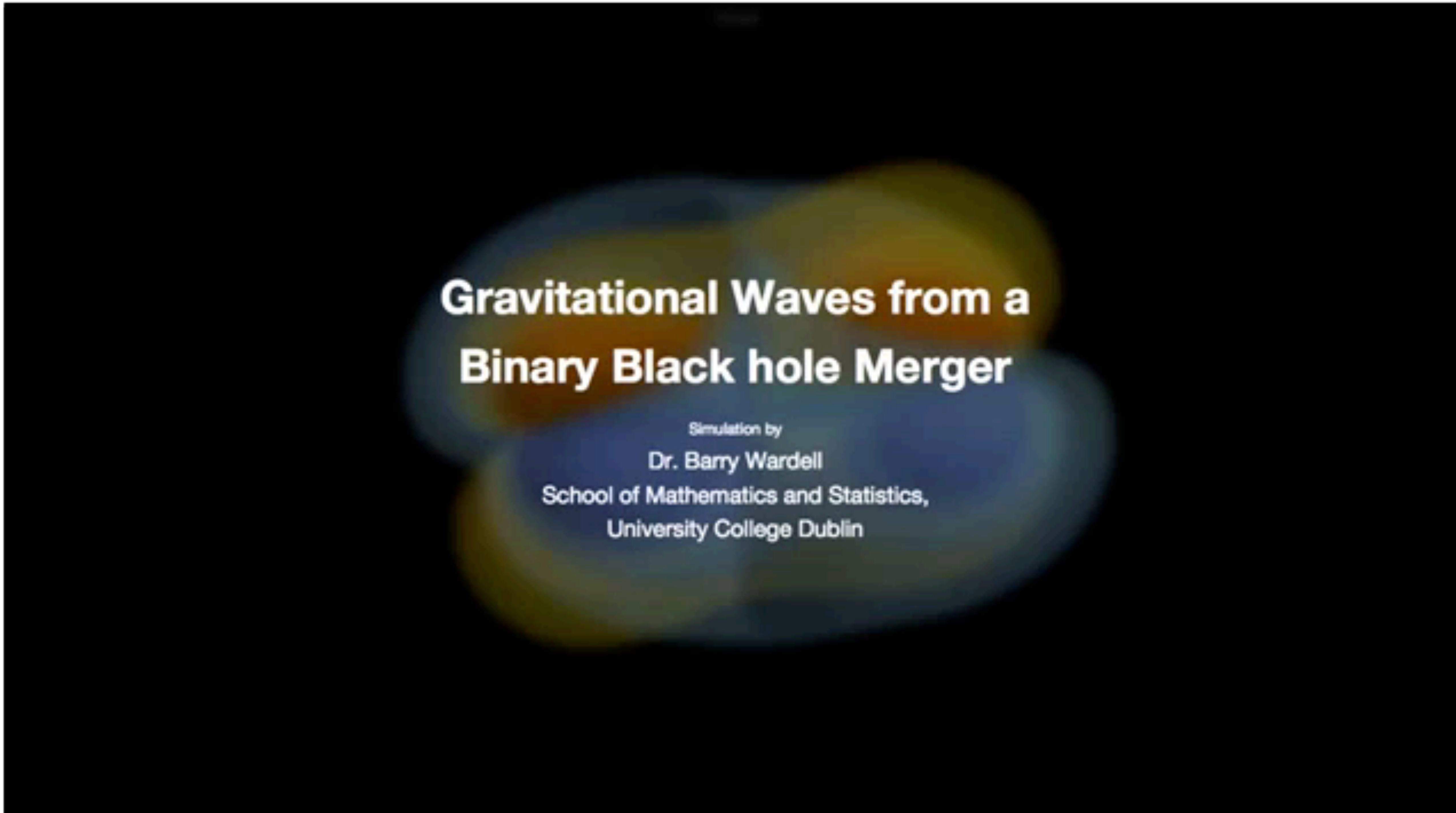
Horizons



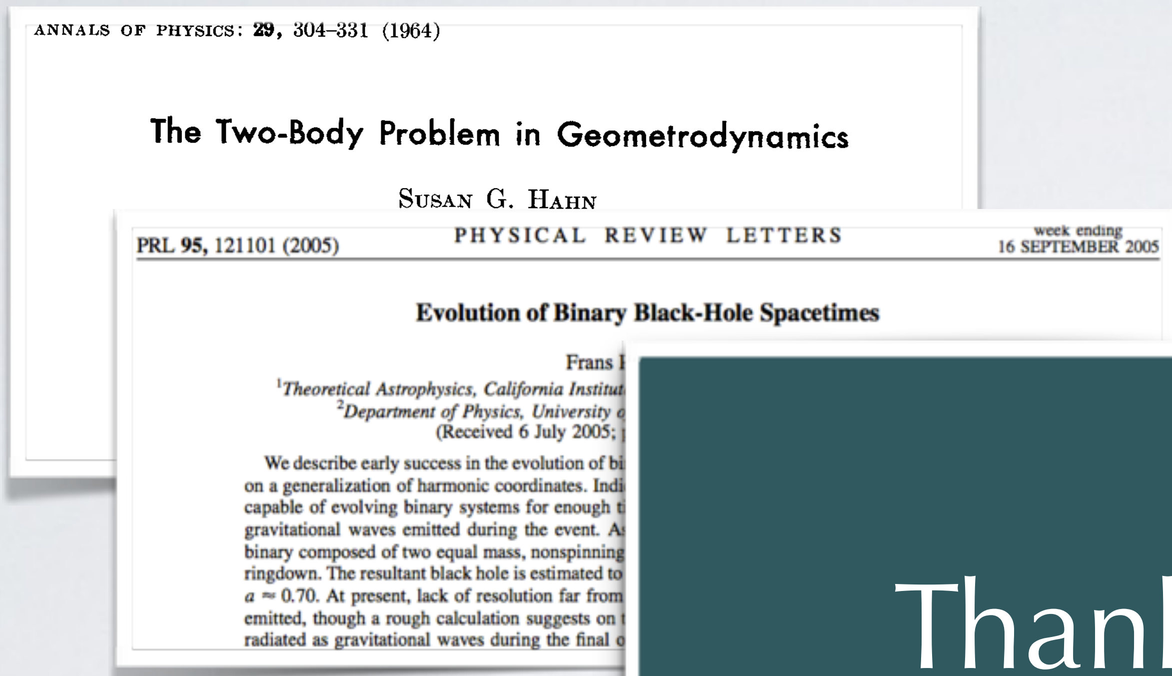
Common horizon



You can simulate the GW150914 event yourself!



These were some of the main points


ANNALS OF PHYSICS: 29, 304–331 (1964)

The Two-Body Problem in Geometrodynamics

SUSAN G. HAHN

PRL 95, 121101 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Evolution of Binary Black-Hole Spacetimes

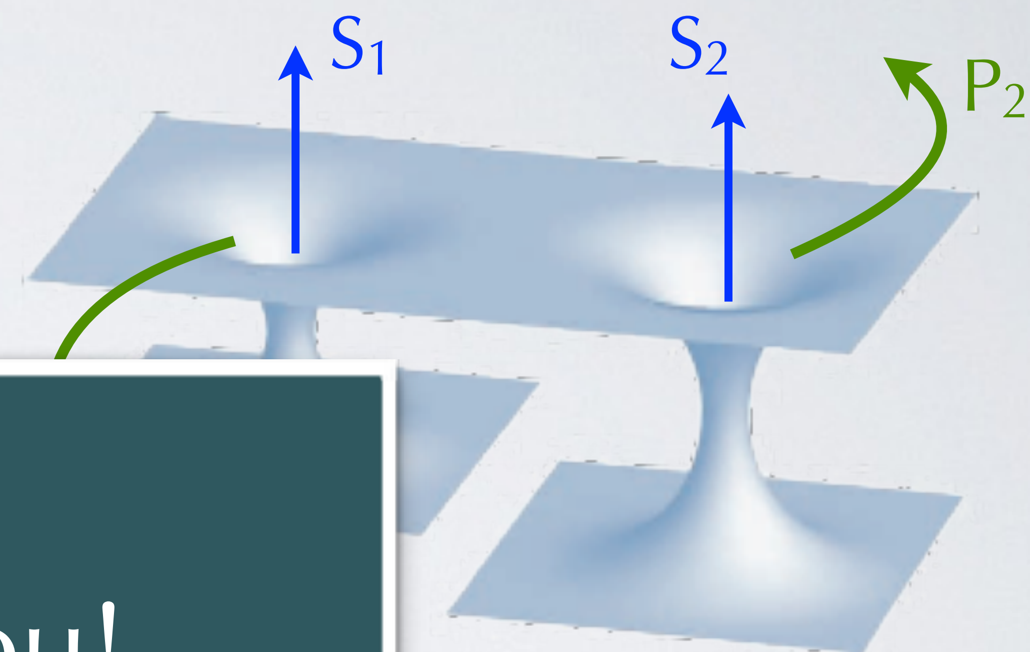
Frans P

[1]Theoretical Astrophysics, California Institut

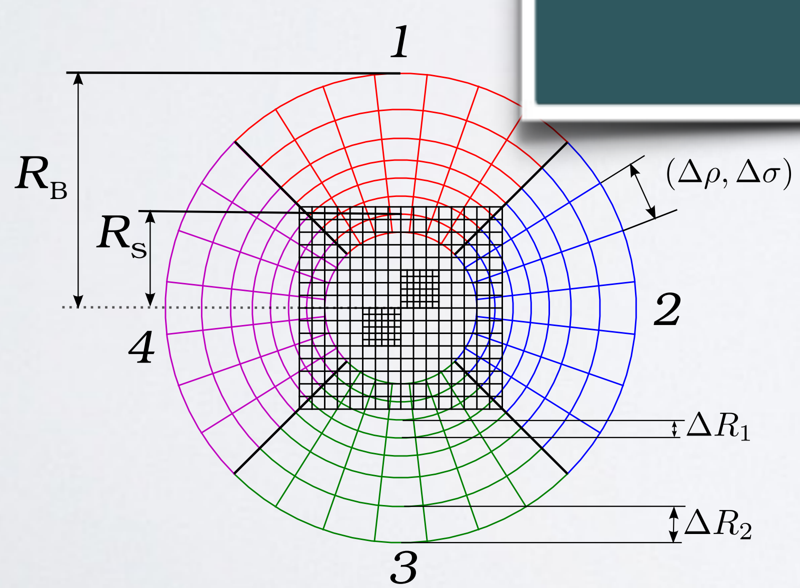
[2]Department of Physics, University o

(Received 6 July 2005;

We describe early success in the evolution of bi on a generalization of harmonic coordinates. Indi capable of evolving binary systems for enough t gravitational waves emitted during the event. A binary composed of two equal mass, nonspinning ringdown. The resultant black hole is estimated to $a \approx 0.70$. At present, lack of resolution far from emitted, though a rough calculation suggests on radiated as gravitational waves during the final



Thank you!



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