Parton Distributions at Hadron Colliders

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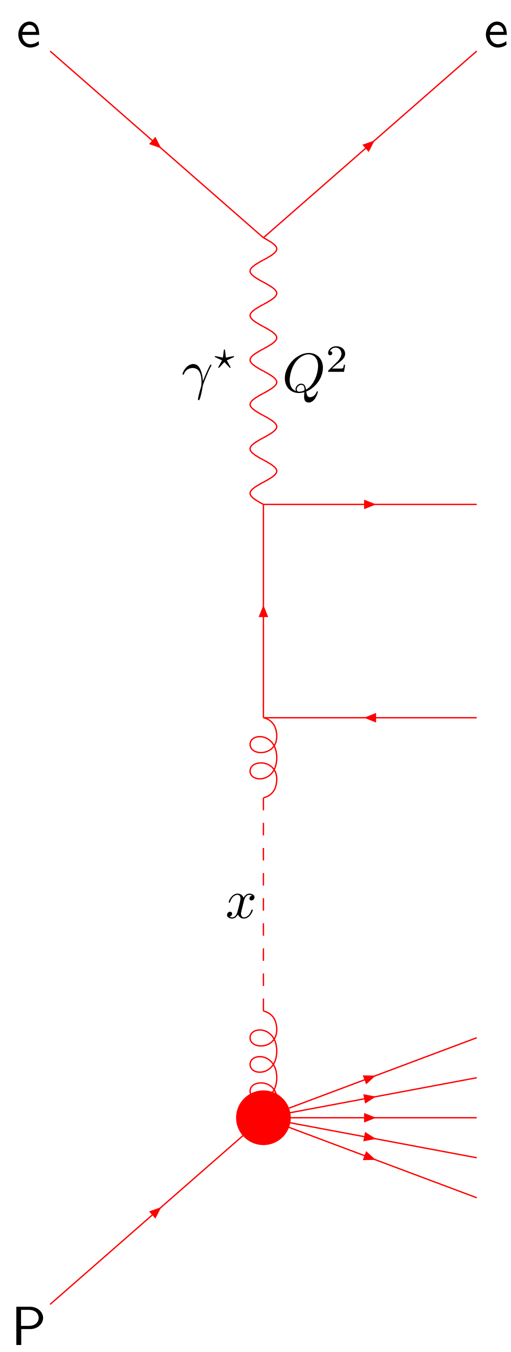
Strong force makes it difficult to perform analytic calculations of scattering processes involving hadronic particles.

The weakening of $\alpha_S(\mu^2)$ at higher scales $\longrightarrow$ the **Factorization Theorem**.

Hadron scattering with an electron factorizes.

Q^2 – Scale of scattering

$x = \frac{Q^2}{2m\nu}$ – Momentum fraction of Parton (ν=energy transfer)

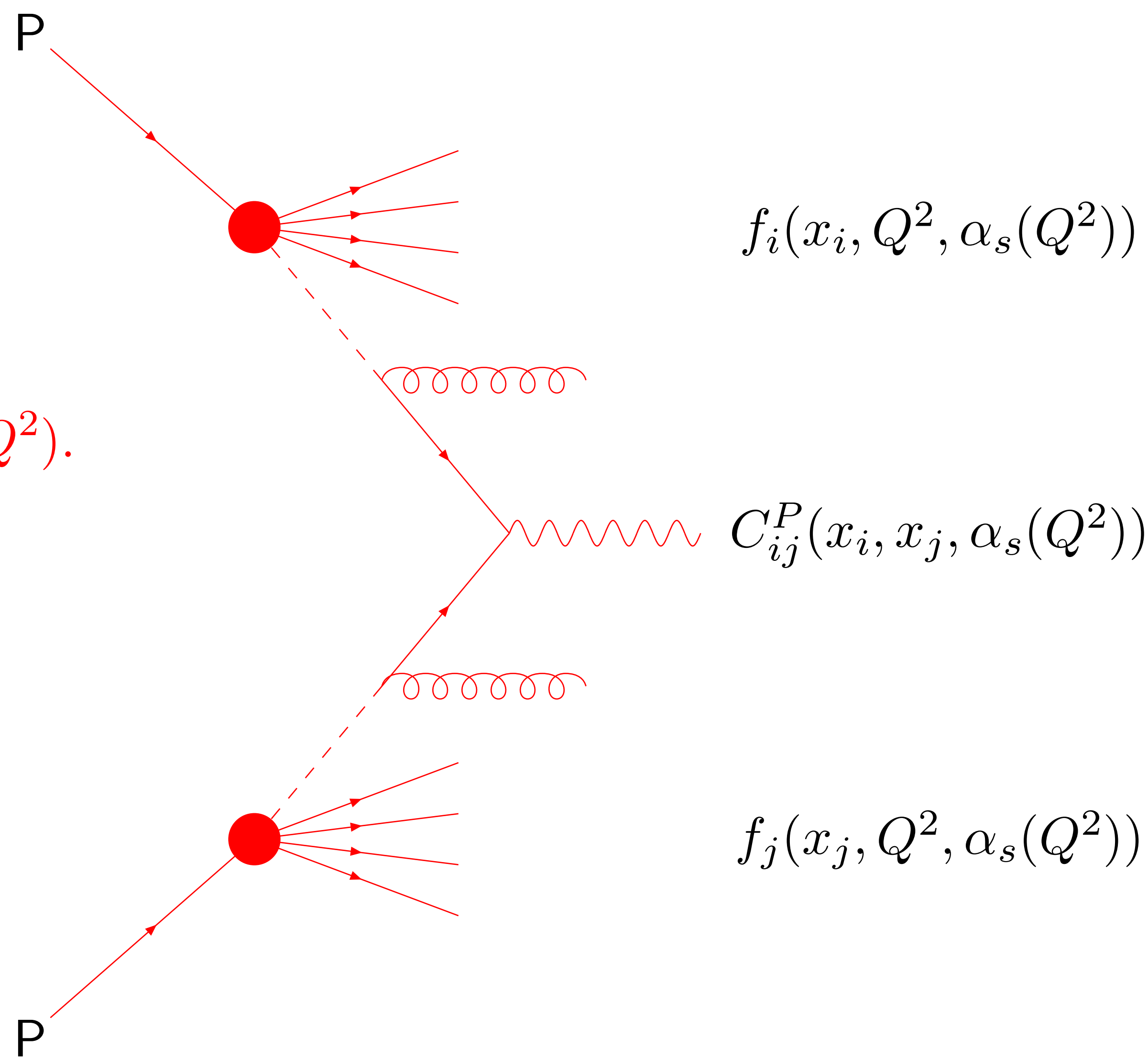


perturbative
calculable
coefficient function
$C_i^P(x, \alpha_s(Q^2))$

nonperturbative
incalculable
parton distribution
$f_i(x, Q^2, \alpha_s(Q^2))$

The coefficient functions $C_i^P(x, \alpha_s(Q^2))$ are process dependent (new physics) but are calculable as a power-series in $\alpha_s(Q^2)$.

$$C_i^P(x, \alpha_s(Q^2)) = \sum_k C_i^{P,k}(x)\alpha_s^k(Q^2).$$



Since the parton distributions $f_i(x, Q^2, \alpha_s(Q^2))$ are process-independent, i.e. **universal**, and evolution with scale is calculable, once they have been measured at one experiment, one can predict many other scattering processes.

General procedure.

Start parton evolution at low scale $Q_0^2 \sim 1\text{GeV}^2$. In principle 11 different partons to consider.

$$u, \bar{u}, \quad d, \bar{d}, \quad s, \bar{s}, \quad c, \bar{c}, \quad b, \bar{b}, \quad g$$

$m_c, m_b \gg \Lambda_{\text{QCD}}$ so heavy parton distributions determined perturbatively. Leaves 7 independent combinations, or 6 if assume $s = \bar{s}$.

$$u_V = u - \bar{u}, \quad d_V = d - \bar{d}, \quad \text{sea} = 2 * (\bar{u} + \bar{d} + \bar{s}), \quad s + \bar{s} \quad \bar{d} - \bar{u}, \quad g.$$

Input partons parameterised as, e.g.

$$xf(x, Q_0^2) = (1-x)^\eta (1 + \epsilon x^{0.5} + \gamma x) x^\delta.$$

For non-singlet combinations, valence quarks, $\bar{d} - \bar{u}$, δ expected to be ~ 0.5. For singlet combinations, sea and gluon, δ expected to be ~ 0.

Evolve partons upwards using LO, NLO (or NNLO) DGLAP equations.

$$\frac{df_i(x, Q^2, \alpha_s(Q^2))}{d \ln Q^2} = \sum_j P_{ij}(x, \alpha_s(Q^2)) \otimes f_j(x, Q^2, \alpha_s(Q^2))$$

Fit data for scales above $2 - 10\mathrm{GeV}^2$. Need many different types of experiment for full determination.

- Lepto-proton collider HERA – (DIS) $\longrightarrow$ small-x quarks. Also gluons from evolution, and $F_L(x, Q^2)$. Also, jets $\longrightarrow$ moderate-x gluon.

- Fixed target DIS – higher x – leptons (BCDMS, NMC, ...) $\longrightarrow$ up quark (proton) or down quark (deuterium) and neutrinos (CHORUS, NuTeV, CCFR) $\longrightarrow$valence or singlet combinations.

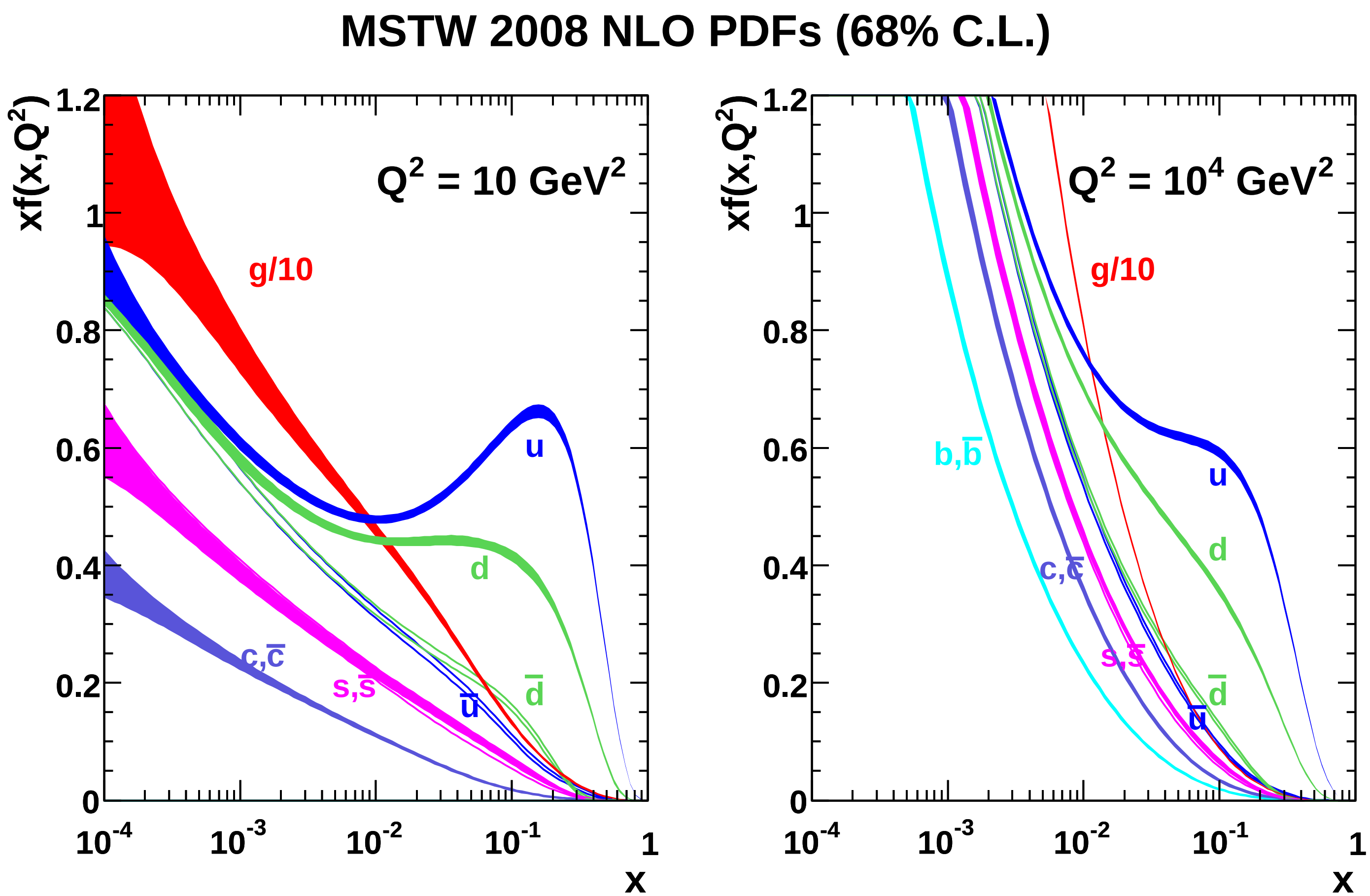
- Di-muon production in neutrino DIS – strange quarks and neutrino-antineutrino comparison $\longrightarrow$ asymmetry .

- Drell-Yan production of dileptons – quark-antiquark annihilation (E605, E866) – high-x sea quarks.

- High-p_T jets at colliders (Tevatron) – high-x gluon distribution.

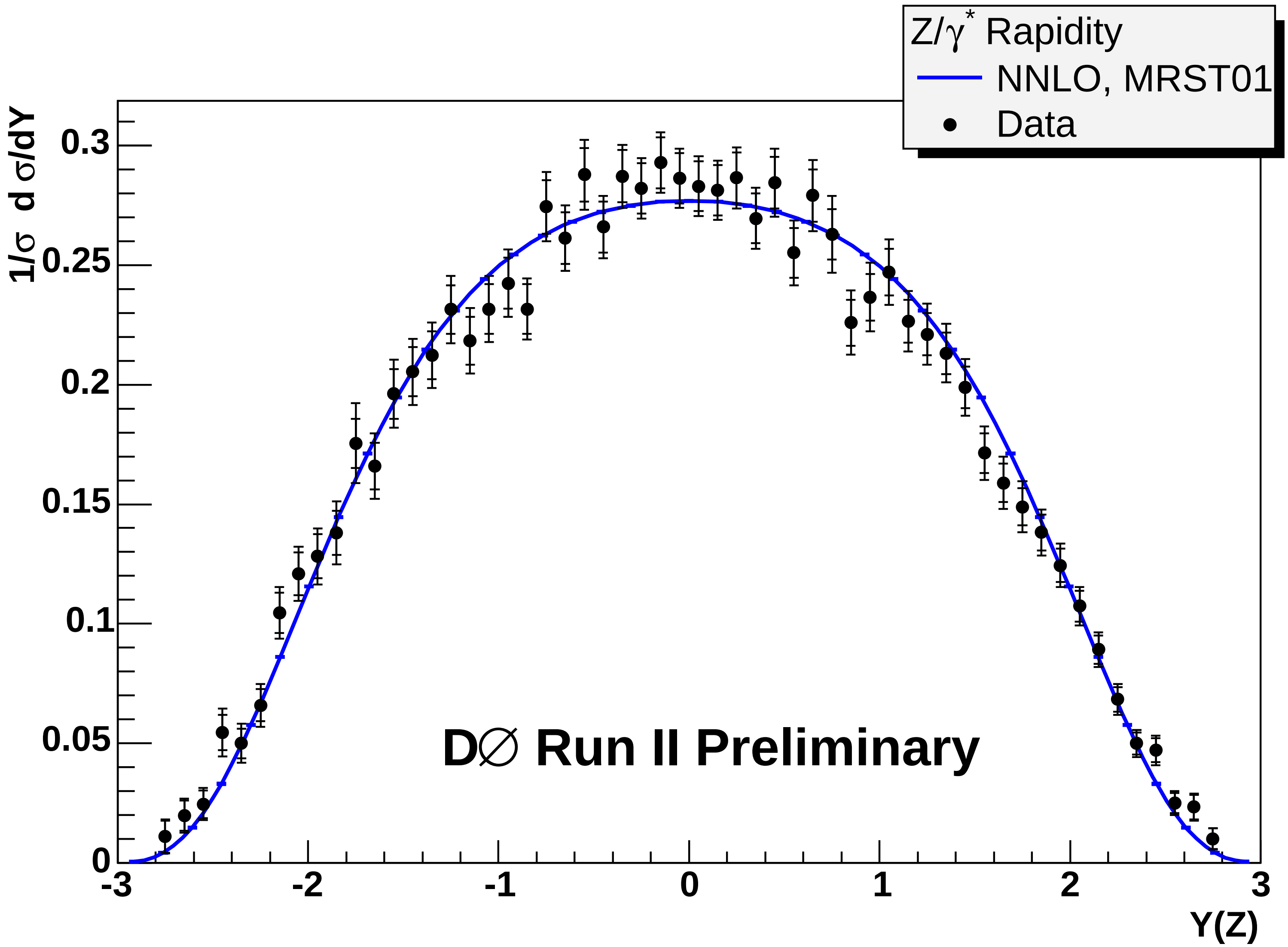
- W and Z production at colliders (Tevatron) – different quark contributions to DIS.

This procedure is generally successful and is part of a large-scale, ongoing project. Results in partons of the form shown.



Various choices of partons – MSTW, CTEQ, NNPDF, Alekhin, ZEUS, H1 and others. All LHC cross-sections rely on our understanding of these partons.

Excellent predictive power – comparison of MRST prediction for Z rapidity distribution with preliminary data.



Interplay of LHC and pdfs/QCD

Make predictions for all processes, both SM and BSM, as accurately as possible given current experimental input and theoretical accuracy.

Check against well-understood processes, e.g. central rapidity W, Z production (luminosity monitor), lowish-E_T jets, ...

Compare with predictions with more uncertainty and lower confidence, e.g. high-E_T jets, high rapidity bosons or heavy quarks .....

Improve uncertainty on parton distributions by improved constraints, and check understanding of theoretical uncertainties, and determine where NNLO, electroweak corrections, resummations $etc.$ needed.

Make improved predictions for both background and signals with improved partons and surrounding theory.

Spot new physics from deviations in these predictions. As a nice by-product improve our understanding of the strong sector of the Standard Model considerably.

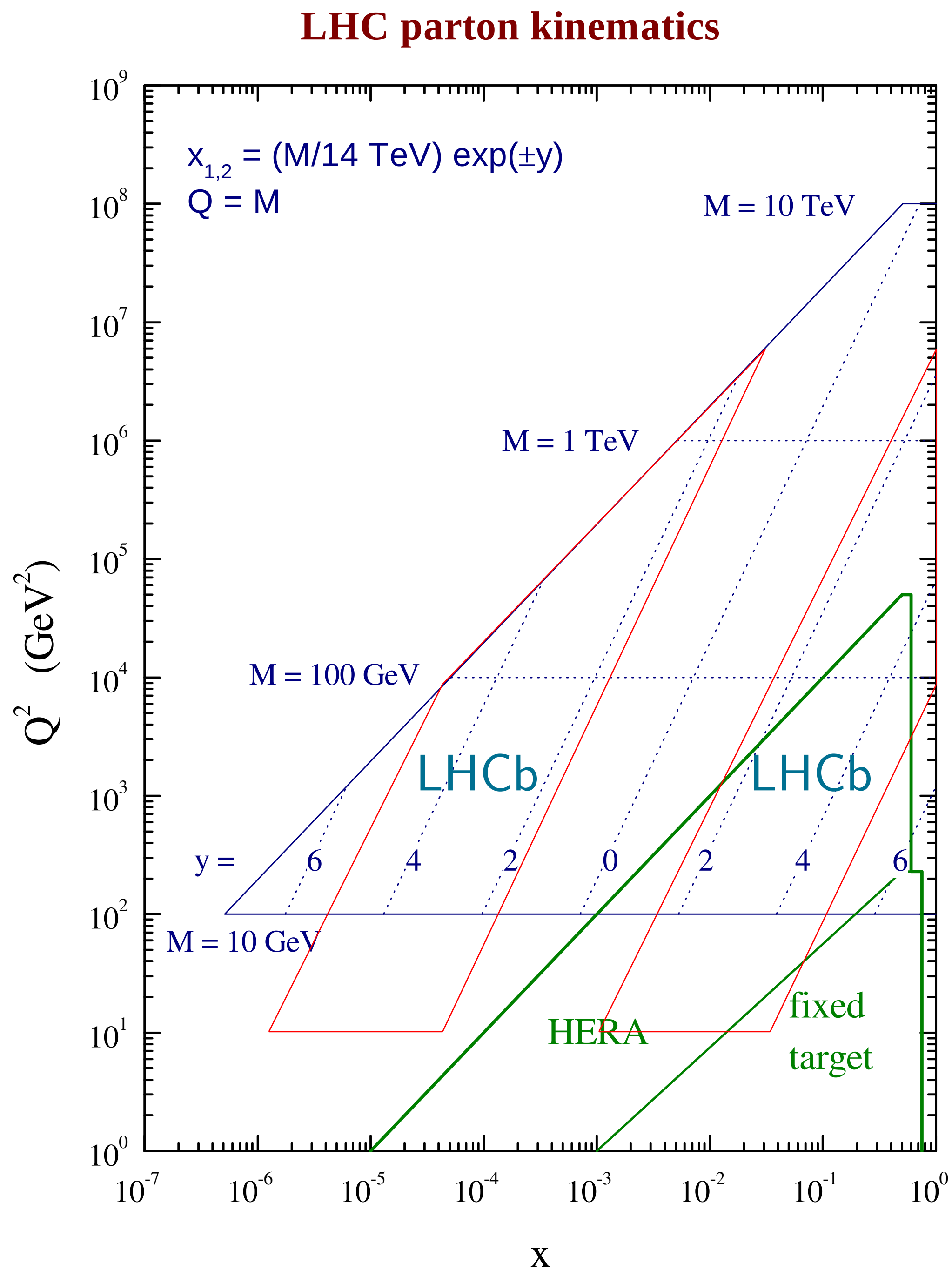
Remainder of talk describes this process in more detail.

