CERN TH, 9 December 2016

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Novel approach to QCD factorization

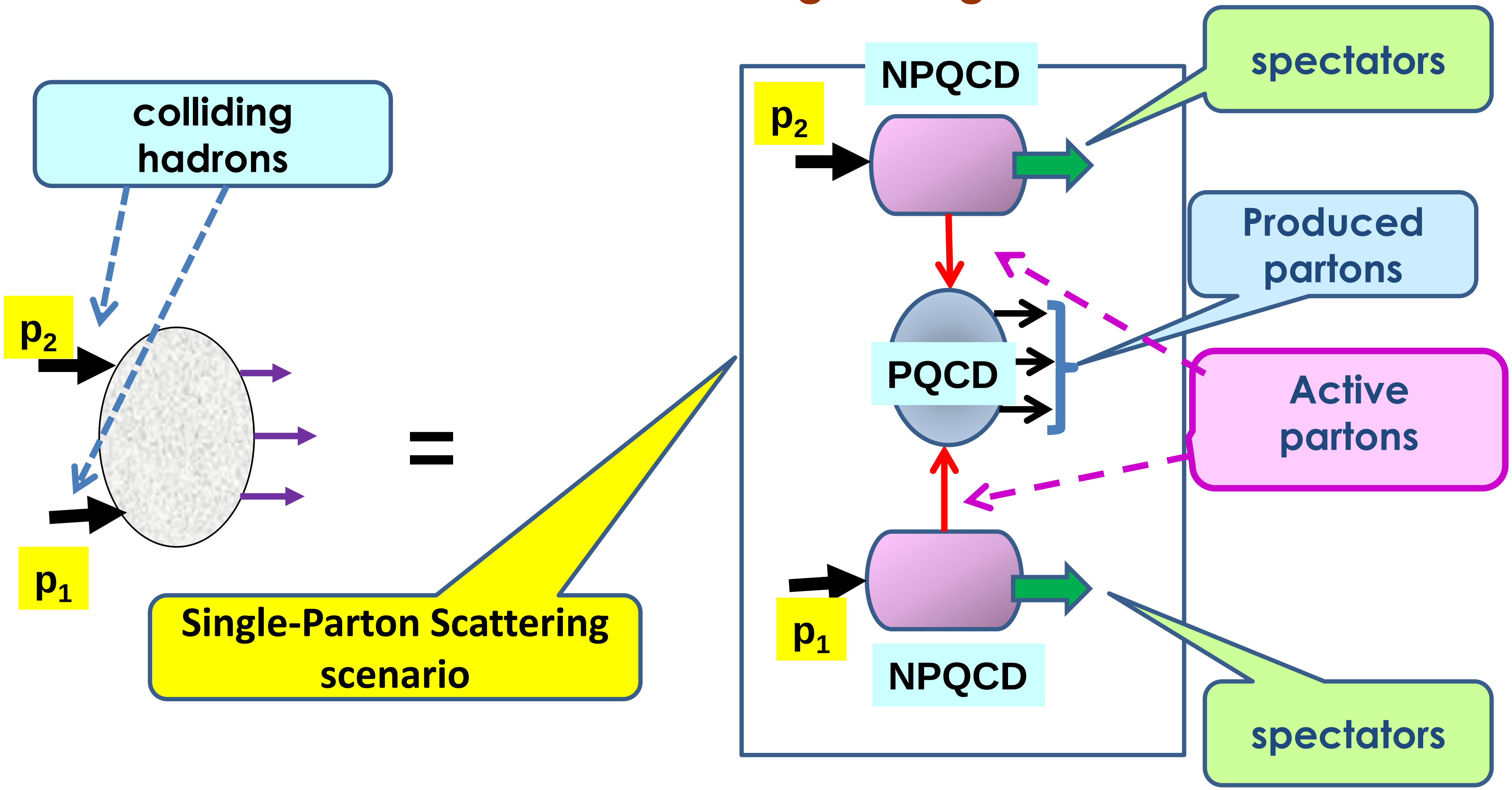
talk based on results obtained in collaboration with
M. Greco and S.I. Troyan

Need for QCD factorization:
Description of hadronic reactions involves QCD calculations at both high and low energies. However, QCD is poorly known at low energies; the confinement problem has not been solved, so approximation methods are needed to mimic the straightforward QCD calculations at low energies. QCD factorization is the most popular approximation method.

Essence of QCD factorization:
Step 1: non-perturbative inputs are introduced through either models or fits.
Step 2: the inputs are evolved with perturbative means (evolution equations).

Comment: non-perturbative inputs are introduced without theoretical grounds. I focus on theoretical restrictions on the inputs

