CERN

CERN-FERMILAB
HADRON COLLIDER PHYSICS
Summer School

Hadron Accelerators

Part 2 of 2

Rende Steerenberg BE-OP
CERN - Geneva

Topics

- A Brief Recap and Transverse Optics
- Longitudinal Motion
- Main Diagnostics Tools
- Possible Limitations
- CERN Upgrade Projects: LIU & HL-LHC

A brief recap and then we continue on transverse optics

Magnetic Element & Rigidity

Dipole magnets

$$\theta = \frac{LB}{(B\rho)}$$

$$k = \frac{K}{B\rho}[m^{-2}]$$

Quadrupole magnets

$$F = q\vec{v} \times \vec{B} = \frac{mv^2}{\rho}$$

$$B\rho[\mathrm{Tm}] = \frac{mv}{q} = \frac{p[\mathrm{GeV/c}]}{q}$$

$$B\rho = 3.3356\, p$$

- Increasing the energy requires increasing the magnetic field with Bρ to maintain radius and same focusing
- The magnets are arranged in cell, such as a FODO lattice

Hill's Equation

- Hill's equation describes the horizontal and vertical **betatron oscillations**

$$\frac{d^2x}{ds^2} + K(s)x = 0$$

Position:

$$x(s) = \sqrt{\varepsilon\beta_s}\cos(\varphi(s) + \varphi)$$

Angle:

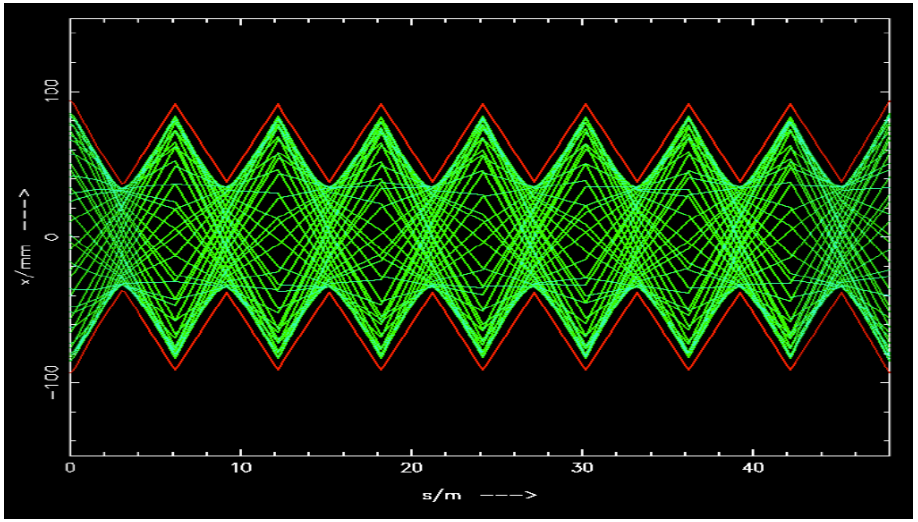
$$x' = -\alpha\sqrt{\varepsilon/\beta}\cos(\varphi) - \sqrt{\varepsilon/\beta}\sin(\varphi)\varphi$$

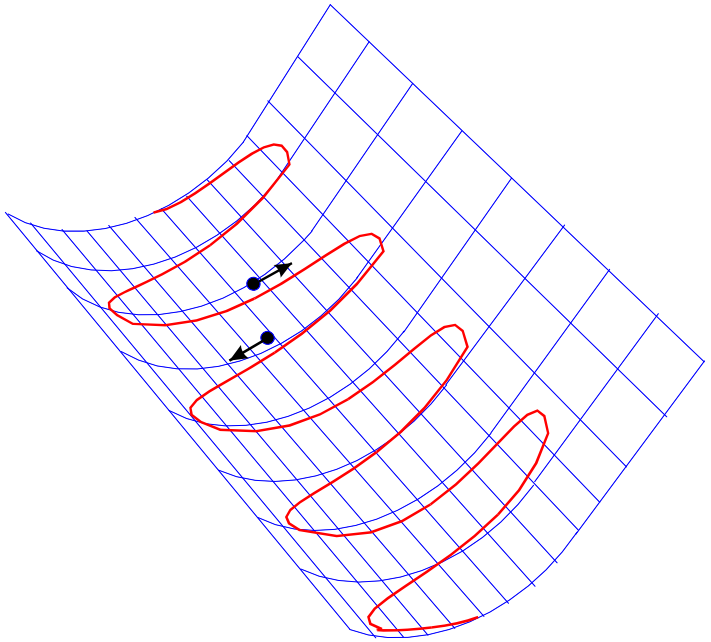
- ε and φ are constants determined by the **initial conditions**
- β**(s)** is the periodic **envelope function** given by the lattice configuration

$$Q_{x/y} = \frac{1}{2\pi}\int_0^{2\pi}\frac{ds}{\beta_{x/y}(s)}$$

- Q_x and Q_y are the **horizontal and vertical tunes**: the number of oscillations per turn around the machine

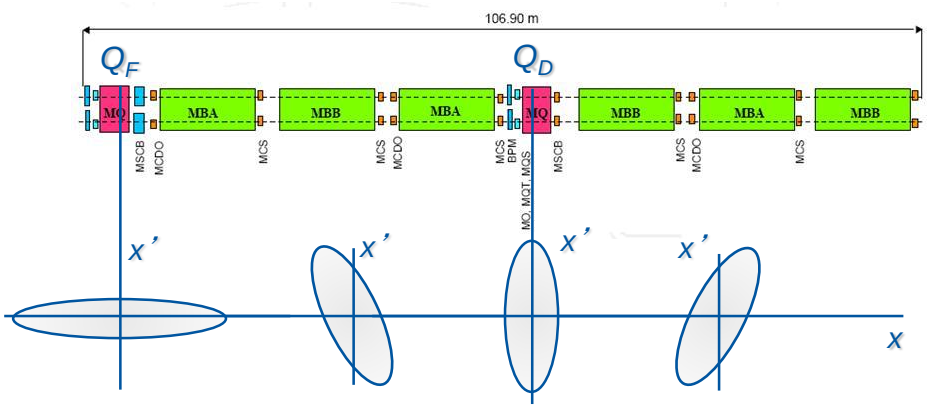
Betatron Oscillations & Envelope





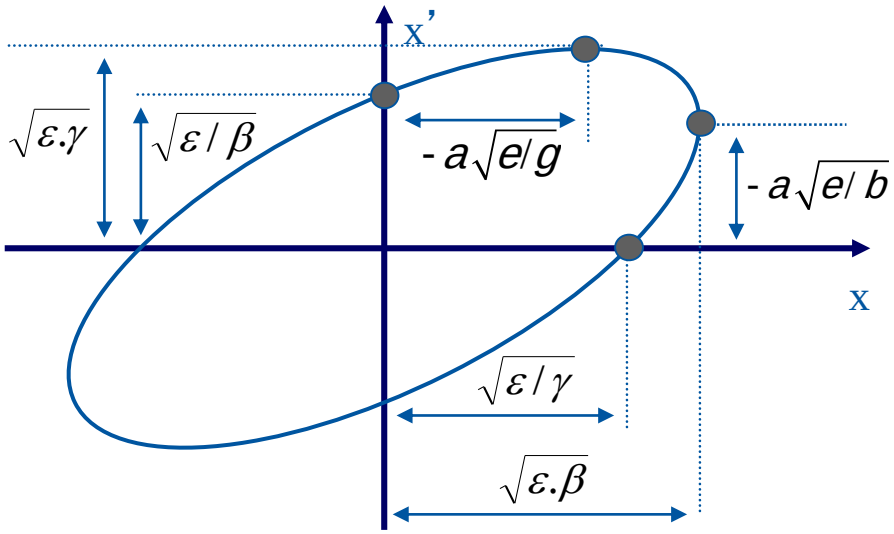
- The β function is the envelope function within which all particles oscillate
- The shape of the β function is determined by the lattice

FODO Lattice & Phase Space



- Calculating a single FODO Lattice and repeat it several time
- Make adaptations where you have insertion devices e.g. experiment, injection, extraction etc.

- Horizontal and vertical phase space
- Q_h = 3.5 means 3.5 horizontal betatron oscillations per turn around the machine, hence 3.5 turns on the phase space ellipse
- Each particle, depending on it's initial conditions will turn on it's own ellipse in phase space



Momentum Compaction Factor

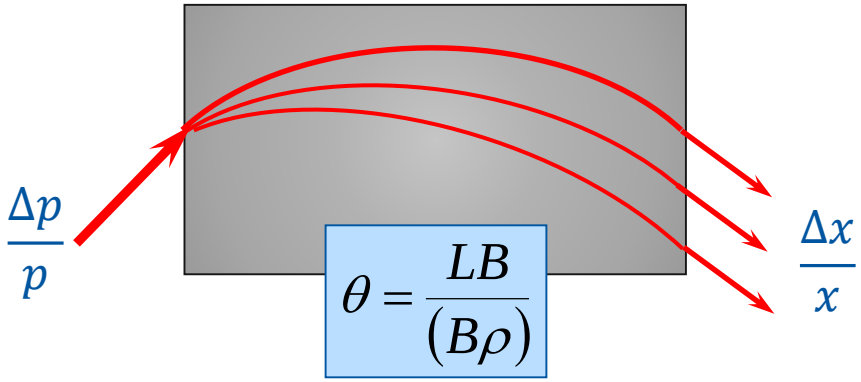
- The change in orbit with the changing momentum means that the average length of the orbit will also depend on the beam momentum.
- This is expressed as the momentum compaction factor, α_p, where:

$$\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$$

- α_p expresses the change in the radius of the closed orbit as a a function of the change in momentum

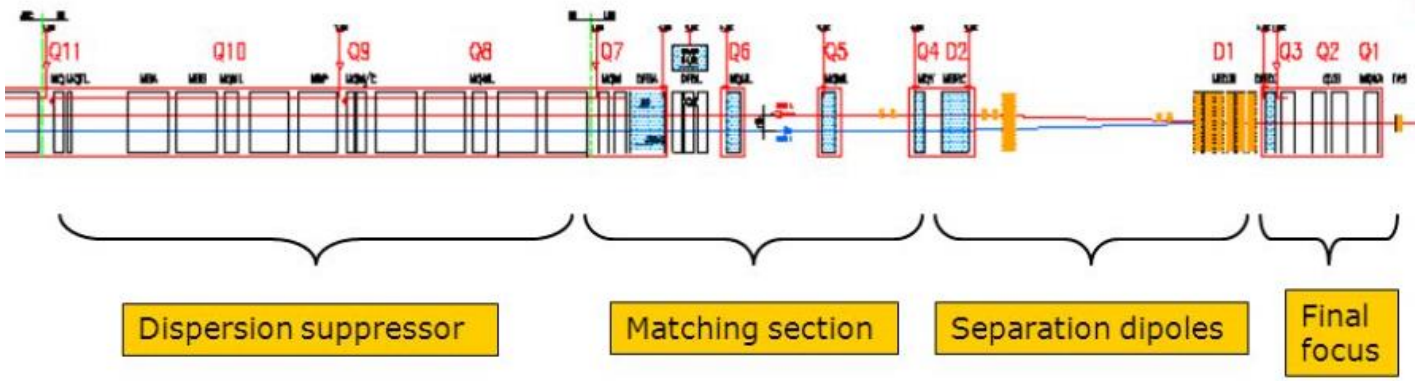
Dispersion

- Our particle beam has a momentum spread that in a homogenous dipole field will translate in a beam position spread at the exit of the magnet



$$\frac{\Delta x}{x} = D(s)\frac{\Delta p}{p}$$

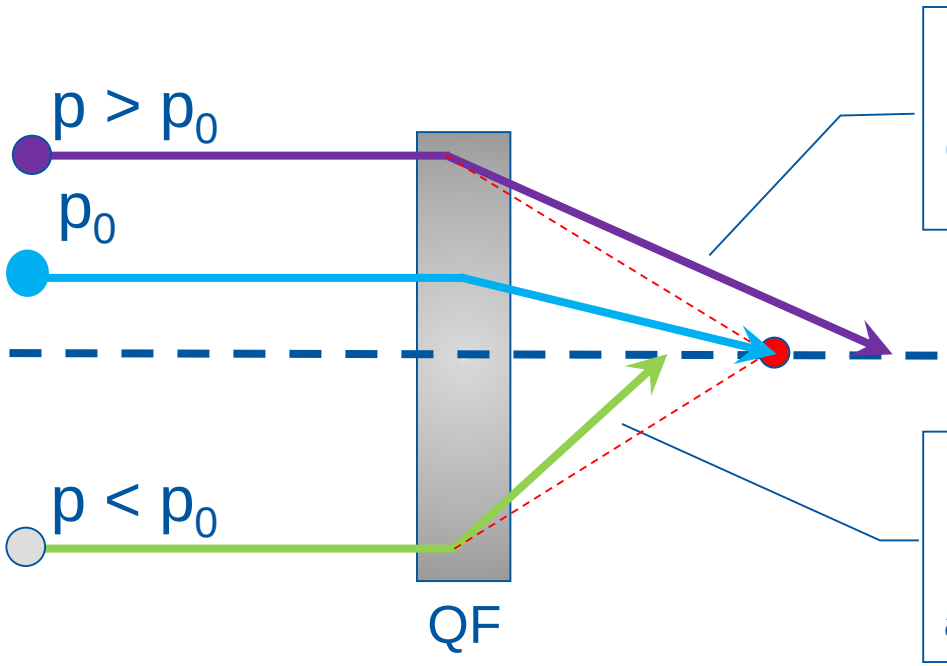
- The beam will have a finite horizontal size due to it's momentum spread, unless we install and dispersion suppressor to create dispersion free regions e.g. long straight sections for experiments



Chromaticity

- The chromaticity relates the tune spread of the transverse motion with the momentum spread in the beam.

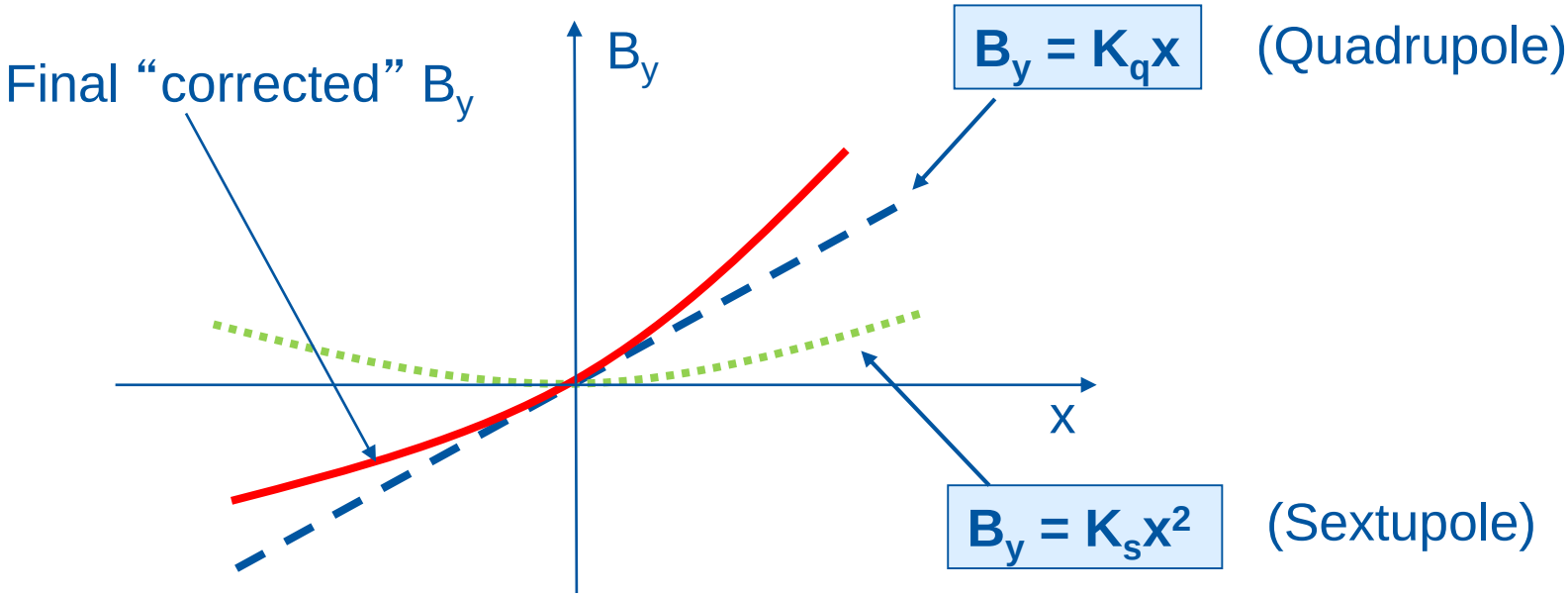
$$\frac{\Delta Q_{h/v}}{Q_{h/v}} = \xi_{h/v} \frac{\Delta p}{p}$$

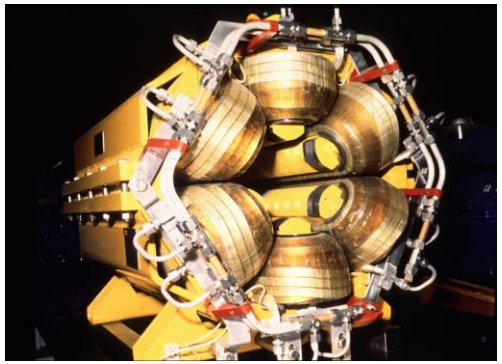


A particle with a higher momentum as the central momentum will be deviated less in the quadrupole and will have a lower betatron tune

A particle with a lower momentum as the central momentum will be deviated more in the quadrupole and will have a higher betatron tune

Chromaticity Correction





$$\frac{\Delta Q}{Q} = \frac{1}{4\pi} l\beta(s) \frac{d^2 B_y}{dx^2} \frac{D(s)}{(B\rho)Q} \frac{\Delta p}{p}$$

Chromaticity Control through sextupoles

Longitudinal Motion

Motion in the Longitudinal Plane

- What happens when particle momentum increases in a constant magnetic field?
  - Travel faster (initially)
  - Follow a longer orbit
- Hence a momentum change influence on the revolution frequency

$$\frac{df}{f} = \frac{dv}{v} - \frac{dr}{r}$$

- From the momentum compaction factor we have: $\frac{\Delta r}{r} = \alpha_p \frac{\Delta p}{p}$
- Therefore: $\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$

Revolution Frequency - Momentum

$$\frac{df}{f} = \frac{dv}{v} - \alpha_p \frac{dp}{p}$$

$$\frac{dv}{v} = \frac{d\beta}{\beta} \Leftrightarrow \beta = \frac{v}{c}$$

From the relativity theory: $p = \frac{E_0 \beta \gamma}{c}$

We can get: $\frac{dv}{v} = \frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$

Resulting in : $\frac{df}{f} = \left[\frac{1}{\gamma^2} - \alpha_p\right]\frac{dp}{p}$

