

Four dimensional $\mathcal{N} = 3$ field theories




Diego Regalado

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Based on

[1512.06434] and [1611.05769] with I. García-Etxebarria

$\mathcal{N} = 3$ theories in four dimensions

- Two (CPT invariant) multiplets with $s \leq 2$ 
 - Vector
 - Gravity
- Let's focus on non-gravitational theories.
- We can try to construct an $\mathcal{N} = 3$ gauge theory. However, the resulting (renormalizable) Lagrangian has $\mathcal{N} = 4$ supersymmetry.

[e.g. Weinberg's book]

- This is why $\mathcal{N} = 3$ theories were thought not to exist.
- There is a **loophole**: we are assuming the existence of a Lagrangian!
- Four dimensional $\mathcal{N} = 3$ **non-Lagrangian** theories are not ruled out.

[Aharony, Evtikhiev '15; García-Etxebarria, DR '15]

Outline

- First examples of non-Lagrangian $\mathcal{N} = 3$ theories (type A).
 - Field theory construction.
 - Simple embedding in string theory.
 - Holographic dual.
- Exceptional $\mathcal{N} = 3$ theories.
 - Step 1: Class \mathcal{S} construction (theories of type A).
 - Step 2 : Six dimensional (2,0) E-type theories in M-theory.
 - Step 3 : Combine both things.

Field theory construction

Four dimensional $\mathcal{N} = 4$ SYM

- Consider $\mathcal{N} = 4$ SYM with gauge group $U(N)$ and coupling constant τ .
 - It has an **R-symmetry** group $SO(6)_R$.
 - **Montonen-Olive** self-duality:

$$\tau \rightarrow \tau' = \frac{a\tau + b}{c\tau + d} \quad \text{for} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

The duality group is **not a symmetry** of the theory!

A theory T_τ is defined by specifying a particular τ , then: $T_\tau \xrightarrow{SL(2, \mathbb{Z})} T_{\tau'}$

However, if for $\Gamma \in SL(2, \mathbb{Z})$, $\Gamma(\tau^*) = \tau^*$, then Γ is a symmetry of T_{τ^*} ,

$$T_{\tau^*} \xrightarrow{\Gamma} T_{\tau^*}$$

$\mathcal{N} = 3$ from $\mathcal{N} = 4$ SYM

- Enhanced discrete symmetry group $\Gamma \subset SL(2, \mathbb{Z})$ when $\Gamma(\tau^*) = \tau^*$.

- For arbitrary τ , $\Gamma = \mathbb{Z}_2$.
- For $\tau = i$, $\Gamma = \mathbb{Z}_4$.
- For $\tau = e^{i\pi/3}$, $\Gamma = \mathbb{Z}_6 = \mathbb{Z}_2 \times \mathbb{Z}_3$.

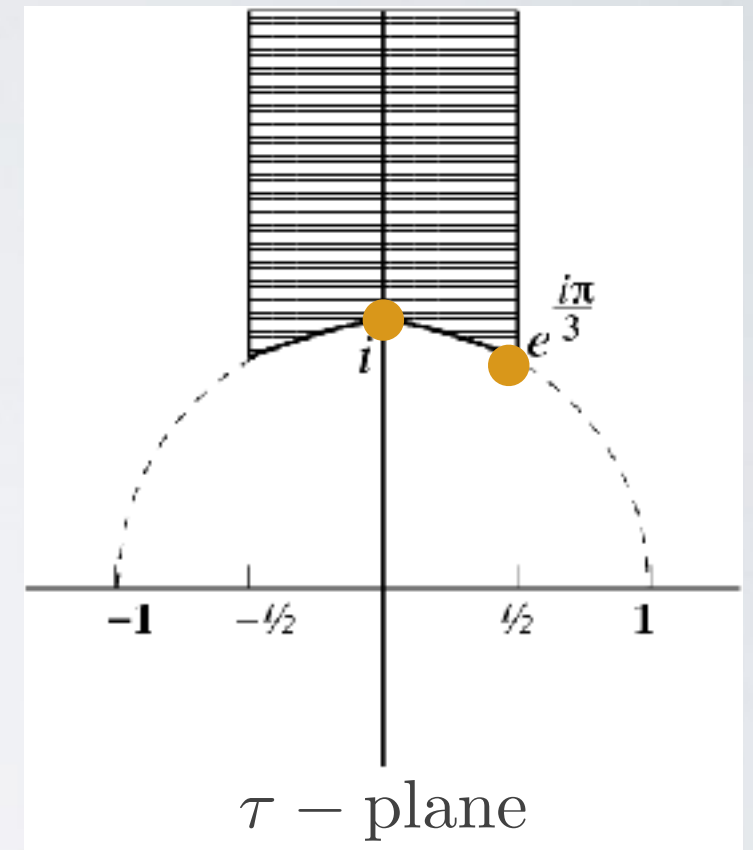
- Given this \mathbb{Z}_k symmetry, we can quotient by it.

