Unitarity constraints on trilinear couplings in the MSSM

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in collaboration with

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outline

- bounds from perturbative unitarity
  - for 2-by-2 scalar scattering
  - to trilinear couplings in a simple scalar particle model example
- MSSM trilinear couplings, scalars and parameters
  - possibly strong trilinear couplings
  - scalar particles and states used
  - needed parameters for heavy squark and Higgs sector
- MSSM trilinear bounds
  - examples
  - 'survey'
- observations and conclusion

asymptotic time evolution operator, transition operator

unitarity $S^\dagger S = 1 \longleftrightarrow$ probability conservation

$$S = 1 + \mathrm{i}\, T$$

$$-\mathrm{i}\left(T - T^\dagger\right) = T^\dagger T$$

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- restrict to scalar 2-by-2 scattering (this is a good approximation in perturbation theory)
- use energy momentum conservation and center of mass system

$$(2\pi)^4\, \delta^{(4)}(P_a - P_b)\, \hat{\mathcal{T}}_{ab}(s, \cos\theta) = \langle a | T | b \rangle \Big|_{\hat{\mathbf{p}}_{a1} = \hat{\mathbf{e}}_z = (1,0,0),\ \hat{\mathbf{p}}_{b1} = (1,\theta,0) \text{ in CMS}}$$

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- use angular momentum conservation (project to partial waves)

$$\mathcal{T}^J_{ab} := \frac{1}{2}\,\frac{\lambda_a^{1/4}\lambda_b^{1/4}}{16\pi\, s}\int_{-1}^{1}\mathrm{d}\cos\theta\,\hat{\mathcal{T}}_{ab}(s,\cos\theta)\,P_J(\cos\theta)\ .$$

the 1/2 is an (standard) convention for partial wave expansion

diagonalizing partial wave transition matrix elements

$$\frac{1}{2\mathrm{i}}\left(\mathcal{T}^J_{fi} - \mathcal{T}^{J*}_{if}\right) = \sum_h \mathcal{T}^{J*}_{hf}\,\mathcal{T}^J_{hi} \quad \text{(sum over intermediate 2-scalar states)}$$

- $\left(\mathcal{T}^J_{fi}\right)$ is a normal matrix $\longrightarrow$ diagonalized by unitary matrix

$$\Longrightarrow \quad \mathrm{Im}\tilde{\mathcal{T}}^J_{ii} = |\tilde{\mathcal{T}}^J_{ii}|^2 \quad \text{eigenvalue equation}$$

$$\Longrightarrow \quad y = x^2 + y^2 \quad \text{circle equation for real and imaginary part}$$

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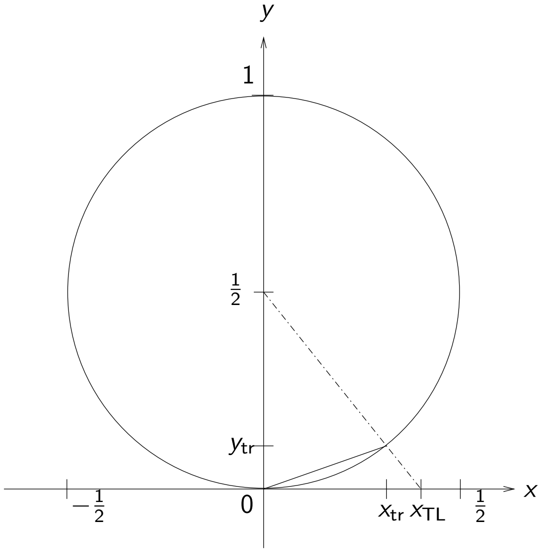
- for bounds only the largest eigenvalue matters
- and only the biggest in scan of center of mass energy $\sqrt{s}$
- tree level (TL) values of the matrix elements $\mathcal{T}^J_{fi}$ are real by proper choice of phases for the scattered states (the needed TL eigenvalues are real anyway)

