$n - \bar{n}$ transitions

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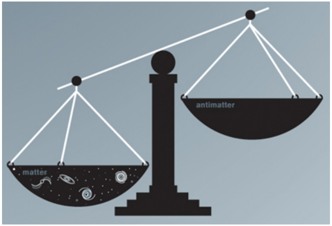
March 4, 2017
Based on Susan Gardner and X. Yan,
Phys. Rev. D93, 096008 (2016) arXiv:1602.00693
and ongoing work

Why $n - \bar{n}$ transitions?

Three ingredients needed for explanation of baryon asymmetry in our universe (BAU):

- Baryon number violation ($\not{B}$);
- C and CP violation;
- departure from thermal equilibrium.

[Sakharov 1967]



Two kinds of $\not{B}$ phenomena :

- $|\Delta B| = 1$: $\Lambda_{p\ decay} \geq 10^{15}$ GeV;
- $|\Delta B| = 2$: $\Lambda_{n\bar{n}} \geq 10^{5.5}$ GeV.

$n - \bar{n}$ oscillations may be also connected to $\not{L}$ [Marshak and Mohapatra (1980), Babu and Mohapatra (2015)]

Challenges of observing $n - \bar{n}$ oscillations

Neutron is spin $\frac{1}{2}$ particle.
CPT and Lorentz symmetry is assumed,

$$H = \begin{pmatrix} M - \mu \cdot \mathbf{B} & \delta \\ \delta & M + \mu \cdot \mathbf{B} \end{pmatrix}$$

$$\Rightarrow \qquad P_{n \to \bar{n}}(t) \simeq \frac{\delta^2}{2(\mu \cdot \mathbf{B})^2}[1 - \cos(2\mu B t)]exp(-\lambda t)$$

where $\lambda^{-1} = \tau_n = 0.88 \times 10^3$s. So external magnetic fields suppress transition, unless $t \ll (2\mu B)^{-1}$ (quasi-free condition).
[Marshak and Mohapatra (1980); Cowsik and Nussinov (1981); Phillips II et al (2014)]
Same conclusion for matter effect. $\Rightarrow$ explore alternative methods.

Using $n - \bar{n}$ oscillation to set limits on the strength of Lorentz invariance violation has also been considered. [Babu and Mohapatra (2015)]

Spin offers a new path?

CPT symmetry guarantees that $n(\uparrow)$ and $\bar{n}(\downarrow)$ have the same energy.
$\Rightarrow$ A hint: spin might be important.

Once the fermion anticommutation relation is taken into account, there are 3 non-trivial lowest mass dimension operators:

- $n^\top C n +$ h.c., $n - \bar{n}$ oscillation operator, always "quenched";
- $n^\top C \gamma^5 n +$ h.c., does not contribute to $n\bar{n}$ oscillation;
  [Berezhiani and Vainshtein, (2015), Fujikawa and Tureanu, (2015)]
- $n^\top C \gamma^\mu \gamma^5 n \partial^\nu F_{\nu\mu} +$ h.c., [Berezhiani and Vainshtein (2015)]

The external source, $j_\mu = \partial^\nu F_{\nu\mu}$, requires the consideration of spin degrees of freedom of n and $\bar{n}$.
Consider the process $n(p_1) + n(p_2) \to \gamma^*(k)$. Crossing yields $n - \bar{n}\gamma^*$ transition.

Phases are restricted!

When we checked the CPT transformation properties of these operators, we found that phases of discrete symmetry transformations, such as **P** and **CPT** are not arbitrary! In fact, $\eta_c \eta_p \eta_t$ is imaginary and η_p is imaginary.

This was noticed before [Feinberg and Weinberg(1959), P. A. Carruthers(1971), **Kayser and Goldhaber(1983), Kayser (1984)**] , but considered as a property of Majorana fields.

We find it is also true for Dirac fields with *B-L violation*, and hence believe it is associated with discrete symmetries themselves.
[SG and Yan (2016)]

Now we focus on the "j_μ" operator.

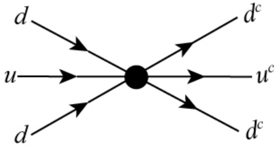
Connect $n - \bar{n}$ conversion with oscillation

Dimension analysis of the j_μ operator shows that

$$\alpha(n^T C\gamma^\mu\gamma_5 n j_\mu + \text{h.c.})$$

with $[\alpha] = -2$.
We want to evaluate the mass scale of this suppression.

