# Extended Higgs Models and the Transition to Exact Susy

L. Clavelli, U of Alabama

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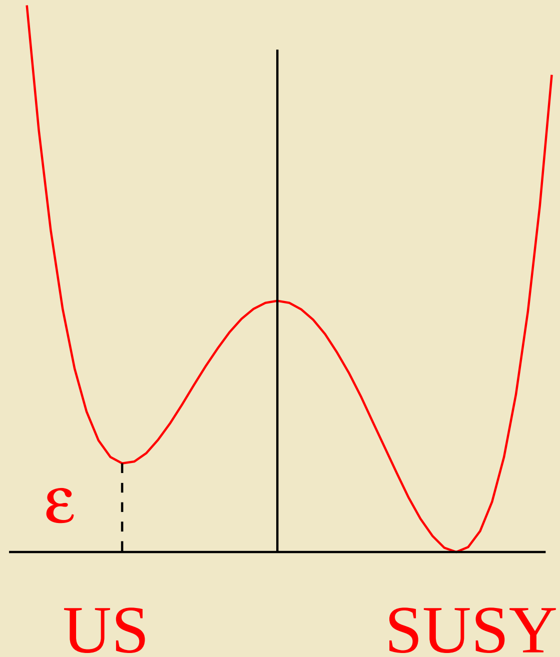
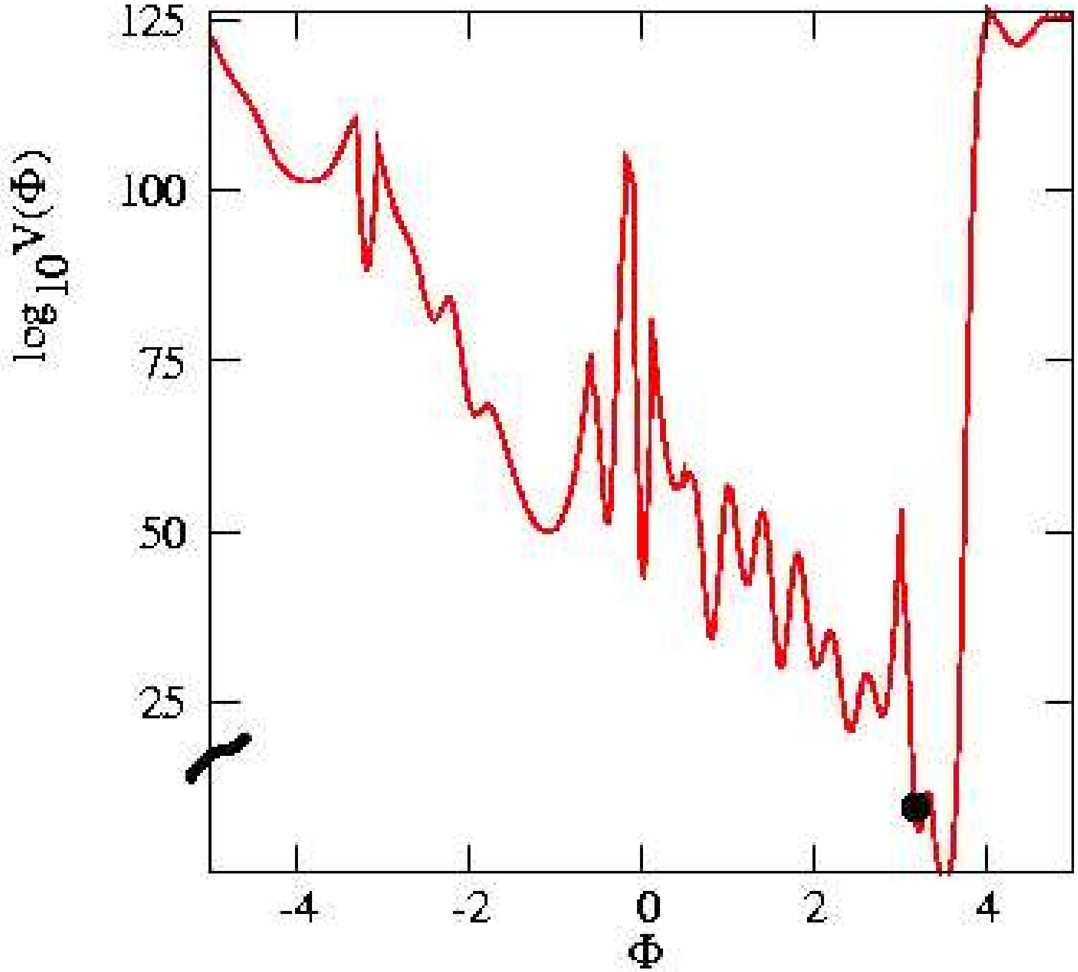



vac energy density ε = 3560 MeV/m$^3$

# The State of the Universe




**half life:** $\tau = \frac{1}{\sqrt{24\pi G_N(\epsilon_i - \epsilon_f)}} = 5.6 \cdot 10^9 yr \left( \frac{(2.3 \cdot 10^{-3} eV)^2}{(\epsilon_i - \epsilon_f)^{1/2}} \right)$

The universe should eventually make a phase transition to an exactly Supersymmetric phase.