Argand diagram / TL value determines minimal correction



circle $y = x^2 + y^2$ for the 'true' element

example minimal correction to get to the 'true' matrix element:

$x_{\mathrm{TL}} = 0.5 \implies |x_{\mathrm{TL}} - (x_{\mathrm{tr}} + \mathrm{i} y_{\mathrm{tr}})|/|x_{\mathrm{TL}}| > 0,414$

$\iff$

$|x_{TL} - (x_{\mathrm{tr}} + \mathrm{i} y_{\mathrm{tr}})|/|x_{\mathrm{tr}} + \mathrm{i} y_{\mathrm{tr}}| > 0,465$

scalar field trilinear couplings, scattering amplitude

example with two real scalar fields, Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\sum_{i=1}^{2} (\partial_\mu \varphi_i)^2 - m_i^2\, \varphi_i^2 \right) - g\, \varphi_1\, \varphi_1\, \varphi_2$$

channels of scattering amplitude $\varphi_1 \varphi_1 \longrightarrow \varphi_1\, \varphi_1$

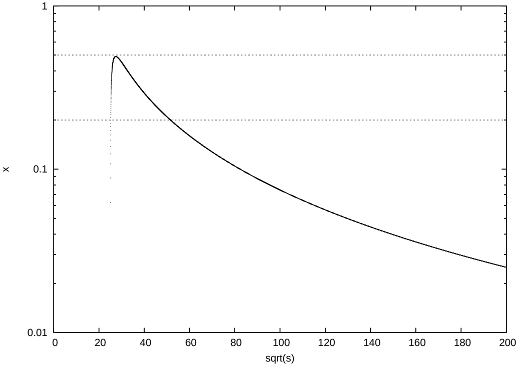
$$\mathcal{T}_{aa}^{J} = \frac{\sqrt{1-4m_1^2/s}}{4 \cdot 16\pi} \int_{-1}^{1} \mathrm{d}\cos\theta \left[\frac{g^2}{s - m_2^2} + \frac{g^2}{t - m_2^2} + \frac{g^2}{u - m_2^2} \right] P_J(\cos\theta)$$

$$\text{maximum } \mathcal{T}_{ab}^{J} \,\tilde{\propto}\, \frac{g^2}{\text{squared masses in the model}}$$

amplitude over center of mass energy

feature of 2-by-2 scalar scattering with only trilinear couplings:

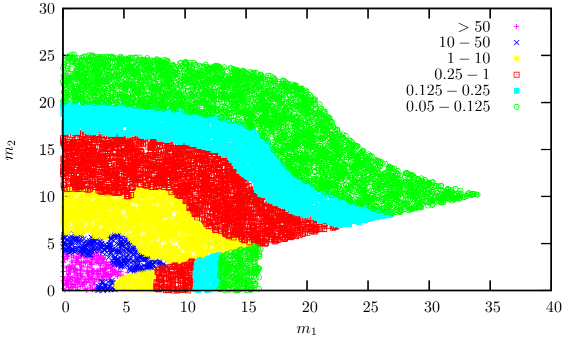
- amplitude drops with $1/s$ for big $\sqrt{s}$.
- example amplitude $\varphi_1\varphi_2 \to \varphi_1\varphi_2$, $g = 100$, $m_1 = 10$, $m_2 = 15$



plotted:
amplitude of the process
over center of mass energy

just at the unitarity limit

2 scalar field model mass bound for $g = 100$



plotted: squared max. T-matrix eigenvalue values over the 2 mass parameters

coupling to masses ratio is bounded

information loss due to u-channel pole in the lower right
unitarity bound at outer end of red region
perturbativity bound at outer end of green region

now to the MSSM, the trilinear couplings

$$\mathcal{L}_{\text{soft}} = \ldots - \left(\lambda_d A_d H_1 \tilde{Q}_L \tilde{d}_R^\dagger + \lambda_u A_u H_2 \tilde{Q}_L \tilde{u}_R^\dagger + \ldots + \text{h.c.}\right)$$

example $h^0, \tilde{t}_1, \tilde{t}_1^*$ here in decoupling scenario (for simpler formula)

vertex $V_{h^0,\tilde{t}_1,\tilde{t}_1^*} = -\,\mathrm{i}\, g' \left(\frac{M_W}{c_w^2} \left(I_3^t c_t^2 - Q_t c_{2t} \right) + \frac{m_t}{M_W} m_t + \frac{m_t}{2M_W} X_t\, s_{2t} \right)$