Predictions at the LHC

New kinematic regime.

PDFs mainly extrapolated via evolution rather than measured directly.

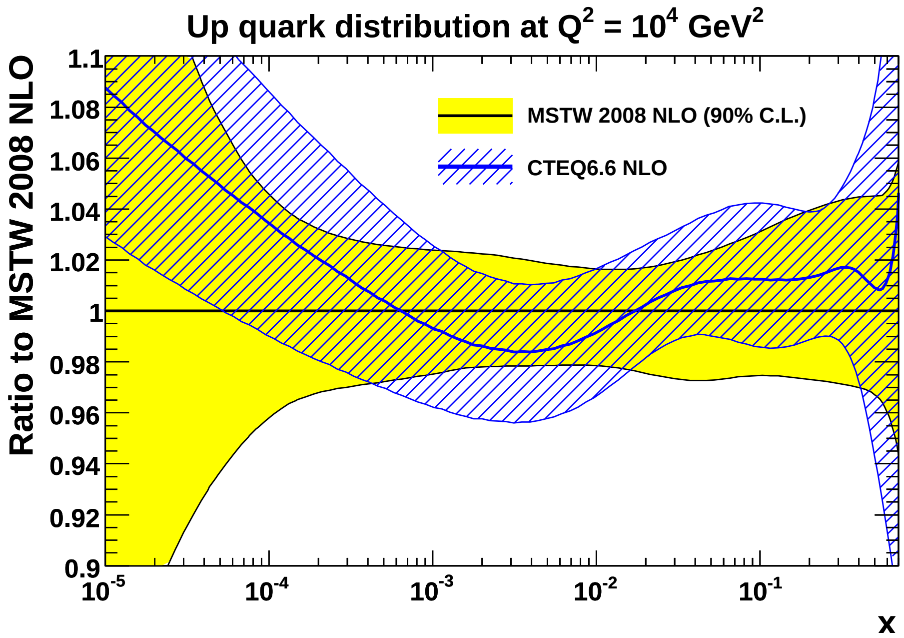
High scale and small-x parton distributions are vital for understanding processes at the LHC.

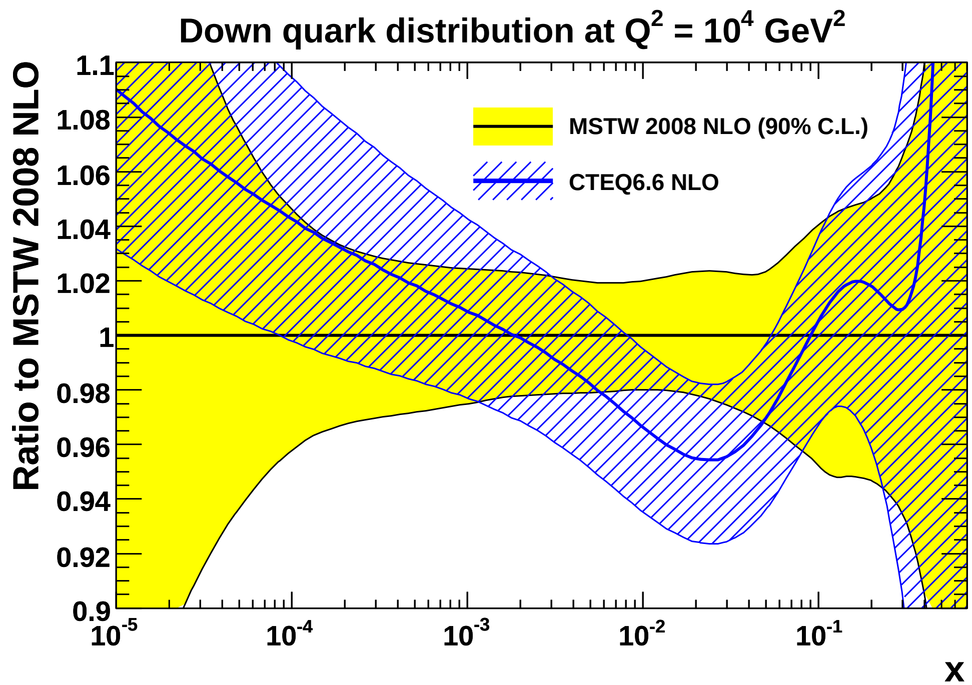


Uncertainty on MSTW u and d distributions, along with CTEQ6.

Reasonable agreement between groups.

Central rapidity $x = 0.006$ is ideal for uncertainty in W, Z (Higgs?) at the LHC.





Predictions for W and Z cross-sections for LHC with common fixed order QCD and vector boson width effects, and common branching ratios.

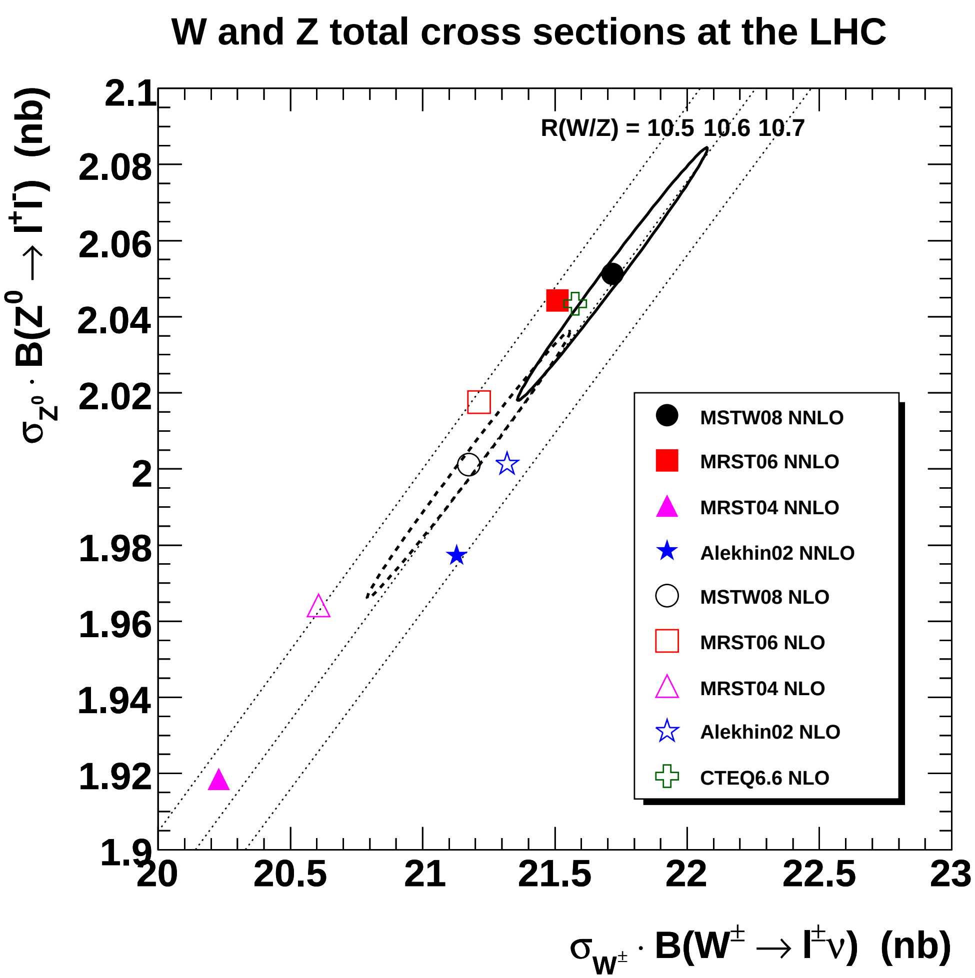
Good agreement at NLO for variety of PDFs.

Fairly significant change from NLO to NNLO mainly due to hard cross-section correction.

Some difference in W/Z ratio.

Generally all fine?

W, Z total cross-sections best-case scenario.



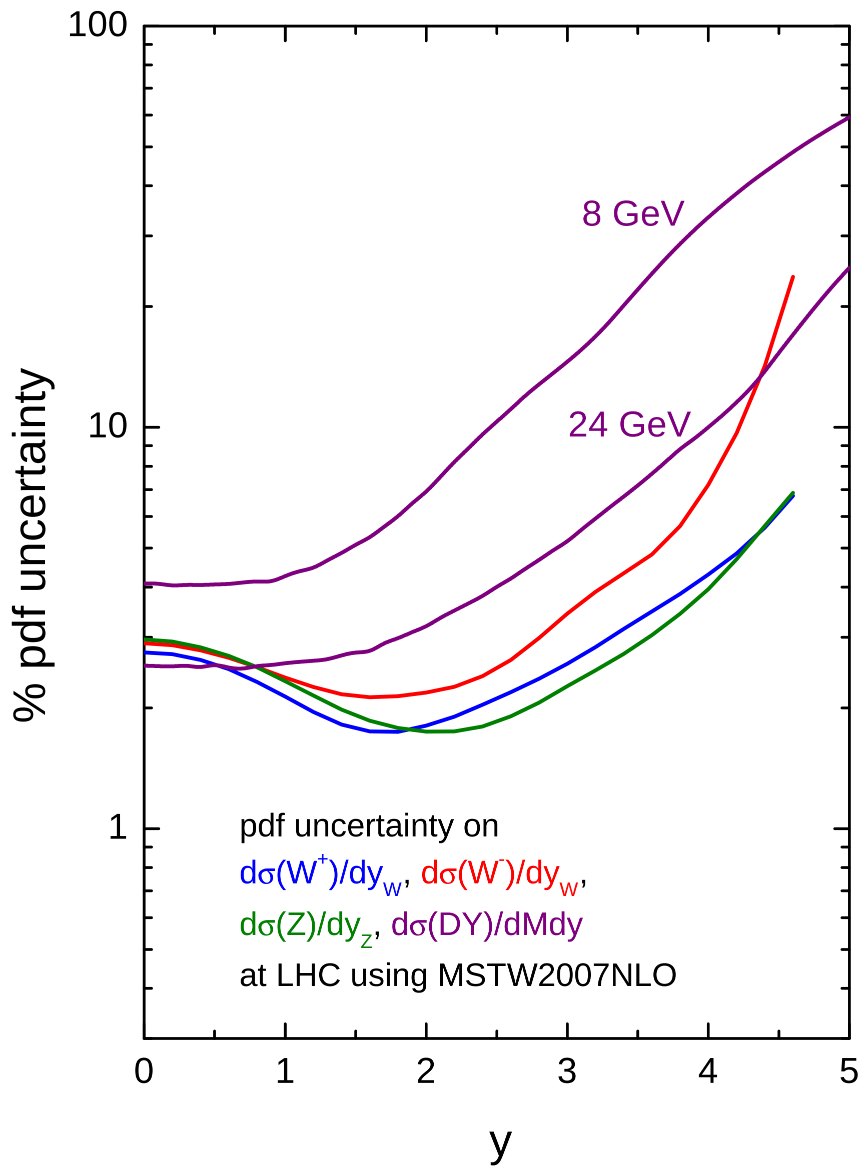
W, Z uncertainty – more details

Uncertainty on $\sigma(Z)$ and $\sigma(W^+)$ grows at high rapidity.

Uncertainty on $\sigma(W^-)$ grows more quickly at very high y – depends on less well-known down quark.

Uncertainty on $\sigma(\gamma^\star)$ is greatest as y increases. Depends on partons at very small x.

Still only uncertainty from data with $perfect$ framework.



Other Sources of Uncertainty

It is vital to consider theoretical/assumption-dependent uncertainties:

- Underlying assumptions in procedure, e.g. parameterisations.
- Treatment of heavy flavours.
- PDF and α_S correlations.
- QED and Weak (comparable to NNLO ?) ($\alpha_s^3 \sim \alpha$). Sometime enhancements.
- Standard higher orders (NNLO)
- Resummations, e.g. small x ($\alpha_s^n \ln^{n-1}(1/x)$)
- or large x ($\alpha_s^n \ln^{2n-1}(1-x)$)
- low Q^2 (higher twist)

Lead to differences in current partons, and to corrections in predicted cross-sections.

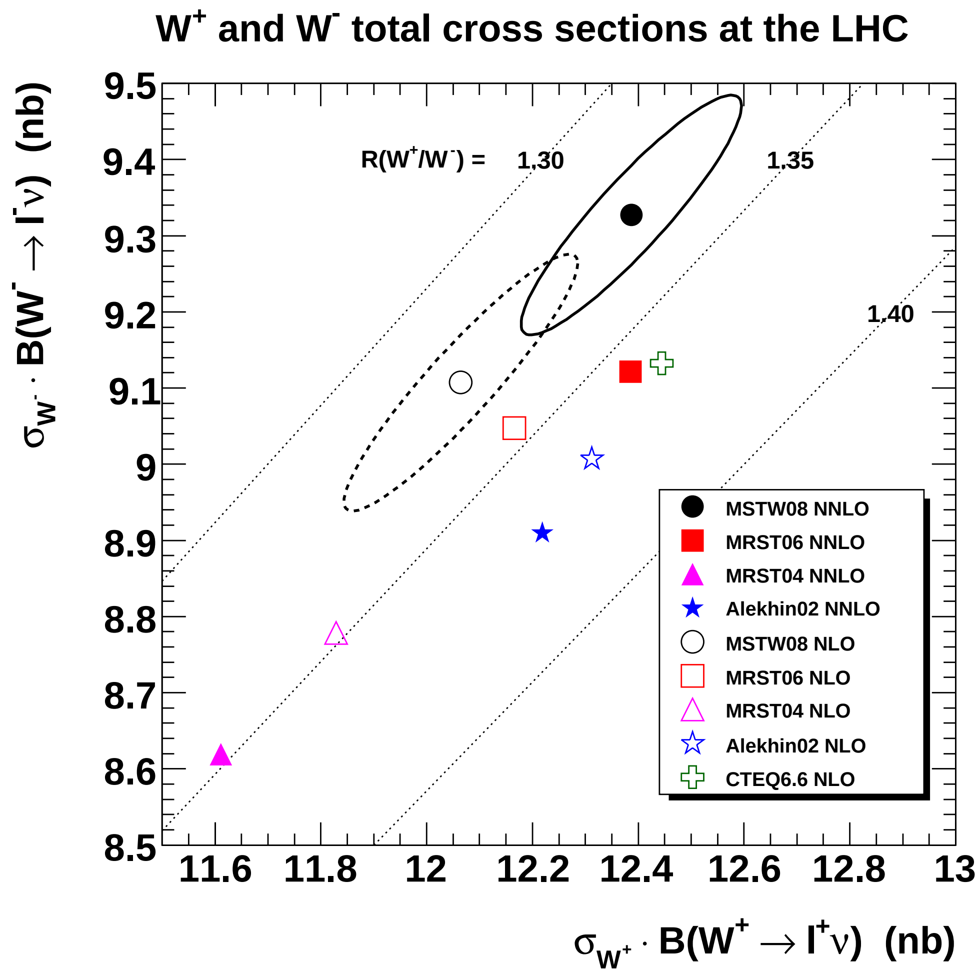
Parameterisations

MSTW predictions for W+ and W- cross-sections for LHC with common fixed order QCD and vector boson width effects, and common branching ratios.

Quoted uncertainty for ratio very small, i.e. $\approx 0.8\%$. Prediction sensitive to u and d quarks.

$$\frac{\sigma(W^+)}{\sigma(W^-)} \approx \frac{u(x)\bar{d}(x)}{d(x)\bar{u}(x)} \approx \frac{u(x)}{d(x)},$$

If $\bar{u}(x) \to \bar{d}(x), x \to 0$, which data implies and most parameterisations assume.



Fit includes most recent neutrino DIS and Tevatron vector boson data. Uncertainties should account for this.

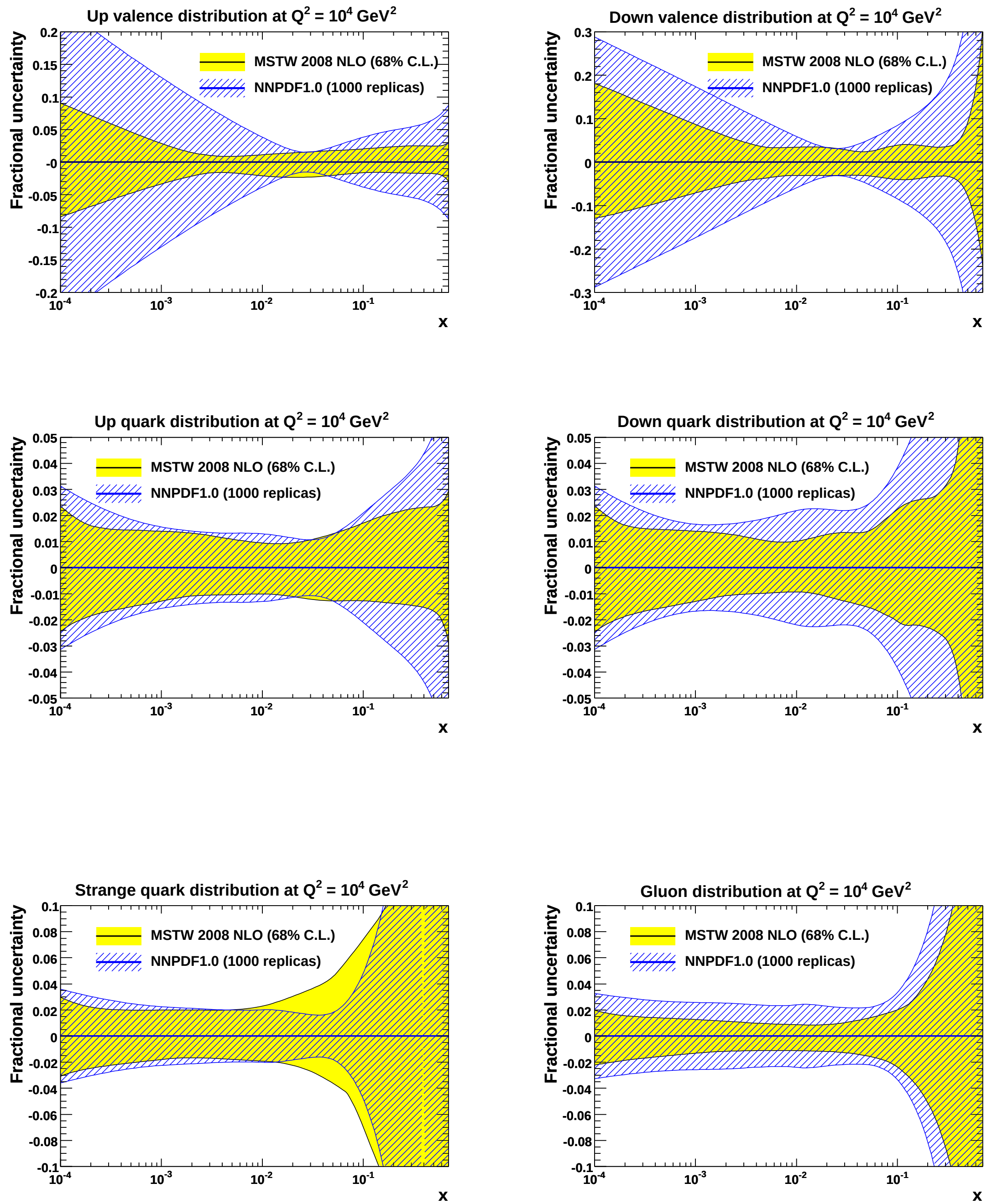
Significantly more difference than uncertainty from other PDFs, including MRST. Very interesting for early data.

Comparison of uncertainties to NLO Neural Network PDF set with effective parameterisation independence (but strange fraction of light quarks).

Often comparable despite input flexibility in NNPDF.

