Leptogenesis and LFV in the left-right symmetric seesaw mechanism

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- introduction
- reconstruction of the right-handed neutrino spectrum
- implications for leptogenesis in SO(10) models
- implications for lepton flavour violation
- conclusions

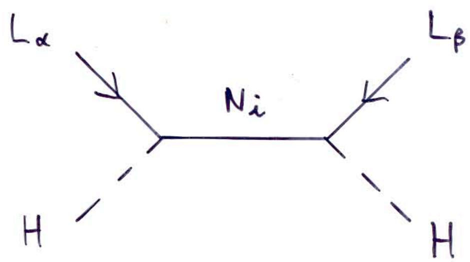
based on:
- P. Hosteins, S. L. and C. Savoy, hep-ph/0606078
- A. Abada, P. Hosteins, F.-X. Josse-Michaux and S. L. (in progress)

SUSY 07
Karlsruhe, Germany, June 26 - August 1, 2007

Introduction

The most popular explanation for the smallness of neutrino masses is the (type I) seesaw mechanism

Minkowski - Gell-Mann, Ramond, Slansky
Yanagida - Mohapatra, Senjanovic



$$\Longrightarrow \qquad m_\nu \sim \frac{y^2 v^2}{M_R}$$

The seesaw mechanism cannot be directly tested, but it has observable consequences:

- leptogenesis [Fukugita, Yanagida]
- lepton flavour violation (LFV) in Susy theories [Borzumati, Masiero]

A lot of efforts have been devoted to their study in the recent years. In particular, conditions for successful leptogenesis have been obtained and many refinements have been added (finite T corrections, flavour effects...)

Most studies have been done in the framework of the type I [heavy right-handed neutrino exchange] seesaw mechanism, or assumed dominance of either type I or type II [heavy scalar $SU(2)_L$ triplet exchange] seesaw

→ it is interesting to investigate whether the generic situation where both contributions are comparable in size can lead to qualitatively different results

In extensions of the SM such as left-right symmetric theories and SO(10) GUTs, the type I and type II seesaw mechanisms are simultaneously present and related by a left-right symmetry

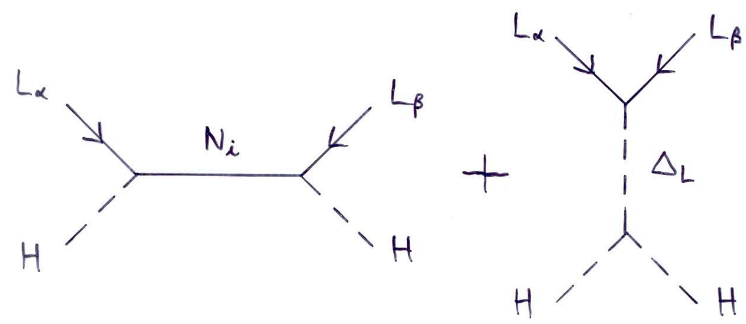
→ appropriate framework to investigate the interplay between the type I and the type II seesaw mechanisms

Further motivation: right-handed neutrinos are suggestive of Grand Unification. However, successful leptogenesis is not so easy to achieve in SO(10) models with a type I seesaw mechanism ($M_D \propto M_u \Rightarrow$ very hierarchical right-handed neutrino masses, with $M_1 << 10^9$ GeV)

The left-right symmetric seesaw mechanism

Type I+II seesaw mechanism:

Δ_L = SU(2)$_L$ triplet with couplings f_{Lij} to lepton doublets



$$M_\nu \;=\; f_L v_L - \frac{v^2}{v_R} Y^T f_R^{-1} Y \;\equiv\; M_\nu^{II} + M_\nu^{I}$$

Right-handed neutrino mass matrix: $M_R \;=\; f_R v_R$

$v_R \equiv \langle \Delta_R \rangle$ scale of B-L breaking

Δ_R = SU(2)$_R$ triplet with couplings f_{Rij} to right-handed neutrinos

v_L is small since it is an induced vev: $v_L \;\equiv\; \langle \Delta_L \rangle \sim\; v^2 v_R / M^2_{\Delta_L}$

In a broad class of theories with underlying left-right symmetry (such as SO(10) with a $\overline{126}_H$), one has $Y = Y^T$ and $f_L = f_R \equiv f$

→ left-right symmetric seesaw mechanism

In addition to the right-handed neutrinos, the $SU(2)_L$ triplet contributes to leptogenesis and LFV

→ in order to study leptogenesis and/or LFV in a theory which predicts the Y_{ij}, need to reconstruct the f_{ij} (which determine both the triplet couplings and the M_i) as a function of the Y_{ij} and of the measured oscillation parameters

Reconstruction of the heavy neutrino mass spectrum

The starting point is the left-right symmetric seesaw formula:

$$M_\nu \;=\; f v_L - \frac{v^2}{v_R} Y f^{-1} Y$$

with f, Y complex and symmetric. The goal is to reconstruct f assuming that Y is known in the basis of charged lepton mass eigenstates

Akhmedov and Frigerio (hep-ph/0509299) showed that there are 2^n solutions for n generations, connected 2 by 2 by a "seesaw duality":

$$f \;\longrightarrow\; \hat{f} \;\equiv\; \frac{M_\nu}{v_L} - f$$

and provided explicit expressions for the fij up to n=3

In hep-ph/0606078, we proposed a simpler reconstruction procedure which employs complex orthogonal matrices

First rewrite the LR symmetric seesaw formula $M_\nu = \alpha f - \beta Y f^{-1} Y$ as

$$Z = \alpha X - \beta X^{-1}$$

with $\alpha \equiv v_L$, $\beta \equiv v^2/v_R$ and

$$Z \equiv N_Y^{-1} M_\nu (N_Y^{-1})^T \qquad X \equiv N_Y^{-1} f (N_Y^{-1})^T$$

where N_Y is such that $Y = N_Y N_Y^T$ (Y invertible)

Z complex symmetric $\Rightarrow$ can be diagonalized by a complex orthogonal matrix O_Z if its eigenvalues z_i are all distinct:

$$Z = O_Z \mathrm{Diag}\,(z_1, z_2, z_3) O_Z^T \,, \qquad O_Z O_Z^T = \mathbf{1}$$

Then X can be diagonalized by the same orthogonal matrix as Z, and its eigenvalues are the solutions of:

$$z_i = \alpha x_i - \beta x_i^{-1} \quad (i = 1, 2, 3)$$

2 solutions x_i^+, x_i^- for each i $\Rightarrow$ $2^3 = 8$ solutions for X, hence for f:

$$f = N_Y O_Z \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix} O_Z^T N_Y^T \,, \qquad x_i = x_i^\pm$$

The corresponding right-handed neutrino masses $M_i = f_i \, v_R$ are obtained by diagonalizing f with a unitary matrix:

$$f = U_f \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U_f^T \,, \qquad U_f U_f^\dagger = \mathbf{1}$$

and the couplings of the N_R mass eigenstates are $U_f^\dagger Y$

Properties of the solutions

We denote the 2 solutions of $z_i = \alpha x_i - \beta x_i^{-1}$ by:

$$x_i^{\pm} \equiv \frac{z_i \pm \sqrt{z_i^2 + 4\alpha\beta}}{2\alpha}$$

(+,+,+) refers to the solution (x_1^+, x_2^+, x_3^+), (+,+,–) to (x_1^+, x_2^+, x_3^-), etc

In the large vR limit ($4\alpha\beta \ll |z_1|^2$):

$$x_i^+ \simeq \frac{z_i}{\alpha} \quad (\text{“type II branch”}) \qquad f^{(+,+,+)} \longrightarrow \frac{M_\nu}{v_L}$$

$$x_i^- \simeq -\frac{\beta}{z_i} \quad (\text{“type I branch”}) \qquad f^{(-,-,-)} \longrightarrow -\frac{v^2}{v_R} Y M_\nu^{-1} Y$$

The remaining 6 solutions correspond to mixed cases in which Mν receives significant contributions from both seesaw mechanisms

In the small vR limit ($|z_3|^2 \ll 4\alpha\beta$):

$$x_i^{\pm} \simeq \pm\sqrt{\beta/\alpha} \qquad f^{(\pm,\pm,\pm)} \longrightarrow \pm\sqrt{\beta/\alpha}\, Y$$

If Y is hierarchical, $f_i \longrightarrow \sqrt{\beta/\alpha}\, y_i$ holds for all 8 solutions