Scenarios of hadronic collisions at high energies



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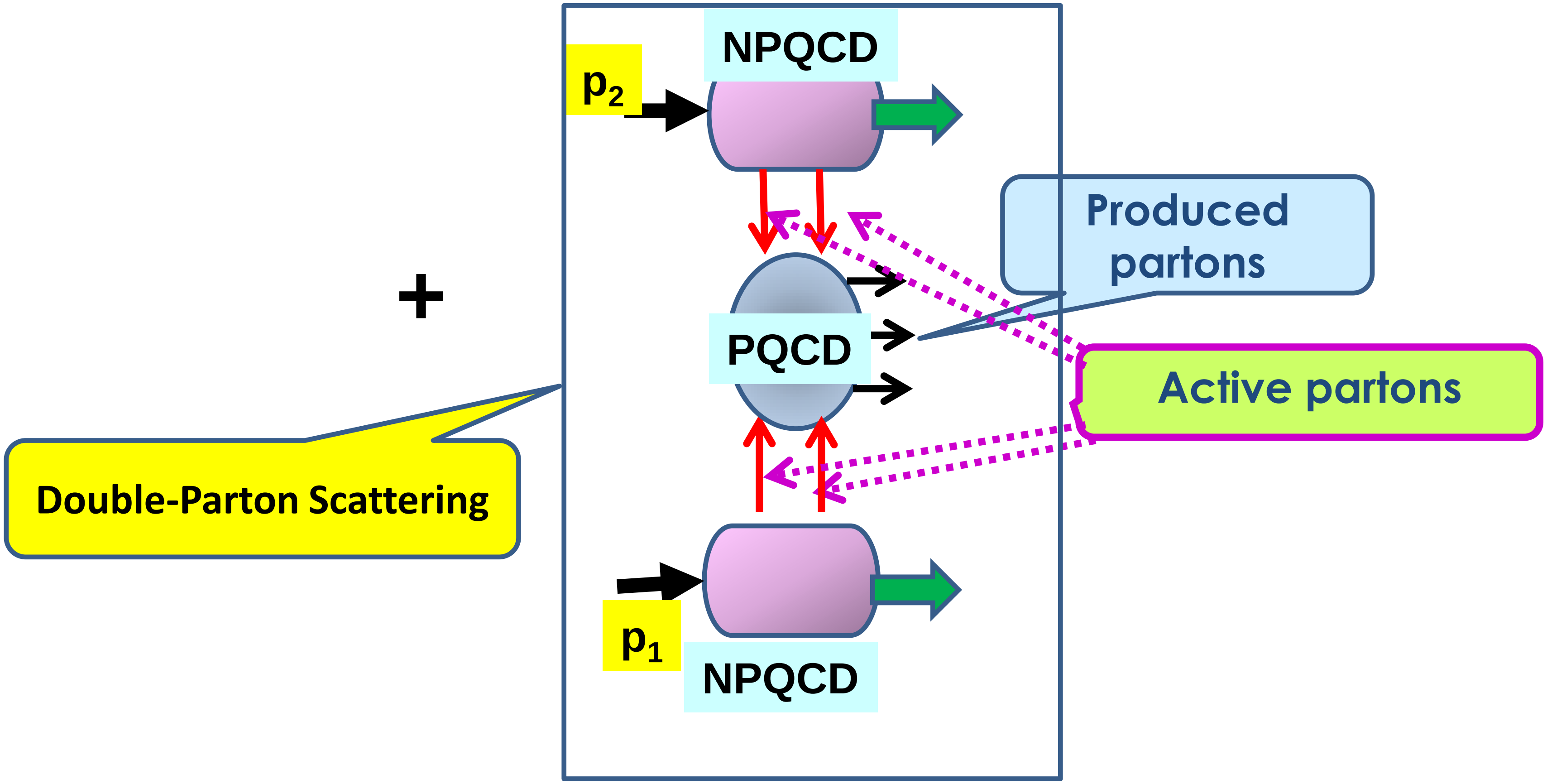
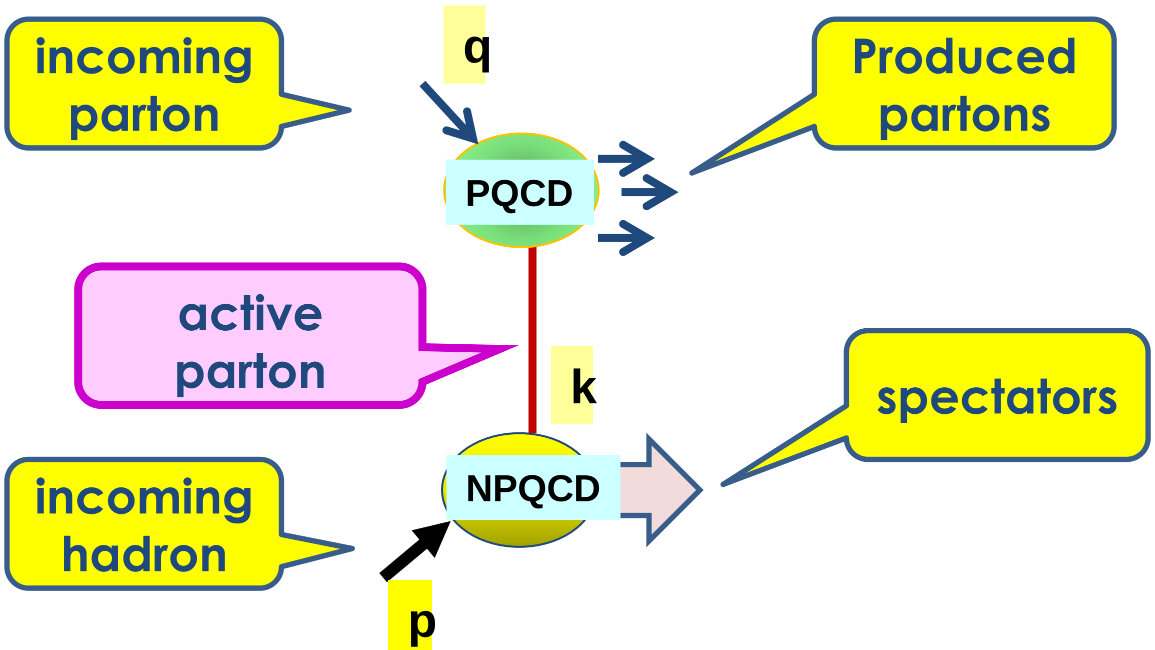


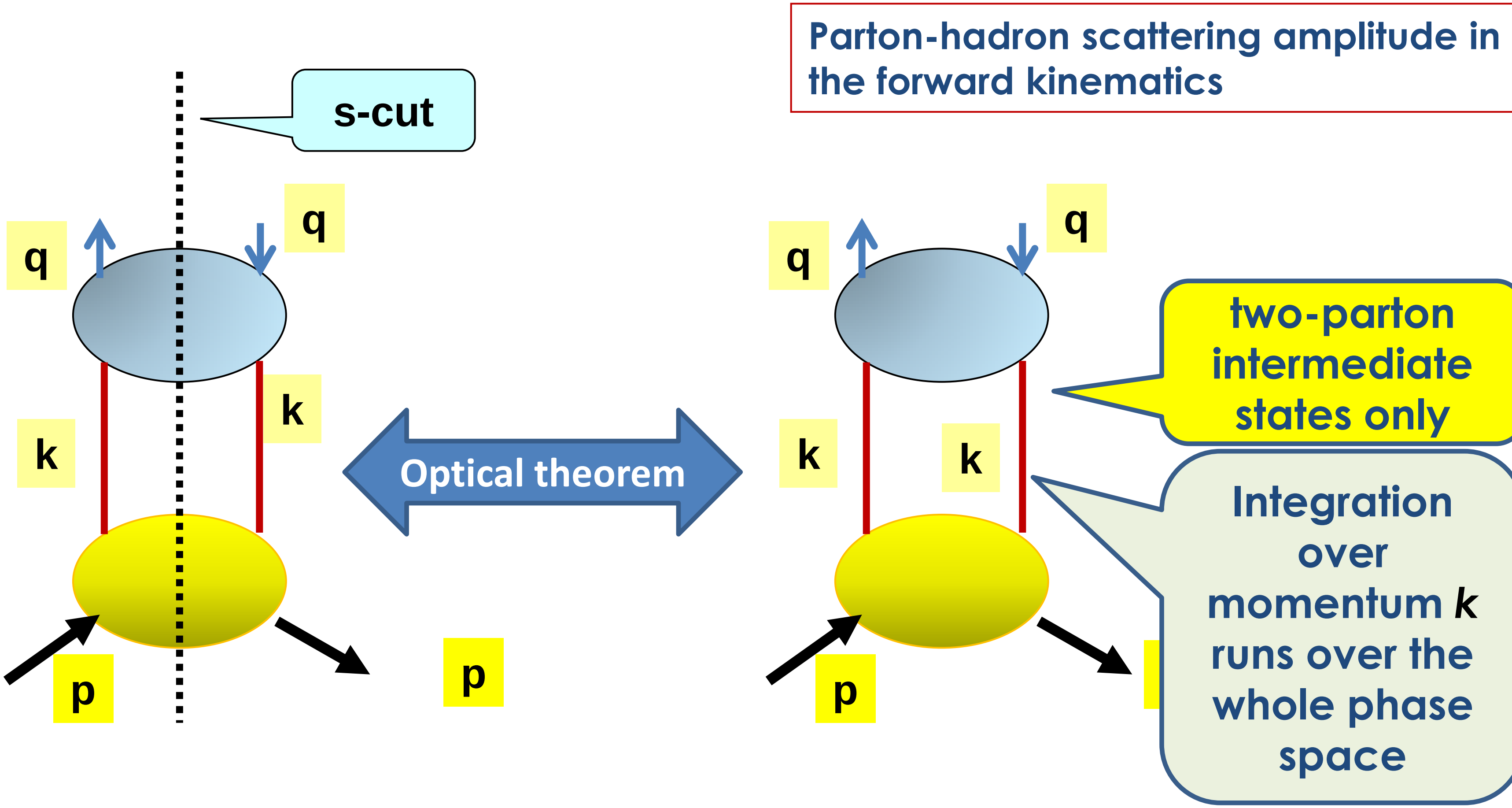
\+ contributions of more complicated Multi-Parton states