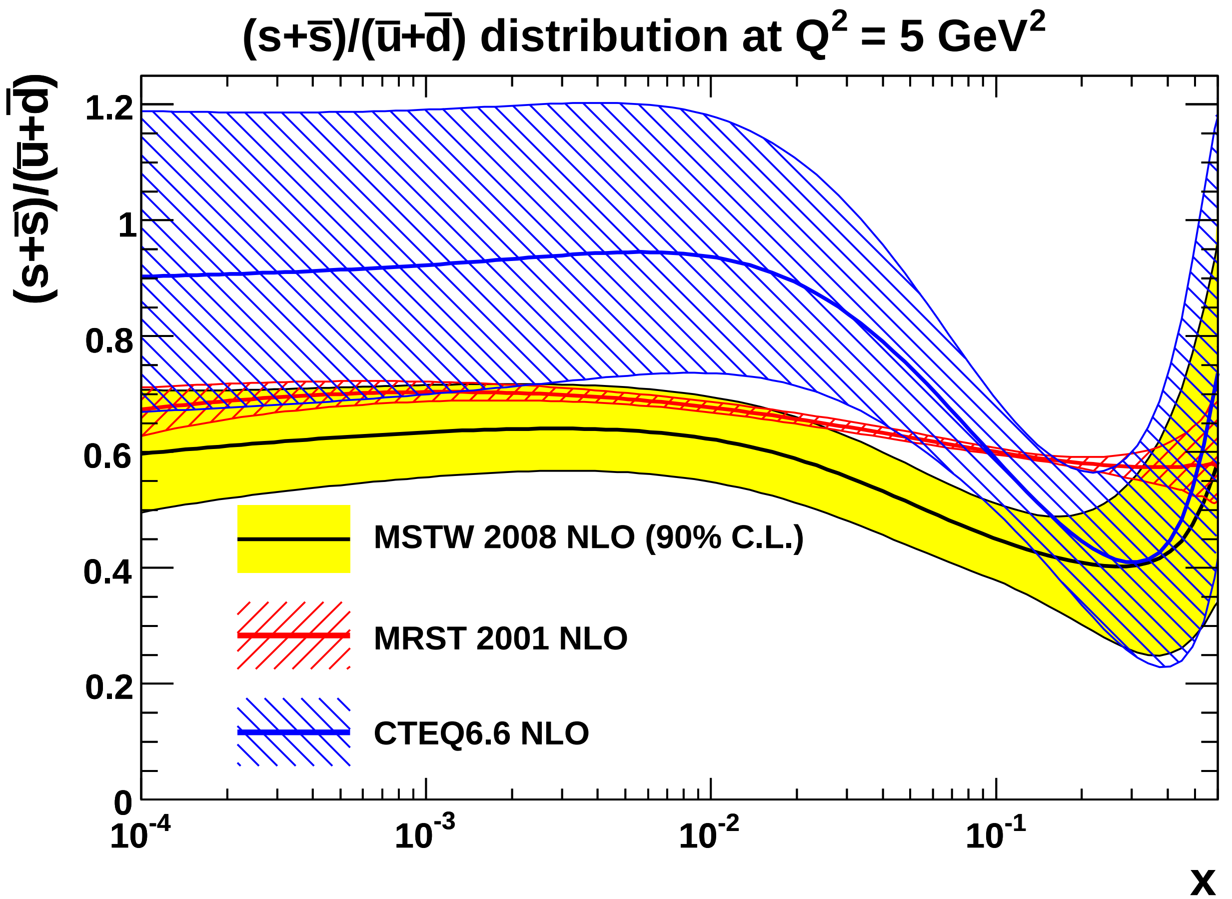
NNPDF currently use only DIS data – less constraint (central values of PDFs not always consistent).

Almost certainly real additional uncertainty on small-x valence and very high x. (Though extra data and sum rules play a role in reducing uncertainties even here.)



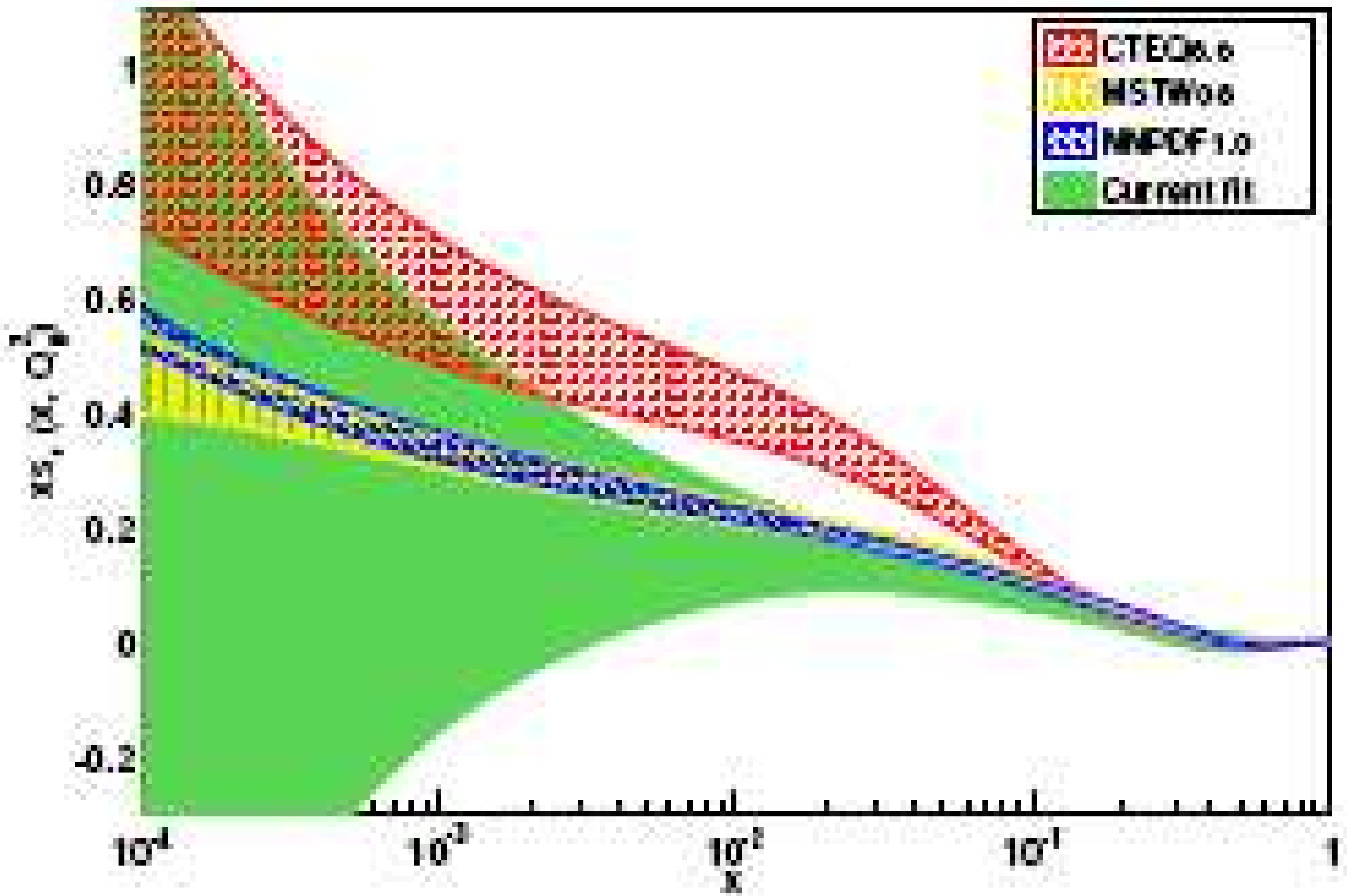
Strange Quarks

Direct fit to $s, \bar{s}$ from dimuon data leads to significant uncertainty increase compared to assumption of fixed fraction of sea.



Significant difference to CTEQ fitting to same data. At small x assume shape of input sea quarks is the same (consistent with mass suppression) whereas CTEQ have different parameterisation.

NNPDF1.1, which includes dimuon data, have no theoretical constraint on strange quark distribution at all at small x.

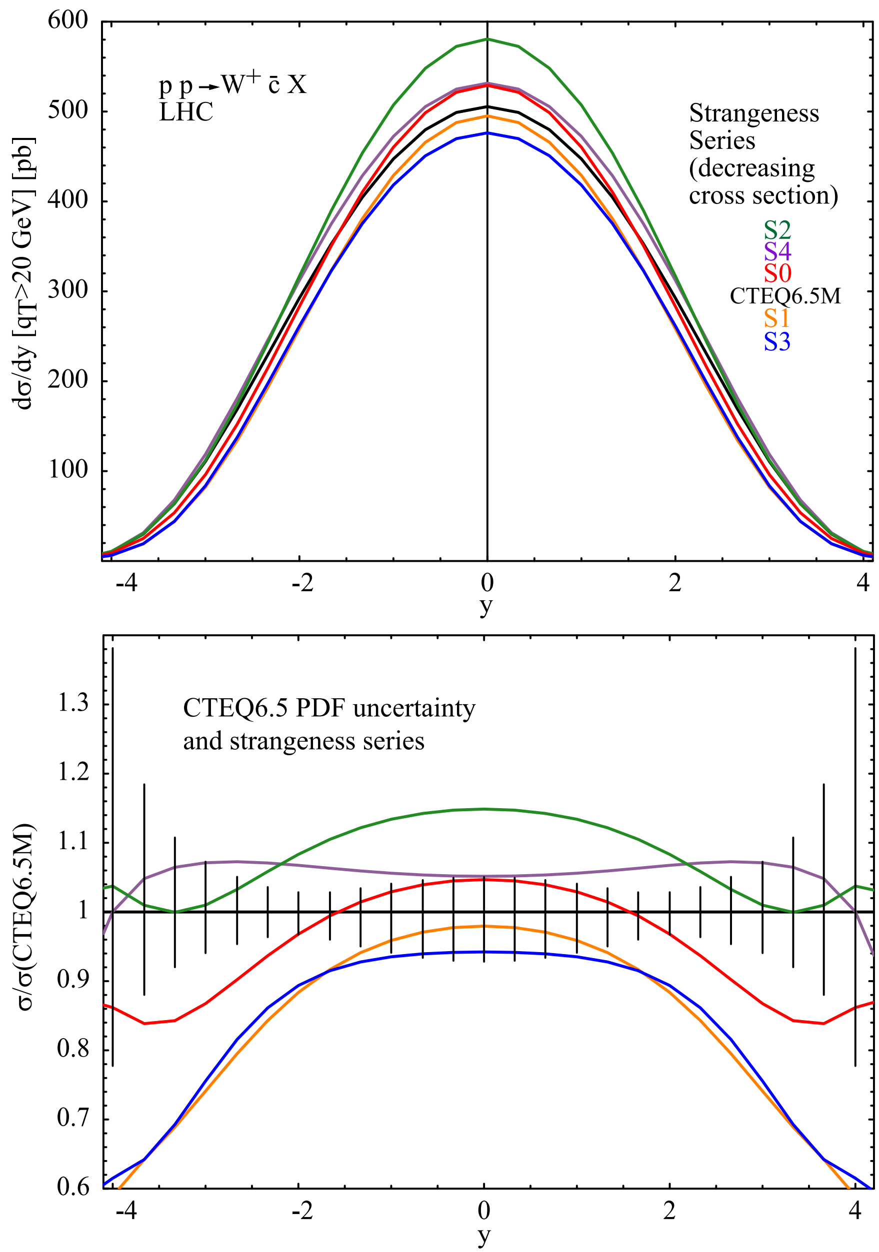


Overestimate of uncertainty?

CTEQ look at special sets with fits to dimuon data and possible (generous) variations.

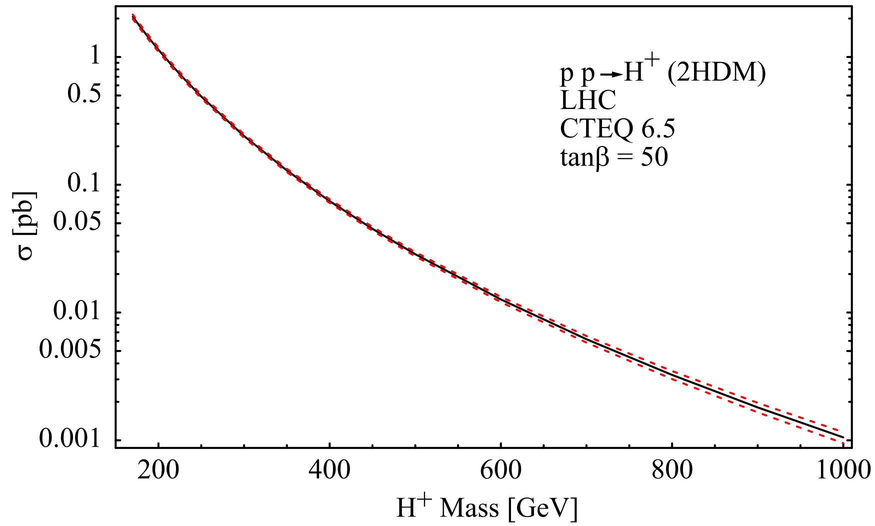
Band represents uncertainty of default CTEQ6.5 set.

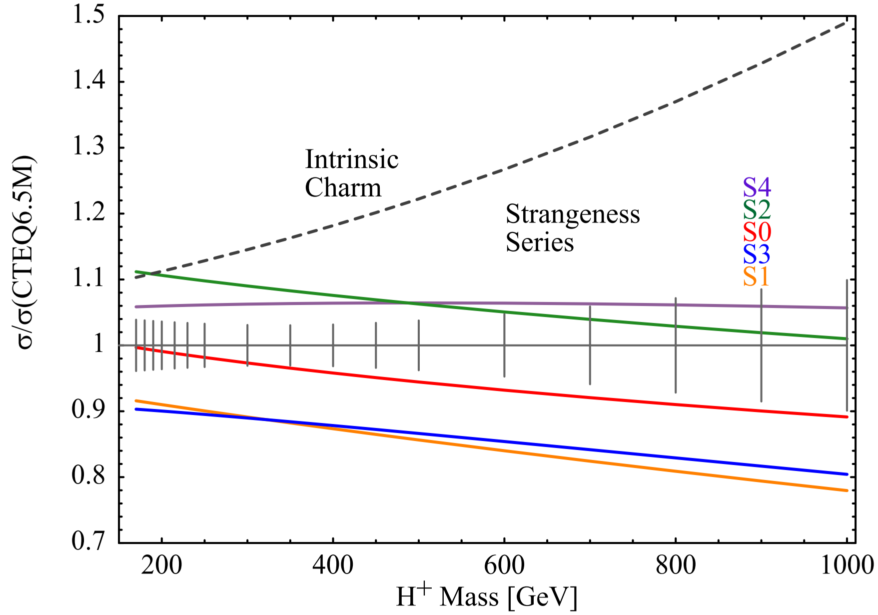
Look at implications for strange sensitive final state, i.e. $W + c$ at LHC.



Also examine uncertainty of predictions for BSM physics, e.g. $c + \bar{s} \to H^+$.

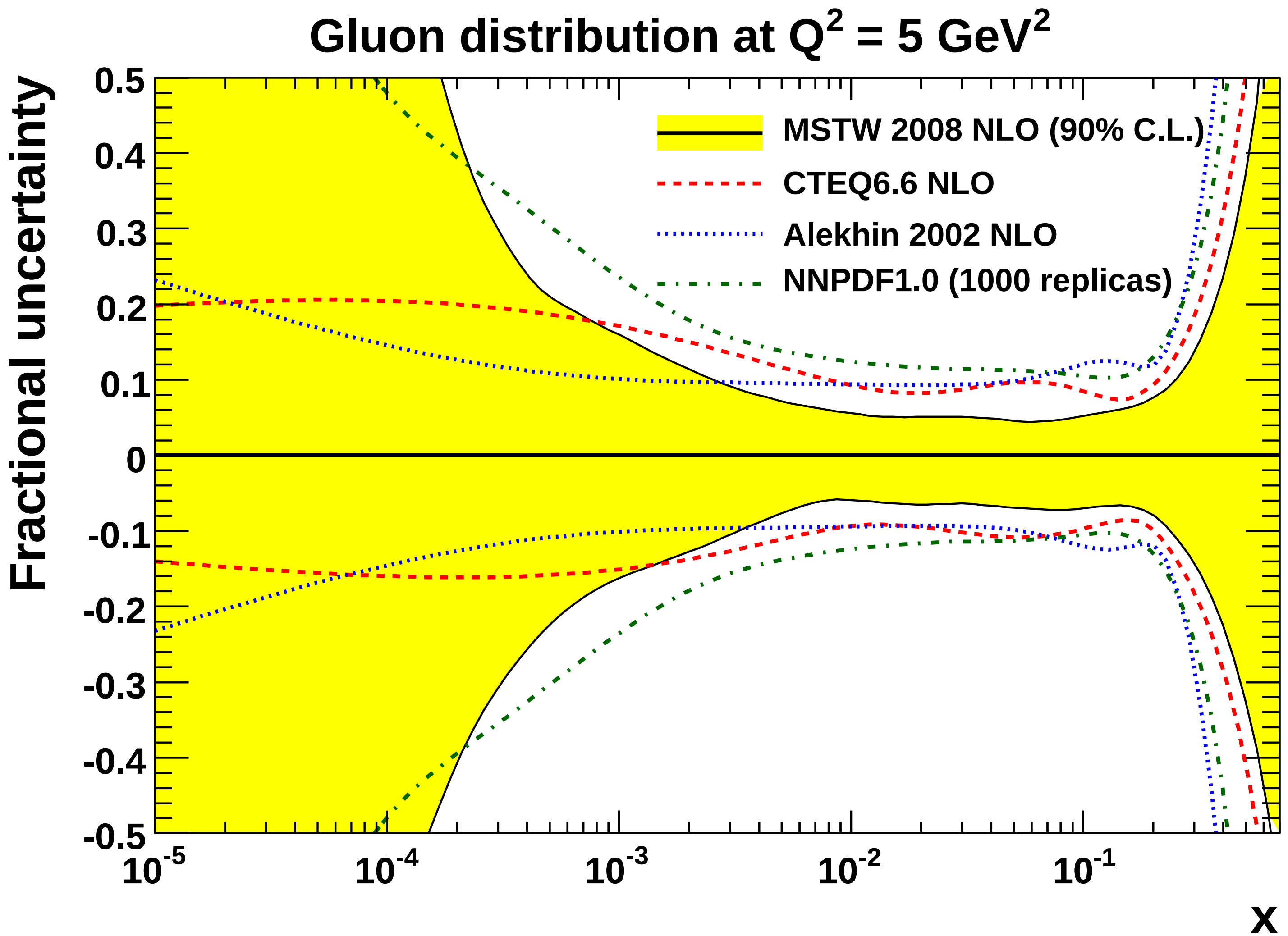
Again allowed sets give wider range of predictions than default uncertainty.





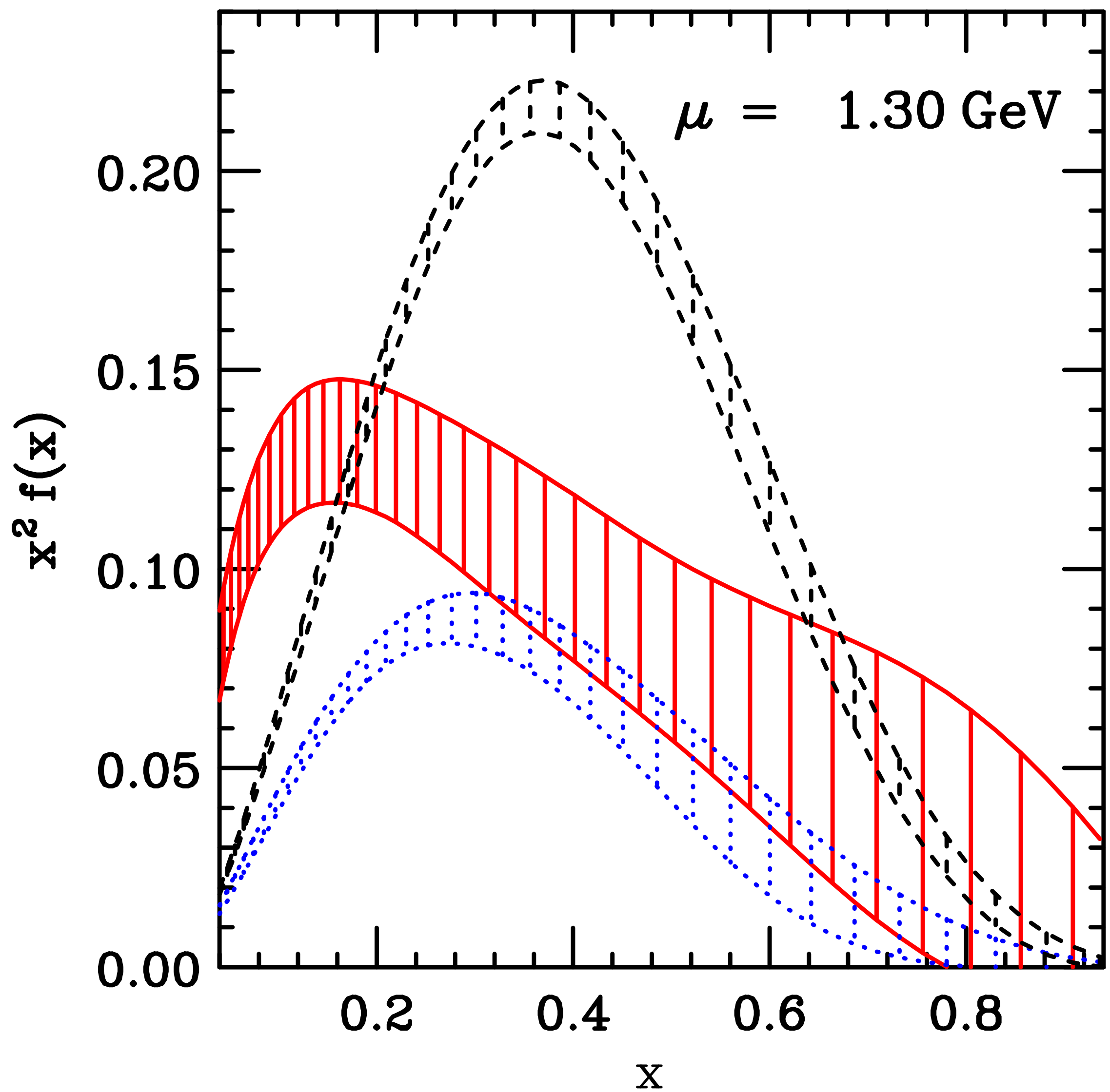
Gluon Parameterisation.

