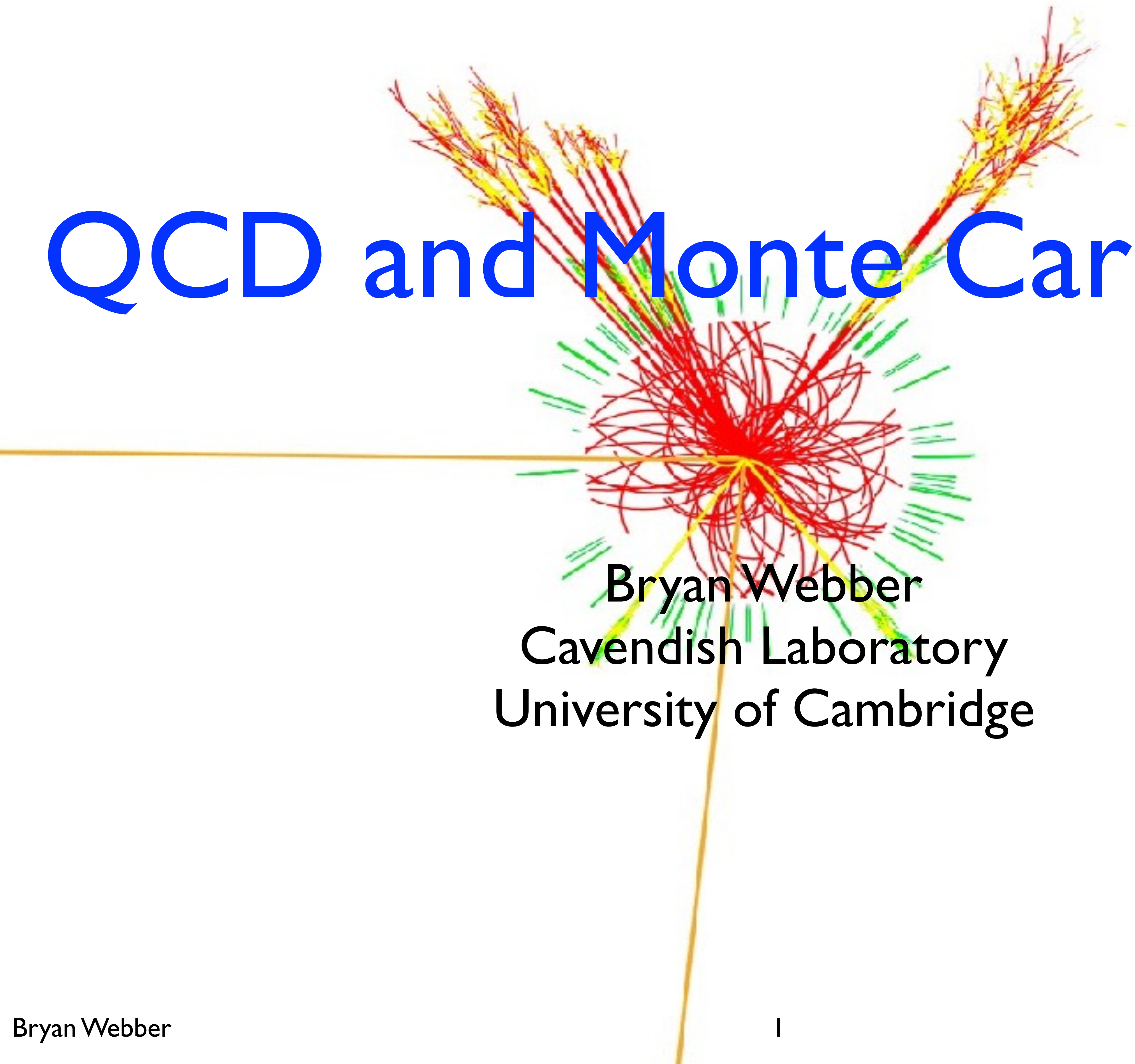
QCD and Monte Carlo Tools



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Outline of lectures

- QCD basics
  - Lagrangian, coupling, quark masses, PDFs
- QCD and the Higgs boson
  - Production, decays, p_T distribution
- Monte Carlo event generation
  - Monte Carlo basics
  - Event generator components
  - Improvements: matching and merging
- Survey of results

References

- R.K. Ellis, W.J. Stirling & B.R. Webber, "QCD and Collider Physics" (C.U.P. 1996)
- A. Buckley et al., "General-purpose event generators for LHC physics", Phys.Rept. 504 (2011) 145 (MCNET-11-01, arXiv:1101.2599)
- P. Nason & P.Z. Skands, "Monte Carlo event generators", in Review of Particle Physics, C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C40,100001 (2016). http://pdg.lbl.gov/2017/reviews/rpp2016-rev-mc-event-gen.pdf

QCD Basics

QCD Lagrangian

$$\boxed{\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + \sum_{q=u\ldots t} \bar{q}_a \left(i\gamma_\mu D^\mu_{ab} - m_q \delta_{ab} \right) q_b}$$

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g f^{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu \,, \quad D^\mu_{ab} = \partial^\mu \delta_{ab} + ig\, t^C_{ab} \mathcal{A}^{C\mu}$$

- a,b = 3 colours of quarks
- A,B,C = 8 (≈3x3) colours of gluons
- t^C = 8 3x3 independent traceless hermitian matrices [generators of colour SU(3) group]
- $[t^A,t^B] = if^{ABC}t^C$ algebra of SU(3)
- All strong interaction physics determined by 7 parameters: $g, m_u, m_d, m_s, m_c, m_b, m_t$ $(\alpha_S = g^2/4\pi)$
  - But these need to be renormalized : $\alpha_S = \alpha_S(\mu^2)$

QCD Coupling

QCD Running Coupling

- Consider a dimensionless quantity R depending on a single hard scale Q, e.g. $R = \sigma(e^+e^- \to \mathrm{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ at c.m. energy Q
  - ★ Dependence on Q can only be via Q/μ
  - ★ But μ is arbitrary, so overall dependence on it must vanish

$$\Rightarrow \quad \mu^2 \frac{d}{d\mu^2} R(Q^2/\mu^2, \alpha_S) \equiv \left[\mu^2 \frac{\partial}{\partial\mu^2} + \mu^2 \frac{\partial\alpha_S}{\partial\mu^2}\frac{\partial}{\partial\alpha_S}\right] R = 0$$

- Define $t = \ln\left(\frac{Q^2}{\mu^2}\right), \quad \beta(\alpha_S) = \mu^2 \frac{\partial\alpha_S}{\partial\mu^2}$

$$\Rightarrow \quad \left[-\frac{\partial}{\partial t} + \beta(\alpha_S)\frac{\partial}{\partial\alpha_S}\right] R(e^t, \alpha_S) = 0$$

- Introduce $\alpha_S(Q^2)$ such that $t = \int_{\alpha_S}^{\alpha_S(Q^2)} \frac{dx}{\beta(x)}, \quad \alpha_S(\mu^2) \equiv \alpha_S$

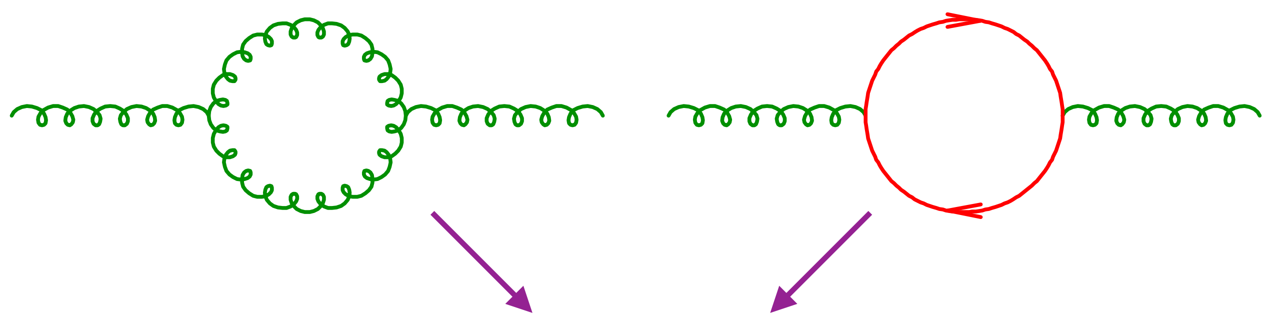
- Then solution is $R(1, \alpha_S(Q^2))$ $\quad \left[\text{Use } \left(\frac{\partial\alpha_S(Q^2)}{\partial t}\right)_{\alpha_S} \left(\frac{\partial t}{\partial\alpha_S}\right)_{\alpha_S(Q^2)} \left(\frac{\partial\alpha_S}{\partial\alpha_S(Q^2)}\right)_t = -1\right]$
  - ★ All scale dependence is absorbed in running coupling $\alpha_S(Q^2)$

QCD Running Coupling

$$\ln\left(\frac{Q^2}{\mu^2}\right) = \int_{\alpha_S(\mu^2)}^{\alpha_S(Q^2)} \frac{d\alpha_S}{\beta(\alpha_S)}, \quad \beta(\alpha_S) = -\alpha_S^2\,(\beta_0 + \beta_1\alpha_S + \ldots)$$

$$\Rightarrow \quad \ln\left(\frac{Q^2}{\mu^2}\right) = \frac{1}{\beta_0}\left[\frac{1}{\alpha_S(Q^2)} - \frac{1}{\alpha_S(\mu^2)}\right] + \ldots$$

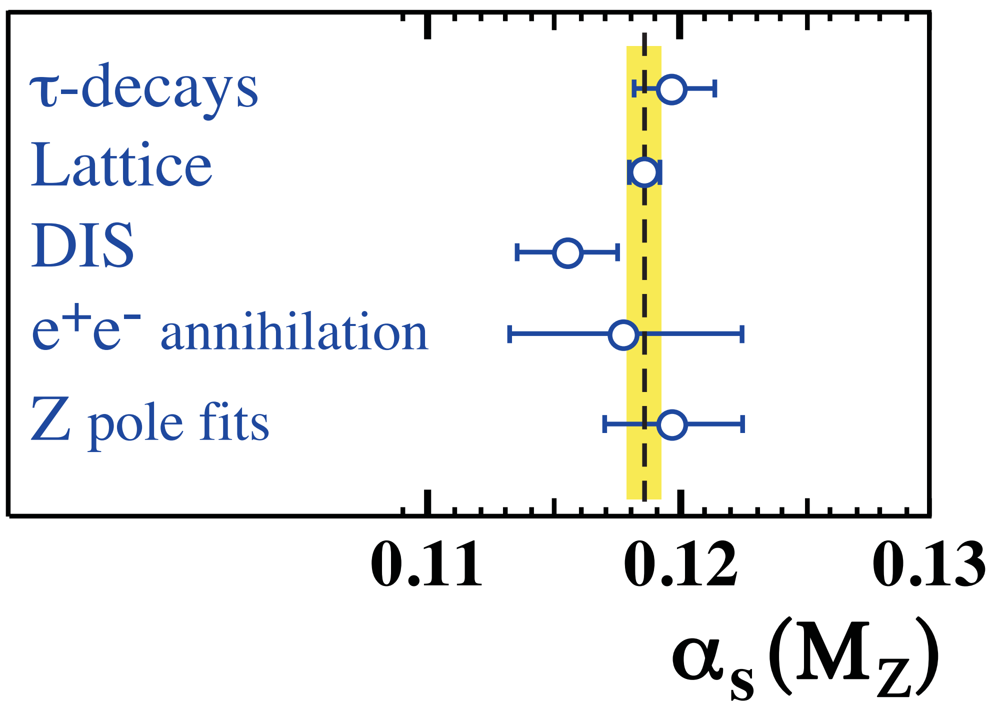
$$\Rightarrow \quad \alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \beta_0\,\alpha_S(\mu^2)\ln\left(Q^2/\mu^2\right)} + \ldots$$

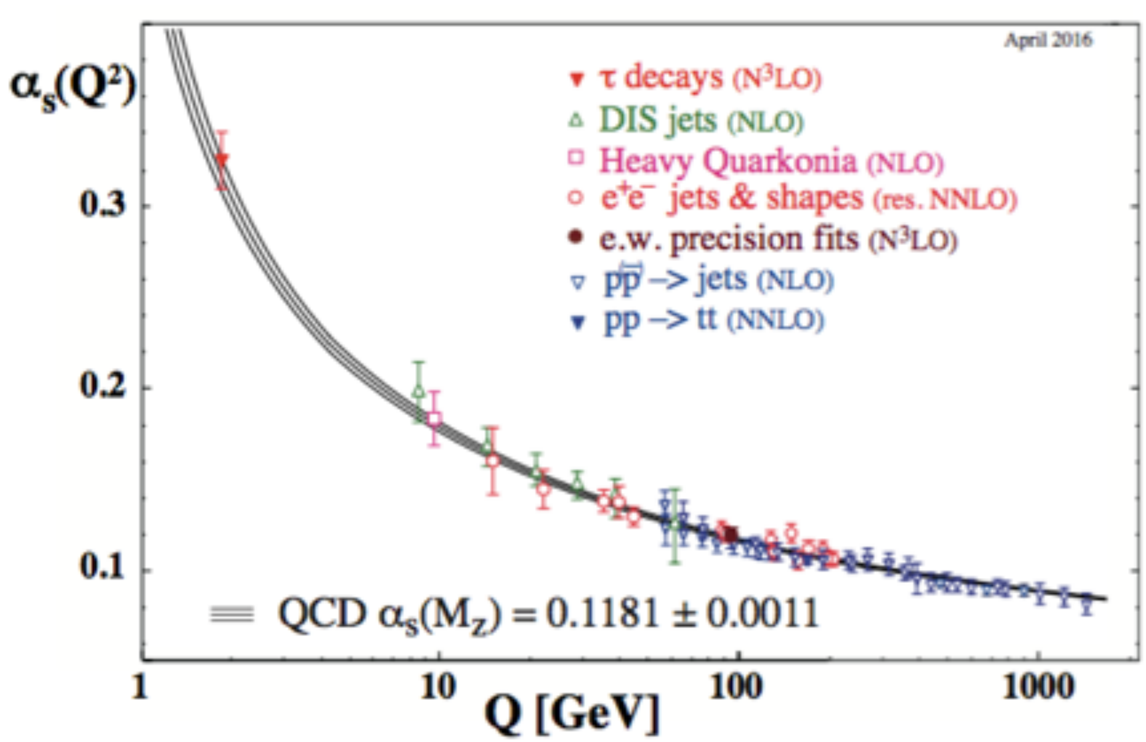


$$\beta_0 = (33 - 2n_f)/12\pi$$

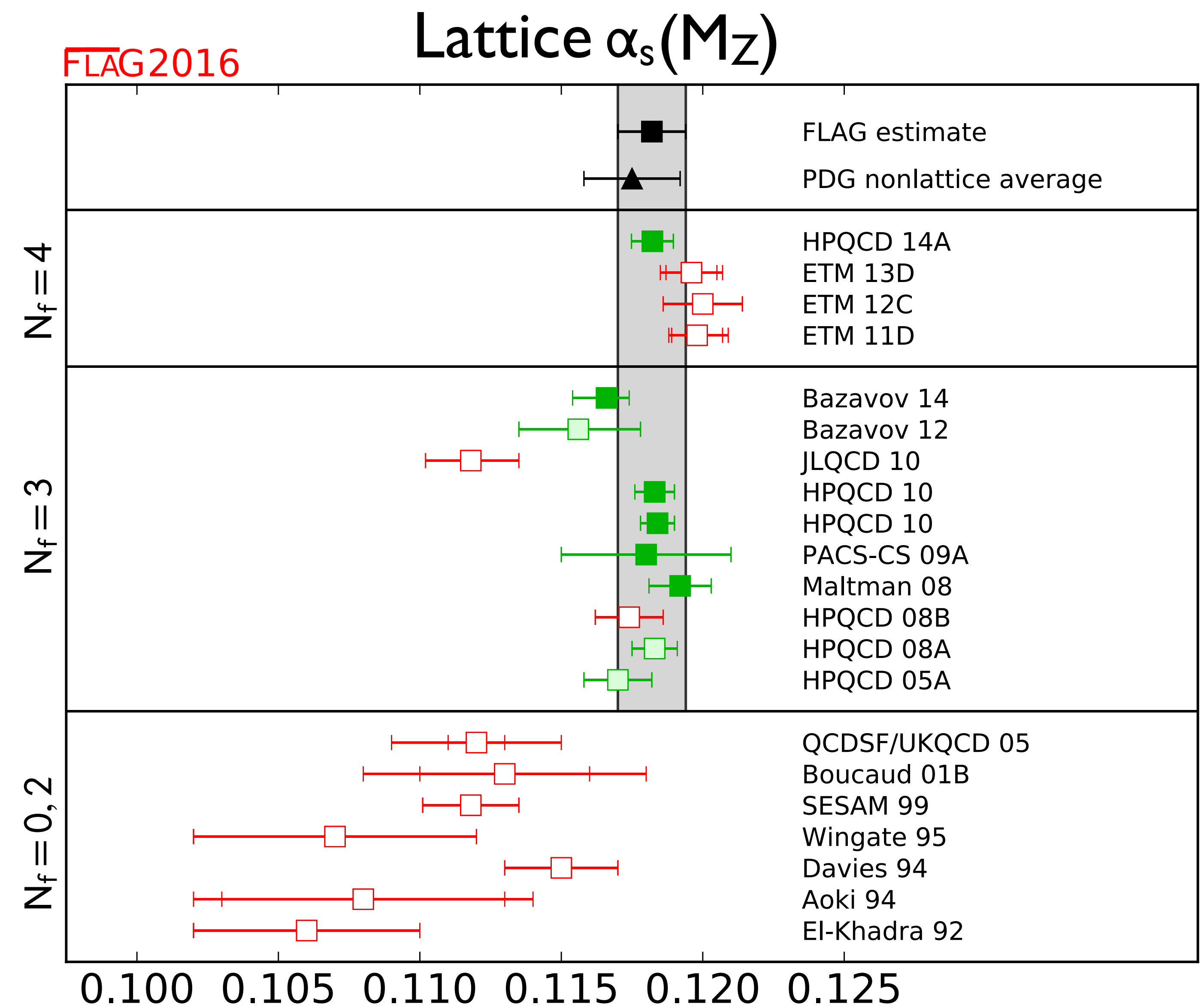
- β_0>0 means asymptotic freedom
- β-function known to 4 loops (β_3)

QCD Running Coupling





Bethke, Dissertori, Salam, RPP 2016



FLAG WG: Aoki et al., 1607.00299

$\alpha_S(M_Z)$=0.1184(12) [lattice]