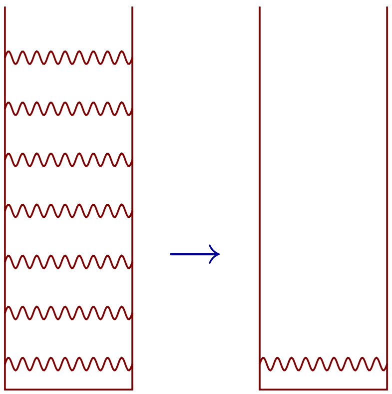
What can we predict for the LHC if we assume that atoms and molecules will form in the future susy phase?

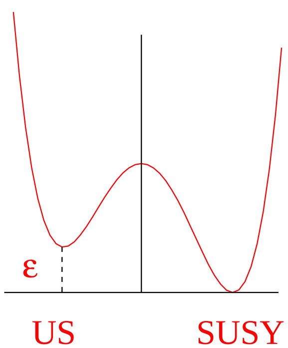
alternatively:

What can we predict about the future susy phase after the higgs structure is revealed at the LHC?

# a supersymmetric universe

## a world of greatly weakened Pauli Principle



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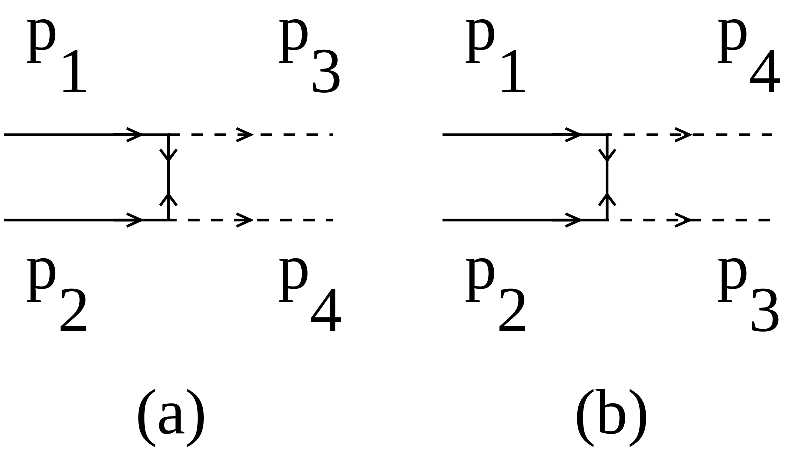



$$f + f \longrightarrow \tilde{f} + \tilde{f}$$

# QM of susy ions including screening

**(LC + Tim Lovorn, IJMPA 2007)**

**(bound state of N electrons, nucleus of atomic number Z)**

$$\psi = \prod_{i=1}^{N} \left( \sqrt{\frac{Z_s^3}{\pi}} e^{-Z_s r_i} \right) \quad .$$

$$\langle H \rangle = N \frac{Z_s^2}{2m} - Z e^2 Z_s N + \frac{N(N-1)}{2} e^2 \frac{5}{8} Z_s \quad .$$

**This is a minimum at** $\qquad Z_s = m e^2 \left( Z - \frac{5}{16}(N-1) \right)$

## Ground state energy:

$$\langle H \rangle = E(N,Z) = -\frac{m\alpha^2}{2} N \left( Z - \frac{5}{16}(N-1) \right)^2 \quad .$$

## mean distance from nucleus:

$$r(N,Z) = 4\pi \frac{Z_s^3}{\pi} \int_0^\infty r^3 dr e^{-2Z_s r} = \frac{3}{2Z_s} = \frac{3}{2m\alpha} (Z - \frac{5}{16}(N-1))^{-1} \quad .$$

| Z | | $I_{susy}$ | $I_{exp}$ | $r_{susy}$ |
|---|---|---|---|---|
| 1 | H | 13.6 | 13.6 | 0.792 |
| 3 | Li | 23.8 | 5.4 | 0.023 |
| 11 | Na | 64.6 | 5.1 | 0.0070 |
| 19 | K | 105 | 4.3 | 0.0041 |
| 37 | Rb | 197 | 4.2 | 0.0021 |
| 55 | Cs | 289 | 3.9 | 0.0014 |