Note that different parameterisations lead to very different types of uncertainty, particularly on small x gluon.



And on high-x gluon distribution.

Is gluon, which is radiated from quarks, harder than the up valence distribution for $x \to 1$?



Heavy Quarks – Essential to treat these correctly. Two distinct regimes:

Near threshold $Q^2 \sim m_H^2$ massive quarks not partons. Created in final state. Described using **Fixed Flavour Number Scheme** (FFNS).

$$F(x,Q^2) = C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2)$$

Does not sum $\ln^n(Q^2/m_H^2)$ terms, and not calculated for many processes beyond LO.

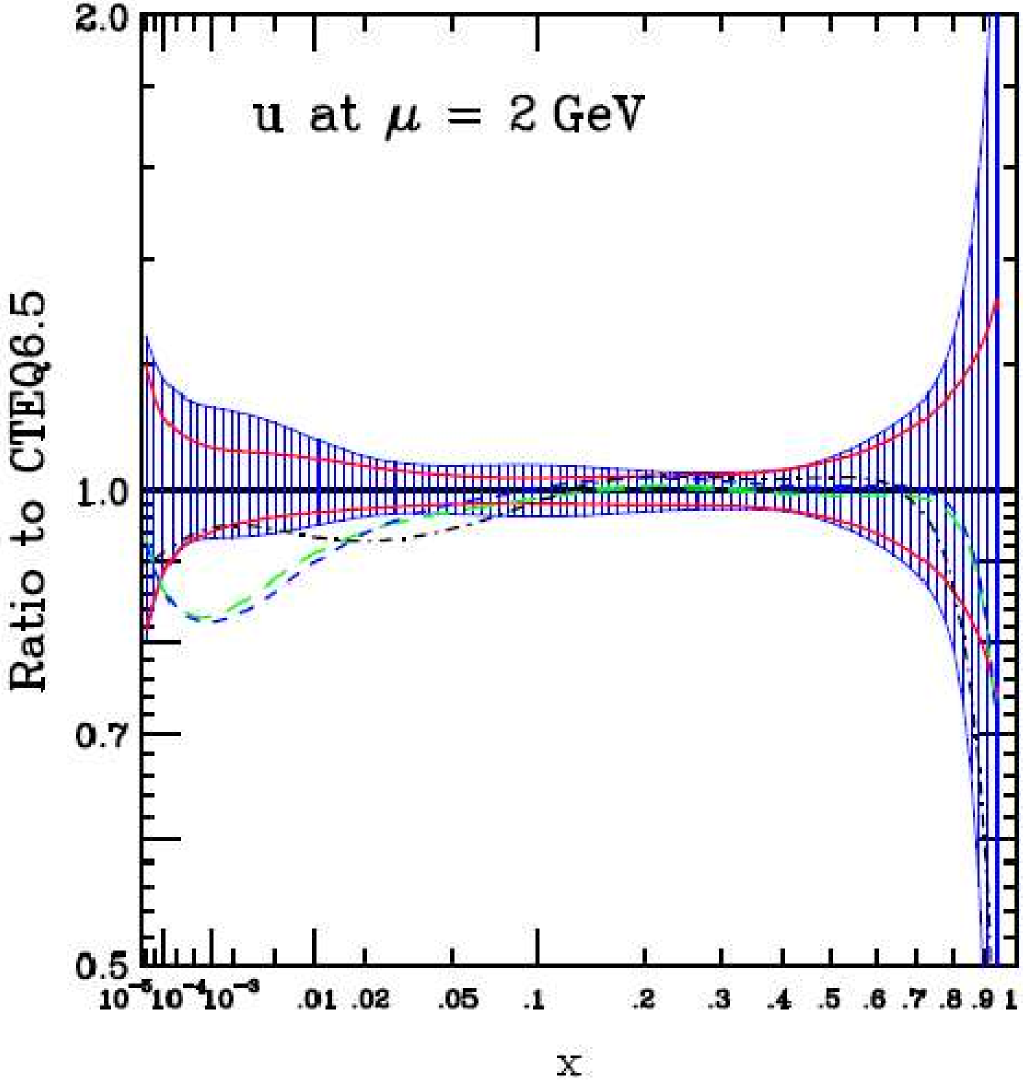
Alternative, at high scales $Q^2 \gg m_H^2$ heavy quarks like massless partons. Behave like up, down, strange. Sum $\ln(Q^2/m_H^2)$ terms via evolution. **Zero Mass Variable Flavour Number Scheme** (ZM-VFNS). Normal assumption in calculations. Ignores $\mathcal{O}(m_H^2/Q^2)$ corrections.

$$F(x,Q^2) = C_j^{ZMVF} \otimes f_j^{n_f+1}(Q^2).$$

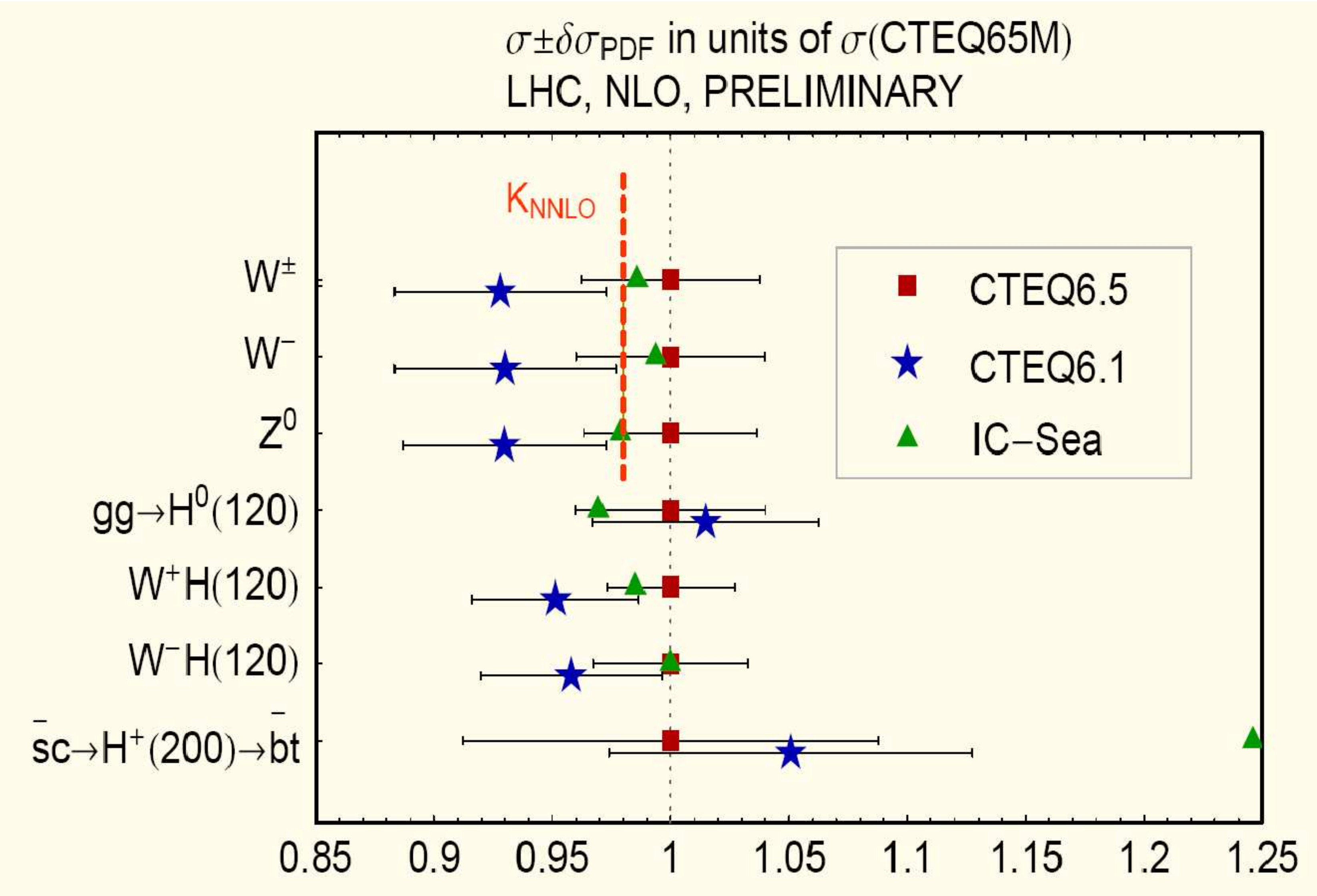
Need a general **Variable Flavour Number Scheme** (VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$. Used by MRST/MSTW and more recently (as default) by CTEQ, and now H1 and ZEUS.

Importance of doing it correctly illustrated by CTEQ6.5 up quark with uncertainties compared with previous versions, e.g. CTEQ6 in green.

MRST in dash-dot line. Reasonable agreement. Already used heavy flavour treatment in default sets.



Leads to large change in predictions using CTEQ partons at LHC of $5 - 10\%$.



Note effects of *intrinsic charm.*

Could also be nonperturbative (intrinsic) heavy flavour.

Suppressed by $\frac{\Lambda^2_{QCD}}{Q^2}$ or possibly $\frac{\Lambda^2_{QCD}}{W^2} \sim \frac{\Lambda^2_{QCD}}{Q^2(1-x)}$.

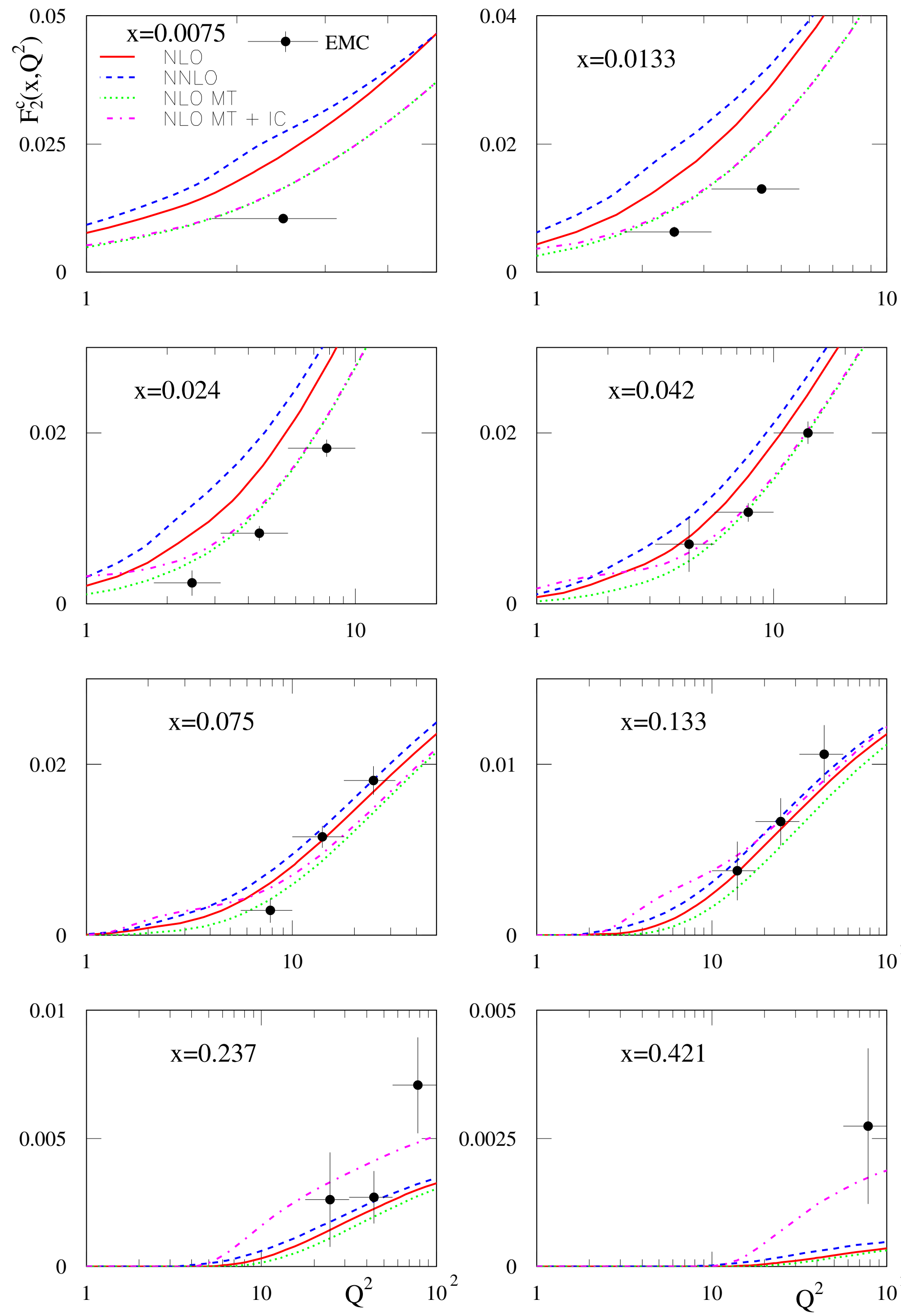
Enhanced at high x (Brodsky *et al*).

CTEQ constrain from normal global fit (and consider large effect at all x).

Check against old EMC data. Suggest at most $\frac{1}{10}th$ this value.

Need to modify threshold physics for good fit.

Large intrinsic $b+\bar{b}$ could dominate Higgs production at $y \geq 5$ at LHC (Brodsky *et al*).



Check effect of change in flavour prescription for NLO.

Compare MRST2004 (with 2001 uncertainties) to unofficial "MRST2006 NLO".

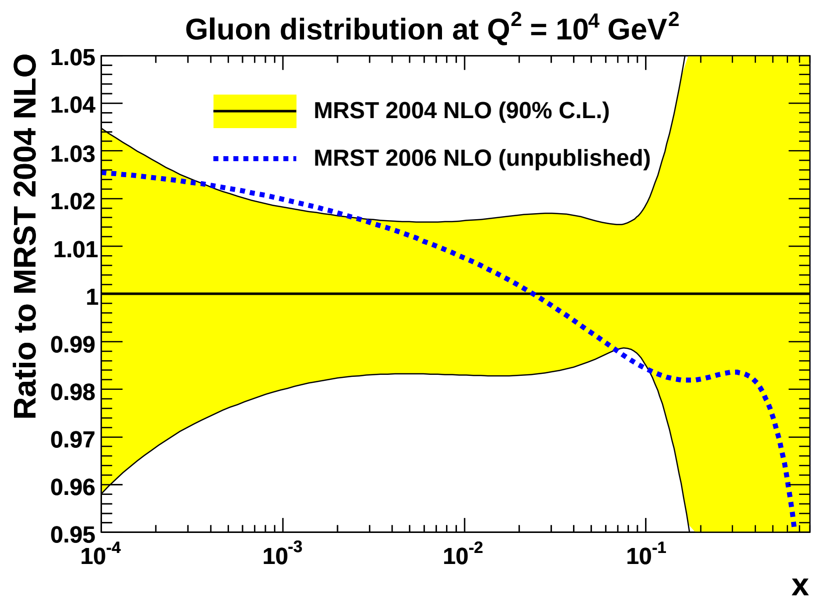
Only difference in flavour schemes (both well-defined).

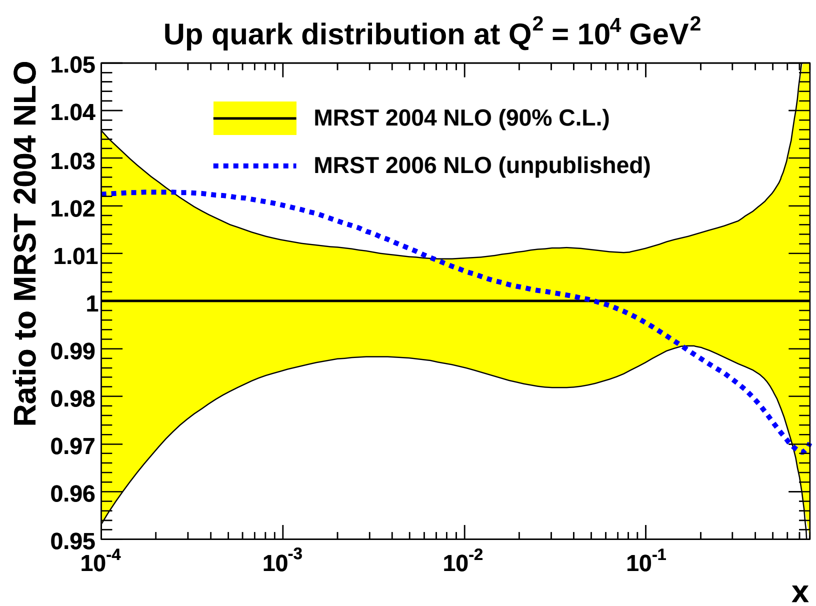
Changes of up to 2% in PDFs.

Up to 3% increase in σ_W and σ_Z at the LHC.

This is a genuine theory uncertainty due to competing but equally valid choices. Ambiguity decreases at higher orders.

Some – but probably quite little – anti-correlation with PDF uncertainties.

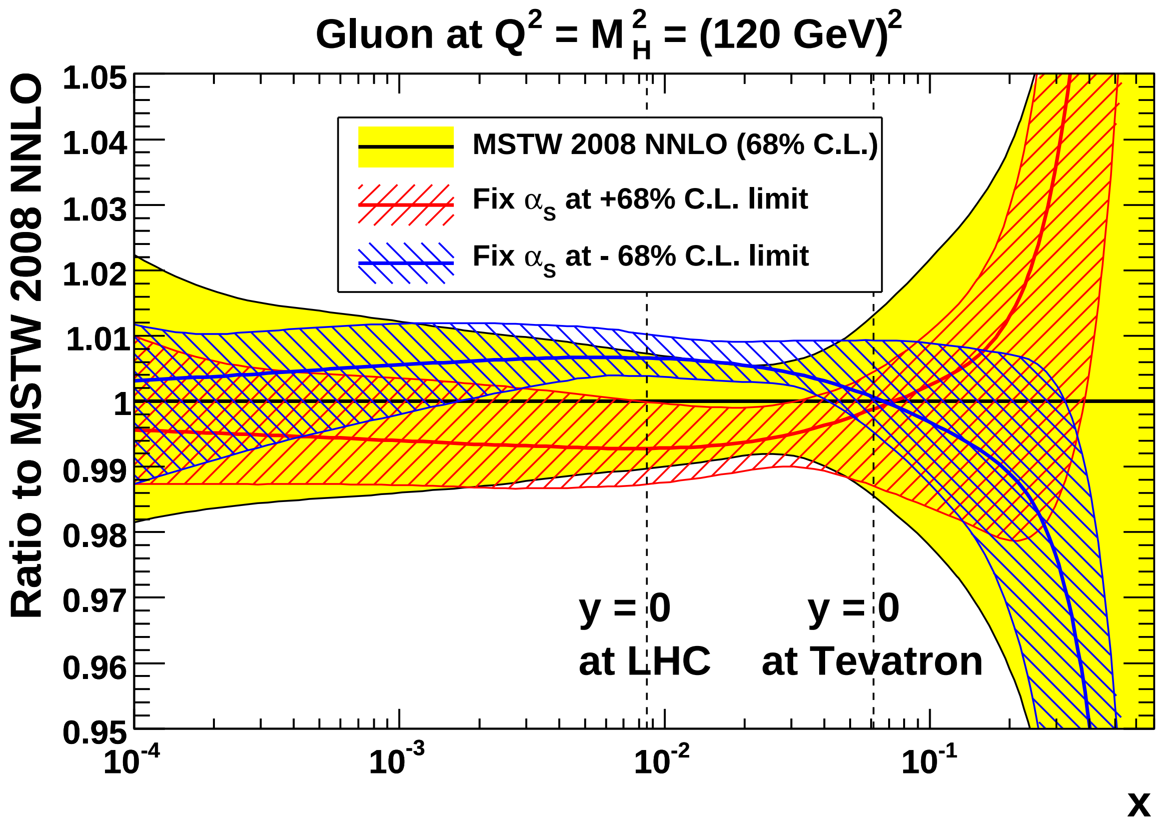




PDF correlation with α_S.

