Noncommutative Higgs and/or Dark Matter Sectors

or

How weird can the Higgs sector be?

Xavier Calmet

University of Brussels (ULB)

Work in collaboration with Martin Kober

Outline

- Why go beyond the standard model?
- The Higgs sector as a gateway to exotic new physics
- Conclusions

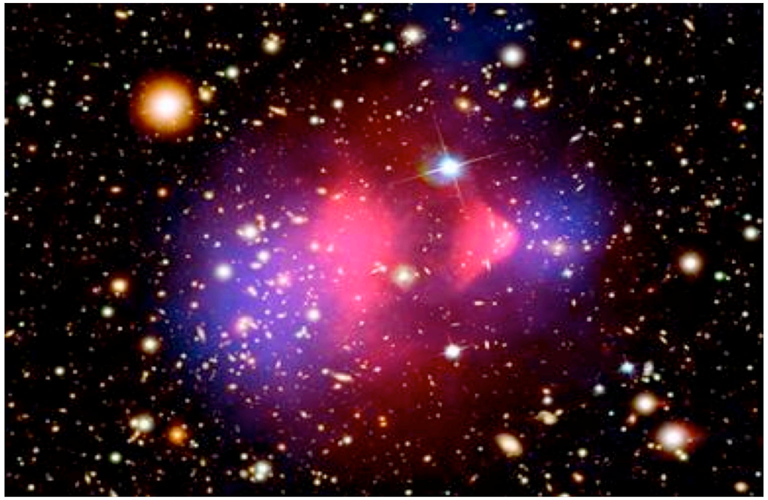
Why go beyond the SM?

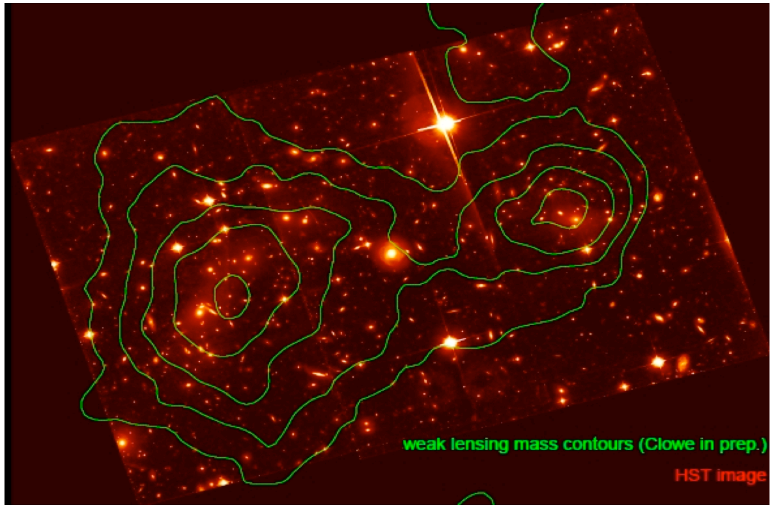
Why go beyond the SM?

- Two experimental facts:

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter





Hot gas seen through X-ray emission (pink) contains most of the baryonic matter in two merging clusters in 1E0657-56, observed against an optical image of the overall cluster. Blue indicates the locations of the total mass as found in an analysis based on gravitational lensing. Most of this mass is clearly separate from the baryonic matter, indicating that it is mainly dark. The bow shape of the hot gas on the right suggests that this cluster has passed through the other. It seems that while the two gas clouds slowed on interacting, the dark matter did not, leading to the separation observed. (X-ray courtesy NASA/CXC/CfA/ M Markevitch *et al*.; optical courtesy NASA/STScI; Magellan/U Arizona/D Clowe *et al*.; lensing map courtesy NASA/STScI; ESO WFI; Magellan/U Arizona/D Clowe *et al*.)

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

No candidate in SM!

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

No candidate in SM!

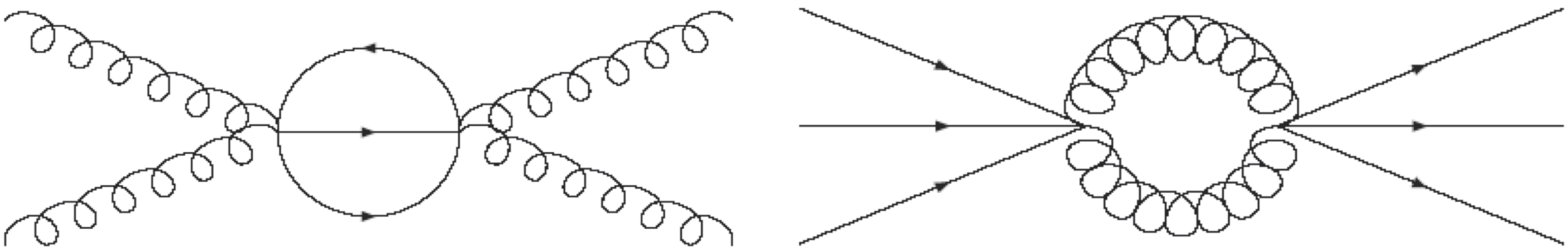
- There is a mathematical problem with a quantum field theory (i.e. SM) coupled to "classical" gravity: gravity has to be quantized.