This breaks supersymmetry: $Q_a \rightarrow e^{i\pi/k} Q_a$

- We can combine it with $\mathbb{Z}_k \subset SO(6)_R$ such that

$$Q_A \rightarrow e^{-i\pi/k} Q_A \quad (A = 1, 2, 3) \quad Q_4 \rightarrow e^{3i\pi/k} Q_4$$

- Under $\mathbb{Z}_k = \mathbb{Z}_k \cdot \mathbb{Z}_k$: $Q_A \rightarrow Q_A \quad Q_4 \rightarrow e^{4i\pi/k} Q_4$



For $k = 2$ we have $\mathcal{N} = 4$ supersymmetry.

For $k = 3, 4, 6$ only $\mathcal{N} = 3$. The theory is stuck at **strong coupling**.

[cf. Aharony, Evtikhiev '15]

String theory embedding

$\mathcal{N} = 4$ from M2 branes

- Let's start with N M2 branes on $\underline{\mathbb{R}^{1,2}} \times \mathbb{C}^3 \times T^2$.

The theory living on the M2's (IR): 3D ABJM at level $k = 1$.

[Aharony, Bergman, Jafferis, Maldacena '08]

- To get a 4D theory, we take the F-theory limit: $T^2 \rightarrow 0$.
 - Reduce along one circle: $\text{M2} \longrightarrow \text{D2 on } \underline{\mathbb{R}^{1,2}} \times \mathbb{C}^3 \times S^1$
 - T-dualize along the other: $\text{D2} \longrightarrow \text{D3 on } \underline{\mathbb{R}^{1,2}} \times \tilde{S}^1 \times \mathbb{C}^3$
 - When $T^2 \rightarrow 0$, we find D3's on $\underline{\mathbb{R}^{1,3}} \times \mathbb{C}^3$.
- The theory on the D3's is $\mathcal{N} = 4$ SYM with gauge group $U(N)$ and coupling constant $\tau = \text{complex structure of } T^2$.
 - Rotations on \mathbb{C}^3 : R-symmetry $SO(6)_R$.
 - Large diffeomorphisms on T^2 : duality $SL(2, \mathbb{Z})$.
- Both $SO(6)_R$ and $SL(2, \mathbb{Z})$ are **geometric** in this picture.

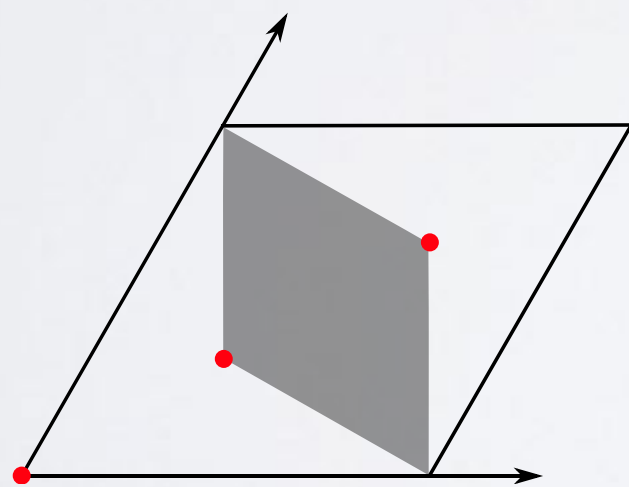
$\mathcal{N} = 3$ from M2 branes

- Field theory argument: quotient $\mathcal{N} = 4$ by $\mathbb{Z}_k = \mathbb{Z}_k^{\text{blue}} \cdot \mathbb{Z}_k^{\text{red}}$.
- Consider now N M2 branes on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$.

$$(z_1, z_2, z_3, u) \rightarrow (\zeta_k z_1, \bar{\zeta}_k z_2, \zeta_k z_3, \bar{\zeta}_k u) \quad \zeta_k = e^{2\pi i/k}$$

