♢ Tops and G_2 mirror symmetry ♢

based on

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- [1701.05202]
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$\Diamond$

1) Mirror Symmetry for Calabi-Yau and G_2 manifolds

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a few words about Calabi-Yau ...

- Mirror symmetry is a feature of compactification of type II strings.
- On Calabi-Yau threefolds, the CFTs associated to IIA and IIB become isomorphic after reversing the left-moving $U(1)$ charge for a pair of appropriatey chosen manifolds

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... this duality has amazing implications ...
[Candelas, de la Ossa, Green, Parkes; ....]

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  for type IIB on $X^\vee$, these are mapped to $D3$-branes wrapped on sLag 3-cycles L, $\dim_{\mathbb{R}}$ of moduli space is $2b_1(L) \stackrel{!}{=} 6$, corresponding to three deformations and three Wilson lines.

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- $\exists$ closed forms Φ_3 and $\Psi_4 = *\Phi_3$ (analogue of Kähler form and $\Omega^{3,0}$); they calibrate 3 and 4-cycles; non-trivial Betti numbers are $b_2 = b_5$ and $b_3 = b_4$.

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- In this case, there are similar automorphisms of the (extended) superconformal algebra, and the CFT can only detect $b_2 + b_3$. [Shatashvili, Vafa; O'Farrill; Becker, Becker, Morrison, Ooguri, Oz, Yin; Roiban, Romelsberger, Walcher; Gaberdiel, Kaste; Papadopoulos, Townsend]

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- Arguments similar to SYZ imply coassociative T^4 fibration for G_2 manifolds. [Acharya]

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This has been studied in detail only for $T^7/(\mathbb{Z}_2)^3$ orbifolds [Joyce] and their smoothings Y_l, $l = 0..8$ [Gaberdiel, Kaste]. Here

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- There are four possible choices of T-dualities $\mathcal{I}_3^{\pm}$ and $\mathcal{I}_4^{\pm}$ corresponding to automorphisms of the extended chiral algebra.

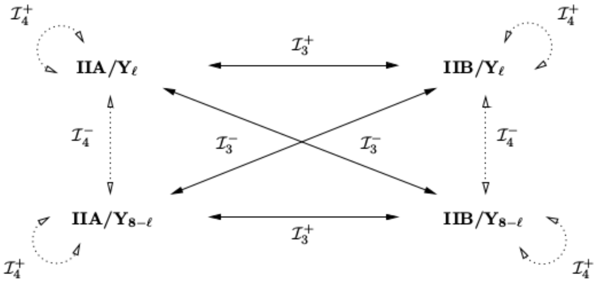

Figure taken from [Gaberdiel, Kaste]

How to construct the mirror: CY manifolds

Classic construction of CY mirror manifolds (besides orbifolds) works for hypersurfaces in weighted projective space [related to superconformal minimal models]:

$$P_{\sum k_i}(x_i) = 0 \quad \text{in} \quad \mathbb{P}_{k_1,\cdots,k_n} \, .$$

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$$X : \qquad x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 - 5\psi x_1 x_2 x_3 x_4 x_5 = 0$$

has a mirror obtained by modding out (and resolving) a $(\mathbb{Z}_5)^3$ acting with weights

$$(1,0,0,0,4)$$
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Indeed $h^{1,1}(X) = h^{2,1}(X^\vee) = 1$ and $h^{2,1}(X) = h^{1,1}(X^\vee) = 101$.

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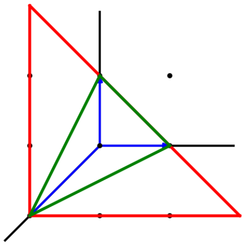
- Each lattice point ν_i on Δ° except the origin gives rise to a homogeneous coordinate x_i and a divisor D_i.
- Each lattice point m on Δ gives a $\mathbf{M}$onomial and the hypersurface equation is

$$\mathbf{X}_{(\boldsymbol{\Delta},\boldsymbol{\Delta}^\circ)}: \quad \sum_{m\in\Delta} c_m \prod_{\nu_i\in\Delta^\circ} x_i{}^{\langle m,\nu_i\rangle+1} = 0$$

More abstract point of view: Δ defines a toric variety $\mathbb{P}_{\Sigma_n(\Delta)}$ via its normal fan $\Sigma_n(\Delta) = \Sigma_f(\Delta^\circ)$ as well as a divisor (our CY !) on it.



