Dynamical breakdown of Abelian gauge chiral symmetry by strong Yukawa interactions

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based on P.B., Tomáš Brauner, Jiří Hošek: Phys.Rev. D75 (2007) 056003

(arXiv:hep-ph/0605147)

Motivations

The Standard Model:

- EW symmetry breaking by means of the Higgs mechanism
- Phenomenologically highly succesfull description

However:

- Difficult to interpret the wrong sign of the Higgs ‘mass’ squared ($M^2 < 0$)
- Fermion masses $\propto$ Yukawa couplings:

$$\frac{m_e}{m_t} \cong 10^{-5} \quad \Leftrightarrow \quad \frac{y_e}{y_t} \cong 10^{-5}$$

Our proposal

The EW symmetry breaking can be achieved dynamically:

- Scalars and Yukawa couplings are still present in theory
- However scalars do not condensate ($M^2 > 0$)
- Fermion masses are nonlinear functions of Yukawa couplings, such that:

$$\frac{m_e}{m_t} \cong 10^{-5} \quad \Leftrightarrow \quad \frac{y_e}{y_t} \cong 1$$

In this presentation we

- do not discuss the full $SU(2) \times U(1)$ theory of EW interactions
- present merely a simple $U(1)$ toy model

The model

The Lagrangian:

$$
\begin{aligned}
\mathcal{L} \;=\;& \bar{\psi}_1 \mathrm{i}\not{D}\psi_1 + \bar{\psi}_2 \mathrm{i}\not{D}\psi_2 + (\mathrm{D}_\mu\phi)^\dagger(\mathrm{D}^\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& - M^2\phi^\dagger\phi - \frac{1}{2}\lambda(\phi^\dagger\phi)^2 \\
& + y_1(\bar{\psi}_{1L}\psi_{1R}\phi + \bar{\psi}_{1R}\psi_{1L}\phi^\dagger) + y_2(\bar{\psi}_{2R}\psi_{2L}\phi + \bar{\psi}_{2L}\psi_{2R}\phi^\dagger)
\end{aligned}
$$

$U(1)_A$ symmetry:

$$
\begin{aligned}
\psi_1 &\longrightarrow \mathrm{e}^{+\mathrm{i}\theta\gamma_5}\psi_1 \\
\psi_2 &\longrightarrow \mathrm{e}^{-\mathrm{i}\theta\gamma_5}\psi_2 \\
\phi &\longrightarrow \mathrm{e}^{-2\mathrm{i}\theta}\phi
\end{aligned}
$$

The case of $M^2 < 0$

Standard Higgs mechanism takes place:

- Scalar develops VEV:

$$\langle\phi\rangle_0 = \sqrt{\frac{-M^2}{\lambda}}$$

- The spectrum:

fermions:

$$\begin{aligned} m_1 &= y_1\langle\phi\rangle_0 \\ m_2 &= y_2\langle\phi\rangle_0 \end{aligned}$$

scalars: ($\phi = \frac{1}{\sqrt{2}}(\phi_1 + \mathrm{i}\phi_2)$)

$$\begin{aligned} M_1 &= \sqrt{2\lambda}\langle\phi\rangle_0 \\ (M_2 &= 0) \end{aligned}$$

gauge boson:

$$M_A = 2\sqrt{2}g\langle\phi\rangle_0$$

The case of $M^2 > 0$

Perturbatively, nothing interesting happens:

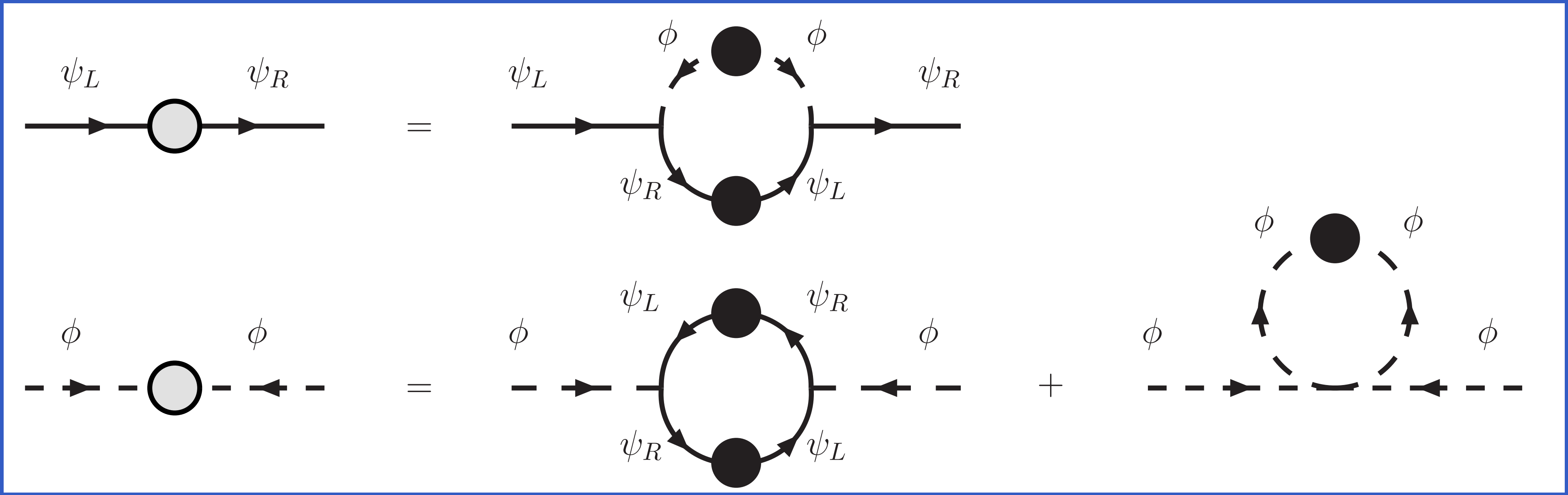
- $\langle\phi\rangle_0 = 0$
- Fermions remain massless
- No mass splitting in the scalar sector
- $\Rightarrow$ the $U(1)_A$ symmetry remains unbroken

But what about some *nonperturbative* treatment?

- Make use of the Schwinger-Dyson equations
- Consider only symmetry-breaking 2-point Green functions:

$$\begin{aligned} \Pi &\sim \langle\phi\phi\rangle \\ \Sigma_1 &\sim \langle\psi_{1L}\bar{\psi}_{1R}\rangle \\ \Sigma_2 &\sim \langle\psi_{2L}\bar{\psi}_{2R}\rangle \end{aligned}$$

Schwinger-Dyson equations



$$\Sigma_{i,p} = \mathrm{i}y_i^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\Sigma_{i,k}}{k^2 - \Sigma_{i,k}^2} \frac{\Pi_{k-p}}{[(k-p)^2 - M^2]^2 - \Pi_{k-p}^2}$$

$$\Pi_p = -\sum_{i=1,2} 2\mathrm{i}y_i^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\Sigma_{i,k}}{k^2 - \Sigma_{i,k}^2} \frac{\Sigma_{i,k-p}}{(k-p)^2 - \Sigma_{i,k-p}^2}$$

$$+\mathrm{i}\lambda \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{\Pi_k}{(k^2 - M^2)^2 - \Pi_k^2}$$

The spectrum

- Fermions: $m_1^2 = \Sigma_1^2(m_1^2)$
  $m_2^2 = \Sigma_2^2(m_2^2)$
- Scalars: $M_1^2 = M^2 + \Pi(M_1^2)$
  $M_2^2 = M^2 - \Pi(M_2^2)$
- Gauge boson:
  1. writing down $U(1)_A$ Ward identities
  2. analysing the pole structure of the corresponding $\psi_i\psi_i A_\mu$ and $\phi\phi A_\mu$ vertices:
     $\rightarrow$ massless pole $\sim$ Nambu-Goldstone boson
  3. calculating the gauge boson mass as the residue of the massless pole of its (1PI) propagator

$$M_A^2 \;=\; g^2(I_{\psi_1} + I_{\psi_2} + I_\phi)$$

Definitions of I's

$$
\begin{aligned}
I_{\psi_1} &= -8\mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\Sigma_{1,k}[\Sigma_{1,k} - 2k^2\frac{\mathrm{d}}{\mathrm{d}k^2}\Sigma_{1,k}]}{[k^2 - \Sigma_{1,k}^2]^2} \\
I_{\psi_2} &= -8\mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\Sigma_{2,k}[\Sigma_{2,k} - 2k^2\frac{\mathrm{d}}{\mathrm{d}k^2}\Sigma_{2,k}]}{[k^2 - \Sigma_{2,k}^2]^2} \\
I_{\phi} &= 16\mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k^2\Pi_k[\Pi_k - (k^2 - M^2)\frac{\mathrm{d}}{\mathrm{d}k^2}\Pi_k]}{[(k^2 - M^2)^2 - \Pi_k^2]^2}
\end{aligned}
$$