Note that quarks are charged under QED and QCD. So the simplest way to explore the connection is through QED.
Also a difference between u and d is necessary to make $B - L$ violation appear in a physically consistent way. [S.G and Yan (2016)]



Quark-level $n - \bar{n}$ oscillation:
$\Lambda_{QCD} \ll \Lambda \ll \Lambda_{BSM}$

6-fermion $n - \bar{n}$ oscillation operators

The observable comes from three inputs:

$$\frac{1}{\tau_{n\bar{n}}} = \delta = c_{BSM} c_{QCD} \langle \bar{n} | O | n \rangle, \text{ [M. Buchoff et al (2012)]}$$

where c_{BSM} is the running of the BSM theory to the weak interaction scale, c_{QCD} is the QCD running from weak to the nuclear scale, and $\langle \bar{n} | O | n \rangle$ is the matrix element of the 6-fermion $n - \bar{n}$ oscillation operators. Both c_{BSM} and c_{QCD} have been analyzed in, e.g., [Winslow and Ng (2010), Buchoff and Wagman (2016)].

The operator $O = \sum_{i,\chi} \lambda_{m,\chi} (O_m)_\chi$ and there are 18 independent operators if $U(1)_{em}$ and $SU(3)_{color}$ symmetries are considered.

$$\begin{aligned}
(O_1)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C u_{\chi_1}^{\beta}][d_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \\
(O_2)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C d_{\chi_1}^{\beta}][u_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_s)_{\alpha\beta\gamma\delta\rho\sigma}, \\
(O_3)_{\chi_1\chi_2\chi_3} &= [u_{\chi_1}^{\top\alpha} C d_{\chi_1}^{\beta}][u_{\chi_2}^{\top\gamma} C d_{\chi_2}^{\delta}][d_{\chi_3}^{\top\rho} C d_{\chi_3}^{\sigma}](T_a)_{\alpha\beta\gamma\delta\rho\sigma},
\end{aligned}$$

with $(T_s)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}$

and $(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}$. [Rao and Shrock (1982)]

6-fermion $n - \bar{n}$ oscillation operators

The number of independent operators can be reduced to 6, since they are expected to be invariant under $SU(2)_L \times U(1)_Y$.

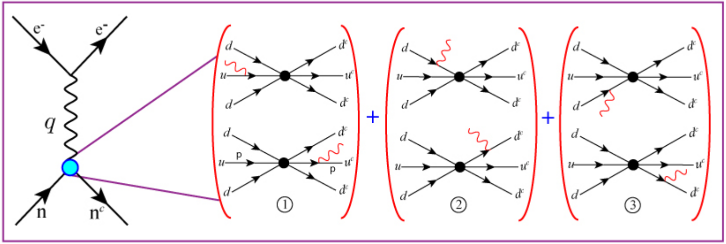
These are

$$\begin{aligned}&(O_1)_{RRR}, \quad (O_2)_{RRR}, \quad (O_3)_{RRR}\\ &2(O_3)_{LRR}, \quad 4(O_3)_{LLR}, \quad 4((O_1)_{LLR} - (O_2)_{LLR}).\end{aligned}$$

The matrix element $\langle \bar{n}|O|n\rangle$ can be calculated in MIT bag model [Rao and Shrock (1982)] or through lattice QCD [M. Buchoff et al (2012)] .

EM dressing

Consider the EM interaction with these quark-level operators:



e.g. consider $(O_1)_{\chi_1\chi_2\chi_3}$: Calculate the amplitude of this process and write down the associated effective operator.

