

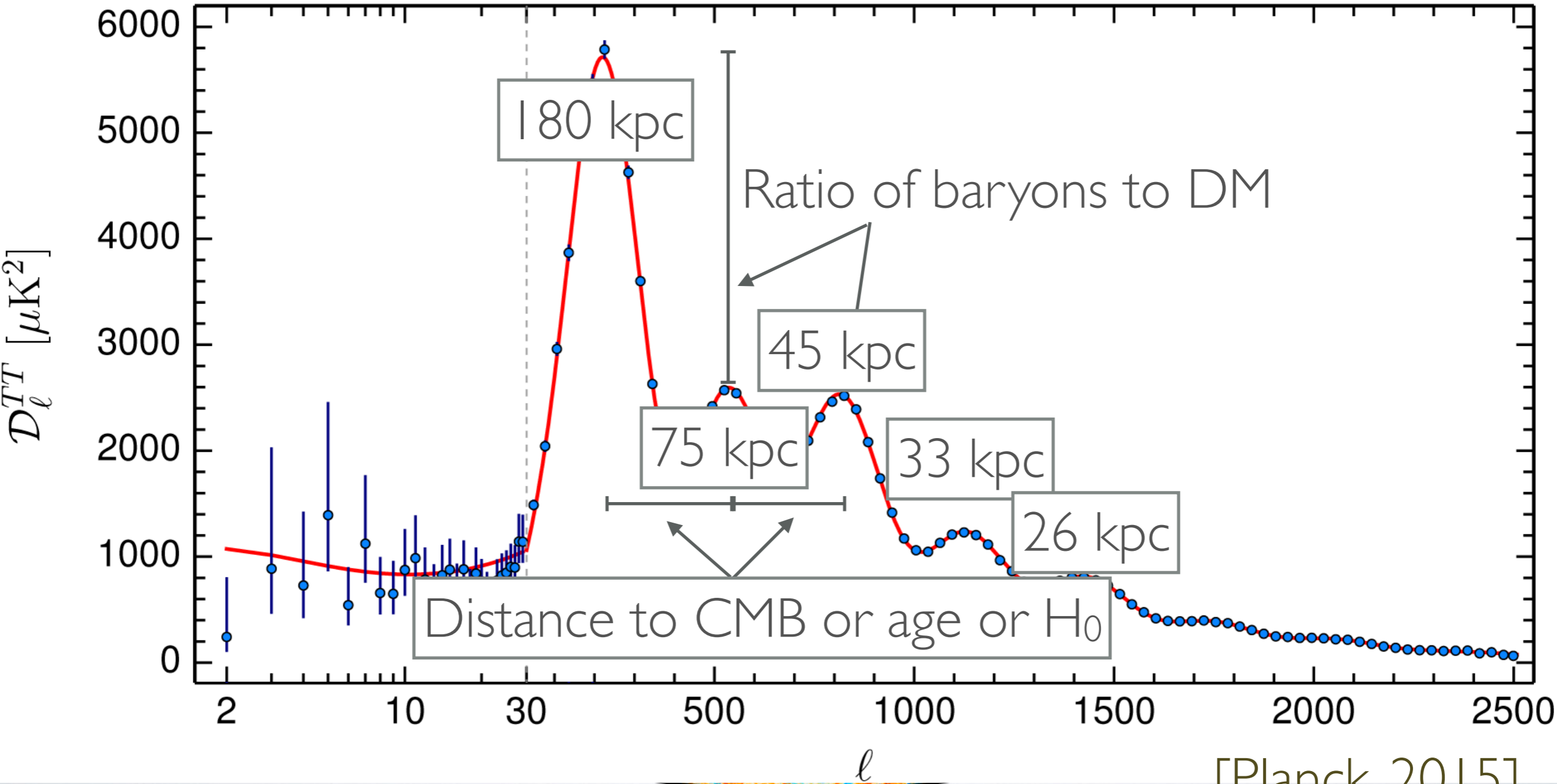
FROM  
LARGE COLD SIMULATIONS  
TO  
FUNDAMENTAL PARTICLE PROPERTIES

Wessel Valkenburg

# OUTLINE

- What we know about Dark Matter (lower bounds)
- Gravitational evolution of structure in Universe
- General relativity and Newtonian gravity
- Dark Matter mass from simulations?
- Setup with multiple matter types: source of inaccuracy

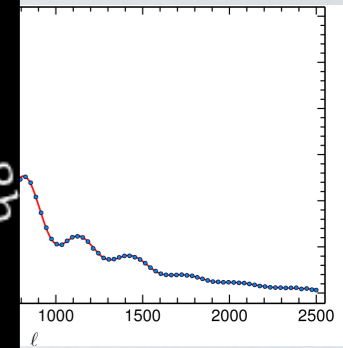
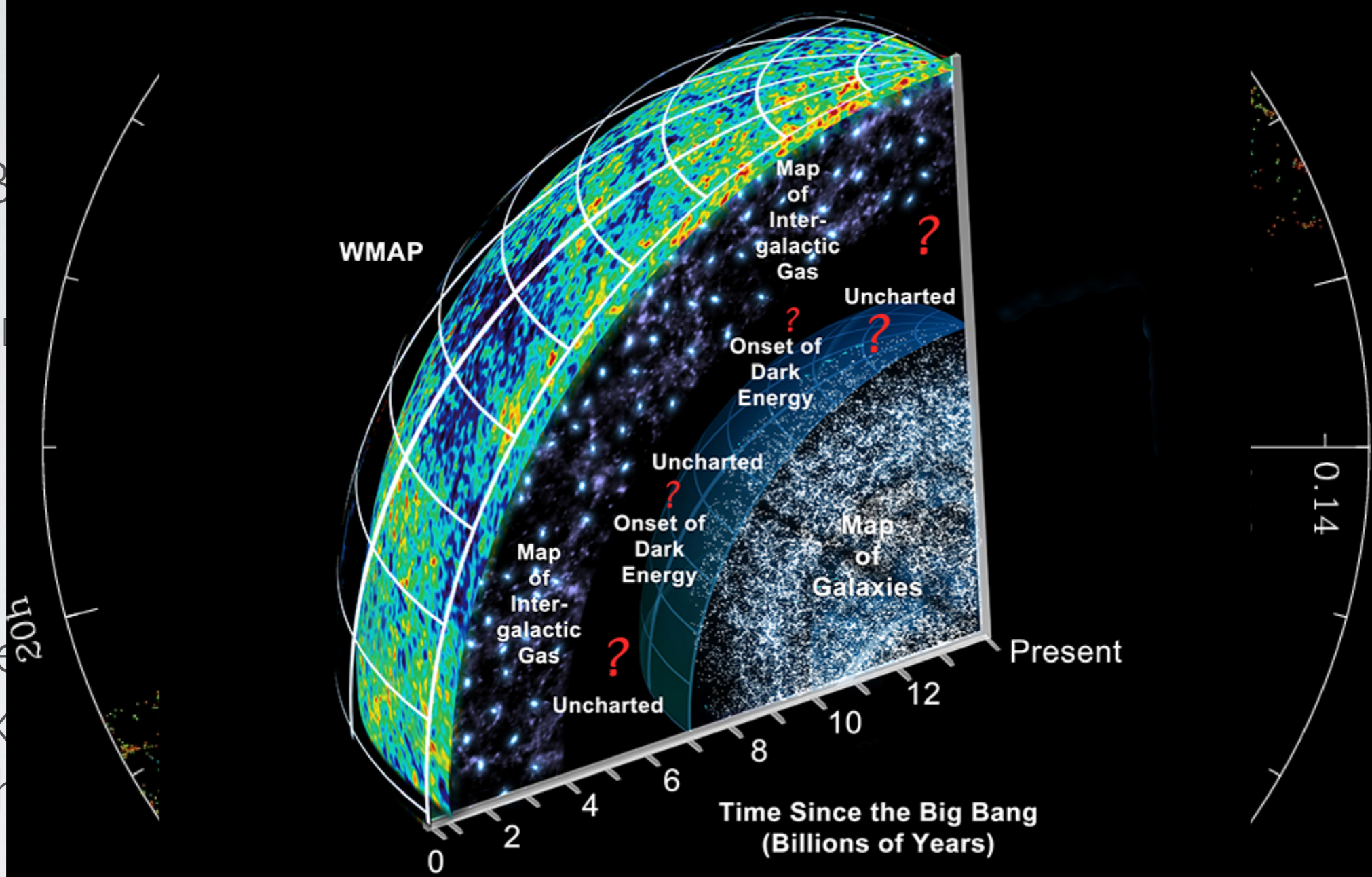
# DARK MATTERS?



[Planck, 2015]

# SDSS-IV Catches the Rise of Dark Energy

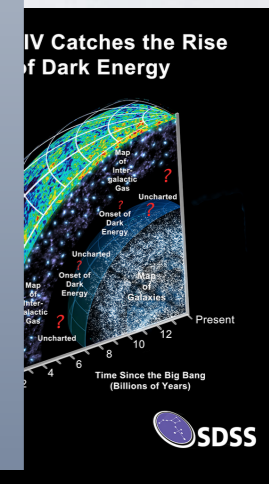
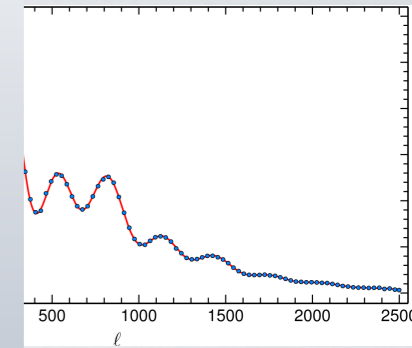
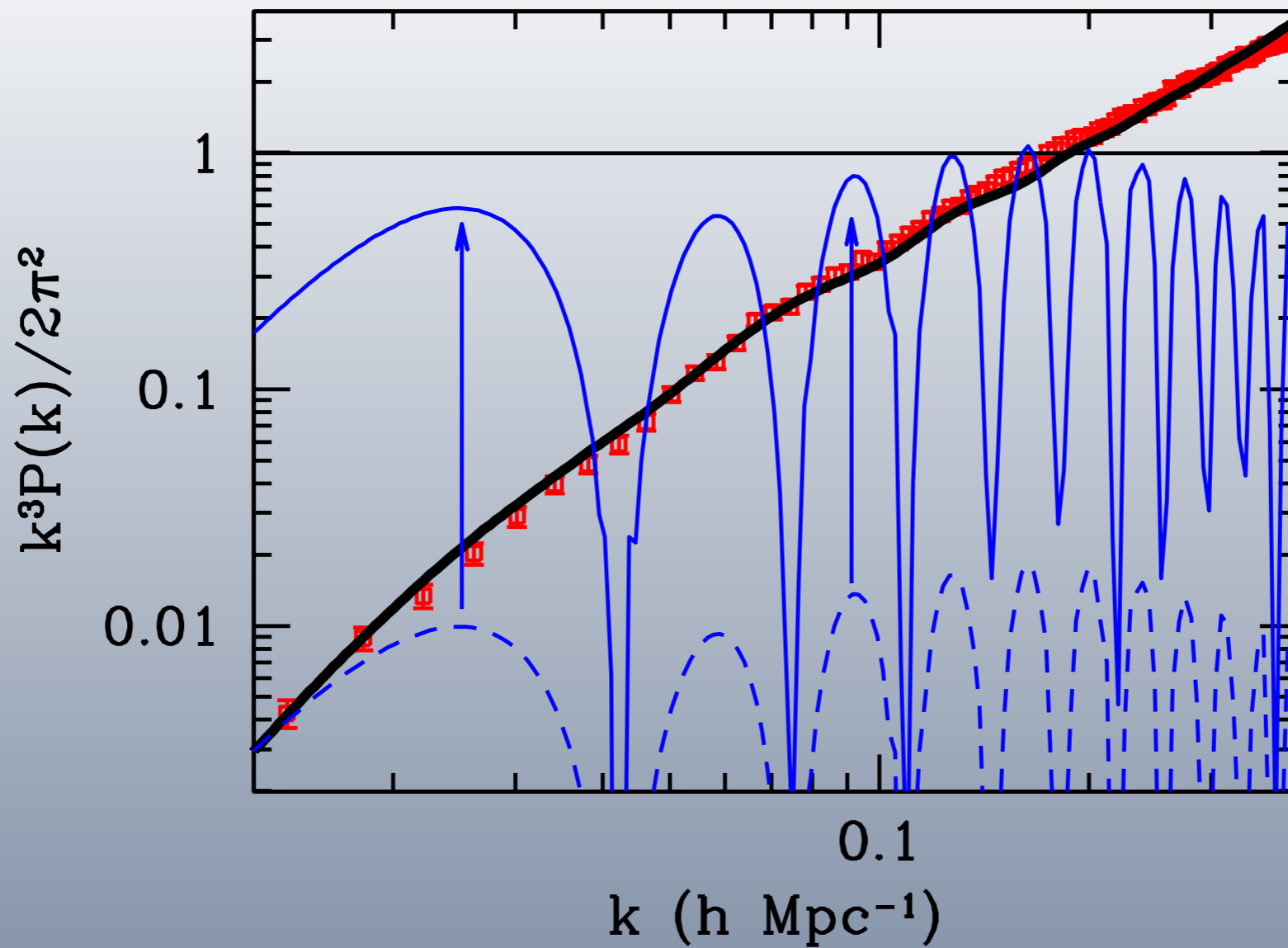
- CMB
- $\Omega$  at
- $\Sigma$
- Large galaxy redshift
- $f\sigma_8$