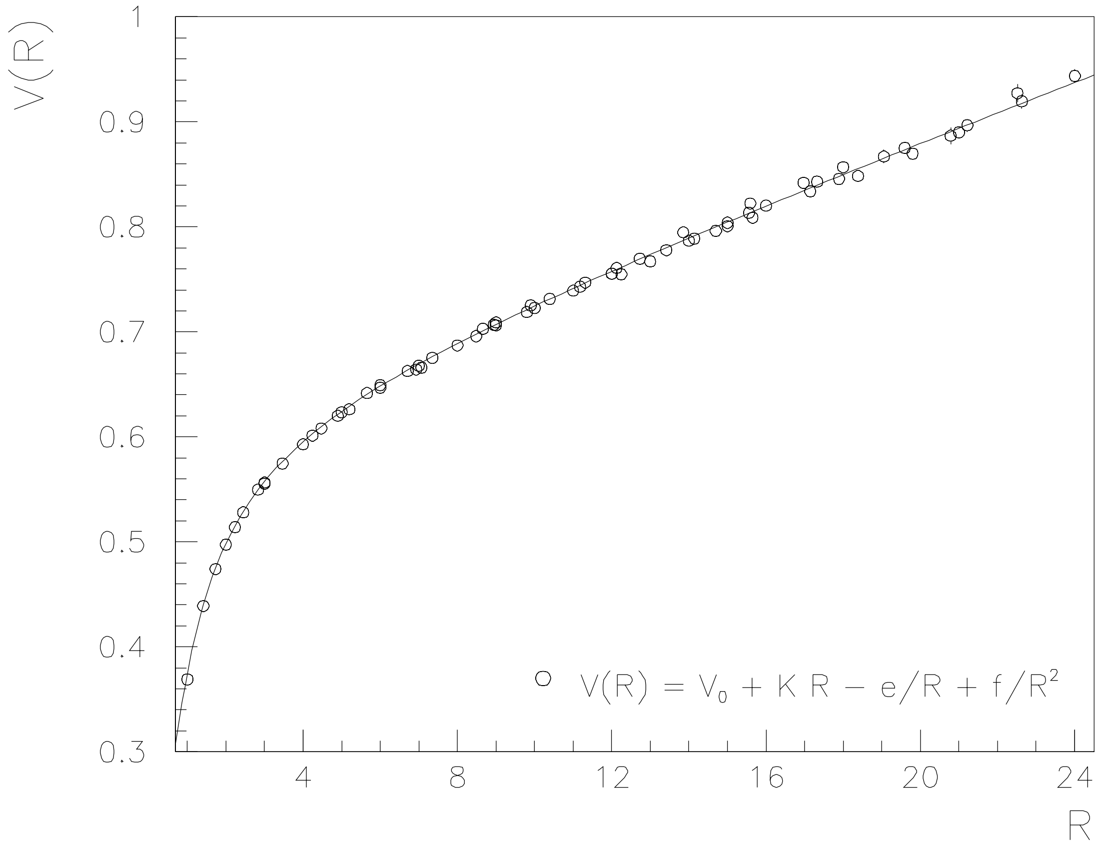
$\alpha_S(M_Z)$=0.1174(16) [non-lattice]

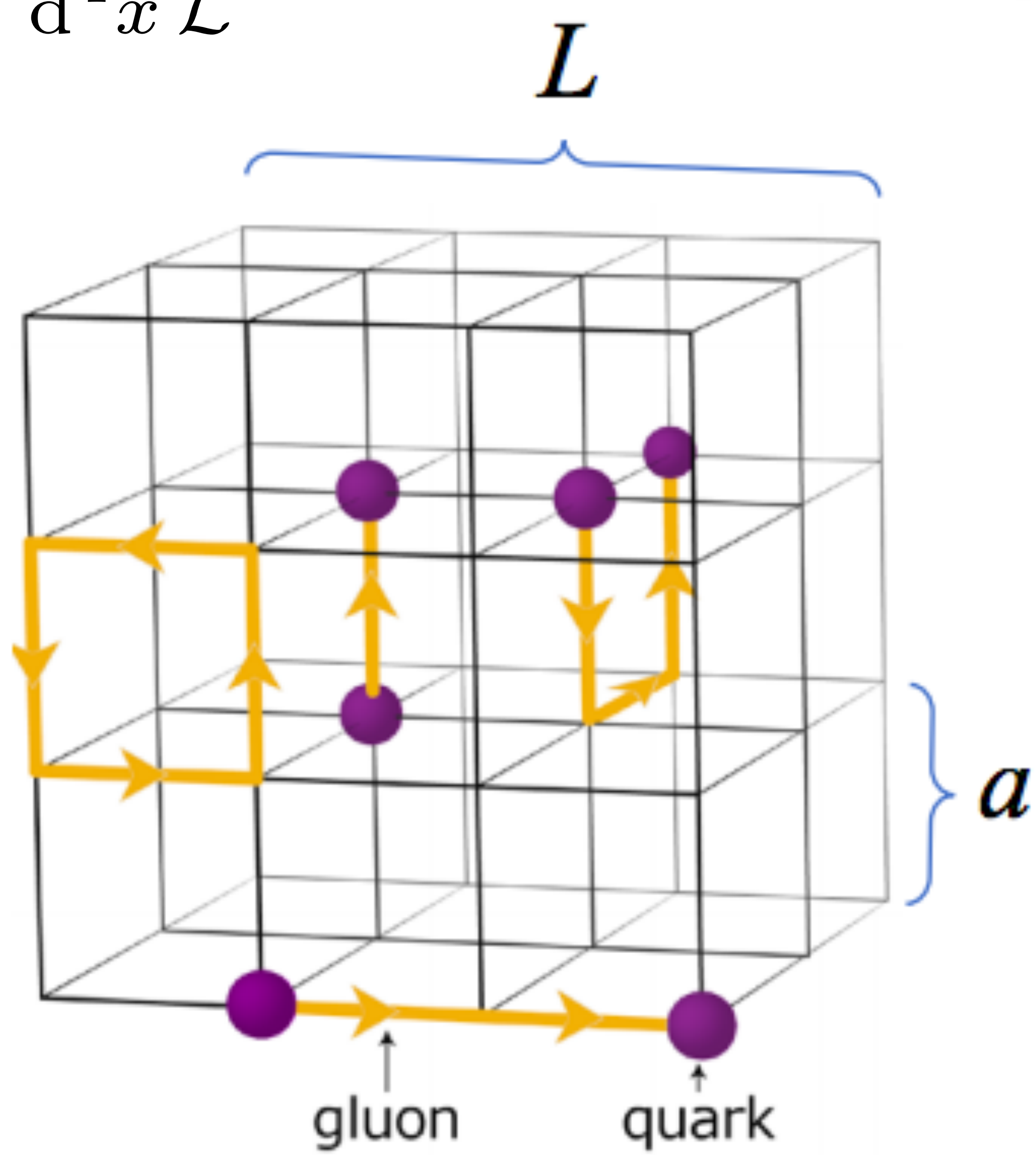
Lattice QCD

- QCD on a (hyper)cubic lattice

$$\langle \mathcal{O} \rangle = \int [\mathrm{d}\mathcal{A}][\mathrm{d}q][\mathrm{d}\bar{q}]\mathcal{O}\,\mathrm{e}^{-\int \mathrm{d}^4x\,\mathcal{L}}$$

- Ideally $a \to 0\,,\quad L \to \infty$
- Quark-antiquark potential:





Lattice QCD Coupling

FLAG WG: Aoki et al., 1607.00299

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	Method
HPQCD 14A	[5]	2+1+1	A	○	★	○	0.11822(74)	current two points
ETM 13D	[645]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex
ETM 12C	[646]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex
ETM 11D	[647]	2+1+1	A	○	○	■	$0.1198(9)(5)(^{+0}_{-5})$	gluon-ghost vertex
Bazavov 14	[61]	2+1	A	○	★	○	$0.1166(^{+12}_{-8})$	Q-$\bar{Q}$ potential
Bazavov 12	[600]	2+1	A	○	○	○	$0.1156(^{+21}_{-22})$	Q-$\bar{Q}$ potential
HPQCD 10	[9]	2+1	A	○	★	○	0.1183(7)	current two points
HPQCD 10	[9]	2+1	A	○	★	★	0.1184(6)	Wilson loops
JLQCD 10	[609]	2+1	A	■	■	■	$0.1118(3)(^{+16}_{-17})$	vacuum polarization
PACS-CS 09A	[62]	2+1	A	★	★	○	$0.118(3)^{\#}$	Schrödinger functional
Maltman 08	[63]	2+1	A	○	○	★	0.1192(11)	Wilson loops
HPQCD 08B	[152]	2+1	A	■	■	■	0.1174(12)	current two points
HPQCD 08A	[613]	2+1	A	○	★	★	0.1183(8)	Wilson loops
HPQCD 05A	[612]	2+1	A	○	○	○	0.1170(12)	Wilson loops
QCDSF/UKQCD 05	[621]	$0, 2 \to 3$	A	★	■	★	0.112(1)(2)	Wilson loops
Boucaud 01B	[640]	$2 \to 3$	A	○	○	■	0.113(3)(4)	gluon-ghost vertex
SESAM 99	[619]	$0, 2 \to 3$	A	★	■	■	0.1118(17)	Wilson loops
Wingate 95	[620]	$0, 2 \to 3$	A	★	■	■	0.107(5)	Wilson loops
Davies 94	[618]	$0, 2 \to 3$	A	★	■	■	0.115(2)	Wilson loops
Aoki 94	[617]	$2 \to 3$	A	★	■	■	0.108(5)(4)	Wilson loops
El-Khadra 92	[616]	$0 \to 3$	A	★	■	○	0.106(4)	Wilson loops

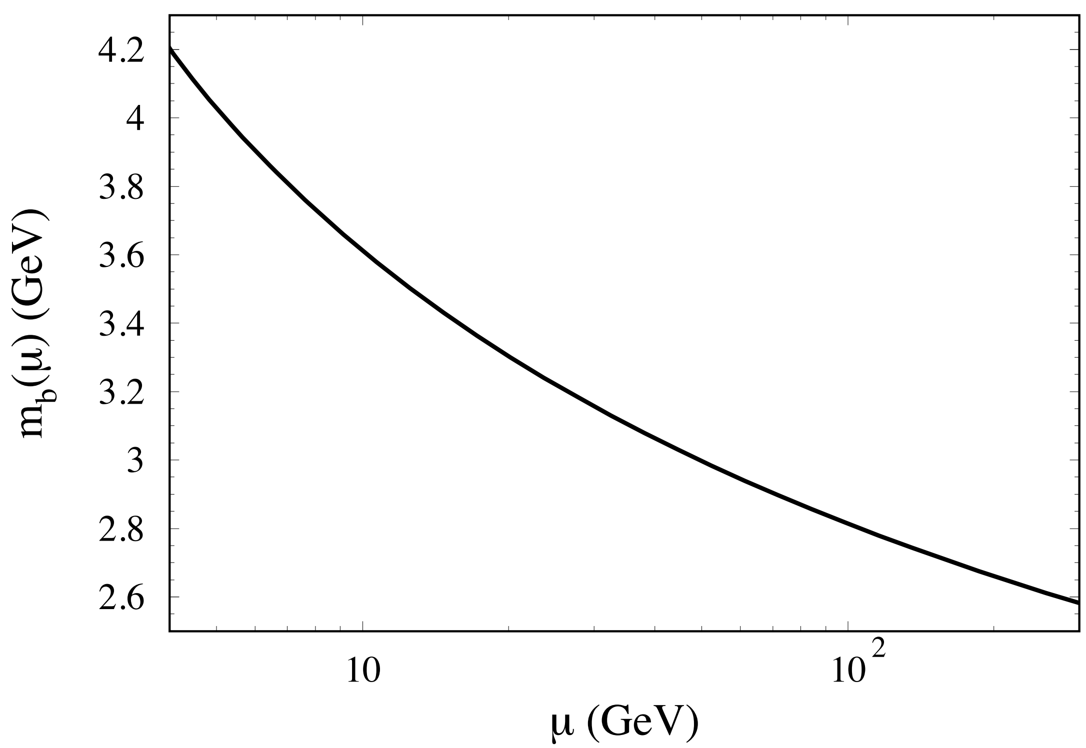
Quark Masses

Running quark mass

- Couplings and masses (parameters in Lagrangian) must all be renormalised, hence masses also scale dependent

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s)\alpha_s = -\alpha_s^2(\beta_0 + \beta_1\alpha_s + \ldots)$$

$$\mu^2 \frac{dm_q}{d\mu^2} = \gamma(\alpha_s)m_q = -\alpha_s(\gamma_0 + \gamma_1\alpha_s + \ldots)m_q \rightarrow \frac{dm_q}{m_q} = \frac{d\alpha_s}{\alpha_s}\frac{\gamma(\alpha_s)}{\beta(\alpha_s)}$$



$$\gamma_0 = \frac{1}{\pi}$$

$$\frac{\gamma_0}{\beta_0} = \frac{12}{33 - 2n_f} \approx \frac{1}{2}$$

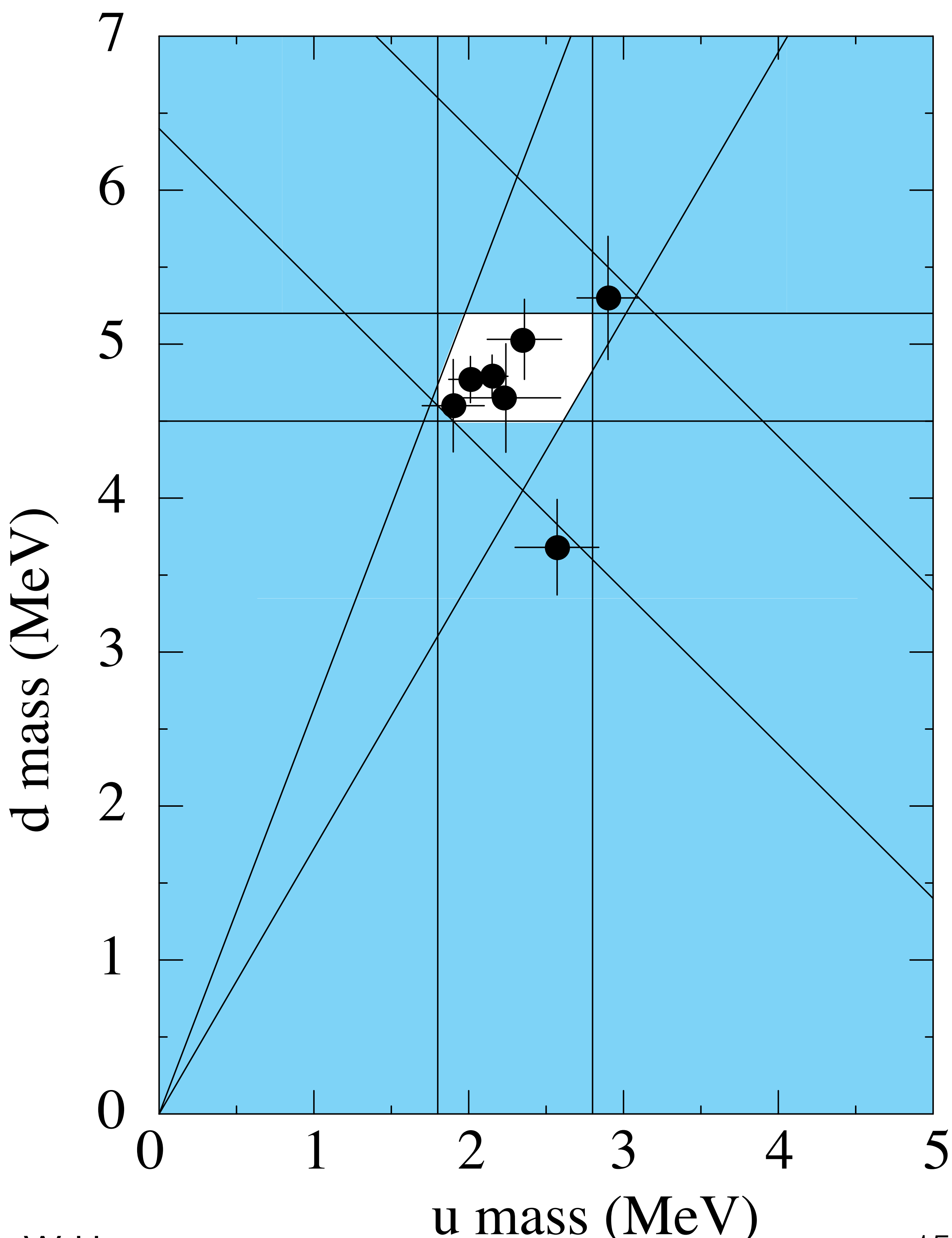
$$\boxed{m_q(\mu) = m_q(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{\gamma_0}{\beta_0}} \left\{1 + \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2}\right)[\alpha_s(\mu) - \alpha_s(\mu_0)] + \ldots\right\}}$$

Lattice light quark masses

FLAG WG: Aoki et al., 1310.8555

Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_u	m_d	m_u/m_d
PACS-CS 12*	[76]	A	★	■	■	★	a	2.57(26)(7)	3.68(29)(10)	0.698(51)
Laiho 11	[77]	C	○	★	★	○	–	1.90(8)(21)(10)	4.73(9)(27)(24)	0.401(13)(45)
HPQCD 10‡	[73]	A	○	★	★	★	–	2.01(14)	4.77(15)	
BMW 10A, 10B$^+$	[22, 23]	A	★	★	★	★	b	2.15(03)(10)	4.79(07)(12)	0.448(06)(29)
Blum 10†	[32]	A	○	■	○	★	–	2.24(10)(34)	4.65(15)(32)	0.4818(96)(860)
MILC 09A	[37]	C	○	★	★	○	–	1.96(0)(6)(10)(12)	4.53(1)(8)(23)(12)	0.432(1)(9)(0)(39)
MILC 09	[15]	A	○	★	★	○	–	1.9(0)(1)(1)(1)	4.6(0)(2)(2)(1)	0.42(0)(1)(0)(4)
MILC 04, HPQCD/ MILC/UKQCD 04	[36, 82]	A	○	○	○	■	–	1.7(0)(1)(2)(2)	3.9(0)(1)(4)(2)	0.43(0)(1)(0)(8)
RM123 13	[45]	A	○	★	○	★	c	2.40(15)(17)	4.80 (15)(17)	0.50(2)(3)
RM123 11$^\oplus$	[104]	A	○	★	○	★	c	*2.43(11)(23)*	*4.78(11)(23)*	*0.51(2)(4)*
Dürr 11*	[61]	A	○	★	○	–	–	2.18(6)(11)	4.87(14)(16)	
RBC 07†	[34]	A	■	■	★	★	–	3.02(27)(19)	5.49(20)(34)	0.550(31)

