Detectable Gravitational Waves in Multi M5-Brane Inflation

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Outline

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MBI from DBI [A.K. & D. Lüst, in progress]

- choose N 4d space-filling branes (resp. antibranes) and distribute them over the internal compact space
- provide mechanism for susy-breaking, allowing branes to interact with each other (e.g. anti-branes, fluxes, non-perturbative effects)
- kinetic terms for inflaton components from multi-brane DBI action

$$S = \sum_{i=1}^{N(+1)} T_p \int d^4x \int d^{p-3}y \sqrt{-\det P_i[G]}$$

General Case

- simplest cases = symmetric cases: underlying symmetry of brane arrangement allows for identification (can be generalized)

$$\Delta X_1^q = \ldots = \Delta X_N^q \equiv \Delta X$$

  where q counts internal directions along which the MBI branes are distributed

- the latter identification means we have both a multi-inflaton and an effective single inflaton description
- split the non-dynamical CM position from the dynamical inflaton field ΔX

$$X_i^q = X_{CM}^q + f_i^q \Delta X \ ,$$

  with constant coefficients f_i^q capturing brane distribution

General Case

- working in static gauge, we have for the 4d pullback

$$P_i[G]_{\mu\nu} = g_{\mu\nu} + \Big(\sum_{q,s} f_i^q f_i^s\, g_{qs}\Big)\partial_\mu \Delta X\, \partial_\nu \Delta X$$

  Without loss of generality, take internal metric g_{qs} to be diagonal.

- Expand DBI action up to quadratic order in $\partial_\mu \Delta X$ and extract inflaton's kinetic term

$$\mathcal{L}_{kin} = -\frac{1}{2} T_p V_{\|} \Big(\sum_q \sum_{i=1}^{N} \left(f_i^q\right)^2 g_{qq}\Big)\partial_\mu \Delta X\, \partial^\mu \Delta X$$

General Case

- read off proper normalization of inflaton φ

$$\varphi = c_N \Delta X$$

$$c_N = \left(T_p V_{\|} \sum_q \sum_{i=1}^{N} \left(f_i^q\right)^2 g_{qq}\right)^{1/2} \sim N^b$$

- stronger N-scaling than in 4d assisted inflation possible!

$$\epsilon_N = \frac{M_{Pl}^2}{2}\left(\frac{dU/d\varphi}{U}\right)^2 \sim \frac{1}{N^{2b}}\,, \quad |\eta_N| = M_{Pl}^2 \frac{|d^2U/d\varphi^2|}{U} \sim \frac{1}{N^{2b}}$$

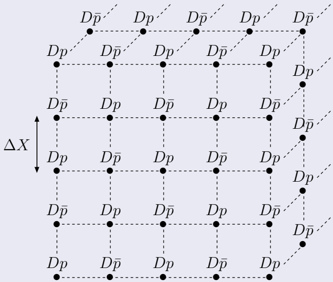
- N-dependence of U drops out: N-scaling of slow-roll parameters depends only on geometry of brane distribution!

Towards “Observing” Brane Distribution

We will now show how exponent b (if measured) contains information about the distribution geometry of the branes

1. Hypercubic Arrangements

consider d-dimensional hypercubic lattice arrangements for $N = n^d$ branes ($q = 1, \ldots, d$). For instance, $N = n^d$ Dp-$D\bar{p}$ pairs with n pairs along each row or column



Hypercubic Geometrical Factor

- choose $X^q_{CM} = 0$; individual brane positions are given by (n even)

$$X^q_{i_q} = f^q_{i_q} \Delta X \,, \qquad f^q_{i_q} = i_q = -\frac{n}{2}, \ldots, \frac{n}{2} \,.$$

$\Rightarrow$ reduction to effective single field model due to discrete translational symmetry

- geometrical factor becomes (flat background)

$$\sum_q \sum_{i_q=-n/2}^{n/2} \left(f^q_{i_q}\right)^2 = d\,\frac{(n+2)(n+1)n}{12} \overset{!}{\sim} N^{2b}$$

- scales like $n^3 = N^{3/d}$, hence scaling exponent

$$b = \frac{3}{2d}$$

Significance of Linear “Throats”