[BOSS/SDSS, 2014]

# DARK MATTER

- CMB
- $\Omega_m$  at
- $\Sigma$
- Large galaxy redshift
- $f\sigma_8$



[Dodelson, 2011]



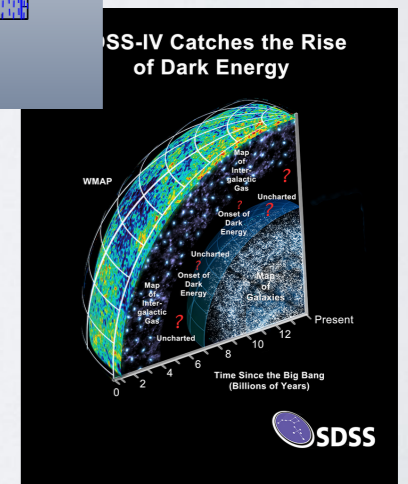
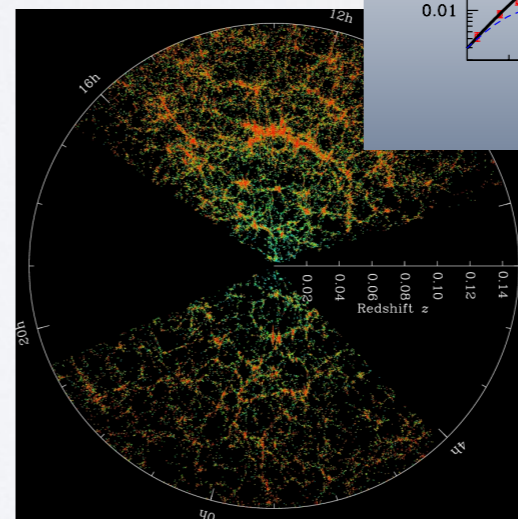
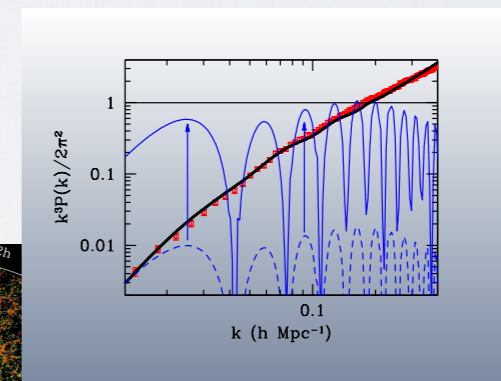
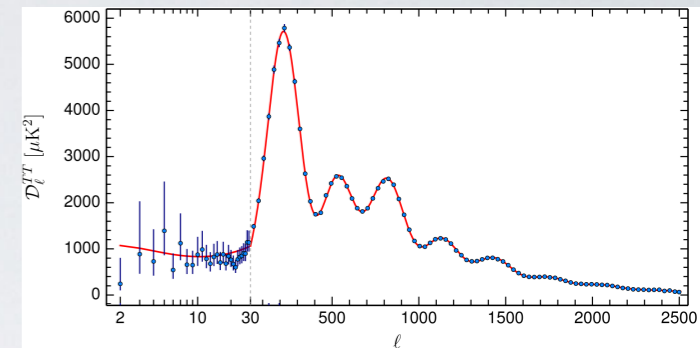
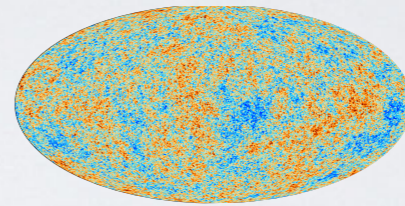
# MOND

MOdified Newtonian Dynamics

- Just kidding.
- But compute! Don't discard on religious grounds.
- Bullet cluster, and this is true, the best experts say it, falsifies  $\Lambda$ CDM at  $5\sigma$ .

# PERTURBATION THEORY

- Both CMB & LSS sensitive to linear density fluctuations.
- CMB is insensitive to CDM properties
- (other than coupling to baryons/photons)

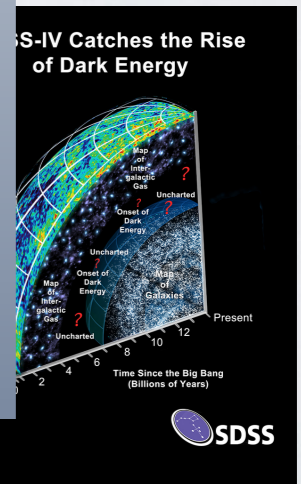
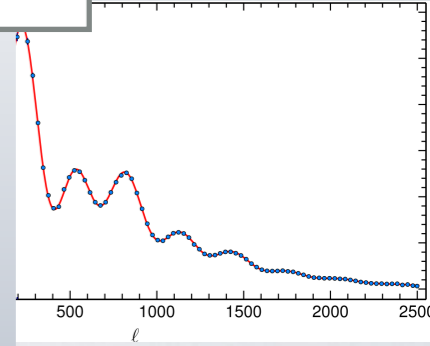
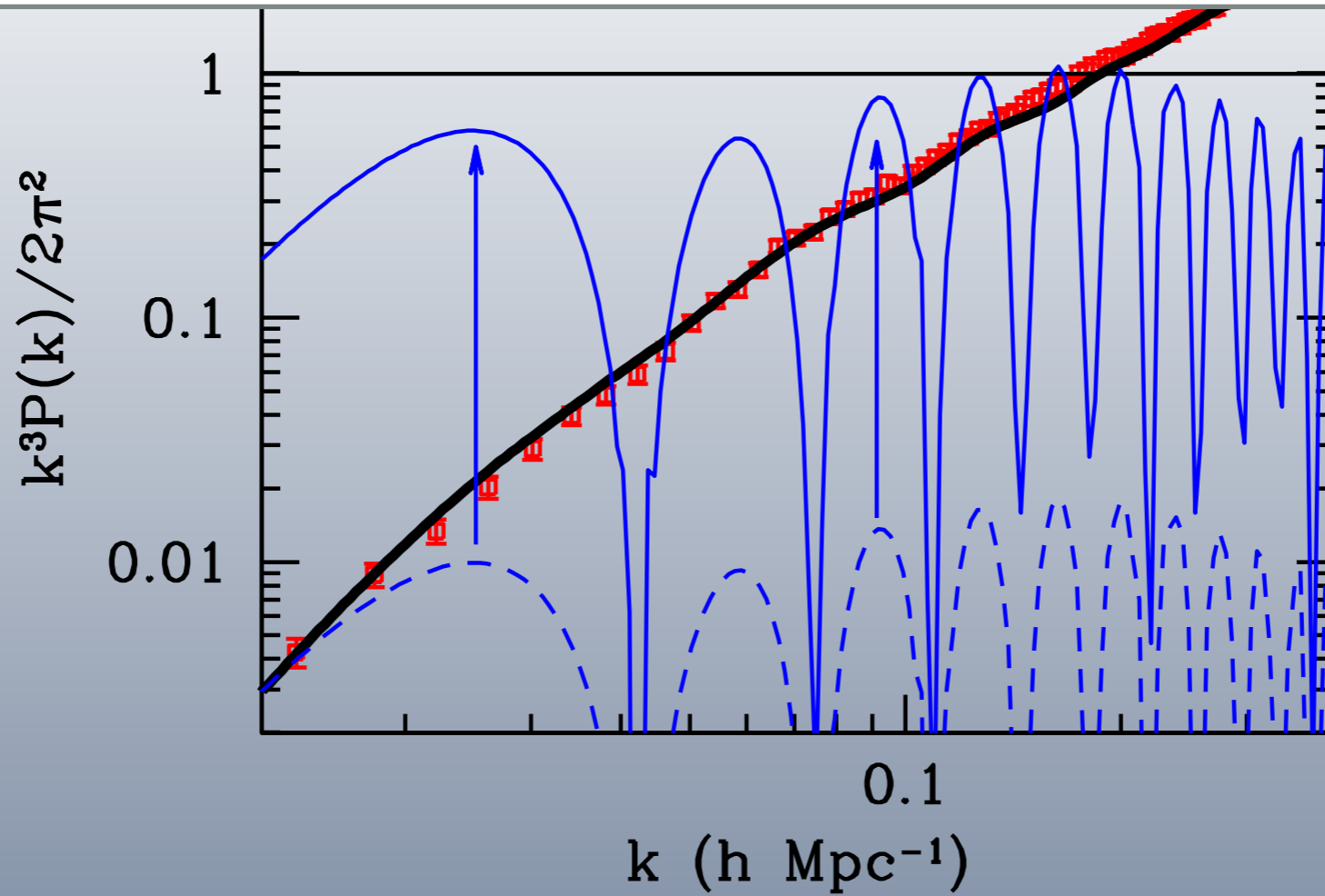




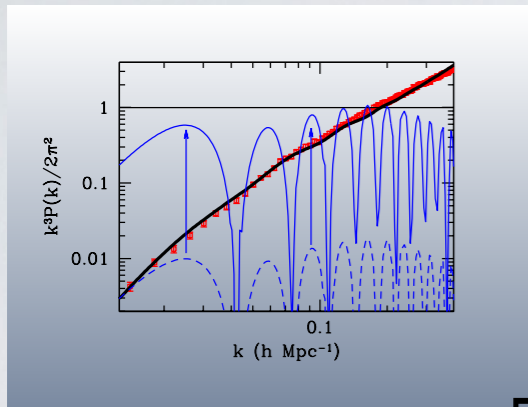
# PERTURBATION THEORY

crosses  $l$  at  $k \sim 0.2 \text{ h/Mpc}$  or roughly  $30 \text{ Mpc/h}$

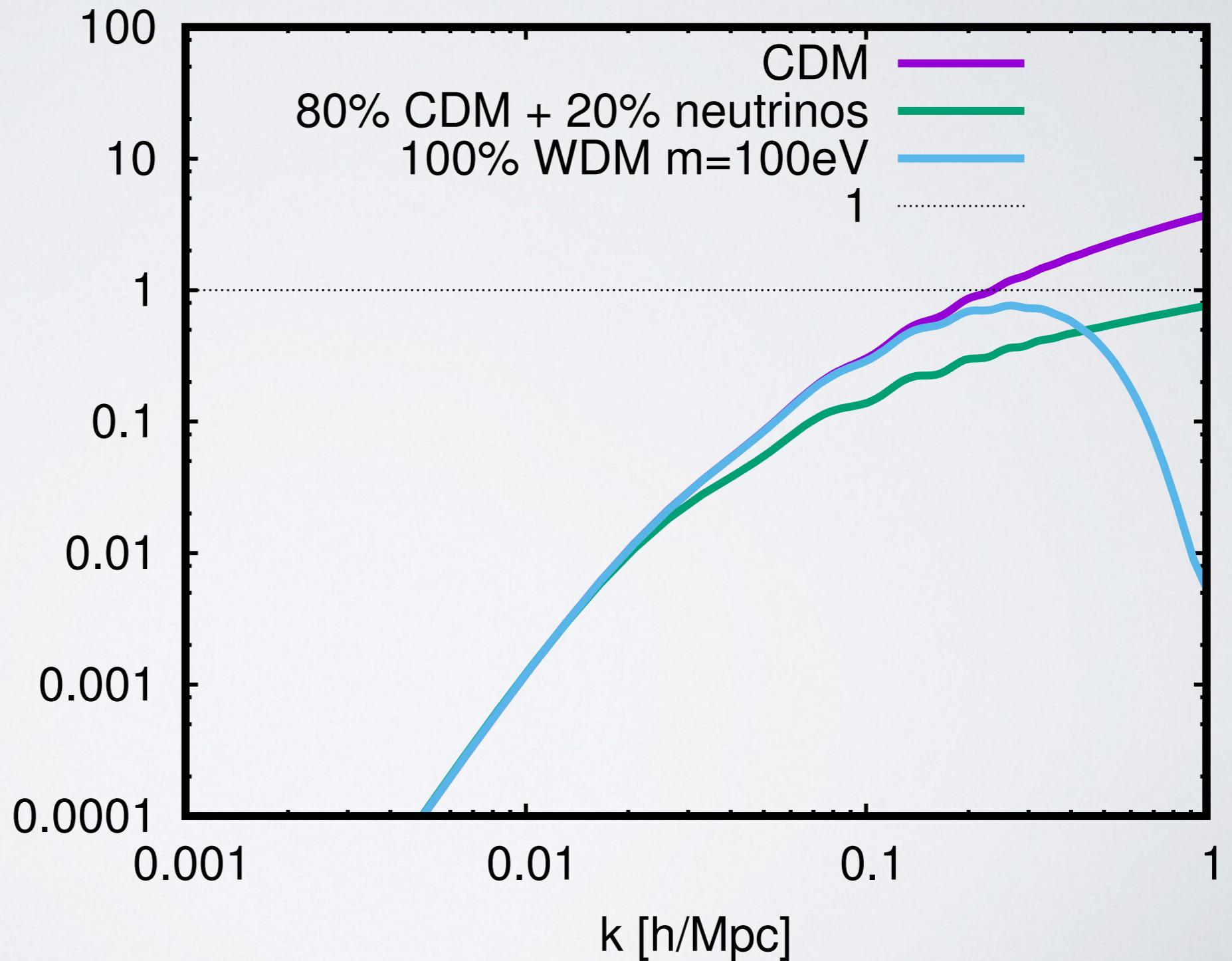
- Both  $C$  to *linear*
- CMB is proper
- (other bary)