Light quark masses



Manohar, Sachrajda, Barnett, RPP 2016

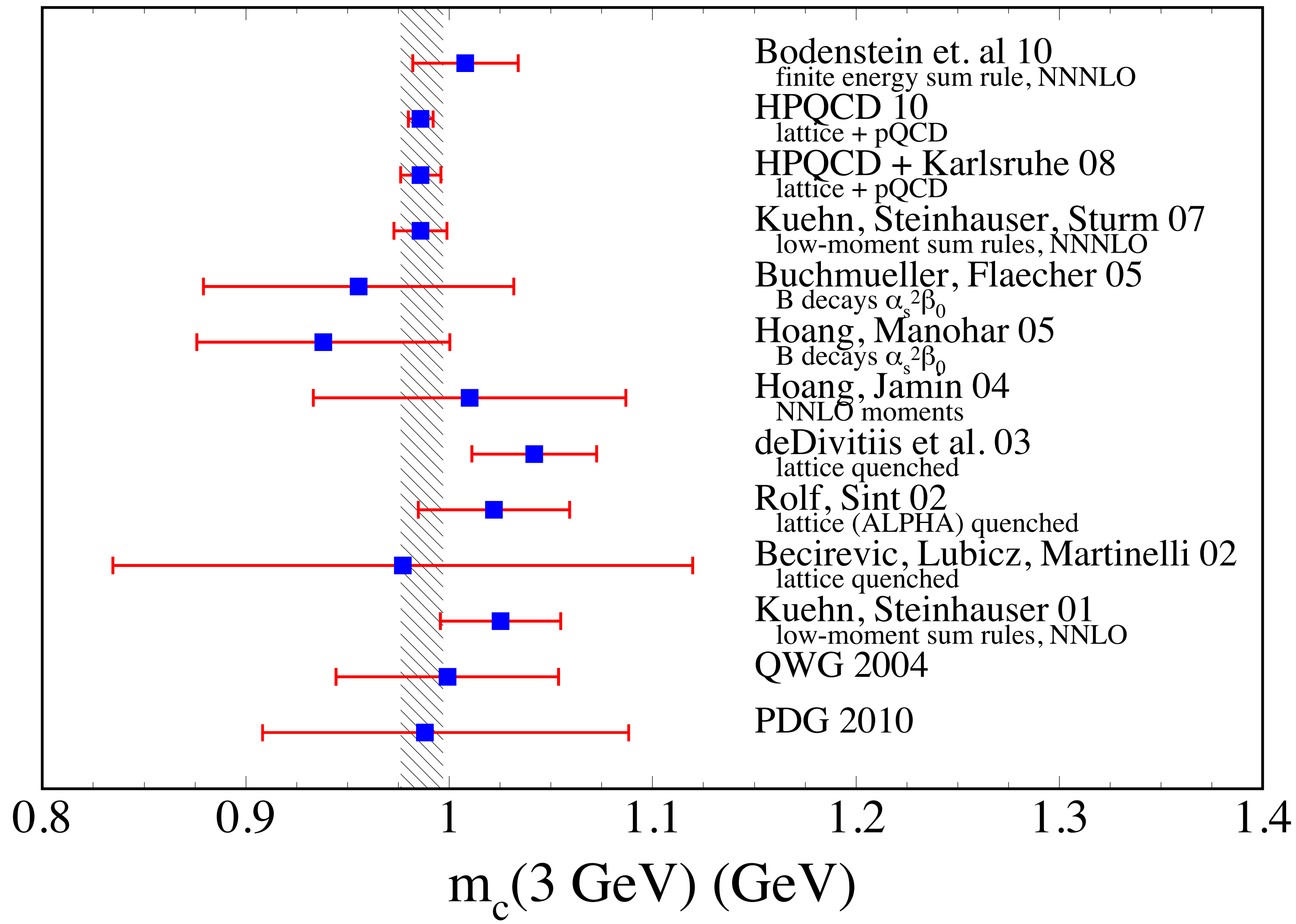
$\overline{\rm MS}$ masses at 2 GeV:

$\overline{m}_u = 2.15 \pm 0.15$ MeV

$\overline{m}_d = 4.70 \pm 0.20$ MeV

$\overline{m}_s = 93.5 \pm 2.0$ MeV

Charm quark mass



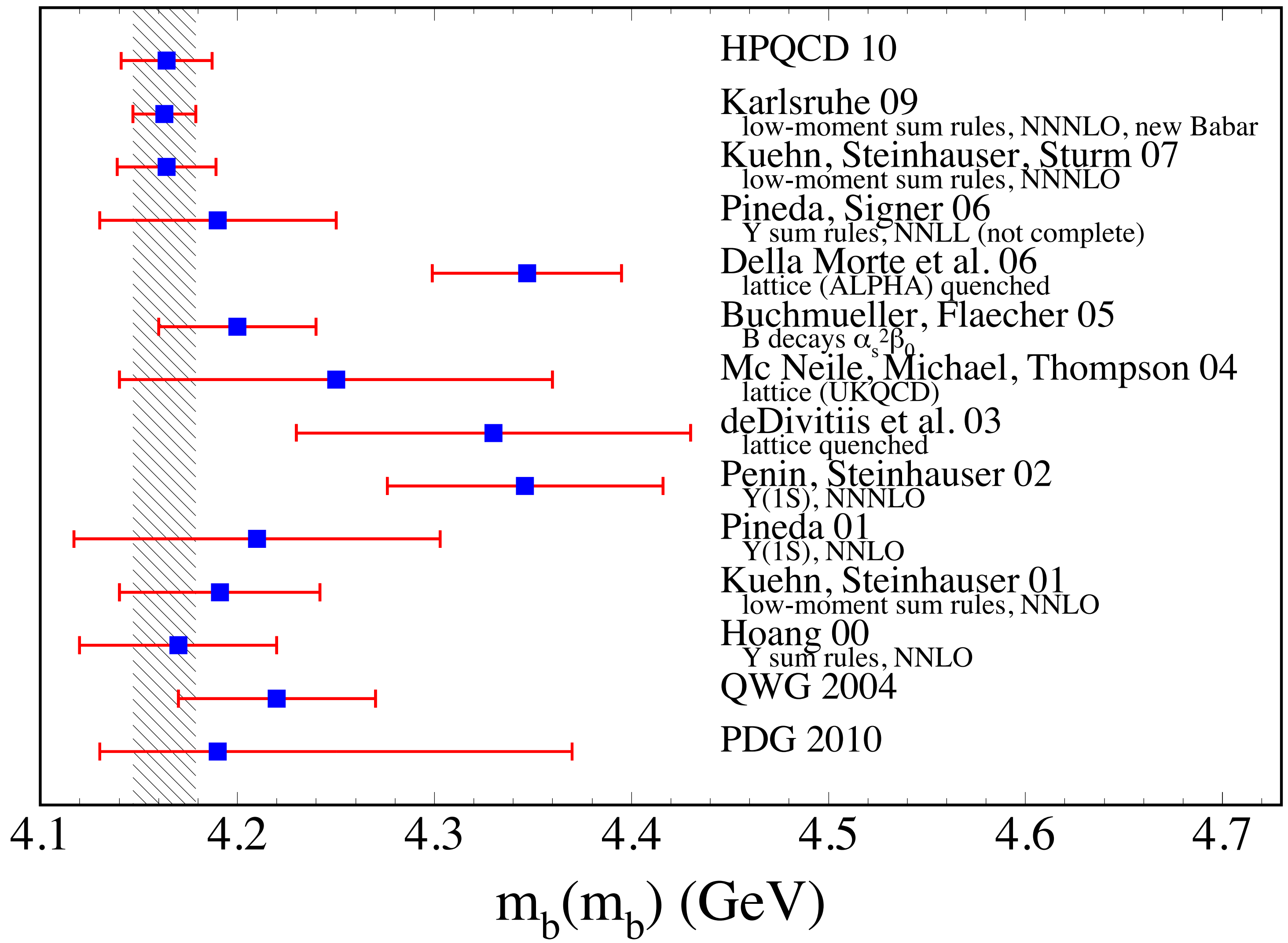
J Kühn, 2013

m_c(3 GeV) = 0.986(6) GeV

➡ $m_c(m_c)$ = 1.268(9) GeV

➡ $m_c(M_H)$ = 0.612(5) GeV

Bottom quark mass



J Kühn, 2013

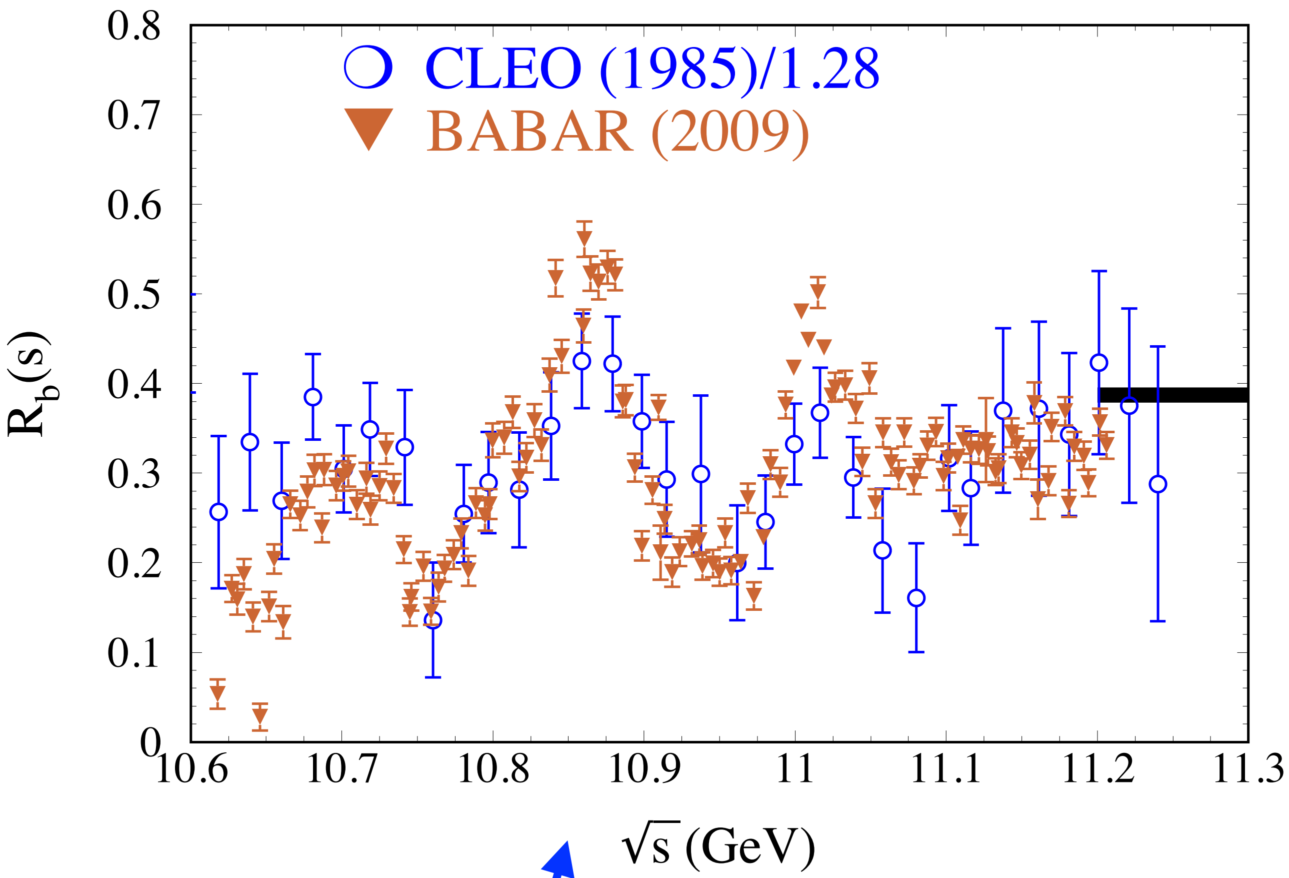
$m_b(10\ \mathrm{GeV}) = 3.617(25)\ \mathrm{GeV}$

→ $m_b(m_b) = 4.164(30)\ \mathrm{GeV}$

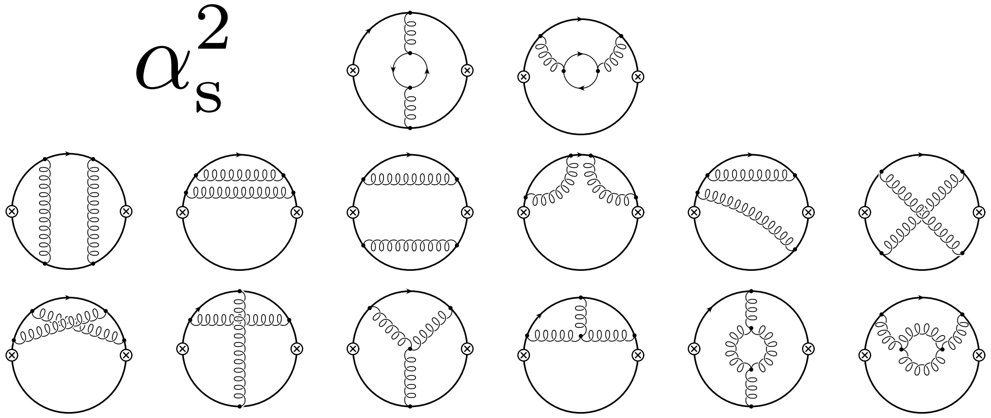
→ $m_b(M_H) = 2.768(21)\ \mathrm{GeV}$

m_b from QCD sum rules

Chetyrkin et al., PRD80(2009)074010



α_s^2



$\ldots + \alpha_s^3$ (~700 diagrams)

$$\mathcal{M}_n = \int \frac{ds}{s^{n+1}} R_b(s) = \frac{9}{4} e_b^2 \left(\frac{1}{4m_b^2(\mu)} \right)^n C_n(\alpha_s, \mu) \longrightarrow m_b(\mu) = \frac{1}{2} \left(\frac{9e_b^2 C_n(\alpha_s, \mu)}{4\mathcal{M}_n} \right)^{\frac{1}{2n}}$$

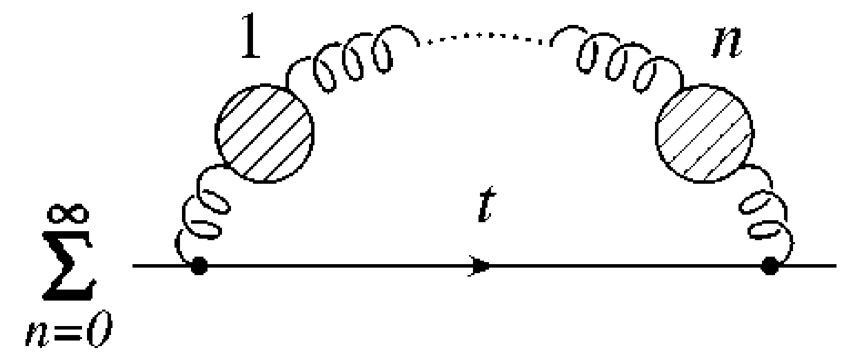
n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

$\boxed{m_b(10\text{ GeV}) = 3.610(16)\text{ GeV}}$

Pole quark mass

$$D(\not{p}) = \frac{i}{\not{p} - m_q - \Sigma(\not{p})}$$

$$\not{p}_{\text{pole}} = m_q + \Sigma(\not{p}) = m_q + \Sigma^{(1)}(m_q) + \ldots$$



$$c_n \sim 2^n n! \sim (2n/e)^n$$

$$\Sigma^{(1)}(m_q) = \frac{16 m_q}{3\beta_0} \sum_{n=0}^{\infty} c_n\, a^{n+1} \qquad a = \frac{\beta_0 \alpha_{\text{s}}(m_q)}{4\pi} \sim \frac{1}{\log(m_q^2/\Lambda^2)} \equiv \frac{1}{L}$$

Asymptotic expansion: sum to smallest term (n~L/2)

Ambiguity ~ smallest term ($c_n\, a^{n+1} \sim e^{-L/2} \sim \Lambda/m_q$)

$$m_{\text{pole}} = m_q(m_q)\left\{1 + 0.4244\,\alpha_{\text{s}}(m_q) + 0.835\,\alpha_{\text{s}}^2(m_q) + 2.375\,\alpha_{\text{s}}^3(m_q)\right] + \mathcal{O}(\Lambda)$$

Renormalon ambiguity
(There is no pole!)