- maximal scaling exponent when $d = 1$

$$d = 1 : \qquad b_{max} = \frac{3}{2} ,$$

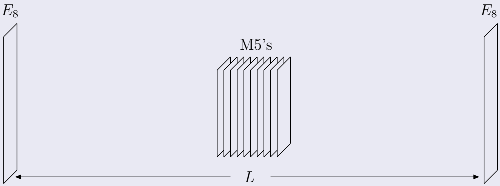
that means for linear brane arrangements. In this case we achieve maximal suppression of the slow-roll parameters

$$d = 1 : \qquad \epsilon_N \sim \eta_N \sim \frac{1}{N^{2b_{max}}} = \frac{1}{N^3} .$$

- One-dimensional distributions of brane/antibranes (“throats”) are thus most effective in generating a prolonged period of inflation (catalyzers for inflation)

M5-MBI in Heterotic M-Theory [2×Becker, A.K. '05]

- Tadpole cancellation generically requires to add N M5-branes to heterotic M-theory setup



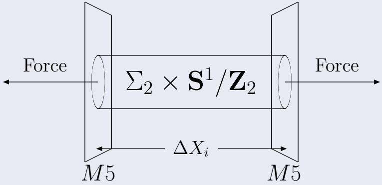
- $N-1$ moduli ΔX_i measure the nearest-neighbor distances between adjacent M5-branes. Identification

$$\Delta X_1 = \ldots = \Delta X_{N-1} \equiv \Delta X$$

turns out to be stable attractor solution!

M5-Brane MBI

- open membrane instantons generate repulsive potential between neighboring M5-branes and break supersymmetry



- exponential superpotentials for ΔX_i

$$W_i(\Delta X_i) \sim e^{-\Delta X_i}$$

M5-Brane MBI

- leads to exponential potential for canonically normalized inflaton

$$U(\varphi) \sim (N-1)^2 e^{-\sqrt{\frac{2}{p_N}}\frac{\varphi}{M_{Pl}}}$$

- linear brane configuration, hence $b = b_{max} = 3/2$ and

$$\varphi \sim N^{3/2} \Delta X$$

- implies parametrically most suppressed slow-roll parameters

$$\epsilon_N = \frac{1}{p_N} \sim \frac{1}{N^3}$$
$$\eta_N = \frac{2}{p_N} \sim \frac{1}{N^3}$$

M5-Brane MBI

- in fact, 4d effective FRW cosmology with this potential has exact solution power-law inflation [Lucchin & Matarrese ’95]

$$a(\mathrm{t}) = a_0 \mathrm{t}^{p_N}$$

with inflaton evolution

$$\varphi(\mathrm{t}) = \sqrt{2p_N} M_{Pl} \ln\left(\frac{\mathrm{t}}{\mathrm{t}_i}\right)$$

- solution is valid for $p_N > 1/3$ and inflation arises only if $p_N > 1$ which is satisfied

$$p_N \sim N^3 \gg 1$$

Outline

1. Multi Brane Inflation
   - General Idea
   - Multi M5-Brane Inflation
2. Gravitational Waves
   - Tensor Fraction
   - Two Puzzles and Their Resolution
3. Cascade Inflation

Scalar and Tensor Perturbations

- negative spectral tilt ($P_s(k) =$ curvature perturbation spectrum)

$$n_s = 1 + \frac{d \ln P_s}{d \ln k}|_{k=aH=\text{horizon exit}} = 1 - 6\epsilon_N + 2\eta_N$$
$$= 1 - \frac{2}{p_N} < 1$$

- tensor fraction ($P_t(k) =$ primordial tensor perturbation spectrum)

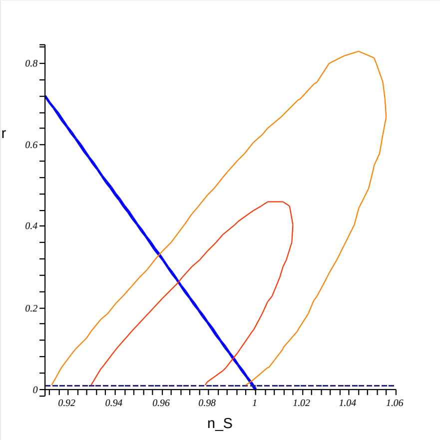
$$r = \frac{P_t}{P_s}|_{k=aH} = 16\epsilon_N = \frac{16}{p_N}$$