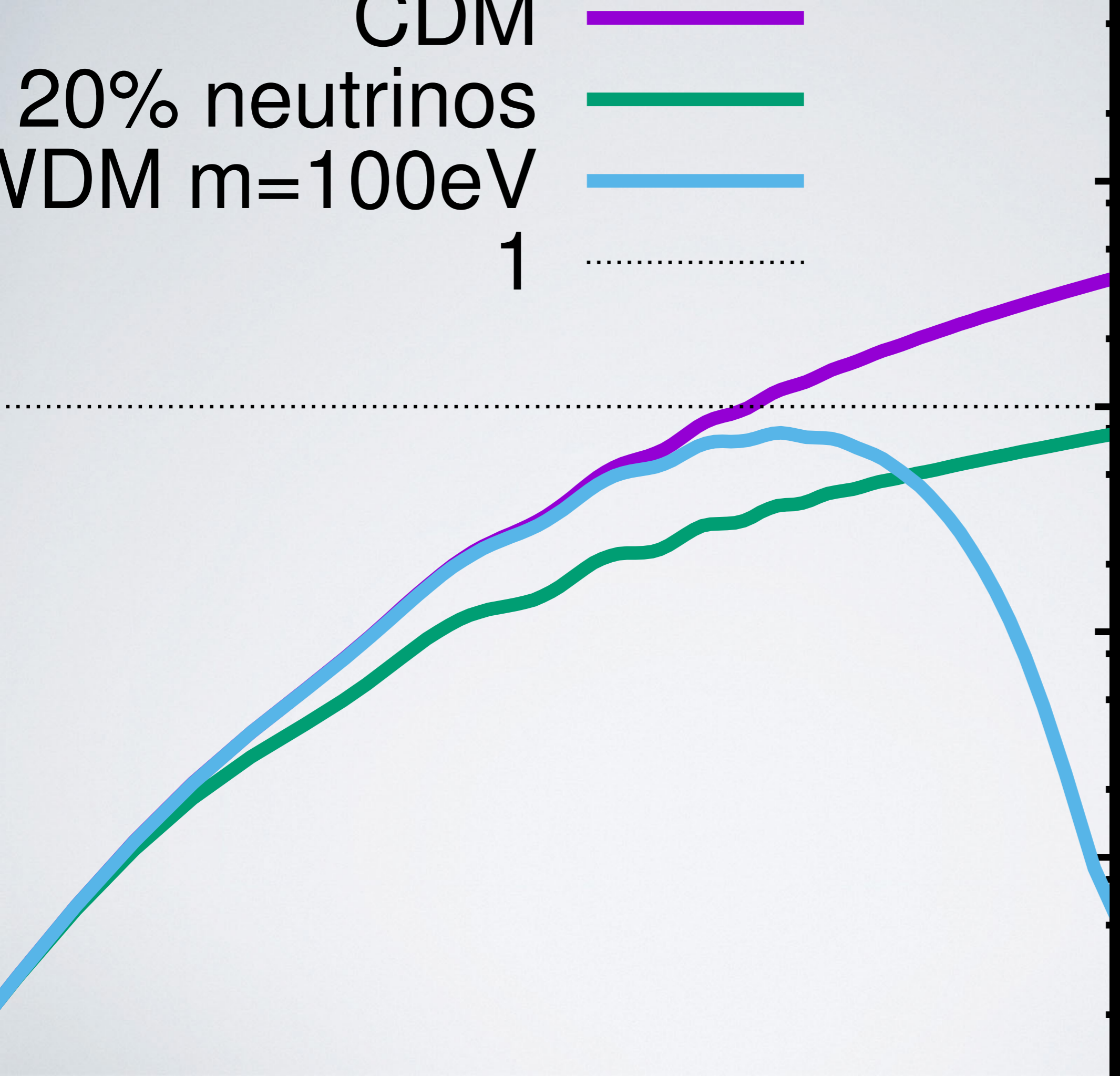


# PERTURBATION THEORY

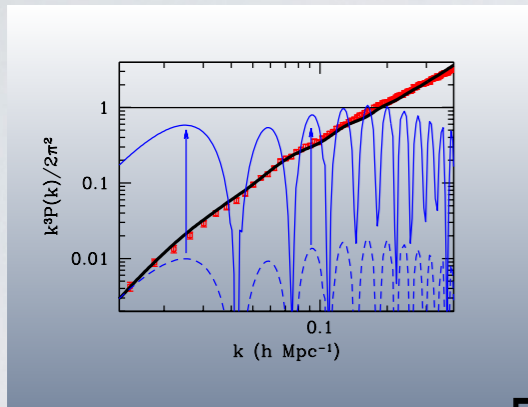


$k^3 P(k) / 2\pi^2$  [dimless]

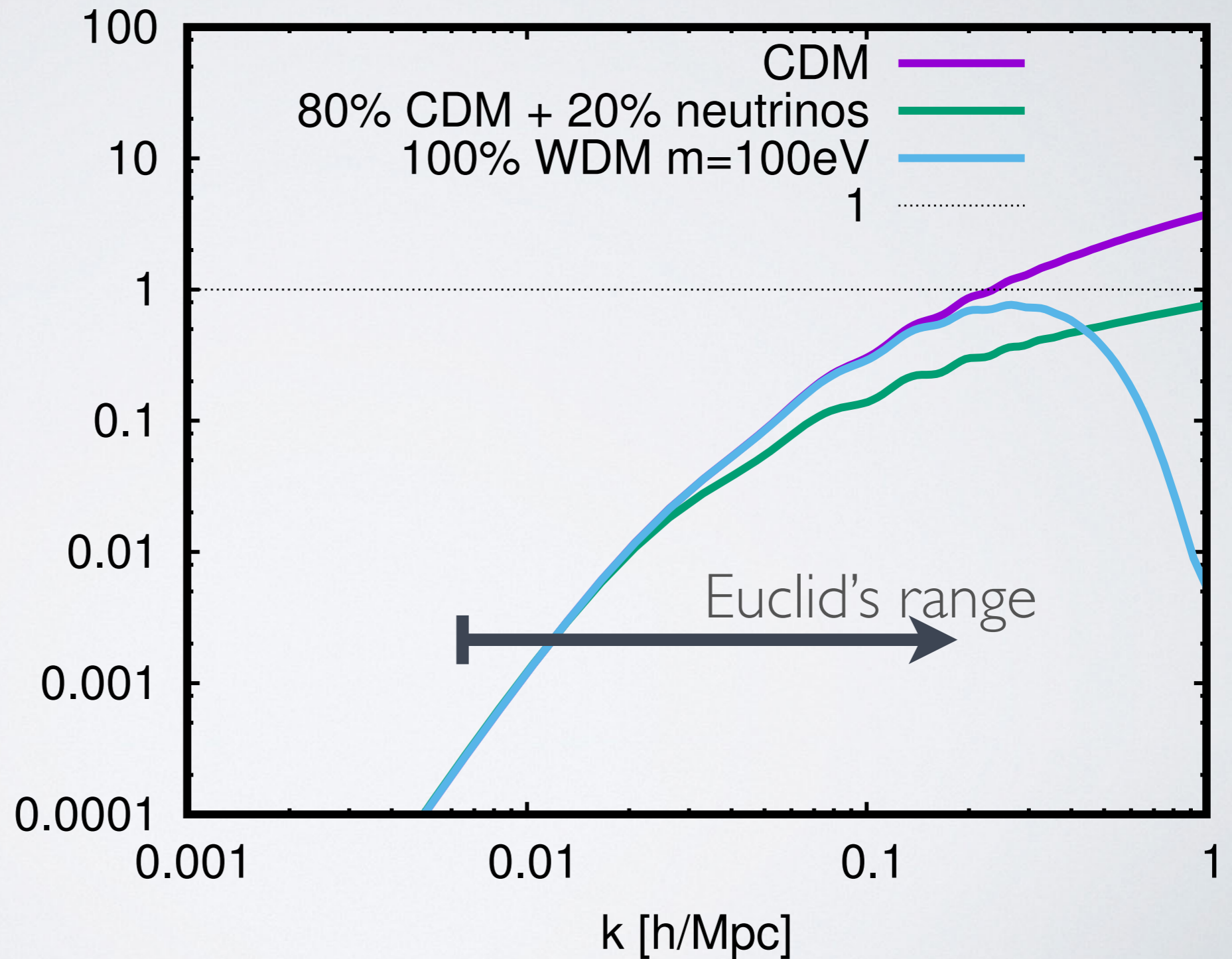




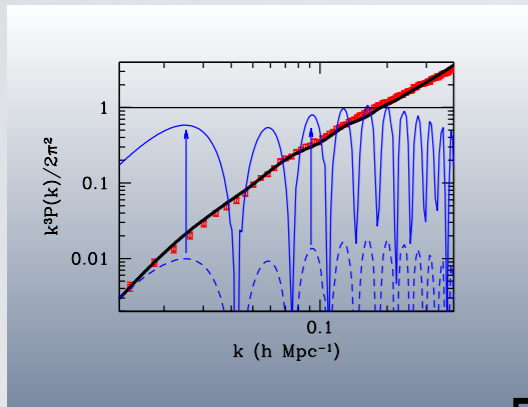
# PERTURBATION THEORY



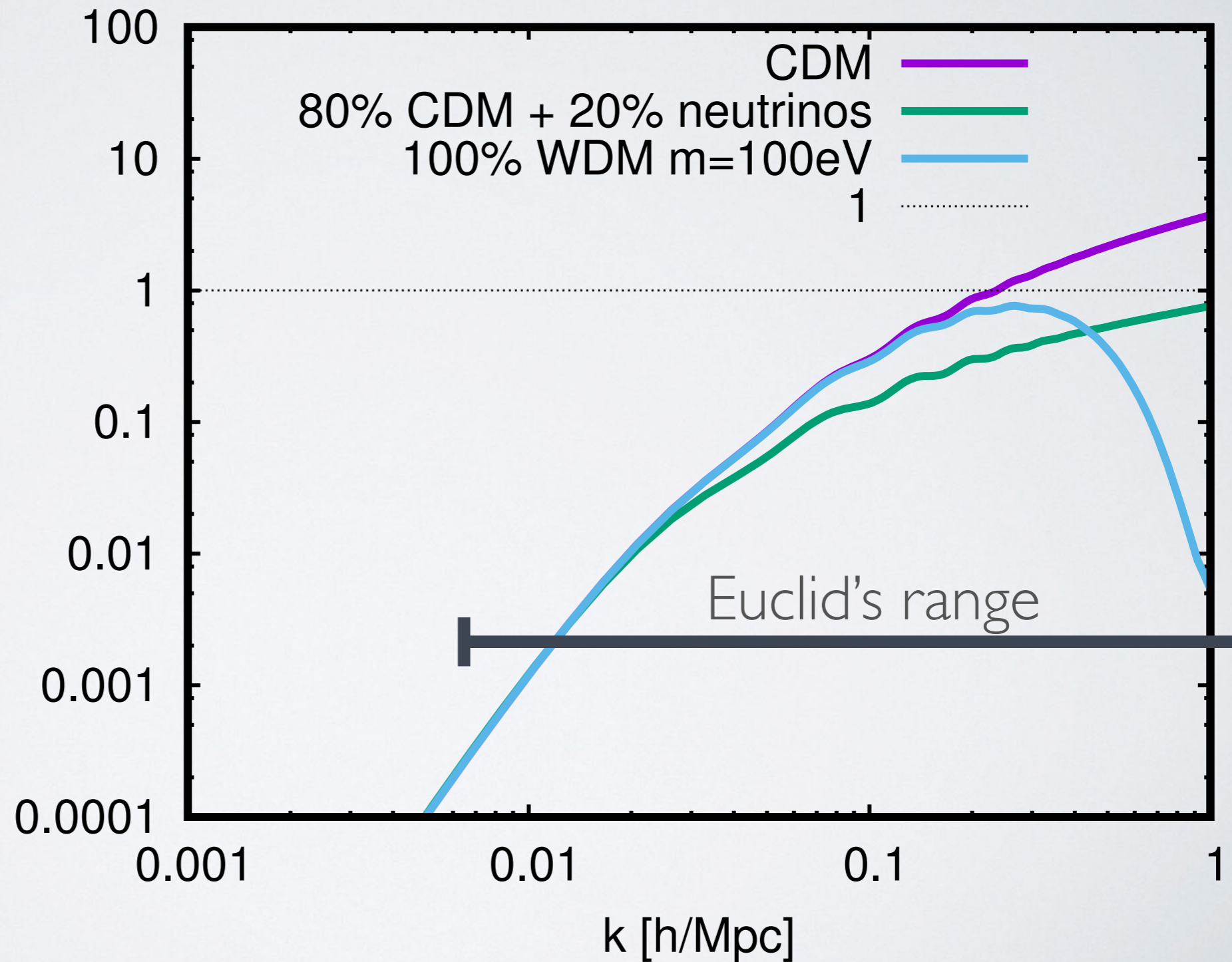
$k^3 P(k) / 2\pi^2$  [dimless]