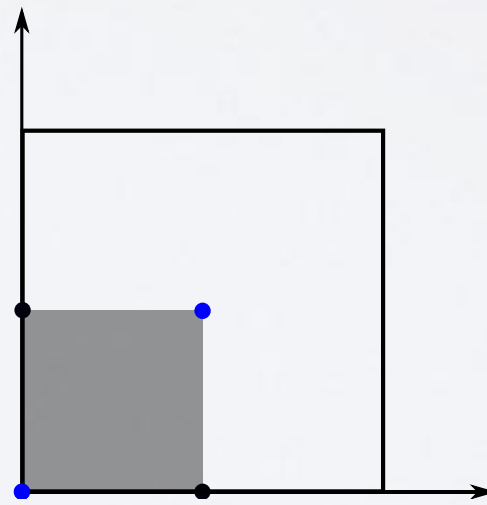
- The action is only well defined for certain values of τ and k .
- Preserves the expected amount of supersymmetry.
- Related to ABJM at level k .

T^2/\mathbb{Z}_k :



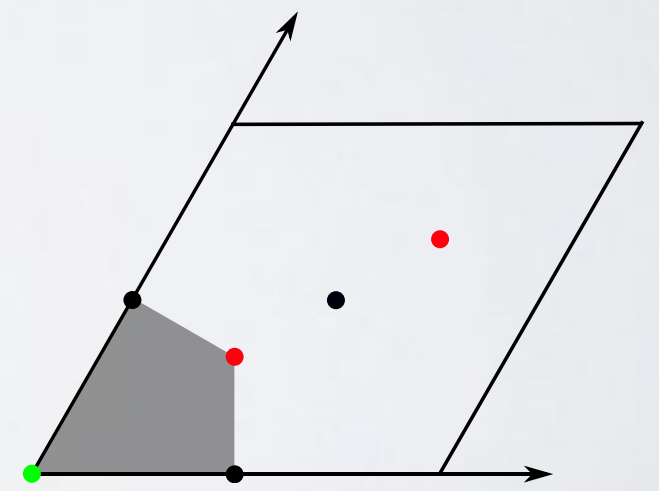
$$k = 3$$

$$\tau = e^{i\pi/3}$$



$$k = 4$$

$$\tau = i$$



$$k = 6$$

$$\tau = e^{i\pi/3}$$

Generalized orientifold planes

- For $k = 2$, we have that after taking the F-theory limit: [Hanany, Kol '00]

$$\begin{array}{ccc} \text{M-theory on} & \xrightarrow{T^2 \rightarrow 0} & \text{Type IIB on} \\ \mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2) / \mathbb{Z}_2 & & \mathbb{R}^{1,3} \times \mathbb{C}^3 / \mathcal{I} \cdot (-1)^{F_L} \cdot \Omega \\ & & \text{(orientifold O3)} \end{array}$$

- \mathcal{I} acts on \mathbb{C}^3 by $z^i \rightarrow -z^i$.
- $(-1)^{F_L} \cdot \Omega$ corresponds to $\mathbb{Z}_2 \subset SL(2, \mathbb{Z})$.
- The theory on the probe D3 branes is 4D $\mathcal{N} = 4$ SYM $\mathfrak{so}(N) / \mathfrak{usp}(N)$.
- There are different variants: $O3^-, O3^+, \widetilde{O3}^-, \widetilde{O3}^+$.
- For $k = 3, 4, 6$, we find **generalized orientifold** 3-planes (S-folds).
 - They preserve 12 supercharges.
 - Only exist at **strong coupling**. (No worldsheet description)
 - Come in different variants. [García-Etxebarria, DR '15 ; Aharony, Tachikawa '16]

Holographic dual

- The holographic dual can be obtained from the IIB construction.
- Before introducing the S-fold, we have $N \gg 1$ D3 branes, whose near horizon limit is $AdS_5 \times S^5$.
- The geometric action \mathbb{Z}_k gives IIB on $AdS_5 \times S^5/\mathbb{Z}_k$ (which is not supersymmetric by itself).
- There is also an $SL(2, \mathbb{Z})$ bundle on S^5/\mathbb{Z}_k (now supersymmetric).
 \uparrow
 \mathbb{Z}_k
- The near horizon geometry in F-theory is $AdS_5 \times (S^5 \times T^2)/\mathbb{Z}_k$.
- For $k = 2$, [Witten '98].
- For $k = 3, 4, 6$:
 - Smooth, weakly curved geometry.
 - Stuck at strong string coupling. No marginal deformation in the CFT.