$$
\begin{aligned}
(O^1_{cov})_{\chi_1\chi_2\chi_3} &= j_\mu \frac{\lambda^1_{\chi_1\chi_2\chi_3}}{q^2}\Big[\frac{-4e}{3}\frac{m_u}{p^2-m_u^2}[u^{\top\alpha}_{-\chi_1} C\gamma^\mu u^\beta_{\chi_1}][d^{\top\gamma}_{\chi_2} Cd^\delta_{\chi_2}][d^{\top\rho}_{\chi_3} Cd^\sigma_{\chi_3}] \\
&+\frac{2e}{3}\frac{m_d}{p^2-m_d^2}[u^{\top\alpha}_{\chi_1} Cu^\beta_{\chi_1}][d^{\top\gamma}_{-\chi_2} C\gamma^\mu d^\delta_{\chi_2}][d^{\top\rho}_{\chi_3} Cd^\sigma_{\chi_3}] \\
&+\frac{2e}{3}\frac{m_d}{p^2-m_d^2}[u^{\top\alpha}_{\chi_1} Cu^\beta_{\chi_1}][d^{\top\gamma}_{\chi_2} Cd^\delta_{\chi_2}][d^{\top\rho}_{-\chi_3} C\gamma^\mu d^\sigma_{\chi_3}]\Big](T_s)_{\alpha\beta\gamma\delta\rho\sigma}
\end{aligned}
$$

Matrix elements in MIT bag model

Calculate the matrix elements in the MIT bag model
($m_u = m_d = 0.108$GeV):
Factor out the common factor $N^6 p^{-3}/(4\pi)^2$ and list the matrix elements of $n - \bar{n}$ oscillation and conversion operators below:

Table 1: Matrix element of $n - \bar{n}$ oscillation operators

$\langle O_1 \rangle_{RRR}$	$\langle O_1 \rangle_{LLR}$	$\langle O_1 \rangle_{RLL}$	$\langle O_2 \rangle_{RRR}$	$\langle O_2 \rangle_{LLR}$	$\langle O_2 \rangle_{RLL}$	$\langle O_3 \rangle_{RRR}$	$\langle O_3 \rangle_{LRR}$	$\langle O_3 \rangle_{LLR}$
-5.33	-4.17	-0.666	1.33	1.92	0.167	2.22	-2.72	2.03

The pattern of these matrix elements is consistent with Lattice QCD calculation.

Table 2: Matrix element of $n - \bar{n}$ conversion operators

$\langle \mathcal{O}_1^z \rangle_{RRR}$	$\langle \mathcal{O}_1^z \rangle_{LLR}$	$\langle \mathcal{O}_1^z \rangle_{RLL}$	$\langle \mathcal{O}_2^z \rangle_{RRR}$	$\langle \mathcal{O}_2^z \rangle_{LLR}$	$\langle \mathcal{O}_2^z \rangle_{RLL}$	$\langle \mathcal{O}_3^z \rangle_{RRR}$	$\langle \mathcal{O}_3^z \rangle_{LRR}$	$\langle \mathcal{O}_3^z \rangle_{LLR}$
37.2	32.27	14.04	-8.27	-11.16	-6.82	-10.04	16.59	-12.11

The $n - \bar{n}$ conversion operator

Recall the relations before:

$$\delta = c_{BSM} c_{QCD} \langle \bar{n} | O | n \rangle, \; O = \sum_{i,\chi} \lambda_{m,\chi} (O_m)_\chi$$

[M. Buchoff et al (2012), Rao and Shrock (1982)] .
Assumption: Only keep the one associated with the biggest matrix element, i.e.,

$$c_{BSM} c_{QCD} \lambda^1_{RRR} \frac{N^6 p^{-3}}{(4\pi)^2} \langle O_1 \rangle_{RRR} \approx \delta$$

Similarly for $n - \bar{n}$ conversion:

$$2\alpha j^z = c_{BSM} c_{QCD} \langle \bar{n} | O_{cov} | n \rangle \approx \frac{j^z}{q^2} c_{BSM} c_{QCD} \lambda^1_{RRR} \frac{N^6 p^{-3}}{(4\pi)^2} \frac{e}{3} \frac{m}{p^2 - m^2} \langle \mathcal{O}^z_1 \rangle_{RRR}$$

$$\Rightarrow \alpha = \delta \frac{\mathbf{e}}{\mathbf{6q^2}} \frac{\mathbf{m}}{\mathbf{p^2 - m^2}} \frac{\langle \mathcal{O}^z_1 \rangle_{\mathbf{RRR}}}{\langle \mathbf{O_1} \rangle_{\mathbf{RRR}}}.$$

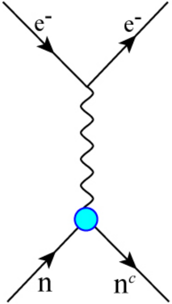
- **The mass scale of the suppression needs not come from BSM theory.**
- **If $m = 0$, $n - \bar{n}$ oscillation can be non-zero, but no $n - \bar{n}$ conversion.**

$n - \bar{n}$ conversion and scattering experiments

We can consider process, such as $e + n \to \bar{n} + e$,
or more practically

- $e + {}^3He \to e + \bar{n} + X(n, p)$,
- $n + {}^1H \to \bar{n} + p + X(e)$,

where X is an unspecified final state.