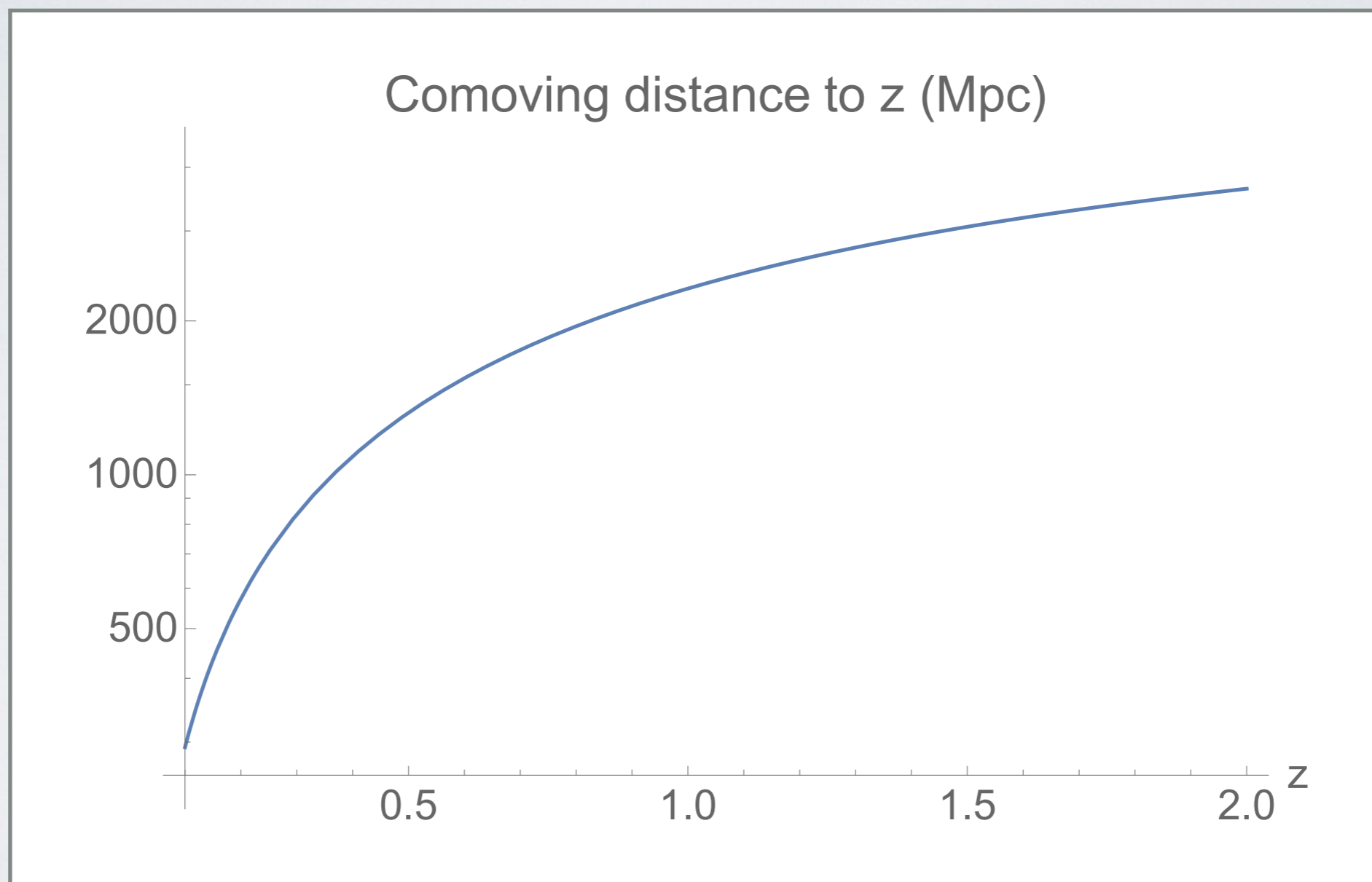
# PERTURBATION THEORY



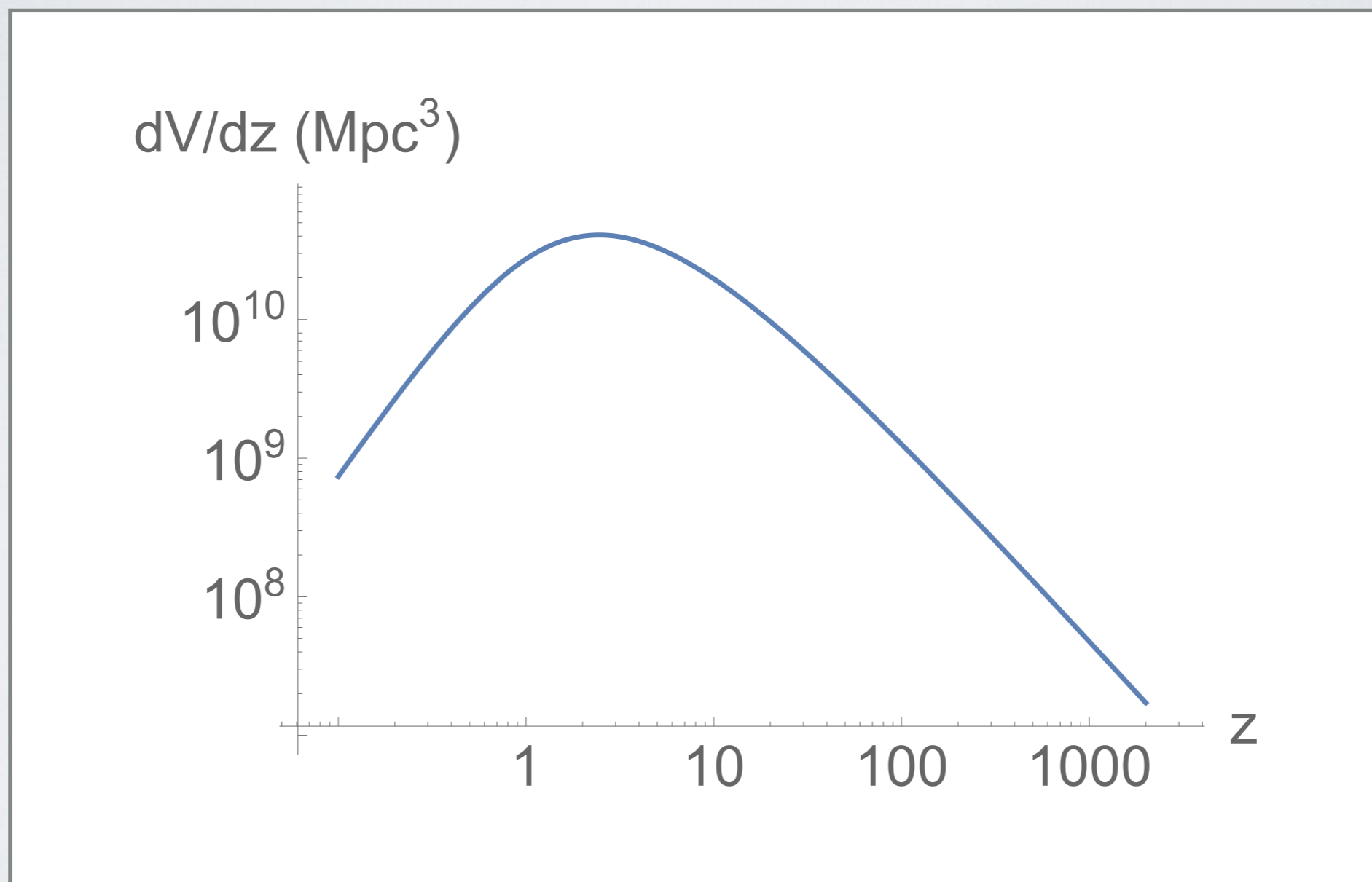
$k^3 P(k) / 2\pi^2$  [dimless]



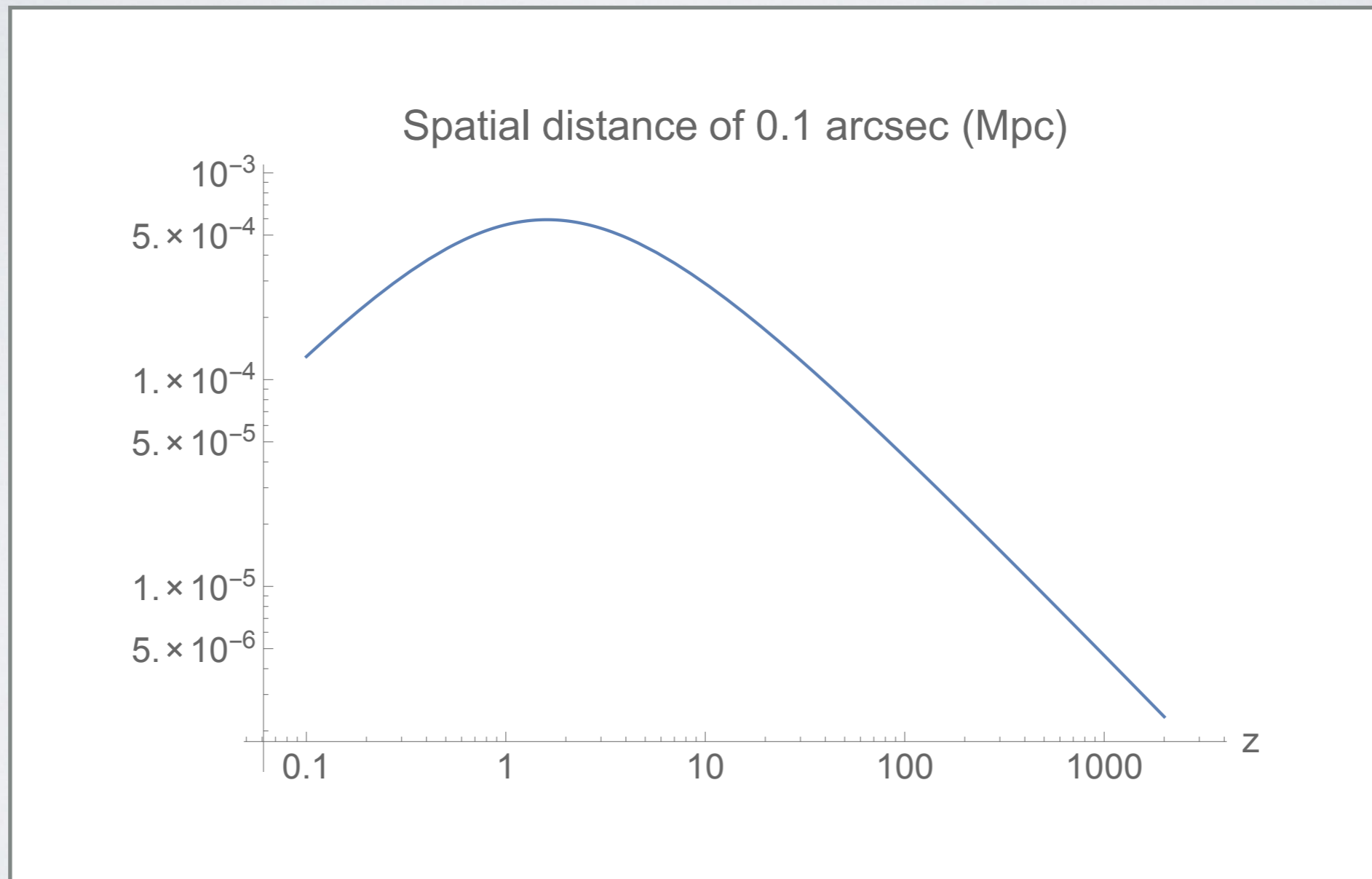
# OBSERVING PERTURBATIONS



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# OBSERVING PERTURBATIONS

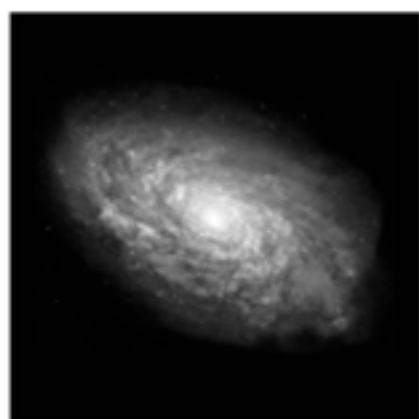




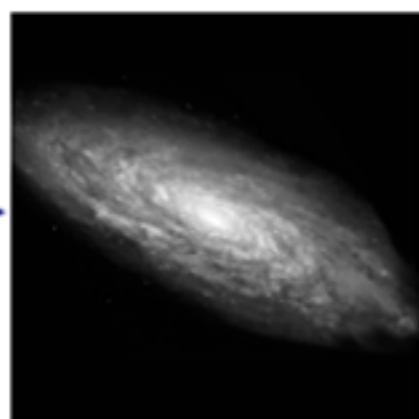
# OBSERVING PERTURBATIONS

## The Forward Process.

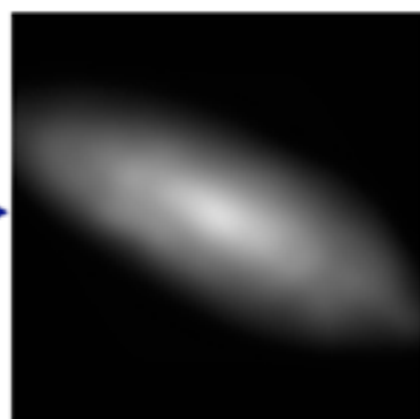
**Galaxies:** Intrinsic galaxy shapes to measured image:



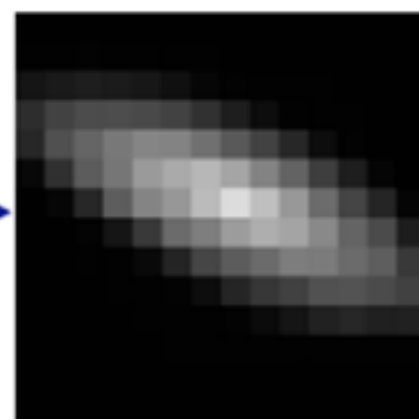
Intrinsic galaxy  
(shape unknown)