- hence predicted relation

$$r = 8(1 - n_s)$$

Tensor Fraction

M5-Brane MBI Gravitational Waves



1. Large Energy Scale of Inflation

- amplitude of gravitational wave CMB anisotropy fixes energy-scale of slow-roll inflation [Lyth, '84]

$$U^{1/4} \approx 3.3 \times 10^{16} r^{1/4}\,\mathrm{GeV}$$

- detectable gravitational waves with $r > 0.01$ imply large inflation energy-scale $U^{1/4} > 10^{16}$ GeV
- seemingly difficult to reconcile with particle theory models
- in M5 MBI

$$U = (N-1)^2 \tilde{U}(\varphi)$$

- thus true inflationary energy-scale $\tilde{U}$ parametrically smaller

$$\tilde{U}^{1/4} \approx 3.3 \times 10^{16} \frac{r^{1/4}}{(N-1)^{1/2}}\,\mathrm{GeV}$$

2. The Lyth Bound

- slow-roll inflation provides a lower bound on inflaton variation during inflation [Lyth, '96] (Efstathiou and Mack '05 find even tighter relation $\sim r^{1/4}$ from stochastic analysis)

$$\frac{\Delta\varphi}{M_{Pl}} \gtrsim \sqrt{2r}$$

- hence large field models with $\Delta\varphi \geq M_{Pl}$ give detectable tensor modes but effective field theory description

$$V = \left(\text{const.} + \frac{1}{2}m^2\varphi^2 + \lambda\varphi^4\right) + \sum_{i=3,4,\ldots} \frac{\lambda_{2i}}{M_{Pl}^{2i-4}}\varphi^{2i}$$

  becomes unreliable if non-renormalizable couplings λ_{2i} are of $\mathcal{O}(1)$

- way out: small couplings λ_{2i} as in chaotic inflation [Linde, '04]

2. M5 MBI – the Lyth Bound

- relation between canonically normalized inflaton φ and microscopically relevant modulus ΔX

$$\frac{\varphi}{M_{Pl}} = \mathcal{V}_{M2}\sqrt{2p_N}\frac{\Delta X}{L}$$

$\mathcal{V}_{M2}$ is open M2-instanton volume, L is $\mathbf{S}^1/\mathbf{Z}_2$ size

- Lyth bound translates into $(\Delta X_f - \Delta X_i \approx \Delta X_f)$

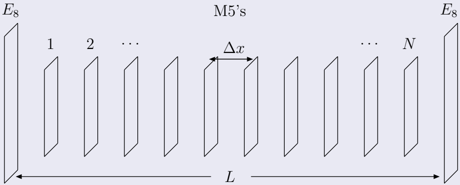
$$\frac{\Delta X_f}{L} \gtrsim \frac{\sqrt{r}}{\sqrt{p_N}\mathcal{V}_{M2}}$$

- thus Lyth bound becomes parametrically suppressed $\sqrt{p_N} \sim N^{3/2}$
- E.g. $\Delta X_f/L \gtrsim \sqrt{r}/70$ for concrete data, which is perfectly consistent with geometric constraint $\Delta X_f \leq L$

Outline

Cascade Inflation [Ashoorioon & A.K. '06]

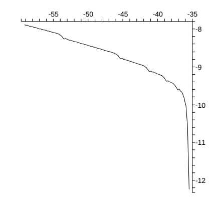
- Since M5-branes repel each other, they will ultimately hit the boundaries

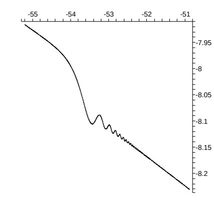


- the outermost M5-branes are absorbed by the boundaries and change their topological data (number of chiral families!)

Power Spectrum

- results in jumps in inflaton potential causing damped oscillations in the power spectrum

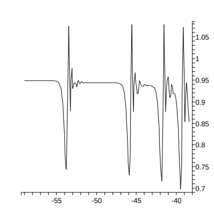


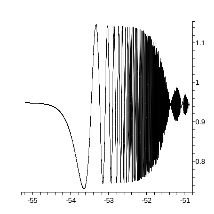


- left graph: dependence of $\log P_s(k)$ on $\log k$
- right graph: zoom in on the first transition.

Spectral Index

- running of spectral index





- left graph: dependence of n_s on $\log k$ for the first five inflationary bouts
- right graph: zoom in on the first transition.