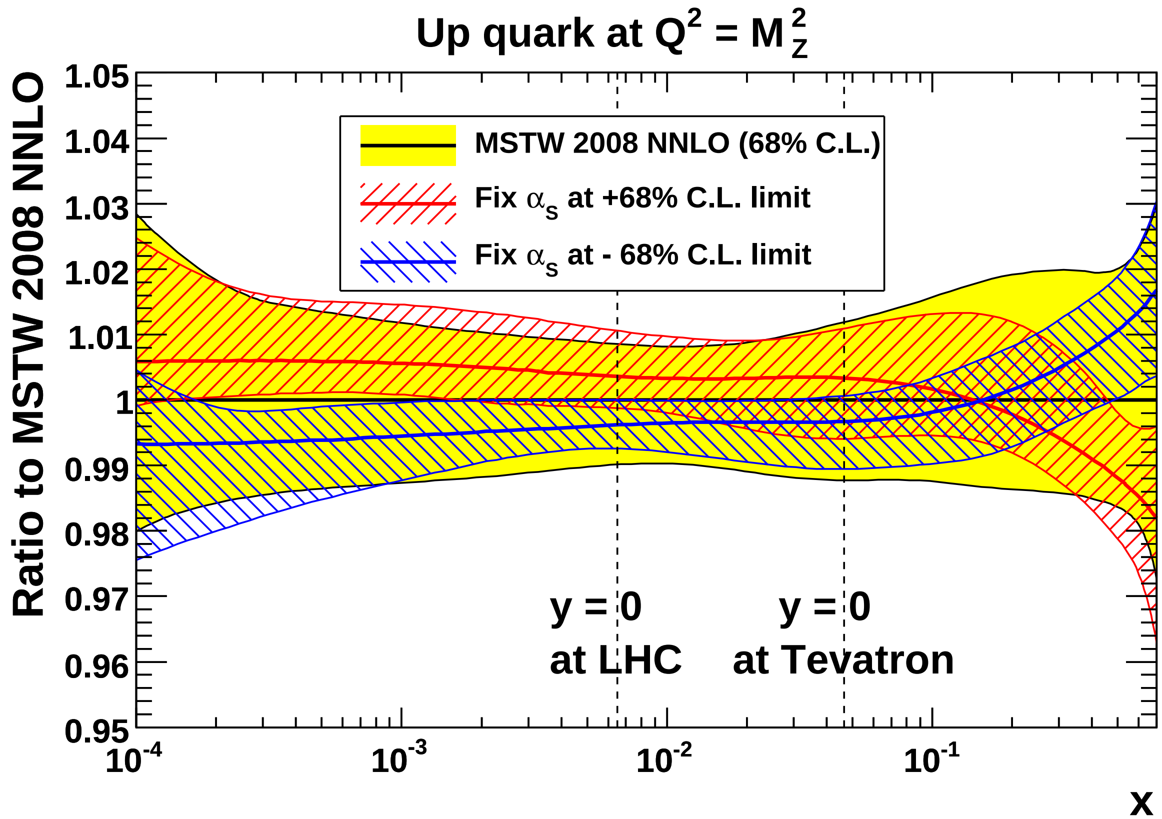
Can also look at PDF changes and uncertainties at different $\alpha_S(M_Z^2)$. Latter usually only for one fixed $\alpha_S(M_Z^2)$.

PDF uncertainties reduced since quality of fit already worse than best fit.



Expected gluon–$\alpha_S(M_Z^2)$ small–x anti-correlation $\rightarrow$ high-x correlation from sum rule.

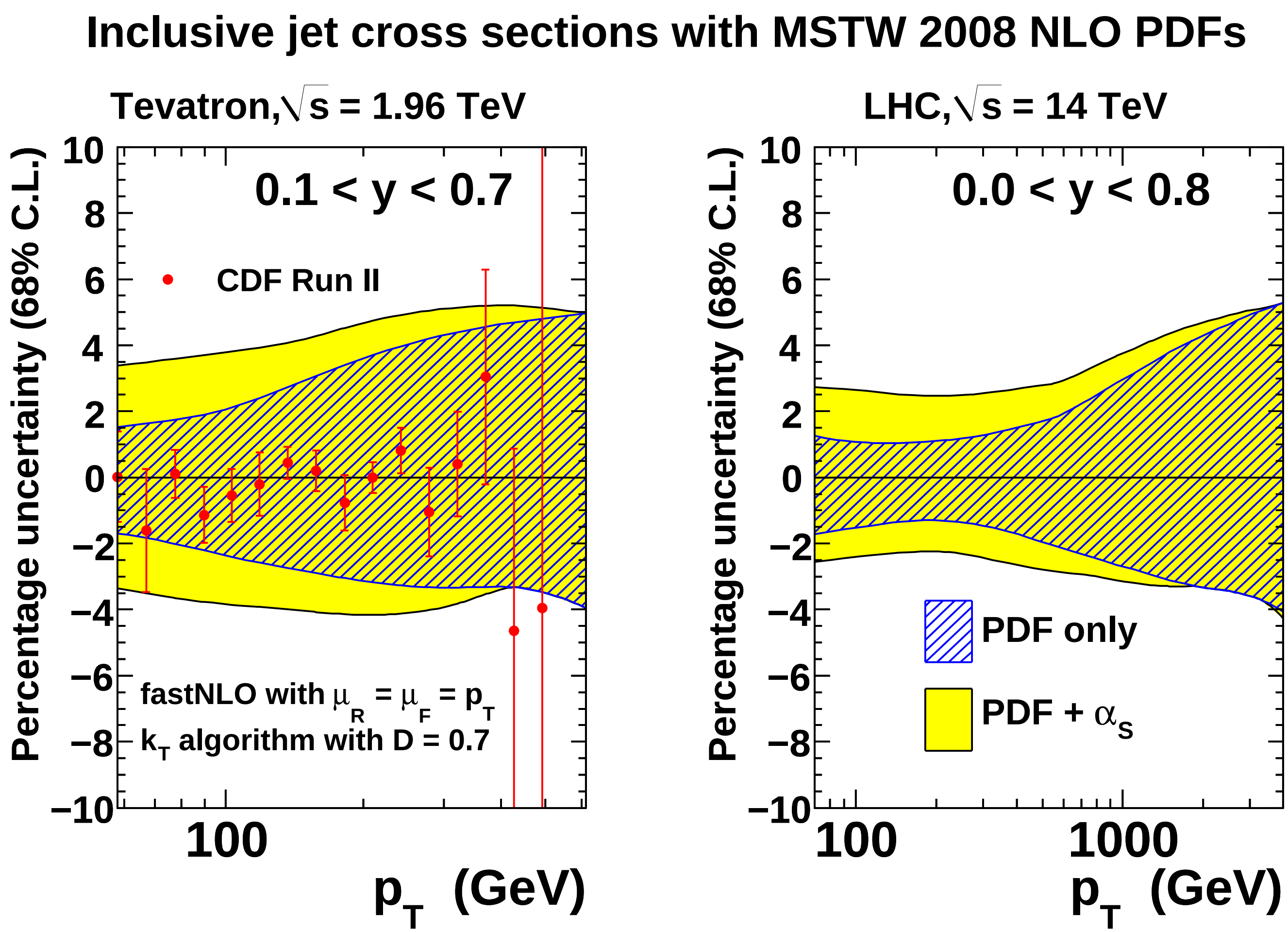
Gluon feeds into evolution of quarks, but change in $\alpha_S(M_Z^2)$ just outweighs gluon change, i.e. larger $\alpha_S(M_Z^2) \rightarrow$ slightly more evolution.



Strong anti-correlation at high-x due to evolution and positive coefficient functions.

Quarks roughly opposite to gluons.

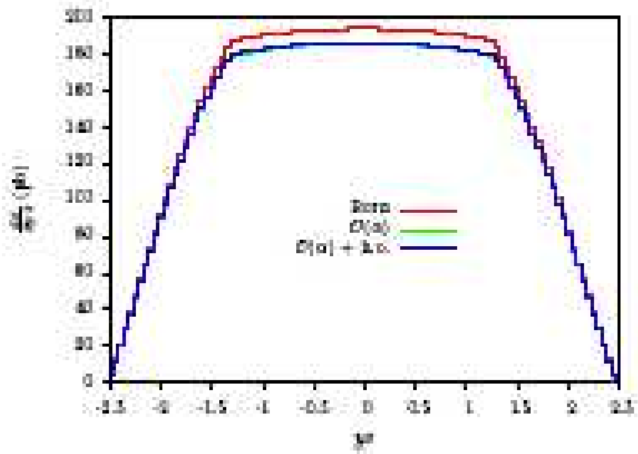
Can also investigate uncertainties for inclusive jets at Tevatron and at LHC.

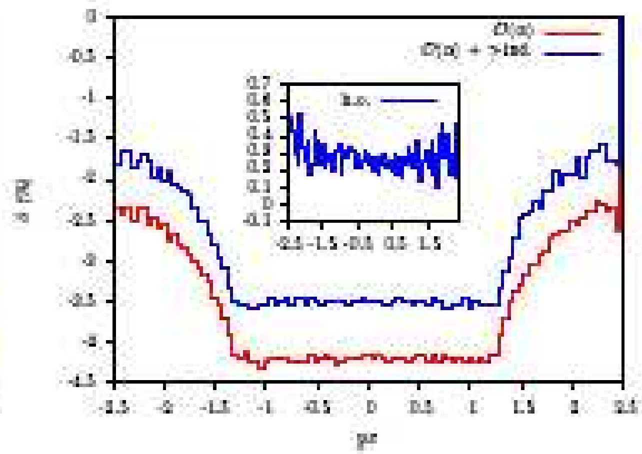


At lower p_T gluons dominate and α_S correlated. At higher p_T quarks become more important and high-x quarks anti-correlated to α_S so no additional α_S uncertainty.

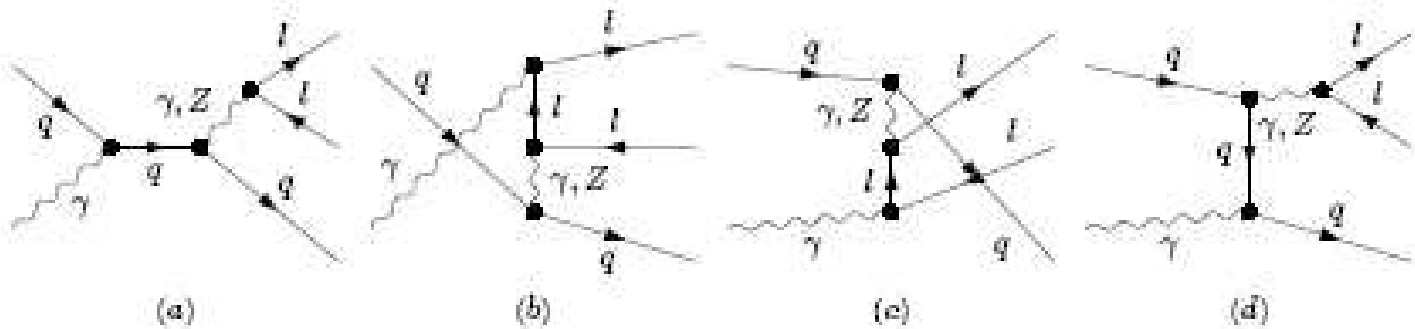
Electroweak corrections

Electroweak corrections typically a few percent, e.g. Calone Calame *et al* who look at Drell-Yan type processes.





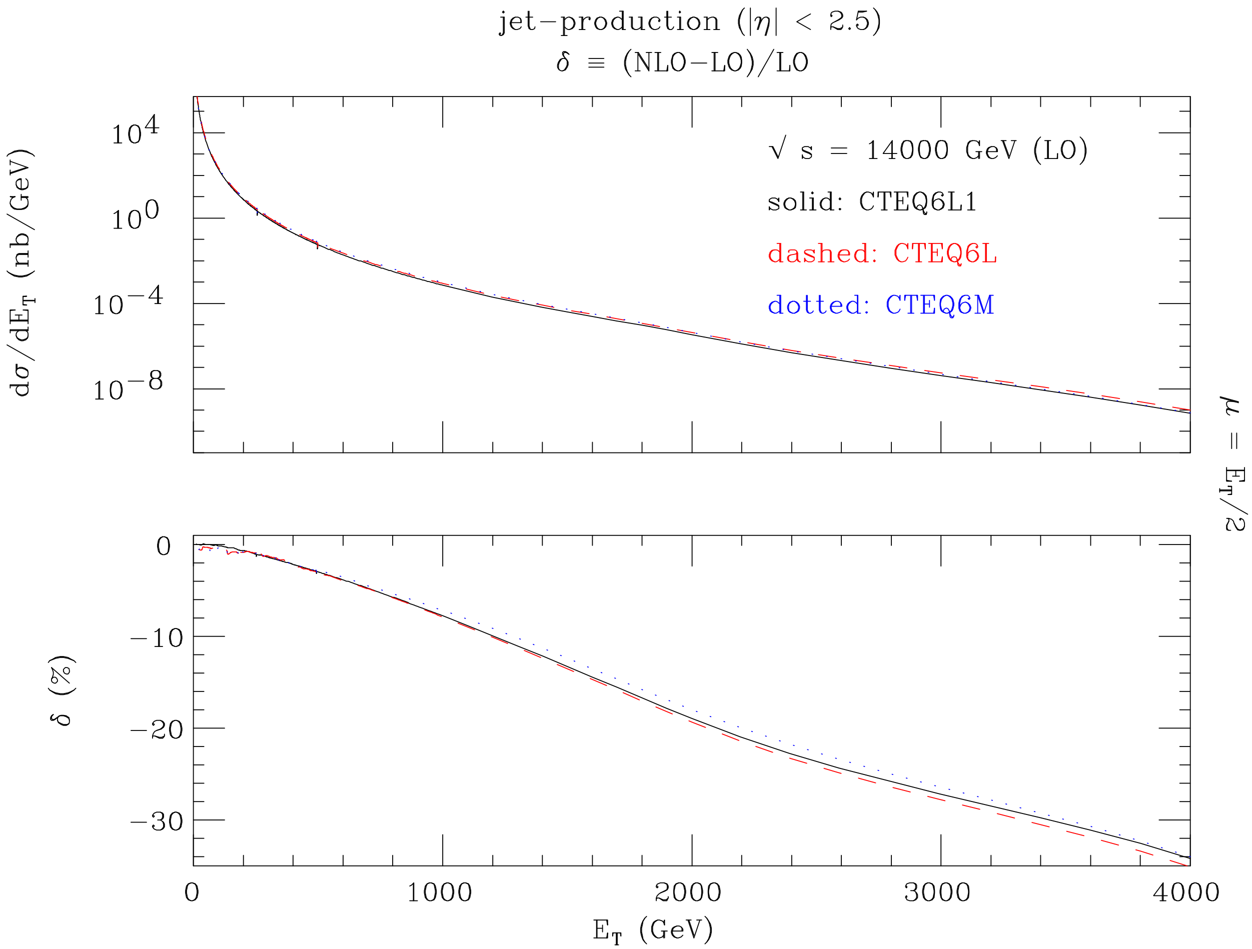
Also consider photon-induced processes. Requires the photon distribution of the proton. Currently only available for one pdf (MRST2004) set.



Can also be a couple of percent (here in opposite direction).

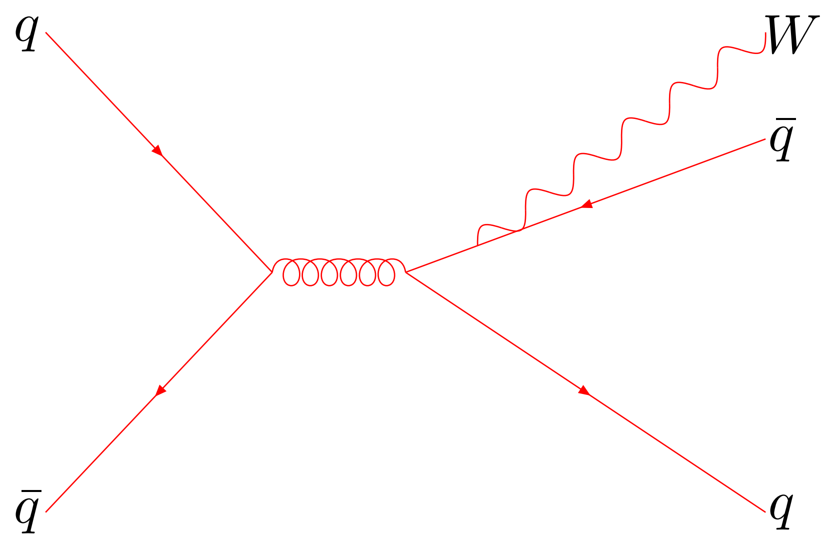
Large Electroweak corrections

Jet cross-section a major example – calculation by Moretti, Nolten, Ross, goes like $(1 - \frac{1}{3} C_F \frac{\alpha_W}{\pi} \log^2(E_T^2/M_W^2))$.



Big effect at LHC energies – $\log^2(E_T^2/M_W^2)$ a very large number. Up to 30%. Bigger than NLO QCD.

Only virtual corrections. Must have contributions of the form



Some electroweak bosons included with jets – some almost collinear with quark, and many decaying into hadrons.

Opposite sign, potentially large contribution. However, perfect cancellation will not happen. Total effect very possibly still large. Similar situation in variety of processes.

Needs calculation and decisions on experimental definitions. Very sensitive to jet veto in di-boson production.

Perhaps want partons with Weak as well as QED corrections, (splitting functions derived – P Ciafaloni and Comelli).

NNLO

Default has long been NLO. Essentially well understood. Now starting to go further.

NNLO coefficient functions for structure functions know for many years.

Splitting functions now complete. (Moch, Vermaseren and Vogt). Improve consistency of fit very slightly (MSTW), and reduces α_S.

Essentially full NNLO determination of partons. Surely this is best, i.e. most accurate.

Yes, but ... only know some hard cross-sections at NNLO.

Processes with two strongly interacting particles largely completed

DIS coefficient functions and sum rules

$pp(\bar{p}) \rightarrow \gamma^\star, W, Z$ (including rapidity dist.), H, A^0, WH, ZH.

But for many other final states NNLO not known. NLO still more appropriate.

NNLO tells us more about the convergence of perturbation theory.

Resummations may be important even beyond NNLO in some regions.

Stability order-by-order.

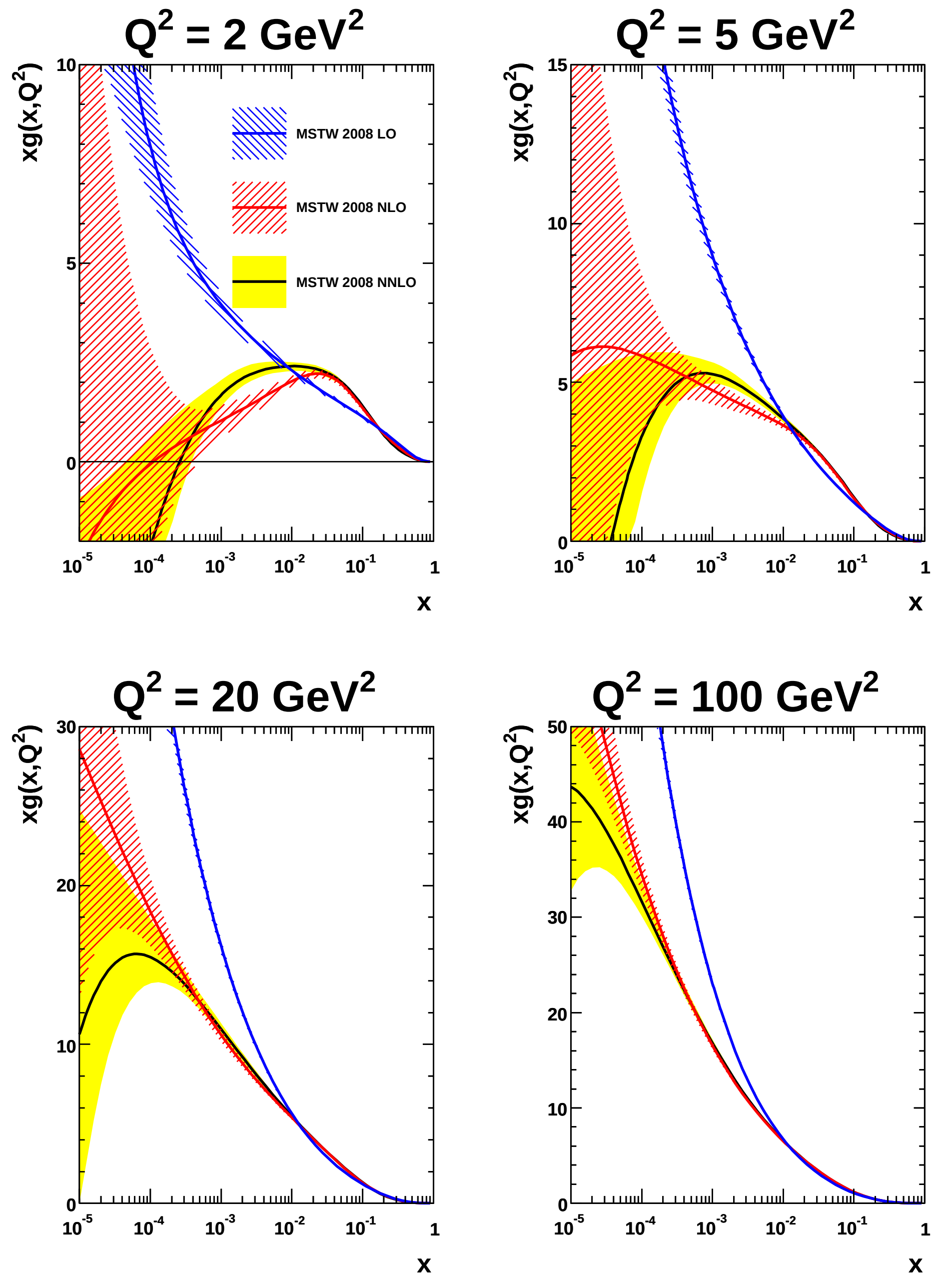
Start by looking at fixed order QCD.

The gluon extracted from the global fit at LO, NLO and NNLO.

Additional and positive small-x contributions in P_{qg} at each order leads to smaller small-x gluon at each order.

Clearly poor stability.