Gravitational lensing  
causes a **shear ( $g$ )**



Atmosphere and telescope  
cause a convolution



Detectors measure  
a pixelated image

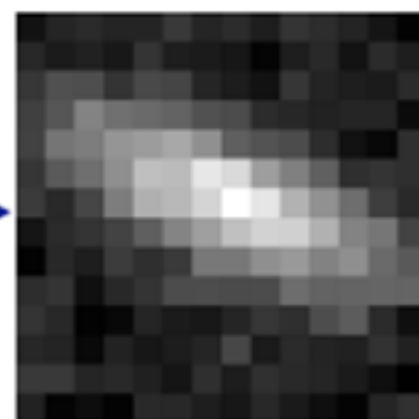


Image also  
contains noise

[Bridle et al., 2008]

## RELATIVISTIC PERTURBATIONS

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$h_{ij}(\mathbf{x}, \tau) = \int d^3k e^{i\mathbf{k} \cdot \mathbf{x}} [\hat{k}_i \hat{k}_j h(\mathbf{k}, \tau) + (\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij}) 6\eta(\mathbf{k}, \tau)]$$

$$k^2 \eta - \frac{1}{2} \frac{\dot{a}}{a} \dot{h} = 4\pi G a^2 \delta T^0_0(\text{Syn})$$

density perturbation

$$k^2 \dot{\eta} = 4\pi G a^2 (\bar{\rho} + \bar{P}) \theta(\text{Syn}),$$

(scalar) velocity perturbation

$$\ddot{h} + 2 \frac{\dot{a}}{a} \dot{h} - 2k^2 \eta = -8\pi G a^2 \delta T^i_i(\text{Syn})$$

pressure perturbation

$$\ddot{h} + 6\ddot{\eta} + 2 \frac{\dot{a}}{a} (\dot{h} + 6\dot{\eta}) - 2k^2 \eta = -24\pi G a^2 (\bar{\rho} + \bar{P}) \sigma(\text{Syn})$$

shear perturbation

## RELATIVISTIC PERTURBATIONS

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\dot{\delta}_v = -\frac{4}{3}\theta_v - \frac{2}{3}\dot{h},$$

Example: massless neutrinos

$$\dot{\theta}_v = k^2\left(\frac{1}{4}\delta_v - \sigma_v\right),$$

$$\dot{F}_{v2} = 2\dot{\sigma}_v = \frac{8}{15}\theta_v - \frac{3}{5}kF_{v3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$

$$\dot{F}_{vl} = \frac{k}{2l+1} [lF_{v(l-1)} - (l+1)F_{v(l+1)}], \quad l \geq 3.$$

$$\delta_v = -\frac{4}{3}\theta_v - \frac{2}{3}\dot{h},$$

$$\dot{\theta}_v = k^2\left(\frac{1}{4}\delta_v - \sigma_v\right),$$

$$\dot{F}_{v2} = 2\dot{\sigma}_v = \frac{8}{15}\theta_v - \frac{3}{5}kF_{v3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$

$$\dot{F}_{vl} = \frac{k}{2l+1} [lF_{v(l-1)} - (l+1)F_{v(l+1)}], \quad l \geq 3.$$

# RELATIVISTIC

# SIMULATIONS

$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

- Very nice.
- So now, let's just do that at the full nonlinear glory: go to lagrangian description and simulate particles.
- Eulerian: describe quantities at fixed coordinates
- Lagrangian: moving labels, EOM for labels.
  - If invertible map between labels and spacetime: LPT
  - Otherwise: infer eulerian quantities by measuring / counting.
- The two can mix: in Lagrangian picture, flows can exist between in other quantities. Choose your frame wisely.

$$\delta_v = -\frac{4}{3}\theta_v - \frac{2}{3}\dot{h},$$

$$\dot{\theta}_v = k^2\left(\frac{1}{4}\delta_v - \sigma_v\right),$$

$$\dot{F}_{v2} = 2\dot{\sigma}_v = \frac{8}{15}\theta_v - \frac{3}{5}kF_{v3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$

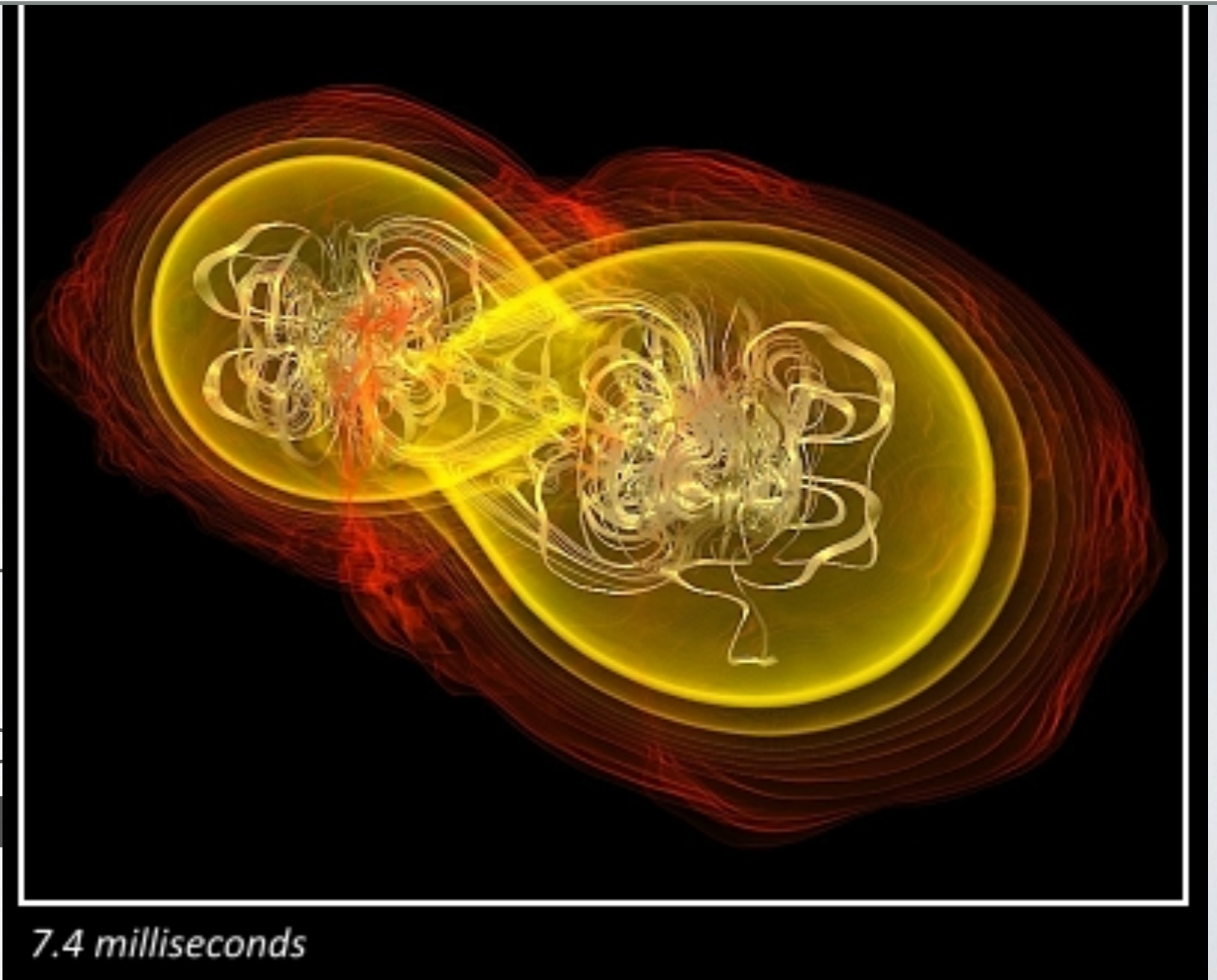
$$\dot{F}_{vl} = \frac{k}{2l+1}[lF_{v(l-1)} - (l+1)F_{v(l+1)}], \quad l \geq 3.$$

# RELATIVISTIC SIMULATIONS

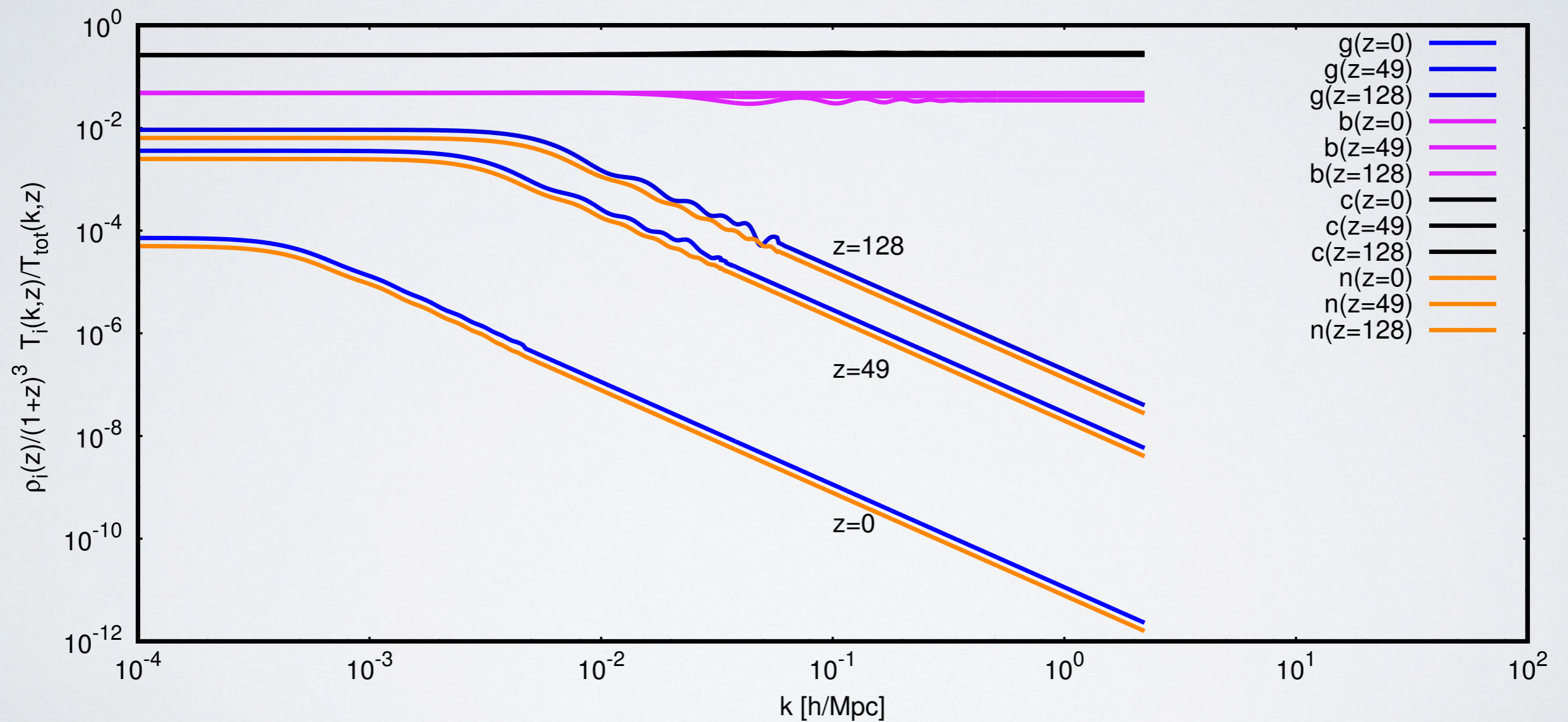
$$ds^2 = a^2(\tau)[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

Colliding neutron stars [Koppitz, Rezzolla, 2011]

- Wave hands:
  - GR Simulations are
  - Solve full 10 Einstein
  - State-of-the-art sim
  - Coordinate freed
  - very close though



# DO YOU NEED RELATIVISTIC SIMULATIONS?



# DO YOU NEED RELATIVISTIC SIMULATIONS?

$$ds^2 = a^2 \left( - (1 + 2\tilde{A}) d\tilde{\eta}^2 - 2\partial_i \tilde{B} d\tilde{x}^i d\tilde{\eta} + \left[ \delta_{ij} + \tilde{\gamma}_{ij} + \tilde{\Gamma}_{ij} \right] d\tilde{x}^i d\tilde{x}^j \right)$$

In comoving gauges

$$\Phi = H_L + \frac{\nabla^2}{3} H_T - \frac{\dot{a}}{a} (v - \dot{H}_T) \quad \nabla^2 \Phi = -4\pi G \bar{\rho} a^2 \delta.$$

$$\nabla^2 \left[ H_L + \frac{\nabla^2}{3} H_T - \frac{\dot{a}}{a} (v - \dot{H}_T) \right] = -4\pi G \bar{\rho} a^2 \delta,$$

$$\frac{\dot{a}}{a} \xi - \dot{H}_L - \frac{\nabla^2}{3} \dot{H}_T = 0,$$

$$\xi + H_L + \frac{\nabla^2}{3} H_T - \left[ \frac{\partial}{\partial \eta} + 2\frac{\dot{a}}{a} \right] (v - \dot{H}_T) = 8\pi G a^2 p \Pi.$$

$$\dot{\delta}_\alpha + \nabla \cdot \mathbf{v}_\alpha = -3\dot{H}_L,$$

$$\left( \frac{\partial}{\partial \eta} + \frac{\dot{a}}{a} \right) \mathbf{v}_\alpha = \nabla \Phi + \nabla \gamma$$

# DO YOU NEED RELATIVISTIC SIMULATIONS?

In comoving gauges

$$\Phi = H_L + \frac{\nabla^2}{3} H_T - \frac{\dot{a}}{a} (v - \dot{H}_T) \quad \nabla^2 \Phi = -4\pi G \bar{\rho} a^2 \delta .$$

$$\begin{aligned} \dot{\delta}_\alpha + \nabla \cdot \mathbf{v}_\alpha &= -3\dot{H}_L \\ \left( \frac{\partial}{\partial \eta} + \frac{\dot{a}}{a} \right) \mathbf{v}_\alpha &= \nabla \Phi - \nabla \gamma \end{aligned}$$

Possible when pressure  
and shear vanish.

