Higher Derivative Corrections in M-theory Via Local Supersymmetry

Yoshifumi Hyakutake (Osaka Univ.)

Based on the works with S. Ogushi

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- hep-th/0601092; JHEP0602(2006)068
- hep-th/0703154; PTP118(2007)

1. Introduction

- Divergences in 4 dim. quantum gravity

It is important to formulate gravitational interaction in a way consistent with quantum mechanics.

Perturbative approach to quantum gravity, however, is problematic. Loop corrections give divergences in UV.

Non-susy gravity: divergence at 1 or 2 loop
Supergravity : divergence at 3 loop (except N=8)

- Superstring theory as a quantum gravity

In order to cure the divergences, we consider not a point particle object but a string object.

Then interactions are smeared around the string scale, and no UV divergence appears in superstring theory.

- Superstring theory in low energy region

Superstring theory is perturbatively defined around 10 dim. flat space-time.

And the low energy limit of superstring theory is described by supergravity.

$$\text{Superstring} \xrightarrow{\ell_s \sim 0} \text{Supergravity}$$

- Quantum correction to supergravity

Loop calculations in superstring theory give corrections to the low energy supergravity.

$$\text{Superstring} \longrightarrow \text{Supergravity + higher derivative corrections}$$

$$\sim (R_{abcd}R^{abcd})^2$$

- In this talk

I will discuss higher derivative corrections in M-theory which is expected as a strong coupling limit of type IIA superstring theory.

In order to determine the structure of higher derivative terms, I will impose local supersymmetry.

Noether's procedure + computer program

As a result I found that the local supersymmetry is powerful and will determine the structure completely.

Plan :

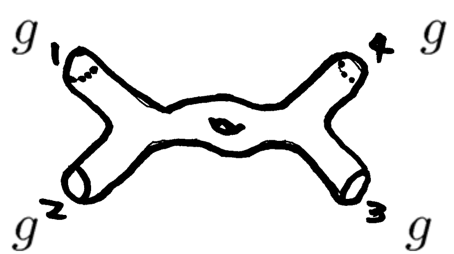
1. Introduction
2. Higher Derivative Corrections in Type IIA
3. Higher Derivative Corrections in M-theory
4. Conclusions and Discussions

2. Higher Derivative Corrections in Type IIA

Higher derivative corrections in string theories are considerably investigated in various ways

- String scattering amplitude
- Non linear sigma model
- Superfield method
- Duality
- Noether's method ... and so on

Higher derivative corrections arise from 4pt amplitudes at one-loop. (0,1,2,3pt amplitudes at 1-loop = 0)



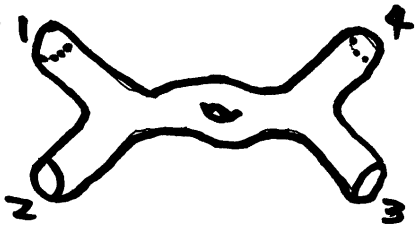
4pt 1-loop

- **Structure of R^4 terms** $\sim t_8 t_8 R^4$

Let us consider on-shell 4 graviton amplitude at 1-loop.

Green, Schwarz

$k_i^1, \zeta_i^1 \otimes \tilde{\zeta}_j^1$ $k_i^4, \zeta_i^4 \otimes \tilde{\zeta}_j^4$



$k_i^2, \zeta_i^2 \otimes \tilde{\zeta}_j^2$ $k_i^3, \zeta_i^3 \otimes \tilde{\zeta}_j^3$

$$\sim \int dp \mathrm{Tr}(\Delta V_1 \Delta V_2 \Delta V_3 \Delta V_4)$$

Δ : propagator,

V_n : graviton vertex op.

Trace over zero modes gives a kinematical term K which linearly depend on each $k_i^n, \zeta_i^n, \tilde{\zeta}_i^n$.

Gross, Witten

Gross, Sloan

$$K = t_8^{ijklmnop} t_8^{abcdefgh} f_{ij}^1 f_{kl}^2 f_{mn}^3 f_{op}^4 \otimes \tilde{f}_{ab}^1 \tilde{f}_{cd}^2 \tilde{f}_{ef}^3 \tilde{g}_{gh}^4$$
$$= t_8^{ijklmnop} t_8^{abcdefgh} R_{ijab} \cdots R_{opgh}$$

$$f_{ij}^n \equiv \frac{1}{2}(k_i^n \zeta_j^n - k_j^n \zeta_i^n)$$
$$f_{ij}^n \otimes \tilde{f}_{ab}^n = R_{ijab}$$

t_8 is a product of 4 Kronecker deltas.

Thus 4pt amplitude at 1-loop gives higher derivative corrections of $t_8 t_8 R^4$. 4pt amplitude at tree level also gives the same kinematical term.