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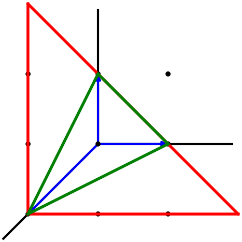

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Combinatorial formulas for Hodge numbers

$$h^{1,1}(X_{(\Delta,\Delta^\circ)}) = \ell(\Delta^\circ) - 5 - \sum_{\Theta^{\circ[3]}} \ell^*(\Theta^{\circ[3]}) + \sum_{\Theta^{\circ[2]}} \ell^*(\Theta^{[1]})\ell^*(\Theta^{\circ[2]})$$

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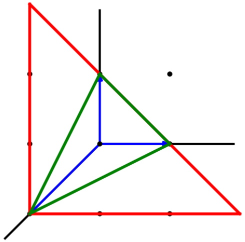

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Combinatorial formulas for Hodge numbers satisfy:

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$$!!!! \rightarrow \quad \mathbf{X}_{(\boldsymbol{\Delta},\boldsymbol{\Delta}^\circ)} = \mathbf{X}^\vee_{(\boldsymbol{\Delta}^\circ,\boldsymbol{\Delta})} \quad \leftarrow !!!!$$

example: the quintic

$$\Delta^\circ \sim \begin{pmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad \Delta \sim \begin{pmatrix} -1 & -1 & -1 & -1 & 4 \\ -1 & -1 & -1 & 4 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ -1 & 4 & -1 & -1 & -1 \end{pmatrix}$$

For the mirror, patches of $\mathbb{P}_{\Sigma_n(\Delta^\circ)} = \mathbb{P}_{\Sigma_f(\Delta)}$ are $\mathbb{C}^3/(\mathbb{Z}_5)^3$: like [Greene, Plesser] !

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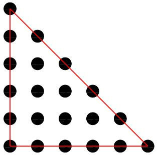
e.g. two-dimensional faces of Δ° looks like this:



Extra points $\sim$ refinement $\Sigma \to \Sigma_f \sim$ resolution of orbifold singularities

2) G_2 manifolds as twisted connected Sums

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- Glue a G_2 manifold from algebraic ‘building blocks’

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- Glue a G_2 manifold from algebraic ‘building blocks’
- $\mathcal{O}$(100s of millions) of examples available

Twisted Connected Sum G_2 manifolds (TCS)

[Kovalev; Corti, Haskins, Nordström, Pacini] Take a $K3$-fibred Kähler threefold* Z (with base $\mathbb{P}^1$) such that

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easiest example: hypersurface of degree $(4,1)$ in $\mathbb{P}^3 \times \mathbb{P}^1$

$$z_0 P_4(x) + z_1 Q_4(x) = 0$$

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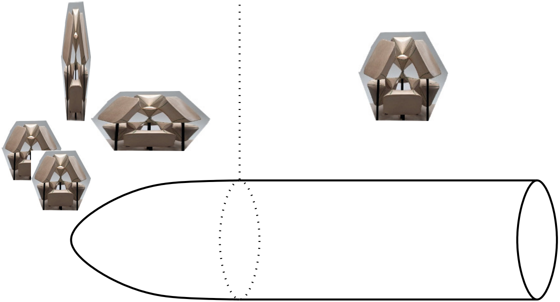
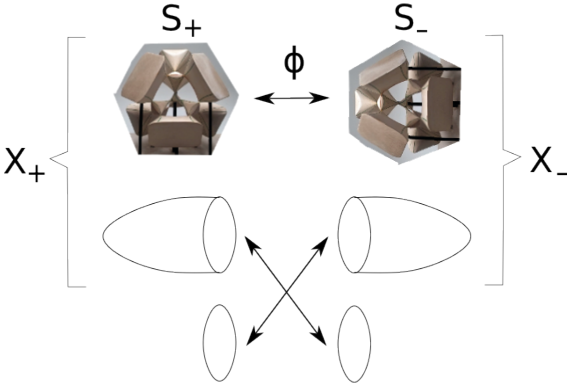


Twisted Connected Sum G_2 manifolds (TCS)

- cut out a generic fibre $X = Z/S_0$
  these are open asymptotically cylindrical Calabi-Yau threefolds !
- take two building blocks Z_+ and Z_- and glue $X_+ \times S^1$ and $X_- \times S^1$ like this:



congratulations:
here is your shiny new G_2 manifold J !