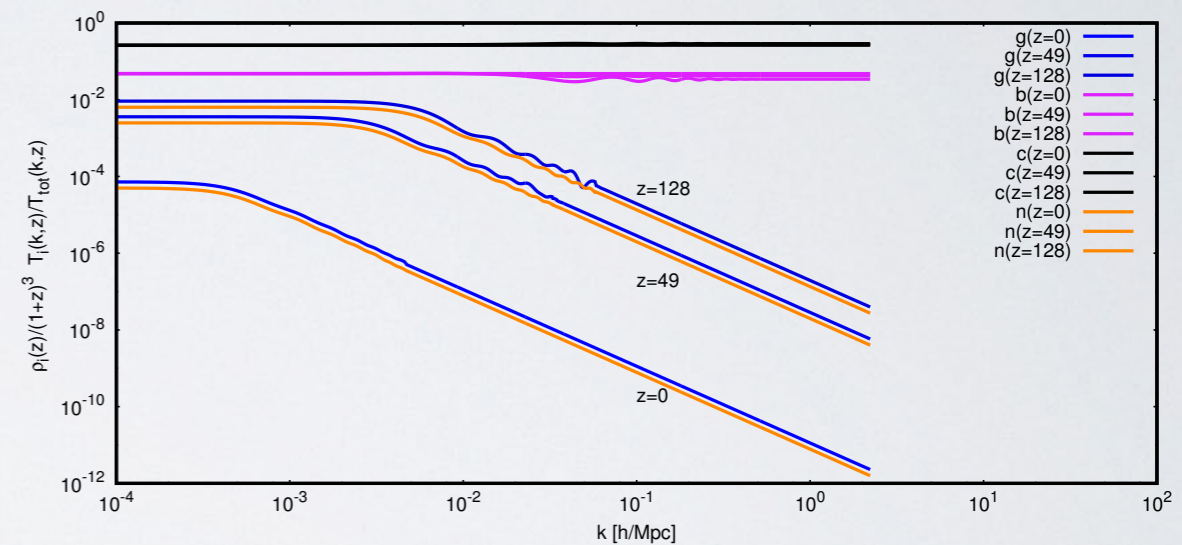


# DO YOU NEED RELATIVISTIC SIMULATIONS?

- With solely shear- and pressureless matter: not at linear level.
- Hubble radius: no thing.
- Sound horizon: a thing.
- No pressure - no sound horizon.
- Nonlinear scales: newtonian equations describe planetary motion "well enough"
  - Don't tell Mercury
- Dynamic range of computer does not go from galaxy clustering down to black holes.

# DO YOU NEED RELATIVISTIC SIMULATIONS?

- But if you care about 1% effect, you do.
- 2nd-order PT GR simulations at UniGe: CDM and neutrinos. [Durrer, Kunz, Adamek, Daverio].
- Newtonian CDM + linear radiation perturbations [Brandbyge et al.].



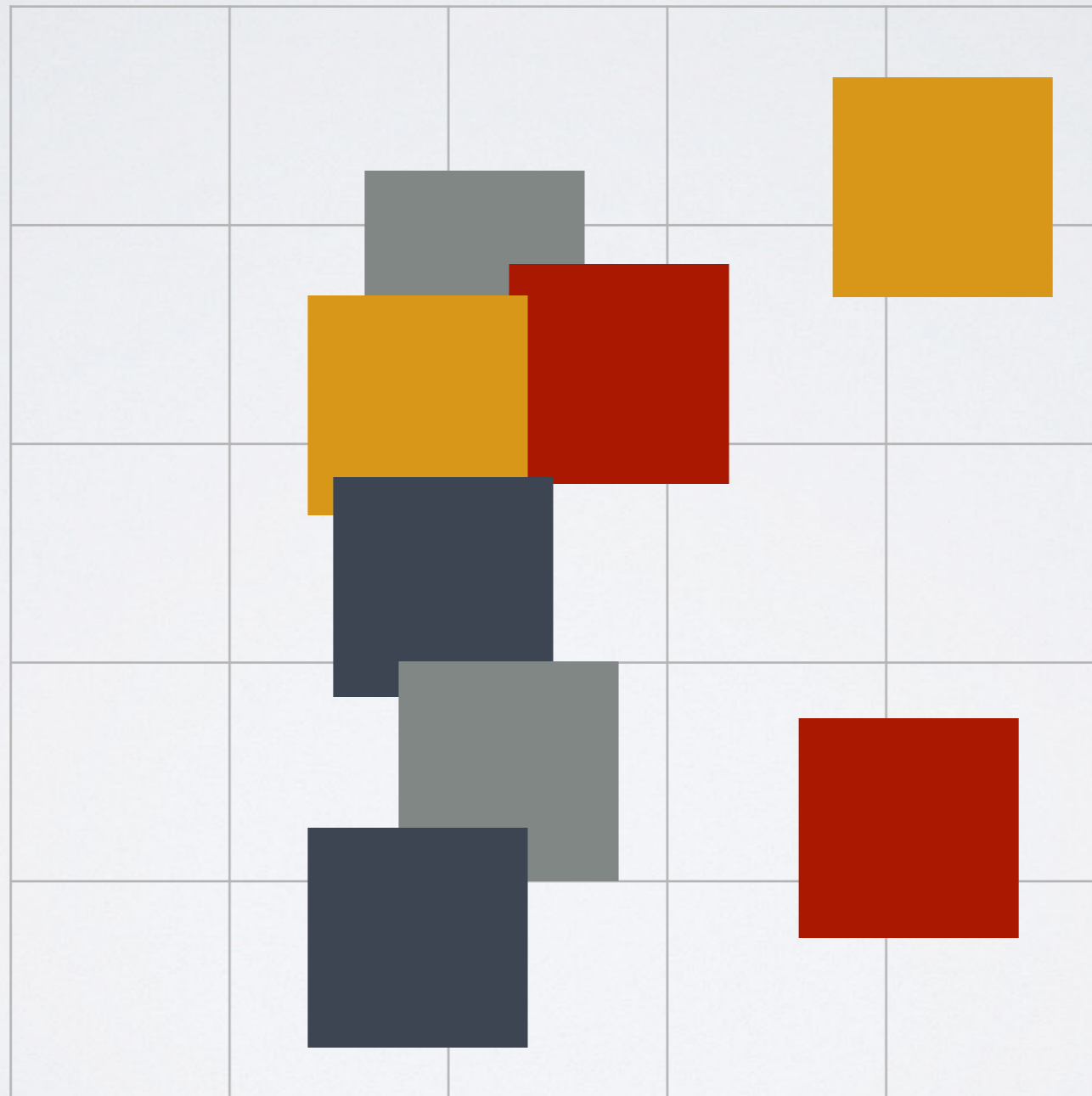
# STATE OF NEWTONIAN SIMULATIONS

- I promised to talk about fundamental properties of DM...
- Linear cosmology sensitive to mass or thermal velocity at best.

# STATE OF NEWTONIAN SIMULATIONS

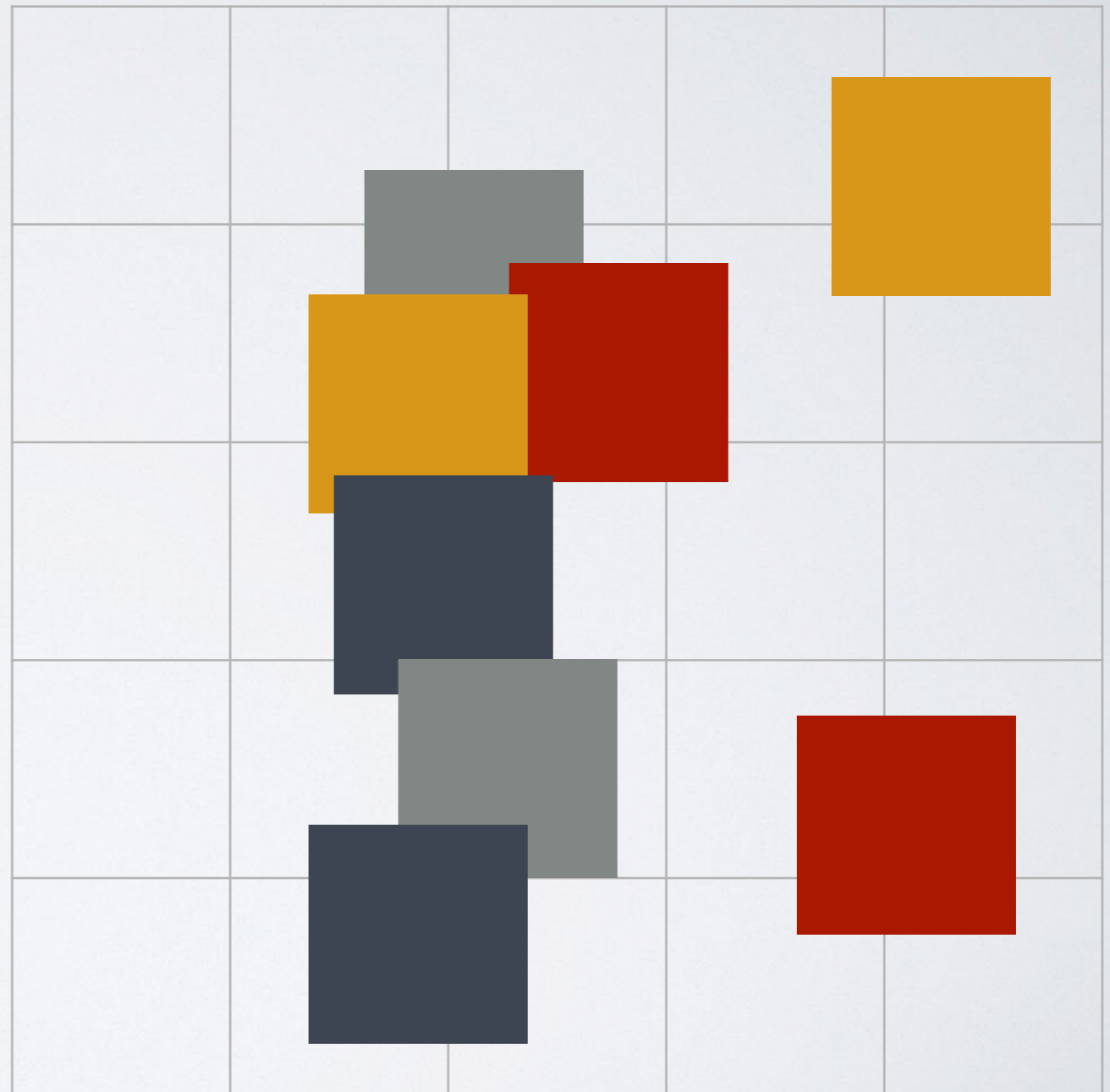
- Next up: a quick introduction to simulations