A case study: SO(10) models with two 10's and a $\overline{126}$ in the Higgs sector

$$W \ \ni \ Y^{(1)}_{ij}\, 16_i 16_j 10_1 + Y^{(2)}_{ij}\, 16_i 16_j 10_2 + f_{ij}\, 16_i 16_j \overline{126}$$

$Y^{(1)}, Y^{(2)}$ symmetric $\qquad \overline{126} \ni \Delta_L, \Delta_R$ with $f_L = f_R = f$

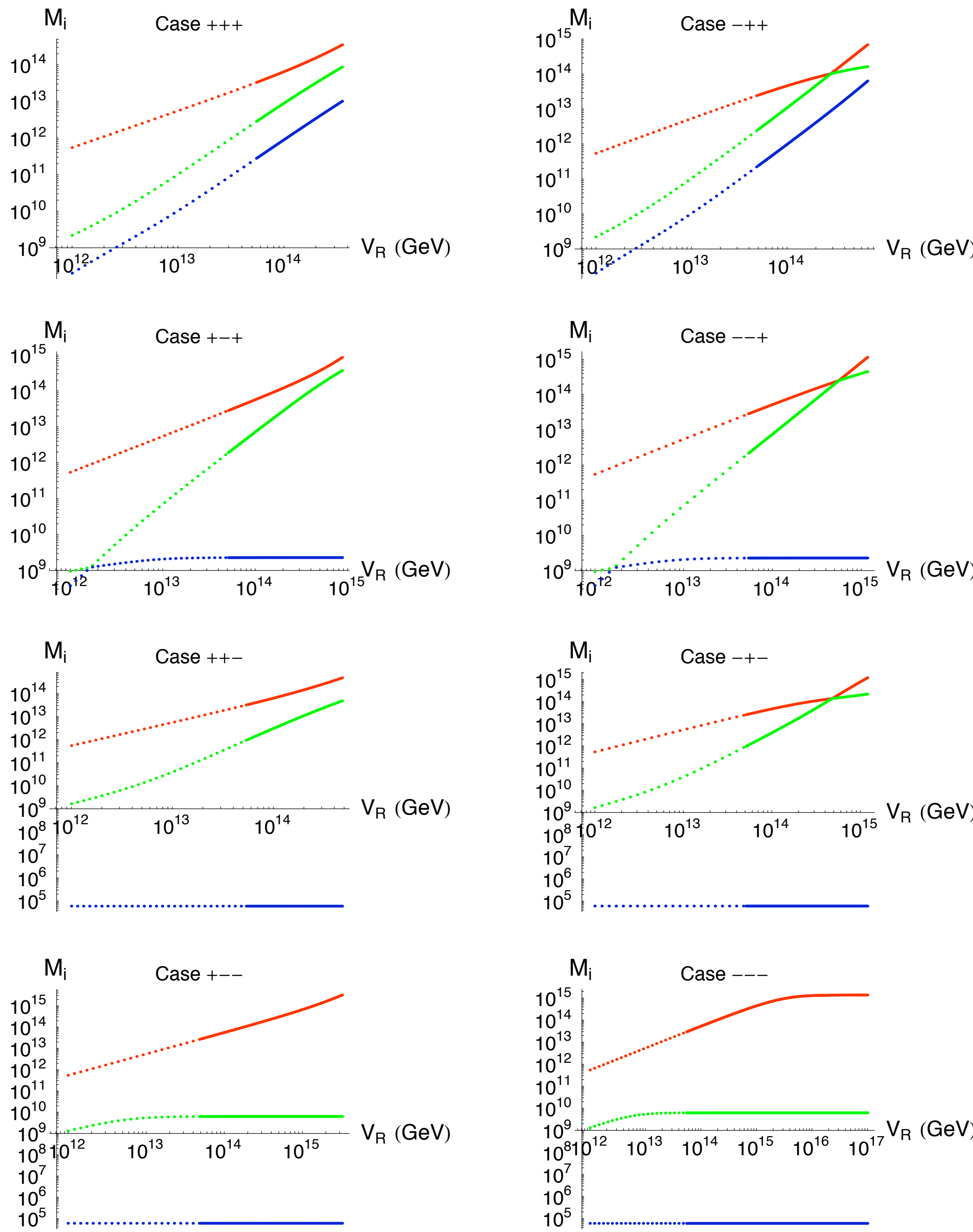
Assuming that the doublets in the $\overline{126}$ have no vev, one has:

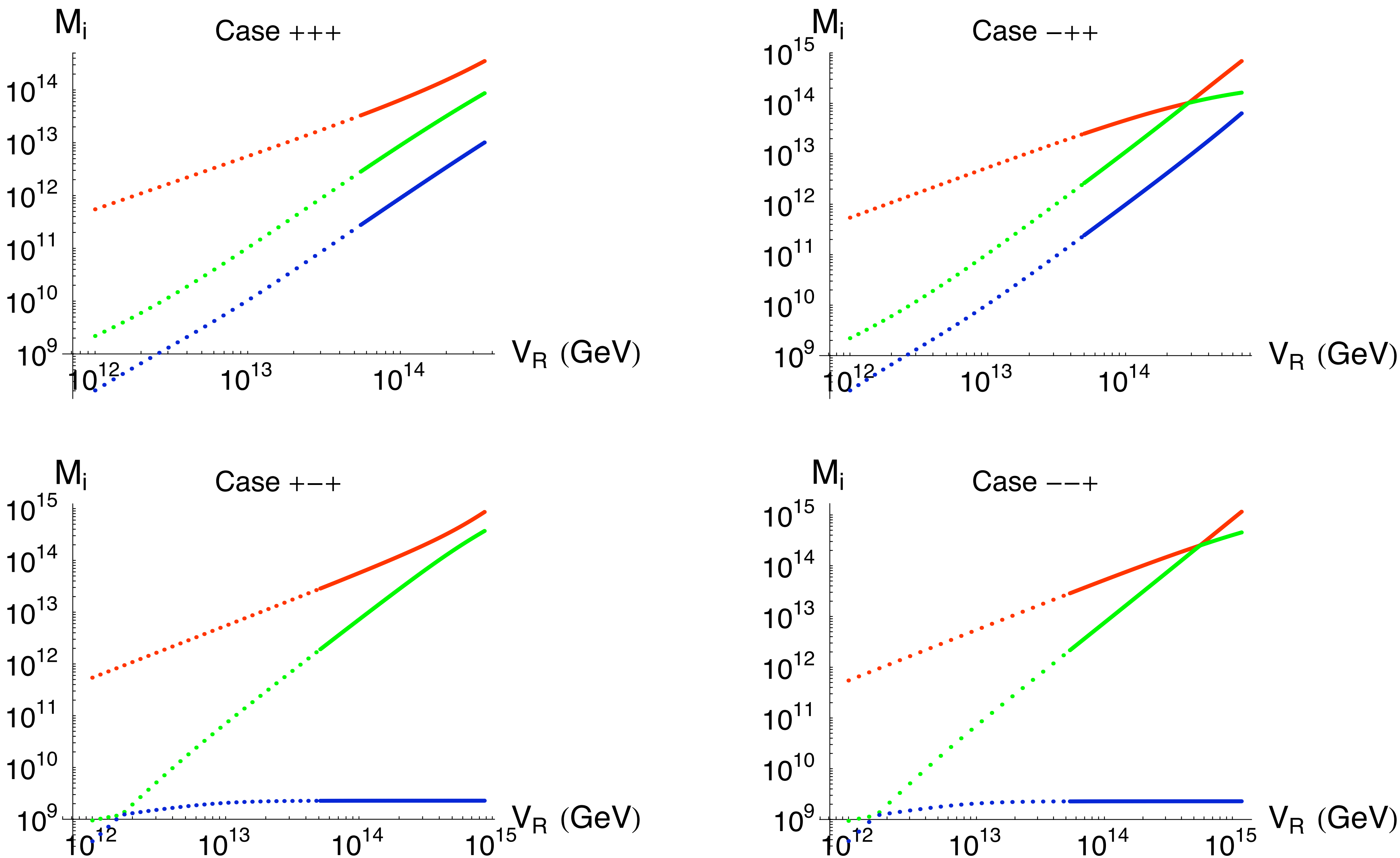
$$Y = M_u / v \qquad\qquad M_d = M_e$$

Then, for a given choice of the neutrino parameters and of the high energy phases contained in M_u, Y and M_ν are known and f can be reconstructed as a function of the B-L breaking scale v_R and of β/α

$\beta/\alpha = v^2 / v_L v_R$ depends on the model. Perturbativity of the f_{ij} couplings constrains β/α ≤ O(1) and restricts the range of v_R