Single-Parton Scenario is much more popular than Multi-Parton one, so in the present talk I will focus on **SINGLE-PARTON COLLISIONS** though a generalization of our results to Multi-Parton Scattering is easy to obtain

Single-Parton Scenario for the parton-hadron scattering



Getting it squared, we arrive at the parton distribution



The forms of QCD factorization available in the literature:

Collinear Factorization

Amati-Petronzio-Veneziano, Efremov-Ginzburg-Radyushkin, Libby-Sterman, Brodsky-Lepage, Collins-Soper-Sterman

K_T- Factorization/High-Energy Factorization

S. Catani - M. Ciafaloni – F. Hautmann;

J.C. Collins- R.K. Ellis

These two conventional forms of factorization were introduced from different considerations and are used for different perturbative approaches

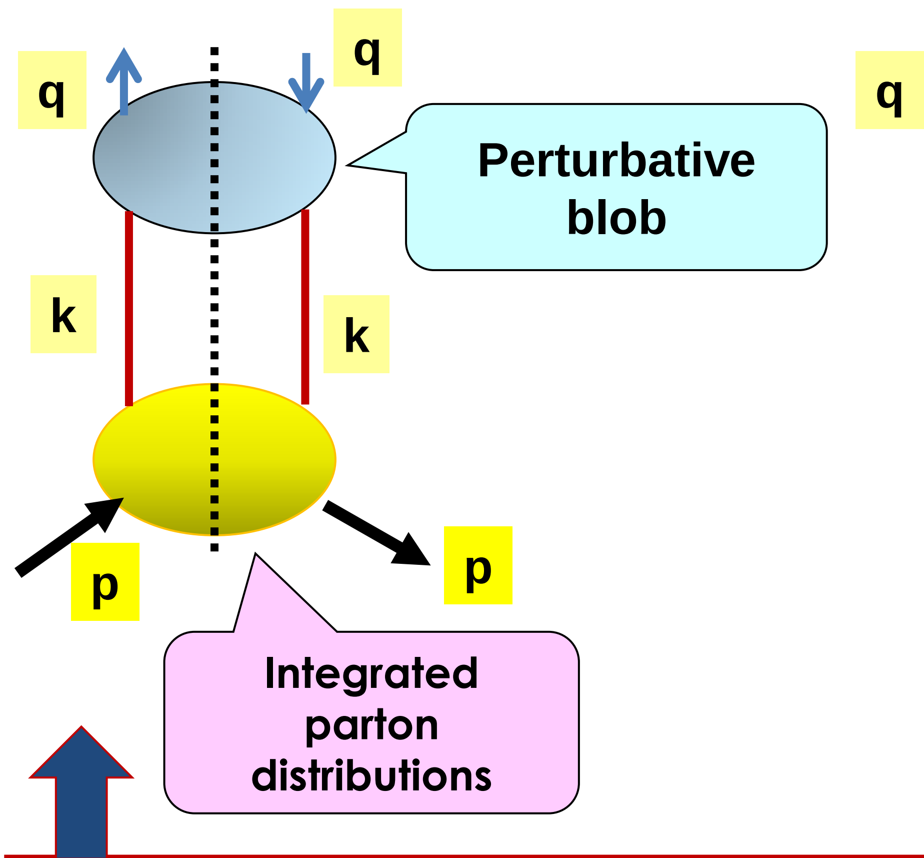
Recently we suggested a new, more general kind of factorization:

Basic Factorization

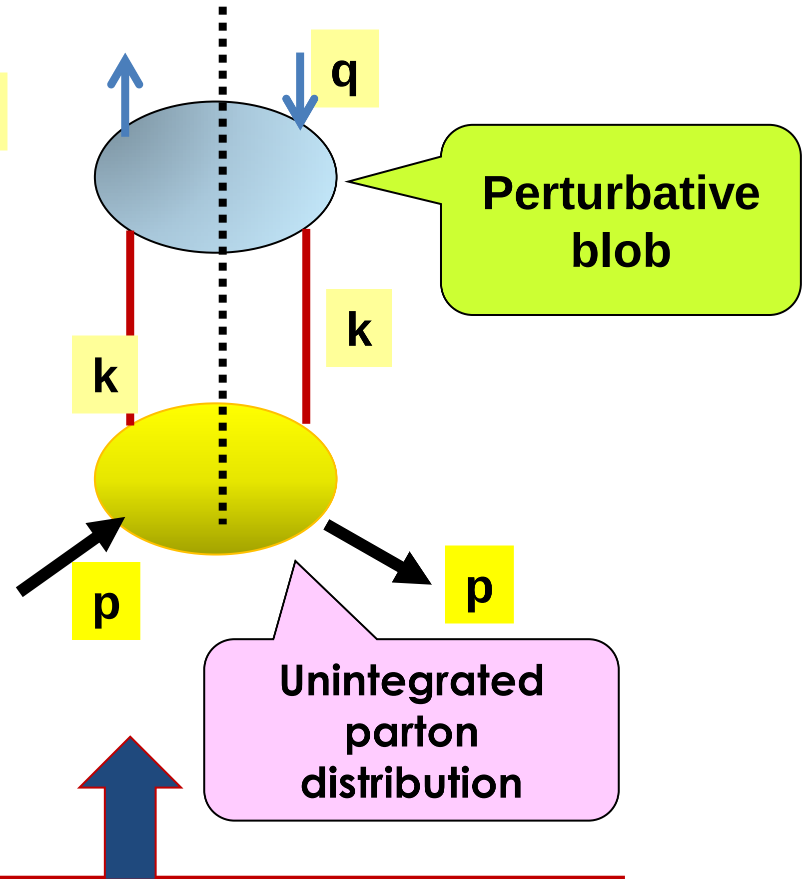
We showed how to reduce it step-by-step to K_T and Collinear Factorizations, keeping the non-perturbative inputs in a general form