[Ferrara, Porrati, Zaffaroni '98 ; García-Etxebarria, DR '15]

Outline

- First examples of non-Lagrangian $\mathcal{N} = 3$ theories (type A) .
 - Field theory construction.
 - Simple embedding in string theory.
 - Holographic dual.
- Exceptional $\mathcal{N} = 3$ theories from non-geometry (type E).
 - Class \mathcal{S} construction.
 - Six dimensional (2,0) E-type theories in M-theory.

Towards exceptional $\mathcal{N} = 3$ theories

- This class of $\mathcal{N} = 3$ theories comes from taking a \mathbb{Z}_k quotient of $\mathcal{N} = 4$ SYM with gauge group $U(N)$.
 - R-symmetry group $SO(6)_R$.
 - Montonen-Olive self-duality.
- We do not really need to have $U(N)$, any ADE gauge group works.
(R-symmetry + self-duality).
- However, the stringy construction we have does not seem to work for the exceptional cases.
- Alternative approach
 - Class \mathcal{S} (for Type A)
 - E-type 6D (2,0) theories from M-theory
 - Combine both things

Step 1: Class \mathcal{S} construction
(previous case)

4D $\mathcal{N} = 3$ from 6D (2,0)?

- 6D (2,0) theory. [Witten '95; ...]
 - Type IIB on \mathbb{C}^2/Γ , with Γ a finite subgroup of $SU(2) \longrightarrow$ ADE.
 - R-symmetry group $SO(5)_R$.
 - When compactified on T^2 : 4D $\mathcal{N} = 4$ SYM.
 - More generally, compactification on a Riemann surface leads to a large class of $\mathcal{N} = 2$ field theories (class \mathcal{S}).
- Is there a way to get the $\mathcal{N} = 3$ theories from the 6D (2,0) theory?
 - Naively, start with the (2,0) on T^2 and try to quotient by $\mathbb{Z}_k = \mathbb{Z}_k \cdot \mathbb{Z}_k$.
 - However, $\mathbb{Z}_k \in SO(6)_R$ and $\mathbb{Z}_k \notin SO(5)_R$.
 - $SO(5)_R$ enhances to $SO(6)_R$ only in the deep IR.
 - We need to make \mathbb{Z}_k manifest in the UV.

From D3 branes to M5 branes

- The 6D (2,0) theory of type A_{N-1} is the worldvolume theory of a stack of N M5-branes.
- Since we have a construction in terms of D3 branes, let us dualize to a system of M5 branes.
- Consider N D3 branes on $\underline{\mathbb{R}^{1,3}} \times \tilde{S}_T^1 \times S_E^1 \times \mathbb{C}^2$.
 - T-dualize along $\tilde{S}_T^1 \longrightarrow$ D4 branes on $\underline{\mathbb{R}^{1,3}} \times S_T^1 \times S_E^1 \times \mathbb{C}^2$.
 - M-theory lift: M5 branes on $\underline{\mathbb{R}^{1,3}} \times S_T^1 \times S_M^1 \times S_E^1 \times \mathbb{C}^2$.
- For generic $\tilde{\tau}_E$ of $\tilde{T}_E^2 = \tilde{S}_T^1 \times S_E^1$, $SO(6)_R$ is broken to $SO(4)$. However, for special $\tilde{\tau}_E$, we have that $\mathbb{Z}_k \in SO(6)_R$ survives.
- The key is to consider M5 branes on $\underline{\mathbb{R}^{1,3}} \times T^2 \times S^1 \times \mathbb{C}^2$ instead of taking $\underline{\mathbb{R}^{1,3}} \times T^2 \times \mathbb{R}^5$.

M-theory on T^3

- 11D supergravity on T^3 : 8D maximal supergravity.
 - U-duality group $SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$. [Hull, Townsend '94]
- $SL(2, \mathbb{Z})$ is an electric-magnetic duality in 8D. [Aharony '96]
 - $d\hat{C}_3 \rightarrow \star_{8d} d\hat{C}_3$ where \hat{C}_3 has all the legs in 8D.
 - It maps wrapped M5 branes to unwrapped M2 branes.
 - It acts on the moduli as $\rho \rightarrow \frac{a\rho + b}{c\rho + d}$, with $\rho = \int_{T^3} C_3 + i \text{Vol}(T^3)$.
- A different perspective, Type IIA on T^2 .
 - T-duality group $O(2, 2; \mathbb{Z}) = (SL(2, \mathbb{Z})_\tau \times SL(2, \mathbb{Z})_\rho) \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$.
 - $SL(2, \mathbb{Z})$ is the M-theory lift of $SL(2, \mathbb{Z})_\rho$ (non-geometry).