# STATE OF NEWTONIAN SIMULATIONS



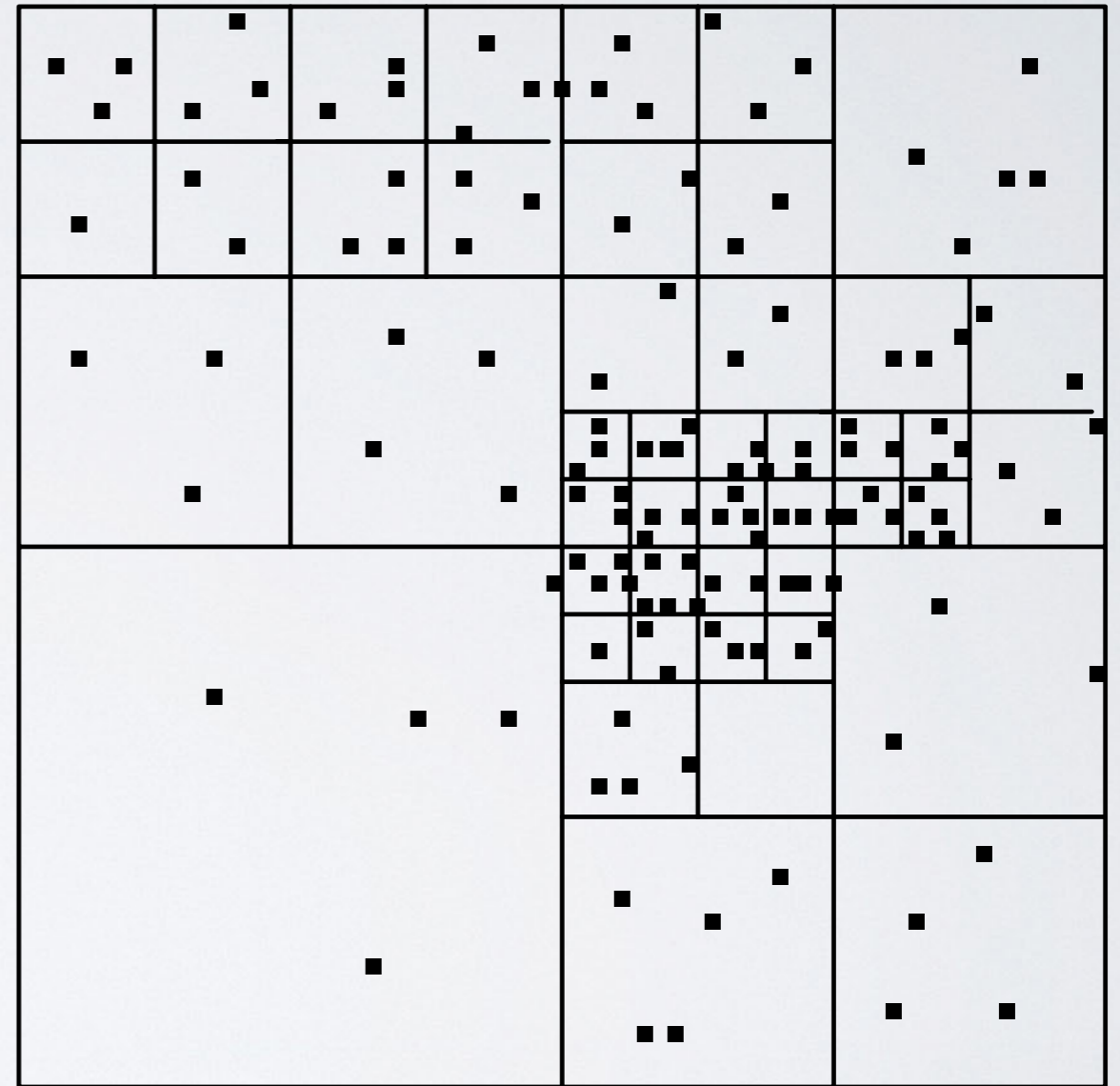
# STATE OF NEWTONIAN SIMULATIONS

- Particle - Particle:
  - $N^2$  operations
  - Slow but exact
- Particle - Mesh:
  - Use FFT for poisson-solver
  - $N \log N$  operations.
  - Fast but low resolution



# STATE OF NEWTONIAN SIMULATIONS

- Particle - Particle:
  - $N^2$  operations
  - Slow but exact
- Particle - Mesh:
  - Use FFT for poisson-solver
  - $N \log N$  operations.
  - Fast but low resolution
- Particle - Tree:
  - Tunable resolution
  - Fast multipole method:  $N$  operations (linear!)



[e.g. Beatson, Greengard, 1997]

# NONLINEAR STRUCTURE

- 2pt correlations no longer carry much information
- What are the parameters that describe output and discriminate models?

Halo masses

Halo shapes

- It's a mess...

Halo substructure

Filament distribution

Delaunay-tessellation  
field estimator

Halo velocities

Void abundance

Origami

many more...



# NONLINEAR STRUCTURE

- My favourite measure:

Abundance of high-velocity cluster collisions

## **Bullet Cluster: A Challenge to $\Lambda$ CDM Cosmology**

Jounghun Lee

*Department of Physics and Astronomy, FPRD, Seoul National University, Seoul 151-747,  
Korea: jounghun@astro.snu.ac.kr*

Eiichiro Komatsu

*Texas Cosmology Center and Department of Astronomy, The University of Texas at  
Austin, 1 University Station, C1400 Austin, TX 78712, USA*

# BARYON PHYSICS

- I will not go into detail.
- Lagrangian: Smoothed Particle Hydrodynamics (SPH)
- Eulerian: Adaptive Mesh Refinement
- Mix: moving meshes.

# WHAT'S IN IT FOR YOU?

- Crucial to get handle on accuracy.
  - Various test cases agreed on in community
  - Physics is too complex to model from first principles
  - Quite ad hoc.
- One big drawback:
  - IR-end know analytically, UV end NOT
  - As opposed to e.g. black-hole mergers.

shock  
treatment

removal of  
spurious structure

AGN feedback:  
huge effect

# WHAT'S IN IT FOR YOU?

## The galaxy population in cold and warm dark matter cosmologies

Lan Wang<sup>1\*</sup>, Violeta Gonzalez-Perez<sup>2,3</sup>, Lizhi Xie<sup>4</sup>, Andrew P. Cooper<sup>2</sup>,  
Carlos S. Frenk<sup>2</sup>, Liang Gao<sup>1</sup>, Wojciech A. Hellwing<sup>3,5</sup>, John Helly<sup>2</sup>,  
Mark R. Lovell<sup>6,7</sup>, and Lilian Jiang<sup>2</sup>

regions within them that are as empty as the observed Local Void. Thus, a highly complete census of the Local Volume and future surveys of void regions could provide constraints on the nature of dark matter.

# WHAT'S IN IT FOR YOU?

- It is as accurate as the last error you found.
  - That propagates into all conclusions you draw.
- I found one!

PT: GR

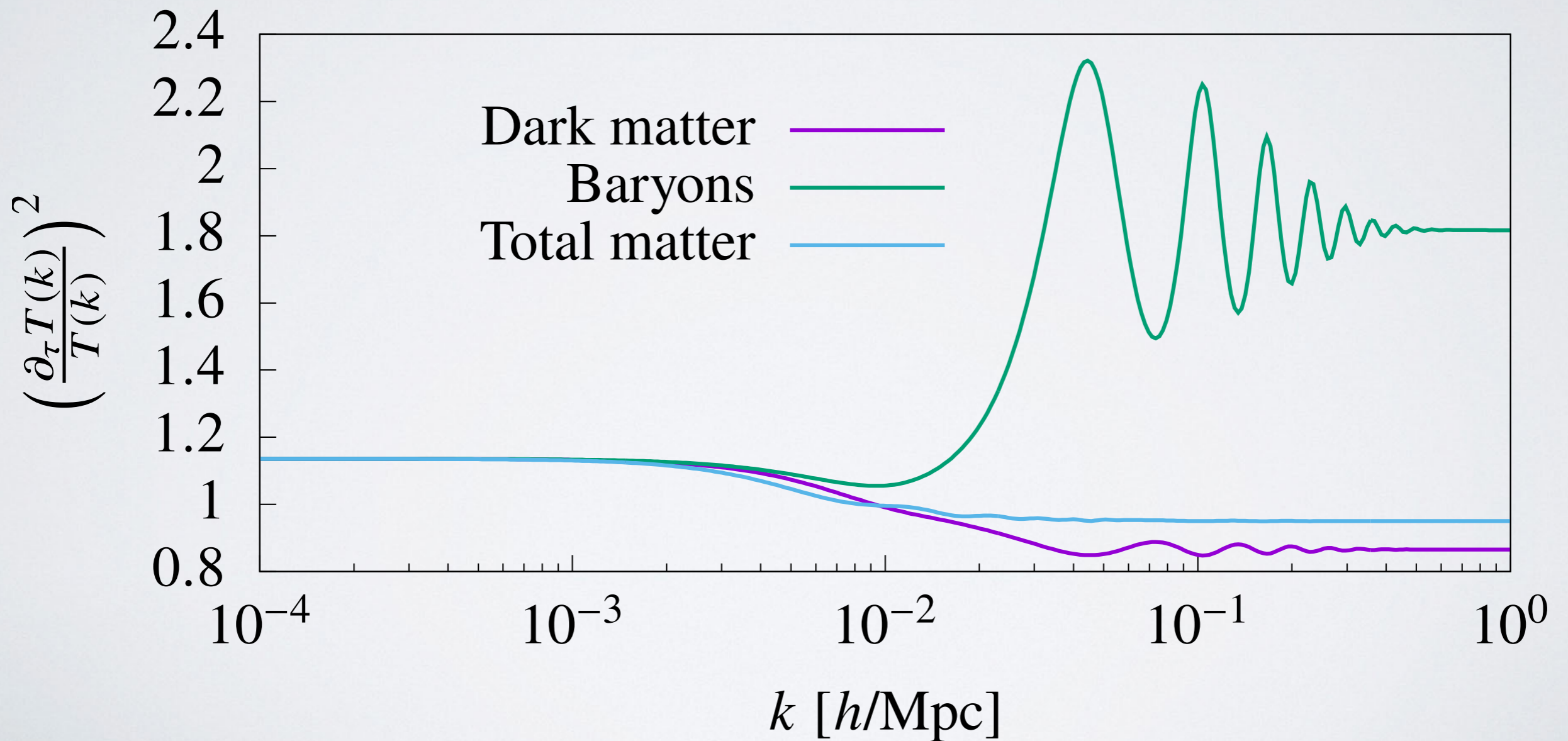
# SIMULATION: NEWTONIAN

- Interface from GR to start of simulation has long been hand-waved.
- N-body gauge gives clarity