3rd column: calculated single electron ionization energy (exact susy)
4th column: exp. single electron ionization energy (broken susy)
5th column: calculated mean atomic radius in Angstroms (exact susy)

From the variational principle an estimate for the energy and mean radius of a system of N electrons and Z protons is

$$E(N,Z) = -\frac{Nme^4}{2}\left(Z - \frac{5}{16}(N-1)\right)^2$$

$$r(N,Z) = \frac{3}{2me^2}\left(Z - \frac{5}{16}(N-1)\right)^{-1}$$

As $m \to 0$ atomic binding energies vanish and the radii of atoms goes to $\infty$.

Could advanced life forms re-evolve after a transition to exact susy?

One might imagine that life would have difficulty arising in a world consisting only of elementary particles with no electromagnetic bound states.

**The problem then arises that in the**

**Minimal Supersymmetric Standard Model (MSSM)**

**and in most of its extensions,**

**electroweak symmetry breaking (EWSB)**

**vanishes in the exact susy limit**

**leaving all fermions massless.**

**superpotential of MSSM:** $W = \mu H_u \cdot H_d = \mu(H_u^{0\dagger} H_d^0 - H_u^{+\dagger} H_d^+)$

**Scalar potential F terms:**

$$V_F = \sum_\phi \left| \frac{\partial W}{\partial \phi} \right|^2 = |\mu|^2 \left( |H_u|^2 + |H_d|^2 \right)$$

**Scalar potential D terms:**

$$V_D = \frac{g_1^2 + g_2^2}{8} \left( |H_d|^2 - |H_u|^2 \right)^2 + \frac{g_2^2}{2} \left( |H_d|^2 |H_u|^2 - |H_u \cdot H_d|^2 \right)^2 \quad .$$

**"Soft" terms:** $V_{soft} = m_d^2 |H_d|^2 + m_u^2 |H_u|^2 + A\mu H_u \cdot H_d$

**In the absence of the soft mass terms, the minimum of the potential is at**

$$< H_u >=< H_d >= 0$$

**Zero vevs implies no EWSB (massless fermions).**
**Massless fermions implies no electromagnetically bound atoms.**

**The next to minimal supersymmetric standard model (NMSSM) introduces a singlet superfield S.**

**P. Fayet, Nucl.Phys. 1976**

**Superpotential:** $W = \lambda S H_u \cdot H_d + \frac{\kappa}{3} S^3$

**Corresponding scalar potential, restricted to neutral fields:**

$$V_F = |\lambda S|^2 \left(|H_u|^2 + |H_d|^2\right) + \left|\lambda H_u H_d + \kappa S^2\right|^2$$

**The D terms are, as in the MSSM,**

$$V_D = \frac{g_1^2 + g_2^2}{8} \left(|H_d|^2 - |H_u|^2\right)^2 + \frac{g_2^2}{2} \left(|H_d|^2 |H_u|^2 - |H_u H_d|^2\right)^2$$

**The potential in the absence of soft mass terms becomes symmetric in $H_u$ and $H_d$ so that**

$$< H_u >=< H_d >= v_0$$

**This ensures that the D terms vanish at the minimum. The minimum also defines a vacuum expectation $S_0$ of the S field.**

$$< S >= S_0$$

**Minimizing the potential with respect to S requires that**

$$S_0 \left( v_0^2 \lambda (\lambda + \kappa) + \kappa^2 S_0^2 \right) = 0$$

**Minimizing the potential with respect to $H_u$ or $H_d$ requires that**

$$v_0 \left( \lambda v_0^2 + (\kappa + \lambda) S_0^2 \right) = 0$$

**The only physical solution of the NMSSM in the absence of soft mass terms is: $v_0 = S_0 = 0$ exact susy but no EWSB**

## nMSSM , “nearly minimal susy standard model”

$$W = \lambda S \left( H_u \cdot H_d - v^2 \right)$$

The F terms in the scalar potential are then

$$V_F = \lambda^2 \left( |S|^2 \left( |H_u|^2 + |H_d|^2 \right) + \left( |H_u H_d| - v^2 \right)^2 \right)$$

The D terms are the same as in the MSSM, vanishing at the generic potential minima:

$$< H_u > = < H_d > = v_0 \quad , \quad < S > = S_0$$

In the exact susy limit of the nMSSM

$$V = V_F = \lambda^2 \left( |S|^2 \left( |H_u|^2 + |H_d|^2 \right) + \left( |H_u H_d| - v^2 \right)^2 \right)$$

the extrema are defined by

$$\frac{\partial V}{\partial S}|_0 = 2\lambda^2 S_0 v_0^2 = 0$$

and

$$\frac{\partial V}{\partial H_u}|_0 = \lambda^2 v_0 (v_0^2 - v^2 + S_0^2) = 0$$

In the susy limit the absolute minimum of the scalar potential is at

$$v_0 = v \quad , \quad S_0 = 0$$

Thus for the nMSSM in the susy limit there is vanishing vacuum energy and a broken electroweak symmetry.