From D3 branes to M5 branes (again)

- Consider N D3 branes on $\underline{\mathbb{R}^{1,3}} \times \tilde{S}_T^1 \times S_E^1 \times \mathbb{C}^2$.
 - T-duality group $O(2, 2; \mathbb{Z}) = (SL(2, \mathbb{Z})_{\tilde{\tau}_E} \times SL(2, \mathbb{Z})_{\tilde{\rho}_E}) \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$.
 - For special values of $\tilde{\tau}_E$, $\mathbb{Z}_k \in SL(2, \mathbb{Z})_{\tilde{\tau}_E}$ becomes a symmetry.
- T-dualize along $\tilde{S}_T^1 \longrightarrow$ D4 branes on $\underline{\mathbb{R}^{1,3}} \times S_T^1 \times S_E^1 \times \mathbb{C}^2$.
 - This acts as $\tilde{\tau}_E \rightarrow \rho_E$. For special ρ_E , $\mathbb{Z}_k \in SL(2, \mathbb{Z})_{\rho_E}$ symmetry.
 - **Non-geometric**: momentum \leftrightarrow winding (closed string sector).
 - Position on $S_E^1 \leftrightarrow$ Wilson line along S_T^1 (open string sector).
- M-theory lift: M5 branes on $\underline{\mathbb{R}^{1,3}} \times S_T^1 \times S_M^1 \times S_E^1 \times \mathbb{C}^2$.

$$\rho_E = \int_{T_E^2} B + i \text{Vol}(T_E^2) \quad \rightarrow \quad \rho = \int_{T^3} C + i \text{Vol}(T^3)$$

- Position on $S_E^1 \leftrightarrow$ Holonomy of self-dual 2-form over $S_T^1 \times S_M^1$.

M5 branes probing a U-fold

- 4D $\mathcal{N} = 3$ theories on the worldvolume of N M5 branes on

$$\underline{\mathbb{R}^{1,3} \times (S_T^1 \times S_M^1 \times S_E^1 \times \mathbb{C}^2) / \mathbb{Z}_k}$$

with $\mathbb{Z}_k = \mathbb{Z}_k \cdot \mathbb{Z}_k$.

- R-symmetry: $\mathbb{Z}_k = \mathbb{Z}_k^R \cdot \tilde{\mathbb{Z}}_k^R$ $\mathbb{Z}_k^R \in SO(4)$
 $\tilde{\mathbb{Z}}_k^R \in SL(2, \mathbb{Z})_\rho$ (condition on ρ)
- S-duality: $\mathbb{Z}_k \in SL(2, \mathbb{Z}) \subset SL(3, \mathbb{Z})$ (condition on τ)
- We know how the supercharges transform under $SL(2, \mathbb{Z})_\rho \times SL(3, \mathbb{Z})$.
 We recover $\mathcal{N} = 3$ (for $k = 3, 4, 6$) independently.

- Example: $k = 4$. $\rho = i \implies R_T R_M R_E = 1$
 $\tau = i \implies R_T = R_M$

UV \longrightarrow IR when $R \rightarrow 0$: $R_M = R, \quad R_T = R, \quad R_E = R^{-2}$

Relation to class \mathcal{S} theories

[Gaiotto '09]

- Typically, a theory of class \mathcal{S} is constructed by taking the 6D (2,0) theory and compactifying it on a Riemann surface Σ .
- In order to preserve supersymmetry, a topological twist is needed. (Include an R-symmetry bundle on the Riemann surface.)
 - When $\Sigma = T^2$, we find $\mathcal{N} = 4$ in four dimensions.
 - Otherwise, only $\mathcal{N} = 2$ survives.
- This can be interpreted as taking a stack of M5 branes wrapped on a cycle Σ in a Calabi Yau.
- In order to get a four dimensional theory with $\mathcal{N} = 3$ supersymmetry in this way, we have to consider a stack of M5 branes inside a **U-fold**.