PT: GR

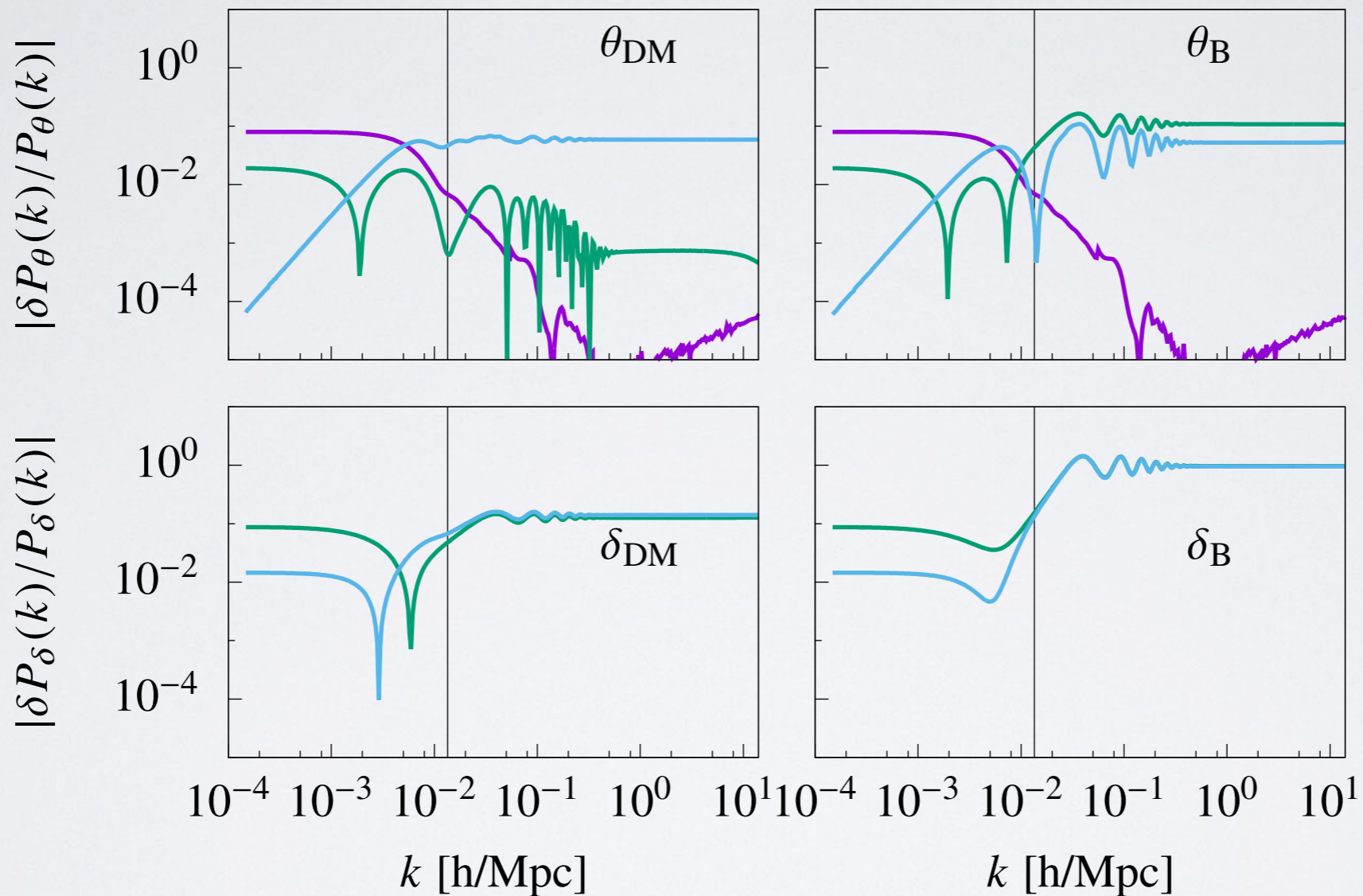
# SIMULATION: NEWTONIAN

Linear growth rate at  $z = 127$



# OLD APPROXIMATIONS

Linear spectra at  $z = 127$

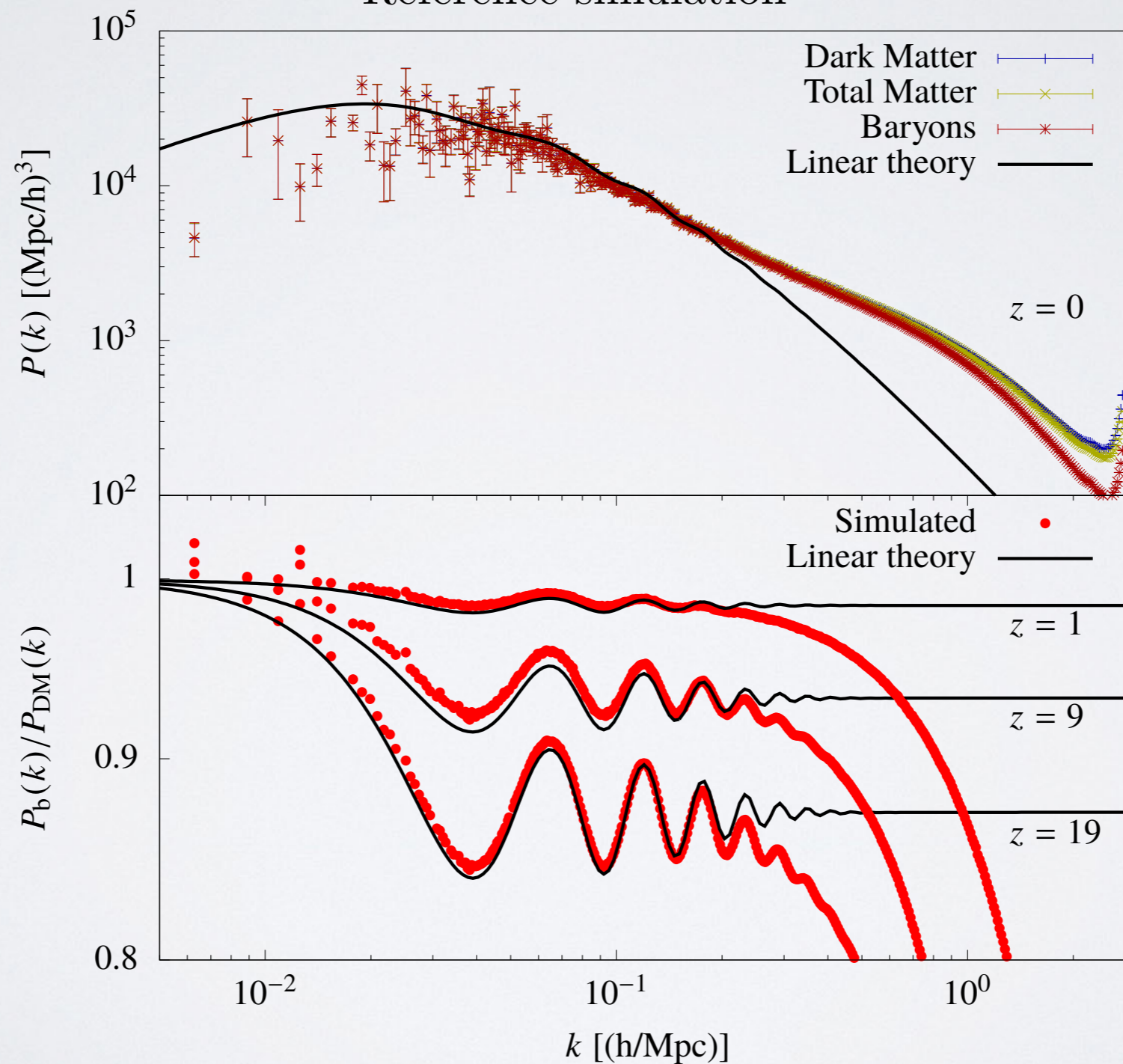


Longitudinal-gauge velocities ———  
Rescaling  $P(k, z = 0)$  ———  
All as total matter ———

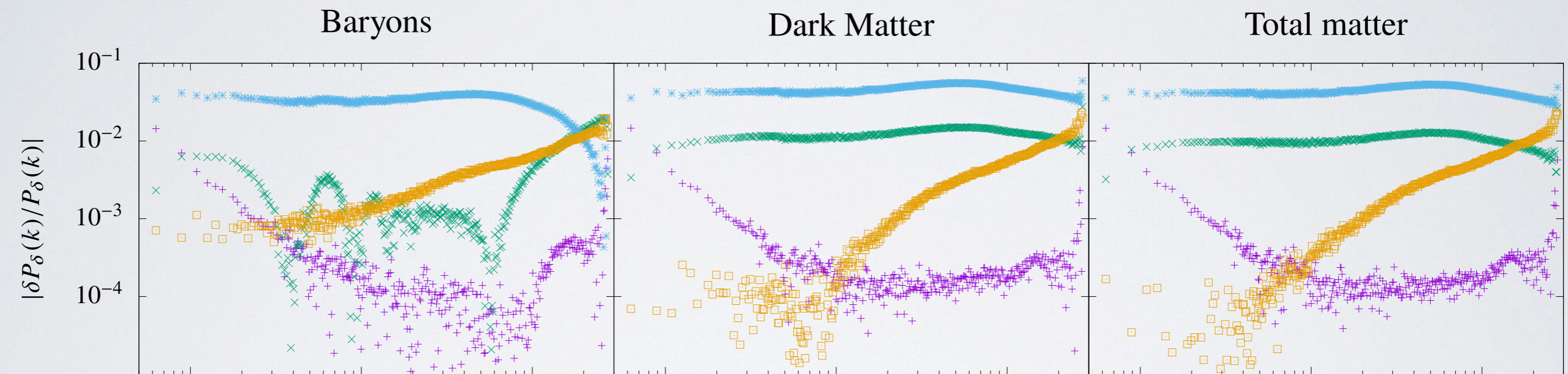


# LONG LIVE THE HPC-QCD GRID AT CERN

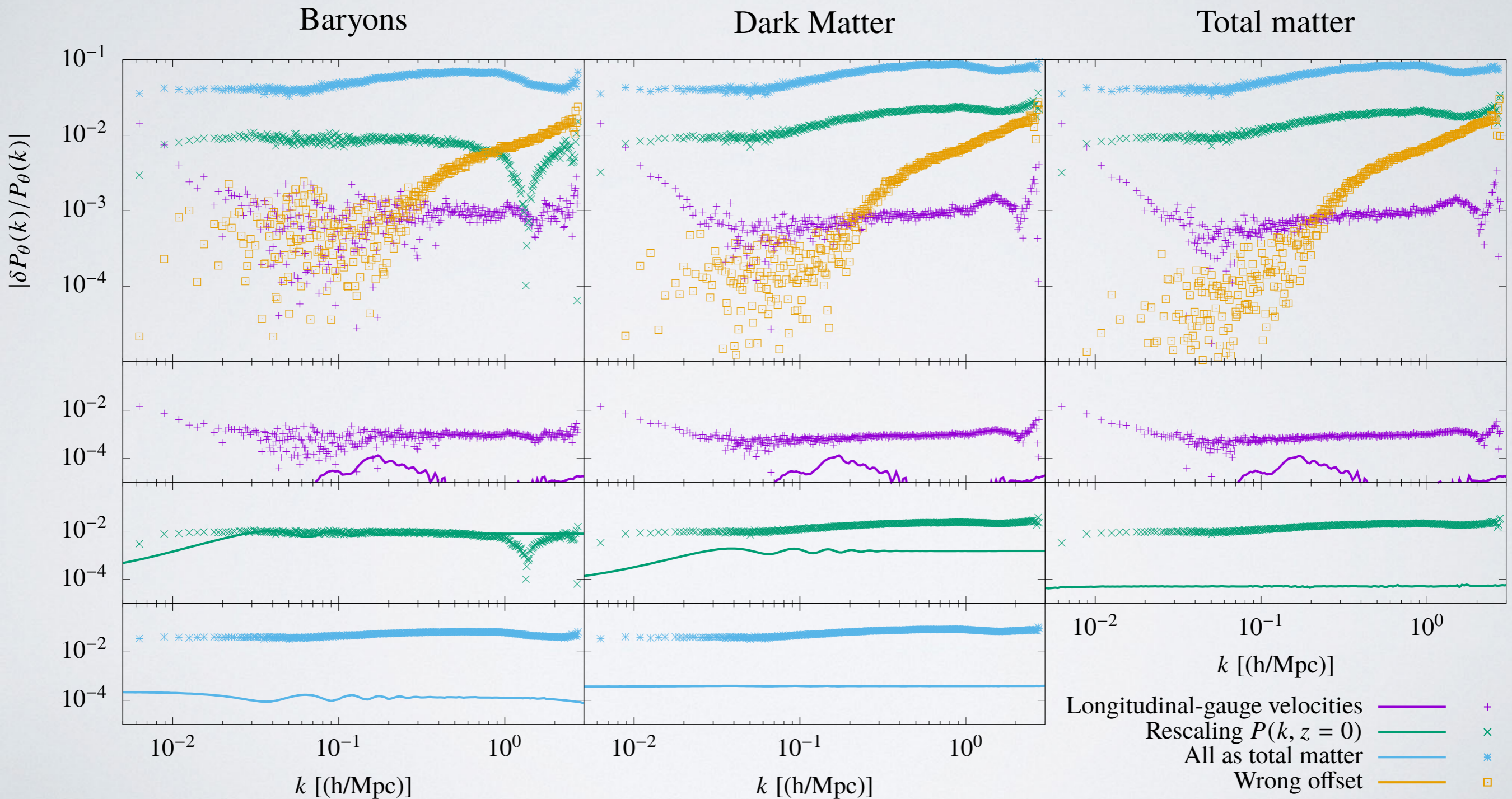
Reference simulation



# ERRORS IN CONTEMPORARY SIMULATIONS



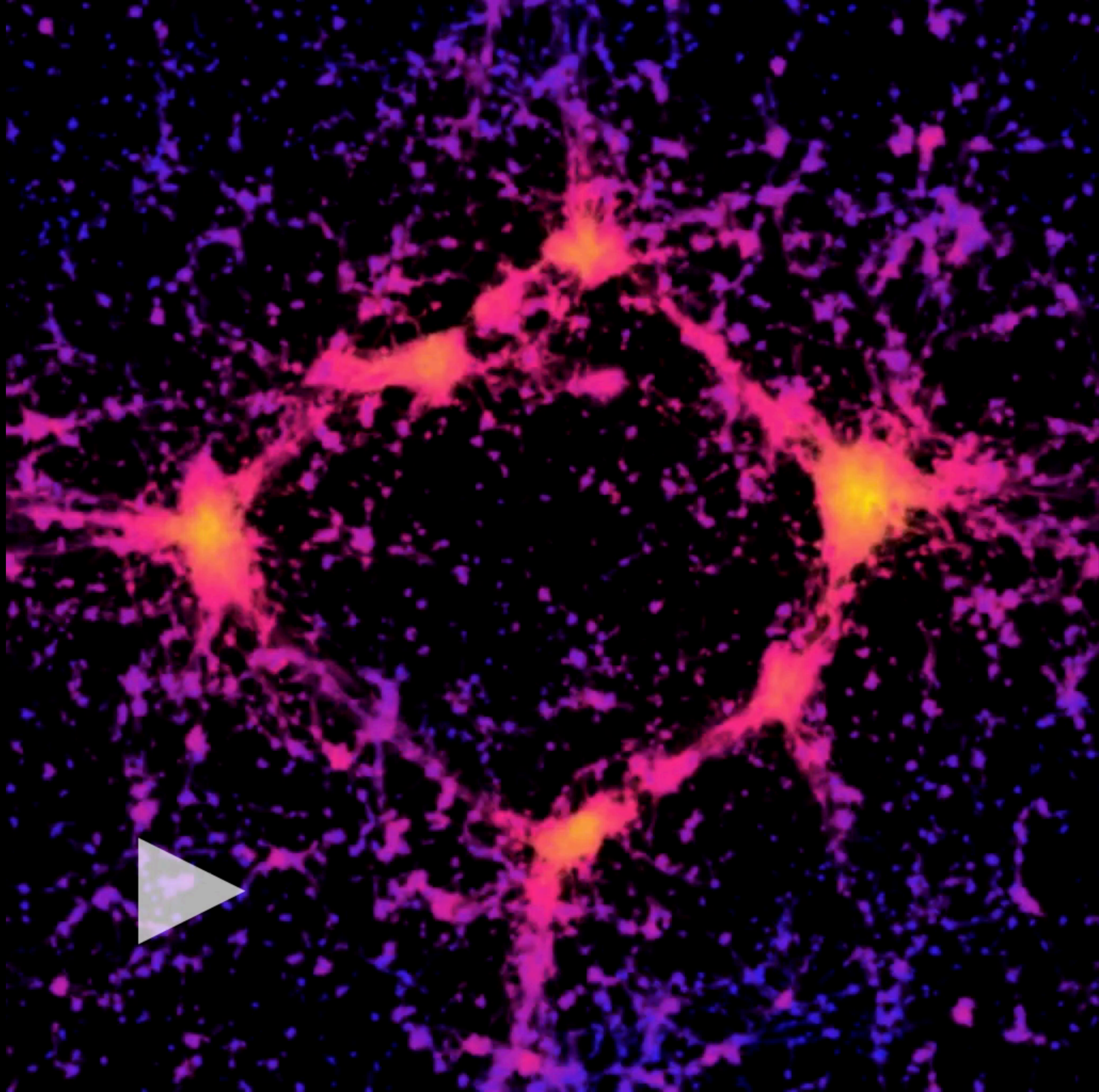
# ERRORS IN CONTEMPORARY SIMULATIONS



# ERRORS IN CONTEMPORARY SIMULATIONS

- up to 10% is however still smaller than the ad-hoc modelled effect of AGN's, starting at 10%.

THE PROMISED  
PRETTY PICTURE



# CONCLUSIONS

- I do not want you to think that simulations are dirty
- but to respectfully think that there is some cleaning left to be done
- DM only simulations differ from reality by up to 10%
- Baryon-DM simulations idem
- Newtonian simulations miss 1% radiation