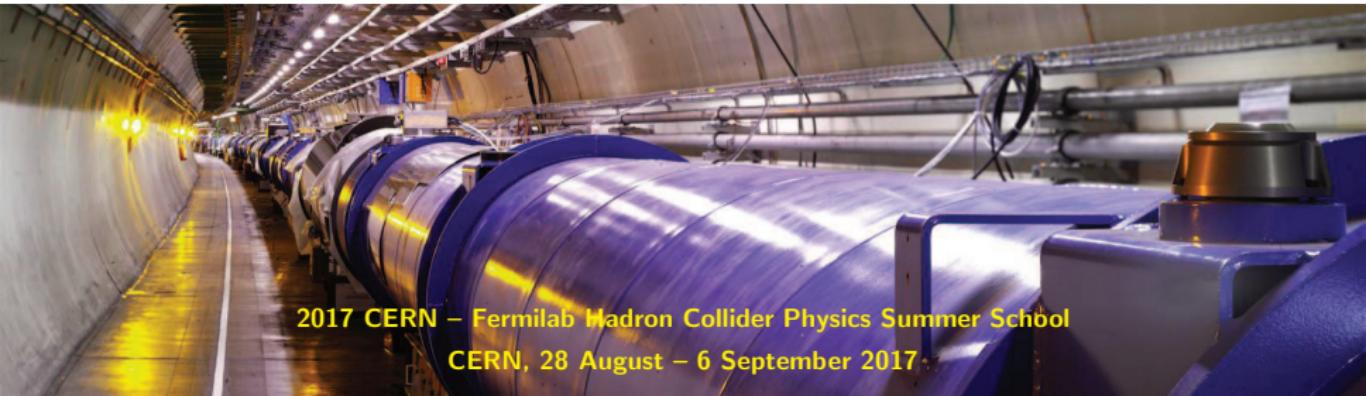


Electroweak & Higgs Physics

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CERN, 28 August – 6 September 2017

Outline

1) The Standard Theory of Electroweak Interactions

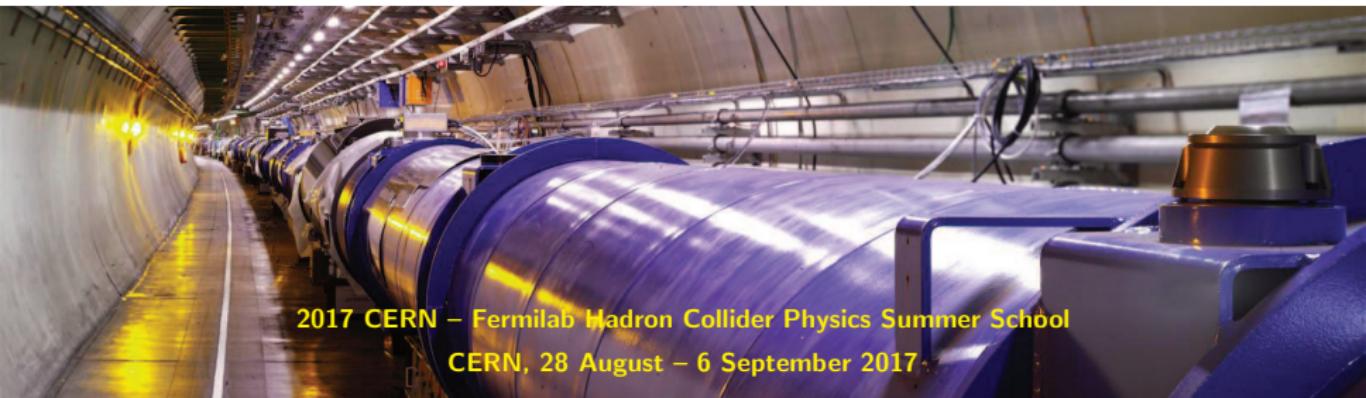
Symmetries & Lagrangian. Higgs Mechanism. Precision Tests

2) The Higgs Boson

SM Higgs. LHC Tests

3) Dynamical Aspects of EWSB

Naturalness. Custodial Symmetry. Unitarity. Flavour



Experimental Facts

- Three Families: $\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix}, \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix}, \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$
- Family Structure $\begin{bmatrix} \nu_\ell & q_u \\ \ell^- & q_d \end{bmatrix} \equiv \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L, \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, l_R^-, q_{u,R}, q_{d,R}$
- Charged Currents: W^\pm $\left\{ \begin{array}{l} \text{Left-handed Fermions only} \\ \text{Flavour Changing: } \nu_\ell \leftrightarrow \ell, q_u \leftrightarrow q_d \end{array} \right.$
- Neutral Currents: γ, Z Flavour Conserving
- Universality: Family-independent Couplings

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- **Neutral Currents:** γ, Z Flavour Conserving
- **Universality:** Family-independent Couplings
- **Scalar Interaction:** H Coupling proportional to mass

$SU(2)_L \otimes U(1)_Y$

Gauge Theory

Fields	ψ_1	ψ_2	ψ_3
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	u_R	d_R
Leptons	$\begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L$	$\nu_{\ell,R}$	ℓ_R^-