Similar for $F_L(x, Q^2)$

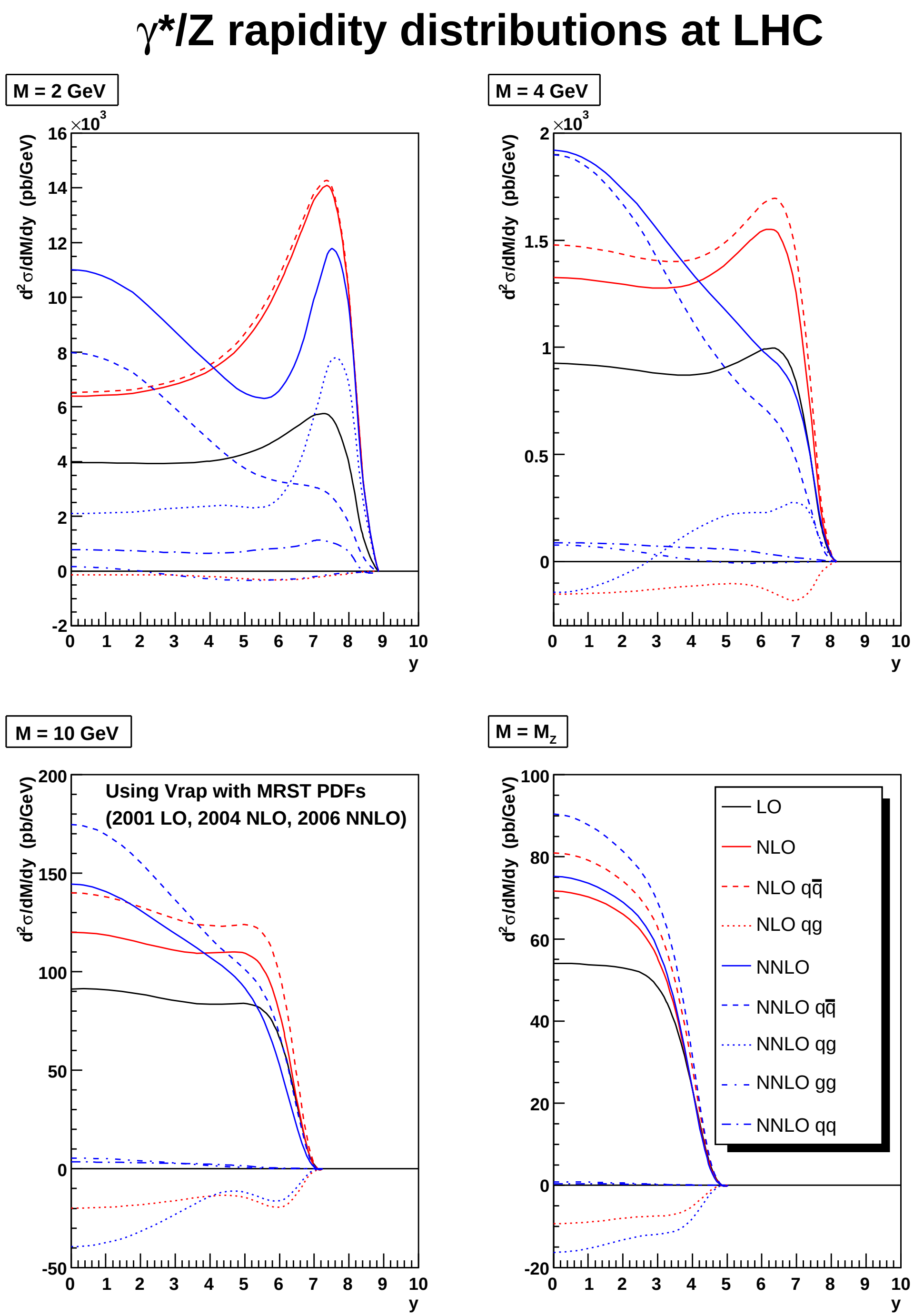


Consequences for LHC

Now have QCD calculations at LO, NLO and NNLO in the coupling constant α_S for Z, W and $\gamma^\star$ production Anastasiou, Dixon, Melnikov, Petriello).

Good stability in predictions for e.g. Z and $\gamma^\star$ cross-sections for very high virtuality.

Becomes worse at lower scales where α_S larger and large $\ln(s/M^2)$ terms appear in expansion (equivalent to $\ln(1/x)$).



Small-x Theory

Reason for this instability.

It is known that at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x).$$

$\rightarrow$ no guarantee of convergence at small x!

$$x < 0.01, \qquad \ln(1/x) > 5, \qquad \rightarrow \alpha_S \ln(1/x) > 1.$$

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

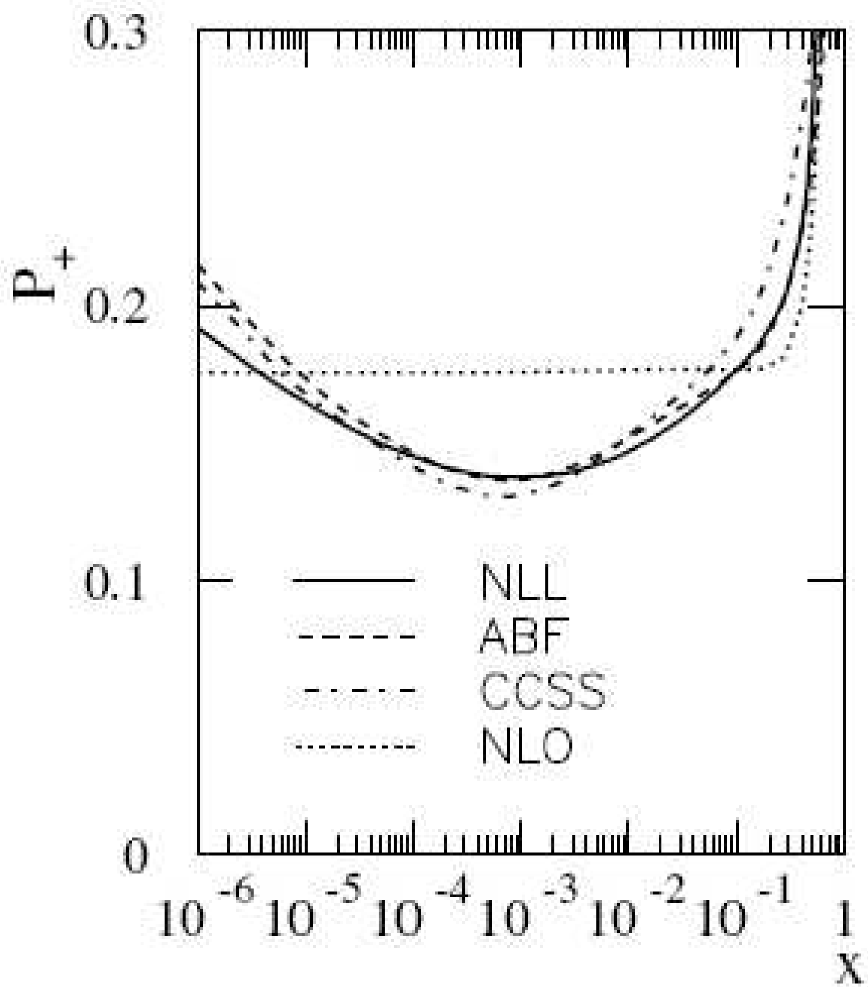
Fits work fairly well at small x, but could be better.

Good recent progress in incorporating $\ln(1/x)$ resummation Altarelli-Ball-Forte, Ciafaloni-Colferai-Salam-Stasto and White-RT.

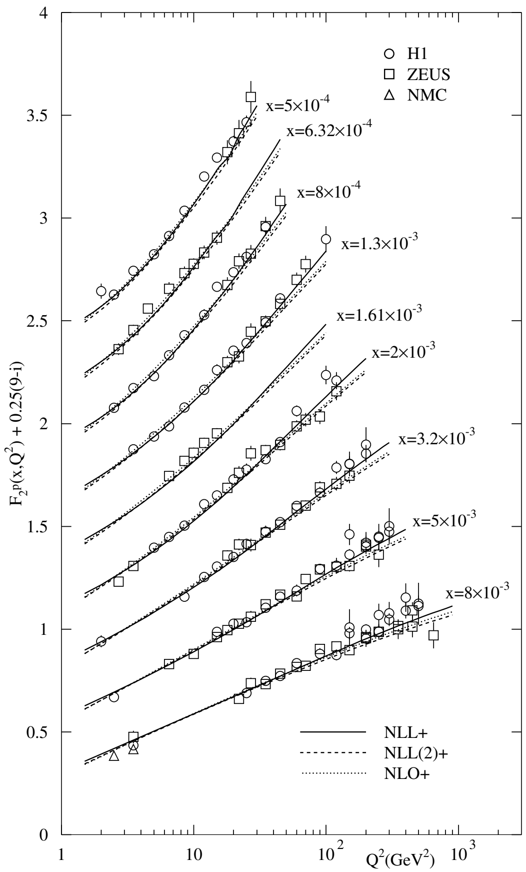
By 2008 very similar results coming from the White-RT, Ciafaloni-Colferai-Salam-Stasto and Altarelli-Ball-Forte procedures, despite some differences in technique.

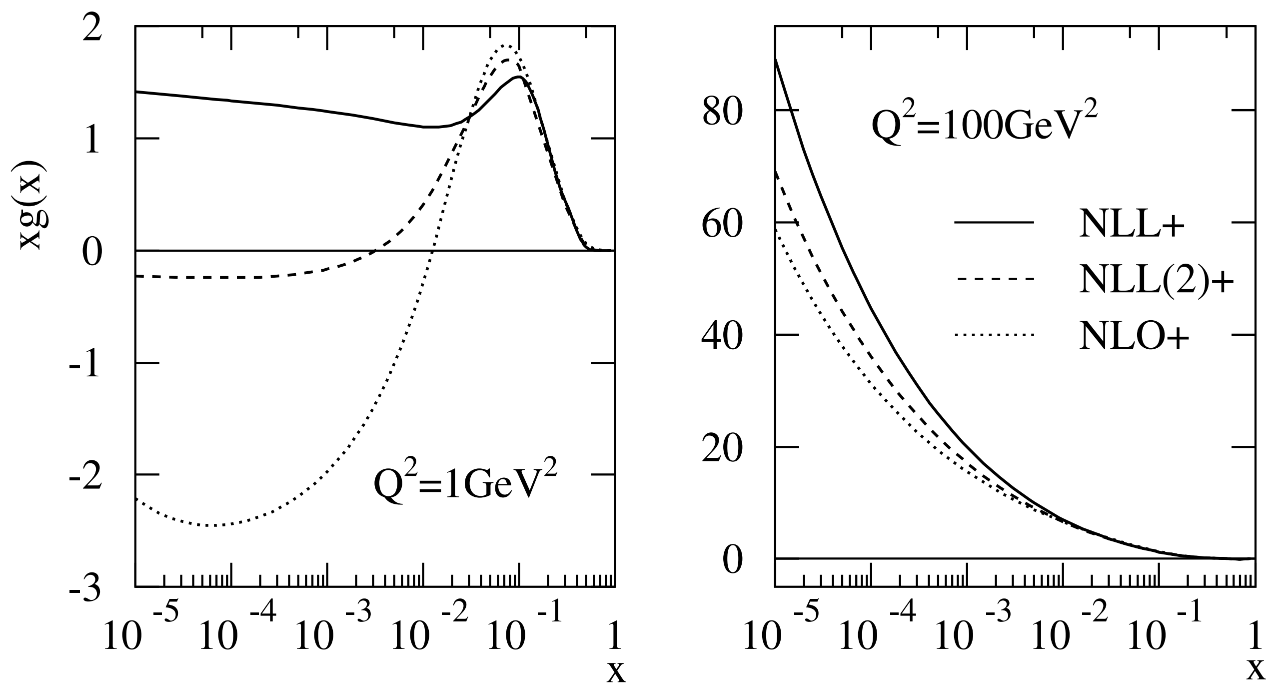
Full set of coefficient functions still to come in some cases, but splitting functions comparable.

Note, in all cases NLO corrections lead to dip in functions below fixed order values until slower growth (running coupling effect) at very small x.



A fit to data with NLO plus NLO resummation, with heavy quarks included (White,RT) performed.

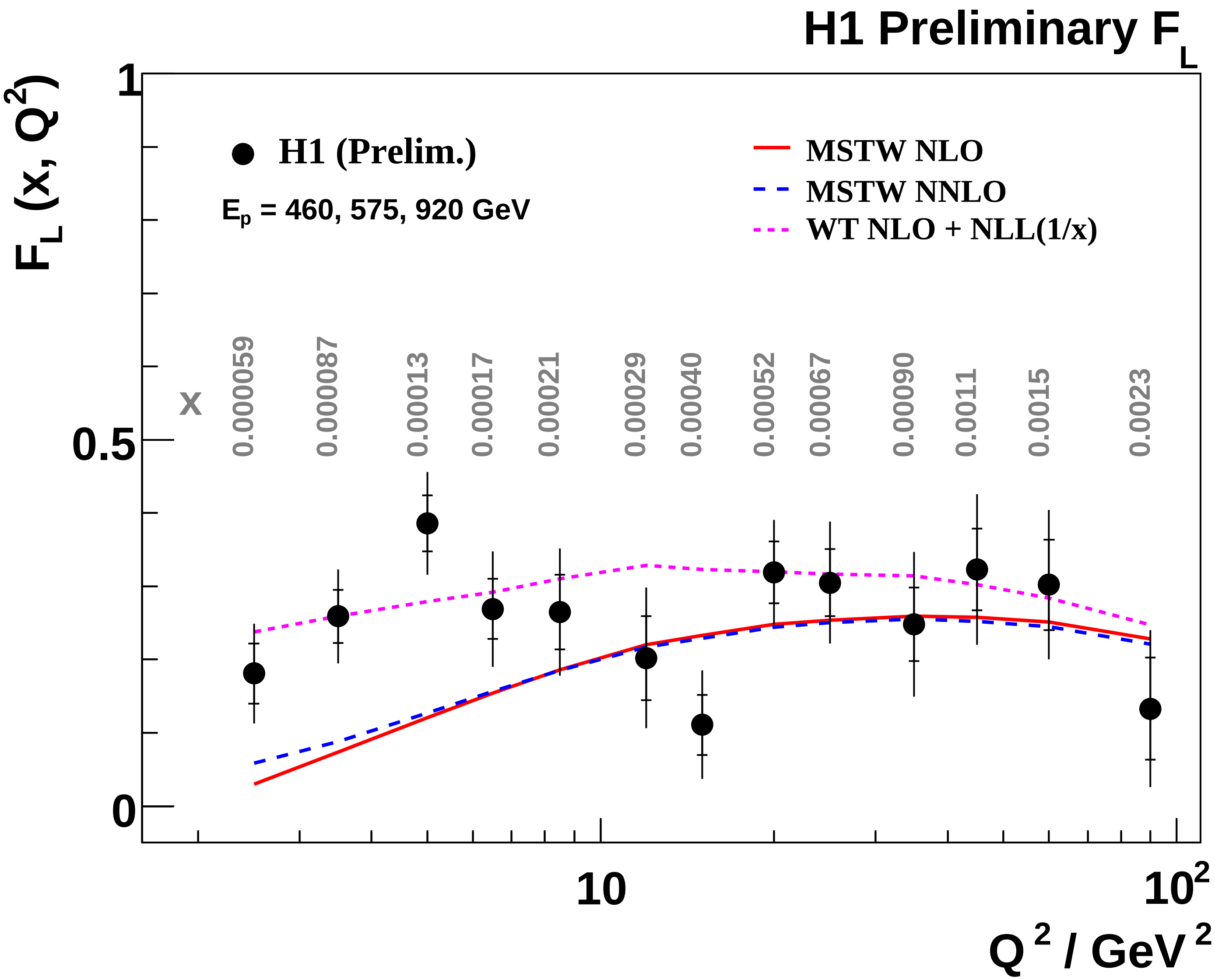




$\rightarrow$ moderate improvement in fit to HERA data within global fit, and change in extracted gluon (more like quarks at low Q^2).

Together with indications from Drell Yan resummation calculations (Marzani, Ball) few percent effect quite possible.

Comparison to H1 prelim data on $F_L(x,Q^2)$ at low Q^2 suggests resummations may be important.

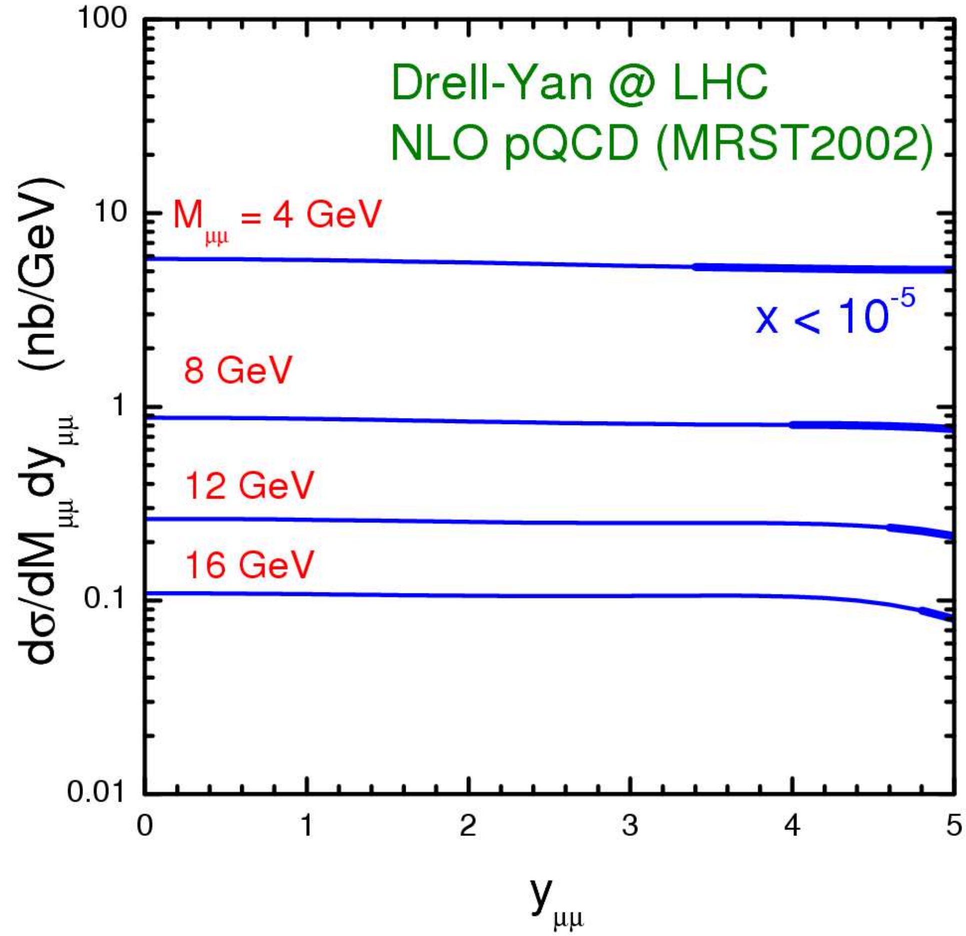


Other possible (sometimes related) explanations.

Possible to get to very low values of x at the LHC, particularly LHCb.

Can probe below $x = 10^{-5}$ - beyond range tested at HERA.

Effects possibly much larger here.



PDFs for LO Monte Carlo generators.

Often need to use generators which calculate only at LO in QCD.

LO matrix elements $+$ LO PDFS often very inaccurate.

Using NLO PDFS suggested – sometimes better, sometimes even worse (particularly small x, important for underlying event etc).

Leads to introduction of new type of LO* PDF.

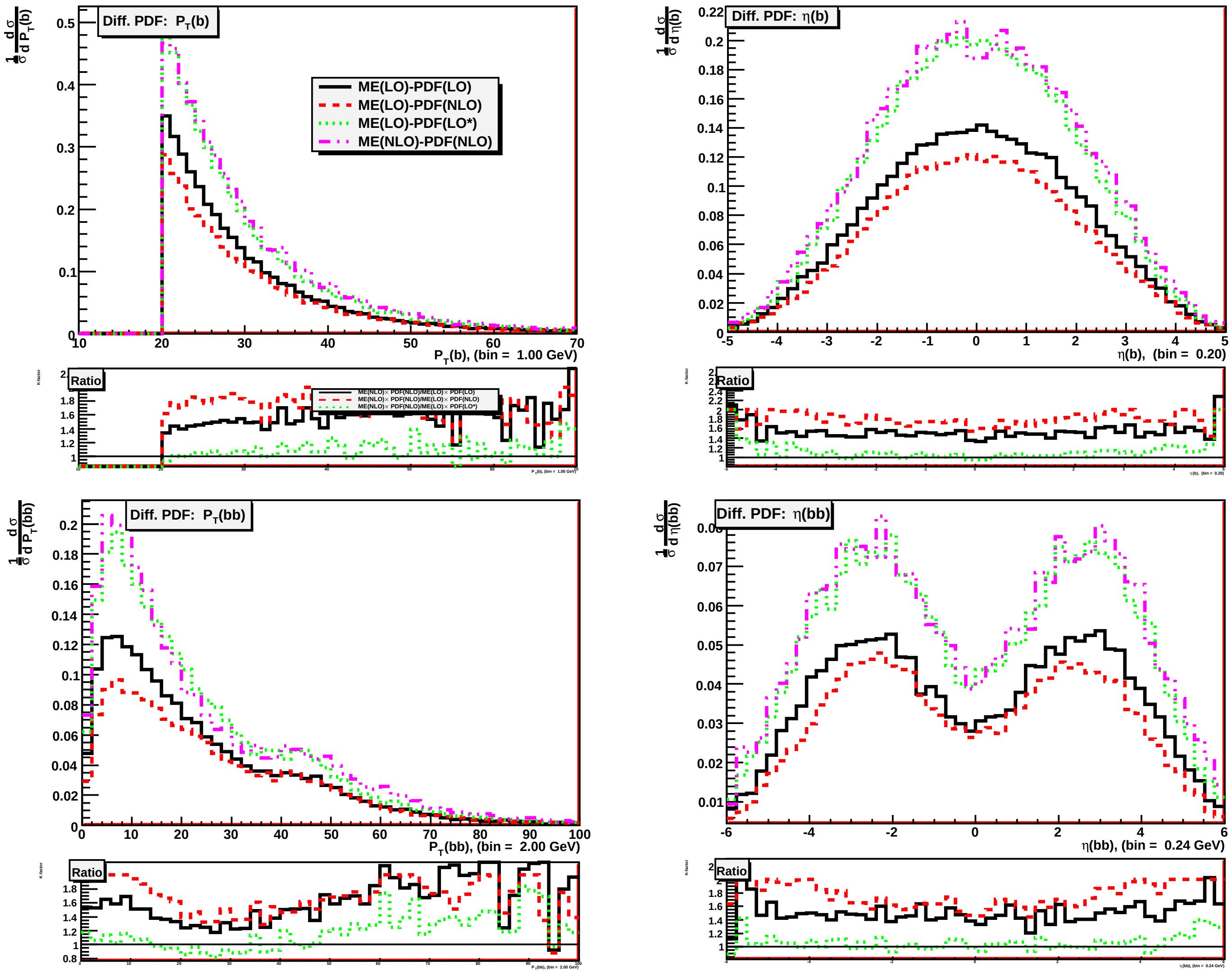
NLO corrections to cross-section usually positive $\rightarrow$ LO PDFs bigger by allowing momentum violation in global fits, using NLO α_S, fit LHC pseudo-data ......