Plots: normal hierarchy with $m_1 = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.009$, $\delta = 0$ and all Majorana and high-energy phases vanish – β = α


Case +++
Case −++
Case +−+
Case −−+
Case ++−
Case −+−
Case +−−
Case −−−
M_i
V_R (GeV)
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}


M_i
Case +++
10^14
10^13
10^12
10^11
10^10
10^9
V_R (GeV)
10^12
10^13
10^14
M_i
Case −++
10^15
10^14
10^13
10^12
10^11
10^10
10^9
V_R (GeV)
10^12
10^13
10^14
M_i
Case +−+
10^15
10^14
10^13
10^12
10^11
10^10
10^9
V_R (GeV)
10^12
10^13
10^14
10^15
M_i
Case −−+
10^15
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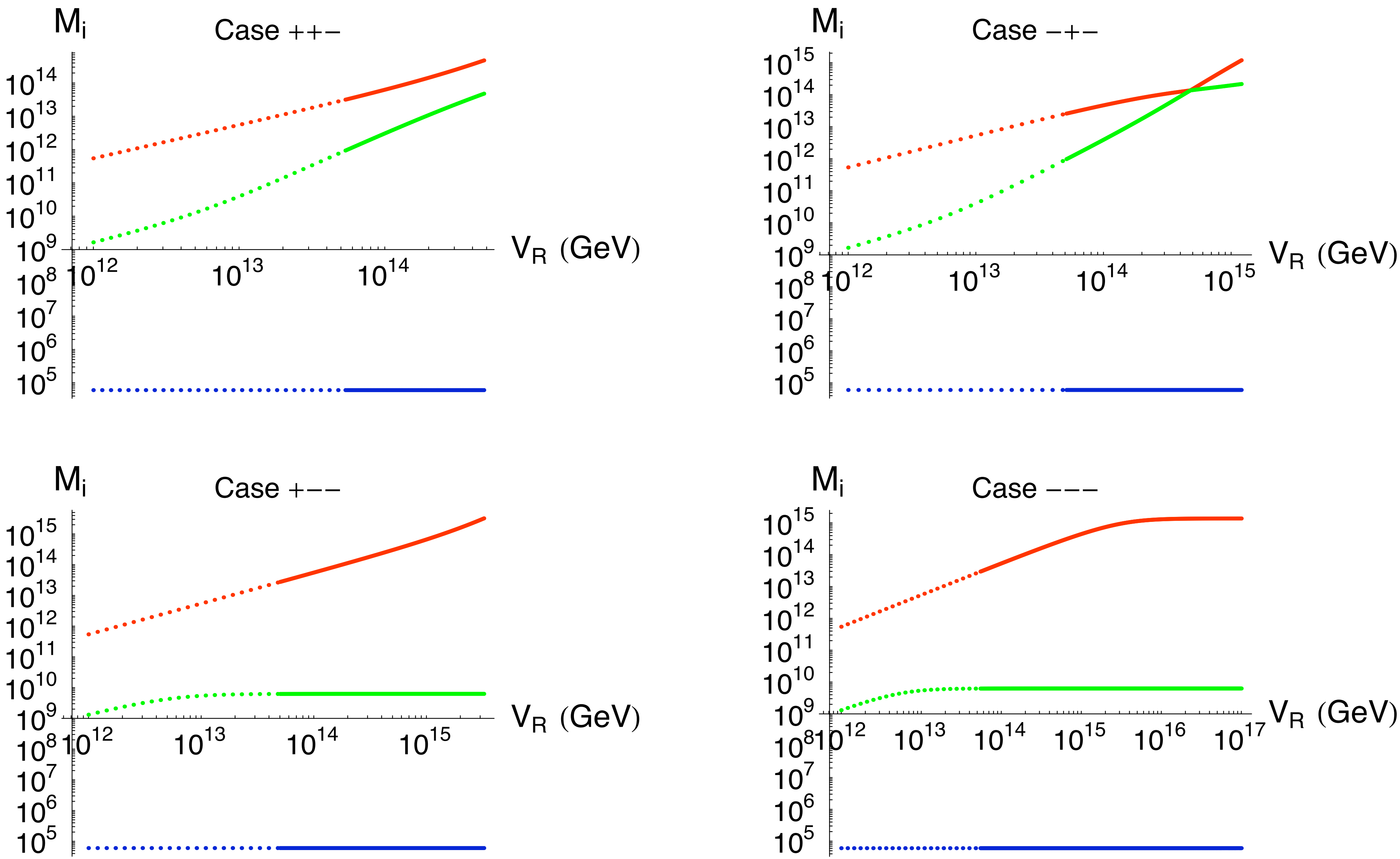
10^9
V_R (GeV)
10^12
10^13
10^14
10^15



M_i
Case $++-$
V_R (GeV)
Case $-+-$
Case $+--$
Case $---$
10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15}
10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17}

Features of the right-handed neutrino spectrum

– at large v_R, the solutions (+,+,+) and (–,–,–) tend to the type II (triplet exchange) and type I (heavy neutrino exchange) cases, respectively:

$$(+,+,+): \qquad M_1 : M_2 : M_3 \sim m_1 : m_2 : m_3$$
$$(-,-,-): \qquad M_1 : M_2 : M_3 \sim m_u^2 : m_c^2 : m_t^2$$

– at small v_R, the type I and type II contribution compensate for each other in such a way that $M_1 : M_2 : M_3 \sim m_u : m_c : m_t$

– 4 solutions are characterized by $M_1 \sim 10^5\,\mathrm{GeV}$

– 2 solutions are characterized by $M_1 \sim 10^9\,\mathrm{GeV}$

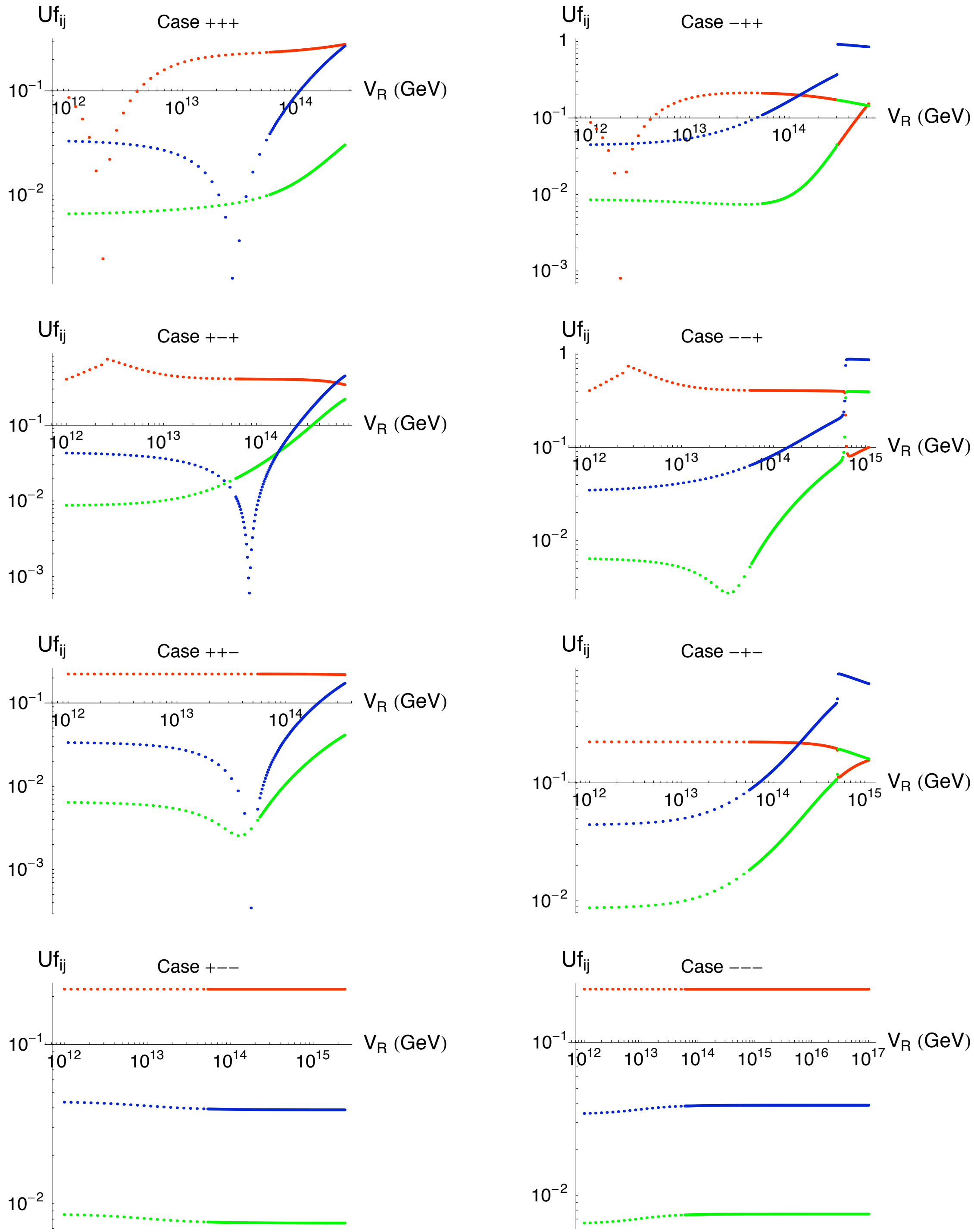
Mixing angles

$$f = U_f \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix} U_f^T \quad \Longrightarrow \quad U_f^\dagger Y$$