Summary and Outlook

- We argued that spin effect can be important to $n - \bar{n}$ transition process and considered the $n - \bar{n}$ conversion operators.
- We find that there exist phase constraints associated with discrete symmetry transformations of fermions with $B - L$ violation.
- Due to the connection between $n - \bar{n}$ oscillation and $n - \bar{n}$ conversion, we can determine the low energy "constant" of this operator through EM interaction and find that the additional mass scale of suppression needs not come from BSM physics.
- This operator offers us an opportunity to realize $n - \bar{n}$ transition through scattering experiments.

Backup slides

Majorana phase constraints

The plane-wave expansion of a general Majorana field ψ_m is

$$\psi_m(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_s \left\{ f(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + \lambda f^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\} .$$

where λ is the creation phase factor and can be chosen arbitrarily. Now applying C transformation and Majorana condition,

$$i\gamma^2\psi_m^*(x) = \lambda^*\psi_m(x),$$

yields

$$\mathbf{C}\psi_m(x)\mathbf{C}^{-1} \;=\; \eta_c\lambda^*\psi_m(x),$$

i.e. $\mathbf{C}f(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f(\mathbf{p},s)$ and $\mathbf{C}f^\dagger(\mathbf{p},s)\mathbf{C}^{-1} = \eta_c\lambda^* f^\dagger(\mathbf{p},s)$.
Since C is a unitary operator, Hermitian conjugate shows $\eta_c^*\lambda$ is real.

Majorana phase constraints

Under **CP**, we find $\eta_p^* \eta_c^* \lambda$ must be imaginary, or η_p^* must be imaginary.
Under **T**, we have $\eta_t \lambda$ must be real.
Under **CPT**, we have

$$\mathbf{CPT}\psi_m(x)(\mathbf{CPT})^{-1} = -\eta_c \eta_p \eta_t \gamma^5 \psi_m^*(-x)\,,$$

or

$$\begin{aligned} \mathbf{CPT} f(\mathbf{p}, s)(\mathbf{CPT})^{-1} &= s\lambda^* \eta_c \eta_p \eta_t f(\mathbf{p}, -s)\,, \\ \mathbf{CPT} f^\dagger(\mathbf{p}, s)(\mathbf{CPT})^{-1} &= -s\lambda \eta_c \eta_p \eta_t f^\dagger(\mathbf{p}, -s)\,. \end{aligned}$$

Notice **CPT** is **antiunitary** and define $\mathbf{CPT} = KU_{cpt}$, where U_{cpt} denotes a unitarity operator. We find $\eta_c \eta_p \eta_t$ is pure imaginary!.

- **C**: $\eta_c^* \lambda$ is real;
- **CP**: $\eta_p^* \eta_c^* \lambda$ is imaginary or η_p^* is imaginary;
- **T**: $\eta_t \lambda$ is real;
- **CPT**: $\eta_c \eta_p \eta_t$ is imaginary. $\Rightarrow \eta_c \eta_t$ is real.

Notice order does not matter and no constraint for $\eta_c \eta_p$.

Phase constraints for Dirac field in B-L violation theories

The plane-wave expansion of a Dirac field $\psi(x)$ is given by

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \left\{ b(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} + d^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\}$$

Construct a Majorana field from Dirac fields:

$$\psi_{m\pm}(x) = \frac{1}{\sqrt{2}}(\psi(x) \pm \mathbf{C}\psi(x)\mathbf{C}^\dagger)$$

then plane-wave expansion is

$$\psi_{m\pm}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s} \left\{ w_\pm(\mathbf{p},s)u(\mathbf{p},s)e^{-ip\cdot x} \pm \eta_c w_\pm^\dagger(\mathbf{p},s)v(\mathbf{p},s)e^{ip\cdot x} \right\}.$$

where $w_{m\pm}(\mathbf{p},s) \equiv \frac{1}{\sqrt{2}}[b(\mathbf{p},s) \pm \eta_c d(\mathbf{p},s)]$ and $\lambda = \pm\eta_c$. We find the **same** phase constraints for Dirac fields as Majorana fields.