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similar $h^0, \tilde{b}_1, \tilde{b}_1^*$ vertex $\quad V_{h^0,\tilde{b}_1,\tilde{b}_1^*} = -\,\mathrm{i}\, g' \left(\ldots - \frac{m_b}{2M_W} X_b\, s_{2b} \right)$

with $X_t = A_t - \mu \cot\beta\,, \quad X_b = A_b - \mu \tan\beta$

$g' = \sqrt{4\pi\alpha_{\text{QED}}} / \sin\theta_W\,, \quad s_{2t} = \sin(2\theta_{\tilde{t}})\,, \quad s_{2b} = \sin(2\theta_{\tilde{b}})$

needed particles for 'heavy' squark and Higgs sector

using Feynman ($R_\xi = 1$) gauge and Goldstone equivalence theorem

- G^0, $G^\pm$ for longitudinal degree of freedom of Z^0, $W^\pm$
- this part has the (potentially strong) trilinear coupling

particle content

- scalar Higgs: h^0, H^0, A^0, $H^\pm$, G^0 and $G^\pm$
- 'heavy' squarks: $\tilde{t}_1$, $\tilde{t}_2$, $\tilde{b}_1$, $\tilde{b}_2$

to get the scattering states just put them in pairs

mixing blocks in the T-Matrix

use charge, color and baryon number to form blocks, examples:

- charge zero, color singlet:

$\tilde{t}_1\,\tilde{t}_1^*$	$\tilde{t}_1\,\tilde{t}_2^*$	$\tilde{t}_2\,\tilde{t}_1^*$	$\tilde{t}_2\,\tilde{t}_2^*$	$\tilde{b}_1\,\tilde{b}_1^*$	$\tilde{b}_1\,\tilde{b}_2^*$	$\tilde{b}_2\,\tilde{b}_1^*$
$\tilde{b}_2\,\tilde{b}_2^*$	$h^0\,h^0$	$h^0\,H^0$	$\mathrm{i}\,h^0\,A^0$	$\mathrm{i}\,h^0\,G^0$	$H^0\,H^0$	$\mathrm{i}\,H^0\,A^0$
$\mathrm{i}\,H\,G^0$	$A^0\,A^0$	$A^0\,G^0$	$G^0\,G^0$	H^+H^-	H^+G^-	G^+G^-

mixing blocks in the T-Matrix

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$\tilde{b}_2\,\tilde{b}_2^*$	$h^0\,h^0$	$h^0\,H^0$	$\mathrm{i}\,h^0\,A^0$	$\mathrm{i}\,h^0\,G^0$	$H^0\,H^0$	$\mathrm{i}\,H^0\,A^0$
$\mathrm{i}\,H\,G^0$	$A^0\,A^0$	$A^0\,G^0$	$G^0\,G^0$	H^+H^-	H^+G^-	G^+G^-

- charge 1/3, color 6 : $\tilde{t}_1\,\tilde{b}_1$ | $\tilde{t}_1\,\tilde{b}_2$ | $\tilde{t}_2\,\tilde{b}_1$ | $\tilde{t}_2\,\tilde{b}_2$
- charge 1/3, color $\bar{3}$: $\tilde{t}_1\,\tilde{b}_1$ | $\tilde{t}_1\,\tilde{b}_2$ | $\tilde{t}_2\,\tilde{b}_1$ | $\tilde{t}_2\,\tilde{b}_2$
- charge 1/3, color $\bar{3}$:

$\tilde{b}_1^*\,h^0$	$\tilde{b}_1^*\,H^0$	$\mathrm{i}\,\tilde{b}_1^*\,A^0$	$\mathrm{i}\,\tilde{b}_1^*\,G^0$	$\tilde{b}_2^*\,h^0$	$\tilde{b}_2^*\,H^0$
$\mathrm{i}\,\tilde{b}_2^*\,A^0$	$\mathrm{i}\,\tilde{b}_2^*\,G^0$	$\tilde{t}_1^*\,H^+$	$\tilde{t}_1^*\,G^+$	$\tilde{t}_2^*\,H^+$	$\tilde{t}_2^*\,G^+$

all together 15 independent blocks

needed parameters for 'heavy' squark and Higgs sector

- t_β : ratio of the two Higgs vacuum expectation values
- M_A : mass of pseudo-scalar Higgs
- μ : Higgsino mass parameter
- A_t : u-type-squark-squark-Higgs coupling parameter for $\tilde{t}$
- A_b : d-type-squark-squark-Higgs coupling parameter for $\tilde{b}$
- $M^2_{\tilde{Q}}$: mass parameter for 'heavy' lefthanded squarks
- $M^2_{\tilde{t}}$: mass parameter for the righthanded Stop
- $M^2_{\tilde{b}}$: mass parameter for the righthanded Sbottom

the choice is to use real parameters
(possible CP violation effects neglected)

calculating the unitarity bound

need maximum eigenvalue of $\mathcal{T}^J$

- $\mathcal{T}^J$ is now block diagonal
- take the maximum eigenvalue from all blocks and all center of mass energies
- this biggest eigenvalue (independent of scattering energy) is now treated as a function x with the MSSM parameters as its variables
- in the following plots it appears squared x^2
- only the $J=0$ partial wave matters ($J \neq 0$ is suppressed)

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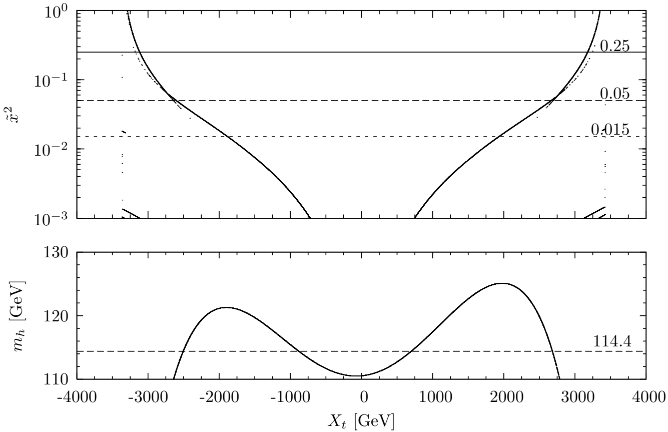
the Higgs mass, example with 2 loop m_h $M_A = M_{\mathrm{SUSY}} = 1\mathrm{TeV}$, $\mu = -200\mathrm{GeV}$, $\tan\beta = 30$

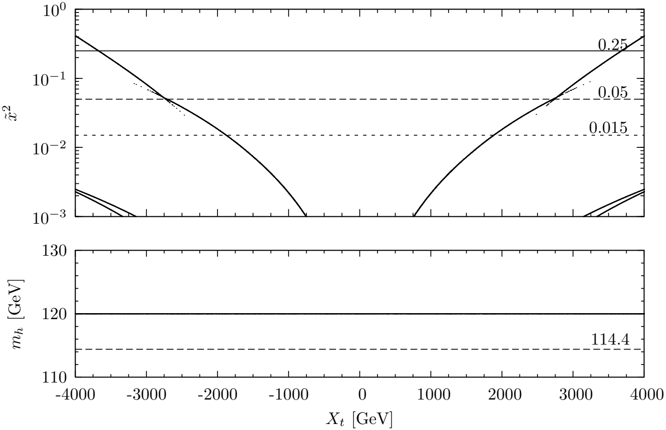


squared max. T-matrix eigenvalue / $X_t = A_t - \cot\beta$

some doubled values (scan over $\sqrt{s}$ hasn't found the biggest value)

unitarity bound $-3100/+3200$ GeV, perturbativity bound ± 2750 GeV

the Higgs mass, example with fixed $M_h = 120$GeV $M_A = M_{\mathrm{SUSY}} = 1$TeV, $\mu = -200$GeV, $\tan\beta = 30$