Letting $\rho = H^2(Z,\mathbb{Z}) \to H^2(S,\mathbb{Z})$ and

$$K = ker(\rho)/[S] \qquad N = im(\rho)$$

TCS:Cohomology

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the cohomology of the resuting G_2 manifold J is

$$\begin{aligned}
H^1(J,\mathbb{Z}) &= 0\\
H^2(J,\mathbb{Z}) &= N_+ \cap N_- \oplus K_+ \oplus K_-\\
H^3(J,\mathbb{Z}) &= \mathbb{Z}[S] \oplus \Gamma^{3,19}/(N_+ + N_-) \oplus (N_- \cap T_+) \oplus (N_+ \cap T_-)\\
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H^4(J,\mathbb{Z}) &= H^4(S) \oplus (T_+ \cap T_-) \oplus \ \Gamma^{3,19}/(N_- + T_+) \oplus \Gamma^{3,19}/(N_+ + T_-)\\
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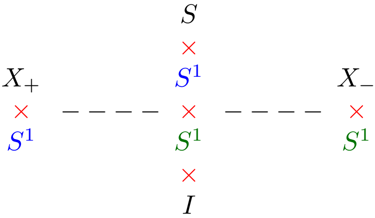
For a specific class of gluings (hyper Kähler rotations) $\phi : S_+ \mapsto S_-$:

$$b_2 + b_3 = 23 + 2\left[h^{2,1}(Z_+) + h^{2,1}(Z_-)\right] + 2\left[|K(Z_+)| + |K(Z_-)|\right] .$$

Mirror Symmetry for TCS G_2 manifolds: heuristics



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together with T-dualities along the S^1 factors.

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$\diamondsuit$

Given J as a TCS, can we construct a mirror in this sense and check that $b_2 + b_3$ stays invariant ?

3) Building Blocks from Tops and G_2 mirrors

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$\rightarrow \quad X_{(\Delta,\Delta^\circ)}$ is fibred by $K3$ surfaces $X_{(\Delta_F,\Delta_F^\circ)}$.
[Candelas, Font; Klemm, Lerche, Mayr; Hosono, Lian, Yau; Avram,Kreuzer,Mandelberg,Skarke]

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- F cuts Δ° into two halves called 'tops', $\Diamond^\circ_a$, $\Diamond^\circ_b$
- If the projection of a top to F sits inside Δ°_F, a top is called 'projecting'.

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$\rightarrow \quad X_{(\Delta,\Delta^\circ)}$ is fibred by $K3$ surfaces $X_{(\Delta_F,\Delta^\circ_F)}$.
[Candelas, Font; Klemm, Lerche, Mayr; Hosono, Lian, Yau; Avram,Kreuzer,Mandelberg,Skarke]

- F cuts Δ° into two halves called 'tops', $\Diamond^\circ_a$, $\Diamond^\circ_b$
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We may think of the tops $\Diamond^\circ_a$, $\Diamond^\circ_b$ as each capturing 'half' of the singular fibres of $X_{(\Delta,\Delta^\circ)}$.

Building Blocks From Projecting Tops

A pair of dual tops is a pair of lattice polytopes which satisfy

$$\langle \Diamond, \Diamond^\circ \rangle \geq -1$$

$$\langle \Diamond, \nu_0 \rangle \geq 0 \qquad\qquad \langle m_0, \Diamond^\circ \rangle \geq 0$$

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$$Z_{(\Diamond,\Diamond^\circ)} : \sum_{m \in \Diamond} c_m x_e^{\langle \nu_0, m \rangle} \prod_{\nu_i \in \Diamond^\circ} x_i^{\langle \nu_i, m \rangle + 1} = 0$$