Implications of the CPT phases

4×4 effective Hamiltonian framework

Work in the basis $|n(\mathbf{p},+)\rangle, |\bar{n}(\mathbf{p},+)\rangle, n(\mathbf{p},-)\rangle, |\bar{n}(\mathbf{p},-)\rangle$.

[SG and Jafari (2015)]

Spin-dependent SM effects involving transverse magnetic fields could realize $n-\bar{n}$ transitions in which the particle spin flips without magnetic quenching.

However, it is sensitive to the CPT phase constraint.

Consider $n-\bar{n}$ oscillate in a static $\mathbf{B}_0$ with $\omega_0 \equiv -\mu_n B_0$. Apply a static $\mathbf{B}_1$ suddenly at $t=0$ and define $\omega_1 \equiv -\mu_n B_1$. The Hamiltonian matrix at $t>0$ is

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 \\ \omega_1 & 0 & M-\omega_0 & -\delta\eta_{cpt}^2 \\ 0 & -\omega_1 & -\delta\eta_{cpt}^2 & M+\omega_0 \end{pmatrix},$$

where δ denotes a $n(+)\to\bar{n}(+)$ transition matrix element.

B-L violation and theories of self-conjugate fermions

In 1967, in attempting to rationalize the spectral pattern of the low-lying, light hadrons, Carruthers discovered a class of theories for which the CPT theorem does not hold. [Carruthers, 1967]

The pions form a self-conjugate isospin multiplet (π^+, π^0, π^-), but the kaons form pair-conjugate multiplets (K^+, K^0) and $(\bar{K}^0, K^-)$.

Carruthers discovered that free theories of self-conjugate bosons with half-integer isospin are nonlocal, that the commutator of two self-conjugate fields with opposite isospin components do not vanish at space-like separations. [Carruthers, 1967]

Same conclusion for theories of arbitrary spin. [Lee, 1967; Fleming and Kazes, 1967; Jin, 1967]

B-L violation and theories of self-conjugate fermions

Failure of weak local communitivity $\Rightarrow$ CPT symmetry is not expected to hold, nor should the CPT theorem of Greenberg apply.
[Carruthers, 1968; Streater and Wightman, 2000; Greenberg, 2002]

The conclusion here is it is possible to have self-conjugate theories of isospin $\mathrm{I} = 0$, but it is not possible to have self-conjugate theories of $\mathrm{I} = 1/2$.
Note neutron and antineutron are members of pair-conjugate $\mathrm{I} = 1/2$ multiplets. In addition, the quark-level operators that generate $n - \bar{n}$ oscillations [Rao & Shrock, 1982] would also produce $p - \bar{p}$ oscillations under the isospin transformation $u \leftrightarrow d$.

Therefore, to study $n - \bar{n}$ oscillations in QCD, Isospin symmetry must be broken.

MIT bag model VS Lattice QCD

Table 2: Preliminarly results for matrix elements of 6-quark operators 2.1 and comparison to the MIT Bag Model results [8]. The first line shows matrix elements for $I = 3_{R,L}$ operators vanishing identically.

	$Z(\text{lat} \to \overline{MS})$	$\mathcal{O}^{\overline{MS}(2\,\text{GeV})}[10^{-5}\text{GeV}^6]$	Bag "A"	$\frac{\text{LQCD}}{\text{Bag "A"}}$	Bag "B"	$\frac{\text{LQCD}}{\text{Bag "B"}}$
$[(RRR)_3]$	0.62(12)	0	0	–	0	–
$[(RRR)_1]$	0.454(33)	45.4(5.6)	8.190	5.5	6.660	6.8
$[R_1(LL)_0]$	0.435(26)	44.0(4.1)	7.230	6.1	6.090	7.2
$[(RR)_1L_0]$	0.396(31)	-66.6(7.7)	-9.540	7.0	-8.160	8.1
$[(RR)_2L_1]^{(1)}$	0.537(52)	-2.12(26)	1.260	-1.7	-0.666	3.2
$[(RR)_2L_1]^{(2)}$	0.537(52)	0.531(64)	-0.314	-1.7	0.167	3.2
$[(RR)_2L_1]^{(3)}$	0.537(52)	-1.06(13)	0.630	-1.7	-0.330	3.2