- Free Massless Lagrangian:
- Global Symmetry:

$$\mathcal{L}_0 = \sum_{j=1}^3 i \bar{\psi}_j(x) \gamma^\mu \partial_\mu \psi_j(x)$$

$$U_L \equiv \exp \left\{ \frac{i}{2} \vec{\sigma} \vec{\alpha} \right\}$$

$$\psi_1(x) \rightarrow e^{iy_1 \beta} U_L \psi_1(x) \quad , \quad \psi_2(x) \rightarrow e^{iy_2 \beta} \psi_2(x) \quad , \quad \psi_3(x) \rightarrow e^{iy_3 \beta} \psi_3(x)$$

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- Local Gauge Symmetry: $\vec{\alpha} = \vec{\alpha}(x)$, $\beta = \beta(x)$ \rightarrow 4 Gauge Fields

$$D_\mu \psi_k(x) \equiv [\partial_\mu + i g' y_k B_\mu(x)] \psi_k(x) \quad \rightarrow \quad e^{iy_k\beta} D_\mu \psi_k(x) \quad (k = 2, 3)$$

$$B_\mu \rightarrow B_\mu - \frac{1}{g'} \partial_\mu \beta$$

$SU(2)_L \otimes U(1)_Y$

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- Local Gauge Symmetry: $\vec{\alpha} = \vec{\alpha}(x)$, $\beta = \beta(x)$ \rightarrow 4 Gauge Fields

$$D_\mu \psi_1(x) \equiv \left[\partial_\mu + i g \widetilde{W}_\mu(x) + i g' y_1 B_\mu(x) \right] \psi_1(x) \rightarrow e^{iy_1\beta} \mathbf{U}_L D_\mu \psi_1(x)$$

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$$\widetilde{W}_\mu \equiv \frac{\vec{\sigma}}{2} \vec{W}_\mu \rightarrow \mathbf{U}_L \widetilde{W}_\mu \mathbf{U}_L^\dagger + \frac{i}{g} \partial_\mu \mathbf{U}_L \mathbf{U}_L^\dagger \quad , \quad B_\mu \rightarrow B_\mu - \frac{1}{g'} \partial_\mu \beta$$

$SU(2)_L \otimes U(1)_Y$ Lagrangian

$$\mathcal{L} =$$

$$\underbrace{\sum_{k=1}^3 i \bar{\psi}_k \gamma^\mu D_\mu \psi_k}_{\mathcal{L}_0 + \mathcal{L}_{CC} + \mathcal{L}_{NC}}$$

$SU(2)_L \otimes U(1)_Y$ Lagrangian

$$\mathcal{L} = \underbrace{-\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr} [\widetilde{W}_{\mu\nu} \widetilde{W}^{\mu\nu}] + \sum_{k=1}^3 i \bar{\psi}_k \gamma^\mu D_\mu \psi_k}_{\mathcal{L}_{\text{Kin}} + \mathcal{L}_3 + \mathcal{L}_4} + \underbrace{\mathcal{L}_0 + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}}}_{\mathcal{L}_0 + \mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}}}$$

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All fields are massless (masses forbidden by symmetry)

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Charged Currents

$$\sum_{k=1}^3 i \bar{\psi}_k \gamma^\mu D_\mu \psi_k \longrightarrow \underbrace{-g \bar{\psi}_1 \gamma^\mu \tilde{W}_\mu \psi_1 - g' B_\mu \sum_{k=1}^3 y_k \bar{\psi}_k \gamma^\mu \psi_k}_{\mathcal{L}_{\text{CC}} + \mathcal{L}_{\text{NC}}}$$

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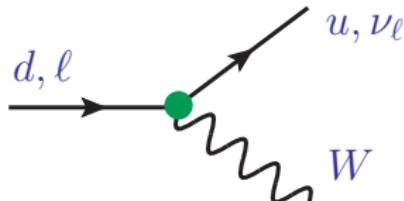
$$\widetilde{W}_\mu = \frac{\sigma^i}{2} W_\mu^i = \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^\dagger \\ \sqrt{2} W_\mu & -W_\mu^3 \end{pmatrix}, \quad W_\mu \equiv (W_\mu^1 + i W_\mu^2)/\sqrt{2}$$

Charged Currents

$$\sum_{k=1}^3 i \bar{\psi}_k \gamma^\mu D_\mu \psi_k \longrightarrow -g \bar{\psi}_1 \gamma^\mu \widetilde{W}_\mu \psi_1 - g' B_\mu \underbrace{\sum_{k=1}^3 y_k \bar{\psi}_k \gamma^\mu \psi_k}_{\mathcal{L}_{CC} + \mathcal{L}_{NC}}$$

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$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} \left\{ W_\mu^\dagger [\bar{u} \gamma^\mu (1 - \gamma_5) d + \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e] + \text{h.c.} \right\}$$



- Quark–Lepton Universality
- Left-handed Interaction

Neutral Currents

Broken symmetry since $M_Z \neq 0$  Mixing of neutral fields

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

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A_μ has the QED interaction if:

$$g \sin \theta_W = g' \cos \theta_W = e \quad , \quad Y = Q - T_3 = Q - \frac{\sigma_3}{2}$$