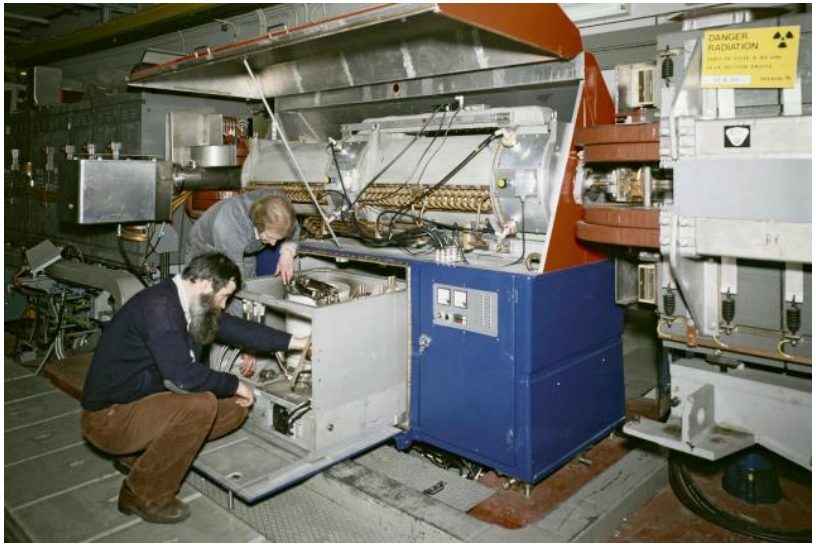
Transition

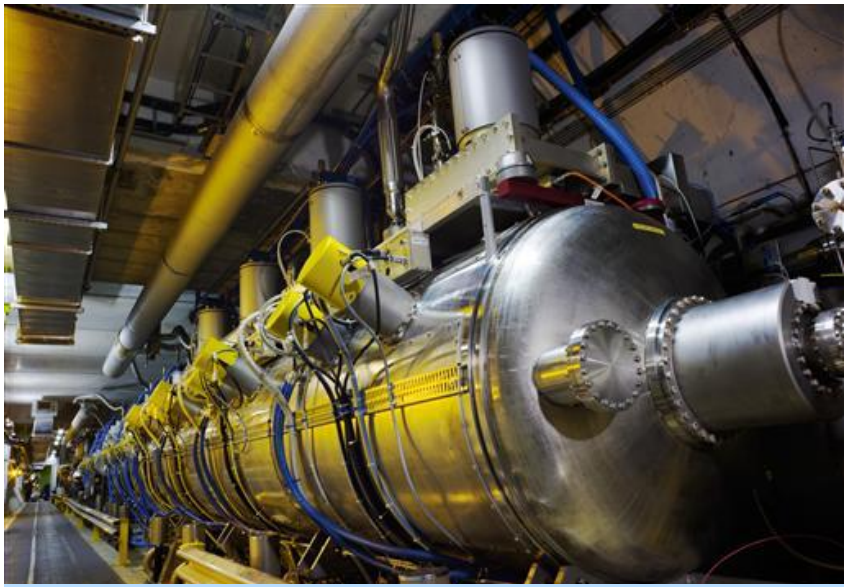
$$\frac{df}{f} = \left[\frac{1}{\gamma^2} - \alpha_p\right]\frac{dp}{p}$$

- Low momentum ($\beta << 1$ & γ is small) → $\frac{1}{\gamma^2} > \alpha_p$
- High momentum ($\beta \approx 1$ & $\gamma >> 1$) → $\frac{1}{\gamma^2} < \alpha_p$
- Transition momentum → $\frac{1}{\gamma^2} = \alpha_p$

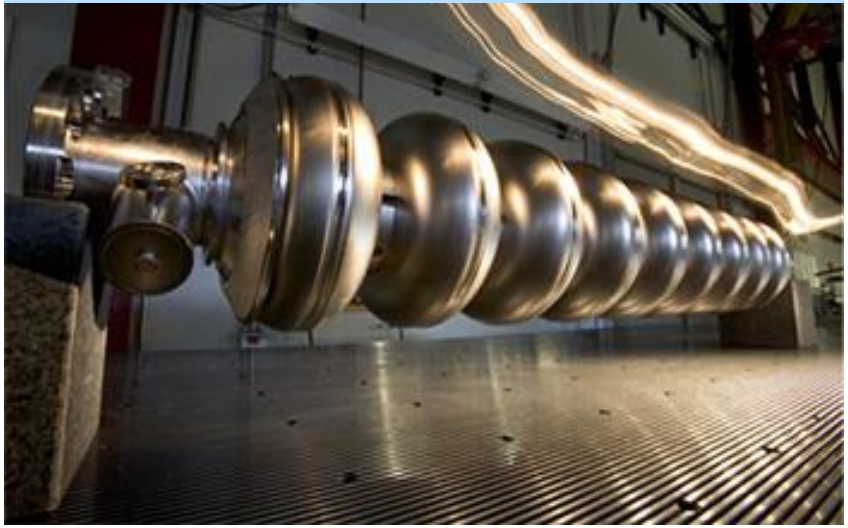
RF Cavities

Variable frequency cavity
(CERN – PS)



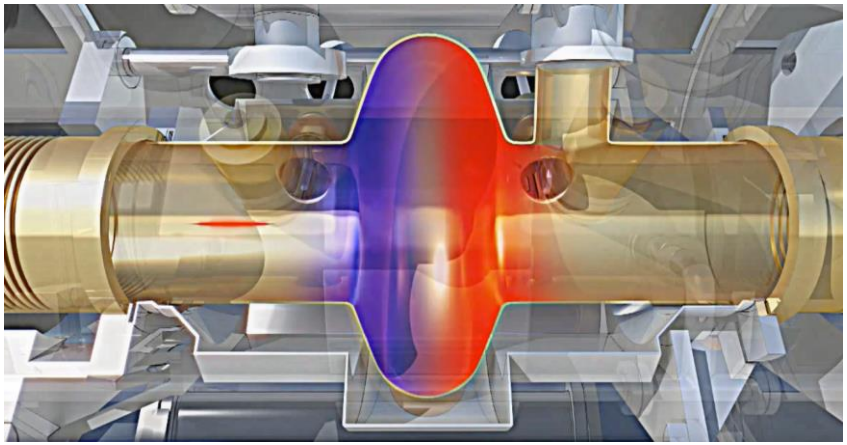


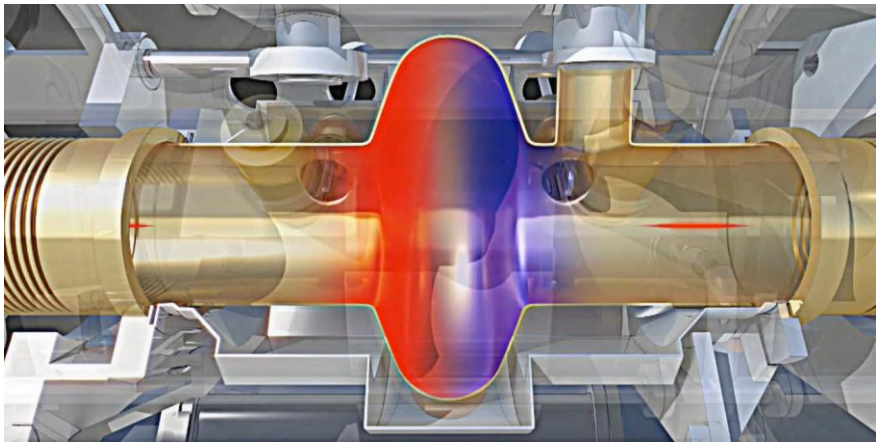
Super conducting fixed frequency cavity
(LHC)



RF Cavity

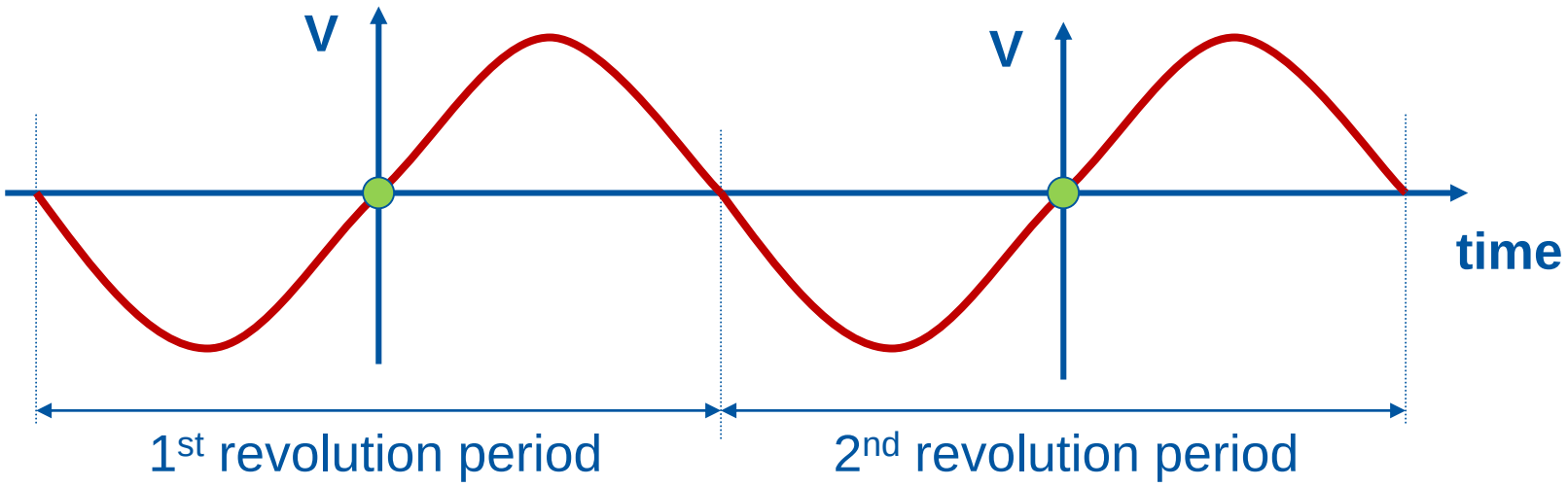
- Charged particles are accelerated by a longitudinal electric field
- The electric field needs to alternate with the revolution frequency





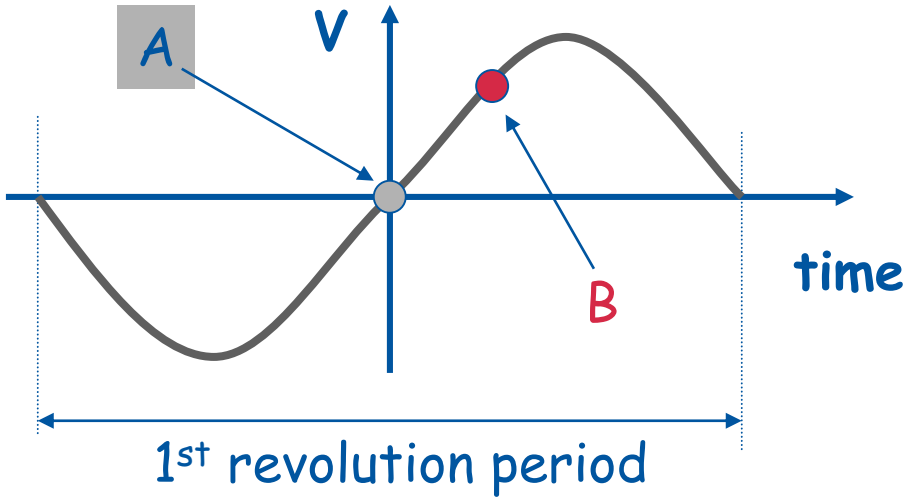
Low Momentum Particle Motion

- Lets see what a low energy particle does with this oscillating voltage in the cavity

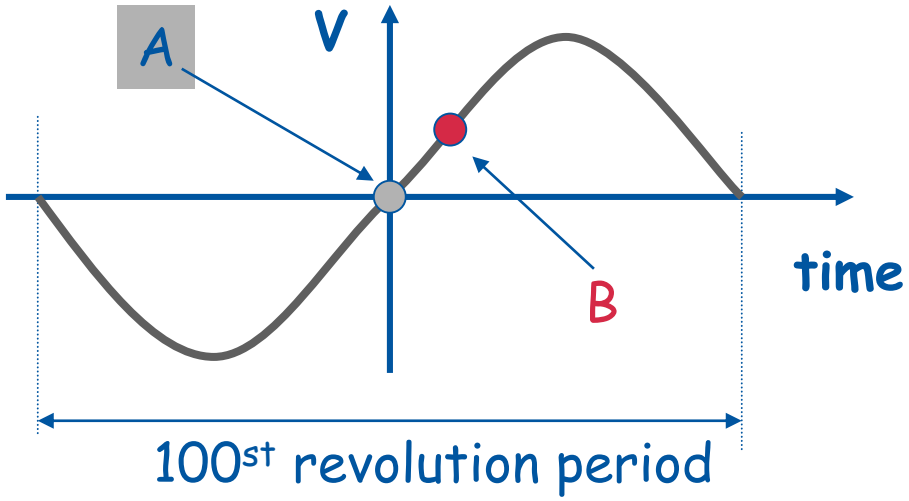


- Lets see what a low energy particle does with this oscillating voltage in the cavity

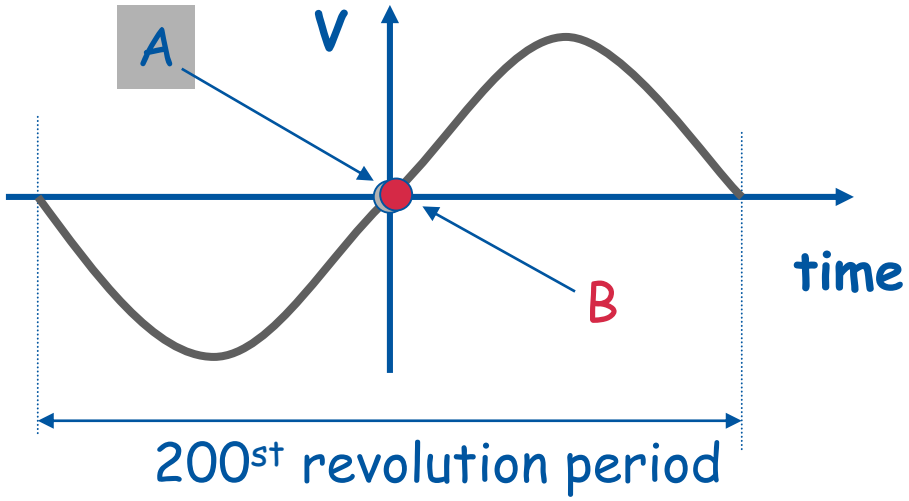
Longitudinal Motion Below Transition



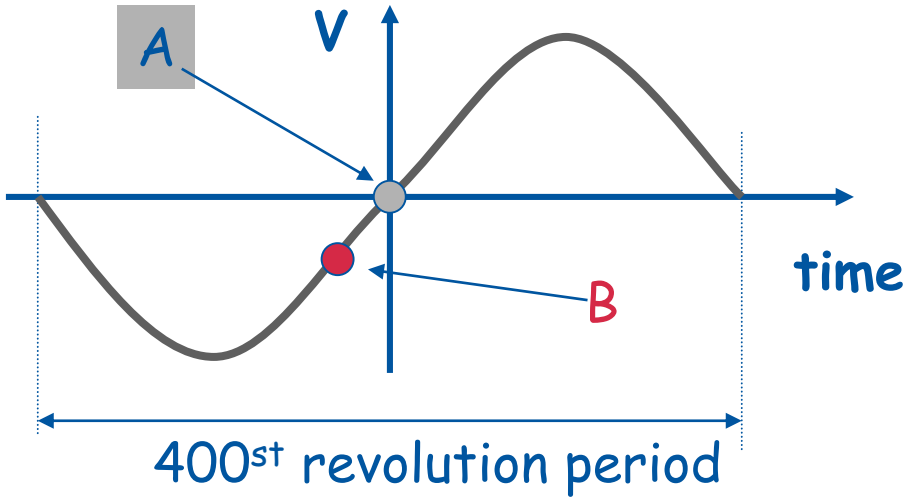
….after many turns…



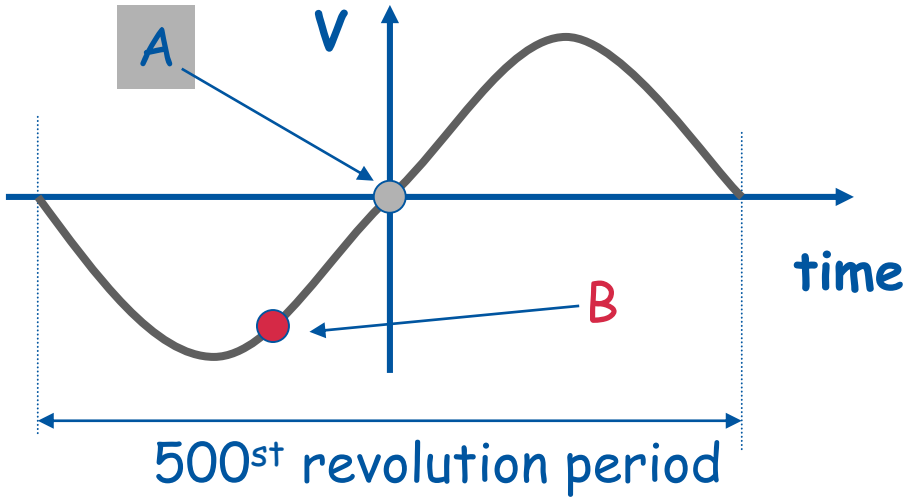
….after many turns…



….after many turns…



….after many turns…



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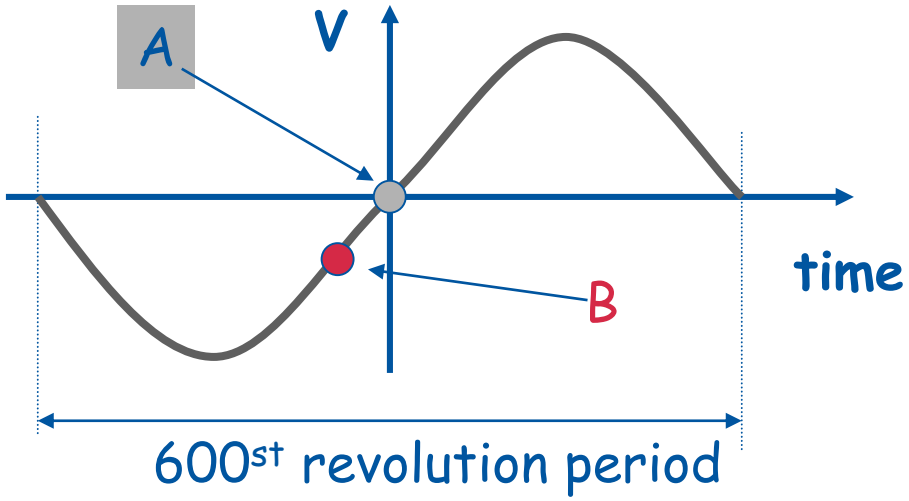
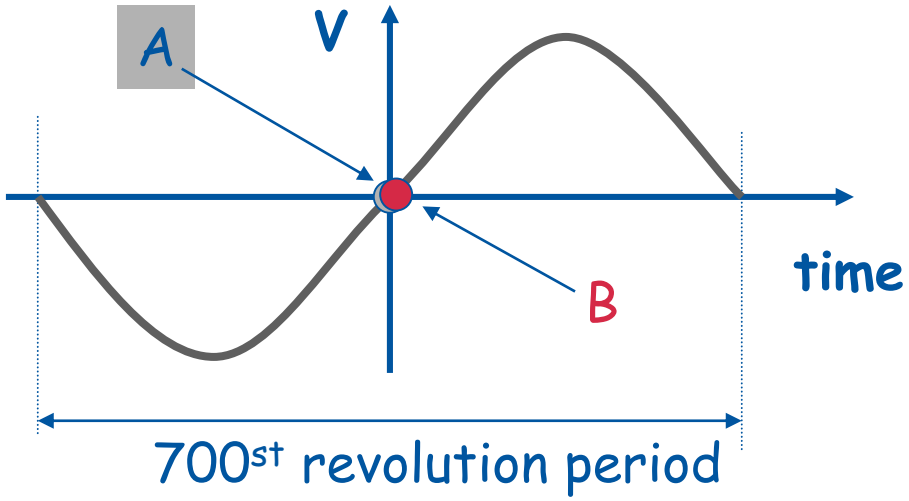
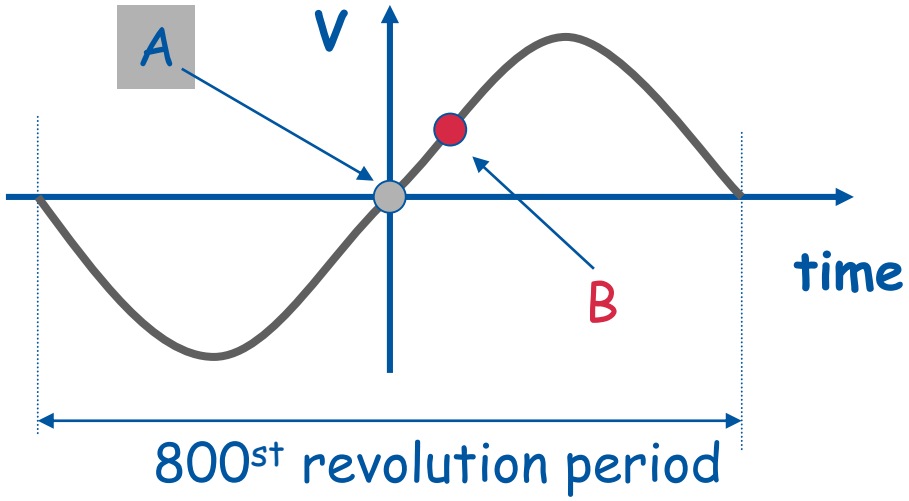