## Most general renormalizable superpotential with a pair of higgs doublets and an extra singlet higgs

$$W = \lambda \left( S(H_u \cdot H_d - v^2) + \frac{\lambda'}{3} S^3 + \frac{\mu_0}{2} S^2 \right)$$

$v\,,\,\mu_0 \to 0: \quad W \to NMSSM$

$\lambda'\,,\,\mu_0 \to 0: \quad W \to nMSSM$

$\lambda'\,,\,\mu_0\,,\,v \to 0: \quad W \to UMSSM$

**The F terms in the scalar potential are**

$$V_F = \lambda^2 \left( \left| H_u \cdot H_d - v^2 + \lambda' S^2 + \mu_0 S \right|^2 + |S|^2 \left( |H_u|^2 + |H_d|^2 \right) \right)$$

**The D terms are as in the MSSM.**

$$V_F = \lambda^2 \left( \left| H_u \cdot H_d - v^2 + \lambda' S^2 + \mu_0 S \right|^2 + |S|^2 \left( |H_u|^2 + |H_d|^2 \right) \right)$$

**The conditions for an extremum of the scalar potential F terms are**

$$\frac{1}{\lambda^2} \frac{\partial V_F}{\partial S}|_0 = 0 = (2\lambda' S_0 + \mu_0)(v_0^2 - v^2 + \mu_0 S_0 + \lambda' S_0^2) + 2 v_0^2 S_0$$

$$\frac{1}{\lambda^2} \frac{\partial V_F}{\partial H_u}|_0 = 0 = v_0 (v_0^2 - v^2 + \mu_0 S_0 + (\lambda' + 1) S_0^2)$$

**Solution 1 (susy+EWSB) :** $v_0 = v \quad , \quad S_0 = 0$

**Solution 2 (susy):** $v_0 = 0 \quad , \quad S_0 = \frac{-\mu_0 \pm \sqrt{\mu_0^2 + 4\lambda' v^2}}{2\lambda'}$

**Solution 3 (saddle pt?):** $v_0 = 0 \quad , \quad S_0 = \frac{-\mu_0}{2\lambda'}$

**Solution 4 (broken susy + EWSB)**

$$2\lambda' S_0^2 + \mu_0 S_0 - 2v_0^2 = 0 \tag{1}$$

$$S_0 = \frac{1}{4\lambda'}\left(-\mu_0 \pm \sqrt{\mu_0^2 + 16\lambda' v_0^2}\right) \tag{2}$$

$$v_0^2 = v^2 - S_0\left(\mu_0 + (\lambda' + 1)S_0\right) \tag{3}$$

**The minimization equations imply a quadratic equation for $v_0^2$:**

$$
\begin{aligned}
v_0^4(2\lambda'+1)^2 - v_0^2(2\lambda' v^2(2\lambda'+1) + \frac{3}{4}\mu_0^2(\lambda'-1)) \\
+ v^2(\lambda'^2 v^2 + (\lambda'-1)\mu_0^2/4) = 0
\end{aligned}
$$

**We write**

$$
\begin{aligned}
x &= \frac{v_0^2}{v^2} \\
y &= \frac{\mu_0^2}{4v^2} && (4) \\
u &= \frac{\lambda'}{2\lambda'+1} + \frac{3y(\lambda'-1)}{2(2\lambda'+1)^2} && (5) \\
w &= \frac{\lambda'^2 + y(\lambda'-1)}{(2\lambda'+1)^2} \quad . && (6)
\end{aligned}
$$

The higgs vev squared in the broken susy phase, $v_0^2$, relative to that in the exact susy phase, $v^2$, is:

$$x = \frac{v_0^2}{v^2} = \frac{< H >^2{}_{brokensusy}}{< H >^2{}_{exactsusy}} = u \pm \sqrt{u^2 - w}$$