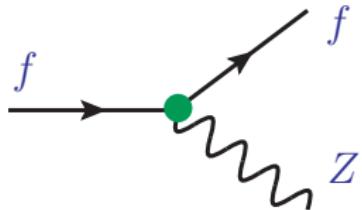


Electroweak Unification

$$\mathcal{L}_{NC} = -e A_\mu \sum_k \bar{\psi}_k \gamma^\mu Q_k \psi_k + \mathcal{L}_Z^{NC}$$

$$Q_1 \equiv \begin{pmatrix} Q_{u/\nu} & 0 \\ 0 & Q_{d/e} \end{pmatrix} \quad , \quad Q_2 = Q_{u/\nu} \quad , \quad Q_3 = Q_{d/e}$$

$$\begin{aligned}\mathcal{L}_{\text{NC}}^Z &= -\frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \sigma_3 \psi_1 - 2 \sin^2 \theta_W \sum_k \bar{\psi}_k \gamma^\mu Q_k \psi_k \right\} \\ &= -\frac{e}{2 \sin \theta_W \cos \theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f\end{aligned}$$

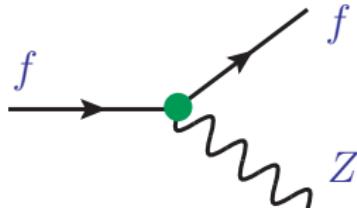


$$a_f = T_3^f$$

$$v_f = T_3^f (1 - 4|Q_f| \sin^2 \theta_W)$$

	u	d	ν_e	e
$2 v_f$	$1 - \frac{8}{3} \sin^2 \theta_W$	$-1 + \frac{4}{3} \sin^2 \theta_W$	1	$-1 + 4 \sin^2 \theta_W$
$2 a_f$	1	-1	1	-1

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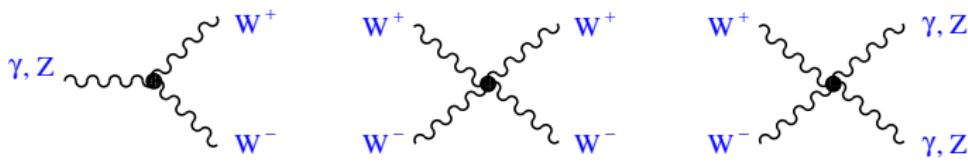
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If ν_R do exist: $y(\nu_R) = Q_\nu = 0$ \rightarrow No ν_R interactions

Gauge Self-interactions



$$\mathcal{L}_3 = ie \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger} \right) W_\mu A_\nu + W_\mu W_\nu^\dagger (\partial^\mu A^\nu - \partial^\nu A^\mu) \right\}$$

$$+ ie \cot \theta_W \left\{ (\partial^\mu W^\nu - \partial^\nu W^\mu) W_\mu^\dagger Z_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger} \right) W_\mu Z_\nu + W_\mu W_\nu^\dagger (\partial^\mu Z^\nu - \partial^\nu Z^\mu) \right\}$$

$$\mathcal{L}_4 = -e^2 \cot \theta_W \left\{ 2W_\mu^\dagger W^\mu Z_\nu A^\nu - W_\mu^\dagger Z^\mu W_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu Z^\nu \right\}$$

$$- e^2 \left\{ W_\mu^\dagger W^\mu A_\nu A^\nu - W_\mu^\dagger A^\mu W_\nu A^\nu \right\} - e^2 \cot^2 \theta_W \left\{ W_\mu^\dagger W^\mu Z_\nu Z^\nu - W_\mu^\dagger Z^\mu W_\nu Z^\nu \right\}$$

$$- \frac{e^2}{2 \sin^2 \theta_W} \left\{ (W_\mu^\dagger W^\mu)^2 - W_\mu^\dagger W^{\mu\dagger} W_\nu W^\nu \right\}$$

There is always a W^+W^- pair

$SU(2)_L$ does not allow for neutral vertices with only γ & Z

Problem with Mass Scales

$$\mathcal{L}_m = \frac{1}{2} m_B^2 B^\mu B_\mu + \frac{1}{2} m_W^2 W_i^\mu W_i^\mu - \sum_f m_f \bar{f} f$$

\mathcal{L}_m not Gauge Invariant \rightarrow All SM fields are massless!



$m_\gamma = 0$ Good!

$M_Z = M_W = m_t = 0$ Bad!

$m_b = m_c = m_s = m_u = m_d = 0$

$m_\tau = m_\mu = m_e = m_{\nu_i} = 0$

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Gauge Symmetry Needed \rightarrow Spontaneous Symmetry Breaking