Dirac couplings in the basis of N_R mass eigenstates

- 2 solutions have RHN mixing angles very close to the CKM angles

- in the other 6 solutions, the mixing angles are close to the CKM angles at small v_R, then take larger values at large v_R

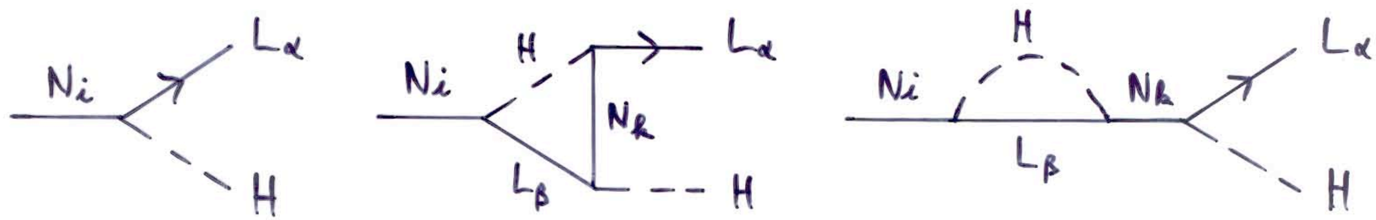


Case +++
Case −++
Case +−+
Case −−+
Case ++−
Case −+−
Case +−−
Case −−−
Uf_{ij}
V_R (GeV)

Implications for leptogenesis

Standard (type I) leptogenesis: out-of equilibrium decays of the heavy RH neutrinos ⇒ lepton asymmetry ⇒ conversion into a baryon asymmetry by sphaleron processes [Fukugita, Yanagida]

CP asymmetry due to interference between tree and 1-loop diagrams:



$$\epsilon_{N_1} \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}H^\star)}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}H^\star)} \simeq \frac{3}{16\pi} \sum_k \frac{\mathrm{Im}[(YY^\dagger)^2_{k1}]}{(YY^\dagger)_{11}} \frac{M_k}{M_1}$$

Covi, Roulet, Vissani
Buchmüller, Plümacher

(assuming $M_1 \ll M_2, M_3$)

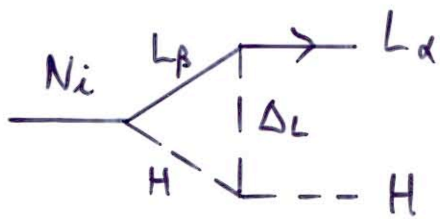
Final baryon asymmetry: $$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = -1.4 \times 10^{-3}\, \eta\, \epsilon_{N_1}$$

(η = efficiency factor)

To generate the observed baryon asymmetry, $Y_B = (8.7 \pm 0.3) \times 10^{-11}$, need $M_1 \gtrsim 10^9\,\mathrm{GeV}$ [Davidson, Ibarra], unless $M_1 \simeq M_2$ (resonant leptogenesis)

The Davidson-Ibarra bound is problematic in SO(10) GUTs with $Y \propto M_u$, which leads to $M_1 << 10^9$ GeV

<u>Type I+II leptogenesis</u>: when $M_1 << M_{\Delta_L}$, the $SU(2)_L$ triplet affects leptogenesis mainly through its contribution to the CP asymmetry of N_1



$$\epsilon^{II}_{N_1} \simeq \frac{3}{8\pi} \sum_{k,l} \frac{\mathrm{Im}\left[Y_{1k} Y_{1l} f^\star_{kl} v_L \right]}{(Y Y^\dagger)_{11} v^2} M_1$$

Hambye, Senjanovic - Antusch, King

The total CP asymmetry $\epsilon_{N_1} = \epsilon^I_{N_1} + \epsilon^{II}_{N_1}$ depends on the reconstructed f_{ij} couplings, and is very sensitive to the high- and low-energy phases

Among the 8 solutions, 3 different patterns emerge for leptogenesis:

- 2 solutions with a rising $M_1 \Rightarrow$ large ϵ_{N_1} for large v_R

- 2 solutions with $M_1 \sim 10^9\,\mathrm{GeV}$

- 4 solutions with $M_1 \sim 10^5\,\mathrm{GeV} \Rightarrow \epsilon_{N_1}$ too small, but $M_2 \sim 10^{10}\,\mathrm{GeV}$ or rises with $v_R \Rightarrow$ the observed baryon asymmetry could be generated from N_2 decays [Di Bari - Vives]

Computation of the baryon asymmetry

Solve the Boltzmann equations with flavour effects and decays of N_1 and N_2

Relevant quantities:

- flavour-dependent CP asymmetries:

$$\epsilon_{N_i}^{\alpha} \equiv \frac{\Gamma(N_i \to L_\alpha H) - \Gamma(N_i \to \bar{L}_\alpha H^\star)}{\Gamma(N_i \to L_\alpha H) + \Gamma(N_i \to \bar{L}_\alpha H^\star)}$$

- wash-out processes: ΔL and N_3 very heavy $\Rightarrow$ associated wash-out processes suppressed. Furthermore, we neglect $\Delta L=2$ processes since we deal with masses M_1 and $M_2 < 10^{12}$ GeV

$\Rightarrow$ only inverse decays and $\Delta L=1$ scatterings associated with N_1 and N_2 enter the Boltzmann equations. The relevant washout parameters are:

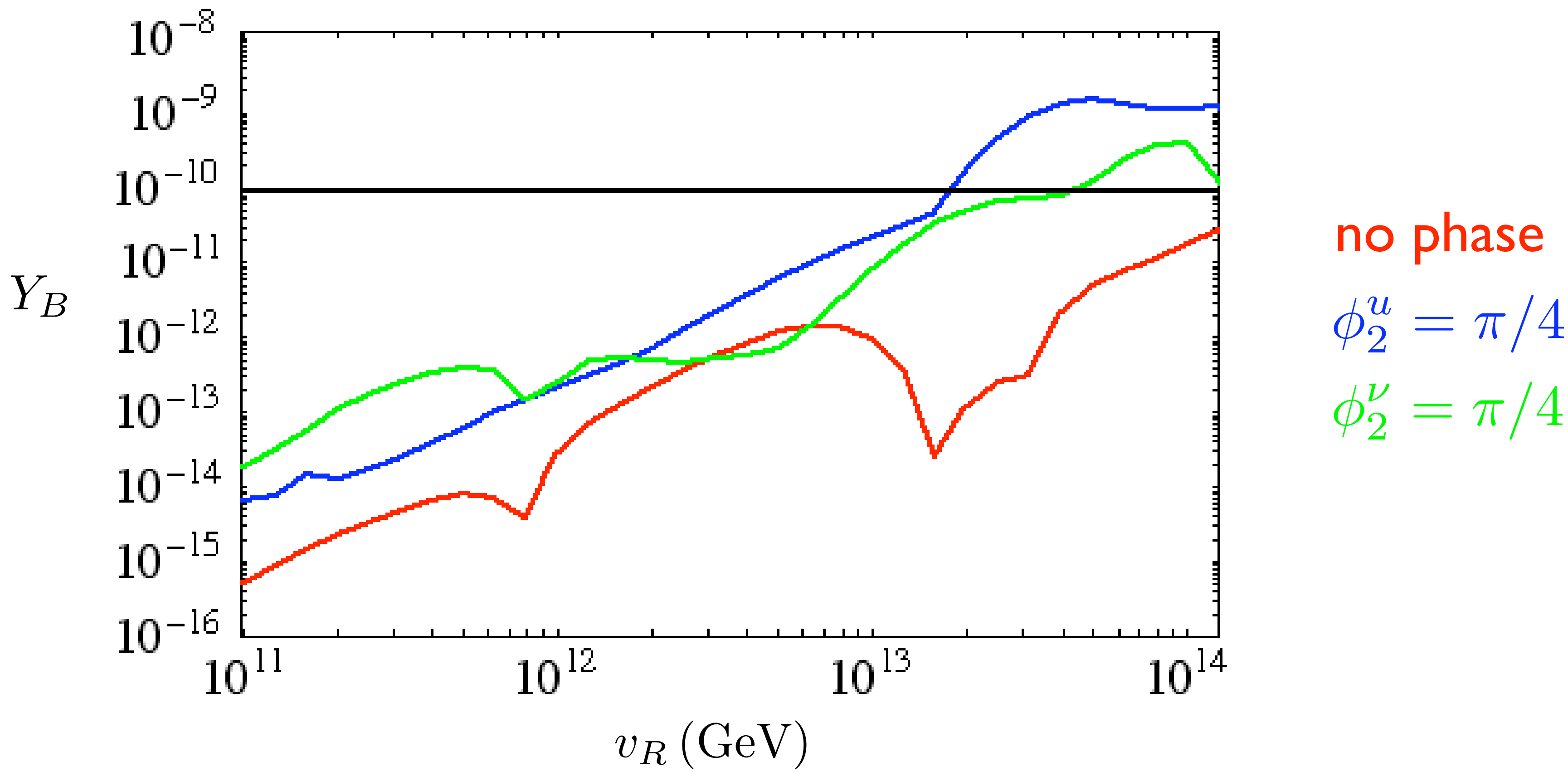
$$\tilde{m}_i^{\alpha} \equiv \frac{|Y_{i\alpha}|^2 v^2}{M_i}$$