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

No candidate in SM!

- There is a mathematical problem with a quantum field theory (i.e. SM) coupled to "classical" gravity: gravity has to be quantized.



Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

No candidate in SM!

- There is a mathematical problem with a quantum field theory (i.e. SM) coupled to "classical" gravity: gravity has to be quantized.

Why go beyond the SM?

- Two experimental facts:
  - There is dark matter
  - Most probably dark energy exits as well.

  No candidate in SM!
- There is a mathematical problem with a quantum field theory (i.e. SM) coupled to "classical" gravity: gravity has to be quantized.

- There are big and small numbers in the standard model without an obvious reason. Why is the Planck scale much bigger than the Fermi scale and why is the Higgs boson mass stable?

$$m_H^2 \approx {m_H^0}^2 + \frac{3g^2\Lambda^2}{32\pi^2 m_W^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4\sum_f \left(\frac{n_f}{3}\right) m_f^2\right)$$

- However, the stability issue only makes sense if you have a Wilsonian approach to Quantum Field Theory.
- Personal point of view: within the framework of a renormalizable quantum field theory, fine-tuning or hierarchy problems make no sense: a parameter is measured at some scale and one can compute its running.
- So what is the meaning of small or big? It's an experimental question.
- My perspective: the standard model will need to be modified at the weak scale to take into account EW breaking (e.g. singlets or doublets etc) and this could be a way to introduce DM in the model.
- Let us look at minimal deviations of the standard model. How weird can these be?
- Higgs sector is fascinating: Higgs mass term is the only super-renormalizable term in the SM: it's a gateway to new physics!

How exotic can the Higgs sector be?

Scalar field in a strong magnetic field

- Let us consider the following action U(1) gauged scalar field:

$$S = \int d^4x (\bar{D}_\mu \phi)^* (\bar{D}^\mu \phi) - V(\phi^* \phi) - \frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu}$$

where $\bar{D}_\mu = \partial_\mu + iq\mathcal{A}_\mu$ and $\bar{F}_{\mu\nu} = -i[\bar{D}_\mu, \bar{D}_\nu]$

- And consider fluctuations around a background field:

$$\bar{D}_\mu = \partial_\mu + iqA_\mu + iqC_\mu = D_\mu + iqC_\mu$$

- We pick: $C_\mu = (0, \frac{By}{2}, -\frac{Bx}{2}, 0)$

$$\begin{aligned} S = & \int d^4x (D_\mu \phi)^* (D^\mu \phi) - V(\phi^* \phi) \\ & -iq\phi^* C_\mu D^\mu \phi + iq(D_\mu \phi)^* C^\mu \phi + q^2 \phi^* C_\mu C^\mu \phi \\ & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{1}{4} F_{\mu\nu} C^{\mu\nu} - \frac{1}{4} C_{\mu\nu} F^{\mu\nu} \end{aligned}$$

- This action has a remaining gauge invariance:

$$\delta\phi = i\alpha\phi, \quad \delta A_\mu = \partial_\mu \alpha$$

- The scalar field describes a particle whose classical canonical momentum is given by:

$$\pi_\mu = p_\mu + qA_\mu + qC_\mu$$

- The first quantization corresponds to the requirement:

$$[x^i, \pi^j] = i\hbar\delta^{ij}$$

- And hence:

$$[x^i, p^j + qA^j + qB\epsilon^{jk}x_k] = i\hbar\delta^{ij}$$

- Let us now consider the limit (First Landau Level) $B \gg m$ $|C^\mu| \gg |A^\mu|$
- We thus find:

$$[x^i, x^j] = i\hbar\frac{1}{qB}\epsilon^{ij} = i\theta^{ij}$$

- In that limit, coordinates do not commute in the x-y plane.
- Gauge transformations of the scalar field involve non commuting coordinates:

$$\delta_\alpha \phi = i\alpha(t, \hat{x}, \hat{y}, z)\phi(t, \hat{x}, \hat{y}, z)$$

- Which implies a non trivial transformation for the gauge potential:

$$\delta_\alpha A_\mu(\hat{x}) = \partial_\mu \alpha(\hat{x}) + i[\alpha(\hat{x}), A_\mu(\hat{x})]$$

- A low energy description is given by the following action:

$$\begin{aligned} S = \int d^4x (D_\mu \phi(\hat{x}))^* (D^\mu \phi(\hat{x})) - V(\phi(\hat{x})^* \phi(\hat{x})) \\ -\frac{1}{4} F_{\mu\nu}(\hat{x}) F^{\mu\nu}(\hat{x}), \end{aligned}$$

- How do we get rid of the non commuting variables? Use the Weyl quantization procedure (see plenary talk of Julius Wess).