Bosonic Degrees of Freedom

Massless W^\pm, Z

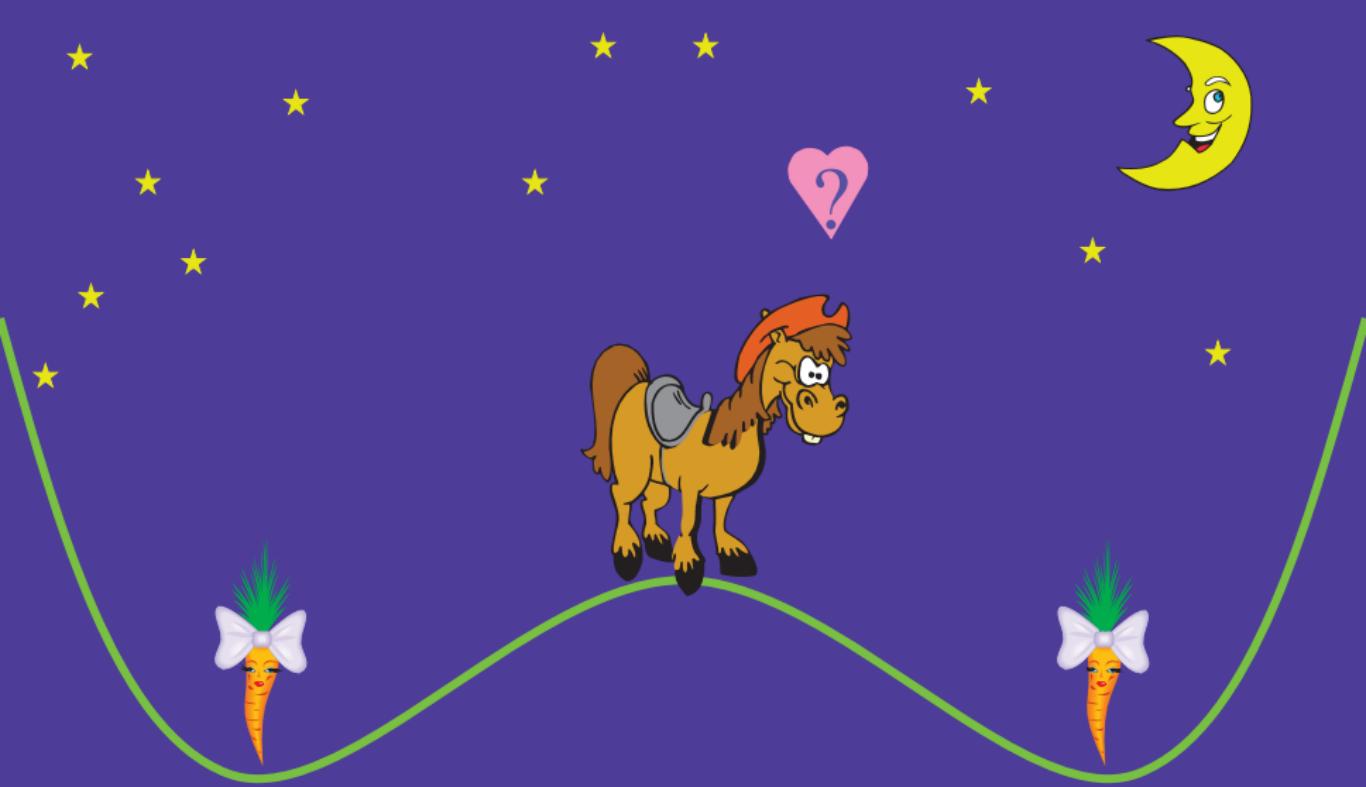
3×2 polarizations = 6



3 d.o.f.
Missing

Massive W^\pm, Z

3×3 polarizations = 9













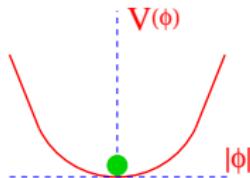
Goldstone Theorem

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - V(\phi) \quad , \quad V(\phi) = \mu^2 \phi^\dagger \phi + h (\phi^\dagger \phi)^2 \quad , \quad h > 0$$

Global Phase Symmetry: $\phi(x) \longrightarrow \phi'(x) \equiv e^{i\theta} \phi(x)$

- $\mu^2 > 0 :$ $m_\phi = \mu$

Single Ground State at $\phi = \phi_0 = 0$



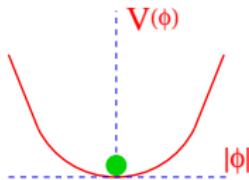
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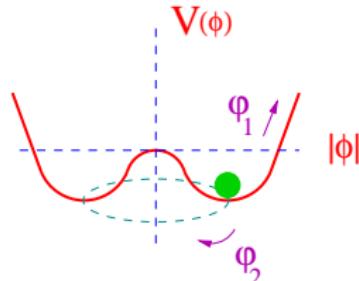
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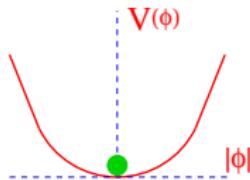
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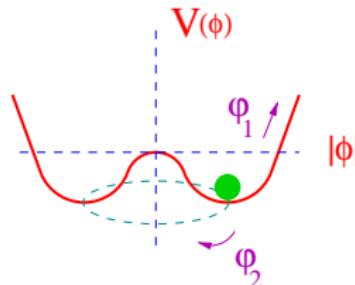
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Vacuum choice: $\phi(x) \equiv \frac{1}{\sqrt{2}} [v + \varphi_1(x)] e^{i\varphi_2(x)}$