[Syritsyn, Buchoff, Schroeder and Wasem, (2015)]

MIT bag model VS Lattice QCD

Table 1: Classification of $N-\bar{N}$ transition operators according to $SU(3)_{L,R}$ flavor symmetry. The other seven operators are obtained by replacing $L \leftrightarrow R$. The operators are built from left/right diquarks denoted as L, R, respectively, in the 1st column. Notation $(\ldots)_{\mathbf{I}}$ denotes projection on representation with total isospin I. The 2nd column shows corresponding operators in terms of Eq.(2.1). The $SU(2)_{L,R}$ representation is shown in the 3rd column. The 1-loop anomalous dimension is given in the 4th column.

	$\mathcal{O}^{6q}$	$\mathbf{I}_R \otimes \mathbf{I}_L$	$\gamma^{\mathcal{O}}$
$[(RRR)_\mathbf{3}]$	$\mathcal{O}^1_{R(RR)} + 4\mathcal{O}^2_{(RR)R}$	$\mathbf{3}_R \otimes \mathbf{0}_L$	$(\alpha_S/4\pi)(-12)$
$[(RRR)_\mathbf{1}]$	$\mathcal{O}^2_{(RR)R} - \mathcal{O}^1_{R(RR)} \equiv 3\mathcal{O}^3_{(RR)R}$	$\mathbf{1}_R \otimes \mathbf{0}_L$	$(\alpha_S/4\pi)(-2)$
$[R_\mathbf{1}(LL)_\mathbf{0}]$	$\mathcal{O}^2_{(LL)R} - \mathcal{O}^1_{L(LR)} \equiv 3\mathcal{O}^3_{(LL)R}$	$\mathbf{1}_R \otimes \mathbf{0}_L$	0
$[(RR)_\mathbf{1}L_\mathbf{0}]$	$3\mathcal{O}^3_{(LR)R}$	$\mathbf{1}_R \otimes \mathbf{0}_L$	$(\alpha_S/4\pi)(+2)$
$[(RR)_\mathbf{2}L_\mathbf{1}]_{(1)}$	$\mathcal{O}^1_{L(RR)}$	$\mathbf{2}_R \otimes \mathbf{1}_L$	$(\alpha_S/4\pi)(-6)$
$[(RR)_\mathbf{2}L_\mathbf{1}]_{(2)}$	$\mathcal{O}^2_{(LR)R}$	$\mathbf{2}_R \otimes \mathbf{1}_L$	$(\alpha_S/4\pi)(-6)$
$[(RR)_\mathbf{2}L_\mathbf{1}]_{(3)}$	$\mathcal{O}^1_{R(LR)} + 2\mathcal{O}^2_{(RR)L}$	$\mathbf{2}_R \otimes \mathbf{1}_L$	$(\alpha_S/4\pi)(-6)$

[Syritsyn, Buchoff, Schroeder and Wasem, (2015)]

CP transformation properties

The CP transformations of non-vanishing operators are:

$$\begin{aligned} \mathcal{O}_1 &= \psi^T C \psi + \mathrm{h.c.} && \stackrel{\mathbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2 \,, \\ \mathcal{O}_2 &= \psi^T C \gamma_5 \psi + \mathrm{h.c.} && \stackrel{\mathbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2 \,, \\ \mathcal{O}_4 &= \psi^T C \gamma^\mu \gamma_5 \psi \, \partial^\nu F_{\mu\nu} + \mathrm{h.c.} && \stackrel{\mathbf{CP}}{\Longrightarrow} -(\eta_c \eta_p)^2 \end{aligned}$$

Even with earlier determined phase constraint that $\eta_p^2 = -1$, CP transformation properties of the operators are not definite and only depend on η_c^2.

CP violation in $n - \bar{n}$ oscillations

$$H = \begin{pmatrix} m_n - \frac{i}{2}\Gamma_n & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m_n - \frac{i}{2}\Gamma_n \end{pmatrix},$$

$$\frac{P_{|n\rangle \to |\bar{n}\rangle}}{P_{|\bar{n}\rangle \to |n\rangle}} - 1 \simeq \frac{2|\Gamma_{12}|}{|M_{12}|} \sin\beta,$$

[McKeen and Nelson, 2015]