Top quark mass

"Direct" (≈pole mass?) measurements:

Liss, Maltoni, Quadt, RPP 2016

m_t (GeV/c^2)	Source	$\int \mathcal{L}$dt	Ref.	Channel
$172.99 \pm 0.48 \pm 0.78$	ATLAS	4.6	[123]	ℓ+jets+$\ell\ell$
$172.04 \pm 0.19 \pm 0.75$	CMS	19.7	[124]	ℓ+jets
$172.47 \pm 0.17 \pm 1.40$	CMS	19.7	[131]	$\ell\ell$
$172.32 \pm 0.25 \pm 0.59$	CMS	19.7	[134]	All jets
$174.34 \pm 0.37 \pm 0.52$	CDF,DØ (I+II)	$\leq$9.7	[145]	publ. or prelim.
$173.34 \pm 0.27 \pm 0.71$	Tevatron+LHC	$\leq$8.7+$\leq$4.9	[3]	publ. or prelim.

m_t(pole) = 173.1±0.9 GeV

$m_t(m_t)$ = 163.35±0.85 GeV

$m_t(m_t)$ = $160^{+4.8}_{-4.3}$ GeV from cross section

Parton Distribution Functions

QCD Factorization

$$\sigma_{pp\to X}(s) = \sum_{i,j} \int_0^1 \underbrace{dx_1\, dx_2}_{\text{momentum fractions}}\, \underbrace{f_i(x_1,\mu_F^2) f_j(x_2,\mu_F^2)}_{\text{parton distribution functions at scale } \mu_F}\, \hat{\sigma}_{ij\to X}\underbrace{\left(x_1 x_2 s, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2\right)}_{\text{hard process cross section, renormalised at scale } \mu_R}$$

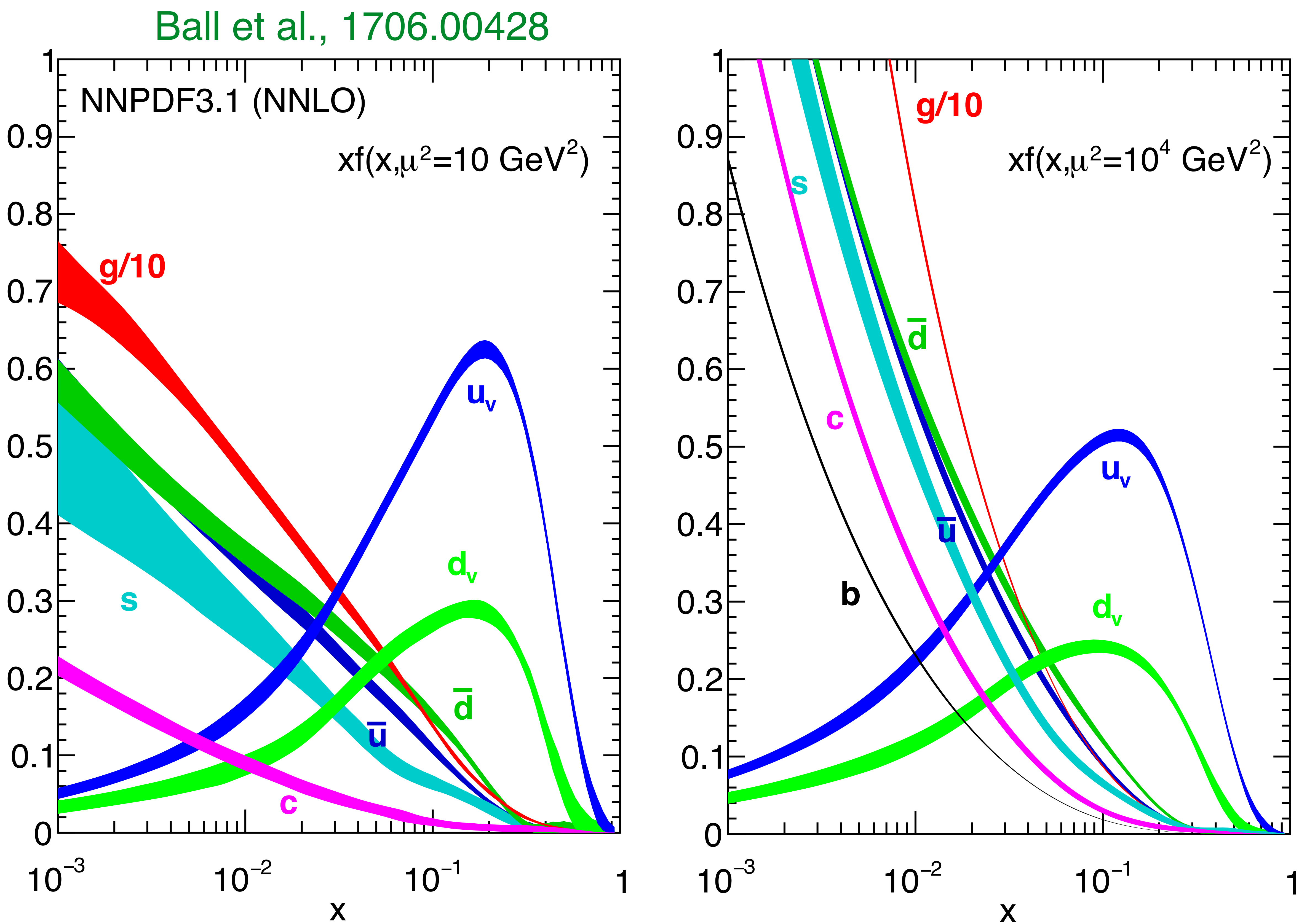
- Non-perturbative physics takes place over a much longer time scale, with unit probability
- Hence it cannot change the cross section
- Scale dependences of parton distribution functions and hard process cross section are perturbatively calculable, and cancel order by order
- Residual scale dependence is (part of) theory uncertainty

PDF Evolution

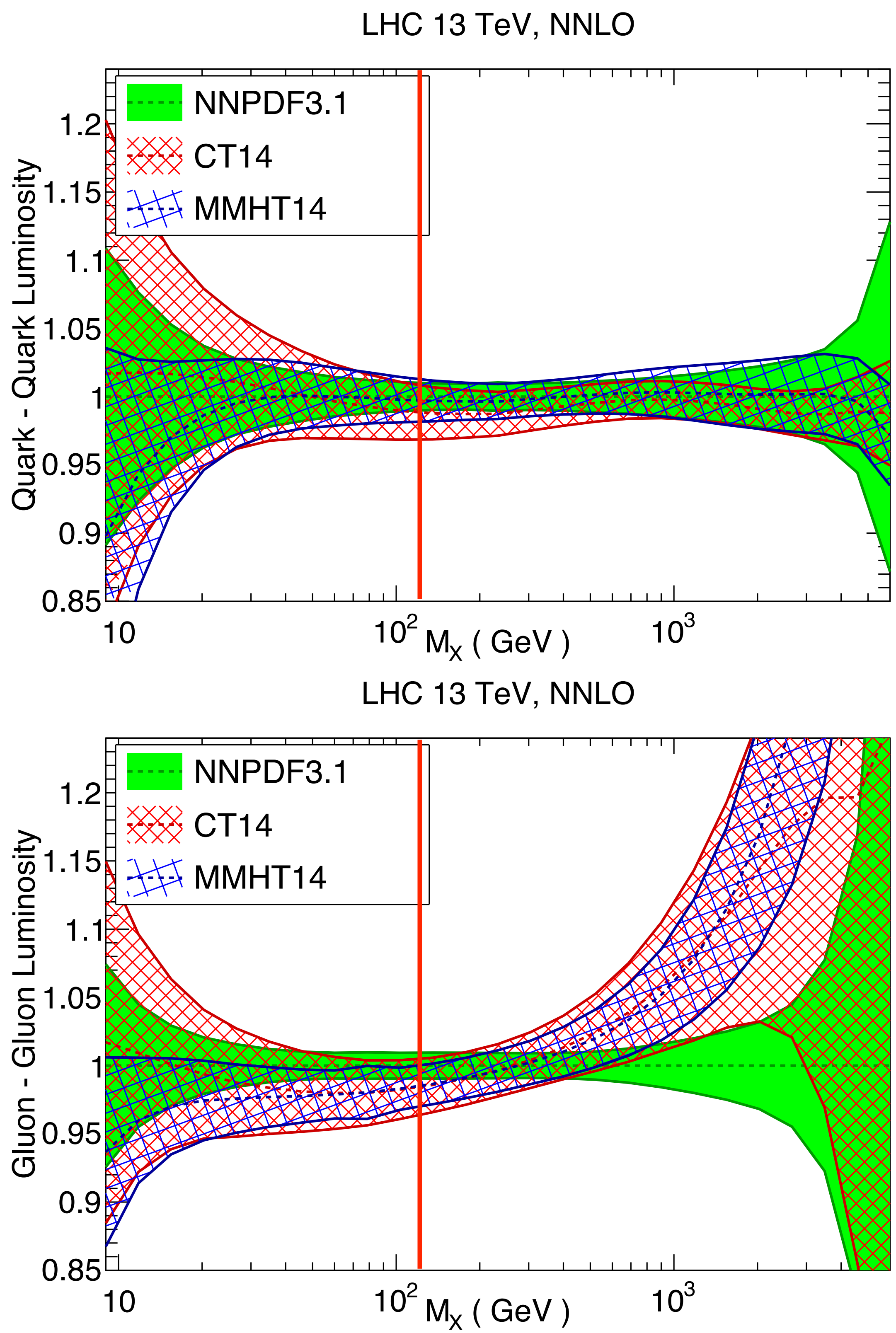
$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x,\mu^2) = \sum_j \int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi}, \alpha_{\mathrm{S}}(\mu^2)\right) f_j(\xi,\mu^2)$$

- PDFs measured in various processes at various scales
- Global fits satisfying evolution equations give PDF sets
- Generally done at NNLO nowadays

PDF Evolution



PDF Uncertainties



Ball et al., 1706.00428

- Parton luminosity $\mathcal{L}_{ij}(M_X^2, s) = \int \mathrm{d}x_1 \mathrm{d}x_2\, f_i(x_1, M_X^2)\, f_j(x_2, M_X^2)\, \delta\left(x_1 x_2 s - M_X^2\right)$
- Relevant PDFs (relatively) well known at x ~ $M_H/\sqrt{s}$
- Still some disagreements in $\mathcal{L}_{gg}$
- Can be improved (in principle)

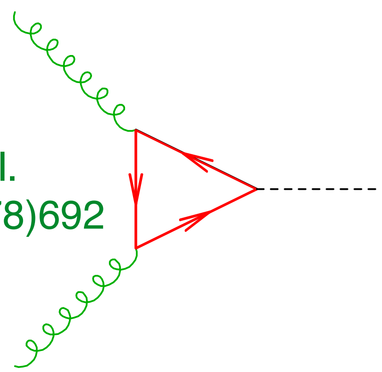
QCD and the Higgs Boson

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHCHXSWG

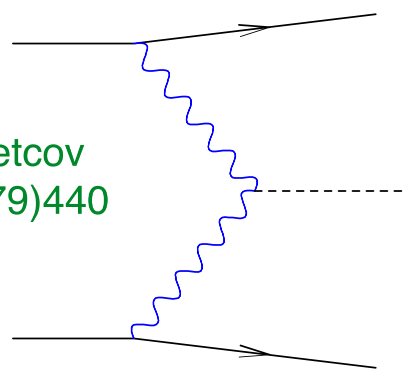
Higgs Production Cross Section

Higgs production cross sections

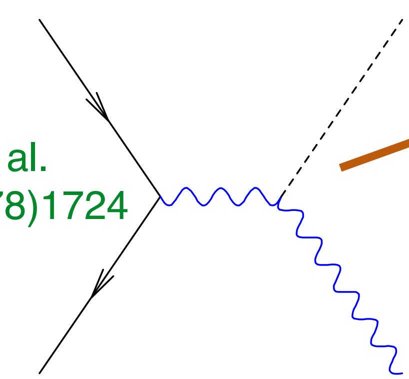
Georgi et al.
PRL40(1978)692



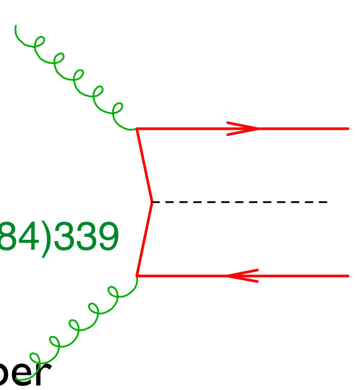
Jones & Petcov
PLB84(1979)440

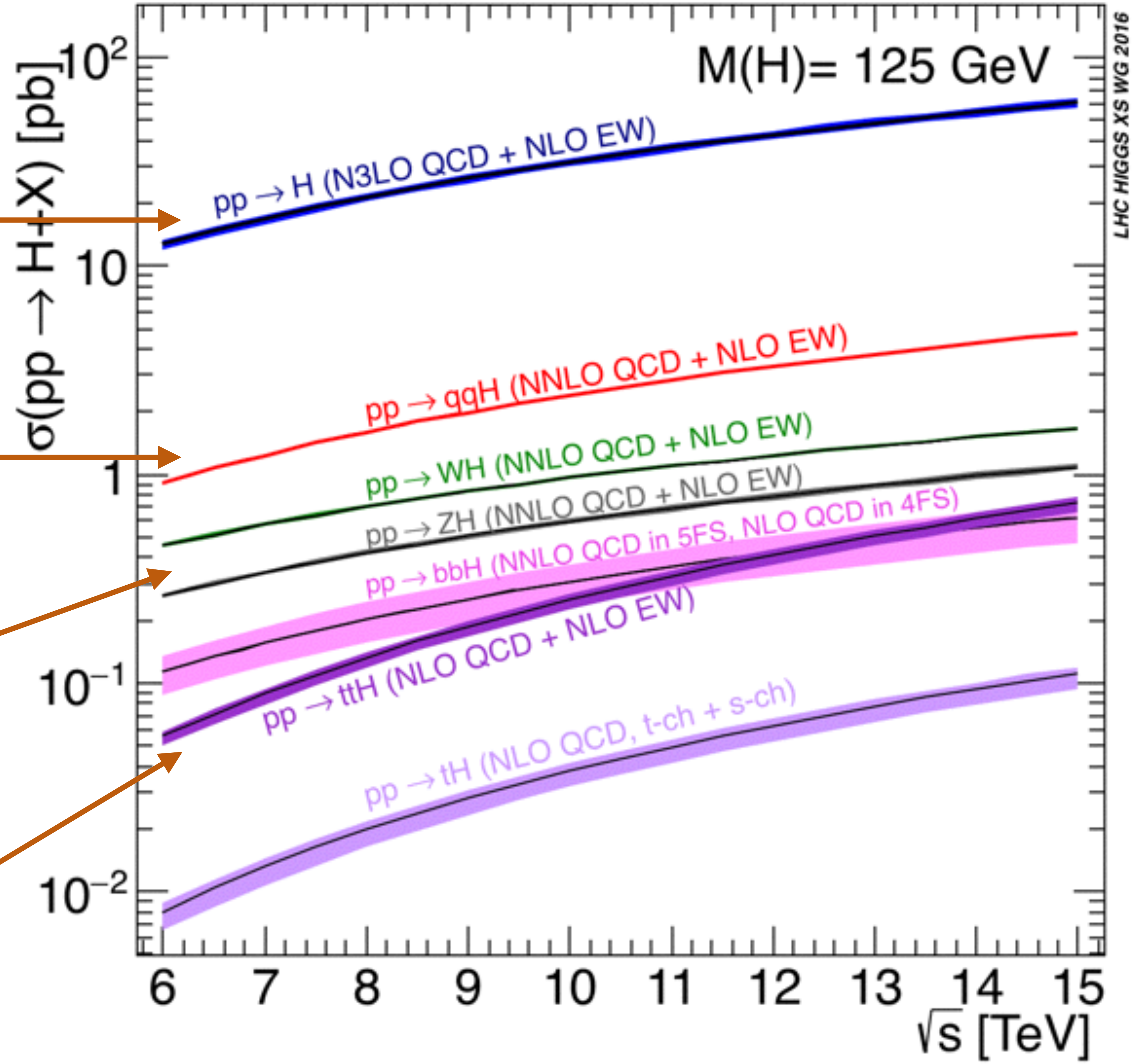


Glashow et al.
PRD18(1978)1724



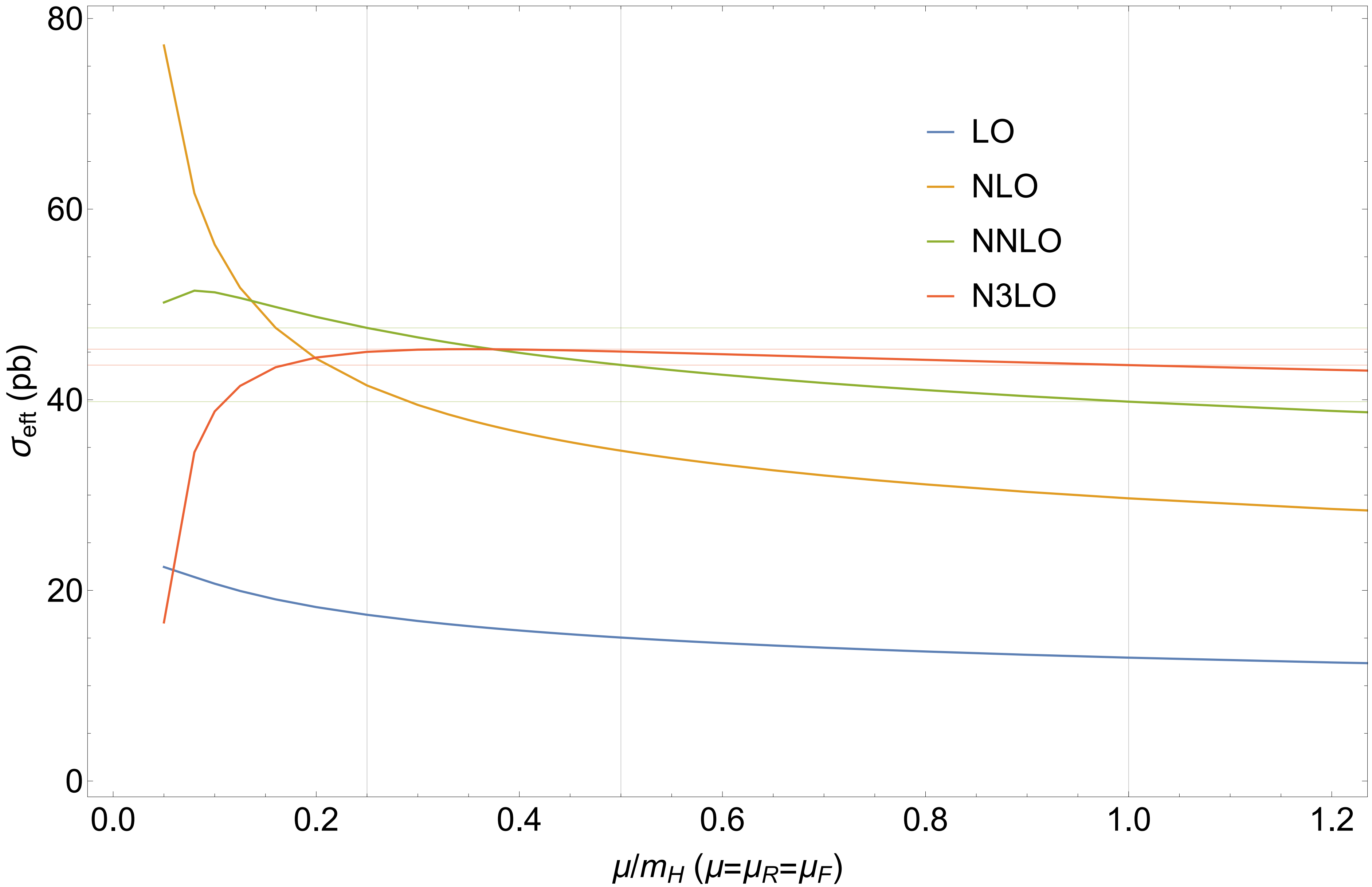
Kunszt
NPB247(1984)339





Gluon fusion cross section

- Scale dependence (13 TeV, $m_t \to \infty$)



$$\sigma = 48.58\,\mathrm{pb}^{+2.22\,\mathrm{pb}\,(+4.56\%)}_{-3.27\,\mathrm{pb}\,(-6.72\%)}\ (\mathrm{theory}) \pm 1.56\,\mathrm{pb}\,(3.20\%)\ (\mathrm{PDF}+\alpha_s)\,.$$

Anastasiou et al., JHEP 05(2016)058 (arXiv:1602.00695)

Gluon fusion cross section

- Quark mass dependence (NLO only)