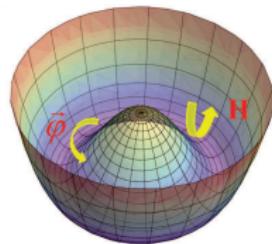
$$V(\phi) = -\frac{h}{4} v^4 - \mu^2 \varphi_1^2 + h v \varphi_1^3 + \frac{h}{4} \varphi_1^4$$

$$m_{\varphi_1}^2 = -2\mu^2 \quad , \quad m_{\varphi_2}^2 = 0 \quad (\text{Goldstone})$$

EWSB: Higgs Mechanism

$$\Phi(x) \equiv \begin{bmatrix} \phi^{(+)} \\ \phi^{(0)} \end{bmatrix}$$

$$y_\phi = Q_\phi - T_3 = \frac{1}{2}$$



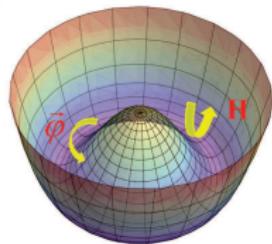
$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$$\mu^2 < 0 \quad , \quad v^2 = -\mu^2/\lambda$$

$$D_\mu \Phi = \left[\partial_\mu + i g \widetilde{W}_\mu(x) + i g' y_\phi B_\mu(x) \right] \Phi$$

EWSB: Higgs Mechanism

$$\Phi(x) \equiv \begin{bmatrix} \phi^{(+)} \\ \phi^{(0)} \end{bmatrix} = \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}}{v} \right\} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H \end{bmatrix} , \quad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$



$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

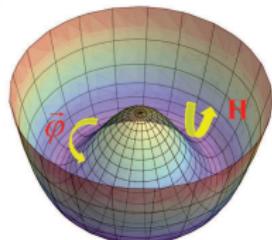
$$\mu^2 < 0 , \quad v^2 = -\mu^2/\lambda$$

$$D_\mu \Phi = \left[\partial_\mu + i g \widetilde{W}_\mu(x) + i g' y_\phi B_\mu(x) \right] \Phi$$

SU(2)_L Invariance: \rightarrow $\vec{\varphi}(x)$ can be gauged away

EWSB: Higgs Mechanism

$$\Phi(x) \equiv \begin{bmatrix} \phi^{(+)} \\ \phi^{(0)} \end{bmatrix} = \exp \left\{ i \vec{\sigma} \cdot \frac{\vec{\varphi}}{v} \right\} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H \end{bmatrix}, \quad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$



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SU(2)_L Invariance: \rightarrow $\vec{\varphi}(x)$ can be gauged away

- Unitary Gauge:** $\vec{\varphi}(x) = 0$

$$(D_\mu \Phi)^\dagger D^\mu \Phi \rightarrow \frac{1}{2} \partial_\mu H \partial^\mu H + \left(1 + \frac{H}{v}\right)^2 \left\{ M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \right\}$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

Bosonic Degrees of Freedom

Massless W^\pm, Z

3×2 polarizations = 6

+

3 Goldstones $\varphi_i(x)$

SSB



Same
Physics

Massive W^\pm, Z

3×3 polarizations = 9

Bosonic Degrees of Freedom

Massless W^\pm, Z
 3×2 polarizations = 6
+
3 Goldstones $\varphi_i(x)$

SSB
↓

Same
Physics

Massive W^\pm, Z
 3×3 polarizations = 9

1 d.o.f. too much



The Higgs Boson

Standard Model Yukawas (1 family)

$$\Phi = \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix}, \quad \langle 0 | \Phi | 0 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad \tilde{\Phi} \equiv i\tau_2 \Phi^*$$

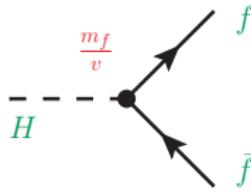
$$\mathcal{L}_Y = -c_1 (\bar{u}_L, \bar{d}_L) \Phi d_R - c_2 (\bar{u}_L, \bar{d}_L) \tilde{\Phi} u_R - c_3 (\bar{\nu}_L, \bar{e}_L) \Phi e_R + \text{h.c.}$$

 SSB

$$\mathcal{L}_Y = - \left(1 + \frac{H}{v}\right) \{ m_d \bar{d}d + m_u \bar{u}u + m_e \bar{e}e \}$$

$$m_d = c_1 \frac{v}{\sqrt{2}}, \quad m_u = c_2 \frac{v}{\sqrt{2}}, \quad m_e = c_3 \frac{v}{\sqrt{2}}$$

Couplings proportional to masses



$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

$$M_Z = 91.1876 \text{ GeV} > M_W = 80.385 \text{ GeV}$$

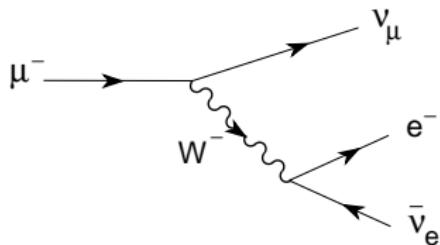


$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

$M_Z = 91.1876 \text{ GeV} > M_W = 80.385 \text{ GeV}$ \rightarrow

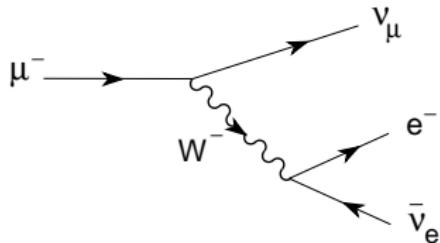
$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$