Conventional illustrations of Factorizations

Collinear Factorization



K_T- factorization



Pictures look identically but formulae differ
NB Standard Feynman diagram technique cannot be applied to these graphs

Analytic expressions for parton distributions

fraction of longitudinal moment

factorization scale

Collinear Factorization

$$D(x,q^2) = \int_x^1 \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, q^2/\mu^2)\,\varphi_{col}(\beta,\mu^2)$$

integrated parton distribution

Perturbative terms are calculated with evolution equations

Parton distributions are found from phenomenological considerations

K_T Factorization

$$D(x,Q^2) = \int \frac{d\beta}{\beta}\frac{dk_\perp^2}{k_\perp^2} D_{KT}^{(pert)}(x/\beta, Q^2/k_\perp^2)\,\varphi_{KT}(\beta,k_\perp^2)$$

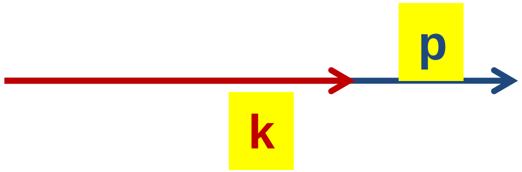
Unintegrated parton distribution

Different Factorizations imply different parameterizations of momenta of the connecting partons

$$\vec{k} = \beta\ \vec{p} \qquad (0 < \beta < 1)$$

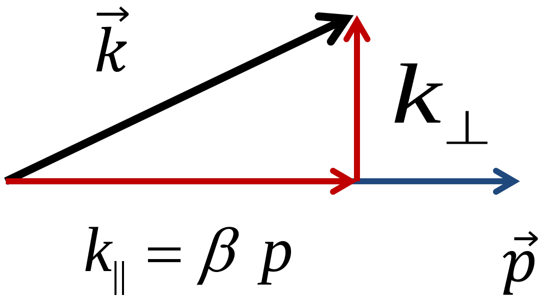
momentum fraction

Collinear Factorization



$$\vec{k} = \beta\ \vec{p} + \vec{k}_\perp$$

K_T Factorization



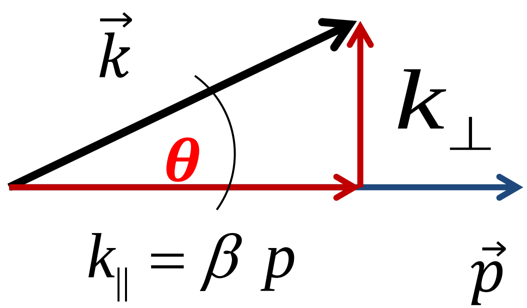
Actual situation with the parametrization is more involved:

k =$[k_0, k_x, k_y, k_z]$ and all components of k should be accounted for

For instance, all of them are present in Sudakov representation

$$k = \alpha q + \beta p + k_\perp$$

Kinematical contents of α and β



$\theta \ll 1$

$$\alpha \approx \frac{\omega}{4E}\theta^2 \qquad \beta \approx \frac{\omega}{E}$$

parton energy

$\alpha << \beta$

hadron energy

by this reason the α -dependence looks less important compared to the β -dependence

When α -dependence is taken into account we arrive at a new form of QCD factorization: Basic Factorization

It is the most general form of QCD factorization

$$D(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta}\, d^2k_\perp\, d\alpha\, D^{(pert)}(S_q, w\beta, q^2, k^2)\left(\frac{B}{k^2k^2}\right)\Phi(S_h, w\alpha, k^2)$$

new integration

perturbative part

non-perturbative totally unintegrated input

parton spin

propagators of active partons

hadron spin

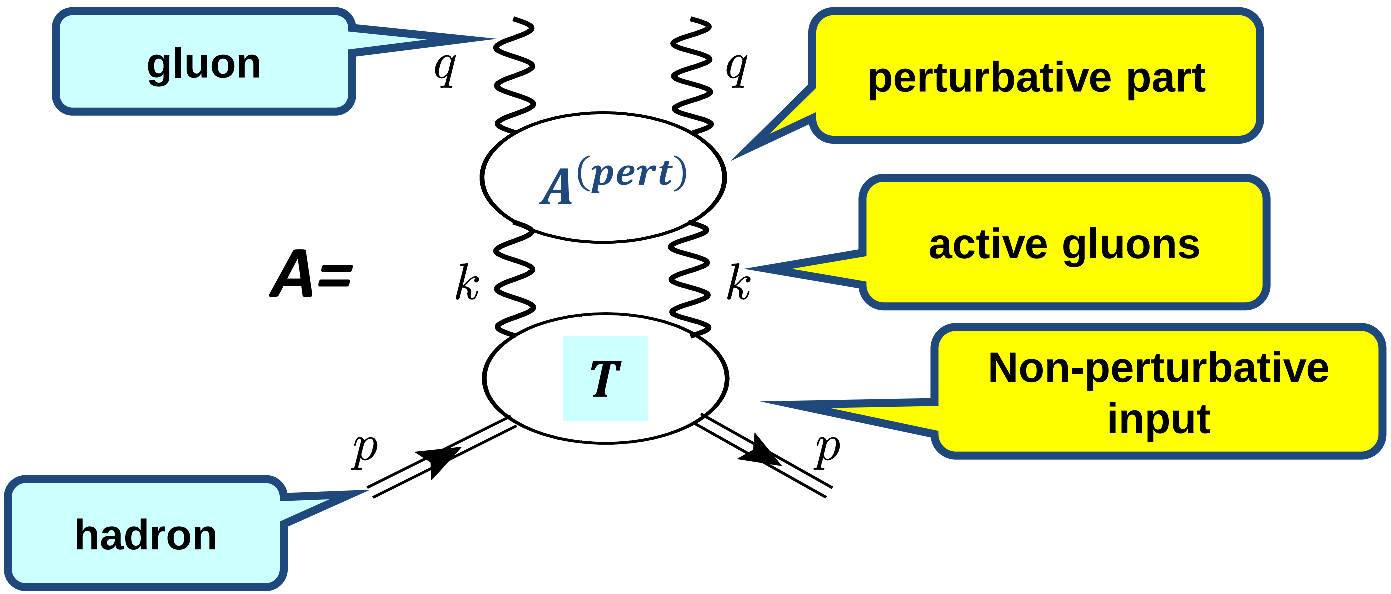
$B = k^2{}_\perp$ for non-singlet distributions, including spin-dependent ones