Can also make evolution more "Monte Carlo like", e.g. change of scale in coupling.

LO* PDFs from MRST/MSTW followed by imminent ones from CTEQ.

Also work on fits using Monte Carlo generators directly (Jung $et\ al$).

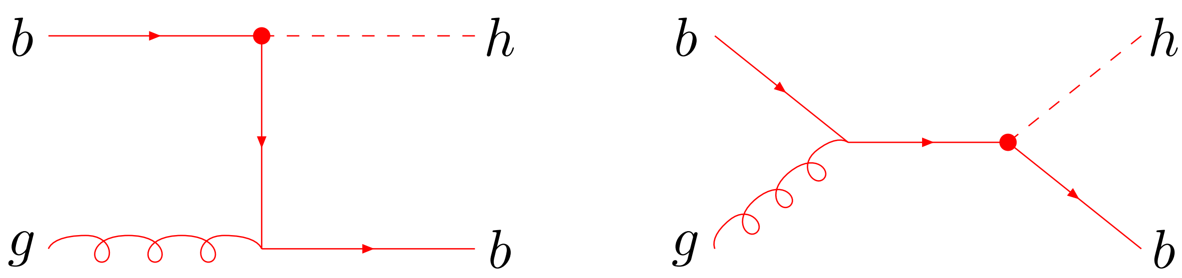
Look at e.g. distributions for single b and $b\bar{b}$ pair (Shertsnev, RT).



Results using LO* partons clearly best in normalization. NLO worst and problems with shape at low scales (i.e. small x).

Final Example

Consider bottom production along with a Higgs boson.



In Standard Model tiny since Higgs-bottom coupling $g_{b\bar{b}h} = m_b/v$, (v Higgs vacuum expectation value.) $m_b = 4.5\mathrm{GeV}$, $v = 246\mathrm{GeV}$.

In Minimal Supersymmetric Standard Model two Higgs doublets coupling separately to d-type and u-type quarks. Expectation values v_d and v_u.

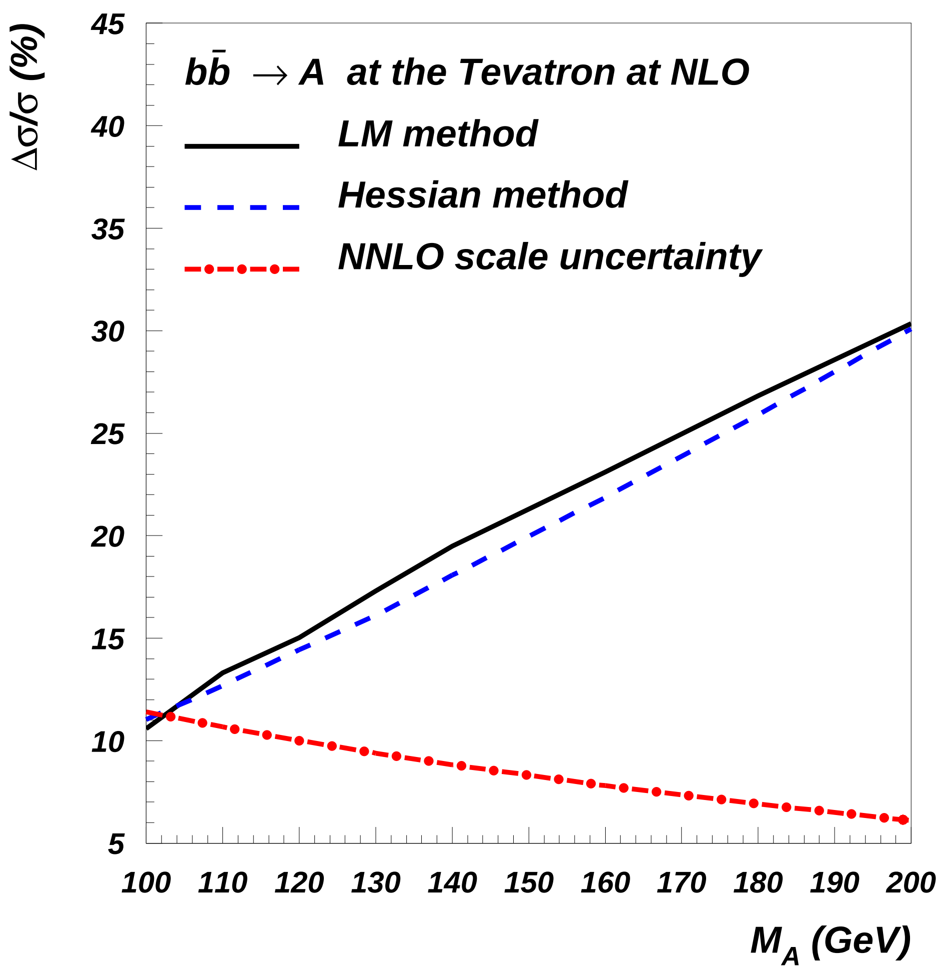
Ratio $\tan\beta = v_u/v_d \longrightarrow$ enhancement of Higgs-bottom coupling

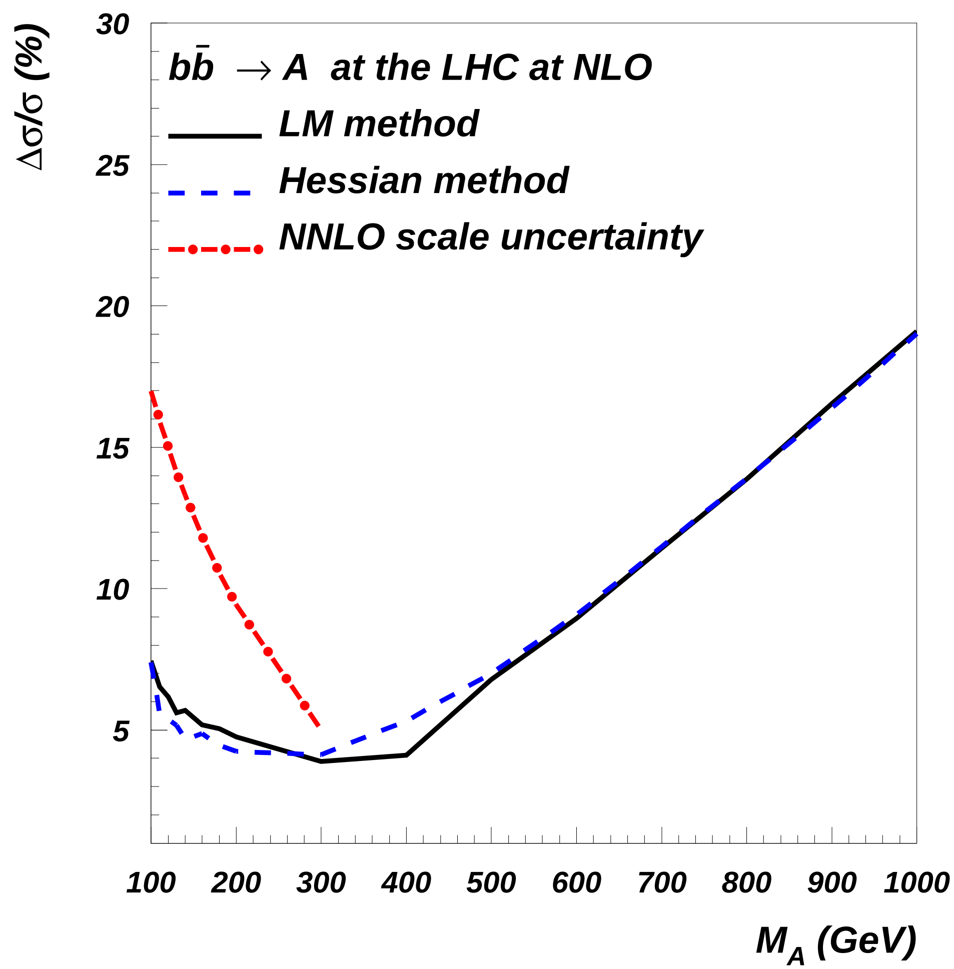
$$g_{b\bar{b}h} \propto \frac{g^{SM}_{b\bar{b}h}}{\cos\beta}.$$

Bounds from LEP, $\tan\beta$ large $\rightarrow$ $\cos\beta$ small. Enhancement of Higgs-bottom coupling.

Example of need to understand both heavy flavours and small-x physics for LHC.

Production of supersymmetric Higgs depends on parton uncertainties (Belyaev, Pumplin, Tung and Yuan), heavy flavour procedure and high-energy (small-x) treatment.





Conclusions

One can determine the parton distributions and predict cross-sections at the LHC, and the fit quality using NLO or NNLO QCD is fairly good.

Various ways of looking at uncertainties due to errors on data. Uncertainties naively rather small – $\sim 1-5\%$ for most LHC quantities. Ratios, e.g. W^+/W^- tight constraint on partons.

Effects from input assumptions e.g. selection of data fitted, cuts and input parameterisation can shift central values of predictions significantly. Also affect size of uncertainties. Want balance between freedom and sensible constraints.

Complete heavy flavour treatments essential in extraction and use of PDFs.

PDFs and α_S heavily correlated.

Electroweak corrections potentially large at very high energies – $\ln^2(E^2/M_W^2)$.

Errors from higher orders/resummation potentially large. Direct measurement of $F_L(x,Q^2)$ at HERA now testing this. At LHC measurement at high rapidities, e.g. W, Z would be useful in testing understanding of QCD, and particularly quantities sensitive to low x at low scales, e.g. low mass Drell-Yan.

Extraction of PDFs from existing data and use for LHC far from a straightforward procedure. Lots of theoretical issues to consider for real precision. Relatively few cases where Standard Model discrepancies will not require some significant input from PDF physics to determine real significance.

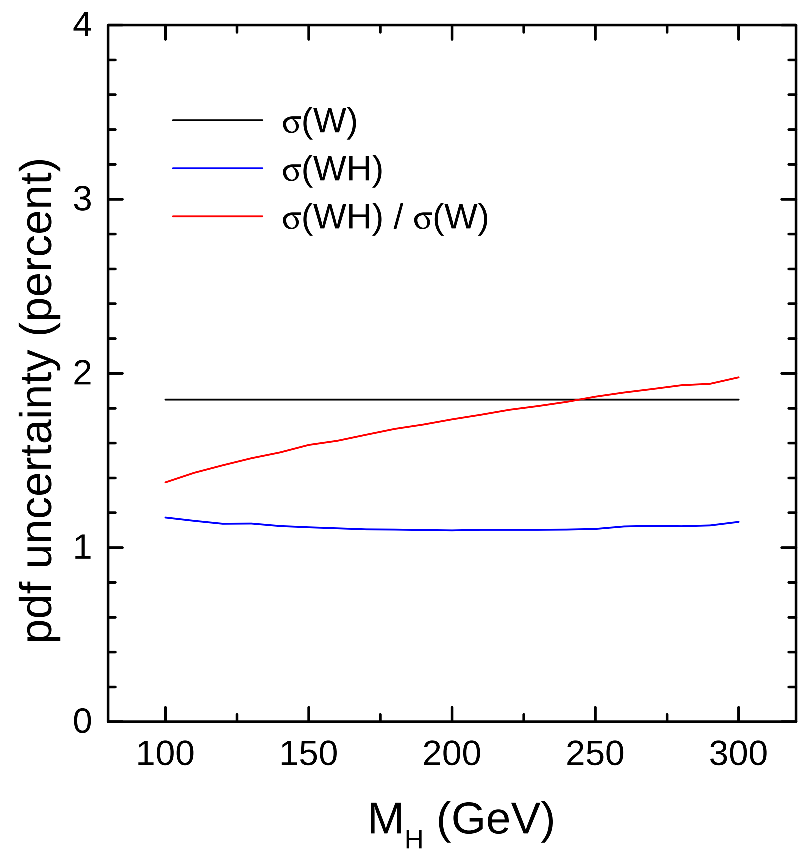
Could $\sigma(W)$ or $\sigma(Z)$ be used to calibrate other cross-sections, e.g. $\sigma(WH)$, $\sigma(Z')$?

$\sigma(WH)$ more precisely predicted because it samples quark pdfs at higher x, and scale, than $\sigma(W)$.

However, ratio shows no improvement in uncertainty, and can be worse.

Partons in different regions of x are often anti-correlated rather than correlated, partially due to sum rules.

pdf uncertainties on W, WH cross sections at LHC (MRST2001E)



Importance of treating heavy flavour correctly illustrated at NNLO with MRST2006 partons.

Previous approximate NNLO sets used (declared) approximate VFNS at flavour thresholds.

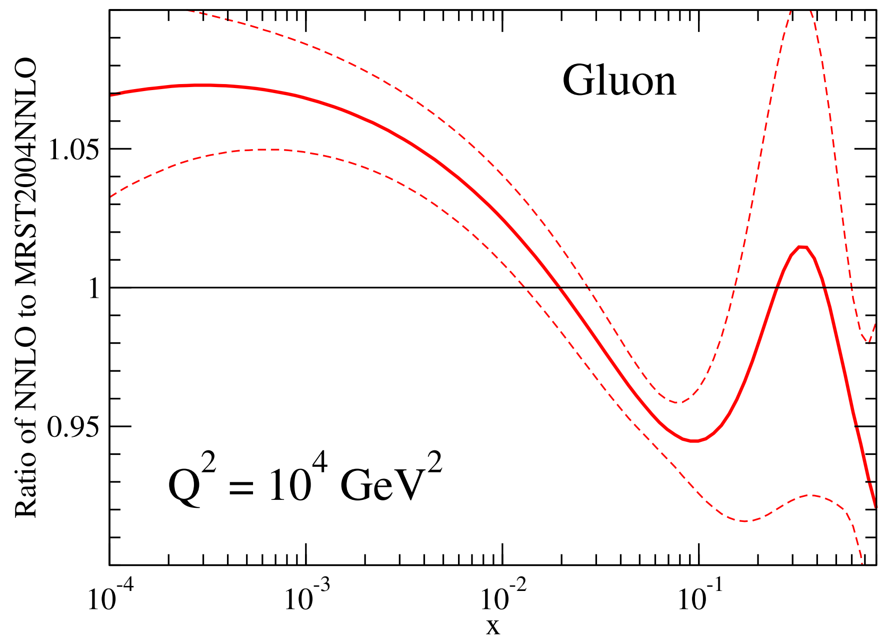
Full VFNS $\rightarrow$ flatter evolution of charm

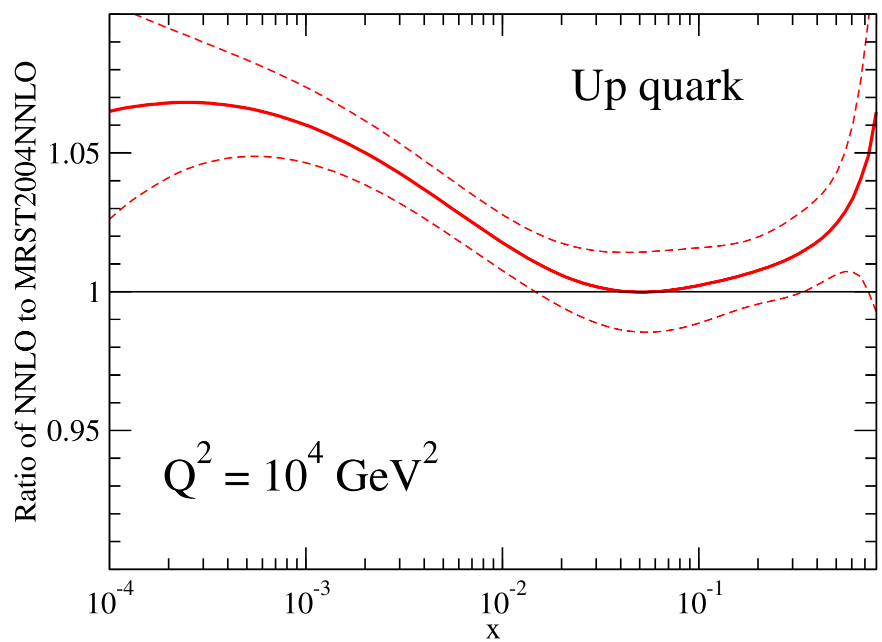
$\rightarrow$ bigger gluon and more evolution of light sea and bigger α_S.

$\rightarrow$ 6% increase in σ_W and σ_Z at the LHC.

This is a correction not uncertainty.

Very important changes nonetheless.





Treatment of errors.

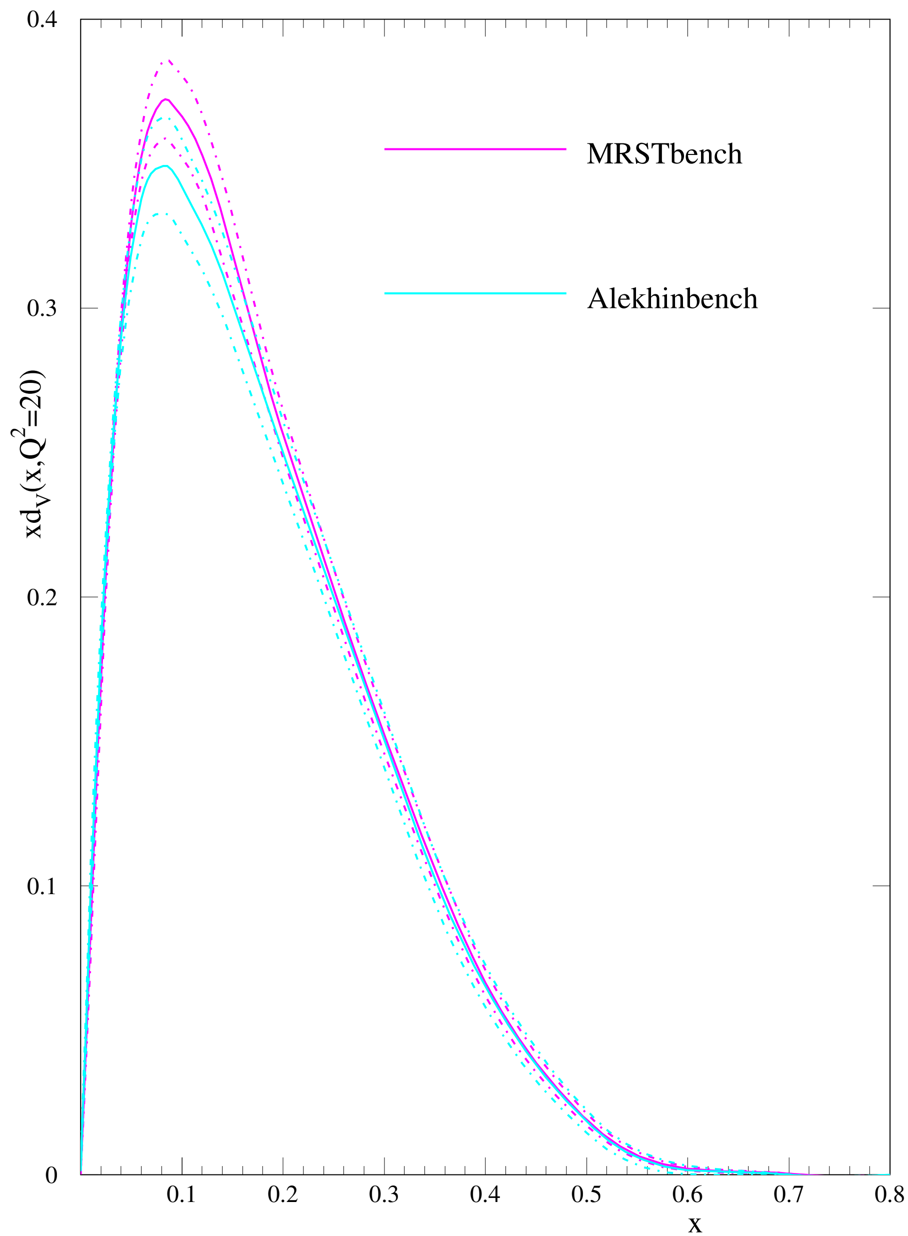
Exercise for $HERA-LHC$ meeting. Fit proton and deuteron structure function data from H1, ZEUS, NMC and BCDMS, for $Q^2 > 9\mathrm{GeV}^2$ using $ZM-VFNS$ and same form of parton inputs at same $Q_0^2 = 1\mathrm{GeV}^2$.