$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2} G_F$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

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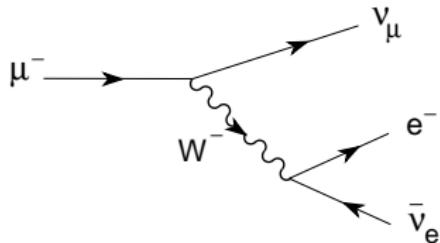
$$\frac{g^2}{M_W^2 - q^2} \approx \frac{g^2}{M_W^2} \equiv 4\sqrt{2} G_F$$

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192 \pi^3} f(m_e^2/m_\mu^2) (1 + \delta_{RC})$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$M_W = M_Z \cos \theta_W = \frac{1}{2} g v$$

$$M_Z = 91.1876 \text{ GeV} > M_W = 80.385 \text{ GeV} \rightarrow \boxed{\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223}$$



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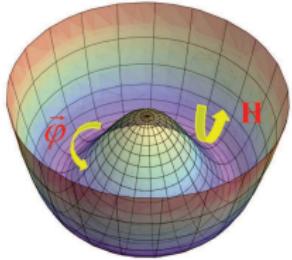
$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 m_\mu^5}{192 \pi^3} f(m_e^2/m_\mu^2) (1 + \delta_{\text{RC}})$$

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$\left. \begin{array}{l} G_F = 1.1663787 \cdot 10^{-5} \text{ GeV}^{-2} \\ M_W \quad , \quad g = e / \sin \theta_W \end{array} \right\}$$

$$\rightarrow \boxed{\begin{aligned} \sin^2 \theta_W &= 0.215 \\ v &= (\sqrt{2} G_F)^{-1/2} = 246 \text{ GeV} \end{aligned}}$$

SM Higgs Potential



$$\Phi(x) = \exp\left\{\frac{i}{v} \vec{\sigma} \cdot \vec{\varphi}(x)\right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

$$V(\Phi) + \frac{\lambda}{4} v^4 = \lambda \left(|\Phi|^2 - \frac{v^2}{2} \right)^2 = \frac{1}{2} M_H^2 H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

$$v = \frac{2M_W}{g} = \left(\sqrt{2} G_F \right)^{-1/2} = 246 \text{ GeV}$$

$$M_H = (125.09 \pm 0.24) \text{ GeV} \quad \rightarrow \quad \lambda = \frac{M_H^2}{2v^2} = 0.13$$

SM Parameters

4 inputs: g, g', μ^2, λ \longleftrightarrow α, G_F, M_Z, M_H

$$\alpha^{-1} = 137.035\,999\,174 \pm 0.000\,000\,035$$

$$G_F = (1.166\,378\,7 \pm 0.000\,000\,6) \cdot 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

$$M_H = (125.09 \pm 0.24) \text{ GeV}$$

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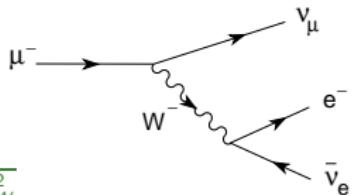
$$M_H = (125.09 \pm 0.24) \text{ GeV}$$

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$M_W^2 \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_F}$$



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$



$$M_W = 80.94 \text{ GeV}$$

[Exp: 80.385 ± 0.015]

$$\sin^2 \theta_W = 0.212$$

SM Parameters

4 inputs: g, g', μ^2, λ \longleftrightarrow α, G_F, M_Z, M_H

$$\alpha^{-1} = 137.035\,999\,174 \pm 0.000\,000\,035$$

$$G_F = (1.166\,378\,7 \pm 0.000\,000\,6) \cdot 10^{-5} \text{ GeV}^{-2}$$

$$M_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

$$M_H = (125.09 \pm 0.24) \text{ GeV}$$

$$\alpha^{-1}(M_Z^2) = 128.947 \pm 0.012$$

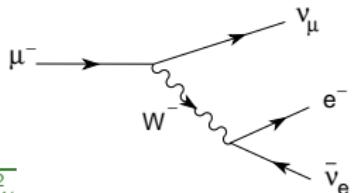
Davier et al, 1706.09436

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$M_W^2 \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_F}$$



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

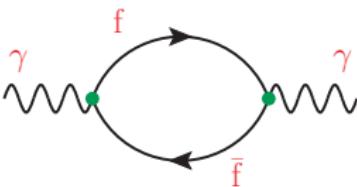


$$M_W = 80.94 \text{ GeV} \quad (79.97)$$

[Exp: 80.385 ± 0.015]

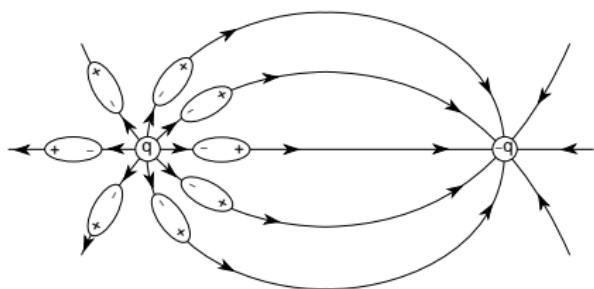
$$\sin^2 \theta_W = 0.212 \quad (0.231)$$

