Higher Derivative Corrections in M-theory via Local Supersymmetry

Yoshifumi Hyakutake[a]

Institute for Higher Education Research and Practice, Osaka University,
Machikaneyama 1-16, Toyonaka, Osaka 560-0043, Japan

Abstract. We investigate higher derivative corrections in M-theory by applying Noether's method. Cancellation of variations, which contain linear terms in 4-form field strength, under local supersymmetry is executed with the aid of computer programming. Structure of R^4 terms is uniquely determined and exactly matches with one-loop effective terms in type IIA superstring theory.

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1 Introduction

Superstring theory is promising as a theory of quantum gravity, and interactions of strings are described by supergravity in the low energy limit[1,2]. Scattering amplitudes of strings also contain higher derivative interactions which give stringy or quantum corrections to supergravity[2].

Among the higher derivative terms, quartic terms of Riemann tensor, R^4 terms, in type II superstring theories are considerably investigated in the last twenty years from various viewpoints. Especially tree level effective action which include R^4 terms is obtained by combining results of scattering amplitude of four gravitons and 4-loop computation in non-linear sigma model[3,4]. Effective action at one-loop order also contains the R^4 terms. We will review some aspects of R^4 terms in sect. 2.

By lifting the effective action in type IIA superstring theory to 11 dimensions, higher derivative corrections in M-theory are revealed. It is also possible to directly confirm the existence of higher derivative terms in eleven dimensions from the scattering amplitudes of superparticles[5,6,7,8] or superspace formalism[9,10,11,12,13].

Without relying on scattering amplitudes, higher derivative corrections in superstring theories are determined by imposing local supersymmetry[14,15,16,17,18,19,20]. This paper is a summary of works of refs. [18,19,20]. The structure of higher derivative terms in M-theory is investigated by applying Noether's method. Variations under local supersymmetry which are linearly dependent on 4-form field strength are cancelled with the aid of a computer programming[1]. We will see that the local supersymmetry is powerful enough to

determine the structure of R^4 terms uniquely. A part of R^3F^2 terms is also determined.

The contents of our paper is as follows. In section 2, we briefly summarize higher derivative corrections in type IIA superstring theory. In section 3, higher derivative corrections in M-theory are investigated via local supersymmetry. We find that the structure of R^4 terms is uniquely determined and is consistent with one-loop scattering amplitudes in type IIA superstring theory. Section 4 is devoted to conclusions and discussions.

2 Higher Derivative Corrections in Type IIA Superstring Theory

It is well known that the low energy limit of superstring theory is described by supergravity. This can be shown explicitly by calculating the scattering amplitudes of massless states. For example, low energy limit of amplitude of three gravitons at tree level correctly reproduces three points vertex of gravitons in the supergravity Lagrangian.

Scattering amplitudes of massless states in superstring theory, however, include corrections to the supergravity Lagrangian. These terms contain positive power of string length ℓ_s and string coupling constant g_s, and are called higher derivative corrections. In type IIA superstring theory, one of those corrections appear in amplitude of four gravitons which is written by

$$\mathcal{M}_4^{\text{tree}} = -\frac{1}{2\kappa_{10}^2} \frac{\ell_s^6}{2^9 \cdot 4!} T(s,t,u) \, t_8 t_8 R^4. \quad (1)$$

Here $2\kappa_{10}^2 = (2\pi)^7 \ell_s^8 g_s^2$, t_8 is composed of 4 Kronecker deltas, and $t_8 t_8 R^4$ is an abbreviation of a product of two t_8 tensors and four Riemann tensors. Explicit expression can be found in ref. [19]. $T(s,t,u)$ is a function

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[a] *Email:* hyaku@het.phys.sci.osaka-u.ac.jp

[1] I employed mathematica code, GAMMA.m, built by U. Gran[21].

of Mandelstam variables and defined by using gamma functions as

$$T(s,t,u) = \frac{\Gamma(-\frac{\ell_s^2}{4}s)\Gamma(-\frac{\ell_s^2}{4}t)\Gamma(-\frac{\ell_s^2}{4}u)}{\Gamma(1+\frac{\ell_s^2}{4}s)\Gamma(1+\frac{\ell_s^2}{4}t)\Gamma(1+\frac{\ell_s^2}{4}u)}$$
$$\sim \frac{2^6}{\ell_s^6 stu} + 2\zeta(3) + \cdots . \tag{2}$$

The second line in the above equation gives the low energy limit of the amplitude. The first term in that line just corresponds to amplitudes of exchanging four gravitons in s, t and u channels in type IIA supergravity[22]. The second term contain a Riemann zeta function $\zeta(3)$, which is irrational, and contributes as a stringy correction to the supergravity. In this way it is basically possible to derive higher derivative corrections from string scattering amplitudes, and terms which involve fourth power of Riemann tensors become

$$\mathcal{L}_{\text{tree}} = -\frac{e^{-2\phi}}{2\kappa_{10}^2}\frac{\ell_s^6}{2^8 \cdot 4!}\zeta(3)$$
$$\times \left(t_8 t_8 R^4 + \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10}R^4\right). \tag{3}$$

The second term is first derived by calculations in non-linear sigma model[4]. By counting mass dimensions, we see that above Lagrangian should be order of ℓ_s^6.