- **Higher derivative corrections in IIA**

Gross, Witten
Grisaru, Zanon
Vafa, Witten

Known bosonic part (R^4 part) in type IIA is written as

$$\mathcal{L}_{R^4} \sim e^{-2\phi}(t_8 t_8 e R^4 + \tfrac{1}{4 \cdot 2!}\epsilon_{10}\epsilon_{10} e R^4 + \cdots) + c(t_8 t_8 e R^4 - \tfrac{1}{4 \cdot 2!}\epsilon_{10}\epsilon_{10} e R^4 - \underbrace{\tfrac{1}{6}\epsilon_{10} t_8 B R^4}_{B\wedge \mathrm{tr}(R^4),\ B\wedge \mathrm{tr}R^2 \wedge \mathrm{tr}R^2} + \cdots)$$

- **Higher derivative corrections in M**

M-theory is described by 11 dim. N=1 Supergravity.

$$2\kappa_{11}^2 S = \int d^{11}x\, e\Big(R - \frac{1}{2}\bar{\psi}_\rho \gamma^{\rho\mu\nu}\psi_{\mu\nu} - \frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\Big) - \frac{1}{3!}\int A\wedge F\wedge F + \cdots$$

$$\delta e^a{}_\mu = \bar{\epsilon}\gamma^a\psi_\mu, \quad \delta\psi_\mu = 2\mathcal{D}_\mu\epsilon, \quad \delta A_{\mu\nu\rho} = -3\bar{\epsilon}\gamma_{[\mu\nu}\psi_{\rho]}$$

Higher derivative corrections in M-theory can be obtained by lifting the IIA result.

$$t_8 t_8 e R^4 + \tfrac{1}{4!}\epsilon_{11}\epsilon_{11} e R^4, \quad t_8 t_8 e R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11} e R^4 - \tfrac{1}{6}\epsilon_{11} t_8 A R^4$$

We will check these forms by local supersymmetry

3. Higher Derivative Corrections in M-theory

- **Strategy**

Since perturbative methods in M-theory are not developed, it is impossible to obtain higher derivative terms by evaluating scattering amplitudes of membranes. The best way to determine this structure is to use the invariance under local supersymmetry

$$\boxed{\text{Noether method + computer programming}}$$

Here we mainly concentrate on bosonic terms and consider cancellation of $O(1)$ and $O(F)$ terms step by step.

Variation	O(1)	O(F)	O(F^2)	…
$O(\psi)$	○	○		
$O(\psi^3)$	×	×	×	

Q. How many R^4 terms ?

For example let us consider a term

$$B_1[1] = R_{abcd}R_{abcd}R_{efgh}R_{efgh},$$

It is possible to assign a matrix by counting number of overlapping indices

$$\begin{pmatrix} 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{pmatrix}$$

Inversely the above matrix will generate R^4 term uniquely

Thus it is important to classify possible matrices. There are 4 matrices up to permutations of 4 Riemann tensors.

$$\begin{pmatrix} 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix}$$

Finally 7 independent terms are assigned for these matrices

$$B_1[1] = R_{abcd}R_{abcd}R_{efgh}R_{efgh}, \quad B_1[2] = R_{abcd}R_{bcdh}R_{efgh}R_{aefg},$$
$$B_1[3] = R_{abcd}R_{efgh}R_{abef}R_{cdgh}, \quad B_1[4] = R_{acbd}R_{egfh}R_{aebf}R_{cgdh},$$
$$B_1[5] = R_{abcd}R_{aefg}R_{befh}R_{cdgh}, \quad B_1[6] = R_{abcd}R_{aefg}R_{bfeh}R_{cdgh},$$
$$B_1[7] = R_{acbd}R_{aefg}R_{befh}R_{cgdh}$$

Q. How many AR^4 terms ?

There are two terms

$$B_{11}[1] = -\frac{1}{3!}A \wedge \mathrm{tr}R^2 \wedge \mathrm{tr}R^2, \quad B_{11}[2] = -\frac{1}{3!}A \wedge \mathrm{tr}R^4$$

(These are related to the anomaly cancellation terms in hetero)

In order to cancel variations of these bosonic terms, it is necessary to add fermionic terms to the ansatz

$$F_1 = [eR^3\bar{\psi}\psi_2]_{92}, \quad F_2 = [eR^2\bar{\psi}_2 D\psi_2]_{25}$$

Variations of the ansatz are expanded by the following terms

$$V_1 = [eR^4\bar{\epsilon}\psi]_{116}, \quad V_2 = [eR^2DR\bar{\epsilon}\psi_2]_{88}, \quad V_3 = [eR^3\bar{\epsilon}D\psi_2]_{40}$$