Vacuum Polarization



The photon couples to virtual $f\bar{f}$ pairs

Vacuum \longleftrightarrow Polarized Dielectric Medium



$$\alpha(Q^2) = \frac{\alpha(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log(Q^2/Q_0^2)}$$

$$\beta_1 = \frac{2}{3} \sum_f Q_f^2 > 0$$

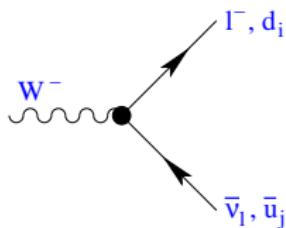
$\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$

Charge SCREENING at Large Distances

$$\alpha^{-1} = \alpha^{-1}(m_e^2) = 137.035\,999\,174\,(35) < \alpha^{-1}(M_Z^2) = 128.947\,(12)$$

Aoyama et al 1205.5368

Davier et al. 1706.09436



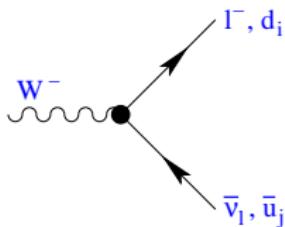
$$W^- \rightarrow \ell \bar{\nu}_\ell , \, d'_i \bar{u}_j$$

$$\bar{u}_j = \bar{u}, \bar{c} \quad , \quad \begin{pmatrix} d' \\ s' \end{pmatrix} \approx \begin{bmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{bmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$\text{Br}(W^- \rightarrow \bar{\nu}_\ell \ell^-) = \frac{1}{3 + 2 N_q} = 10.8 \% \quad [\text{Exp: } (10.86 \pm 0.09) \%]$$

$$\Gamma(W^- \rightarrow \text{all}) = \frac{G_F M_W^3}{6\sqrt{2}\pi} (3 + 2 N_q) \approx 2.10 \text{ GeV} \quad [\text{Exp: } (2.085 \pm 0.042) \text{ GeV}]$$

$$N_q = N_C \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \dots \right\} \approx 3.12$$



$$W^- \rightarrow \ell \bar{\nu}_\ell , d'_i \bar{u}_j$$

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$$\text{Br}(W^- \rightarrow \bar{\nu}_\ell \ell^-) = \frac{1}{3 + 2 N_q} = 10.8 \% \quad [\text{Exp: } (10.86 \pm 0.09) \%]$$

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$$N_q = N_C \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \dots \right\} \approx 3.12$$

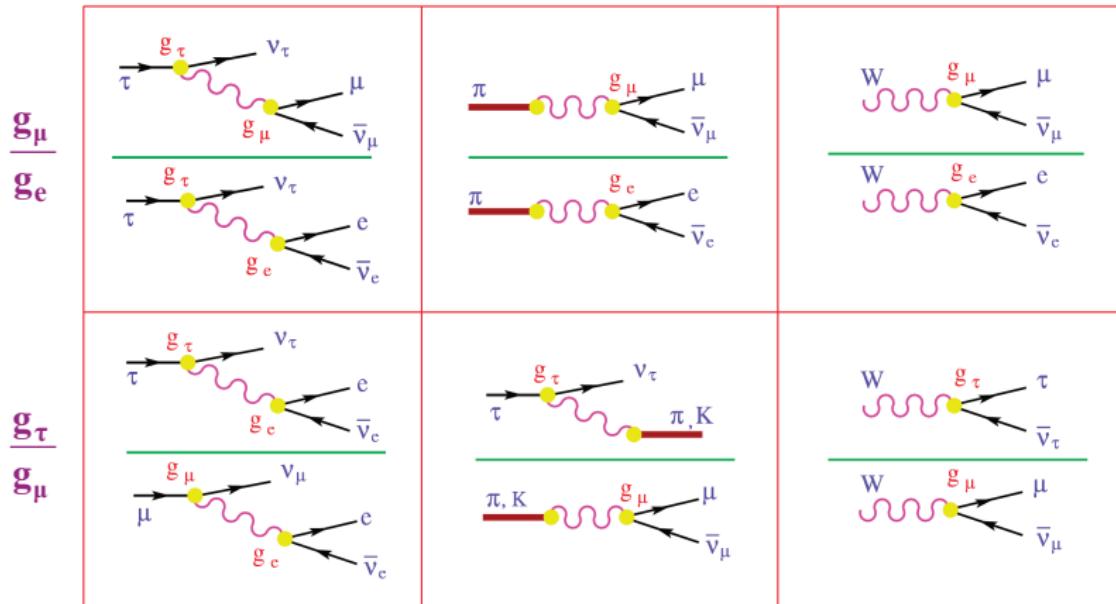
$$\text{Br}(W^- \rightarrow \bar{\nu}_e e^-)_{\text{exp}} = (10.71 \pm 0.16) \%$$

$$\text{Br}(W^- \rightarrow \bar{\nu}_\mu \mu^-)_{\text{exp}} = (10.63 \pm 0.15) \%$$

$$\text{Br}(W^- \rightarrow \bar{\nu}_\tau \tau^-)_{\text{exp}} = (11.38 \pm 0.21) \%$$