At 1-loop level, 0, 1, 2, 3 points amplitudes of massless states are zero, and higher derivative corrections first arise out of amplitude of four gravitons.

$$\mathcal{M}_4^{\text{1-loop}} = \frac{1}{2\kappa_{10}^2}\frac{\pi g_s^2 \ell_s^6}{2^8 \cdot 4!} I(s,t,u)\, t_8 t_8 R^4. \tag{4}$$

$I(s,t,u)$ is an integral of a modular function on a torus and approximated in the low energy limit as

$$I(s,t,u) \sim \frac{\pi}{3}. \tag{5}$$

Finally the effective action which contain fourth power of Riemann tensor at 1-loop level become

$$\mathcal{L}_{\text{1-loop}} = \frac{1}{2\kappa_{10}^2}\frac{\ell_s^6}{2^8 \cdot 4!}\frac{\pi^2}{3}g_s^2 \tag{6}$$
$$\times \left(t_8 t_8 R^4 - \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10}R^4 - \frac{1}{6}\epsilon_{10}t_8 BR^4\right).$$

The last term comes from 5 points amplitude of antisymmetric tensor field and 4 gravitons. Note that the signs in front of $\epsilon_{10}\epsilon_{10}R^4$ are opposite. Those are sensitive to the chirality of the theory.

So far we have concentrated on terms which involve fourth power of the Riemann tensor. Compared with gravitons, our knowledge on higher derivative terms which consist of NS B field and R-R potentials is poor. In order to determine these terms, it is necessary to calculate more than 5 points amplitudes and extract their low energy limit correctly. This is very tough work. In the next section we consider higher derivative corrections in M-theory and try to determine the structure of these terms only by employing local supersymmetry.

3 Higher Derivative Corrections in M-theory

The string coupling constant in type IIA superstring theory is identified with 11th circular direction with a radius $R = g_s \ell_s$, and strong coupling limit of this theory is described by 11 dimensional M-theory. The low energy limit of M-theory is described by 11 dimensional supergravity, and gravity multiplet consists of a vielbein $e^a{}_\mu$, a 3-form field A and a Majorana gravitino ψ_μ. In M-theory a supermembrane plays a fundamental role and often capture nonperturbative aspects of superstring. However, quantization of supermenbrane is incomplete, so it is impossible to obtain higher derivative terms by evaluating scattering amplitudes of supermembranes.

Therefore we should employ another method to determine the structure of higher derivative corrections in M-theory. Since the gravity multiplet is simple, it will be possible to fix the structure by local supersymmetry. In this section we will apply Noether's method with the aid of computer programming, and show that terms which involve fourth power of the Riemann tensor, which are abbreviated as $[eR^4]$, can be determined uniquely.

First let us classify independent terms in $[eR^4]$. Here we only consider terms which do not contain Ricci tensors or scalar curvatures. Because of this restriction, four indices of one Riemann tensor should be contracted with those of other Riemann tensors. Then it is convenient to consider a 4×4 symmetric matrix whose (i,j) component represent a number of contracted indices between i-th and j-th Riemann tensors. The sum of each column should be 4, which is the number of indices in the Riemann tensor, and the symmetric matrix is controlled by two parameters as

$$\begin{pmatrix} 0 & a & b & 4-a-b \\ a & 0 & 4-a-b & b \\ b & 4-a-b & 0 & a \\ 4-a-b & b & a & 0 \end{pmatrix}, \tag{7}$$

where $0 \le a, b, a+b \le 4$. Without loss of generality we set $4-a-b \le b \le a$, since this is always possible by properly exchanging positions of Riemann tensors. Then it is easy to see that there are 4 possible matrices.