$B = 2pq \equiv w$ for the unpolarized gluon distribution

$k^2 = - w\, \alpha\, \beta - k^2{}_\perp$

Optical theorem relates the parton distributions to parton-hadron scattering amplitudes in the forward kinematics

For instance, the gluon-hadron scattering amplitude ***A*** is represented as follows:



NB: in contrast to K_T and Collinear Factorizations, one can apply the standard Feynman rules to the graphs in Basic Factorization in order to obtain analytic expressions . Doing so, we arrive at the expression for the gluon-hadron scattering amplitude ***A***

$$A(S_{q,}\, S_h, w, q^2) =$$

$$-i\int \frac{d\beta}{\beta} d^2k_\perp \; d\alpha \; A^{(pert)}(S_{q,} w\beta, q^2, k^2)\left(\frac{B}{k^2k^2}\right) T\left(S_{h,} w\alpha, k^2\right)$$

Result of dealing with gluon polarizations

For simplicity, I will skip this factor in what follows

Perturbative contribution

gluon propagators

Non-perturbative input

integration over momentum ***k*** covers the whole phase space

Then one should do all the integrations

However, the integrand has singularities which must be regulated before integrating because the integration should yield a finite result

HANDLING THE SINGULARITIES :

Group A:

IR and UV singularities of the perturbative amplitude $A^{(pert)}$ taken alone

Solution: IR singularities are in PQCD regulated by external momenta k^2 and q^2 *Therefore* $A^{(pert)}$ is IR stable as long as k^2 and q^2 are not equal to zero.

UV singularities in Pert QCD are known to be absorbed by redefinitions of the couplings and masses.

Group B: However, after the regularized $A^{(pert)}$ has been substituted into the convolution, the problem of IR and UV singularities appears once again

Solution is more involved

$$A(S_q, S_h, w, q^2) =$$

$$\int \frac{d\beta}{\beta}\, dk^2{}_\perp\, d\alpha\, A^{(pert)}(S_q, w\beta, q^2, k^2)\left(\frac{B}{k^2 k^2}\right) T\,(S_h, w\alpha, k^2)$$

Double pole at $k^2 = 0$

IR divergence:

Integration over k^2 runs through the point $k^2 = 0$ and we cannot introduce a IR cut off because there is no any physical reason to restrict the phase space

UV divergence:

Integration over α runs: $-\infty < \alpha < \infty$

so it can yield a diverging result at large $|\alpha|$

WAY OUT: input *T* should kill both IR and UV divergences in order to ensure IR and UV stability of the factorization convolutions

Requirement of stability of factorization convolutions leads to theoretical restrictions on non-perturbative inputs *T*

IR stability: $T \sim (k^2)^{1+\eta}$ at small k^2, with $\eta > 0$

UV stability $T \sim \alpha^{-\kappa}$ at large $|\alpha|$, with $\kappa > 0$

These restrictions can be regarded as criteria to select models for non-perturbative inputs *T*

Reduction of Basic Factorization to K_T and Collinear Factorizations

Basic Factorization:

$$A(S_q, S_h, w, q^2) = \int \frac{d\beta}{\beta} \frac{dk_\perp{}^2}{k^2} \, d\alpha \; A^{(pert)}(S_q, w\beta, q^2, k^2) \, M \left(S_h, w\alpha, k^2\right)$$

$$T = k^2 M$$

However, such a reduction cannot be done in the straightforward way

$$A(w,q^2) = \int \frac{d\beta}{\beta}\frac{dk_\perp{}^2}{k^2} \boxed{\int d\alpha \;\; A^{(pert)}(x/\beta, q^2/k^2)\, M\,(w\alpha, k^2)}$$

Integral does not correspond to the product $A_{KT}^{(pert)}(x/\beta, q^2/k_\perp^2)\,\varphi_{KT}(\beta, k_\perp^2)$

$$A(w,q^2) = \int \frac{d\beta}{\beta} \boxed{\int \frac{dk_\perp{}^2}{k^2} d\alpha \;\; A^{(pert)}(x/\beta, q^2/k^2)\, M\,(w\alpha, k^2)}$$

Integral does not correspond to $A_{col}^{(pert)}(x/\beta, q^2/\mu^2)\,\varphi_{col}(\beta, \mu^2)$

In order to get the products, the both reductions should not involve integrating the perturbative contributions

Reduction of Basic Factorization to K_T Factorization

$$A = \int \frac{d\beta}{\beta} \frac{dk_\perp{}^2}{k^2} d\alpha \;\; A^{(pert)}(x/\beta, q^2\beta/k^2)\; M\;(w\alpha, k^2)$$

Integration over α should be performed without involving $A^{(pert)}$

Problem: $A^{(pert)}$ depends on α because k^2= -w $\alpha\, \beta - k^2{}_\perp$

This dependence can be neglected if the essential integration region is $\alpha \ll k^2{}_\perp / w\beta$ In this case $k^2 \approx -k^2{}_\perp$ and we arrive at

$$A = \int \frac{d\beta}{\beta} \frac{dk_\perp{}^2}{k_\perp{}^2} \;\; A^{(pert)}(x/\beta, q^2/\zeta)\; M_{KT}\;(\zeta, \beta\;)$$

where ζ = $k_\perp{}^2/\;\beta$

and new the non-perturbative input is

$$M_{KT}\left(\beta,{k_\perp}^2\right) = \int_{-\zeta}^{\zeta} d\,\alpha\, M\left(w\alpha,k^2\right)$$

Applying Optical theorem, we arrive at the gluon distribution in K_T Factorization

$$D_{KT}\left(\beta,{k_\perp}^2\right) \not\int \frac{d\beta}{\beta}\frac{d{k_\perp}^2}{{k_\perp}^2}\, {D_{KT}}^{(pert)}\left(x/\beta,q^2/\zeta\right)\,\varphi_{KT}\left(\beta,{k_\perp}^2\right)$$

with

$$\varphi_{KT}\left(\beta,{k_\perp}^2\right) = Im\int_{-\zeta}^{\zeta} d\,\alpha\, M\left(w\alpha,k^2\right)$$