Both the $\epsilon_{N_i}^{\alpha}$ and the $\tilde{m}_i^{\alpha}$ depend on the M_i and on the $Y_{i\alpha}$, hence on the reconstructed f_{ij} couplings

Results

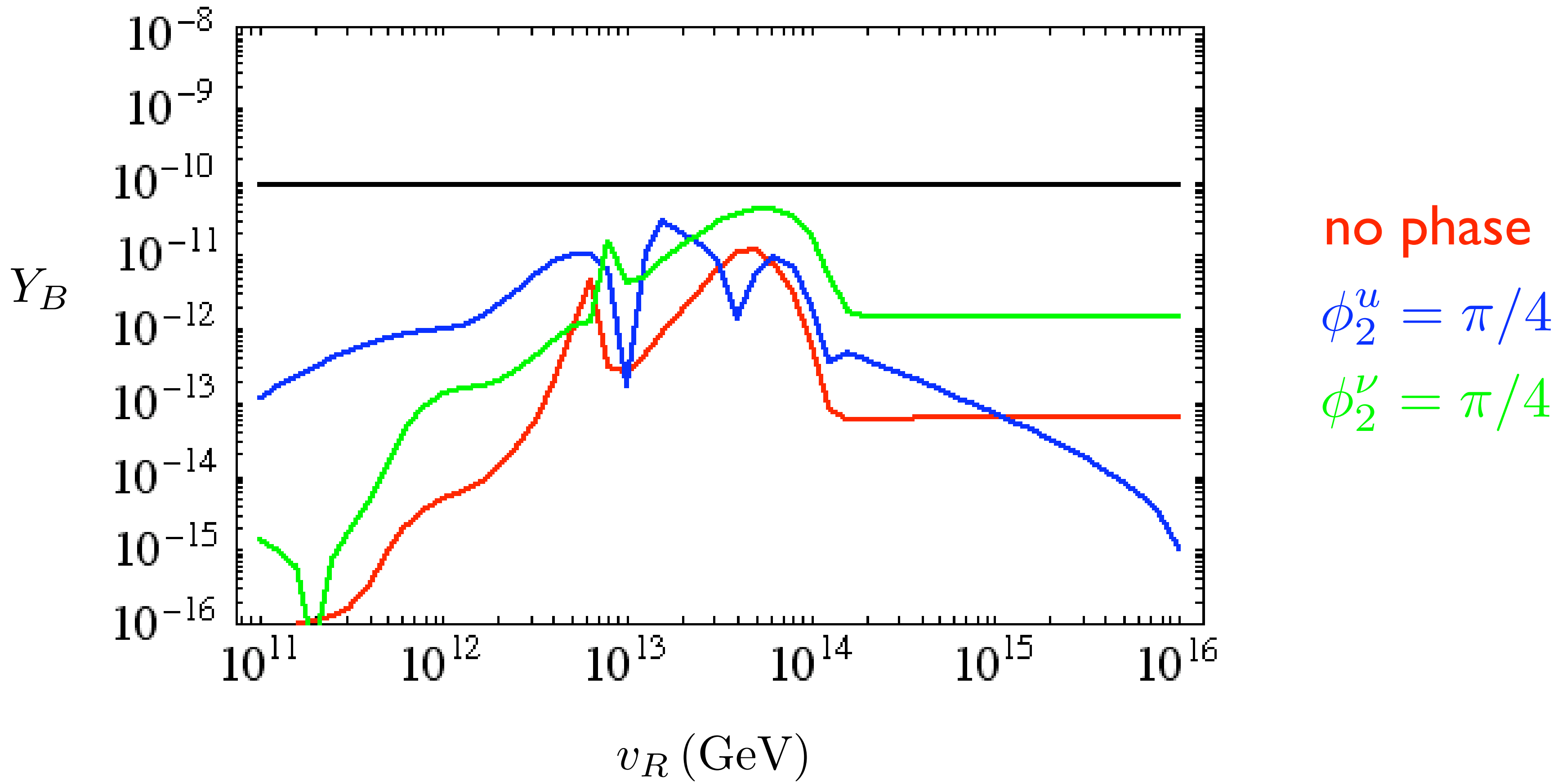
<u>Inputs:</u> normal hierarchy with $m_1 = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.009$, $\delta = 0$ and various choices of the Majorana and high-energy phases – $\beta/\alpha = 0.1$ – Md = Me

<u>solution +++</u>



Tension with gravitino overproduction above $v_R \sim 10^{13}\,\mathrm{GeV}$ ($M_1 > 10^{10}\,\mathrm{GeV}$)