$$\begin{pmatrix} 0&4&0&0 \\ 4&0&0&0 \\ 0&0&0&4 \\ 0&0&4&0 \end{pmatrix}, \begin{pmatrix} 0&3&1&0 \\ 3&0&0&1 \\ 1&0&0&3 \\ 0&1&3&0 \end{pmatrix}, \begin{pmatrix} 0&2&2&0 \\ 2&0&0&2 \\ 2&0&0&2 \\ 0&2&2&0 \end{pmatrix}, \begin{pmatrix} 0&2&1&1 \\ 2&0&1&1 \\ 1&1&0&2 \\ 1&1&2&0 \end{pmatrix}, \tag{8}$$

Patterns of contractions of two Riemann tensors are considerably restricted by the properties of the Riemann tensor, and possible expressions are given by

$$R_{abcd}R_{abcd}, \quad R_{abcx}R_{abcx},$$
$$R_{abxx}R_{abxx}, \quad R_{axbx}R_{axbx}, \tag{9}$$

where x represents a blank and a, b, c and d are contracted indices. By using the properties of the Riemann tensor we see that there are 7 independent terms

in $[eR^4]$.

$$R_{abcd}R_{abcd}R_{efgh}R_{efgh}, \; R_{abcg}R_{abch}R_{defg}R_{defh},$$
$$R_{abcd}R_{abef}R_{cdgh}R_{efgh}, \; R_{acbd}R_{aebf}R_{cgdh}R_{egfh},$$
$$R_{abef}R_{abgh}R_{cedg}R_{cfdh}, \; R_{abef}R_{abgh}R_{cedg}R_{chdf}, \quad (10)$$
$$R_{aebf}R_{agbh}R_{cedg}R_{cfdh}.$$

This procedure is systematic and it is possible to execute this by employing computer programming. In order to supersymmetrize terms in $[eR^4]$, it is necessary to add more possible terms in the effective action, and the result by computer programming is summarized as

$$B_1 = [eR^4]_7, \qquad B_{11} = [e\epsilon_{11}AR^4]_2,$$
$$F_1 = [eR^3\bar\psi\psi_2]_{92}, \quad F_2 = [eR^2\bar\psi_2 D\psi_2]_{25}. \quad (11)$$

Here ϵ_{11} is totally antisymmetric tensor in 11 dimensions, and ψ_2 represents a field strength of the Majorana gravitino. Each number in the subscript shows that of independent terms. Details of these expressions are found in ref. [19].

Supersymmetric transformations of massless fields are the same as those of 11 dimensional supergravity, and are abbreviated as[23]

$$\delta e = [\bar\epsilon\psi], \quad \delta\psi = [D\epsilon], \quad \delta A = [\bar\epsilon\psi]. \quad (12)$$

ϵ is a parameter which belongs to a Majorana spinor representation. Then variations of B_1, B_2, F_1 and F_2 are expanded by terms in 3 classes, V_1, V_2 and V_3.

$$V_1 = [eR^4\bar\epsilon\psi]_{116},$$
$$V_2 = [eR^2 DR\bar\epsilon\psi_2]_{88}, \quad (13)$$
$$V_3 = [eR^3\bar\epsilon D\psi_2]_{40}.$$

Then cancellation mechanism of these variations are summarized in the following table.

$$\delta B_1 = V_1 \oplus V_2,$$
$$\delta B_{11} = V_1,$$
$$\delta F_1 = V_1 \oplus V_2 \oplus V_3, \quad (14)$$
$$\delta F_2 = \qquad V_2 \oplus V_3.$$

Local supersymmetry requires that the right hand side in the above table should be zero. Thus we obtain 244 equations among 126 variables. After miraculous cancellation, the bosonic part is determined as

$$\mathcal{L} = a\big(t_8 t_8 R^4 + \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4\big)$$
$$+ b\big(t_8 t_8 R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4 - \tfrac{1}{6}\epsilon_{11}t_8 AR^4\big), \quad (15)$$

where a and b are free parameters. The first line matches with the result of tree level scattering amplitude (3), and the second line is consistent with 1-loop amplitude (6) in type IIA superstring theory.

So far we have neglected terms which involve 4-form field strength. Thus the next step is to examine the invariance under local supersymmetry up to

$\mathcal{O}(F^2)$. In order to execute the cancellation to this order, we have to add

$$B_{21} = [eR^3F^2]_{30}, \qquad B_{22} = [eR^2(DF)^2]_{24},$$
$$F_{11} = [eR^3F\bar\psi\psi]_{447}, \quad F_{12} = [eR^2F\bar\psi_2\psi_2]_{190}, \quad (16)$$
$$F_{13} = [eR^2DF\bar\psi\psi_2]_{614}, \; F_{14} = [eRDF\bar\psi_2 D\psi_2]_{113}.$$

Variations of these terms are expanded by terms in 6 classes.