….after many turns…



….after many turns…



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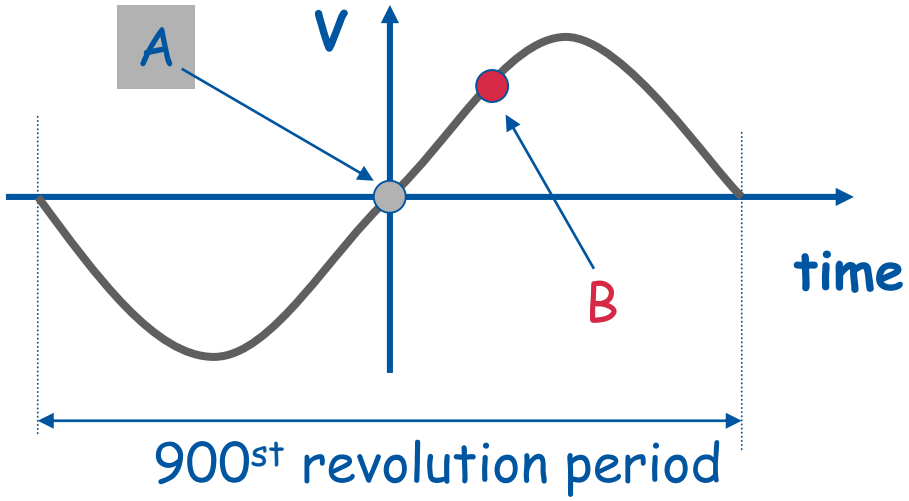

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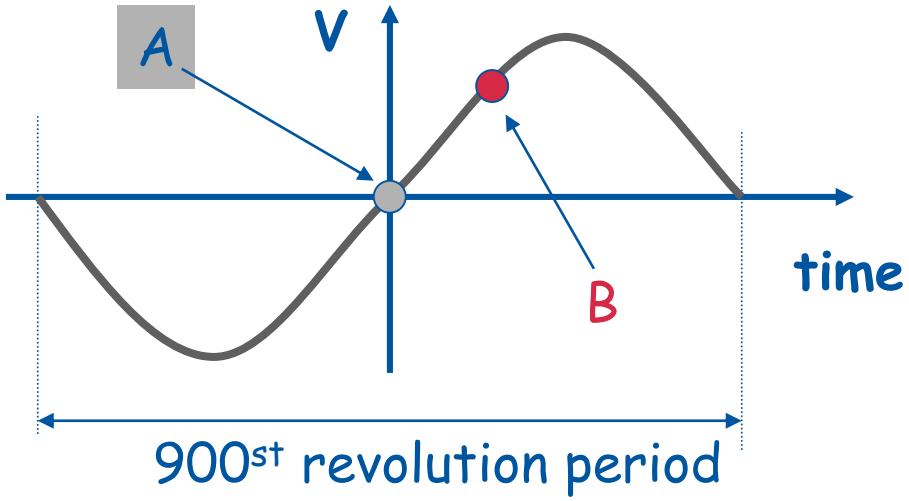
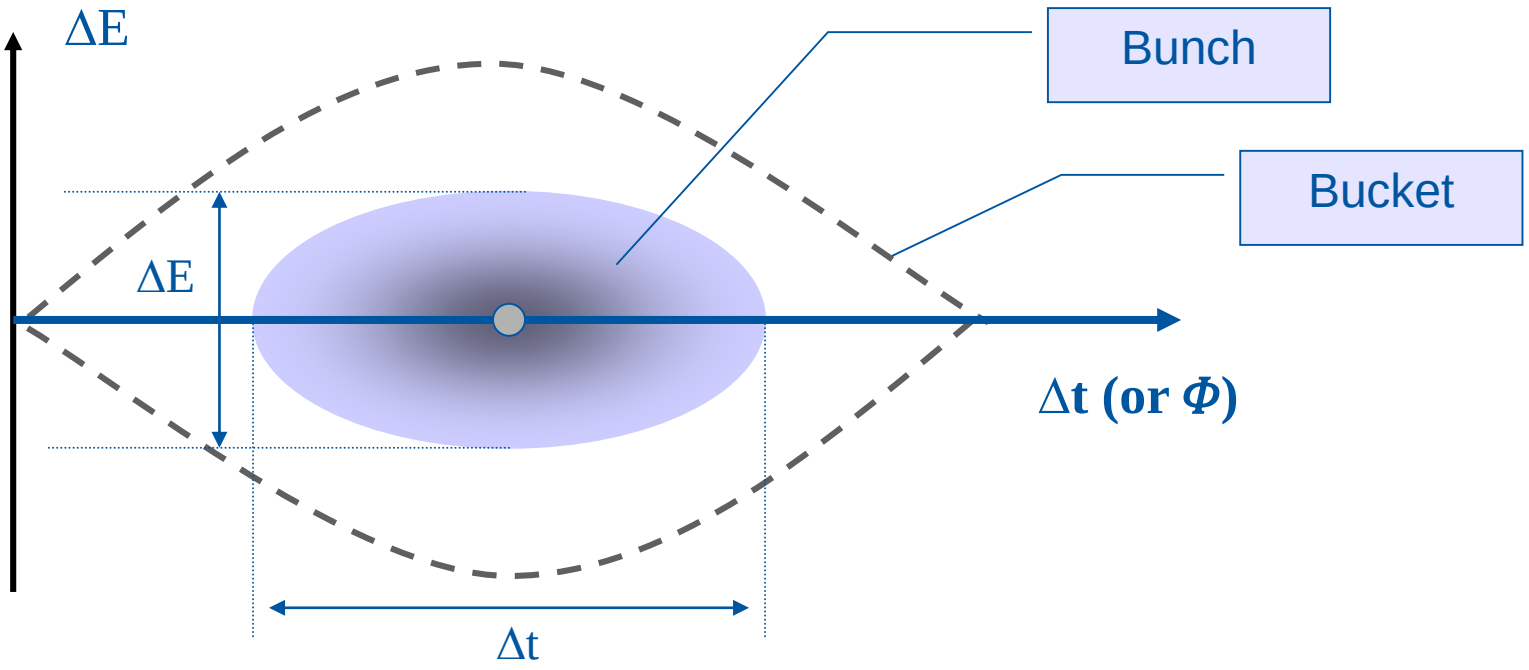


- Particle B has made 1 full oscillation around particle A
- The amplitude depends on the initial phase

- This are Synchrotron Oscillations

Stationary Bunch & Bucket



- Bucket area = **longitudinal Acceptance** [eVs]
- Bunch area = **longitudinal beam emittance** = $\pi.\Delta E.\Delta t/4$ **[eVs]**

What About Beyond Transition

- Until now we have seen how things look like below transition

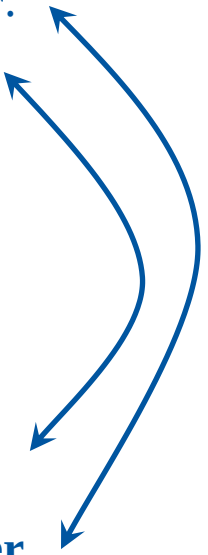
Higher energy ⇨ faster orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.

Lower energy ⇨ slower orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

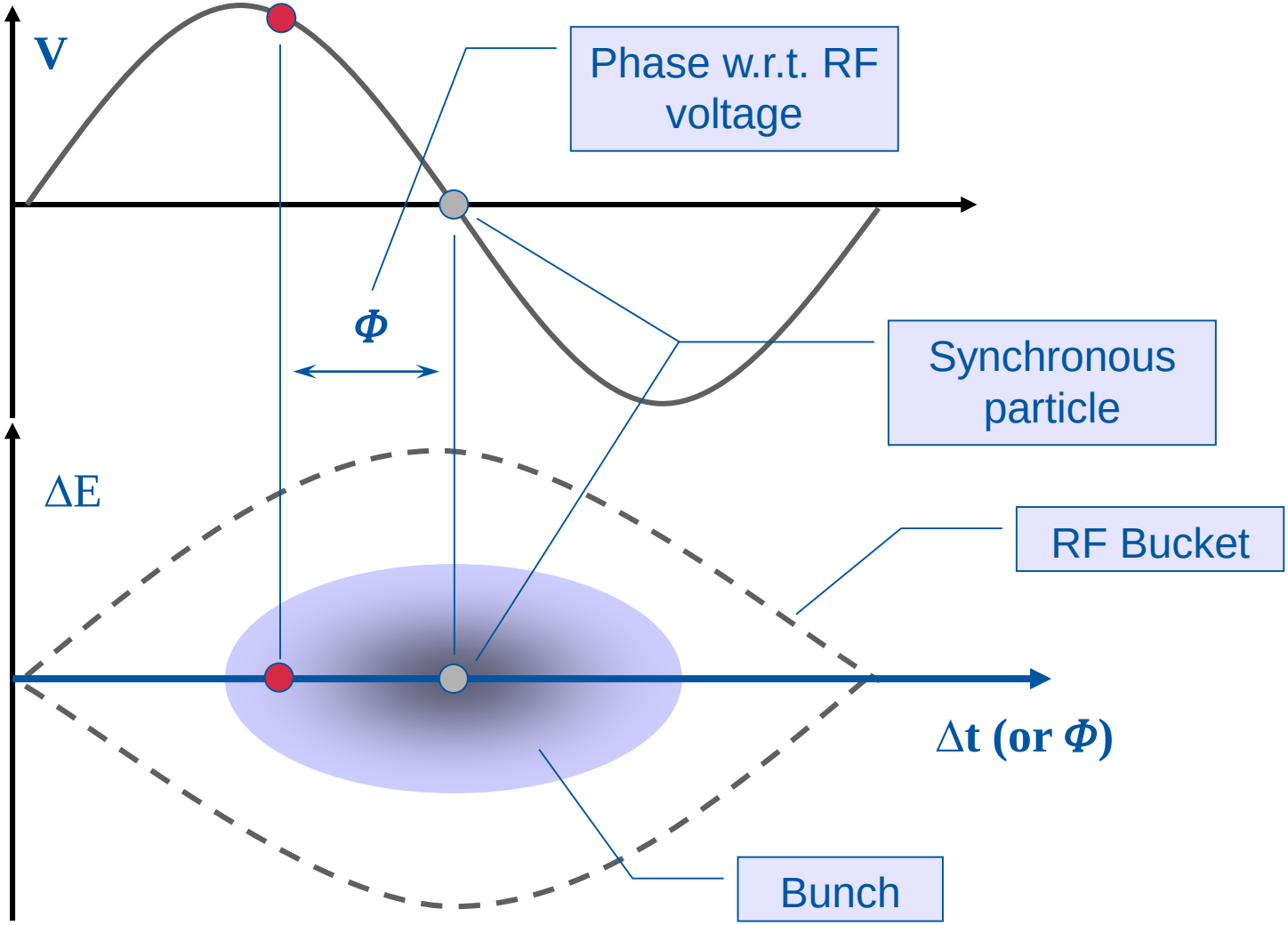
- What will happen above transition ?

Higher energy ⇨ longer orbit ⇨ lower F_{rev} ⇨ next time particle will be **later**.

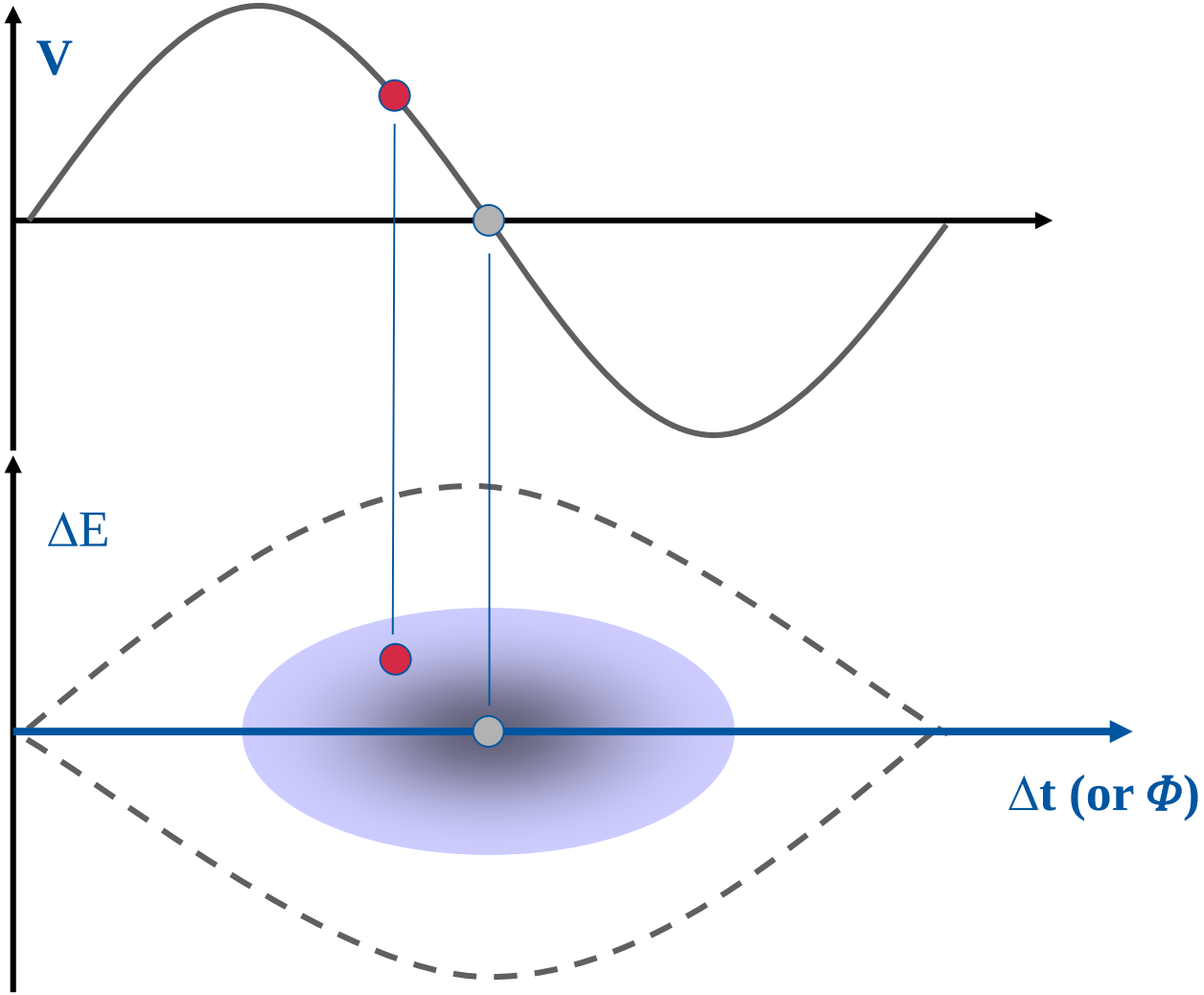
Lower energy ⇨ shorter orbit ⇨ higher F_{rev} ⇨ next time particle will be **earlier**.