Step 2: E-type (2,0) in 6D from M-theory

String construction of 6D (2,0)

- First as Type IIB on \mathbb{C}^2/Γ , with Γ a finite subgroup of $SU(2)$. [Witten '95]
This leads to an ADE classification.
- The theory of type A_{N-1} is also the worldvolume theory of a stack of N M5 branes.
- How can we relate the two descriptions (for type A_{N-1} , $\Gamma = \mathbb{Z}_N$)?
 - Instead of Type IIB on $\mathbb{C}^2/\mathbb{Z}_N$, consider an **elliptic fibration** over \mathbb{C} with the same local singularity.
 - As we go around the origin in \mathbb{C} , we act on the complex structure of the fibre T^2

$$\begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{Z})_\tau \quad \tau \rightarrow \tau + N$$

$$O(2, 2; \mathbb{Z}) = (SL(2, \mathbb{Z})_\tau \times SL(2, \mathbb{Z})_\rho) \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

String construction of 6D (2,0)

- Type IIB on an elliptic fibration over \mathbb{C} with monodromy

$$\begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{Z})_\tau \quad \tau \rightarrow \tau + N$$

$$O(2, 2; \mathbb{Z}) = (SL(2, \mathbb{Z})_\tau \times SL(2, \mathbb{Z})_\rho) \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

- T-duality along one of the circles: Type IIA on a T^2 -fibration acting on the parameter ρ (recall that under T-duality $\tau \rightarrow \rho$).

$$\rho = \int_{T^2} B + i \text{Vol}(T^2) \quad \rho \rightarrow \rho + N \quad \textbf{NS5 branes}$$

- The M-theory lift is a T^3 -fibration with monodromy on ρ_M .

$$\rho_M = \int_{T^3} C + i \text{Vol}(T^3) \quad \rho_M \rightarrow \rho_M + N \quad SL(2, \mathbb{Z})_{\rho_M} \times SL(3, \mathbb{Z})$$

$$\text{Type IIB on } \mathbb{C}^2/\mathbb{Z}_N \longrightarrow \text{M-theory with } N \text{ M5 branes}$$

Exceptional (2,0) from M-theory

- What about the other cases? Namely, Type IIB on \mathbb{C}^2/Γ , for all Γ . We can simply repeat the same chain of dualities.
- Consider a singular elliptic fibration which is locally the same as \mathbb{C}^2/Γ .
 - T-duality along one of the circles.
 - M-theory lift.
- M-theory on a singular T^3 -fibration with monodromy acting on ρ_M .

$$\rho_M = \int_{T^3} C + i \text{Vol}(T^3)$$

- Example: E_7 . Monodromy = $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\rho_M \rightarrow -\frac{1}{\rho_M}$

In this case, the monodromy is acting on the **volume**.

Genuinely **non-geometric** (T-fold or U-fold).

Step 3: Combine both constructions

Exceptional $\mathcal{N} = 3$ theories

- We have an M-theory construction of:

- $\mathcal{N} = 3$ quotient.
- E-type (2,0) theories.

Exceptional $\mathcal{N} = 3$ theories

- M-theory on T^5 : U-duality group $O(5, 5; \mathbb{Z})$.

$$\mathbb{R}^{1,3} \times (\mathbb{C} \times T^5) / (\mathbb{Z}_k^{\mathcal{N}=3} \times \mathbb{Z}_p^E)$$

- Tuning the moduli of T^5 , $\mathbb{Z}_k^{\mathcal{N}=3} \times \mathbb{Z}_p^E \subset O(5, 5; \mathbb{Z})$ becomes a symmetry.
- $\mathbb{Z}_k^{\mathcal{N}=3}$ acts on ρ of $T^3 \subset T^5$.
- \mathbb{Z}_p^E acts on $\tilde{\rho}$ of a different $\tilde{T}^3 \subset T^5$.
- We can check that only 12 supercharges survive the quotient.
- It seems to be intrinsically **non-geometric**.

Summary and outlook

- We have built the first examples of $\mathcal{N} = 3$ field theories in 4D as quotients of $\mathcal{N} = 4$ SYM by particular $SO(6)_R$ and $SL(2, \mathbb{Z})$ symmetries.
- Only works for specific values of the coupling. Isolated field theories.
- The worldvolume theory of D3's probing generalized orientifolds.
- Can be thought of as the 4D version of ABJM (only for some k).
- Large N limit as a quotient of $AdS_5 \times S^5$ acting on the IIB coupling.
- Also as M5 branes probing a U-fold.
- Generalization of class \mathcal{S} . Extra discrete symmetry.
 - Can we use AGT to understand these theories?
- M-theory realization of exceptional (2,0) theories. Non-geometry.
- Exceptional $\mathcal{N} = 3$ theories from non-geometric singularities.
 - Seem to be intrinsically non-geometric.

Thank you!