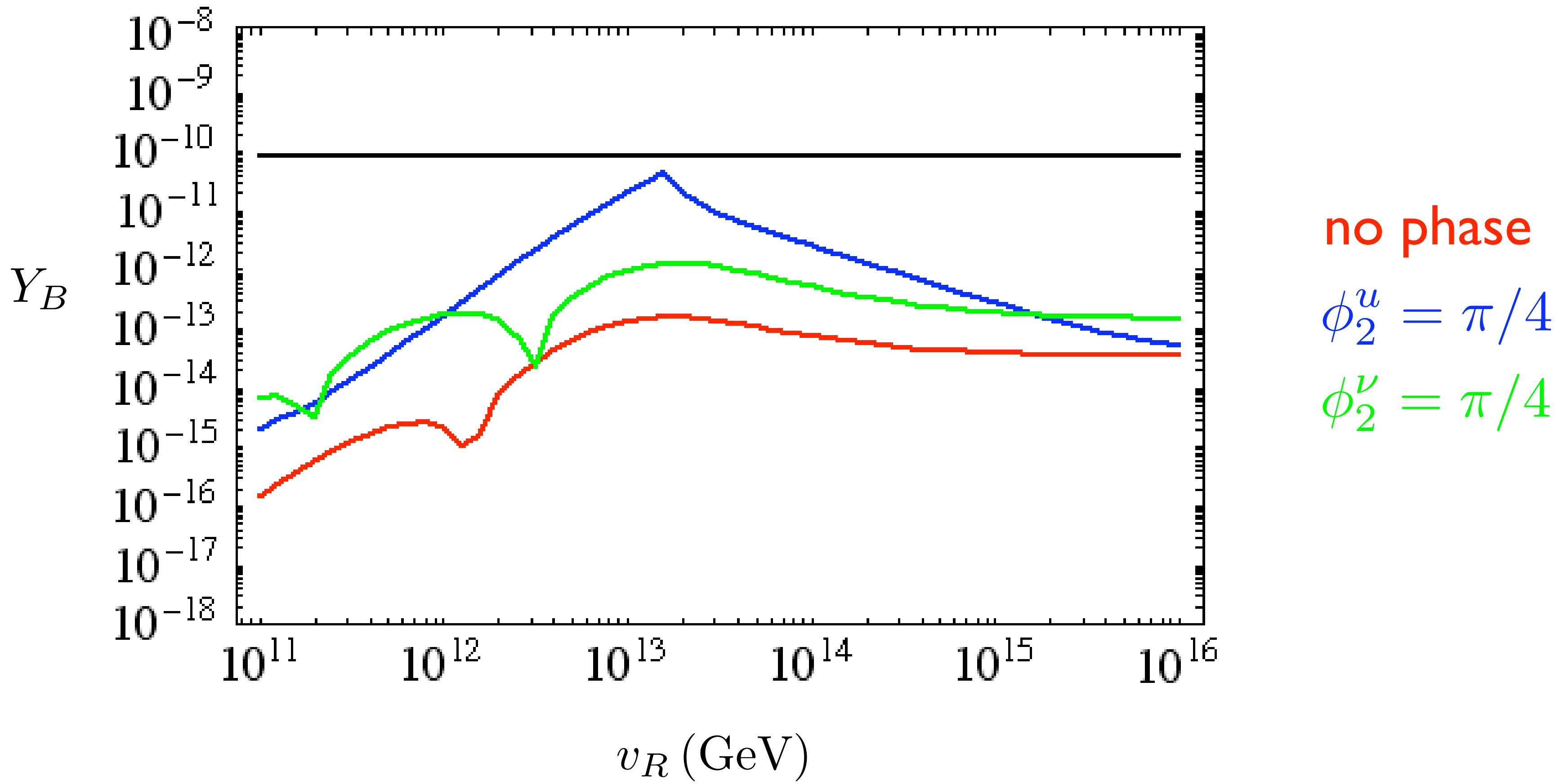
solution +-+



This solution fails to generate the observed baryon asymmetry for hierarchical light neutrino masses and Md = Me

[see Akhmedov et al. (hep-ph/0612194) and the talk by T. Hällgren in the cosmology session for a discussion of the inverted hierarchy case]

solution ---



Flavour effects matter: the most asymmetrically produced flavour in N_2 decays is the least erased by N_1 inverse decays. Still the baryon asymmetry generated from N_2 decays lies below the observed level

Corrections to the mass relation $M_d = M_e$

The above results assumed $M_d = M_e$, but this relation is in conflict with experimental data $\Rightarrow$ must add corrections, e.g. from

$$\frac{\kappa_{ij}}{\Lambda}\; 16_i 16_j 10_H 45_H$$

Assuming that $\langle 45_H \rangle$ is in the B-L direction, and that $\langle 10_H \rangle$ does not contribute to up-type fermion masses, $M_D = M_u$ is preserved but

$$M_d \;=\; \left(Y^d_{10} + \frac{\langle 45_H \rangle}{\Lambda}\, \kappa \right) v_d \qquad\qquad M_e \;=\; \left(Y^d_{10} - 3\, \frac{\langle 45_H \rangle}{\Lambda}\, \kappa \right) v_d$$

This affects the f_{ij} by introducing a mismatch U_m between the bases of charged lepton and down quark mass eigenstates, yielding

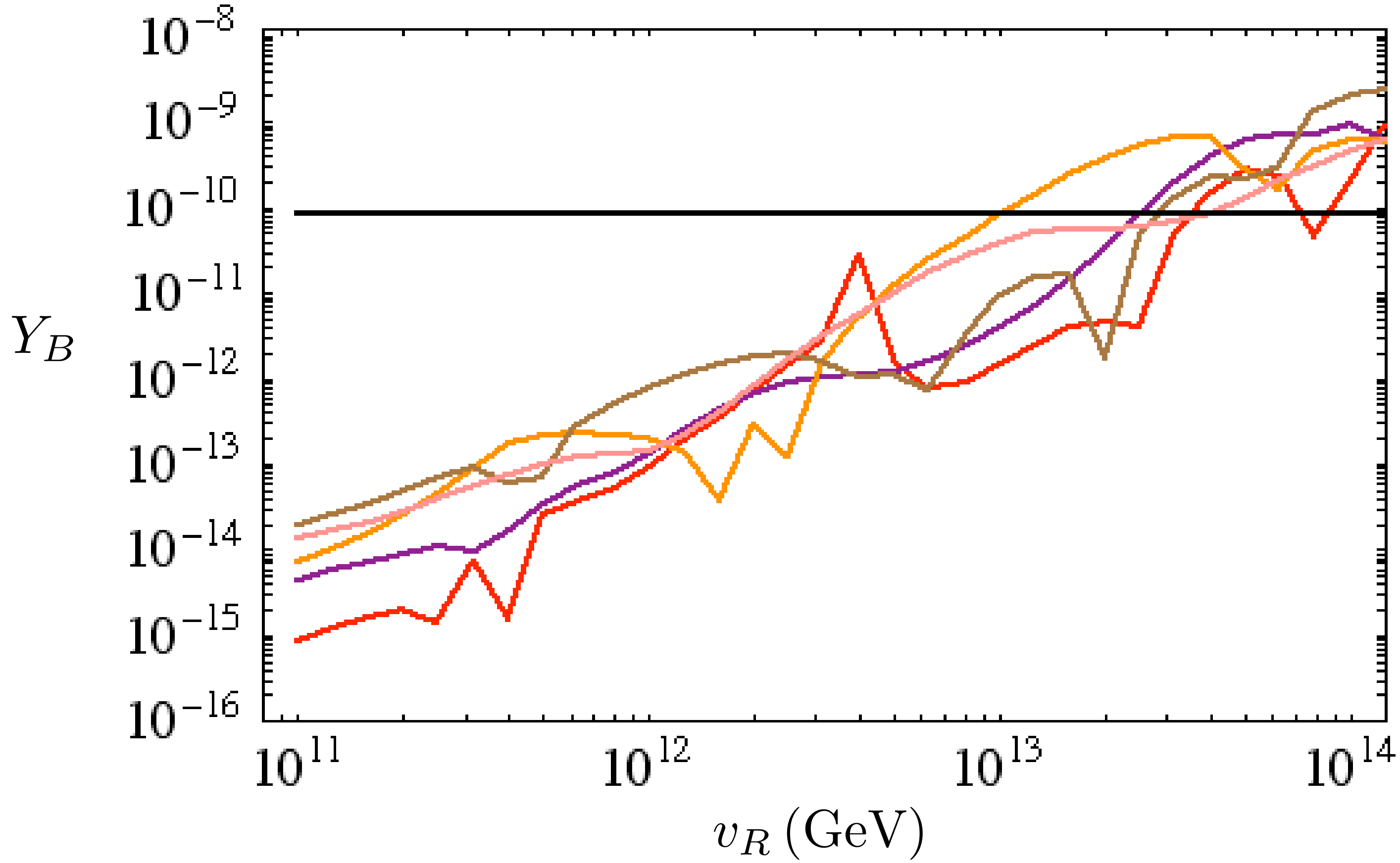
$$M_D \;=\; U_m^T U_q^T \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U_q U_m$$