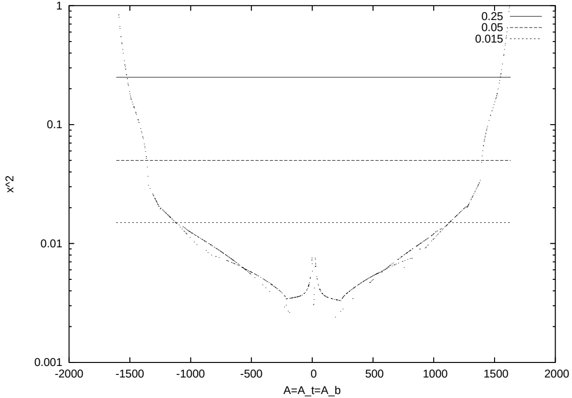


squared max. T-matrix eigenvalue / $X_t = A_t - \cot\beta$

curve(s) around 10^{-3} are from other (width) bounds

unitarity bound ± 3700 GeV, perturbativity bound ± 2750 GeV

the A parameter, $M_A = \mu = M_{SUSY} = 500\text{GeV}$, $\tan\beta = 50$

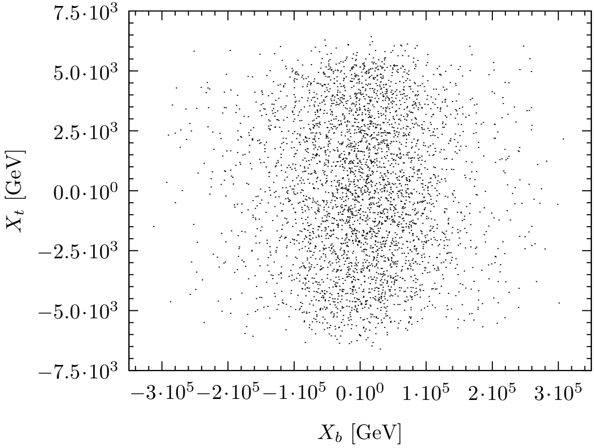


smaller SUSY mass scale example

squared max. T-matrix eigenvalue over $A = A_t = A_b$ parameter

unitarity bound ± 1500 GeV, perturbativity bound ± 1350 GeV

parameterscan of all 8 parameters, projection to X_b - X_t

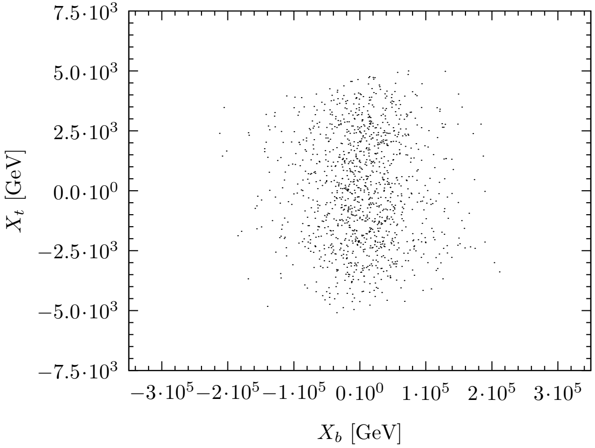


squared maximum T-matrix eigenvalue

$x^2 < 0.25$ unitarity bound

experimental limits (PDG) for squarks, fixed $M_h = 120$GeV
unitarity bound: $|X_t| < 7$TeV, $|X_b| < 320$TeV