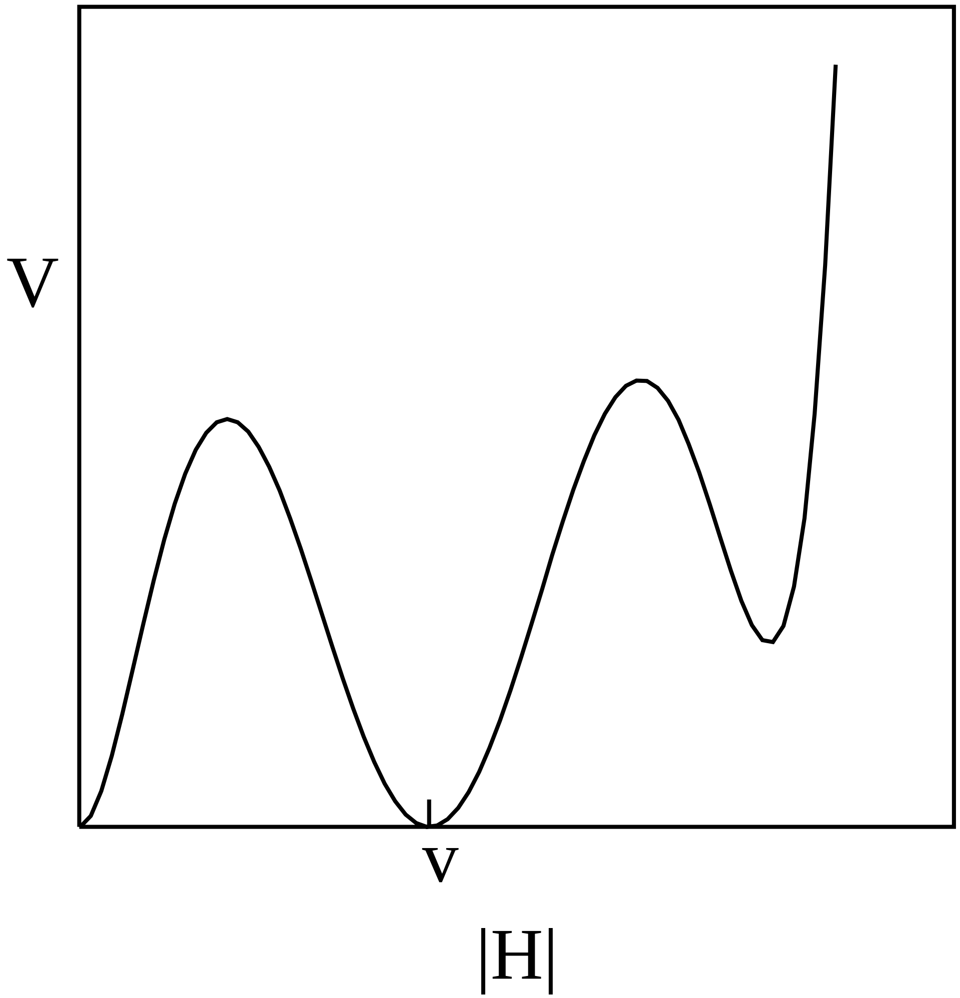
Scanning over the parameter space of $\lambda'$ and $\mu_0$ one finds large regions where $x \geq 1$ thus allowing the possibility of an exothermic transition to exact susy. For example, if $\lambda'$ is near 2 and $y$ is near 4.5, then $x$ is near unity. On the other hand, if we take the limit to the restricted nMSSM ($\lambda', \mu_0 \to 0$), one finds no solutions with $x > 1$.

The value of the higgs potential at the broken susy min is

$$V_F(0) = \lambda^2 S_0^2 (S_0^2 + 2v_0^2)$$

For large $V_F(0)$ the broken susy phase becomes short-lived. Thus, in a realistic model, the values of $\lambda$ and the other parameters become constrained by the 13.7 billion year lifetime of the current universe.

## phase structure of the nMSSM with atomphilic parameters



# Summary

If atoms are to form in the future exact susy universe, the common electron/selectron mass must be non zero (Electroweak Symmetry Breaking, EWSB).

The MSSM, NMSSM, and UMSSM have no EWSB in the exact susy phase.

The "nearly minimal susy standard model", nMSSM does have EWSB in the susy phase.

In the nMSSM, as currently defined, the common electron/selectron mass in the susy phase is greater than the electron mass in the broken phase.

This would make the transition to exact susy endothermic.

**In the most general extended higgs model with a single singlet higgs there is a hitherto unexplored parameter, $\mu_0$.**

**For non-zero values of $\mu_0$ and for a range of values of the other parameters there is an exothermic transition to exact susy with EWSB.**

**If the LHC discovers an extended higgs model with parameters in this range, a future susy universe containing atoms and molecules is possible.**

**If the LHC discovers the MSSM or an extended higgs model with parameters well outside this range, atoms and molecules would not be expected in the future susy universe.**