Top quark			Bottom quark			Charm quark		
$\delta m_t = 1$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_b = 0.03$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77	$\delta m_c = 0.026$ GeV	$\sigma^{NLO}_{ex;t+b+c}$	34.77
$m_t + \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.74	$m_b + \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.76	$m_c + \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.76
$m_t - \delta m_t$	$\sigma^{NLO}_{ex;t+b+c}$	34.80	$m_b - \delta m_b$	$\sigma^{NLO}_{ex;t+b+c}$	34.79	$m_c - \delta m_c$	$\sigma^{NLO}_{ex;t+b+c}$	34.78

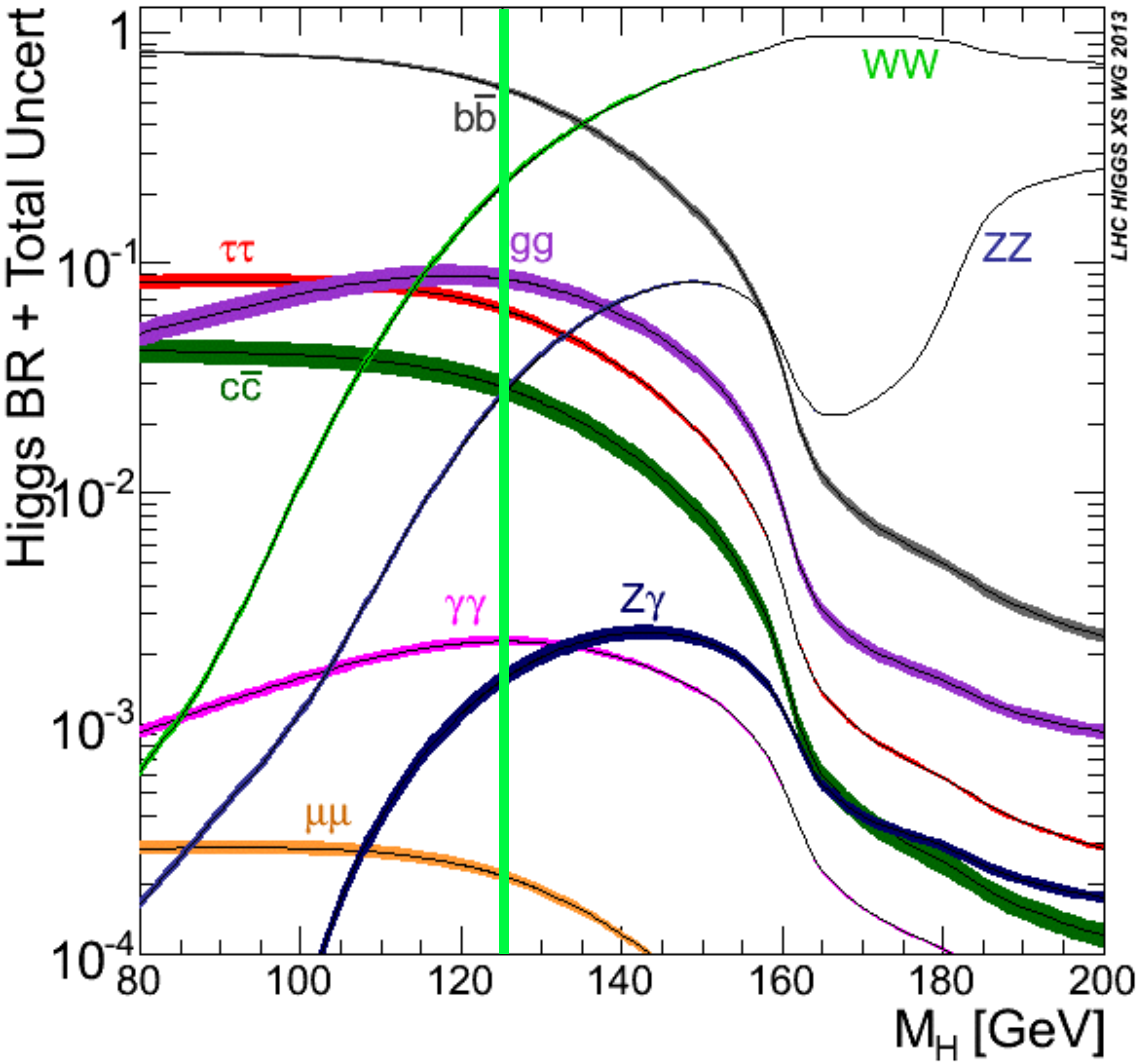
- Energy dependence

E_{CM}	σ	δ(theory)	δ(PDF)	$\delta(\alpha_s)$
2 TeV	1.10 pb	$^{+0.04\text{pb}}_{-0.09\text{pb}}\left(^{+4.06\%}_{-7.88\%}\right)$	$\pm$ 0.03 pb ($\pm$ 3.17%)	$^{+0.04\text{pb}}_{-0.04\text{pb}}\left(^{+3.36\%}_{-3.69\%}\right)$
7 TeV	16.85 pb	$^{+0.74\text{pb}}_{-1.17\text{pb}}\left(^{+4.41\%}_{-6.96\%}\right)$	$\pm$ 0.32 pb ($\pm$ 1.89%)	$^{+0.45\text{pb}}_{-0.45\text{pb}}\left(^{+2.67\%}_{-2.66\%}\right)$
8 TeV	21.42 pb	$^{+0.95\text{pb}}_{-1.48\text{pb}}\left(^{+4.43\%}_{-6.90\%}\right)$	$\pm$ 0.40 pb ($\pm$ 1.87%)	$^{+0.57\text{pb}}_{-0.56\text{pb}}\left(^{+2.65\%}_{-2.62\%}\right)$
13 TeV	48.58 pb	$^{+2.22\text{pb}}_{-3.27\text{pb}}\left(^{+4.56\%}_{-6.72\%}\right)$	$\pm$ 0.90 pb ($\pm$ 1.86%)	$^{+1.27\text{pb}}_{-1.25\text{pb}}\left(^{+2.61\%}_{-2.58\%}\right)$
14 TeV	54.67 pb	$^{+2.51\text{ pb}}_{-3.67\text{ pb}}\left(^{+4.58\%}_{-6.71\%}\right)$	$\pm$1.02 pb ($\pm$ 1.86%)	$^{+1.43\text{pb}}_{-1.41\text{pb}}\left(^{+2.61\%}_{-2.59\%}\right)$

Anastasiou et al., JHEP 05(2016)058 (arXiv:1602.00695)

Higgs Decays

Higgs Branching Ratios



$\Gamma_H = 4.10 \pm 0.09$ MeV

Mode	BR (%)	δBR
$b\bar{b}$	58.1	1.3
WW*	21.5	0.6
gg	8.2	0.7
ττ	6.3	0.2
$c\bar{c}$	2.9	0.2
ZZ*	2.64	0.07
γγ	0.227	0.008

Higgs → $q\bar{q}$

$$\Gamma(H \to q\bar{q}) = \frac{3\sqrt{2}}{8\pi} G_F M_H m_q^2(M_H) \left[1 - \frac{4m_q^2(M_H)}{M_H^2}\right]^{\frac{3}{2}} \left[1 + 1.803\,\alpha_s(M_H) + 2.953\,\alpha_s^2(M_H) + \ldots\right]$$

(known to 4th order)

- Running of masses is enormously important!

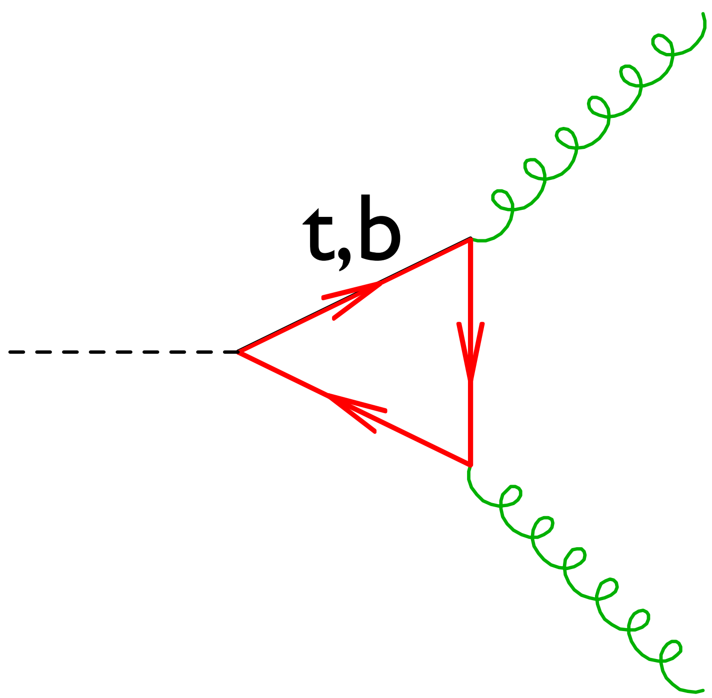
$m_b^2(M_H)/m_b^2(\text{pole}) = (2.77/4.95)^2 = 0.313$

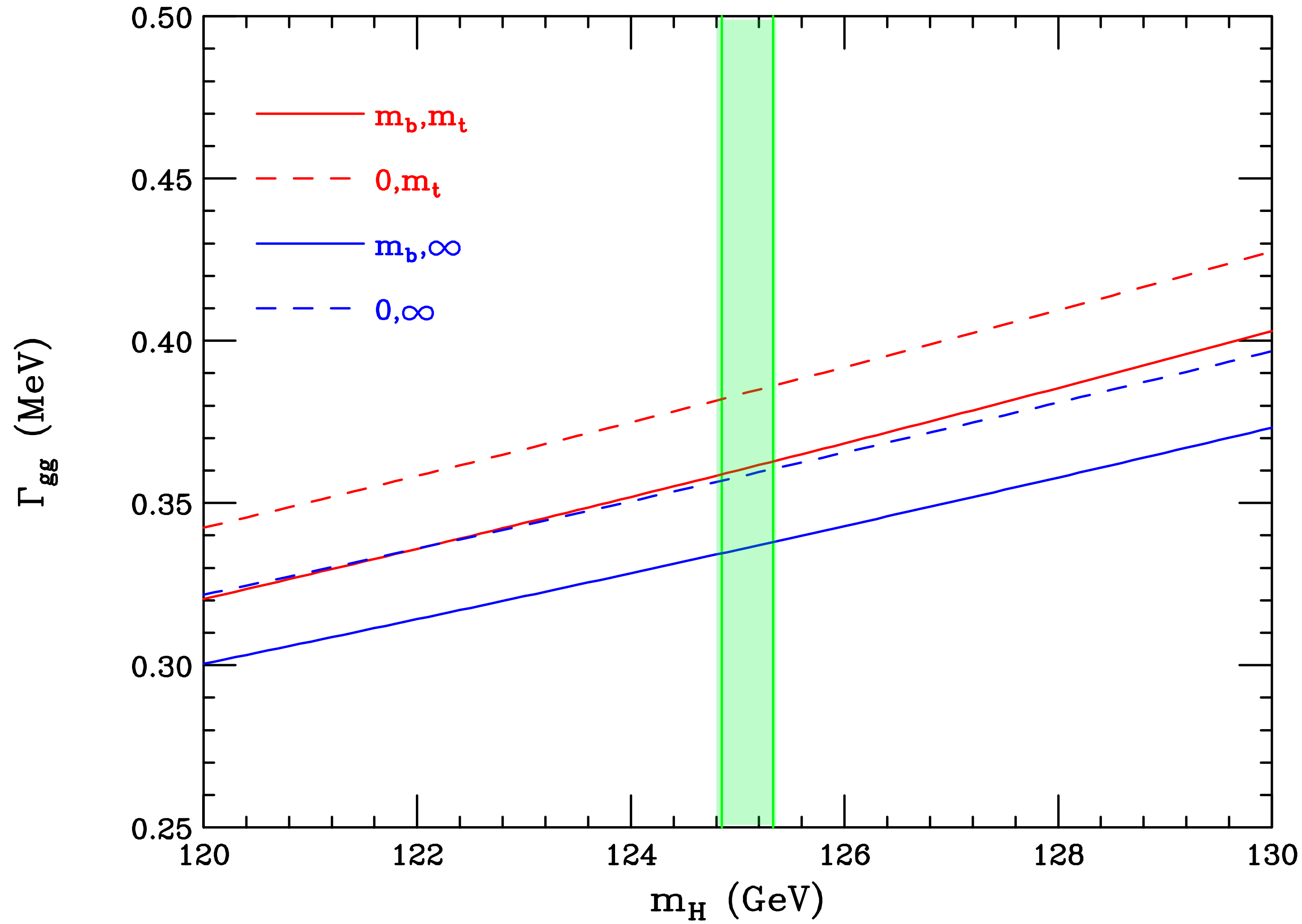
$m_c^2(M_H)/m_c^2(\text{pole}) = (0.612/1.27)^2 = 0.233$

- Γ_b affects all branching ratios!

$$\mathrm{BR(X)} = \frac{\Gamma_\mathrm{X}}{\Gamma_\mathrm{tot}} \quad \longrightarrow \quad \frac{\delta\,\mathrm{BR(X)}}{\mathrm{BR(X)}} = \frac{\delta\,\Gamma_\mathrm{b}}{\Gamma_\mathrm{tot}} = 0.58\frac{\delta\,\Gamma_\mathrm{b}}{\Gamma_\mathrm{b}}$$

Higgs→gg

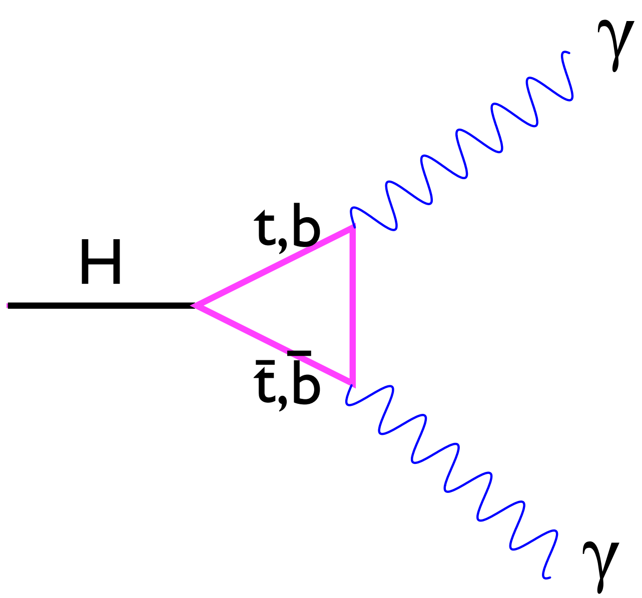


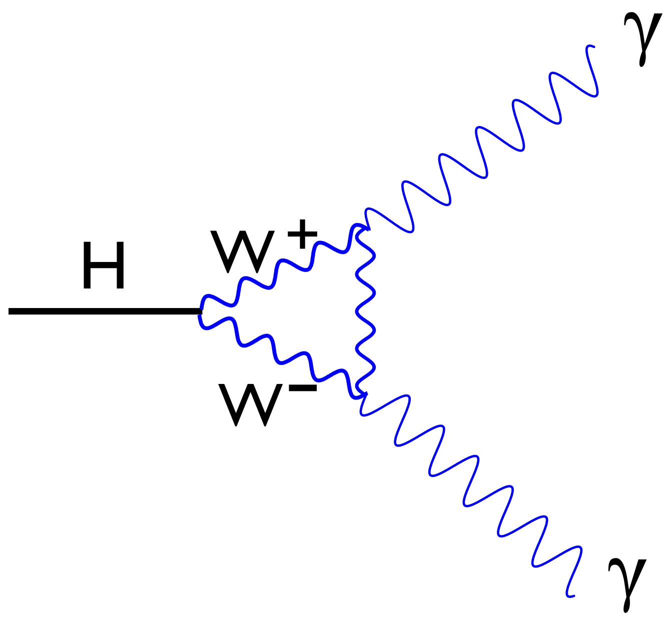


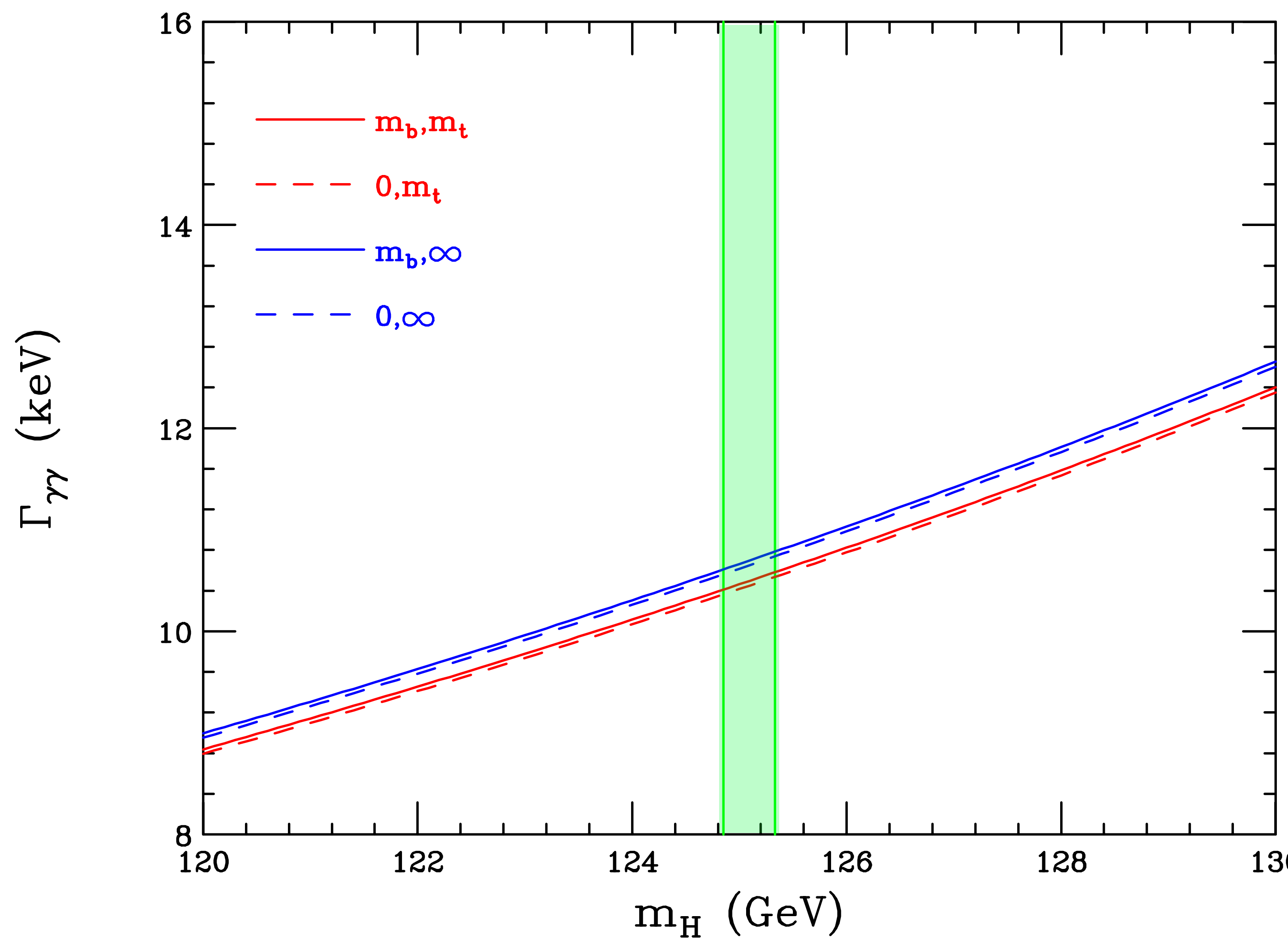
$$\Gamma_{gg} = \frac{\alpha_s^2 G_F M_H^3}{64\sqrt{2}\pi^3} \left| \sum_q I_q \left(\frac{m_q^2(M_H)}{M_H^2} \right) \right|^2 \left(1 + 6.14\,\alpha_s + 17.5\,\alpha_s^2 + 15.1\,\alpha_s^3 + \ldots\right)$$