in the basis of charged lepton mass eigenstates

Results

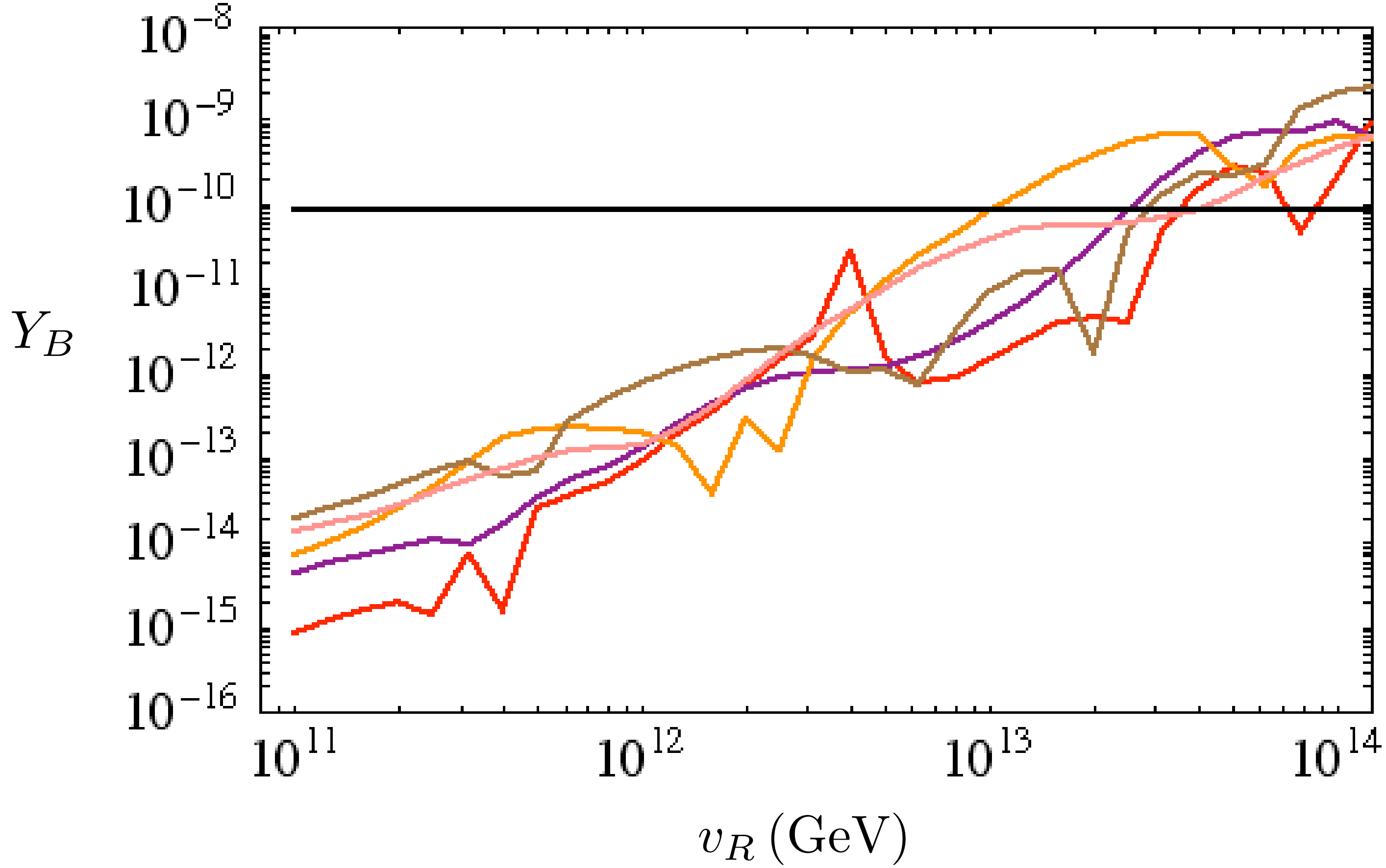
Inputs: normal hierarchy with $m_1 = 10^{-3}$ eV, $\sin^2\theta_{13} = 0.009$, $\delta = 0$ – various choices of Um and of the Majorana and high-energy phases – $\beta/\alpha = 0.1$

solution +++



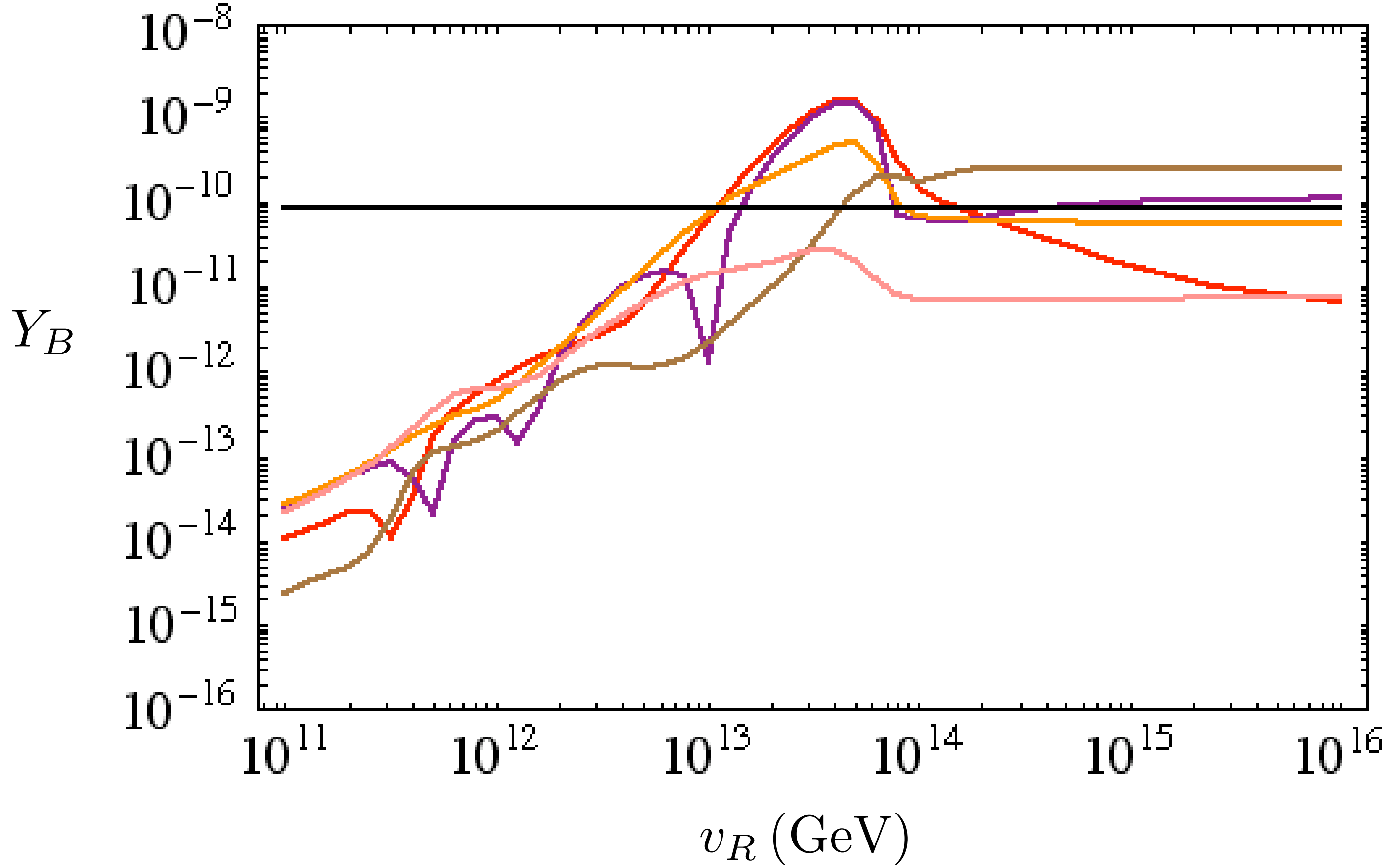
Tension with gravitino overproduction above $v_R \sim 10^{13}\,\mathrm{GeV}$ ($M_1 > 10^{10}\,\mathrm{GeV}$)