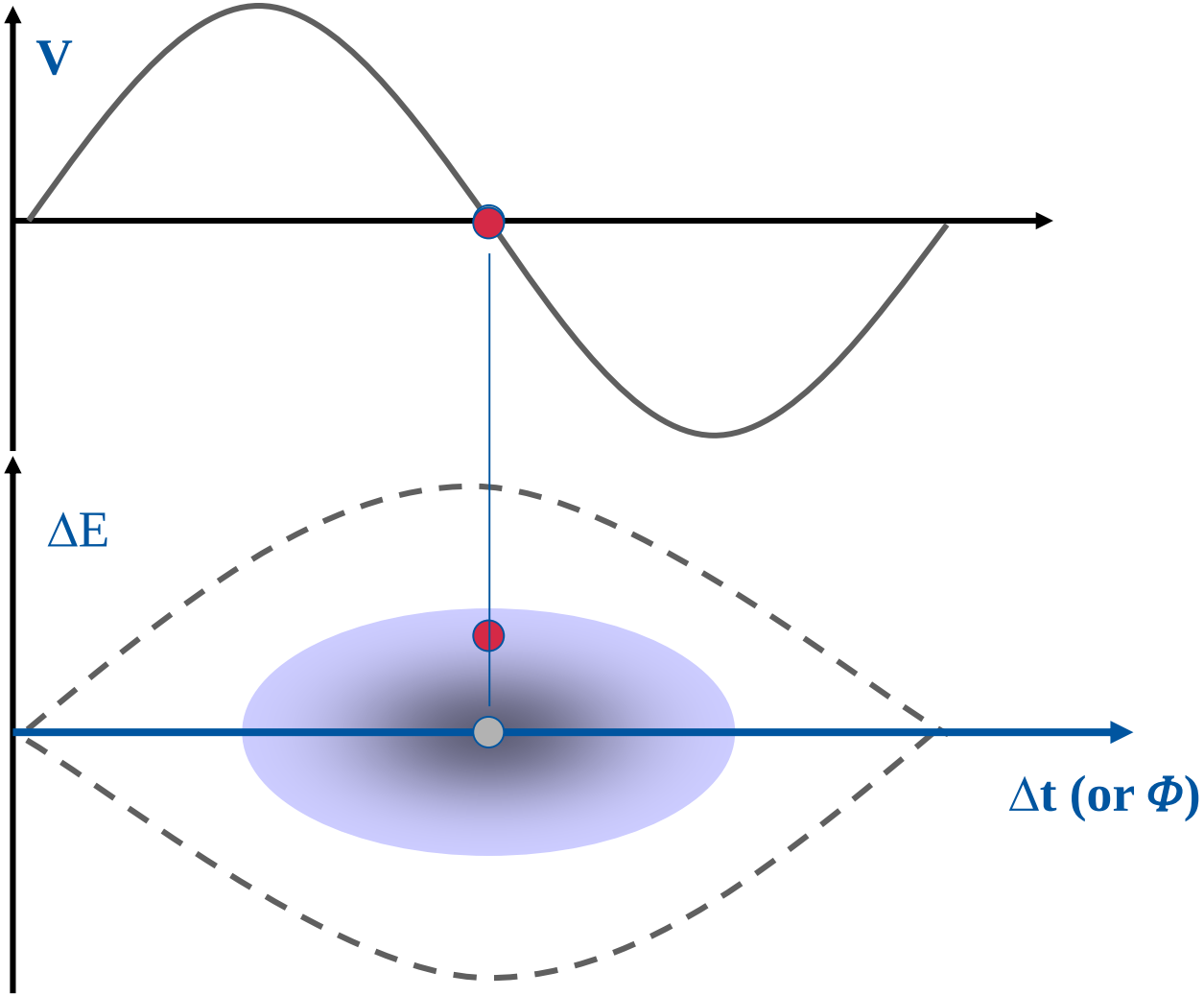
Longitudinal Motion Beyond Transition



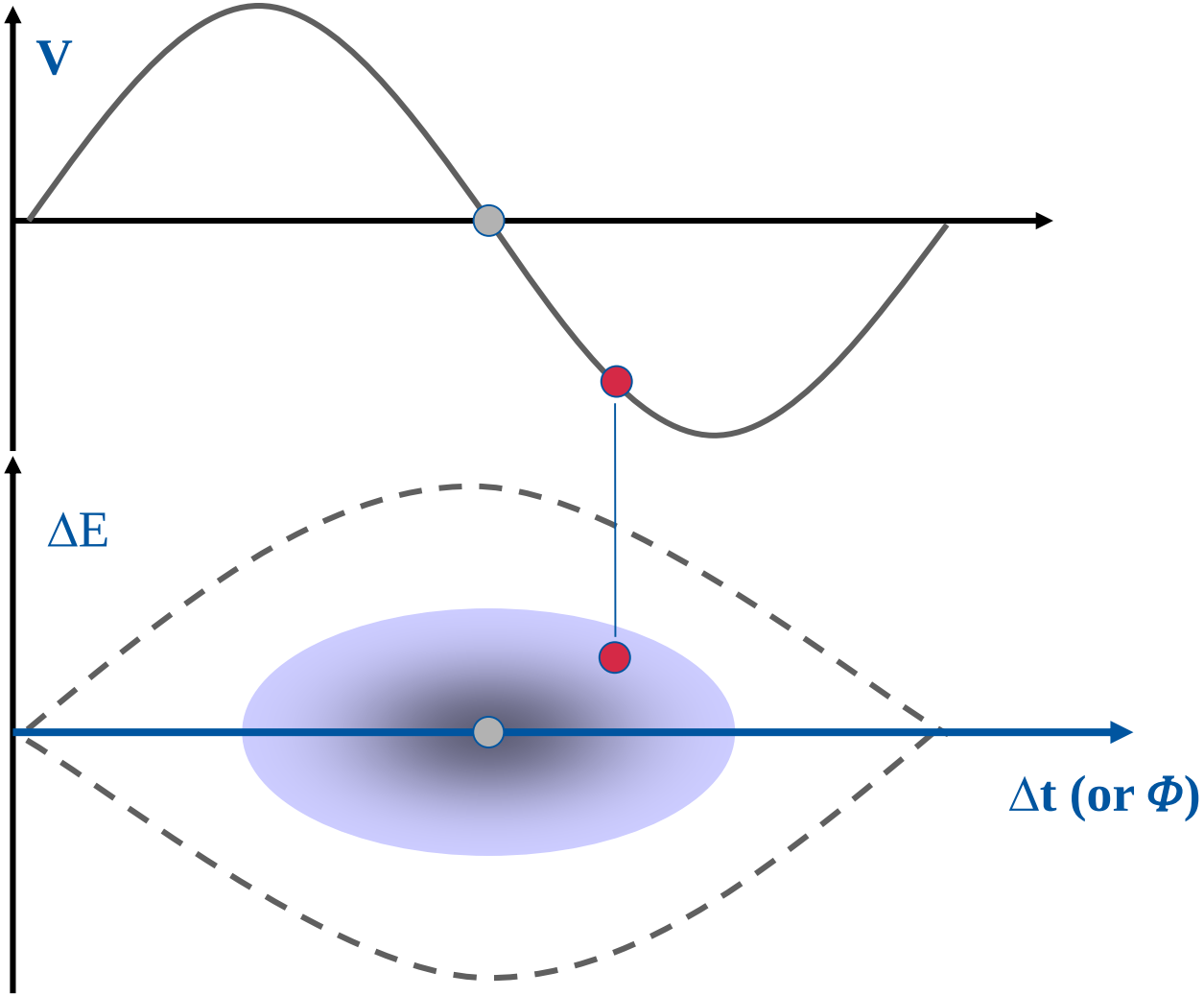
Longitudinal Motion Beyond Transition



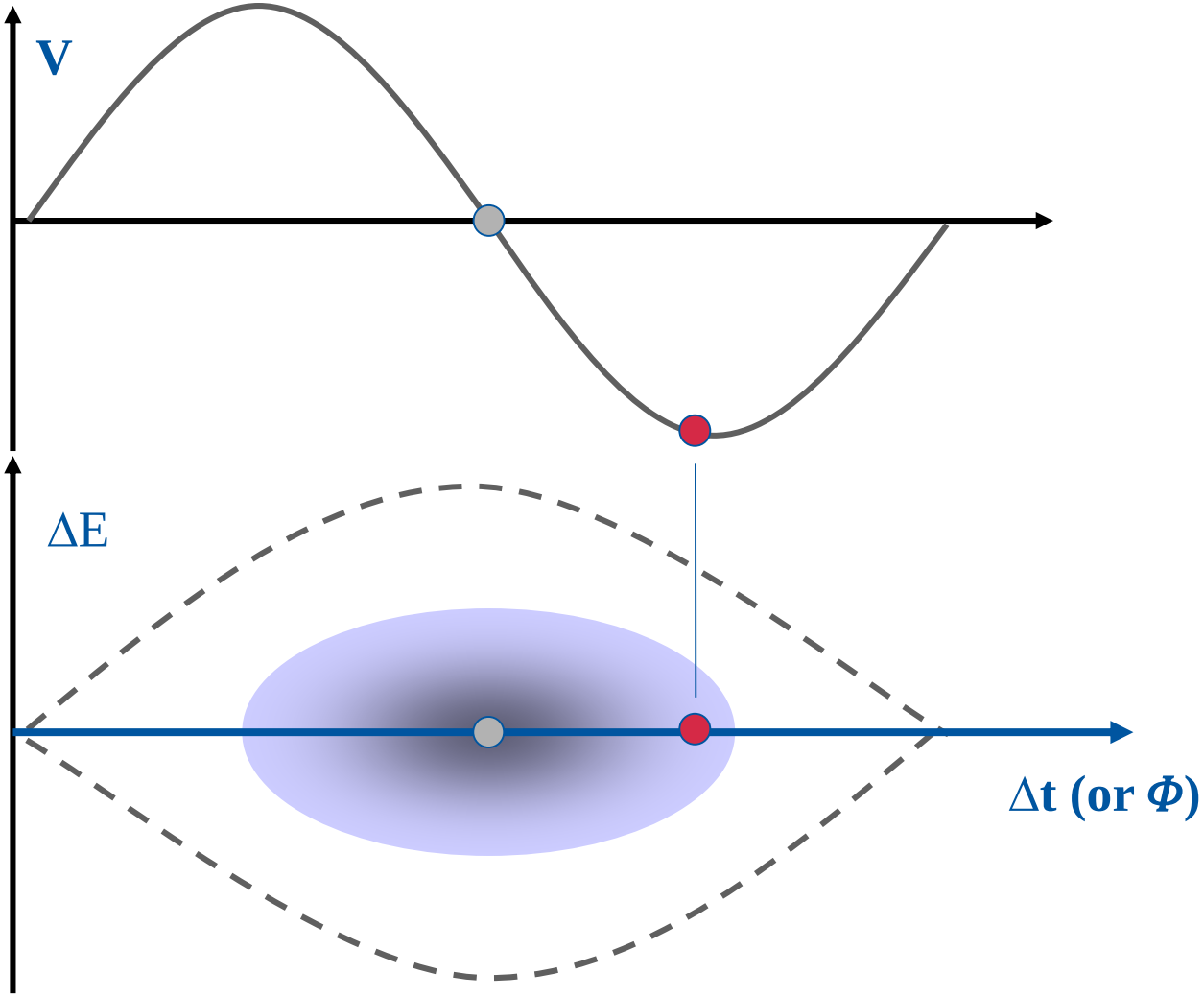
Longitudinal Motion Beyond Transition



Longitudinal Motion Beyond Transition



Longitudinal Motion Beyond Transition



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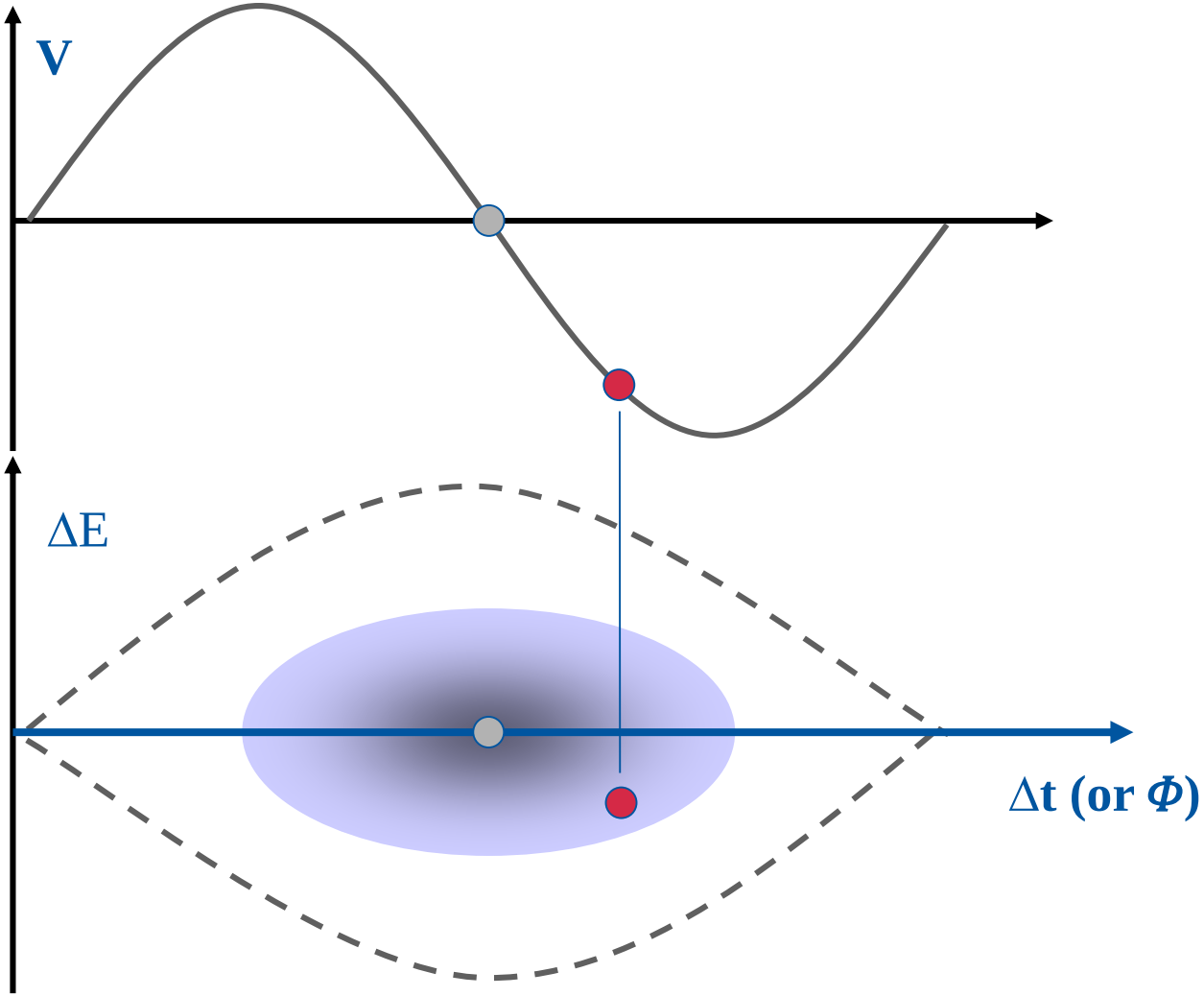

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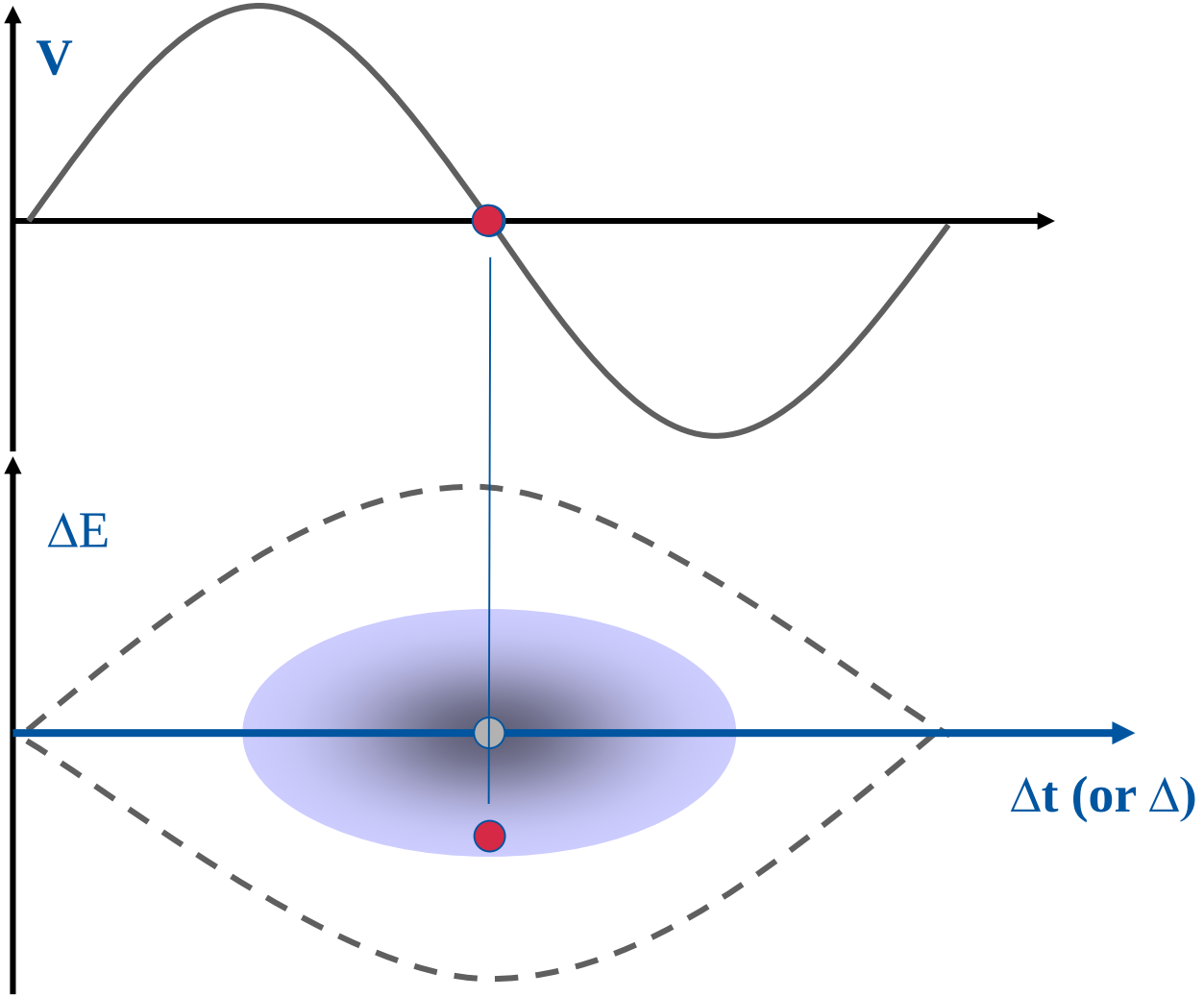
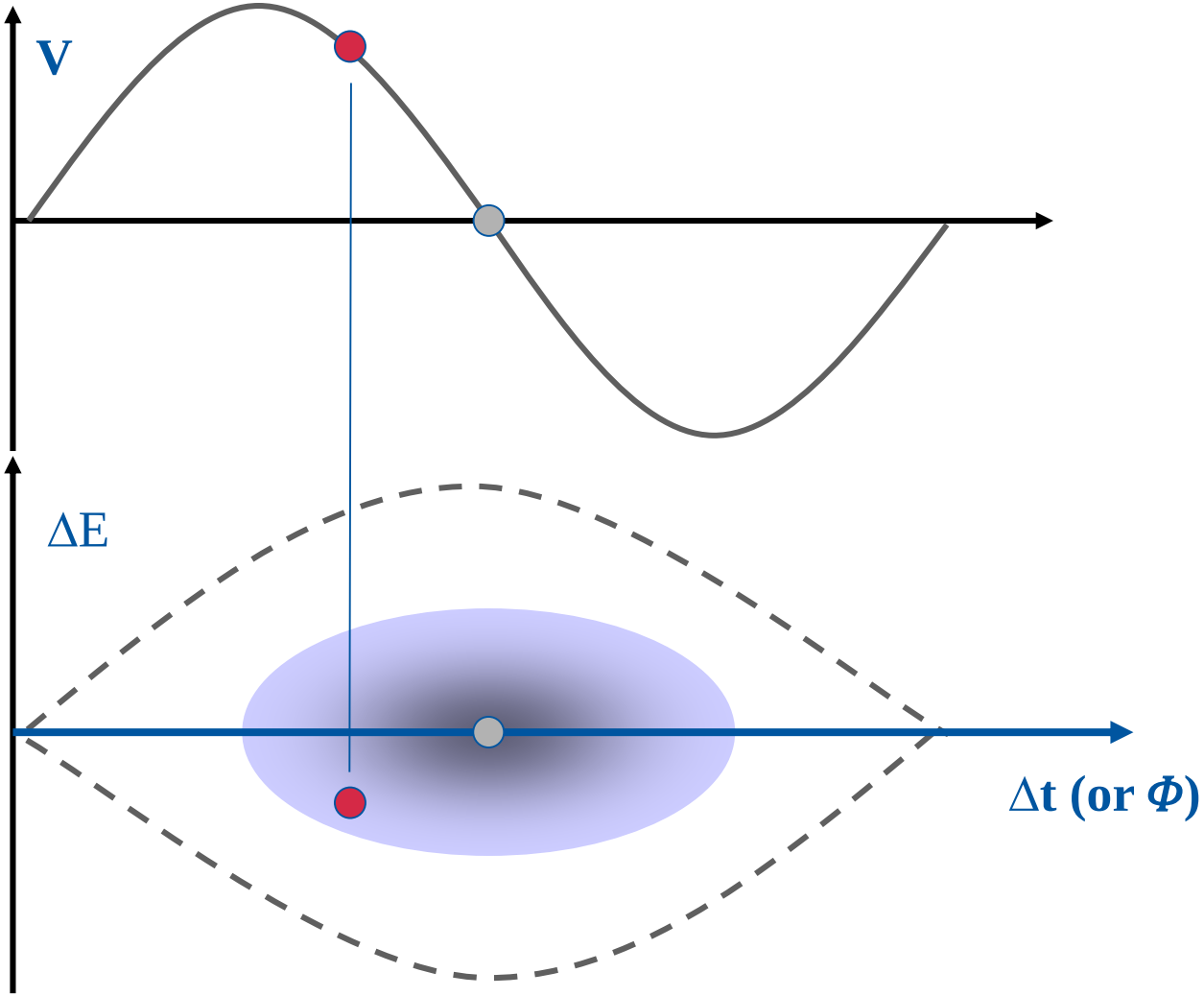
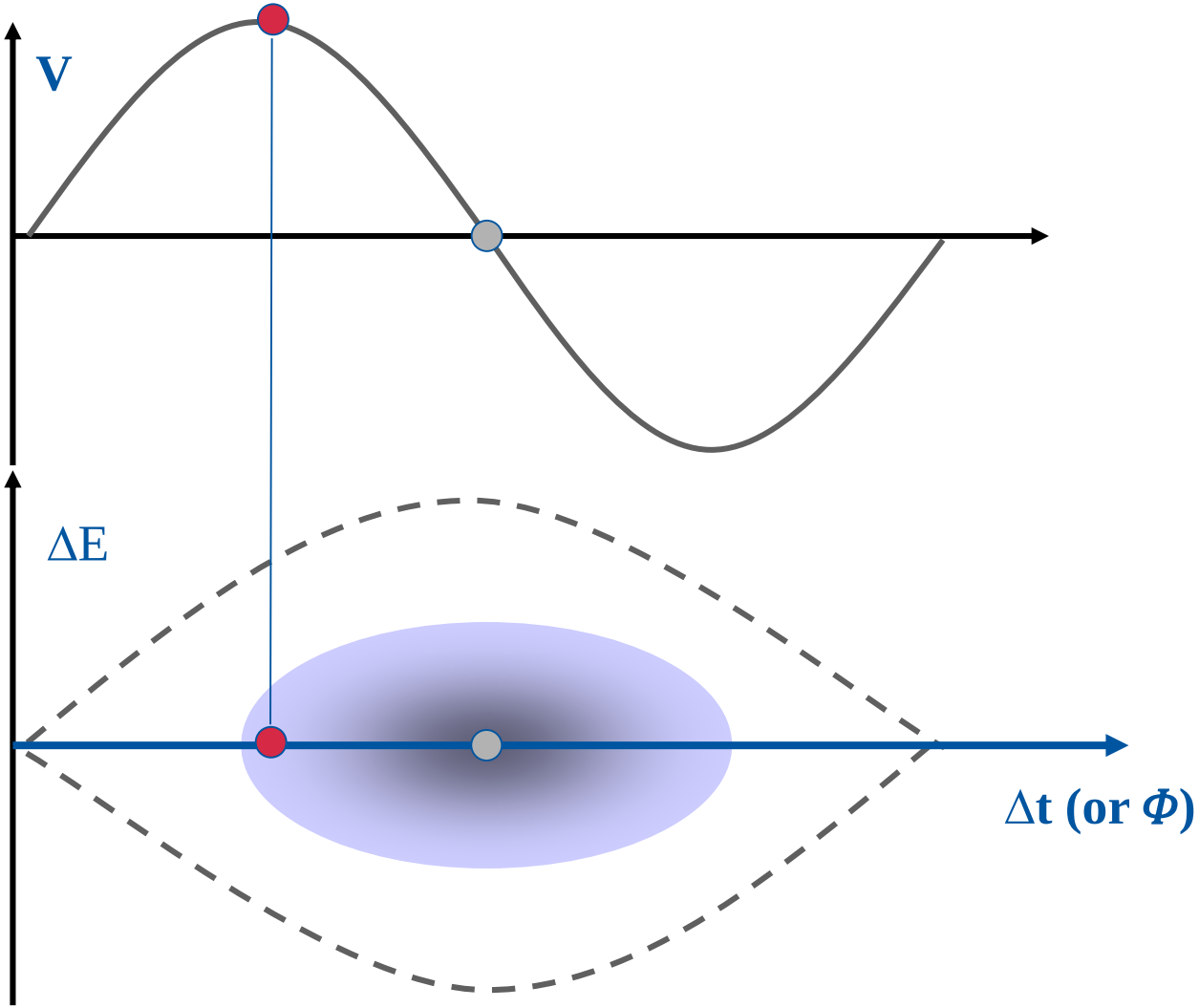