- b contributes ~ -6%, which almost cancels top mass effect

Higgs→γγ





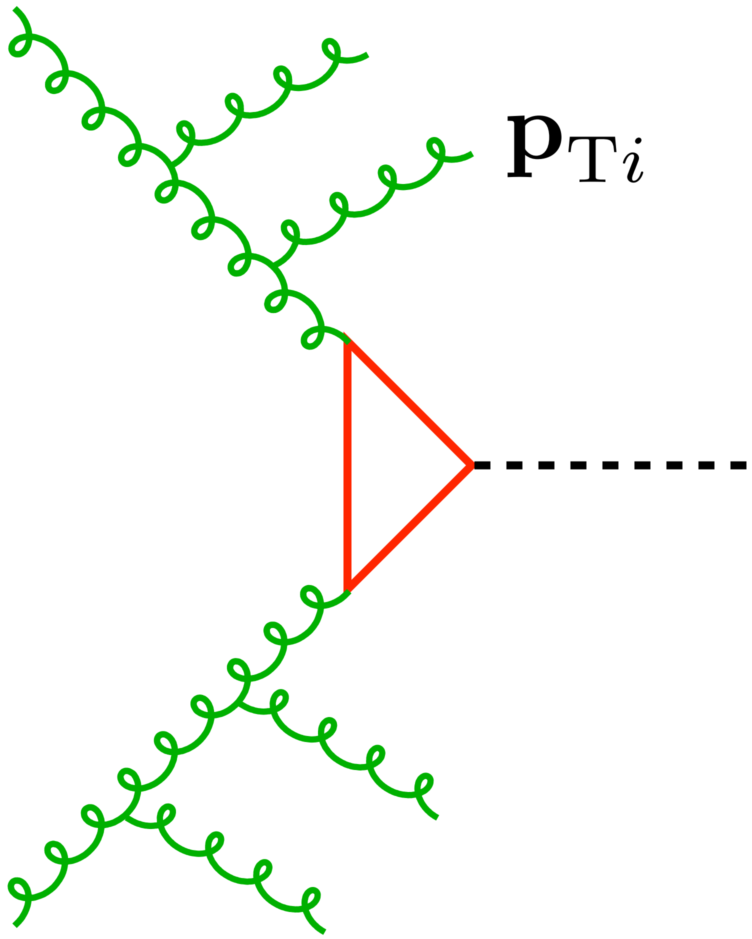


$$\Gamma_{\gamma\gamma} = \frac{\alpha^2 G_F M_H^3}{128\sqrt{2}\pi^3}\left|3\sum_q e_q^2 I_q\left(\frac{m_q^2(M_H)}{M_H^2}\right) + I_W\left(\frac{M_W^2}{M_H^2}\right)\right|^2$$

- W loop dominates
- b contributes less, so top mass effect is significant (~-2%)

Higgs Transverse Momentum

Higgs Transverse Momentum

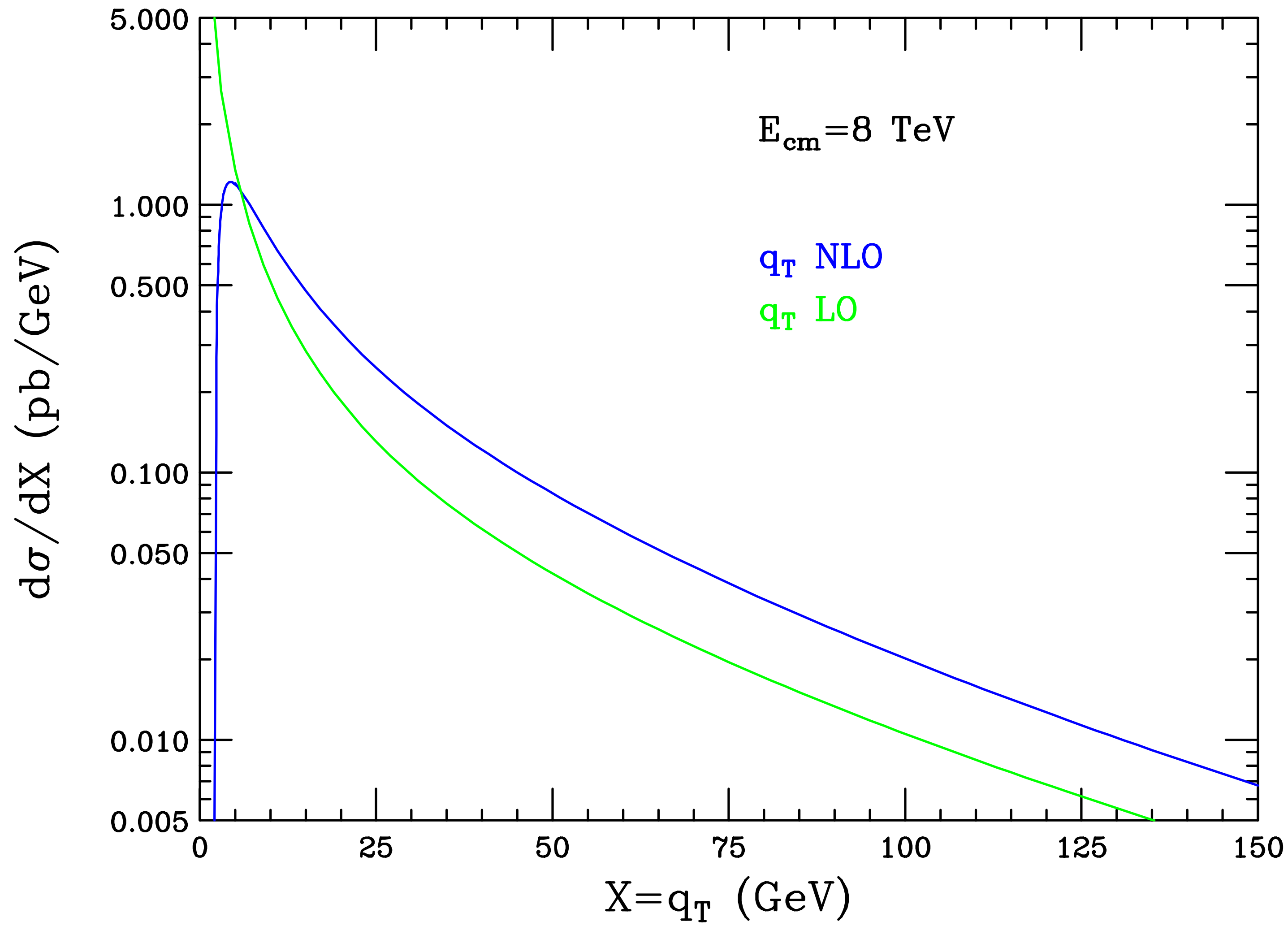


- Resummation of Higgs transverse momentum

$$\mathbf{q}_{\mathrm{T}} = -\sum \mathbf{p}_{\mathrm{T}i}$$

Bozzi et al. 0705.3887
Mantry & Petriello, 0911.4135
Catani & Grazzini, 1011.3918
de Florian et al. 1109.2109

Higgs q_T (fixed order)



- $(\mathrm{N})\mathrm{LO} \underset{q_\mathrm{T}\to 0}{\longrightarrow} (-)\infty$
- Large logs of m_H^2/q_T^2 need resummation

Resummation of Higgs q_T

$$d\sigma = \int dx_1 dx_2\, f_a(x_1,\mu) f_b(x_2,\mu)\, d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \ldots)$$

$$\frac{1}{\hat{\sigma}_{gg}}\frac{d^2\hat{\sigma}_{gg}}{d\,\mathbf{q}_\mathrm{T}^2} \sim \delta^2(\mathbf{q}_\mathrm{T}) + \alpha_\mathrm{S}\int d^2\mathbf{p}_\mathrm{T}\left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2}\ln\frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right]_+ \delta^2(\mathbf{q}_\mathrm{T}+\mathbf{p}_\mathrm{T}) + \ldots$$

$$A_g = C_A = 3\,,\quad B_g = -\frac{1}{6}(11C_A - 2n_f) = -\frac{23}{6}$$

Resummation of Higgs q_T

$$d\sigma = \int dx_1 dx_2\, f_a(x_1,\mu) f_b(x_2,\mu)\, d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \ldots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2\hat{\sigma}_{gg}}{d\,\mathbf{q}_\mathrm{T}^2} \sim \delta^2(\mathbf{q}_\mathrm{T}) + \alpha_\mathrm{S} \int d^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2} \right]_+ \delta^2(\mathbf{q}_\mathrm{T} + \mathbf{p}_\mathrm{T}) + \ldots$$

$$A_g = C_A = 3\,, \quad B_g = -\frac{1}{6}(11 C_A - 2 n_f) = -\frac{23}{6}$$

$$\sim \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}\cdot\mathbf{q}_\mathrm{T}} \left\{ 1 + \alpha_\mathrm{S} \int d^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2} \right] \left(e^{i\mathbf{b}\cdot\mathbf{p}_\mathrm{T}} - 1 \right) + \ldots \right\}$$

Resummation of Higgs q_T

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2\, f_a(x_1,\mu) f_b(x_2,\mu)\, \mathrm{d}\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \ldots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{\mathrm{d}^2\hat{\sigma}_{gg}}{\mathrm{d}\,\mathbf{q}_\mathrm{T}^2} \sim \delta^2(\mathbf{q}_\mathrm{T}) + \alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right]_+ \delta^2(\mathbf{q}_\mathrm{T} + \mathbf{p}_\mathrm{T}) + \ldots$$

$$A_g = C_A = 3\,, \quad B_g = -\frac{1}{6}(11 C_A - 2n_f) = -\frac{23}{6}$$

$$\sim \int \frac{\mathrm{d}^2\mathbf{b}}{(2\pi)^2} \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}_\mathrm{T}} \left\{1 + \alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right] \left(\mathrm{e}^{i\mathbf{b}\cdot\mathbf{p}_\mathrm{T}} - 1\right) + \ldots\right\}$$

$$\sim \int \frac{\mathrm{d}^2\mathbf{b}}{(2\pi)^2} \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}_\mathrm{T}} \exp\left\{\alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right] \left(\mathrm{e}^{i\mathbf{b}\cdot\mathbf{p}_\mathrm{T}} - 1\right)\right\}$$

Resummation & matching of Higgs q_T

$$d\sigma = \int dx_1 dx_2\, f_a(x_1,\mu) f_b(x_2,\mu)\, d\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \ldots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d^2\hat{\sigma}_{gg}}{d\,\mathbf{q}_T^2} \sim \delta^2(\mathbf{q}_T) + \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2}\ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right]_+ \delta^2(\mathbf{q}_T + \mathbf{p}_T) + \ldots$$

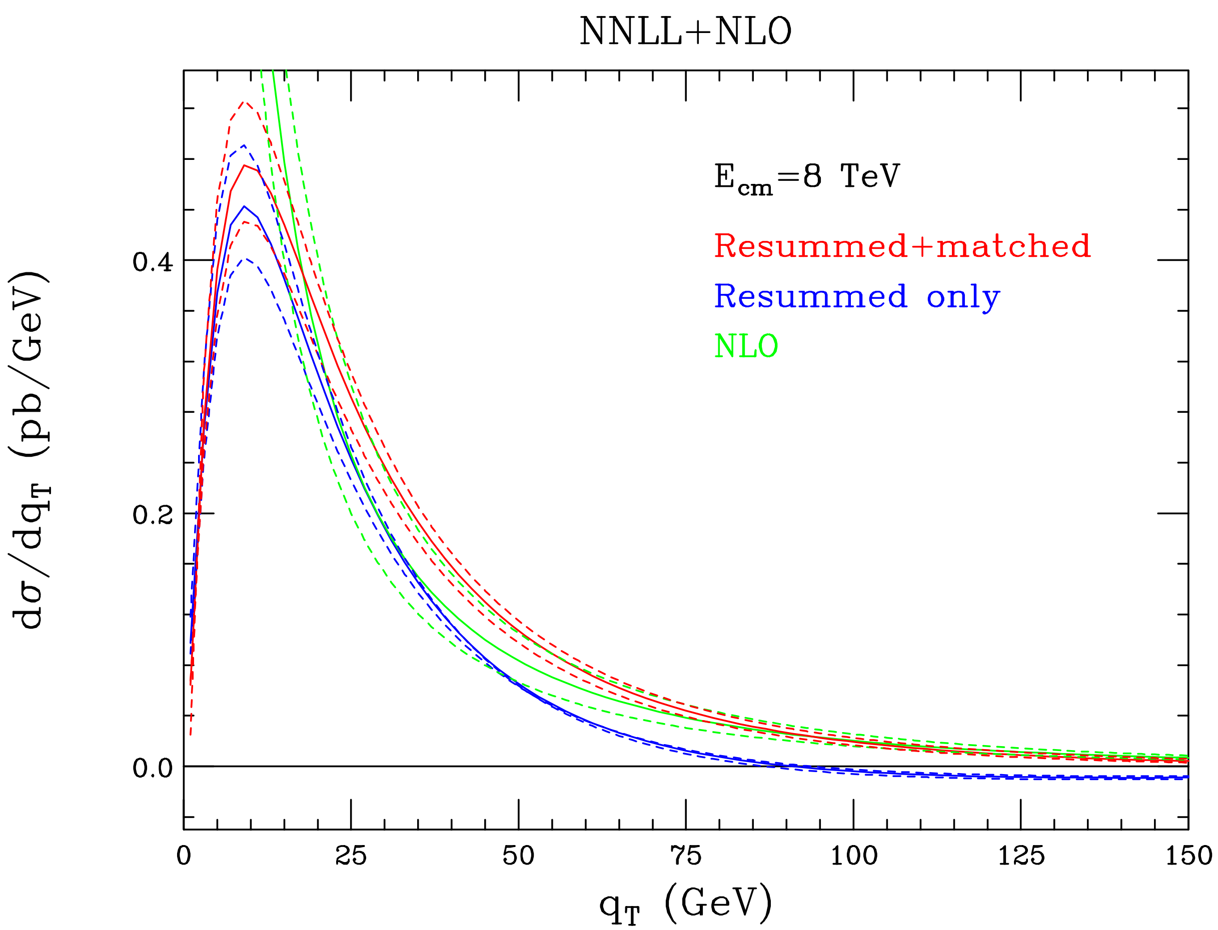
$$A_g = C_A = 3\,, \quad B_g = -\frac{1}{6}(11C_A - 2n_f) = -\frac{23}{6}$$

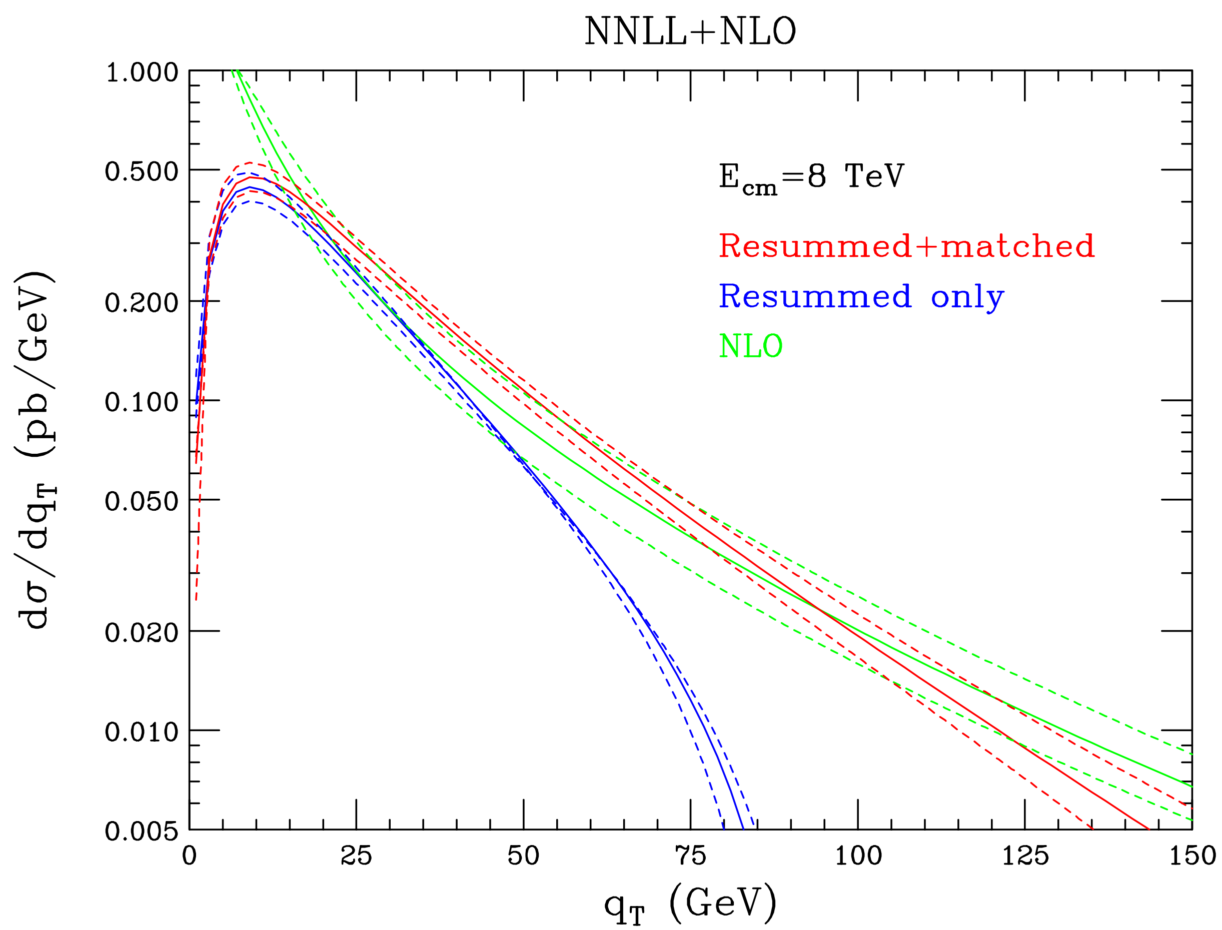
$$\sim \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\mathbf{q}_T} \left\{1 + \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2}\ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right]\left(e^{i\mathbf{b}\cdot\mathbf{p}_T} - 1\right) + \ldots\right\}$$

$$\sim \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\mathbf{q}_T} \exp\left\{\alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2}\ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right]\left(e^{i\mathbf{b}\cdot\mathbf{p}_T} - 1\right)\right\}$$

$$\frac{d\sigma}{dq_T} = \left[\frac{d\sigma}{dq_T}\right]_{\rm resum} - \left[\frac{d\sigma}{dq_T}\right]_{\rm resum,NLO} + \left[\frac{d\sigma}{dq_T}\right]_{\rm NLO}$$