Reduction of K_T Factorization to Collinear Factorization

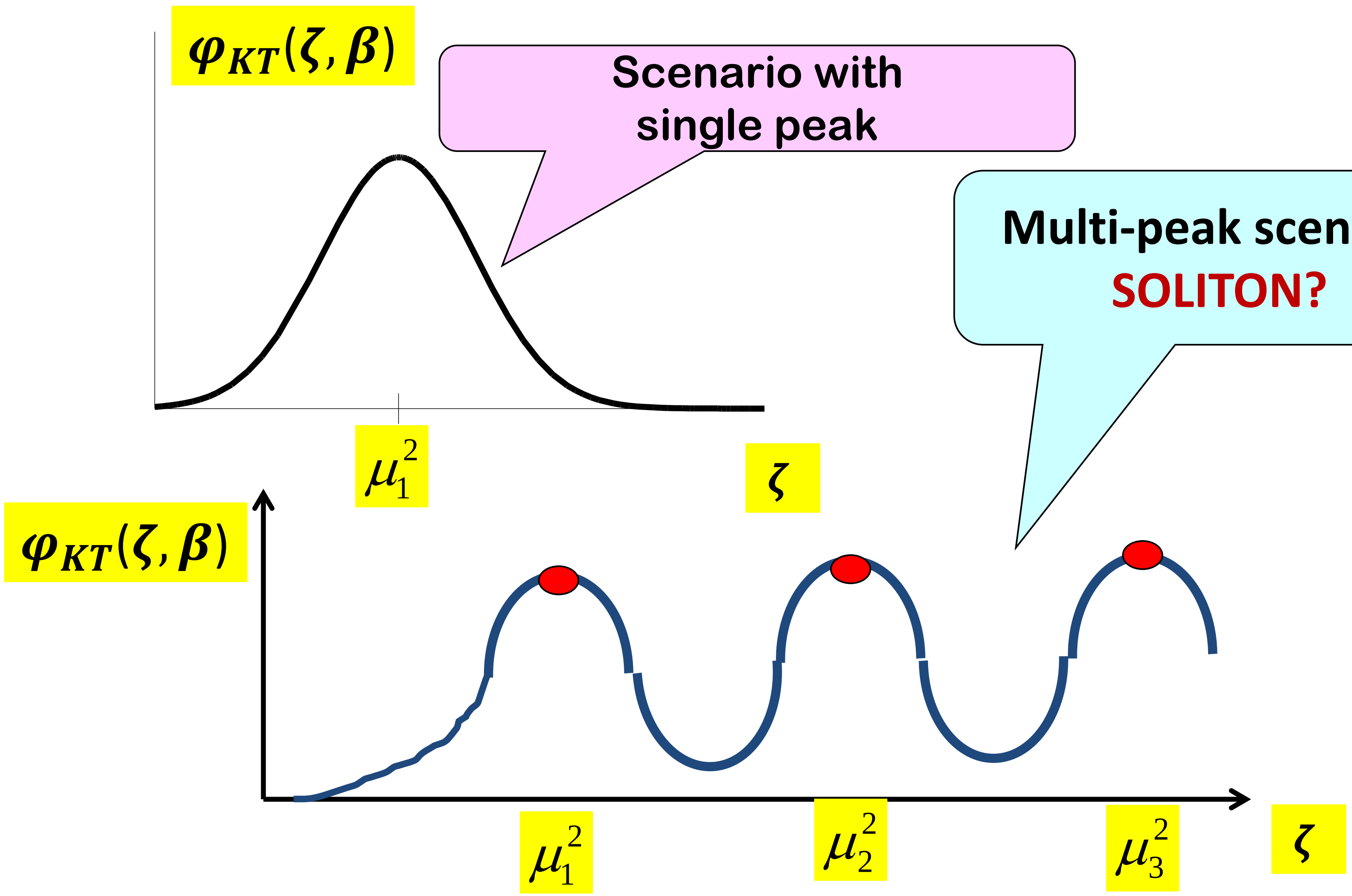
We should integrate out the ζ -dependence in

$$D_{KT}\left(\beta, k_\perp{}^2\right) = \int \frac{d\beta}{\beta}\frac{dk_\perp{}^2}{k_\perp{}^2} D_{KT}{}^{(pert)}\left(x/\beta, q^2/\zeta\right) \varphi_{KT}\left(\beta, k_\perp{}^2\right)$$

$$\boxed{\zeta = k_\perp{}^2/\beta} \Rightarrow \quad = \int \frac{d\beta}{\beta}\frac{d\zeta}{\zeta} D_{KT}{}^{(pert)}\left(x/\beta, q^2/\zeta\right) \varphi_{KT}\left(\zeta, \beta\right)$$

However, it should be done without integrating $D_{KT}{}^{(pert)}$,
which is possible only if φ_{KT} depends on ζ
in the sharp-peaked way

NB: all the peaks are located in domain of Non-Perturbative of QCD



The number of peaks is arbitrary. The maximums can have different heights and widths The sharper the peaks are, the better is accuracy of the reduction

After integration we arrive at the gluon distribution in Collinear Factorization

Single-peak scenario

$$D_{col} = \int \frac{d\beta}{\beta} D_{col}{}^{(pert)}(x/\beta, q^2/\mu_1{}^2)\, \varphi_1\left(\mu_1{}^2, \beta\right)$$

Non-perturbative inputs

Multi-peak scenario

$$D_{col} = \sum_r \int \frac{d\beta}{\beta} D_{col}{}^{(pert)}(x/\beta, q^2/\mu_r{}^2)\, \varphi_r\left(\mu_r{}^2, \beta\right)$$

The mass scale(s) μ_1 (μ_r) cannot be associated with the conventional factorization scale μ_F because they do not vanish in the convolutions. Their values cannot be chosen arbitrary: they correspond to location of the maximums. We call them inherent mass scale(s)

Comparison to the conventional form of Collinear Factorization

Usually the paron distributions in Collinear Factorization are:

$$D_{col} = \int \frac{d\beta}{\beta} D_{col}^{(pert)}(x/\beta, q^2/\mu_F^2)\, \phi \,(\beta, \mu_F^2)$$

factorization scale

Factorization scale is arbitrary. Usually they choose $\mu_F \sim 1$ GeV, i.e. μ is in the domain of perturbative QCD. Although each of $D_{col}^{(pert)}$ and ϕ depends on μ_F their convolutions are μ_F– independent. By definition, ϕ contains both perturbative and non-perturbative contributions

In contrast, the inherent mass scale μ_1 has a fixed position: it corresponds To the maximum location and its value is in the non-perturbative domain. The input $\varphi_{col}(\mu_1^2, \beta)$ is non-perturbative

Comparison to the conventional form of Collinear Factorization

It is convenient to illustrate evolution of the parton distribution from scale ${\mu_r}^2$ to q^2 considering it in the Mellin (momentum) space :