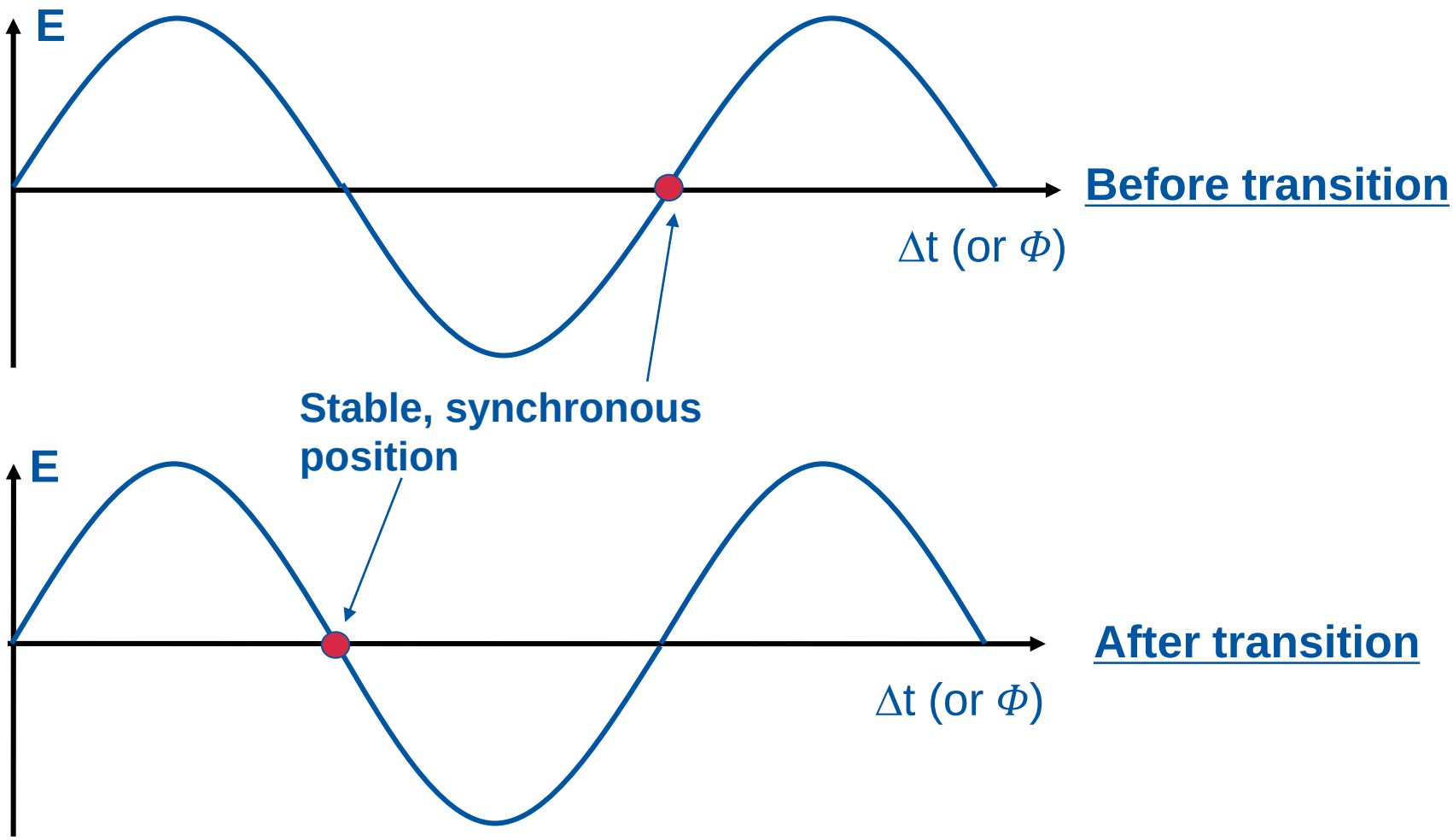
Longitudinal Motion Beyond Transition



Longitudinal Motion Beyond Transition



Before & Beyond Transition



Synchrotron Oscillation

- On each turn the phase, Φ, of a particle w.r.t. the RF waveform changes due to the synchrotron oscillations.

$$\frac{d\phi}{dt} = 2\pi h \Delta f_{rev}$$

Change in revolution frequency

Harmonic number

- We know that $\frac{df_{rev}}{f_{rev}} = -\eta \frac{dE}{E}$

- Combining this with the above $\therefore \frac{d\phi}{dt} = \frac{-2\pi h \eta}{E} \cdot dE \cdot f_{rev}$

- This can be written as:

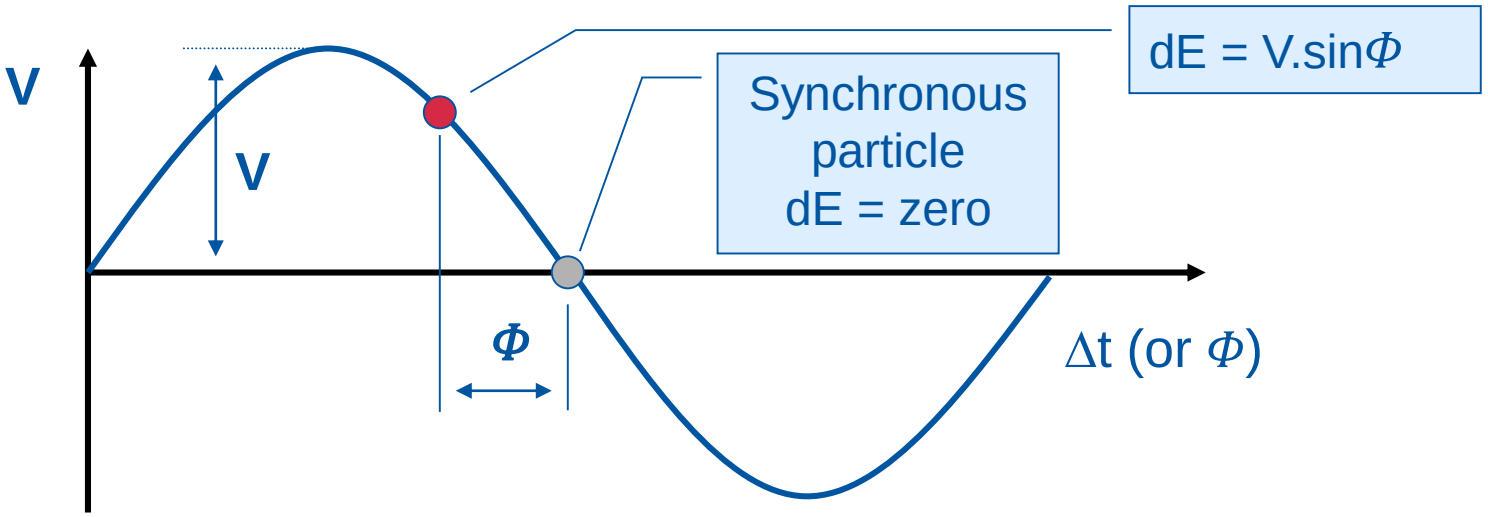
$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h \eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

Change of energy as a function of time

Synchrotron Oscillation

- So, we have: $$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E} \cdot f_{rev} \cdot \frac{dE}{dt}$$

- Where dE is just the energy gain or loss due to the RF system during each turn



Synchrotron Oscillation

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E}\cdot f_{rev}\cdot\frac{dE}{dt}$$

and

$$dE = V\sin\phi \rightarrow \frac{dE}{dt} = f_{rev}V\sin\phi$$

$$\frac{d^2\phi}{dt^2} = \frac{-2\pi h\eta}{E}\cdot {f_{rev}}^2\cdot V.\sin\phi$$

- If Φ is small then $\sin\Phi=\Phi$

$$\frac{d^2\phi}{dt^2} + \left(\frac{2\pi h\eta}{E}\cdot {f_{rev}}^2\cdot V\right)\phi = 0$$

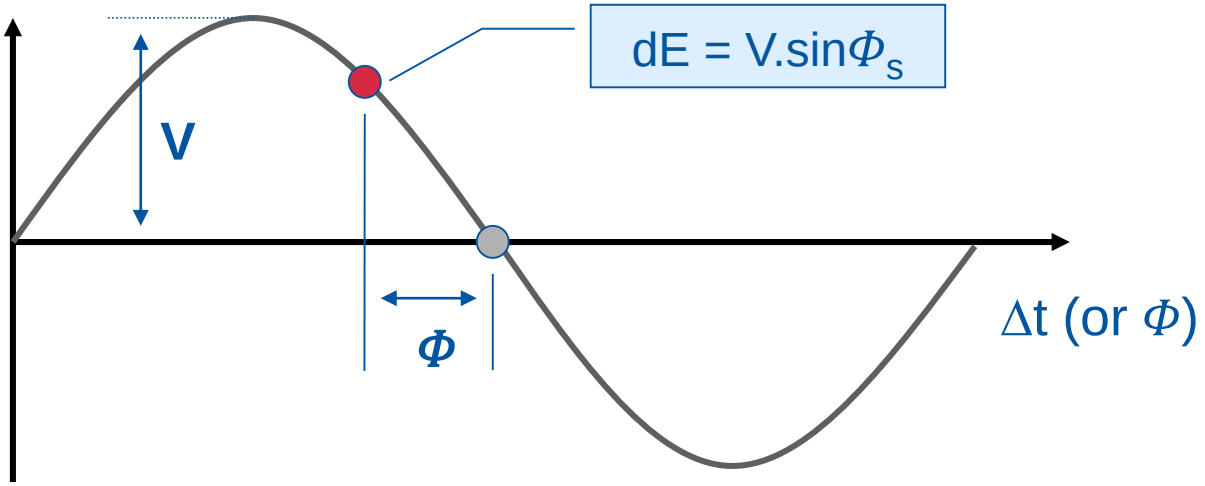
- This is a SHM where the synchrotron oscillation frequency is given by:

Synchrotron tune Qs

$$\left(\sqrt{\frac{2\pi \boldsymbol{h}\eta \boldsymbol{V}}{\boldsymbol{E}}}\right)\cdot \boldsymbol{f}_{rev}$$

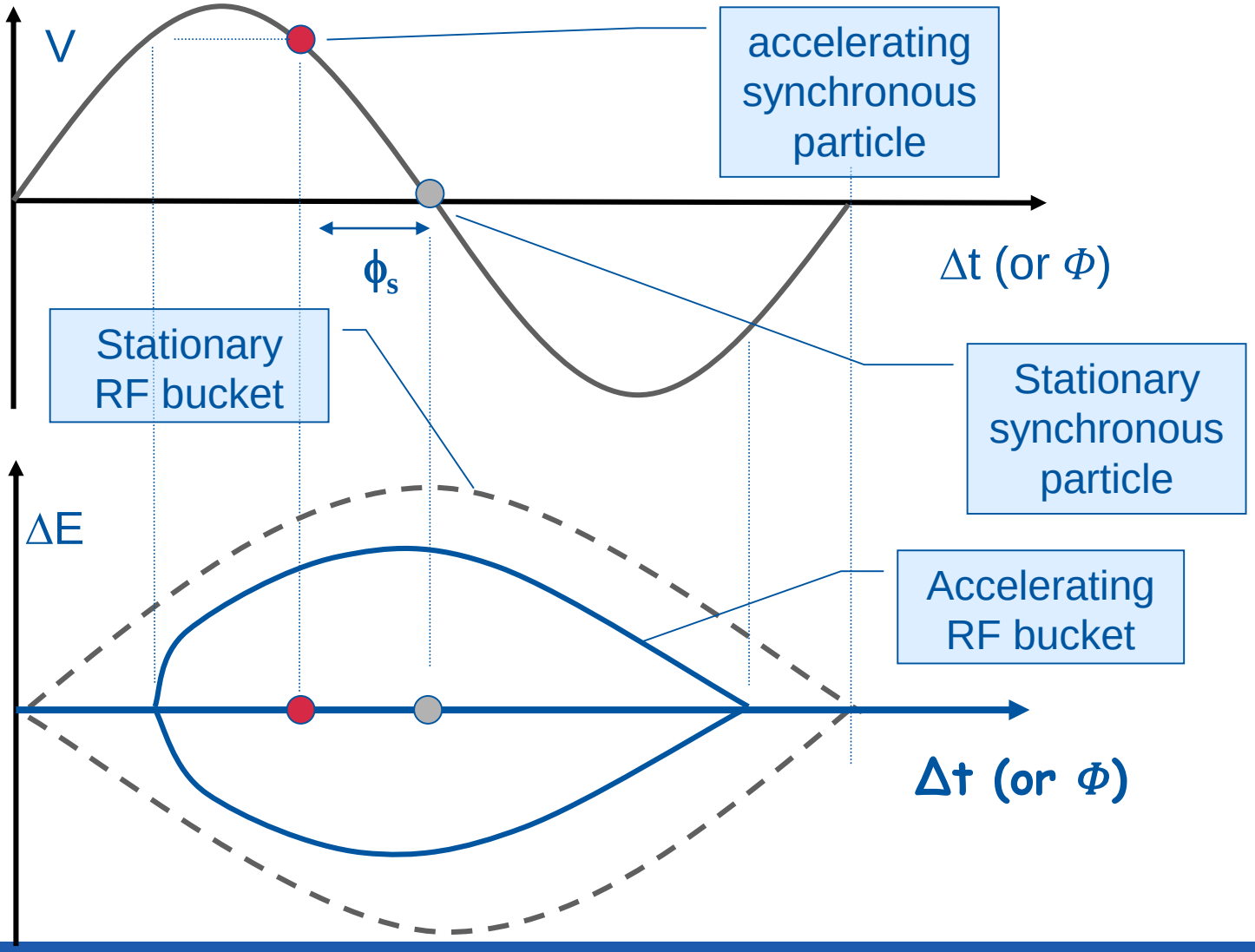
Acceleration

- Increase the magnetic field slightly on each turn.
- The particles will follow a shorter orbit. ($\mathbf{f_{rev} < f_{synch}}$)
- Beyond transition, early arrival in the cavity causes a gain in energy each turn.