parameterscan of all 8 parameters, projection to X_b - X_t

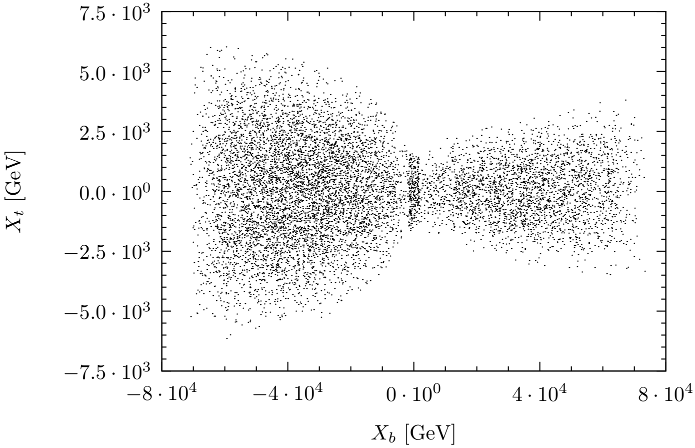


squared maximum T-matrix eigenvalue

$x^2 < 0.05$ perturbativity bound

experimental limits (PDG) for squarks, fixed $M_h = 120$GeV
perturbativity bound: $|X_t| < 5$TeV, $|X_b| < 220$TeV

special case very light $\tilde{b}_1$ without coupling to Z^0

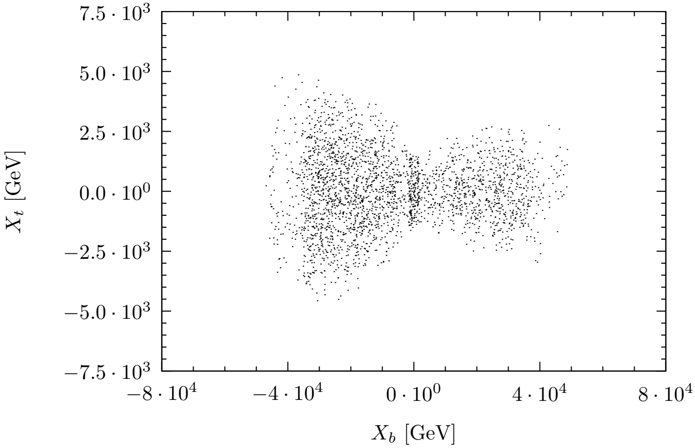


squared maximum T-matrix eigenvalue

$x^2 < 0.25$ unitarity bound

no mass bounds used, fixed $M_h = 120\text{GeV}$, $m_{\tilde{b}_1} = 5\text{GeV}$, $\theta_{\tilde{b}} = 1.17$
unitarity bound: $|X_t| < 6\text{TeV}$, $|X_b| < 75\text{TeV}$

special case very light $\tilde{b}_1$ without coupling to Z^0



squared maximum T-matrix eigenvalue

$x^2 < 0.05$ perturbativity bound

no mass bounds used, fixed $M_h = 120\text{GeV}$ $m_{\tilde{b}_1} = 5\text{GeV}$ $\theta_{\tilde{b}} = 1.17$
perturbativity bound: $|X_t| < 5\text{TeV}$, $|X_b| < 50\text{TeV}$

advantages and disadvantages of the method

advantages

- projection to partial waves is analytic (no need for numerical integration)
- handling the s, t and u channel poles is understood and implemented (the poles would take another talk)

disadvantages

- $\sqrt{s}$ scan is needed
- numerical diagonalization is needed

further observations

- restricting to $J=0$ partial wave is enough
- bounds can derived from very few T-matrix blocks
- bounds from particle widths are less restrictive then amplitude bounds (bounds from widths were not discussed here)

conclusion

- bounds to trilinear couplings can be produced with tree level computation
- example: A_t, A_b, $\mu \tan\beta$, X_t, X_b
- program contains all relevant parameters $\Longrightarrow$ can be used for many scenarios
- you get stronger bounds when varying fewer parameters

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