Mellin variable conjugated to β

Anomalous dimension

Factorization scale

Inherent scale

$$\varphi_{col}\,(q^2,\omega) = \left(\frac{q^2}{{\mu_r}^2}\right)^{\gamma} \varphi_{col}\,({\mu_r}^2,\omega) \qquad \text{as } q^2 > {\mu_F}^2 > {\mu_r}^2$$

$$= \left(\frac{q^2}{{\mu_F}^2}\right)^{\gamma/b} \left[\left(\frac{{\mu_F}^2}{{\mu_r}^2}\right)^{\gamma/b} \varphi_{col}\,({\mu_r}^2,\omega)\right]$$

perturbative

Non-perturbative

Conventional integrated parton density

$$\boxed{\phi\,(\beta, {\mu_F}^2)}$$

Non-perturbative inputs for parton distributions in hadrons are introduced through the models and fits. Alternatively, there are lattice calculations

Models of hadrons:

Dmitri Diakonov, V. Petrov, P. Pobylitsa, Maxim V. Polyakov; H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan;Ivan Vitev, Leonard Gamberg, Zhongbo Kang, Hongxi Xing; Asmita Mukherjee, Sreeraj Nair, Vikash Kumar Ojha;

Brodsky, Hoyer, Peterson, Sakai
K. Golec-Biernat, M. Wustoff; H. Jung;
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov;
Jon Pumplin;

Fits:

G. Altarelli, R. Ball, S. Forte, G. Ridolfi; E. Leader, A.V. Sidorov, D.B. Stamenov;
J. Blumlen, H. Botcher; M. Hirai

Most actively used in the context of factorization

Recent Lattice Calculations:

Yan-Quing Ma, Jian-Wei Qui; Marta Constantinou

I apologize if I have overlooked some name(s) and willingly accept corrections

Any model for input T in the parton-hadron scattering amplitudes must satisfy the following constraints:

(i) Input T should satisfy the IR and UV stability restrictions

(ii) It should have non-zero imaginary part in the s-channel in order to apply the Optical theorem

(iii) Model should ensure the step-by-step reductions of Basic Factorization to other forms of factorization.

In particular, the input in K_T – factorization should have a sharp-peaked form . This ensures reducing to Collinear Factorization

First of all, we fix the spinor part of the input for quark-hadron amplitudes

ASSUMPTION

$$\hat{T} = (\hat{p} + m_h)\ T_U\ -\ (\hat{p} + m_h)\,\gamma_5 \hat{S}\, T_S$$

Hadron mass

Hadron spin

Invariant amplitude for polarized hadron

Invariant amplitude for unpolarized hadron

Such a representation obeys Conformity: When the hadron is replaced by an elementary fermion, $\hat{T}$ is replaced by $\hat{\rho}$

For gluon-hadron amplitudes , we choose the inputs in the following form :

$$T_{\lambda\rho} = \left(2p_\lambda p_\varrho - k_\lambda p_\varrho - pk\, g_{\lambda\rho}\right) T_U + i\, m_h \epsilon_{\lambda\varrho\tau\sigma} k_\tau S_\sigma\, T_S$$

Invariant unpolarized amplitude

Hadron spin

Invariant amplitude for polarized hadron

All such invariant amplitudes are scalars

$$T_{U,S} = T_{U,S}\left(s_1, k^2, {m_h}^2\right)$$

Invariant energy

quark virtuality

hadron mass

$$s_1 = (p-k)^2 = w\alpha + k^2 + {m_h}^2$$

In order to fix $T_{U,S}$ we suggest **RESONANCE MODEL**

In what follows we skip the subscripts U,S

MOTIVATION FOR THE RESONANCE MODEL

After emitting the active quark from the hadron, the ensemble of remaining partons becomes color and therefore it is unstable, so it can be described through resonances.

$$T = R'(k^2) Z_n(s_1) \qquad Z_n(s_1) = \prod_{r=2}^{r=n} \frac{1}{(s_1 - m_r{}^2 + i\Gamma_r)}$$

$$s_1 = (p-k)^2 = w\alpha + k^2 + m_h{}^2$$

It satisfies the requirement of UV stability when $n = 2,3,\ldots$ In the simplest case of $n=2$ and write it as an interference of resonances:

$$T = R'(k^2) \frac{1}{(s_1 - m_1{}^2 + i\Gamma_1)} \frac{1}{(s_1 - m_2{}^2 + i\Gamma_2)} =$$

$$R(k^2)\left[\frac{1}{s_1 - m_1{}^2 + i\Gamma_1} - \frac{1}{s_1 - m_2{}^2 + i\Gamma_2}\right]$$

In terms of Sudakov variables

$$T = R(k^2)\left[\frac{1}{w\alpha - {\mu_1}^2 + i\Gamma_1} - \frac{1}{w\alpha - {\mu_2}^2 + i\Gamma_2}\right]$$

Applying Optical theorem, we arrive at the non-perturbative input for gluon distributions in Basic Factorization:

$$\Phi = R(k^2)\left[\frac{\Gamma_1}{(w\alpha - {\mu_1}^2)^2 + {\Gamma_1}^2} - \frac{\Gamma_2}{(w\alpha - {\mu_2}^2)^2 + {\Gamma_2}^2}\right]$$

Breit-Wigner factors

The only rigorous fact about ***R*** is

IR stability → $R(k^2) \sim (k^2)^{1+\eta}$ at small k^2

In order to fix ***R*** at arbitrary k^2 we complement this factor by the exponential (Gaussian) parametrization which is used by many authors in K_T Factorization
K. Golec-Biernat, M. Wustoff;
H. Jung;
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov;
J. Pumplin