The cancellation mechanism up to O(F) is sketched as

$$\begin{aligned}
\delta B_1 &\sim V_1 \oplus V_2 \\
\delta B_{11} &\sim V_1 \\
\delta F_1 &\sim V_1 \oplus V_2 \oplus V_3 \\
\delta F_2 &\sim \phantom{V_1 \oplus {}} V_2 \oplus V_3
\end{aligned}$$

244 Equations among 126 Variables

After miraculous cancellation, the bosonic part is determined as

$$\mathcal{L}_{\rm boson} = \frac{1}{24\cdot 32}a\Big(t_8 t_8 R^4 + \frac{1}{4!}\epsilon_{11}\epsilon_{11}R^4\Big) + \frac{1}{24}b\Big(t_8 t_8 R^4 - \frac{1}{4!}\epsilon_{11}\epsilon_{11}R^4 - \frac{1}{6}\epsilon_{11}t_8 A R^4\Big)$$

The first term corresponds to tree level and the second does to one-loop part in type IIA superstring

- **The next step is to examine the invariance under local supersymmetry up to O(F^2)**

In order to execute the cancellation to this order, we have to add

$$B_{21} = [eR^3F^2]_{30}, \qquad B_{22} = [eR^2D\hat{F}^2]_{24}$$

$$F_{11} = [eR^3F\bar{\psi}\psi]_{447}, \qquad F_{12} = [eR^2F\bar{\psi}_2\psi_2]_{190}$$

$$F_{13} = [eR^2DF\bar{\psi}\psi_2]_{614}, \qquad F_{14} = [eRDF\bar{\psi}_2D\psi_2]_{113}$$

The variations of this ansatz are expanded by

$$V_{11} = [eR^2DRF\bar{\epsilon}\psi]_{1563}, \qquad V_{12} = [eR^3F\bar{\epsilon}\psi_2]_{513}$$

$$V_{13} = [eR^3DF\bar{\epsilon}\psi]_{995}, \qquad V_{14} = [eRDRDF\bar{\epsilon}\psi_2]_{371}$$

$$V_{15} = [eR^2DF\bar{\epsilon}D\psi_2]_{332}, \qquad V_{16} = [eR^2DDF\bar{\epsilon}\psi_2]_{151}$$

The cancellation mechanism up to O(F^2) is sketched as

$$
\begin{array}{lllllllllllll}
\delta B_1 & \sim & V_1 \oplus V_2 & & \oplus V_{11} & & & & & \\
\delta B_{11} & \sim & V_1 & & \oplus V_{11} & & \oplus V_{13} & & & \\
\delta F_1 & \sim & V_1 \oplus V_2 \oplus V_3 & & & \oplus V_{12} & \oplus V_{13} & & & \\
\delta F_2 & \sim & \quad\; V_2 \oplus V_3 & & & \oplus V_{12} & & \oplus V_{14} & \oplus V_{15} & \\
\delta B_{21} & \sim & & & V_{11} & & \oplus V_{13} & & & \\
\delta B_{22} & \sim & & & & & & V_{14} & & \oplus V_{16} \\
\delta F_{11} & \sim & & & V_{11} & \oplus V_{12} & \oplus V_{13} & & & \\
\delta F_{12} & \sim & & & & V_{12} & & & & \\
\delta F_{13} & \sim & & & & & V_{13} & \oplus V_{14} & \oplus V_{15} & \oplus V_{16} \\
\delta F_{14} & \sim & & & & & & V_{14} & \oplus V_{15} & \oplus V_{16}
\end{array}
$$

4169 Equations among 1544 Variables

From this cancellation we obtain

$$\mathcal{L}_{\text{boson}} = \tfrac{1}{24} b\Big(t_8 t_8 R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11} R^4 - \tfrac{1}{6}\epsilon_{11} t_8 A R^4\Big) + \text{(terms with 2 parameters)}$$

The structure of R^4 terms is completely determined by local supersymmetry

- ## Vanishing theorem

Tree and one-loop amplitudes only contribute to R^4 terms

Proof :

In 11 dim. there is only one superinvariant which contain R^4 terms. These become tree level or 1-loop terms in type IIA by Kaluza-Klein reduction. No terms more than one-loop.

11d		10d	
		$e^{-2\Phi}R^4$	Sum of KK non-zero modes (Green, Vanhove)
R^4	$\nearrow$		
	$\rightarrow$	R^4	KK zero mode
	$\times$	$e^{2(g-1)\Phi}R^4 (g>1)$	

4. Conclusions and Discussions

The higher derivative corrections in M-theory are considered by employing local supersymmetry.

R^4 terms are completely determined, and this result is consistent with the result of type IIA

R^3F^2 and R^2DF^2 terms are governed by two parameters. These can be fixed by using the scattering amplitude in type IIA

After dimensional reduction we can obtain many terms which are not known so far