- We change the phase of the cavity such that the new synchronous particle is at Φ_s and therefore always sees an accelerating voltage
- $V_s = V\sin\Phi_s = V\Gamma$ = energy gain/turn = dE

Accelerating Bucket



Accelerating Bucket

- The modification of the RF bucket reduces the acceptance
- The faster we accelerate (increasing sin Φ_s) the smaller the acceptance
- Faster acceleration also modifies the synchrotron tune.
- For a stationary bucket ($\Phi s = 0$) we had:

$$\left(\sqrt{\frac{2\pi h\eta}{E}}\right)\cdot f_{rev}$$

- For a moving bucket ($\Phi s \neq 0$) this becomes:

$$\left(\sqrt{\frac{2\pi h\eta}{E}}\right)\cdot f_{rev}\cos\phi_s$$

Higher Harmonic RF Voltage

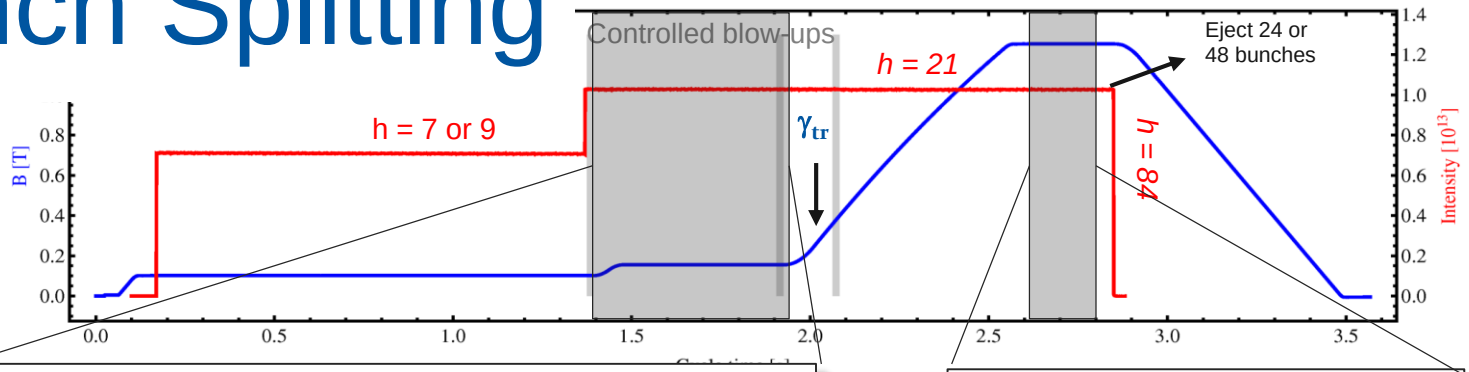
- Until now we have applied an oscillating voltage with a frequency equal to the revolution frequency

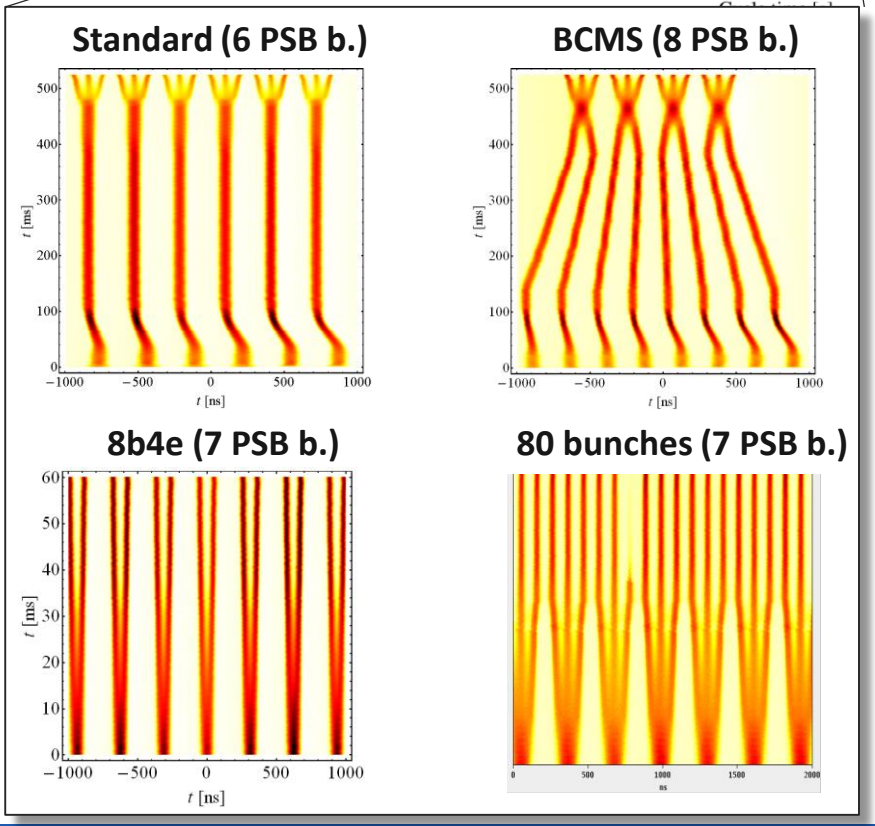
$$f_{rf} = f_{rev}$$

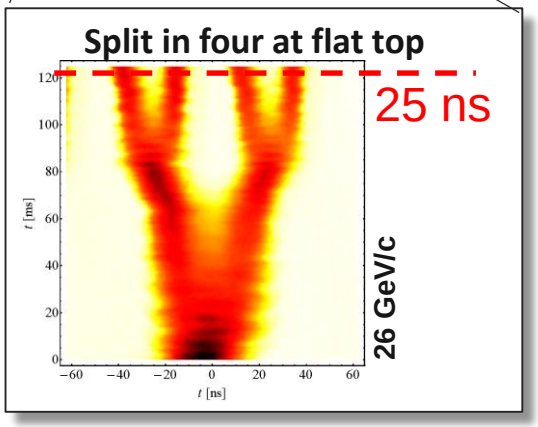
- What will happen when f_{rf} is a multiple of f_{rev} ???

$$f_{rf} = h\, f_{rev}$$

Bunch Splitting



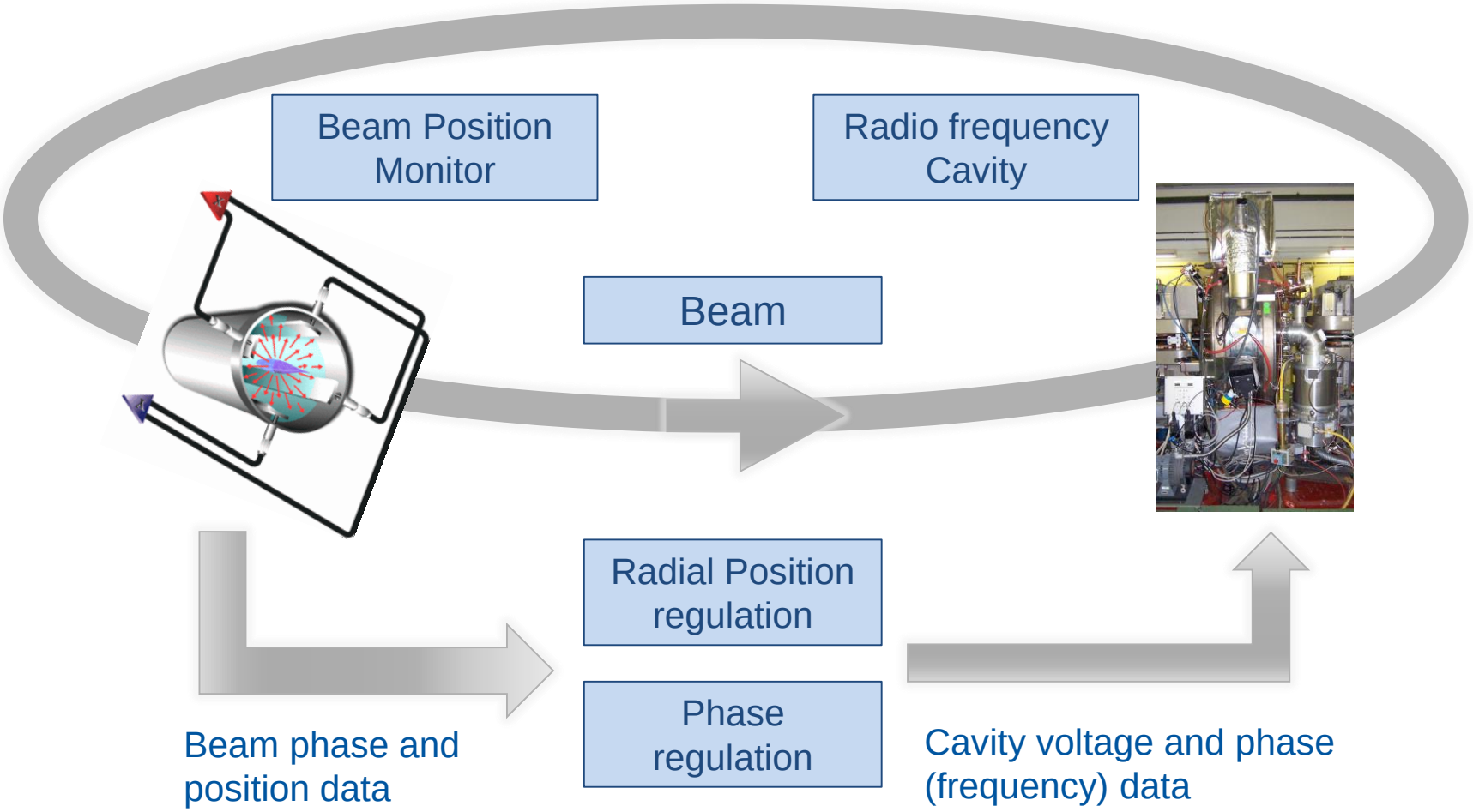




Standard: 72 bunches @ 25 ns
BCMS: 48 bunches @ 25 ns

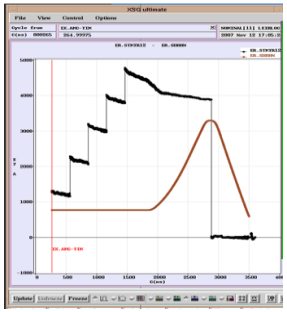
The PS defines the longitudinal beam characteristics

RF Beam Control



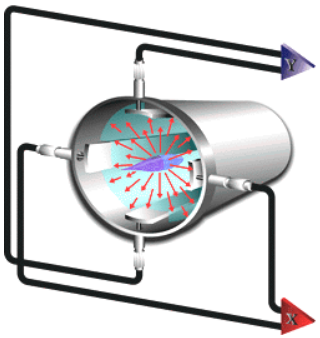
Main Diagnostics Tools

Beam Current & Position



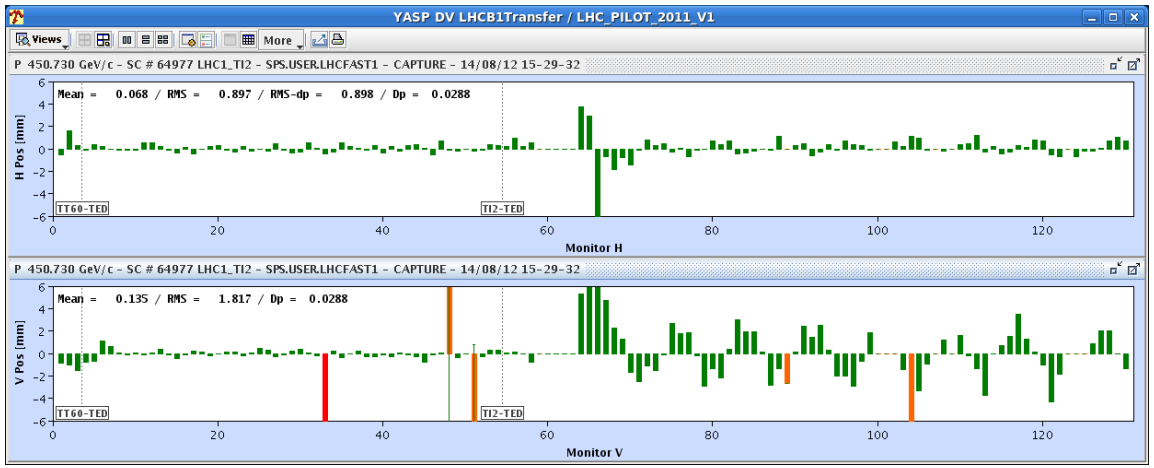
Beam intensity or current measurement:

- Working as classical transformer
- The beam acts as a primary winding



Beam position/orbit measurement:

$$x = a \cdot \frac{U_{right} - U_{left}}{U_{right} + U_{left}} \equiv a \cdot \frac{\Delta}{\Sigma}$$

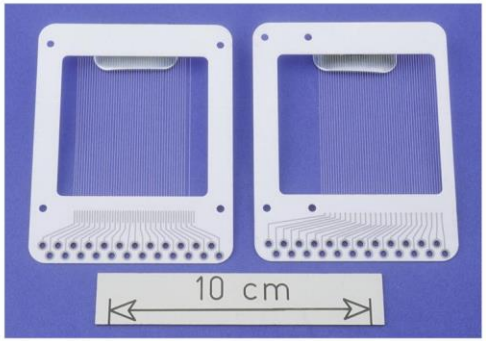


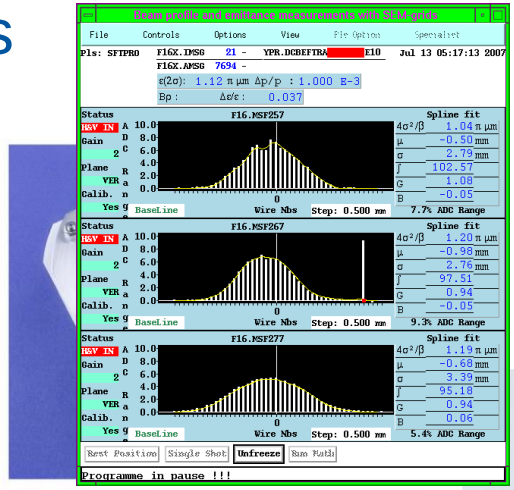
Correcting orbit using automated beam steering

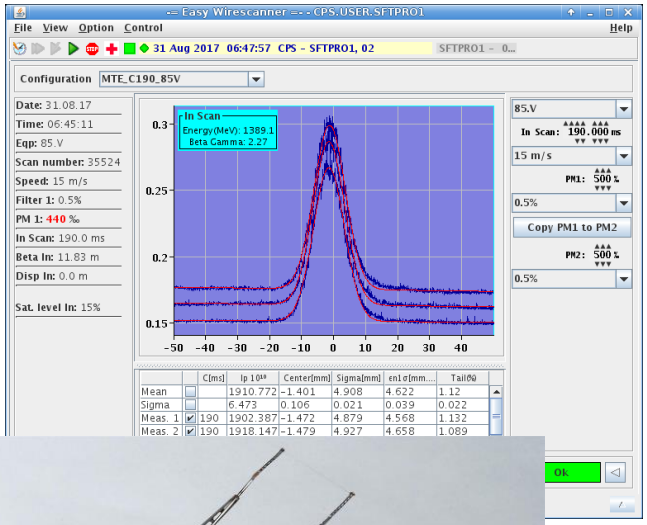
Transverse Beam Profile Monitor

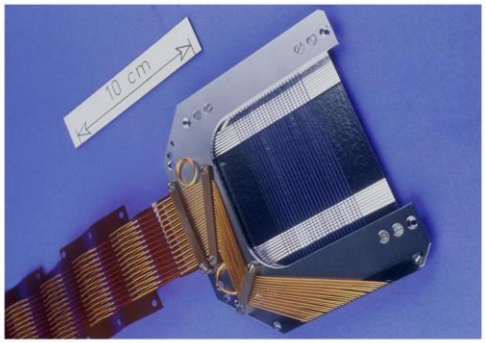
Transverse beam profile/size measurement:

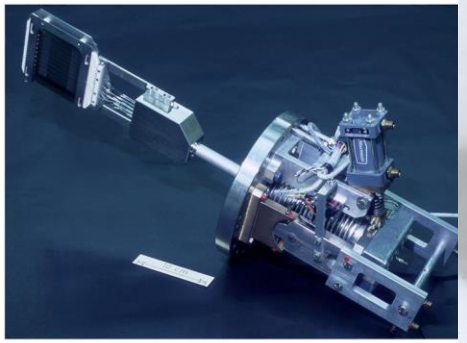
- Secondary Emission Grids
- Wire scanners