- Let us now introduce the star product:

$$S = \int d^4x (D_\mu \phi)^* \star (D^\mu \phi) - V(\phi^* \star \phi) - \frac{1}{4} F_{\mu\nu} \star F^{\mu\nu}$$

$$f \star g = f \exp(i\partial_i \theta^{ij} \partial_j) g \text{ with } \theta^{ij} = \tfrac{\hbar}{qB} \epsilon^{ij} \text{ for } i, j \in \{1, 2\}$$

- Till now we considered a U(1) gauge theory. Let us see what happens for Yang-Mills theory, e.g. SU(2) charged scalar doublet

$$\vec{\phi} = (\phi_1, \phi_2)$$

- If we look at the classical equations of motion and go through the first quantization again we find:

$$\pi_1^i = p_1^i + gB^{3i} + gD^{3i} \qquad \pi_2^i = p_2^i - gB^{3i} - gD^{3i}$$

$$[x_i, p_j + gB_j^3 + gE\epsilon_{jk}x^k] = i\hbar\delta_{ij} \qquad [x_i, p_j - gB_j^3 - gE\epsilon_{jk}x^k] = i\hbar\delta_{ij}$$

$$[x^i, x^j] = i\hbar \tfrac{1}{gE} \epsilon^{ij} \qquad [x^i, x^j] = -i\hbar \tfrac{1}{gE} \epsilon^{ij}$$

Local gauge theories on NC spaces

- In other words there is no non-commutative description (duality) for an SU(2) charged scalar field in a strong external field.
- We recover an old result, but now understand the physical reason: there is a non-commutative dual description for U(N) groups when the strong external field is in the direction of the unite matrix.

- Old argument: let $\hat{\Lambda}$, $\hat{\Sigma}$ be Lie-algebra valued gauge transformations, the commutator:

$$[\hat{\Lambda}, \hat{\Sigma}] = \frac{1}{2}\{\hat{\Lambda}_a(\hat{x}), \hat{\Sigma}_b(\hat{x})\}[T^a, T^b] + \frac{1}{2}[\hat{\Lambda}_a(\hat{x}), \hat{\Sigma}_b(\hat{x})]\{T^a, T^b\}$$

is a gauge transformation only for U(N) gauge transformations in the (anti) fundamental and adjoint representations.

- Furthermore, for U(N) Yang-Mills theories the background field needs to be introduced in the action:

$$
\begin{aligned}
S = \int d^4x (D_\mu\phi)^*(D^\mu\phi) - V(\phi^*\phi) \\
-iq\phi^* C_\mu D^\mu\phi + iq(D_\mu\phi)^* C^\mu\phi + q^2\phi^* C_\mu C^\mu\phi \\
-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{1}{4}F_{\mu\nu}C^{\mu\nu} - \frac{1}{4}C_{\mu\nu}F^{\mu\nu}
\end{aligned}
$$

$$
\delta A^\mu = \partial^\mu\alpha + i[\alpha, A^\mu]
$$

$$
\delta C^\mu = i[\alpha, C^\mu]
$$

- This suggest a generalization of noncommutative gauge theories to:

$$
\begin{aligned}
S = \int d^4x (D_\mu\phi)^\dagger \star (D^\mu\phi) - V(\phi^\dagger \star \phi) \\
-i\phi^\dagger \star C_\mu \star D^\mu\phi + i(D_\mu\phi)^\dagger \star C^\mu \star \phi \\
+\phi^\dagger \star C_\mu \star C^\mu\phi - \frac{1}{4}F_{\mu\nu} \star F^{\mu\nu} - \frac{1}{4}C_{\mu\nu} \star C^{\mu\nu} \\
-\frac{1}{4}F_{\mu\nu} \star C^{\mu\nu} - \frac{1}{4}C_{\mu\nu} \star F^{\mu\nu}.
\end{aligned}
$$

$$
\begin{aligned}
\delta A^\mu &= \partial^\mu\alpha + i\alpha \star A^\mu - iA^\mu \star \alpha \\
\delta C^\mu &= i\alpha \star C^\mu - iC^\mu \star \alpha
\end{aligned}
$$

Application to the Higgs sector

- The scalar field we studied can't be the Higgs boson which is charged under SU(2)
- However, there has been a growing interest in simple extensions of the standard model.
- Let us for example imagine a situation where the SM including the Higgs field is confined to a brane whereas an extra scalar field charged under a new U(1) lives in 5 dimensions.
- The low energy action is given by

$$
\begin{aligned}
S = \int d^4x \Big(& (\bar{D}_\mu \phi)^* (\bar{D}^\mu \phi) - m_\phi^2 \phi^* \phi - \lambda_\phi (\phi^* \phi)^2 \\
& + (D_\mu H)^\dagger (D^\mu H) - m_H^2 H^\dagger H - \lambda_H (H^\dagger H)^2 \\
& + \lambda \phi^* \phi H^\dagger H \Big)
\end{aligned}
$$

Different scenarios!