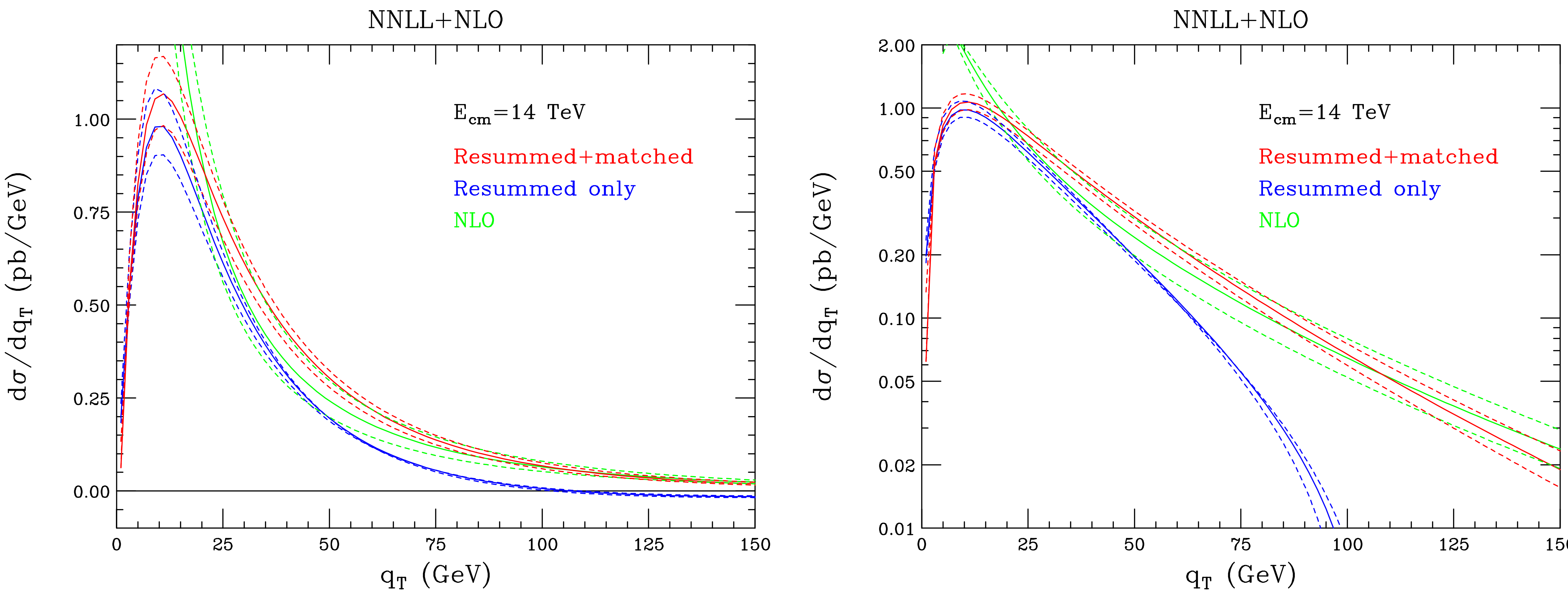
Higgs transverse momentum: 8 TeV





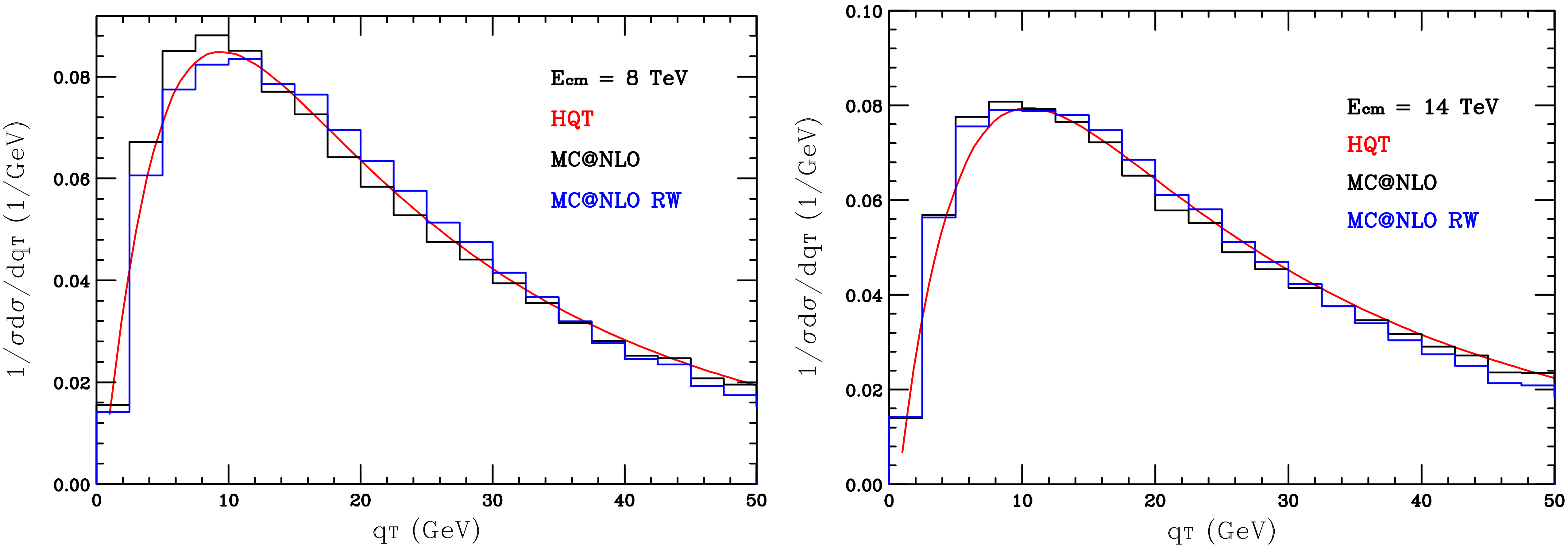
- Peak at ~10 GeV: $\log(m_H^2/q_T^2)\sim 5.1$
- Resummation affects spectrum out to larger q_T

Higgs transverse momentum: 14 TeV



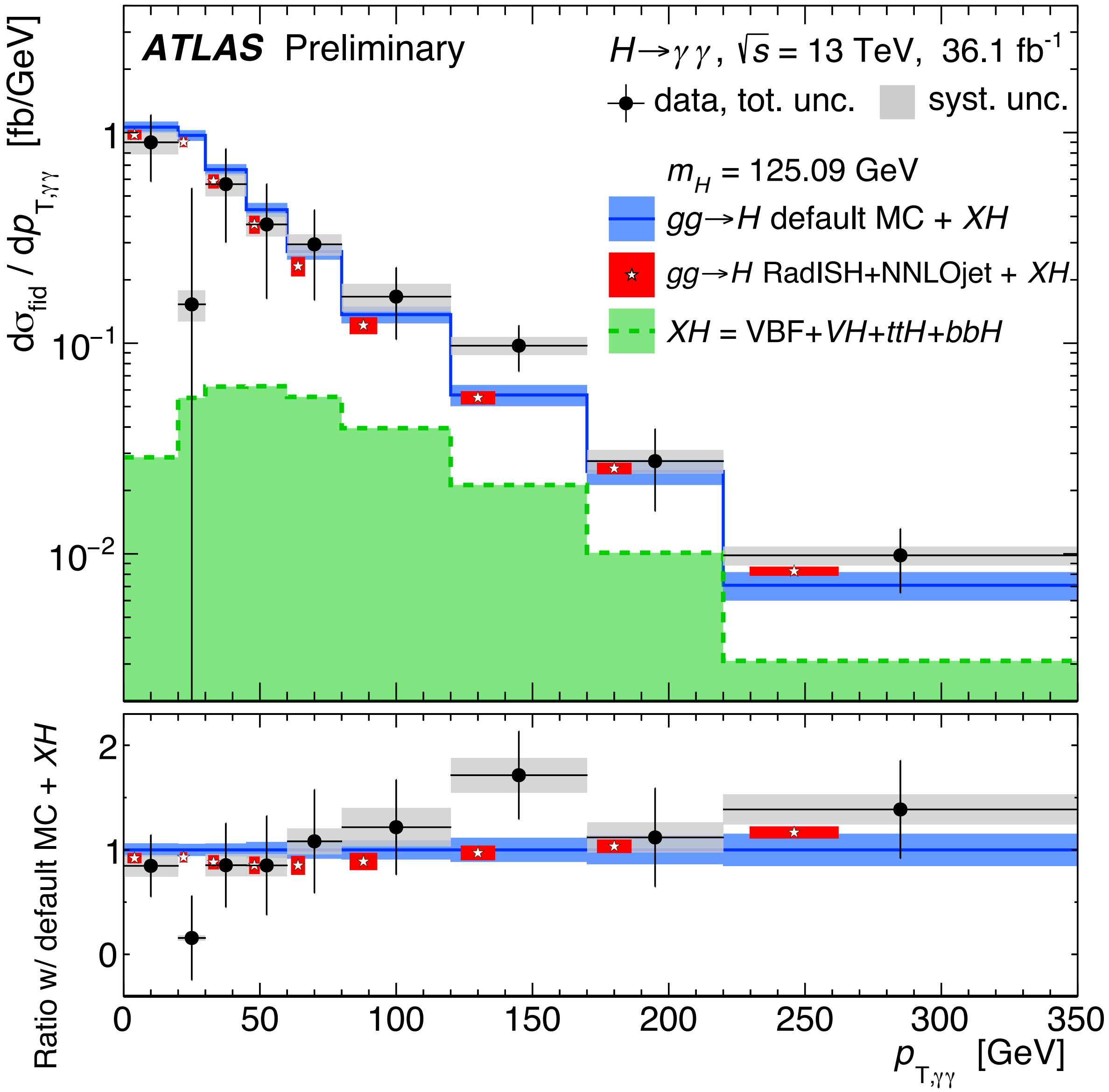
- Peak at ~10 GeV: $\log(m_H^2/q_T^2)$~5.1
- Resummation affects spectrum out to larger q_T

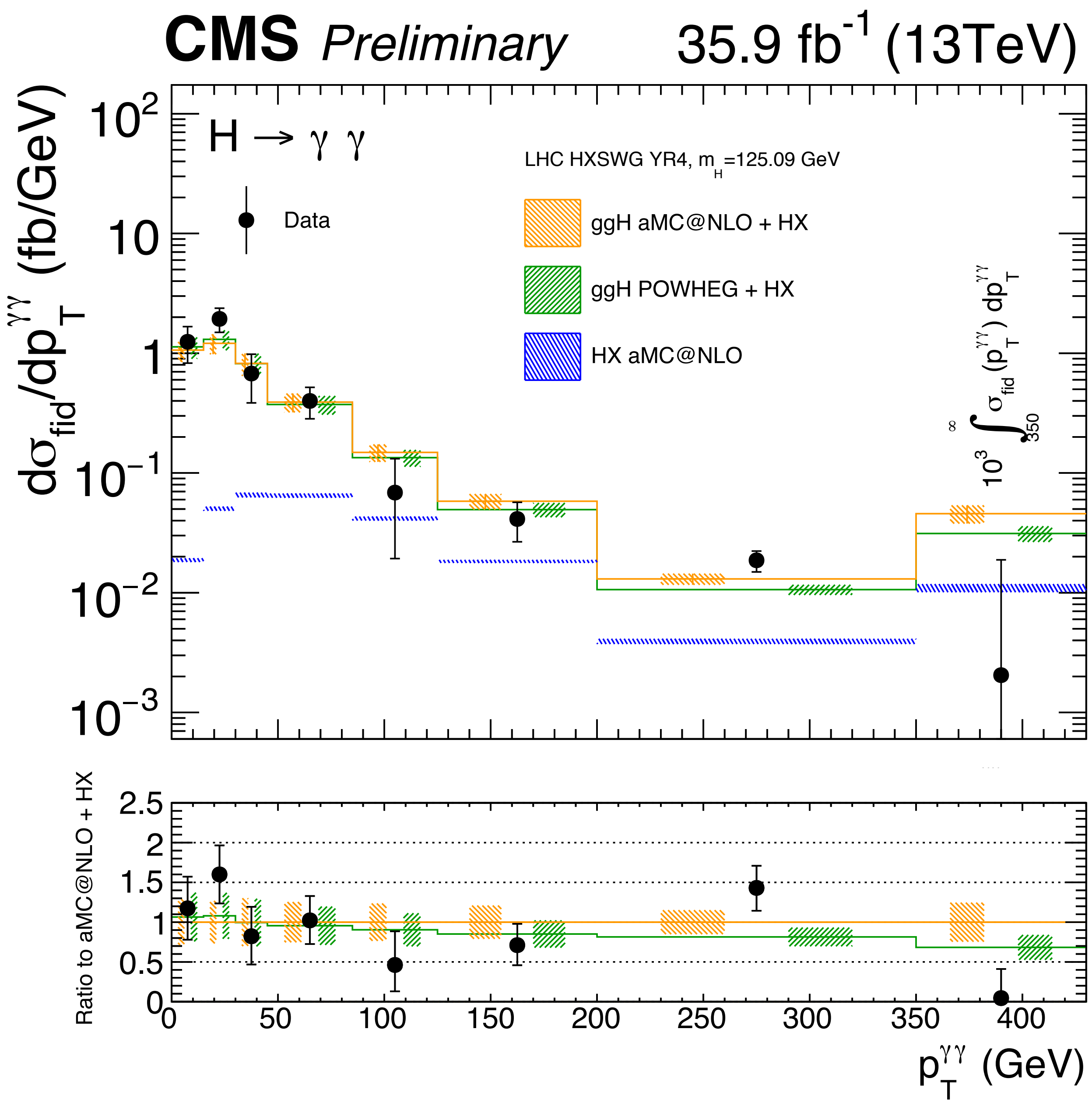
Monte Carlo Higgs q_T



- HQT = resummed+matched q_T (de Florian et al.)
- MC@NLO = Monte Carlo matched to NLO (see later)
- RW = reweighted to resummed+matched scalar E_T

Comparisons to data (γγ mode)





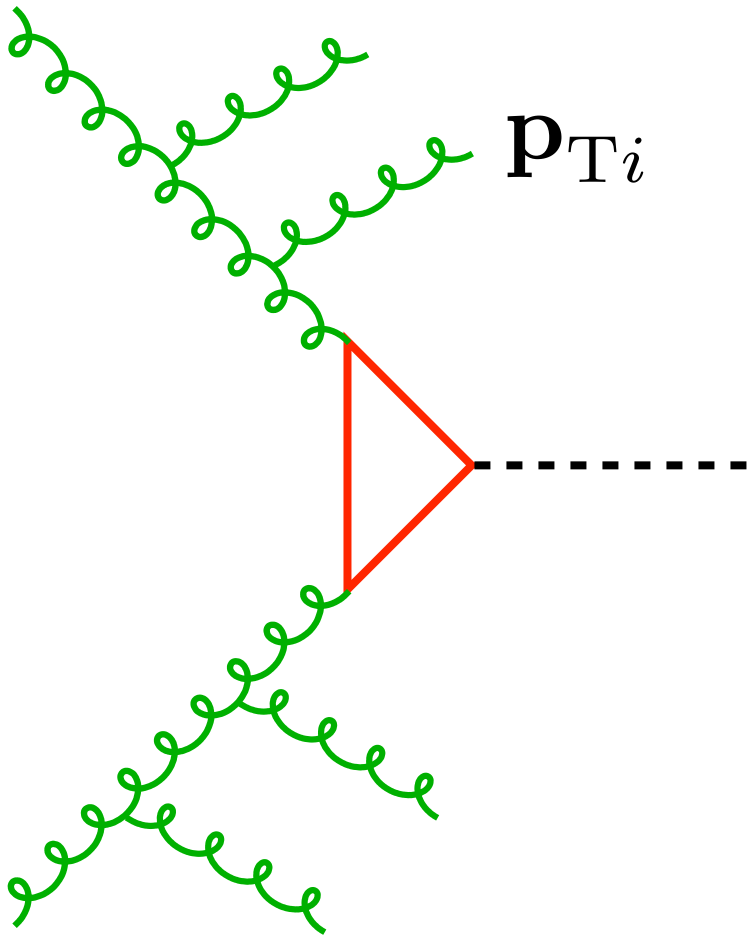
Andy Chisholm, LHCHXSWG, July 2017

Summary

- QCD Basics
  - α_S and m_q uncertainties ≈ 1%
  - PDF uncertainties ≈ few %
- QCD and Higgs
  - Cross section uncertainty ≈10%
  - Decay uncertainties ≈ few to 10%
- QCD and Higgs transverse momentum
  - Large log resummation
  - Matched to NLO