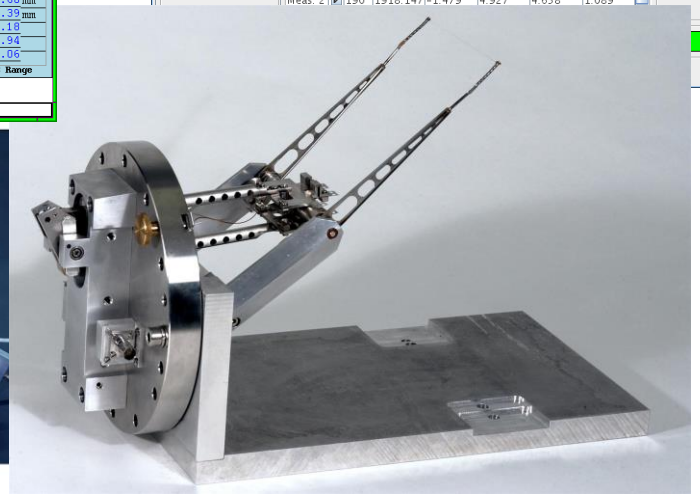






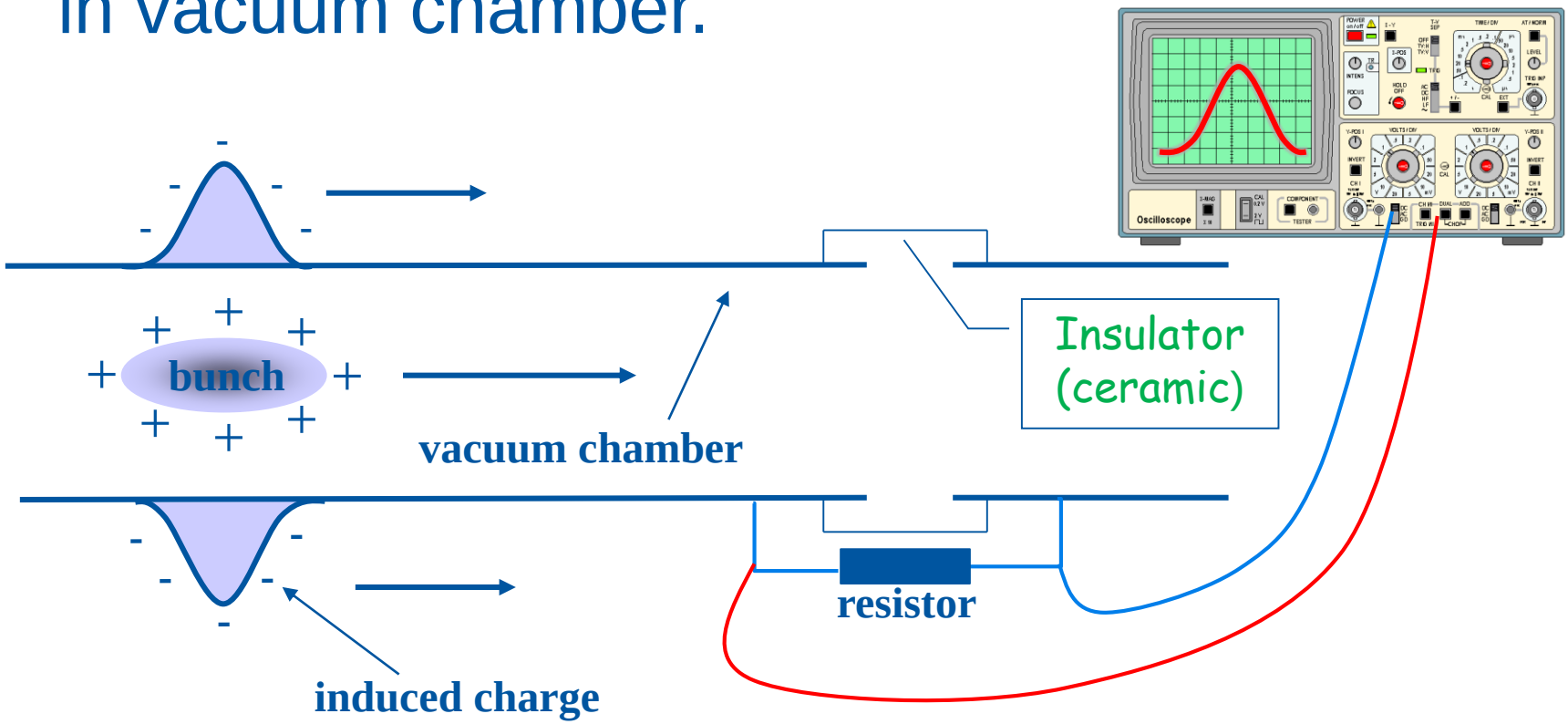






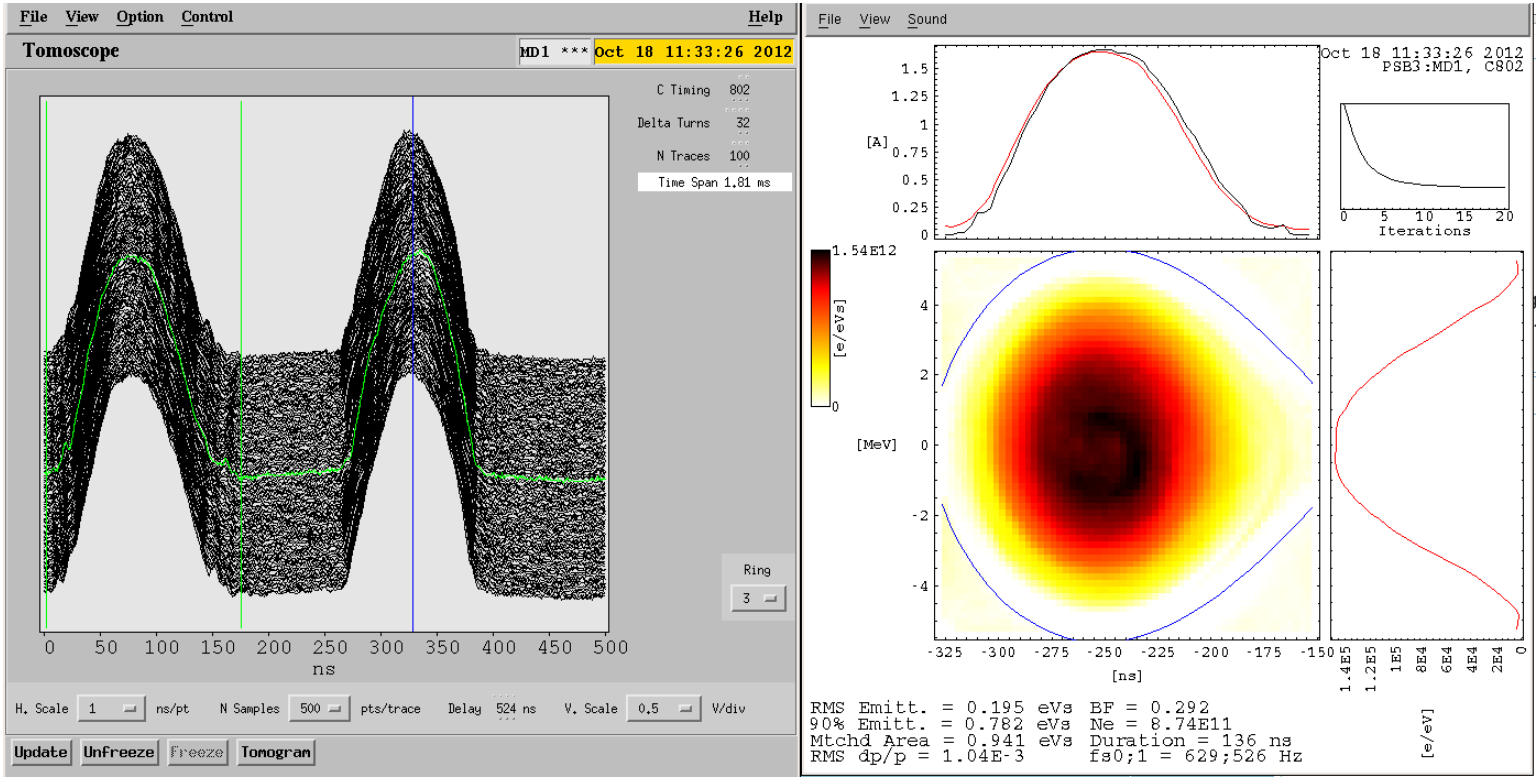
Wall Current Monitor

- A circulating bunch creates an image current in vacuum chamber.



- The induced image current is the same size but has the opposite sign to the bunch current.

Longitudinal TomoScope

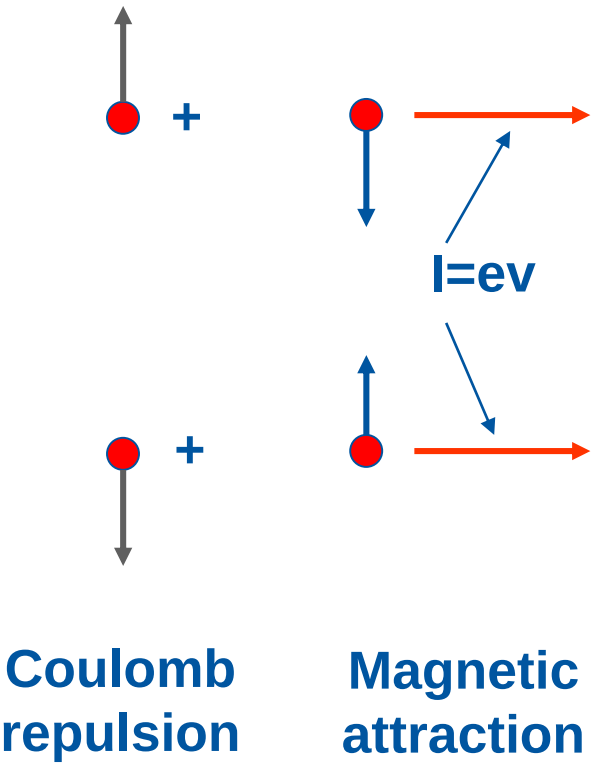


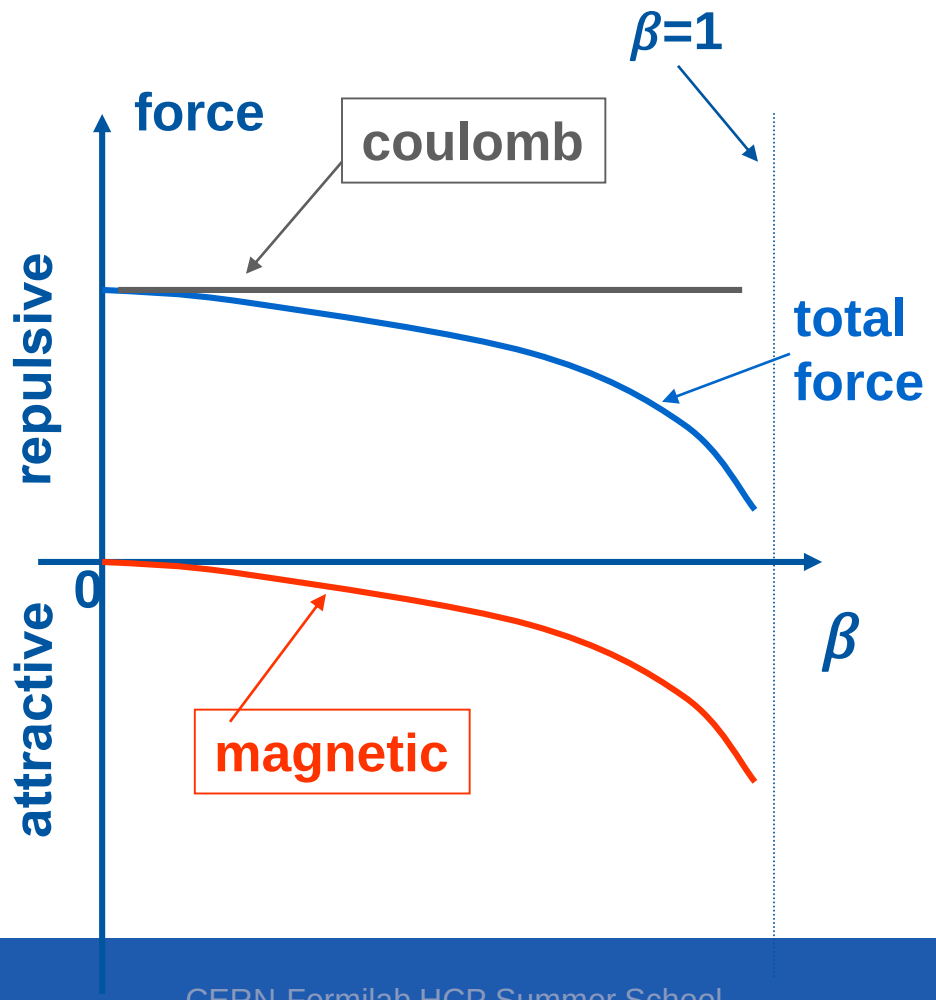
- Make use of the synchrotron motion that turns the “patient” in the Wall Current monitor

Possible Limitations

Space Charge

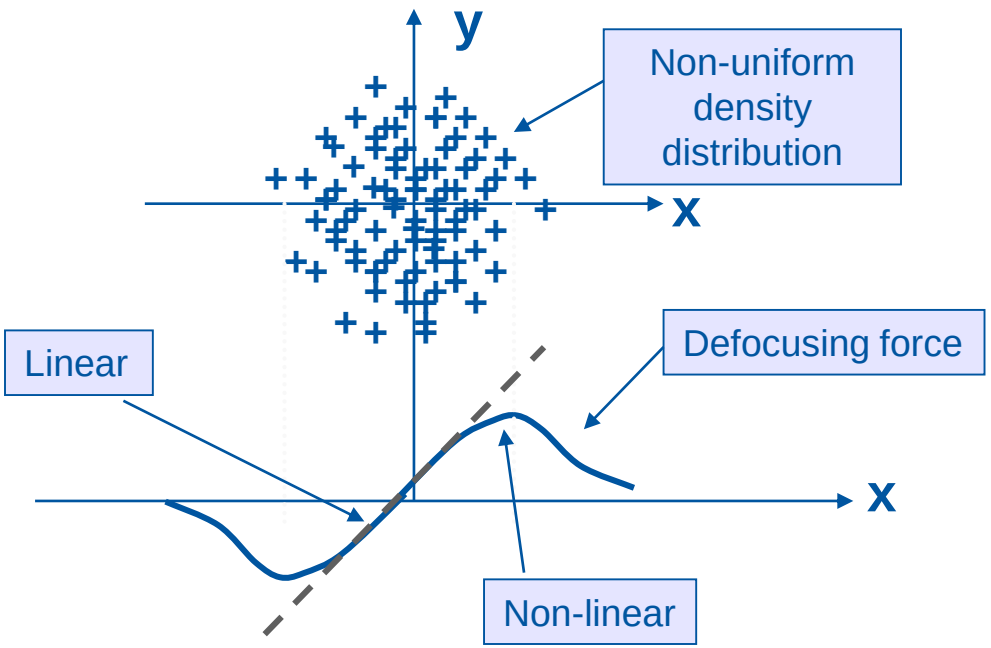
- Between two charged particles in a beam we have different forces:





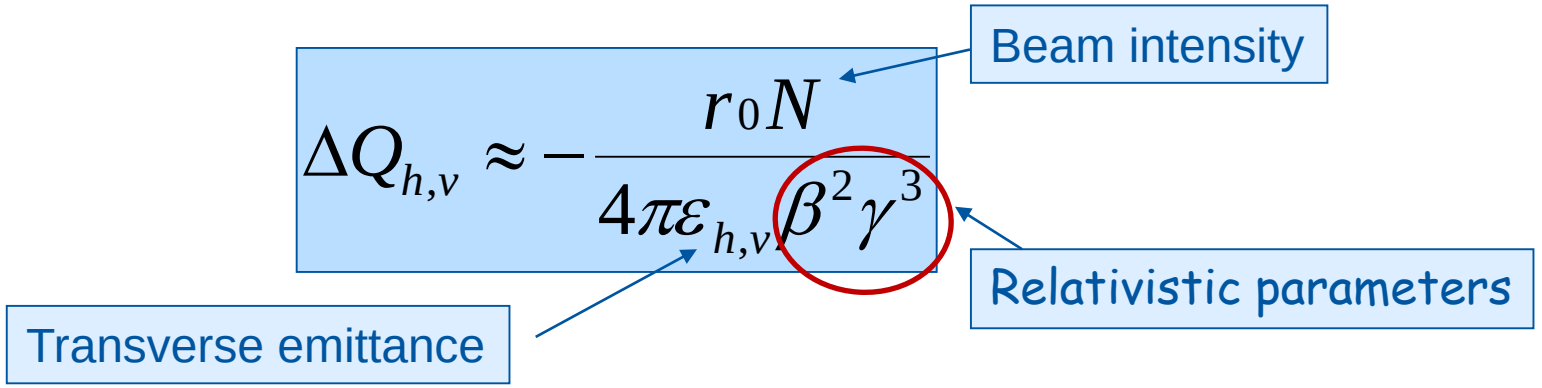
Space Charge

- At low energies, which means $\beta << 1$, the force is mainly repulsive ⇒ defocusing
- It is zero at the centre of the beam and maximum at the edge of the beam



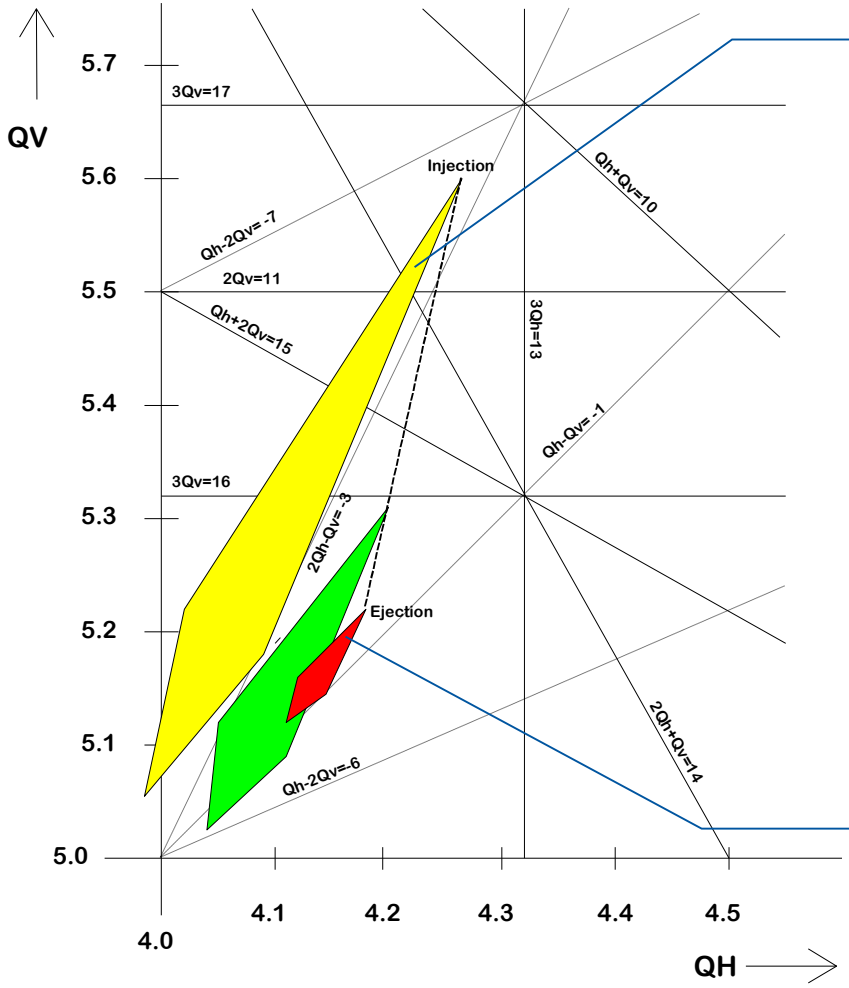
Laslett tune shift

- For the non-uniform beam distribution, this non-linear defocusing means the ΔQ is a function of x (transverse position)
- This leads to a spread of tune shift across the beam
- This tune shift is called the 'LASLETT tune shift'



- This tune spread cannot be corrected and does get very large at high intensity and low momentum

Resonance & Tune Diagram



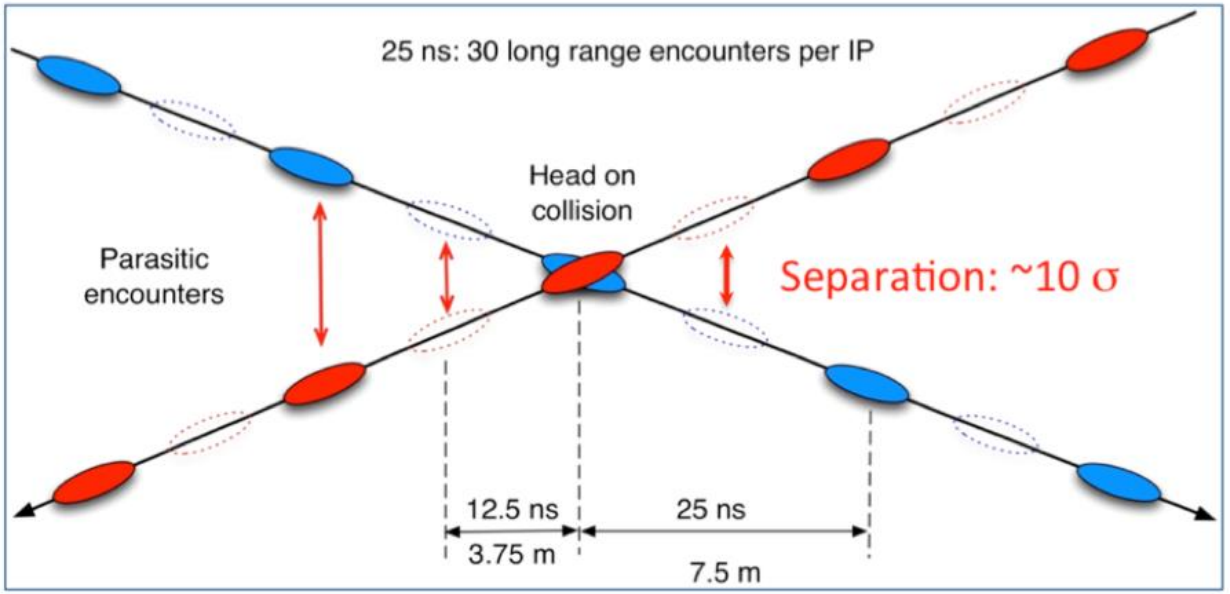
injection

During acceleration we change the horizontal and vertical tune to a place where the beam is the least influenced by resonances.

ejection

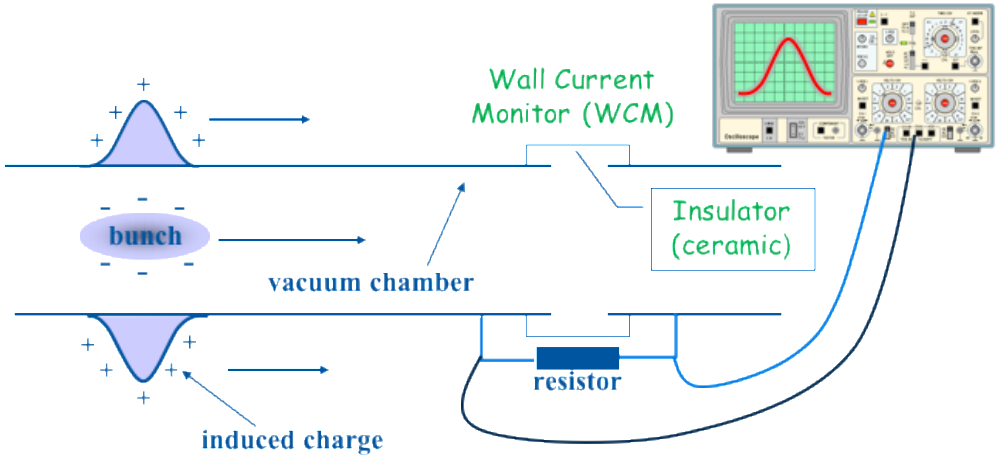
Beam – Beam Effect

- Particle beam are surrounded by magnetic fields
- If the beams “see” each other in colliders these magnetic fields can act on the both beams and can cause defocussing effect and tune shifts