Numerical computation

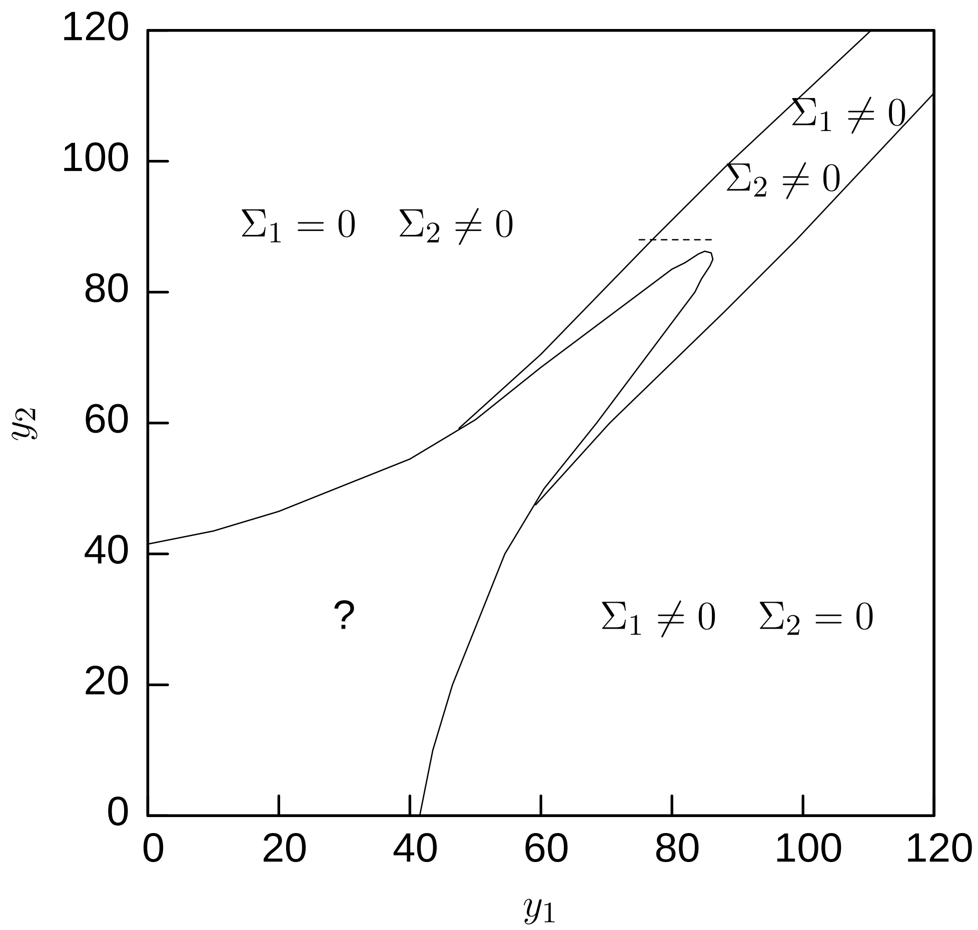
SD equations still too difficult, need some simplifications:

- Switch to Euclidean metric
  $\Rightarrow$ some poles removed (but not all)
- Set $\lambda = 0$

Then:

- The (y_1, y_2)-plane was scanned and for some values of $y_{1,2}$ a non-trivial solution could have been found
- Resulting non-trivial self-energies Σ_1, Σ_2, Π are rapidly decreasing functions
- We have probed the $y_{1,2}$-dependence of the spectrum ...

(y_1, y_2) plane



120
100
80
60
40
20
0
0
20
40
60
80
100
120
y_1
y_2
$\Sigma_1 = 0 \quad \Sigma_2 \neq 0$
$\Sigma_1 \neq 0$
$\Sigma_2 \neq 0$
?
$\Sigma_1 \neq 0 \quad \Sigma_2 = 0$

Bosonic spectrum

Bosonic spectrum is stable upon the change of the Yukawa couplings:

- Scalars:

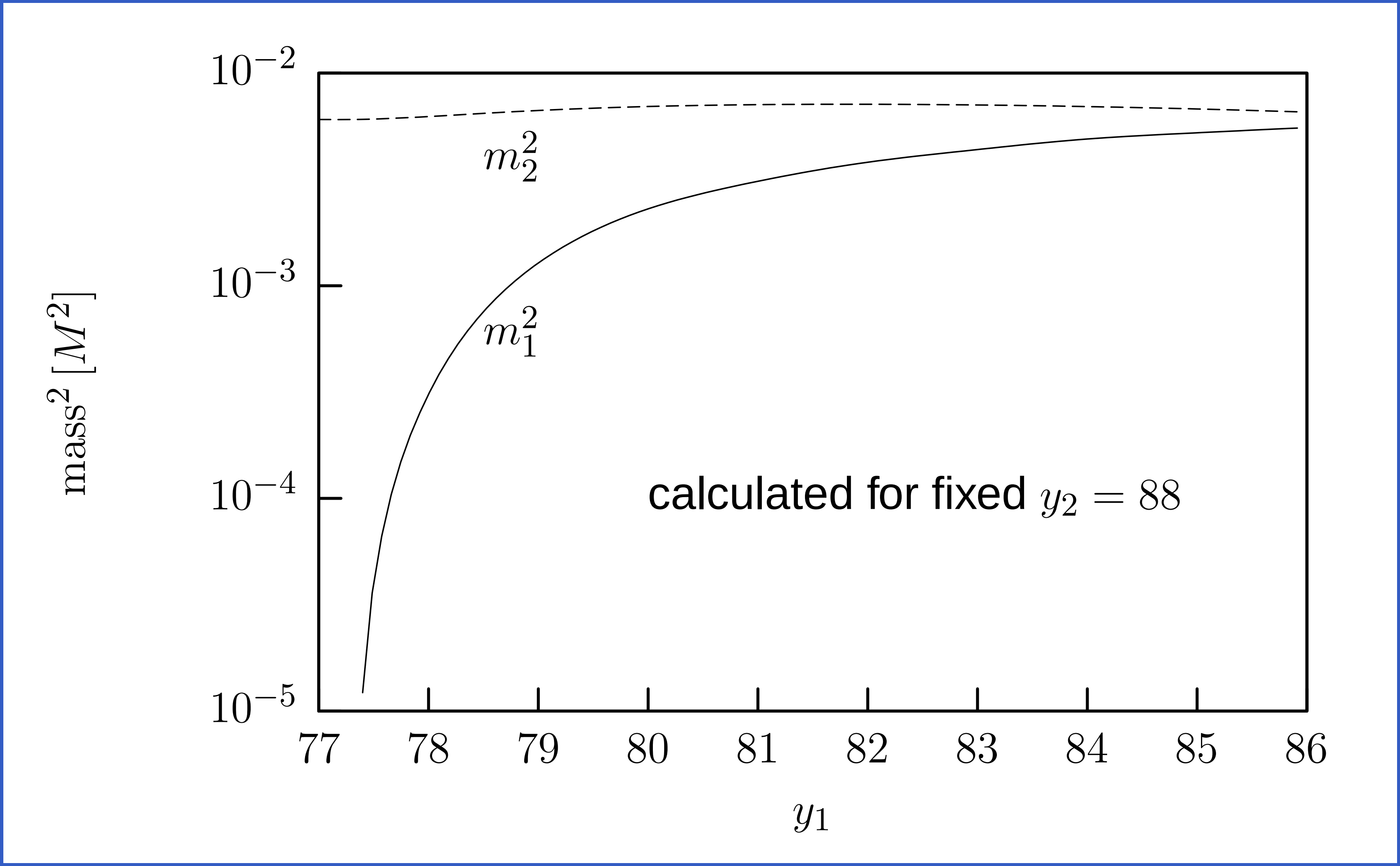
$$M_1^2 - M_2^2 \;\cong\; 10^{-4}\, M^2$$

- Gauge boson:

$$M_A^2 \;\cong\; 10^{-4}\, g^2 M^2$$

Fermionic spectrum

The fermionic spectrum is sensitive to the change of the Yukawa couplings:



Summary and outlook

We have found that

- SSB possible for $M^2 > 0$
- the ratio of dynamically generated masses can be large for not vastly different Yukawa couplings
- would-be Nambu-Goldstone boson is composite
- so far, solutions found only for large Yukawa couplings

But still much to do:

- Get some analytical insight into the above results
- Probe the self-energies in the time-like region
- Work out the realistic $SU(2)_L \times U(1)_Y$ model (sketch already present in `hep-ph/0407339`)

References

- "*Dynamical fermion mass generation by a strong Yukawa interaction*"
  - Phys.Rev. D72 (2005) 045007, `hep-ph/0505231`
  - toy model with global $U(1)_A$ symmetry
- "*Dynamical breakdown of Abelian gauge chiral symmetry by strong Yukawa interactions*"
  - Phys.Rev. D75 (2007) 056003, `hep-ph/0605147`
  - toy model with gauged $U(1)_A$ symmetry
- "*A model of flavors*"
  - `hep-ph/0407339`
  - sketch of the realistic $SU(2)_L \times U(1)_Y$ model