- Let me assume that the new U(1) is unbroken:
  - The Higgs field gets a vev:

$$H = (0, h + v) \quad v^2 = -m_H^2/(2\lambda_H)$$

  - The scalar potential is the given by:

$$V[h, \phi\phi^*] = -2m_H^2 h^2 + \lambda_H (h+v)^4 + \lambda (h+v)^2 \phi\phi^* + m_\phi^2 \phi\phi^* + \lambda_\phi \phi\phi^* \phi\phi^*$$

  - In the first Landau level limit we have:

$$V[h, \phi\star\phi^*] = -2m_H^2 h^2 + \lambda_H (h+v)^4 + \lambda (h+v)^2 \phi\star\phi^* + m_\phi^2 \phi\star\phi^* + \lambda_\phi \phi\star\phi^* \star\phi\star\phi^*$$

  - i.e. we have a noncommutative dark matter candidate. Note that the two photons do not mix.
  - The cross section is similar to the one discussed in the commutative case. But new phenomenology in the dark matter sector: annihilation would show some preferred direction.

$$\Gamma(Higgs \rightarrow \phi\phi^*) = \frac{1}{4\pi}\lambda^2 v^2 \frac{\sqrt{m_H^2 - 4m_\phi^2}}{2m_H^2}$$

Abundance is calculated the same way as in

PHYSICAL REVIEW D VOLUME 50, NUMBER 6 15 SEPTEMBER 1994

Gauge singlet scalars as cold dark matter

John McDonald
CFMC-GTAE, Av. Prof. Gama Pinto 2, Lisboa 1699, Portugal
(Received 22 November 1993)

We consider a very simple extension of the standard model in which one or more gauge singlet scalars S_i couples to the standard model via an interaction of the form $\lambda_S S_i^\dagger S_i H^\dagger H$, where H is the standard model Higgs doublet. The thermal relic density of S scalars is calculated as a function of the coupling λ_S and the S scalar mass m_S. The regions of the (m_S, λ_S) parameter space which can be probed by present and future experiments designed to detect scattering of S dark matter particles from Ge nuclei, and to observe upward-moving muons and contained events in neutrino detectors due to high-energy neutrinos from annihilations of S dark matter particles in the Sun and the Earth, are discussed. Present experimental bounds place only very weak constraints on the possibility of thermal relic S scalar dark matter. The next generation of cryogenic Ge detectors and of large area (10^4 m^2) neutrino detectors will be able to investigate most of the parameter space corresponding to thermal relic S scalar dark matter up to $m_S \approx 50$ GeV, while a 1 km^2 detector would in general be able to detect thermal relic S scalar dark matter up to $m_S \approx 100$ GeV and would be able to detect up to $m_S \approx 500$ GeV or more if the Higgs boson is lighter than 100 GeV.

PACS number(s): 95.35.+d, 12.60.Cn

However it might be possible to see more annihilations in a preferred direction due to the NC nature of the model. Needs to be investigated!

- Let me now assume that the new U(1) is now broken:
  - The physical Higgs boson is a mixture between the singlet and the remaining degree of freedom of the Higgs doublet:

$$h_{\text{phys}} = \cos\alpha\ h + \sin\alpha\ \sigma \qquad \sigma_{\text{phys}} = \cos\alpha\ \sigma - \sin\alpha\ h$$

  - Noncommutative character is transmitted to the Higgs boson.
  - Besides an extra singlet which mixes with the Higgs boson, one would detect non commutative self-interactions of the Higgs boson, however probably difficulty to do at the LHC: we will need to wait for a linear collider.

The Higgs sector can indeed be pretty weird and interesting!

Conclusions

- The Higgs sector is a gateway to hidden sectors and thus dark matter candidates. Electroweak symmetry sector is the only SM one which has not be tested yet.
- Possible connection to hidden sectors/dark matter: LHC will produce DM in most of the scenarios.
- Physics in the Higgs/scalar sector might be quite exotic and thus interesting.
- Higgs and/or Dark Matter sectors might be noncommutative.
- New idea: if NC is not a spacetime property but due to an interaction with a background field, the SM can be partially noncommutative!
- Thanks for your attention.