Extras

Higgs q_T & E_T



- Higgs transverse momentum

$$\mathbf{q}_{\mathrm{T}} = -\sum \mathbf{p}_{\mathrm{T}i}$$

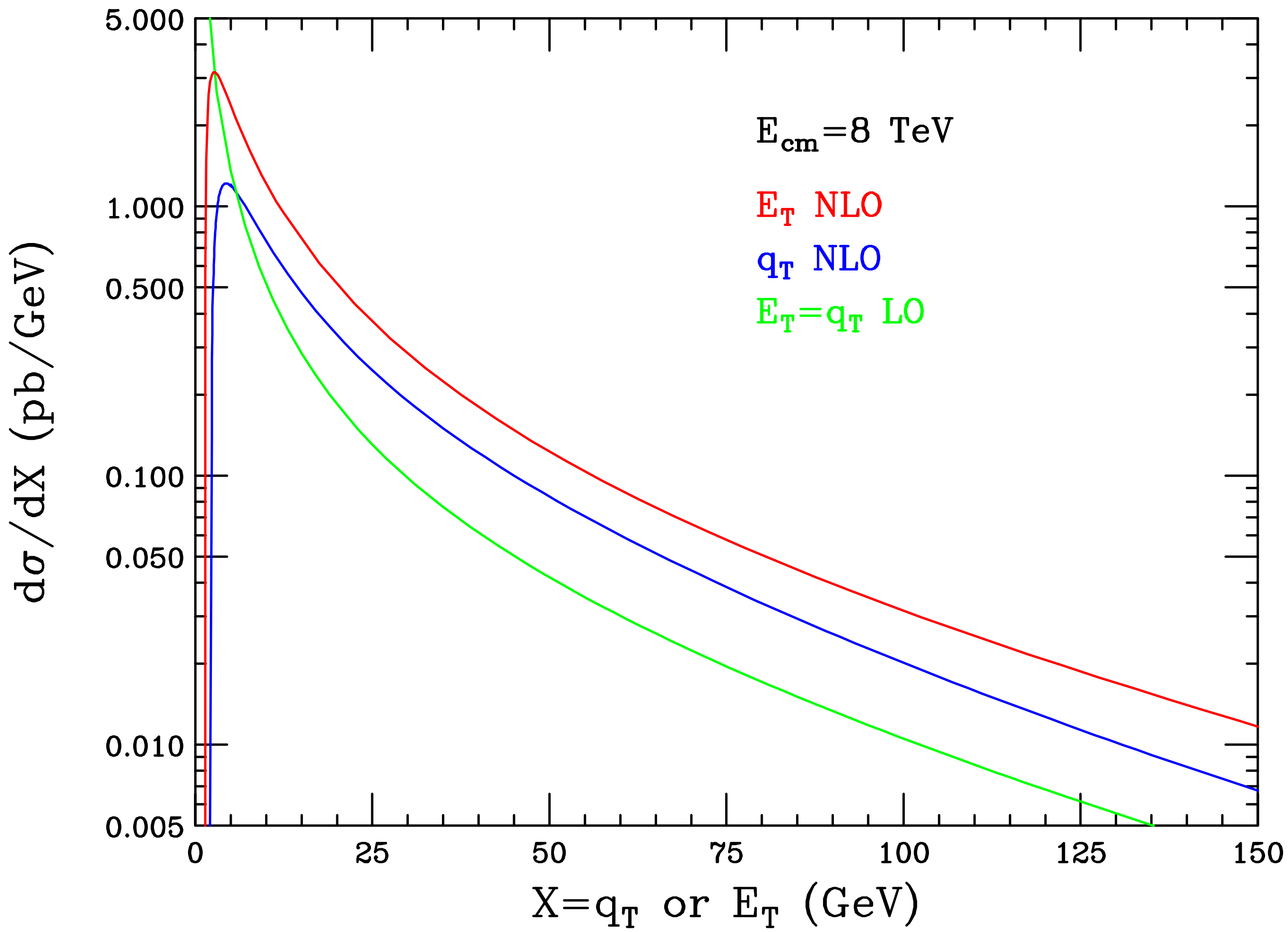
Bozzi et al. 0705.3887
Mantry & Petriello, 0911.4135
Catani & Grazzini, 1011.3918
de Florian et al. 1109.2109

- Radiated transverse energy

$$E_T = \sum |\mathbf{p}_{\mathrm{T}i}|$$

Papaefstathiou, Smillie, BW, 1002.4375
+Grazzini, 1403.3394

Higgs q_T & E_T (fixed order)



- $\displaystyle (\mathrm{N})\mathrm{LO} \underset{E_\mathrm{T}\to 0}{\longrightarrow} (-)\infty$
- Large logs of m_H^2/E_T^2 need resummation

Resummation of Higgs q_T

$$\mathrm{d}\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2\, f_a(x_1,\mu) f_b(x_2,\mu)\, \mathrm{d}\hat{\sigma}_{ab}(x_1 x_2 s, \mu, \ldots)$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{\mathrm{d}^2\hat{\sigma}_{gg}}{\mathrm{d}\,\mathbf{q}_\mathrm{T}^2} \sim \delta^2(\mathbf{q}_\mathrm{T}) + \alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right]_+ \delta^2(\mathbf{q}_\mathrm{T} + \mathbf{p}_\mathrm{T}) + \ldots$$

$$\sim \int \frac{\mathrm{d}^2\mathbf{b}}{(2\pi)^2}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}_\mathrm{T}} \left\{1 + \alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right] \left(\mathrm{e}^{i\mathbf{b}\cdot\mathbf{p}_\mathrm{T}} - 1\right) + \ldots\right\}$$

$$\sim \int \frac{\mathrm{d}^2\mathbf{b}}{(2\pi)^2}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}_\mathrm{T}} \exp\left\{\alpha_\mathrm{S} \int \mathrm{d}^2\mathbf{p}_\mathrm{T} \left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2} \ln \frac{m_H^2}{\mathbf{p}_\mathrm{T}^2} + \frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right] \left(\mathrm{e}^{i\mathbf{b}\cdot\mathbf{p}_\mathrm{T}} - 1\right)\right\}$$

Resummation of Higgs E_T

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$$\sim \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\mathbf{q}_T} \left\{1 + \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right] (e^{i\mathbf{b}\cdot\mathbf{p}_T} - 1) + \ldots\right\}$$

$$\sim \int \frac{d^2\mathbf{b}}{(2\pi)^2}\, e^{i\mathbf{b}\cdot\mathbf{q}_T} \exp\left\{\alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right] (e^{i\mathbf{b}\cdot\mathbf{p}_T} - 1)\right\}$$

$$\frac{1}{\hat{\sigma}_{gg}} \frac{d\hat{\sigma}_{gg}}{d\,E_T} \sim \delta(E_T) + \alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right]_+ \delta(E_T - |\mathbf{p}_T|) + \ldots$$

$$\sim \int \frac{d\tau}{2\pi}\, e^{i\tau E_T} \exp\left\{\alpha_S \int d^2\mathbf{p}_T \left[\frac{A_g}{\mathbf{p}_T^2} \ln\frac{m_H^2}{\mathbf{p}_T^2} + \frac{B_g}{\mathbf{p}_T^2}\right] \left(e^{-i\tau|\mathbf{p}_T|} - 1\right)\right\}$$

Resummation of Higgs E_T

$$\frac{1}{\hat\sigma_{gg}}\frac{d\hat\sigma_{gg}}{d\,E_T} \sim \int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}\,e^{i\tau E_T}\exp\left\{\alpha_S\int d^2\mathbf{p}_T\left[\frac{A_g}{\mathbf{p}_T^2}\ln\frac{m_H^2}{\mathbf{p}_T^2}+\frac{B_g}{\mathbf{p}_T^2}\right]\left(e^{-i\tau|\mathbf{p}_T|}-1\right)\right\}$$

- Defined for $E_T \lesseqgtr 0$

Resummation of Higgs E_T

$$\frac{1}{\hat{\sigma}_{gg}}\frac{\mathrm{d}\hat{\sigma}_{gg}}{\mathrm{d}\,E_T} \sim \int_{-\infty}^{+\infty}\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{i\tau E_T}\,\exp\left\{\alpha_{\mathrm{S}}\int \mathrm{d}^2\mathbf{p}_{\mathrm{T}}\left[\frac{A_g}{\mathbf{p}_{\mathrm{T}}^2}\ln\frac{m_H^2}{\mathbf{p}_{\mathrm{T}}^2}+\frac{B_g}{\mathbf{p}_{\mathrm{T}}^2}\right]\left(\mathrm{e}^{-i\tau|\mathbf{p}_{\mathrm{T}}|}-1\right)\right\}$$

- Defined for $E_T \lesseqgtr 0$
- For $E_T < 0$, can close τ−contour in lower half-plane

Resummation of Higgs E_T

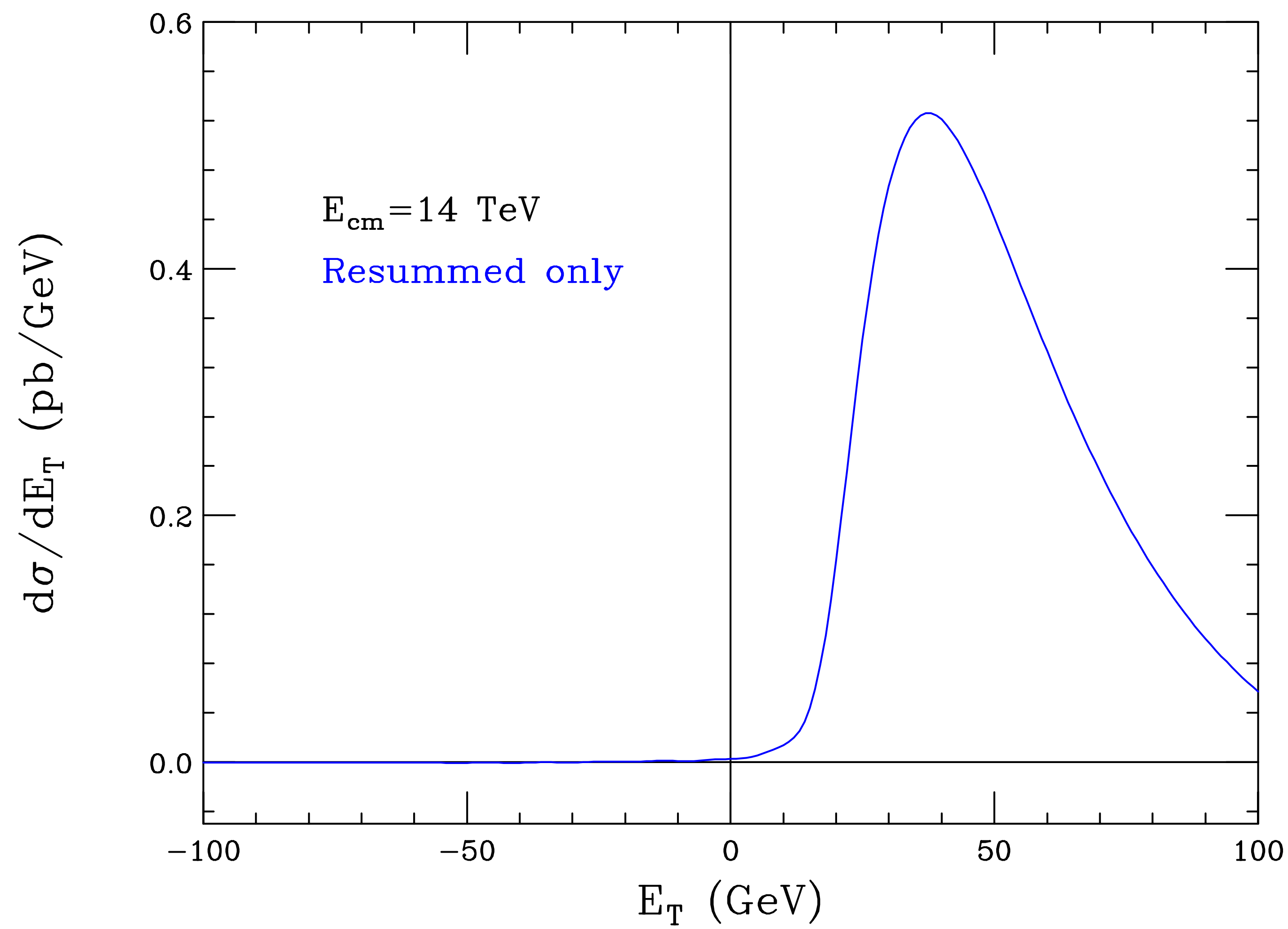
$$\frac{1}{\hat{\sigma}_{gg}}\frac{\mathrm{d}\hat{\sigma}_{gg}}{\mathrm{d}\,E_T} \sim \int_{-\infty}^{+\infty}\frac{\mathrm{d}\tau}{2\pi}\,\mathrm{e}^{i\tau E_T}\exp\left\{\alpha_\mathrm{S}\int \mathrm{d}^2\mathbf{p}_\mathrm{T}\left[\frac{A_g}{\mathbf{p}_\mathrm{T}^2}\ln\frac{m_H^2}{\mathbf{p}_\mathrm{T}^2}+\frac{B_g}{\mathbf{p}_\mathrm{T}^2}\right]\left(\mathrm{e}^{-i\tau|\mathbf{p}_\mathrm{T}|}-1\right)\right\}$$

- Defined for $E_T \lesseqgtr 0$
- For $E_T < 0$, can close $\tau-$contour in lower half-plane
- No singularities in lower half-plane

Resummation of Higgs E_T

$$\frac{1}{\hat{\sigma}_{gg}}\frac{d\hat{\sigma}_{gg}}{d\,E_T} \sim \int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}\,e^{i\tau E_T}\exp\left\{\alpha_S\int d^2\mathbf{p}_T\left[\frac{A_g}{\mathbf{p}_T^2}\ln\frac{m_H^2}{\mathbf{p}_T^2}+\frac{B_g}{\mathbf{p}_T^2}\right]\left(e^{-i\tau|\mathbf{p}_T|}-1\right)\right\}$$

- Defined for $E_T \lessgtr 0$
- For $E_T < 0$, can close τ−contour in lower half-plane
- No singularities in lower half-plane



Resummation & matching of Higgs E$_T$

$$\left[\frac{d\sigma_H}{dQ^2\, dE_T}\right]_{\rm res.} = \frac{1}{2\pi}\sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \int_{-\infty}^{+\infty} d\tau\; {\rm e}^{-i\tau E_T}\; f_{a/h_1}(x_1,\mu)\; f_{b/h_2}(x_2,\mu)\;\; W_{ab}^H(x_1 x_2 s; Q,\tau,\mu)$$

$$W_{ab}^H(s;Q,\tau,\mu) = \int_0^1 dz_1 \int_0^1 dz_2\; C_{ga}(\alpha_{\rm S}(\mu), z_1;\tau,\mu)\; C_{gb}(\alpha_{\rm S}(\mu), z_2;\tau,\mu)\; \delta(Q^2 - z_1 z_2 s)\; \sigma_{gg}^H(Q,\alpha_{\rm S}(Q))\; S_g(Q,\tau)$$

$$S_g(Q,\tau) = \exp\left\{-2\int_0^Q \frac{dq}{q}\left[2A_g(\alpha_{\rm S}(q))\;\ln\frac{Q}{q} + B_g(\alpha_{\rm S}(q))\right]\left(1-{\rm e}^{iq\tau}\right)\right\}$$

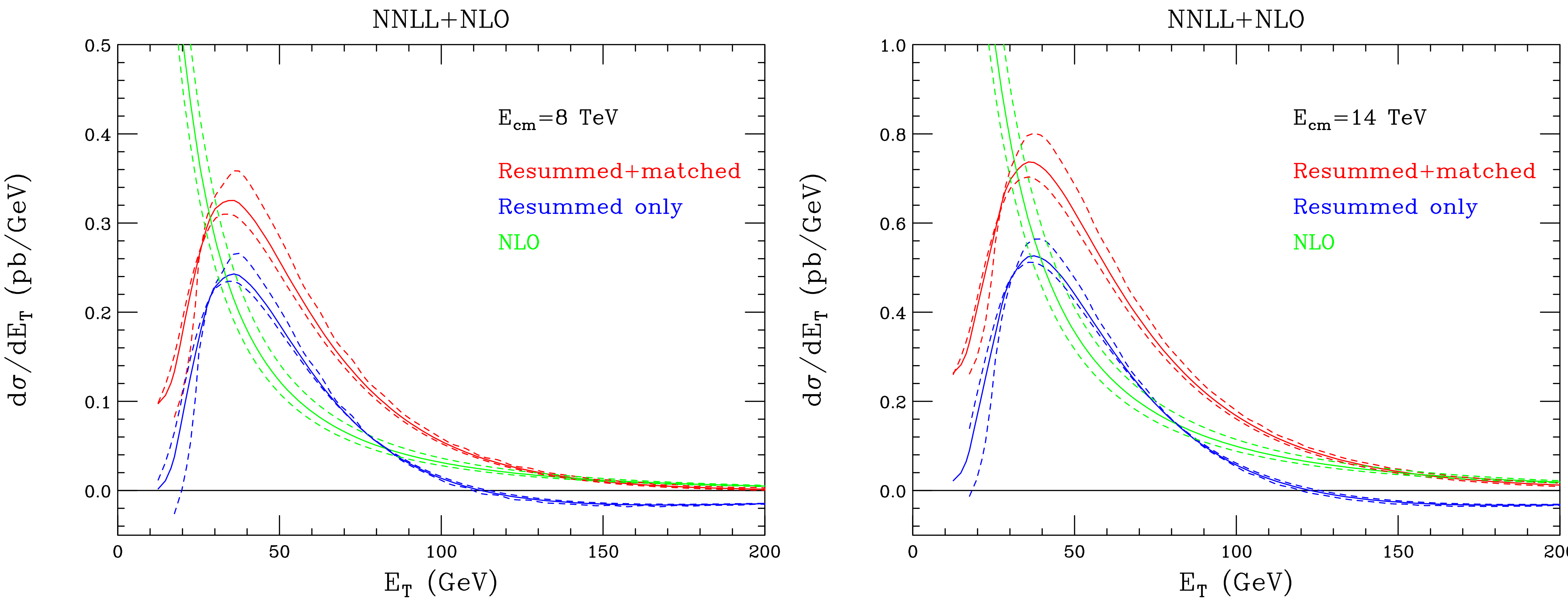
$$A_g(\alpha_{\rm S}) = \sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm S}}{\pi}\right)^n A_g^{(n)} \;\; ,$$

$$B_g(\alpha_{\rm S}) = \sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm S}}{\pi}\right)^n B_g^{(n)} \;\; ,$$

$$C_{ga}(\alpha_{\rm S}, z) = \delta_{ga}\,\delta(1-z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_{\rm S}}{\pi}\right)^n C_{ga}^{(n)}(z)$$

$$\frac{d\sigma_H}{dE_T} = \left[\frac{d\sigma_H}{dE_T}\right]_{\rm resum} - \left[\frac{d\sigma_H}{dE_T}\right]_{\rm resum,NLO} + \left[\frac{d\sigma_H}{dE_T}\right]_{\rm NLO}$$

Transverse energy distribution



- Peak at ~35 GeV: $\log(m_H^2/E_T^2)$~2.6
- Resummation affects spectrum out to much larger E_T
- Unlike q_T, the Underlying Event also contributes…