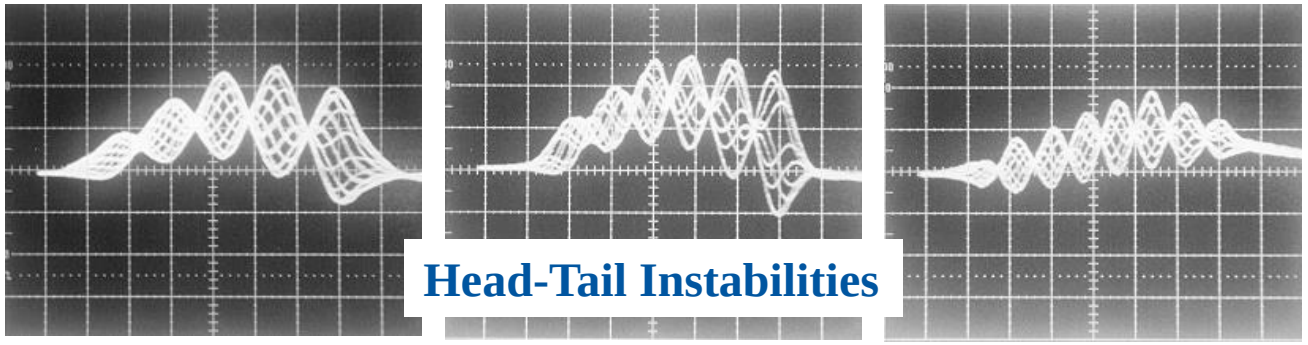


Collective Effects

- Induced currents in the vacuum chamber (impedance) can result in electric and magnetic fields acting back on the bunch or beam



Coupled Bunch Instabilities

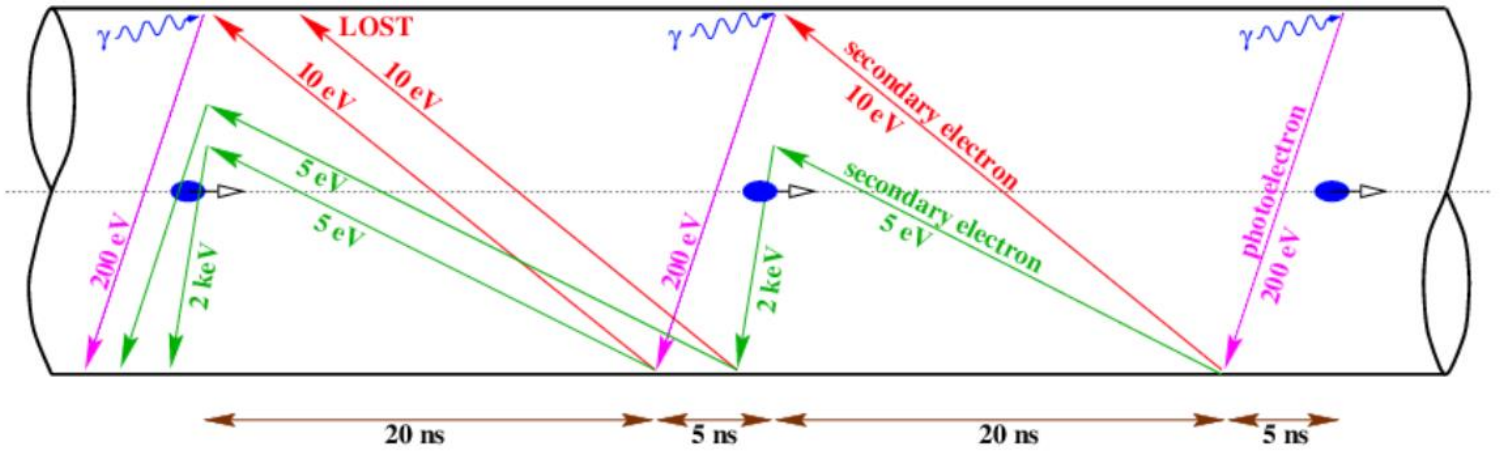


Head-Tail Instabilities

Cures for Collective Effects

- Ensure a spread in betratron/synchrotron frequencies
  - Increase Chromaticity
  - Apply Octupole magnets (Landau Damping)
- Reduce impedance of your machine
- Avoid higher harmonic mode in cavties
- Apply transverse and longitudinal feedback systems

Electron Cloud



- e-cloud when SEY is beyond 2, hence it depends on the vacuum chamber surface
- The electron cloud forms an impedance to the beam and can cause beam instability
- In the SPS and the LHC we use the “scrubbing” method to reduce the SEY
- The SPS vacuum chambers will be Carbon coated to reduce the SEY

LIU & HL-LHC Projects

Luminosity, the Figure of Merit

$$LUMINOSITY = \frac{N_{event/sec}}{s_r} = \frac{N_1 N_2 f_{rev} n_b}{4 \pi s_x s_y} F$$

Intensity per bunch

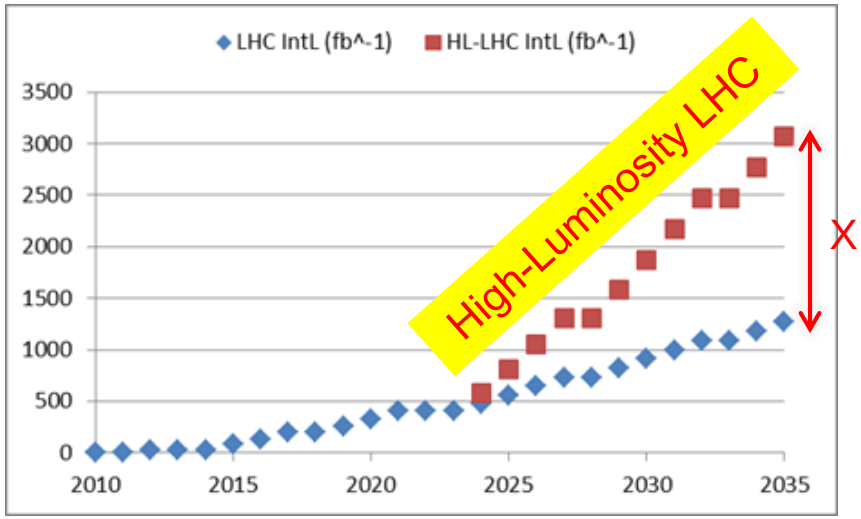
Number of bunches

Geometrical Correction factors

Beam dimensions

- More or less fixed:
  - Revolution period
  - Number of bunches

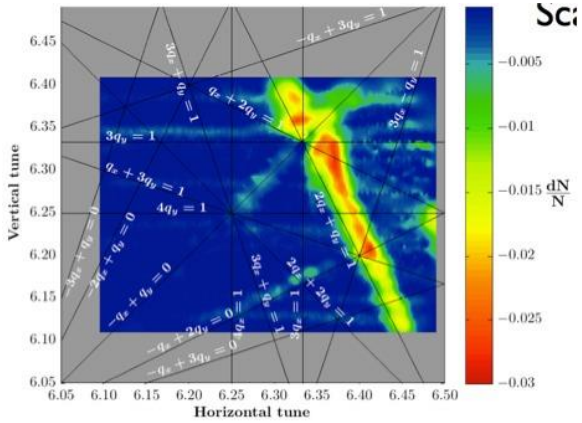
- Parameters to optimise:
  - Number of particles per bunch
  - Beam dimensions
  - Geometrical correction factors



LIU: What will be changed ?

- LINAC4 – PS Booster:
  - New LINAC 4 with H^- injection
  - Higher injection energy
  - New Finemet® RF cavity system
  - Increase of extraction energy

- PS:
  - Injection energy increase from 1.4 GeV to 2 GeV
  - New Finemet® RF Longitudinal feedback system
  - New RF beam manipulation scheme to increase beam brightness

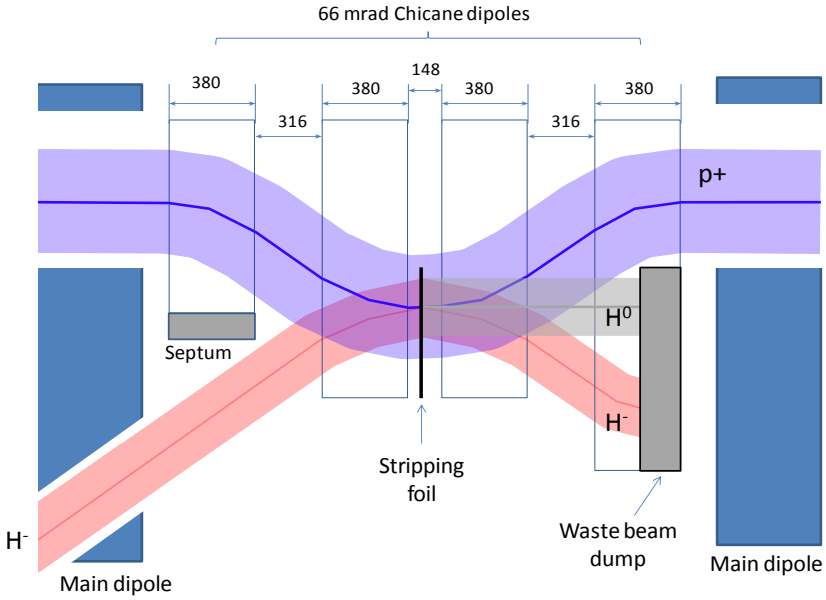


- SPS
  - Machine Impedance reduction (instabilities)
  - New 200 MHZ RF system
  - Vacuum chamber coating against e-cloud

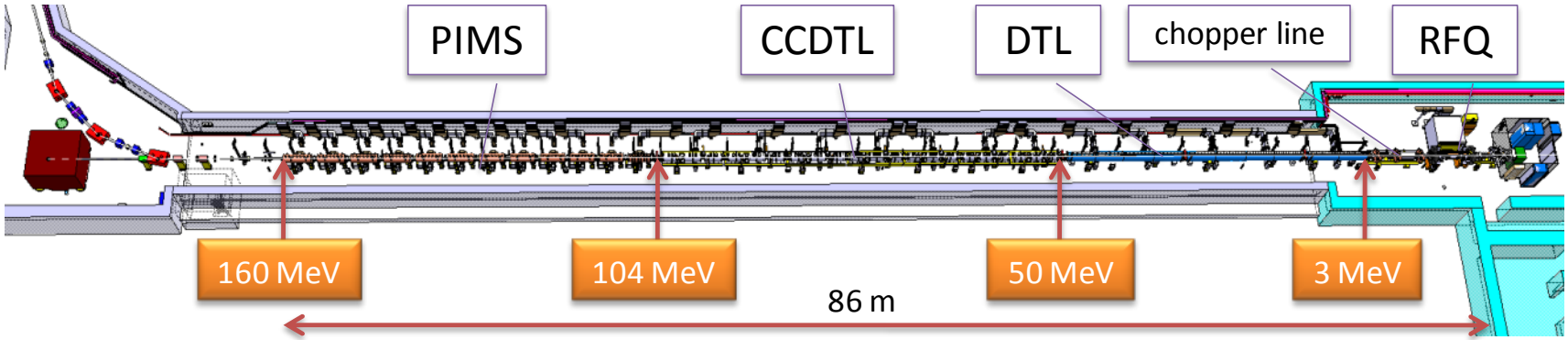
These are only the main modifications and this list is not exhaustive

LINAC4

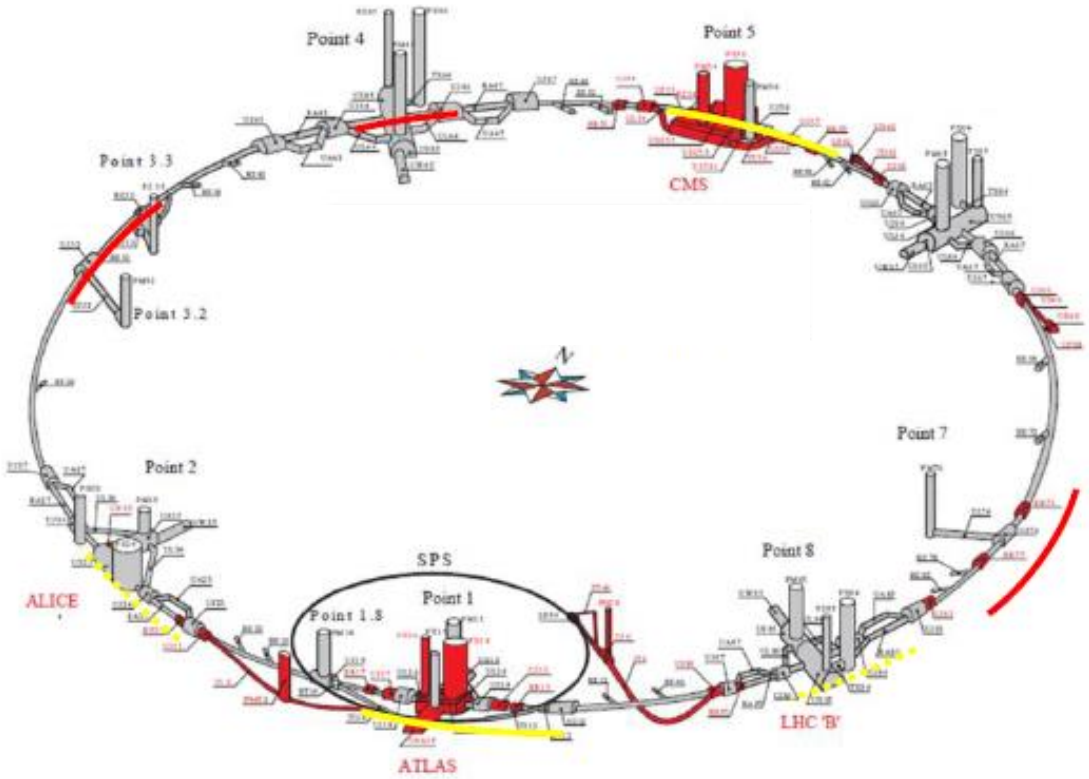
- Produce and accelerate H^- at 160 MeV
- Inject H^- into the PSB and strip the electrons → protons in the PSB
- During the following turns interleave the circulating protons with H- that will be stripped



Injecting multiple turns will increase **intensity** and **density**



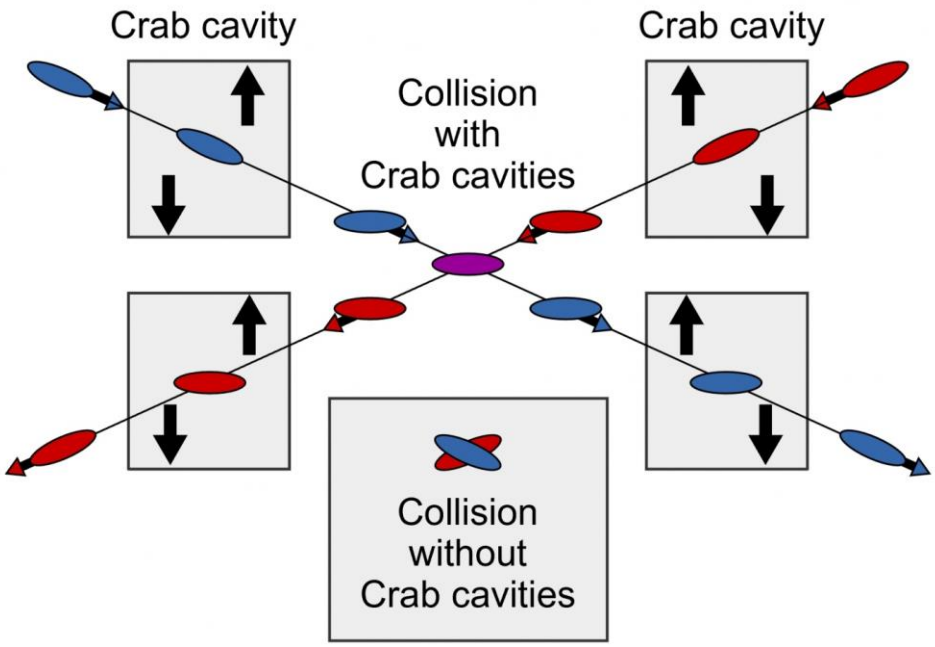
HL-LHC: What will be changed ?



- New IR-quads (inner triplets)
- New 11T short dipoles
- Collimation upgrade
- Cryogenics upgrade
- Crab Cavities
- Cold powering
- Machine protection
- …

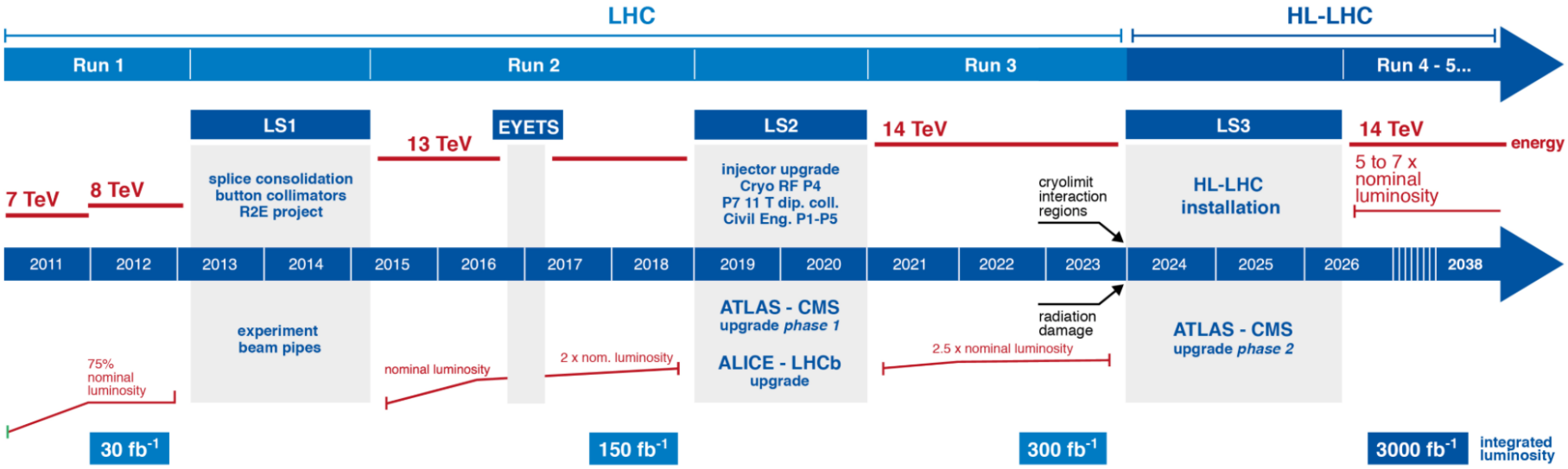
Major intervention on more than 1.2 km of the LHC
These are only the main modifications and this list is not exhaustive

Crab Cavities



Crab cavities will reduce the effect of the geometrical factor on the luminosity

The Schedule



For those who want to learn more

- **Accelerators for Pedestrians**
  - Author: Simon Baird
  - Reference: *CERN-AB-Note-2007-014* (Free from the Web)
- **CERN Accelerator School**
  - Fifth General Accelerator Physics Course
  - Editor: S. Turner
  - Reference: *CERN 94-01* (volume I & II) (Free from the Web)
- **An Introduction to Particle Accelerators**
  - Author: Edmund Wilson
  - Reference: *ISBN 0-19-850829-8* (CERN Book shop)
- **Particle Accelerator Physics (3rd edition)**
  - Author: Helmut Widemann
  - Reference: *ISBN 978-3-540-49043-2* (CERN Book shop)

www.cern.ch