Very conservative fit.

Compare rigorous treatment of all systematic errors (Alekhin) with simple quadratures approach (MRST), both with $\Delta\chi^2 = 1$.

$\rightarrow$ some difference in central values (other possible reasons) and similar errors.

Fairly consistent.



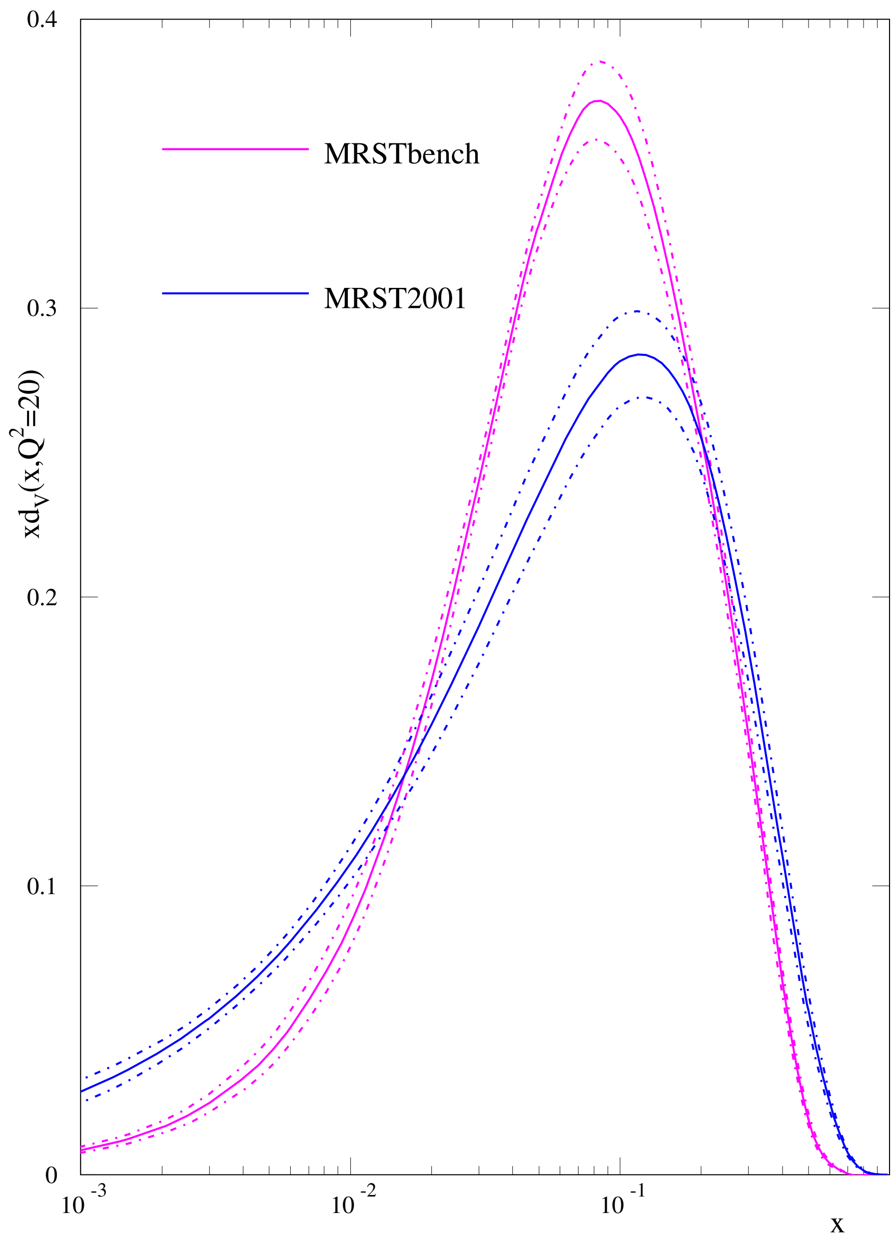
Back to HERA-LHC benchmark partons.

How do partons from very conservative, structure function only data compare to global partons?

Compare to MRST01 partons with uncertainty from $\Delta\chi^2 = 50$.

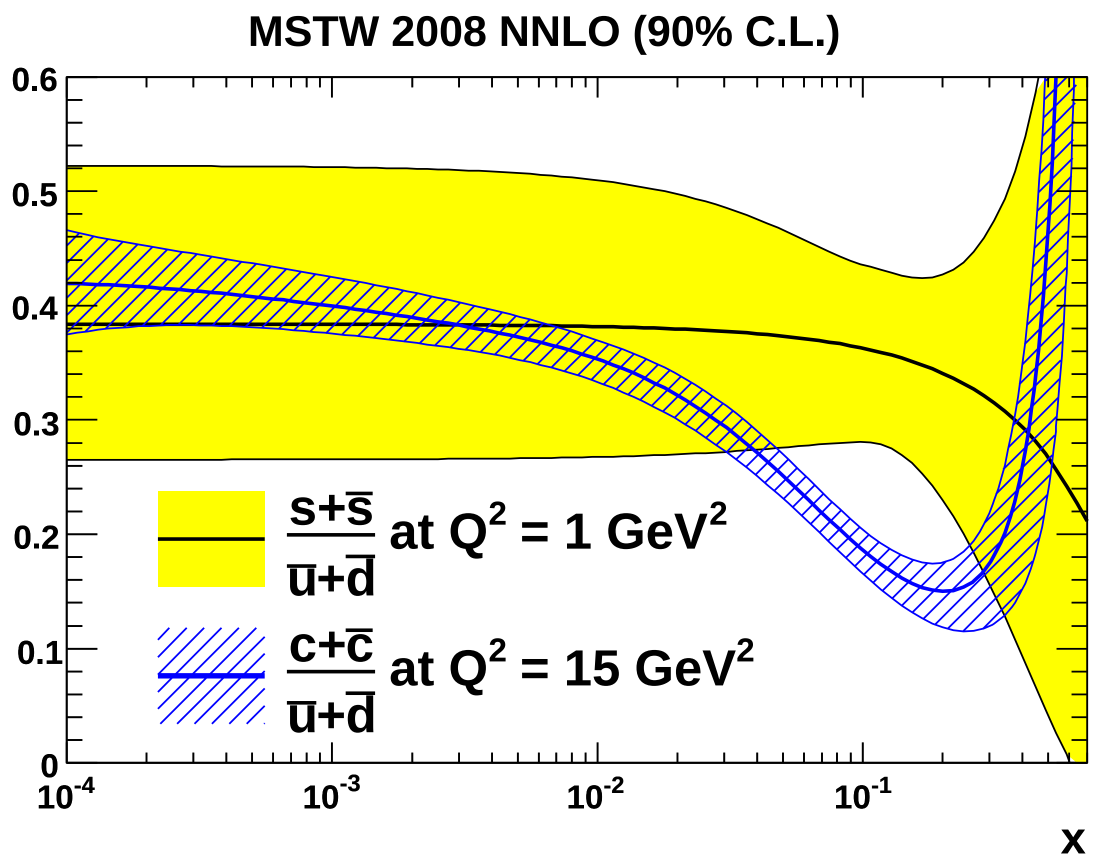
Enormous difference in central values.

Errors similar.



Strange itself has some non-insignificant mass, and this should qualitatively lead to suppression compared to light sea quarks up and down.

When c and b turn on they evolve like massless quarks, but always lag behind. $\rightarrow$ some suppression at all x for finite Q^2.



$c+\bar{c}$ evolved through $\sim 7-8$ times input scale similar to $s+\bar{s}$ at $Q^2 = 1\mathrm{GeV}^2$. Do not expect exact correspondence, but very good except $c+\bar{c}$ more suppressed at $x \sim 0.1$. (Implication for $s+\bar{s}$ from recent HERMES $K^{\pm}$ data).

No obvious advantage in using $\sigma(t\bar{t})$ as a calibration SM cross-section, except maybe for very particular, and rather large, M_H.

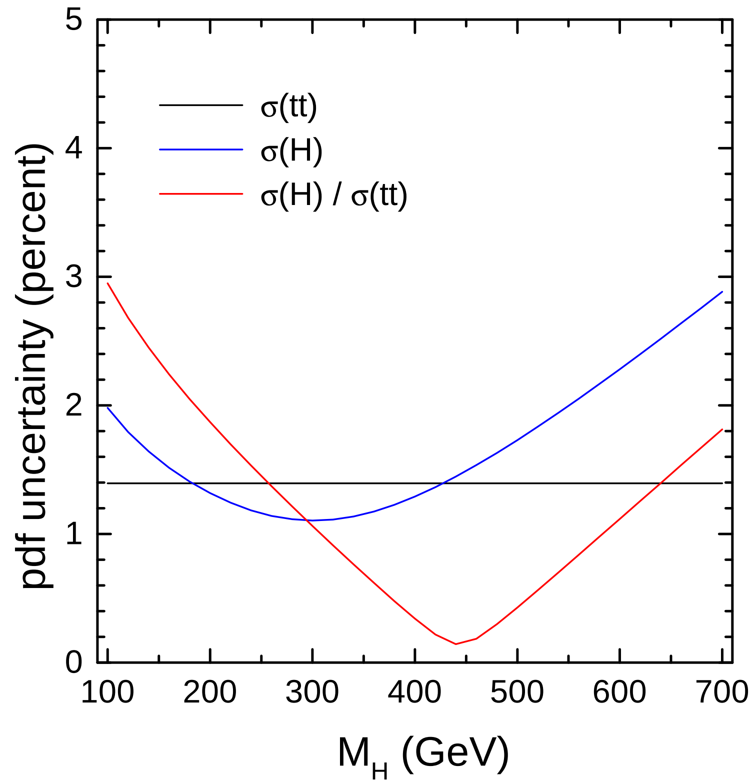
However, a light (SM or MSSM) Higgs dominantly produced via $gg \to H$ and the cross-section has small pdf uncertainty because $g(x)$ at small x is well constrained by HERA DIS data.

Current best (MRST) estimate, for $M_H = 120$ GeV: $\delta\sigma_H^{\rm NLO}(\text{expt pdf}) = \pm 2 - 3\%$ with less sensitivity to small x than $\sigma(W)$.

Much smaller than the uncertainty from higher-order corrections, for example, Catani et al,

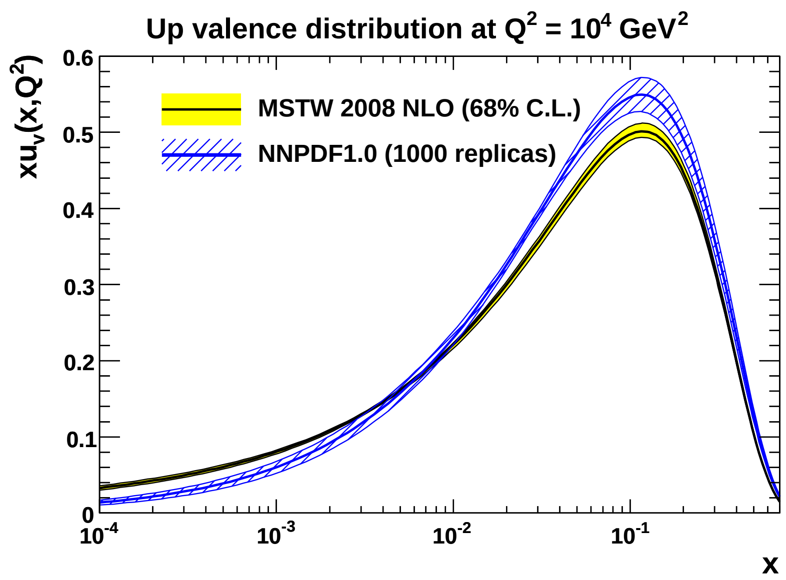
$\delta\sigma_H^{\rm NNLL}(\text{scale variation}) = \pm 8\%$

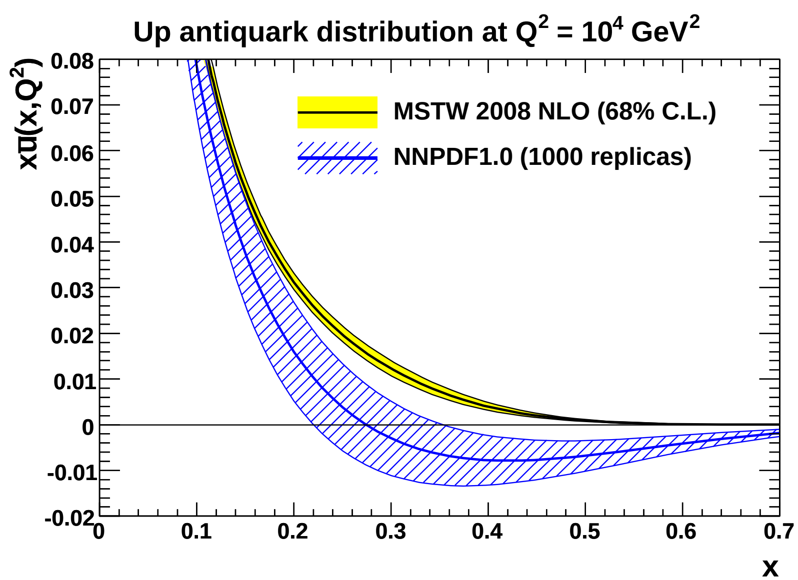
pdf uncertainties on top, (gg→) H cross sections at LHC (MRST2001E)



Note that MSTW2008 and NNPDF sets sometimes differ significantly in central values though.

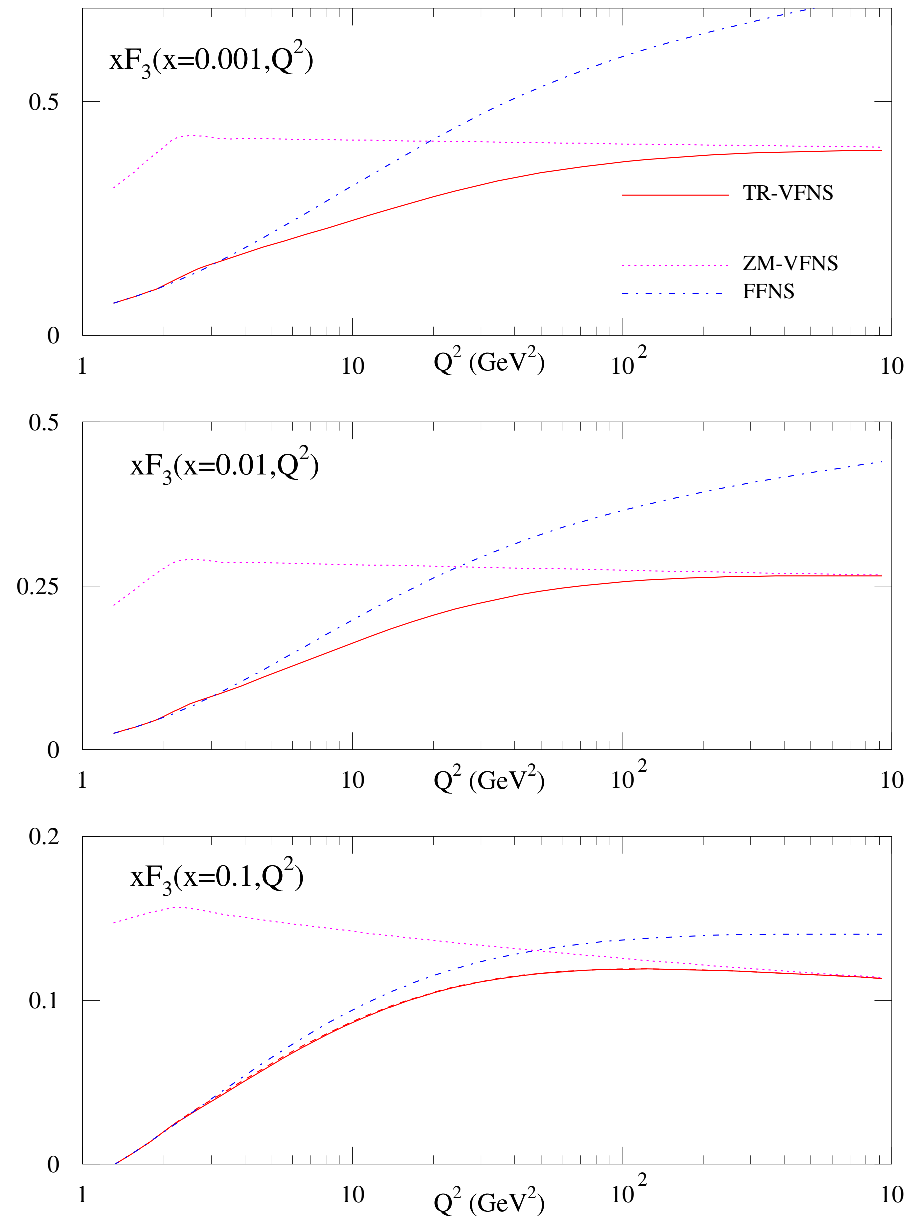
Clearly seen for up valence and anti-up distributions.





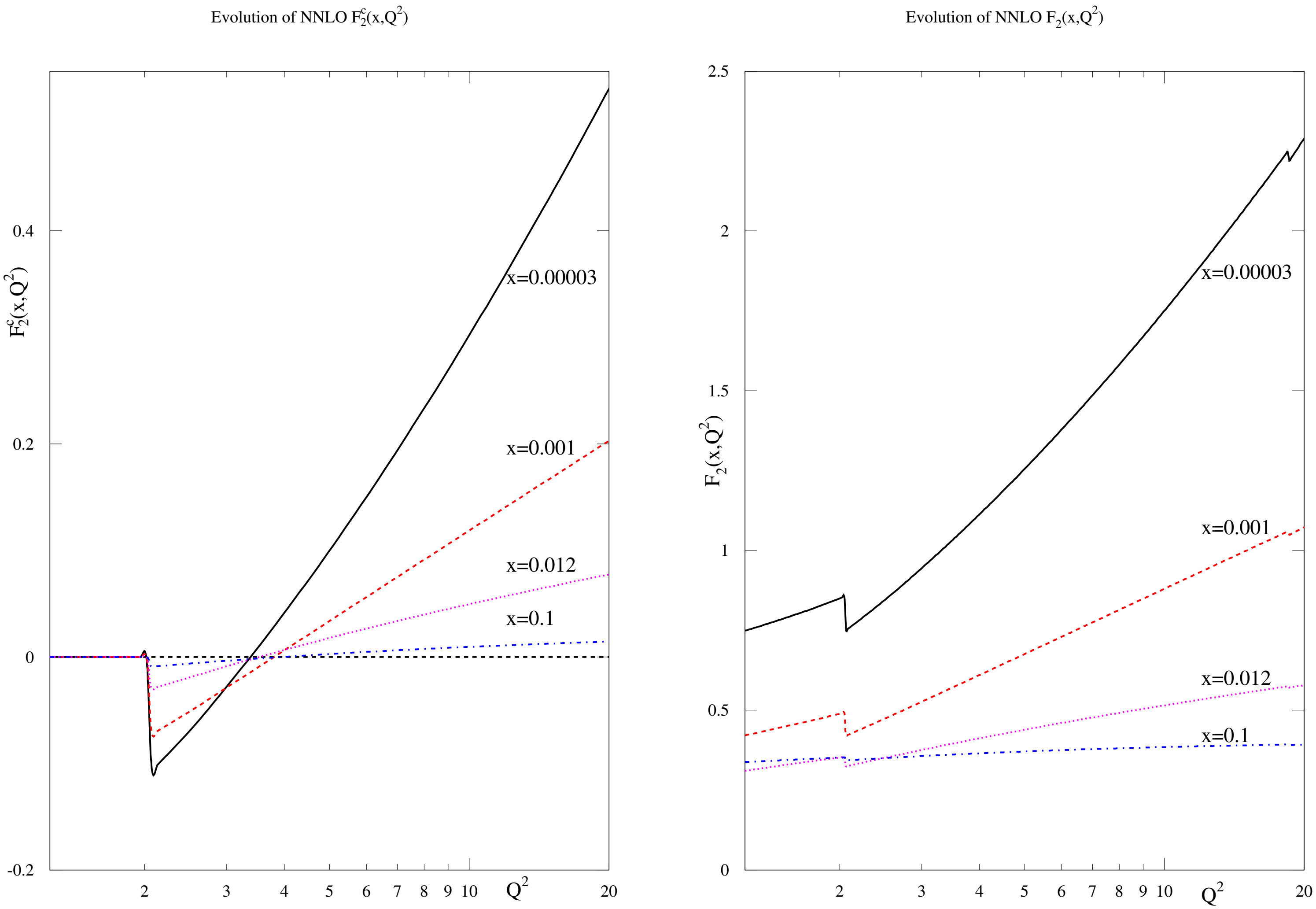
Need a general **Variable Flavour Number Scheme** (VFNS) interpolating between the two well-defined limits of $Q^2 \leq m_H^2$ and $Q^2 \gg m_H^2$.

Conclusion easily reached by looking at the extrapolation between the two simple kinematic regimes for xF_3, measured using neutrino scattering at NuTeV



At NNLO additional complications – partons become discontinuous.

ZM-VFNS leads to peculiar, unphysical results. FFNS not known at this order.



Makes need for **Variable Flavour Number Scheme** more vital but also more difficult to implement.