**Universal
Wℓ̄νℓ Couplings**

Lepton Universality



SM: Same W^\pm couplings for all fermion families

Lepton Universality

$|g_\mu/g_e|$

$B_{\tau \rightarrow \mu}/B_{\tau \rightarrow e}$	1.0018 ± 0.0014
$B_{\pi \rightarrow \mu}/B_{\pi \rightarrow e}$	1.0021 ± 0.0016
$B_{K \rightarrow \mu}/B_{K \rightarrow e}$	0.9978 ± 0.0020
$B_{K \rightarrow \pi \mu}/B_{K \rightarrow \pi e}$	1.0010 ± 0.0025
$B_{W \rightarrow \mu}/B_{W \rightarrow e}$	0.996 ± 0.010

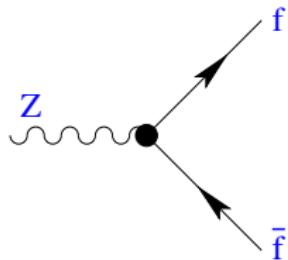
$|g_\tau/g_\mu|$

$B_{\tau \rightarrow e} \tau_\mu/\tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau \rightarrow \pi}/\Gamma_{\pi \rightarrow \mu}$	0.9962 ± 0.0027
$\Gamma_{\tau \rightarrow K}/\Gamma_{K \rightarrow \mu}$	0.9858 ± 0.0070
$B_{W \rightarrow \tau}/B_{W \rightarrow \mu}$	1.034 ± 0.013

$|g_\tau/g_e|$

$B_{\tau \rightarrow \mu} \tau_\mu/\tau_\tau$	1.0030 ± 0.0015
$B_{W \rightarrow \tau}/B_{W \rightarrow e}$	1.031 ± 0.013

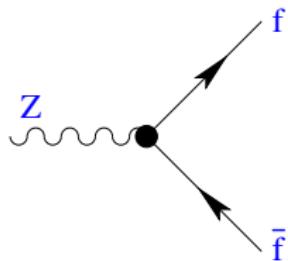
A. Pich, 1310.7922



$$\Gamma(Z \rightarrow \bar{f}f) = N_f \frac{G_F M_Z^3}{6\pi\sqrt{2}} (|v_f|^2 + |a_f|^2)$$

$$N_\ell = 1 \quad , \quad N_q = N_C \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \dots \right\} \approx 3.12$$

$$\frac{\Gamma_{\text{inv}}}{\Gamma_\ell} \equiv \frac{N_\nu \Gamma(Z \rightarrow \bar{\nu} \nu)}{\Gamma_\ell} = \frac{2 N_\nu}{(1 - 4 \sin^2 \theta_W)^2 + 1} = \begin{cases} 1.955 \times N_\nu & [\alpha(m_e^2)] \\ 1.989 \times N_\nu & [\alpha(M_Z^2)] \end{cases}$$

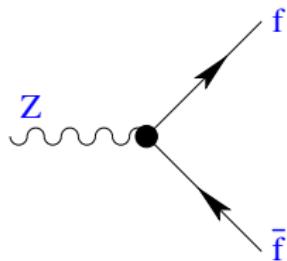


$$\Gamma(Z \rightarrow \bar{f}f) = N_f \frac{G_F M_Z^3}{6\pi\sqrt{2}} (|v_f|^2 + |a_f|^2)$$

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- LEP: $\frac{\Gamma_{\text{inv}}}{\Gamma_\ell} = 5.943 \pm 0.016 \quad \rightarrow \quad N_\nu = \begin{cases} 3.04 \\ 2.99 \end{cases}$



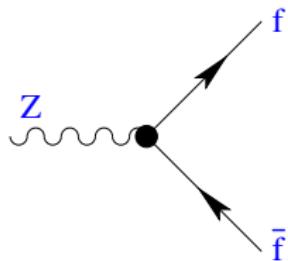
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All (NLO) EW corrections \rightarrow $N_\nu = 2.984 \pm 0.008$



$$\Gamma(Z \rightarrow \bar{f}f) = N_f \frac{G_F M_Z^3}{6\pi\sqrt{2}} (|v_f|^2 + |a_f|^2)$$

$$N_\ell = 1 \quad , \quad N_q = N_C \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \dots \right\} \approx 3.12$$

$$\frac{\Gamma_{\text{inv}}}{\Gamma_\ell} \equiv \frac{N_\nu \Gamma(Z \rightarrow \bar{\nu} \nu)}{\Gamma_\ell} = \frac{2 N_\nu}{(1 - 4 \sin^2 \theta_W)^2 + 1} = \begin{cases} 1.955 \times N_\nu & [\alpha(m_e^2)] \\ 1.989 \times N_\nu & [\alpha(M_Z^2)] \end{cases}$$

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All (NLO) EW corrections $\rightarrow N_\nu = 2.984 \pm 0.008$

$$\frac{\Gamma(Z \rightarrow \text{had})}{\Gamma(Z \rightarrow \ell^- \ell^+)} = 20.767 \pm 0.025 \rightarrow \alpha_s(M_Z^2) = 0.1196 \pm 0.0030$$