$$V_{11} = [eR^2DRF\bar\epsilon\psi]_{1563}, \; V_{12} = [eR^3F\bar\epsilon\psi_2]_{513}, \quad (17)$$
$$V_{13} = [eR^3DF\bar\epsilon\psi]_{995}, \qquad V_{14} = [eRDRDF\bar\epsilon\psi_2]_{371},$$
$$V_{15} = [eR^2DF\bar\epsilon D\psi_2]_{332}, \; V_{16} = [eR^2DDF\bar\epsilon\psi_2]_{151}.$$

Cancellation mechanism of variations which are linear in F is summarized in the following table.

$$\delta B_1 = V_{11},$$
$$\delta B_{11} = V_{11} \qquad\qquad \oplus V_{13},$$
$$\delta F_1 = \qquad V_{12} \oplus V_{13},$$
$$\delta F_2 = \qquad V_{12} \qquad\qquad V_{14} \oplus V_{15},$$
$$\delta B_{21} = V_{11} \qquad \oplus V_{13}, \qquad\qquad (18)$$
$$\delta B_{22} = \qquad\qquad\qquad\qquad V_{14} \qquad \oplus V_{16},$$
$$\delta F_{11} = V_{11} \oplus V_{12} \oplus V_{13},$$
$$\delta F_{12} = \qquad V_{12},$$
$$\delta F_{13} = \qquad\qquad V_{13} \oplus V_{14} \oplus V_{15} \oplus V_{16},$$
$$\delta F_{14} = \qquad\qquad\qquad V_{14} \oplus V_{15} \oplus V_{16}.$$

Requirement of local supersymmetry in (14) and (18) gives 4169 equations among 1544 variables. From this cancellation we obtain only one solution,

$$\mathcal{L} = b\big(t_8 t_8 R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4 - \tfrac{1}{6}\epsilon_{11}t_8 AR^4\big)$$
$$+ \big([R^3F^2] \oplus [R^2(DF)^2]\big). \quad (19)$$

Therefore by imposing local supersymmetry, the structure of $[eR^4]$ is uniquely determined. The second line in the above equation contain 2 parameters and explicit forms will be reported elsewhere[20]. The coefficient b is fixed by comparing with eq. (6).

$$b = \frac{1}{2\kappa_{11}^2}\frac{\ell_p^6}{2^8 \cdot 4!}\frac{\pi^2}{3}, \quad (20)$$

where $2\kappa_{11}^2 = (2\pi)^8\ell_p^9$ and $\ell_p = g_s^{1/3}\ell_s$.

4 Conclusions and Discussions

The higher derivative terms in M-theory are investigated by applying Noether's method. The cancellation of the variations under local supersymmetry is examined to the order linear in the 4-form field strength F. Since the calculations are hard, we heavily employed the computer programming to check the cancellation.

The bosonic part of the ansatz consists of 63 terms, B_1, B_{11}, B_{21} and B_{22}, and the fermionic part of the ansatz does of 1481 terms, F_1, F_2, F_{11}, F_{12}, F_{13} and

F_{14}. The variations of the ansatz are expanded by 4169 terms, V_1, V_2, V_3, V_{11}, V_{12}, V_{13}, V_{14}, V_{15} and V_{16}. By requiring the cancellation of these variations, we obtain 4169 linear equations among 1544 coefficients in the ansatz. Then the coefficients of the bosonic part is solved as eq. (19). Remarkably the structure of the R^4 terms is uniquely determined by the requirement of the local supersymmetry. This result exactly matches with the fact that the one-loop effective action in the type IIA superstring theory survives after taking $g_s \to \infty$. When g_s is finite, 11th direction is compactified on the circle. Tree level R^4 terms in type IIA superstring theory arise after summing over non-zero modes of Kaluza-Klein mass spectrum[6]. From these arguments, we see that there is no term which corresponds to amplitudes at 2-loop or more in type IIA superstring theory. This gives proof of vanishing theorem via local supersymmetry.

The result by computer programming shows that R^3F^2 terms are governed by two parameters. It is interesting to compare the result with that obtained by the scattering amplitudes in type IIA superstring theory[25]. It is also important to check the consistency to the result obtained by the other methods[13, 26,27].

As a conclusion the local supersymmetry seems to determine the structure of the higher derivative corrections in M-theory uniquely. Similar statement can be found in the context of D-particle dynamics[28]. We will succeed the procedure executed in this paper and determine the structure of the higher derivative corrections in M-theory completely. After the determination of the action, corrections to black p-branes or cosmology will become interesting future directions[29, 30,31].

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