$$R(k^2) = N(k^2)^{1+\eta}\, e^{-|k^2|/a}$$

This parametrization is not unique. One can suggest others

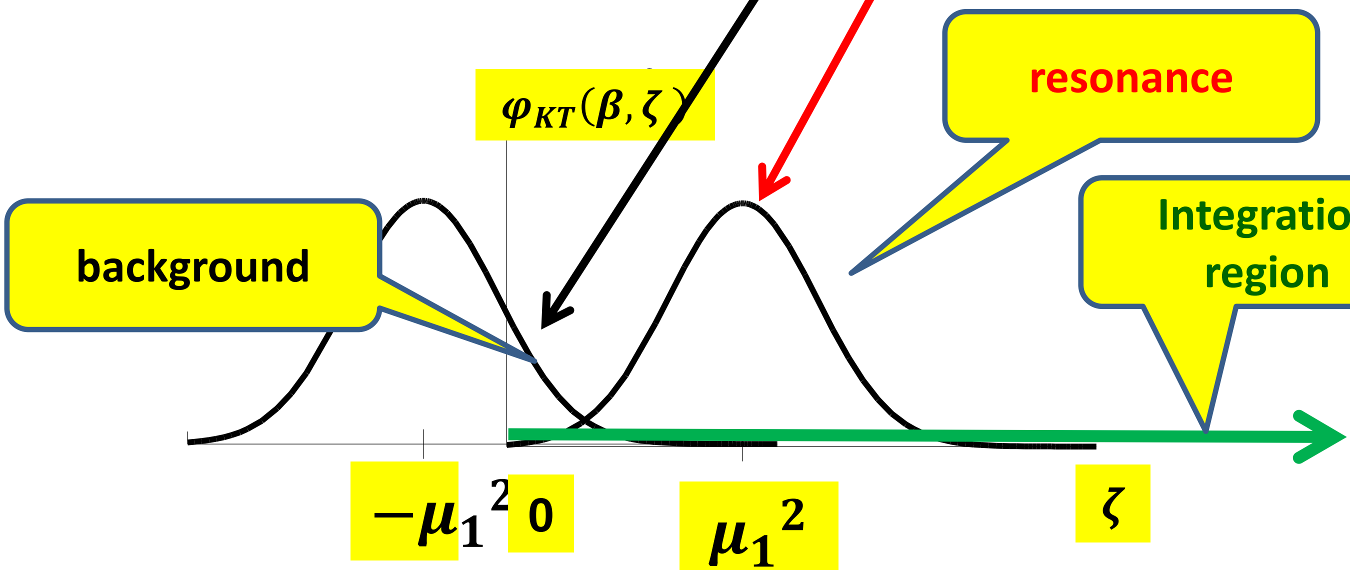
Transition from Basic Factorization to K_T Factorization

It is done according to the prescription I have given above
For simplicity I consider only one resonance term

$$\varphi_{KT} = R(k_\perp{}^2)\left[\frac{\Gamma_1}{(\zeta - \mu_1{}^2)^2 + \Gamma_1{}^2} + \frac{\Gamma_1}{(\zeta + \mu_1{}^2)^2 + \Gamma_1{}^2}\right]$$

$\boxed{\zeta = k_\perp{}^2/\beta > 0}$

One of these terms is within the resonance region while the other is outside so only its tail works



So, the non-perturbative input in K_T Factorization includes the resonance term(s) and background.

Specifying the factor ***R***

IR stability requires that at small $k_\perp{}^2$

$$R(k_\perp{}^2) \sim (k_\perp{}^2)^\eta$$

In many papers ***R*** is chosen in the exponential/Gaussian form:

$$R(k_\perp{}^2) = R_1(k_\perp{}^2) \equiv N e^{-k_\perp{}^2/a}$$

K. Golec-Biernat, M. Wustoff;
Jon Pumplin

Suppressed by the IR stability

$$R(k_\perp{}^2) = R_2(k_\perp{}^2) \equiv N(k_\perp{}^2)^\eta\, e^{-k_\perp{}^2/a}$$

H. Jung
A.V. Lipatov, G.I. Lykasov, A.A. Grinyuk, N.P. Zotov

Agrees with the IR stability

Transition from K_T Factorization to Collinear Factorization

Integrating φ_{KT} over $k_\perp$, we arrive at the non-perturbative input in Collinear Factorization.

$$\varphi_{col}\left(\beta,\mu_1{}^2\right) = N\beta^\eta e^{-\lambda\beta} \approx N\beta^\eta(1-\ \lambda\beta)$$

Comparison to Standard DGLAP fit

normalization

singular factor

two regular terms

$$N, a, b, c, d > 0$$

$$\delta q, \delta g = N \quad x^{-a} \quad (1-x)^b \quad (1+c\, x^d)$$

PROVED: mimics resummation of ln(1/x)

PRESUMEDLY: mimics resummation of ln(1-x)

These two terms must be dropped when the resummations are accounted for

When they are dropped, the DGLAP fit is

$$\delta q, \delta g = N\,(1+c\,x^d)$$

while we predict a more general structure $\varphi_{col} = N\beta^\eta(1 - \lambda\beta)$

where $\eta > 0$ However, keeping $\eta < 0$ is vital for extending DGLAP to the small-x region because it provides the fast growth at small x

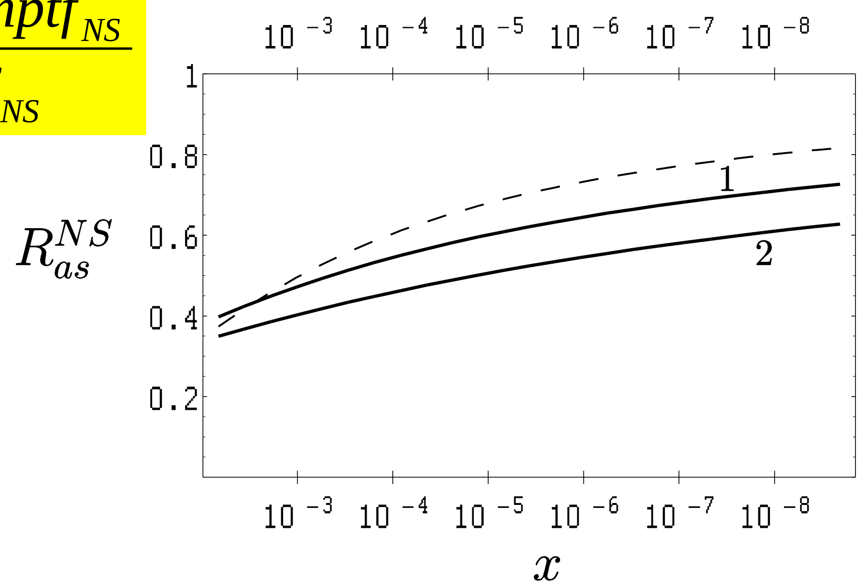
Introducing the term x^{-a} means postulating the Regge asymptotis, though asymptotics should not have been used at available energies

$$R_{as}^{NS} = \frac{Asymptf_{NS}}{f_{NS}}$$

dashed line $Q^2 = 1GeV$

line1 $Q^2 = 10$

line2 $Q^2 = 100$



CONCLUSIONS

We obtained the most general kind of QCD factorization.
We call it Basic Factorization
Basic Factorization can be reduced first to K_T- and then to Collinear Factorizations

Imposing the requirements of IR and UV stability on the convolutions in Basic Factorization allowed us to impose general restrictions on the non-perturbative inputs for parton distributions, without specifying the inputs

Motivated by the simple observation that the ensemble of quarks and gluons in a hadron becomes unstable after the hadron emits an active parton(s) and therefore can be described through resonances, we suggested a model for non-perturbative inputs to the factorization convolutions
We call it Resonance Model. We have constructed it for Single-Parton Scattering but a generalization on Multi-Parton Scattering is easy to obtain.
This model can universally describe the inputs to parton-hadron amplitudes, parton distributions, DIS structure functions, etc., and can universally be used for the polarized and unpolarized hadrons