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Starting from $\Diamond$, this is the same as Batyrev's construction and results in a building block for G_2 manifolds. There is combinatorial computation of Hodge numbers (and lattices N and K) using toric stratification [AB]:

$$h^{1,1} = -4 + \sum_{\Theta^{[3]} \in \Diamond} 1 + \sum_{\Theta^{[2]} \in \Diamond} \ell^*(\sigma_n(\Theta^{[2]})) + \sum_{\Theta^{[1]} \in \Diamond} (\ell^*(\Theta^{[1]}) + 1) \cdot (\ell^*(\sigma_n(\Theta^{[1]}))$$

$$h^{2,1} = \ell(\Diamond) - \ell(\Delta_F) + \sum_{\Theta^{[2]} < \Diamond} \ell^*(\Theta^{[2]}) \cdot \ell^*(\sigma_n(\Theta^{[2]})) - \sum_{\Theta^{[3]} < \Diamond} \ell^*(\Theta^{[3]})$$

More exciting:

- swapping the roles of $\Diamond \leftrightarrow \Diamond^\circ$ gives the SYZ mirror of the open, asymptotically cylindrical Calabi-Yau $X = Z_{(\Diamond,\Diamond^\circ)}/S$.
- in particular: the mirror map exchanges $h^{2,1}$ with $|ker(H^2(Z,\mathbb{Z}) \to H^2(S,\mathbb{Z}))/[S]|$:

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- For a specific class of gluings ϕ:

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We are in business !

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- In fact

$$H^2(J,\mathbb{Z}) \oplus H^4(J,\mathbb{Z})$$

is invariant under this mirror map !

example: the simplest TCS G_2 manifold and its mirror

Consider again a building block Z described by a hypersurface of degree $(4,1)$ in $\mathbb{P}^3 \times \mathbb{P}^1$.

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The associated mirror G_2 manifold has

$$b_2(J^\vee) = 84\,, \qquad b_3(J^\vee) = 71\,.$$

TCS and EFT

The TCS construction is good in the limit in which the cylindrical region in the middle:

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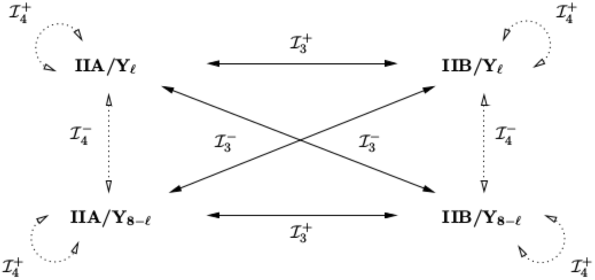
These are coupled together to form an $\mathcal{N} = 2$ theory ... our mirror map acts on all of these ... relation to 3D mirror symmetry ?

Sneak Preview: comparing to Orbifolds

For Joyce orbifold $Y_l = T^7/\mathbb{Z}_2^3$: $b_2 = 8 + l$, $\quad b_3 = 47 - l$, $(l = 0..8)$



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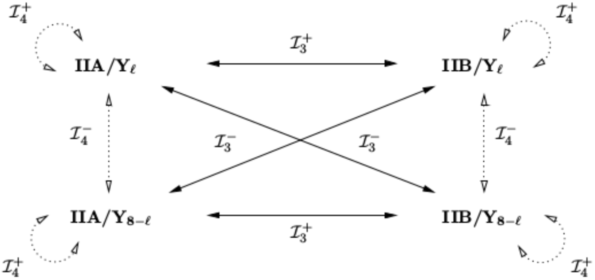

This orbifold and its smoothings can also be decomposed as twisted connected sums [AB, Michele del Zotto: to appear]...

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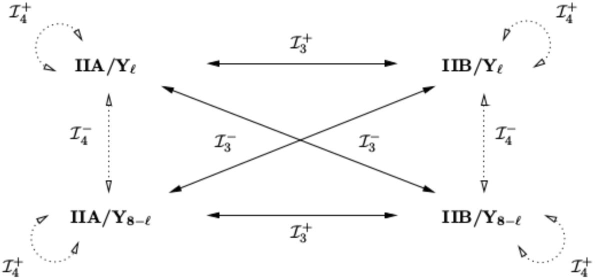

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- Swapping only one $Z_+ \leftrightarrow Z_+^\vee$ (but not Z_-) realizes $\mathcal{I}_3^-$.

– Thank you –