solution +++



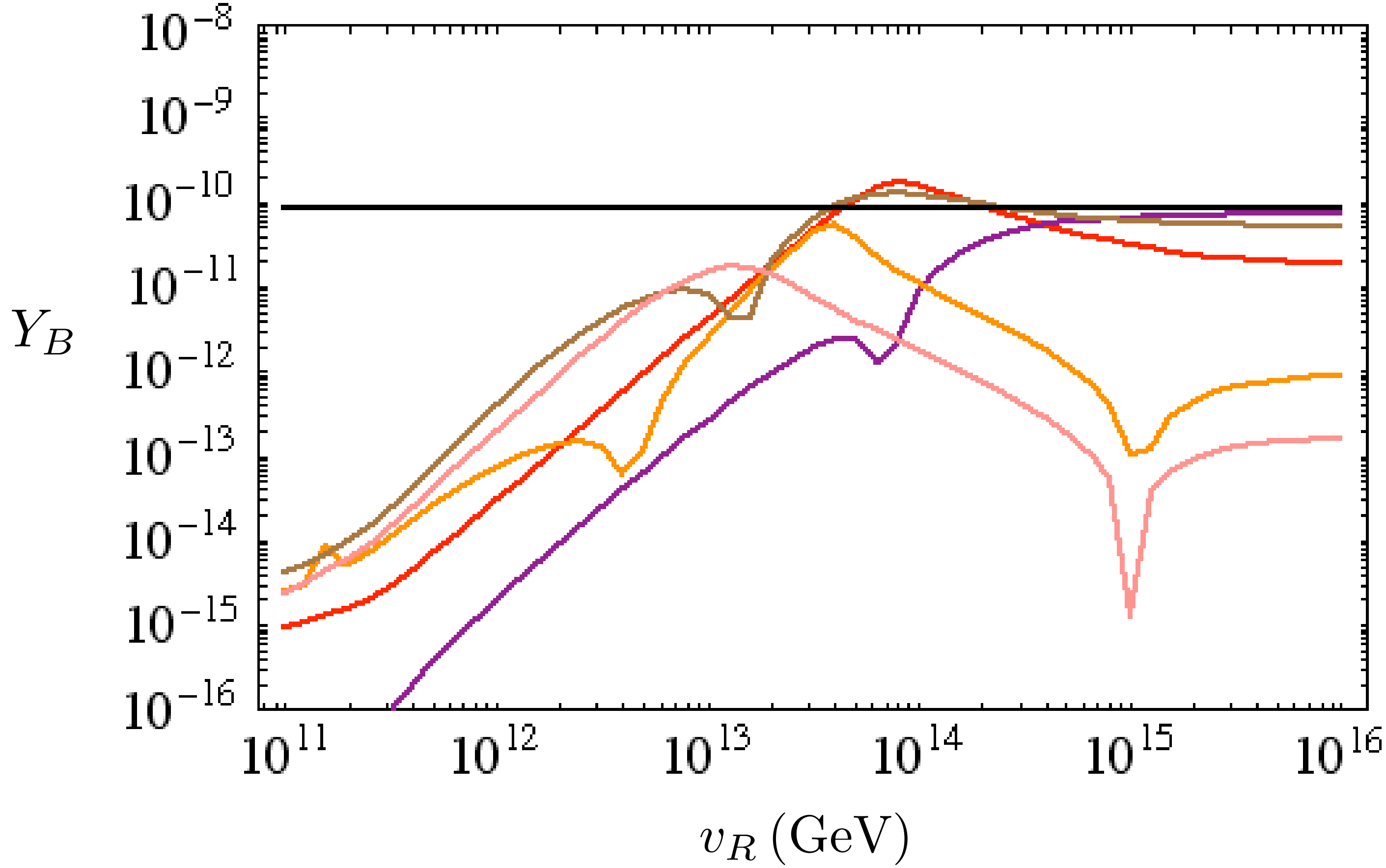
Same inputs as before, but with corrections to $M_d = M_e$ (for different choices of Um reproducing the correct down quark and charged lepton masses). Still conflict with upper bound on T_R above $v_R \sim 10^{13}\,\mathrm{GeV}$

solution +-+



Successful leptogenesis possible for $v_R \gtrsim 10^{13}\,\mathrm{GeV}$ (with $M_1 \lesssim 10^{10}\,\mathrm{GeV}$). The corrections to $M_d = M_e$ play a crucial role here (not enough baryon asymmetry produced for $U_m = 1$) [see Akhmedov et al. (hep-ph/0612194) and the talk by T. Hällgren in the cosmology session for a discussion of the inverted hierarchy case]

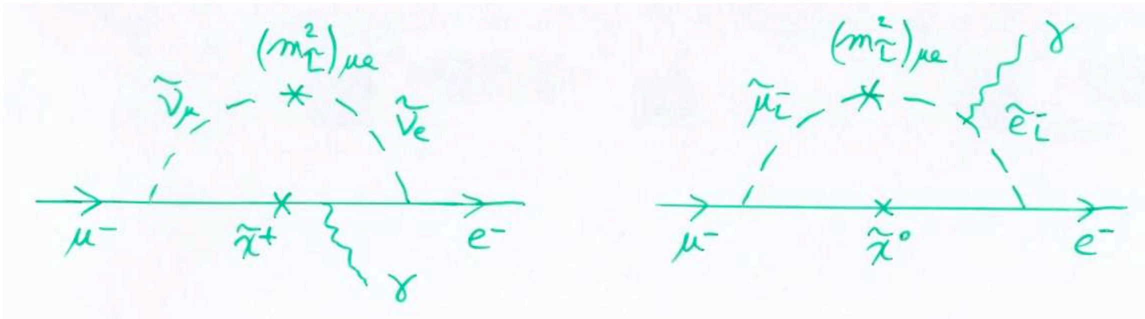
solution ---



The baryon asymmetry generated from N2 decays (with $M_2 \sim 10^{10}\,\mathrm{GeV}$) could marginally explain the observed value. For this solution too the corrections to Md = Me play a crucial role

Implications for lepton flavour violation

Flavour violation in the slepton sector induces LFV processes such as $\mu \to e\,\gamma$ or $\tau \to \mu\,\gamma$



Heavy states with LFV couplings induce flavour-violating slepton mass terms radiatively. The contribution of the RH neutrinos and of the scalar triplet [Borzumati, Masiero; Rossi] can be estimated by (leading-log approximation + universality among soft terms at M_U):

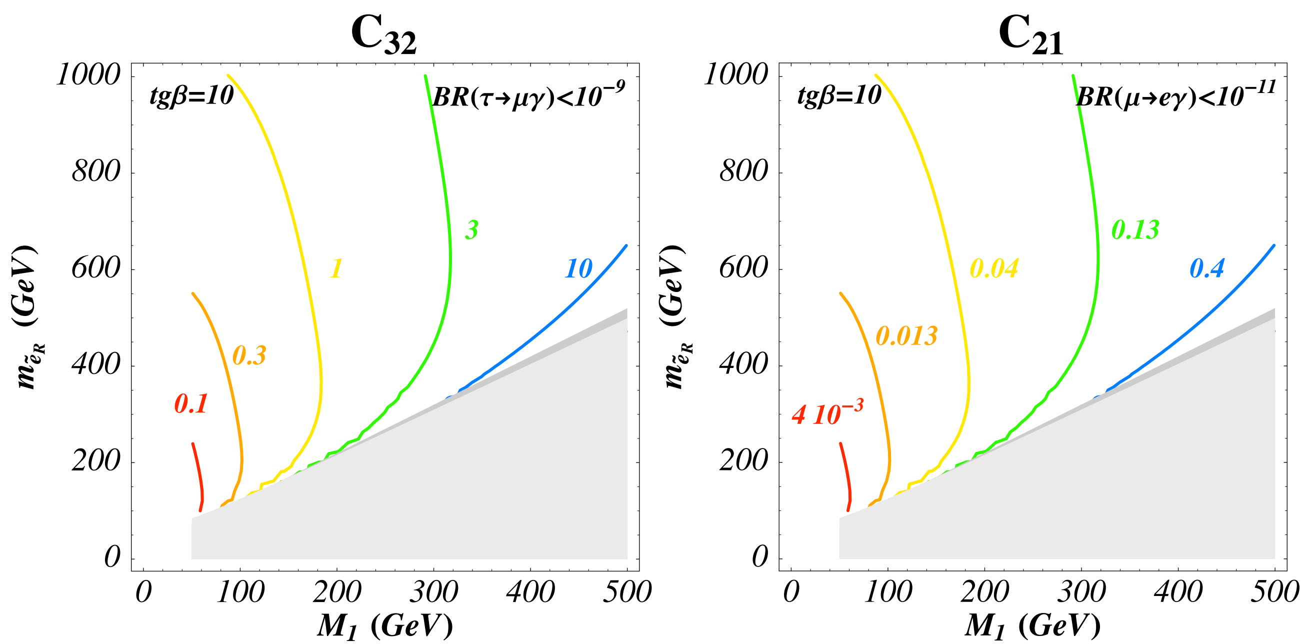
$$(m^2_{\tilde{L}})_{ij} \;\simeq\; -\,\frac{3m_0^2 + A_0^2}{8\pi^2}\,C_{ij}\;, \qquad (m^2_{\tilde{e}_R})_{ij} \;\simeq\; 0\;, \qquad A^e_{ij} \;\simeq\; -\,\frac{3}{8\pi^2}\,A_0 y_{e_i} C_{ij}$$

where the Cij's encapsulate the dependence on the seesaw parameters:

$$C_{ij} \;\equiv\; \sum_k Y^\star_{ki} Y_{kj}\,\ln\left(\frac{M_U}{M_k}\right) + 3\,(f f^\dagger)_{ij}\,\ln\left(\frac{M_U}{M_{\Delta_L}}\right)$$

[in the following, we take $M_U = 10^{17}$ GeV and $M_{\Delta_L} = v_R$]

Experimental upper limits on the LFV decays li → lj γ can be turned into upper bounds on the Cij's as a function of the supersymmetric mass parameters and of tanβ:



S.L., Masina, Savoy

For tanβ = 10 and m_0, $M_{1/2}$ ≤ O(1 TeV), we obtain the "experimental" upper bounds $|C_{23}|$ ≤ 10 (from τ → μ γ) and $|C_{12}|$ ≤ 0.1 (from μ → e γ)

We can then compare the predicted Cij's for a given solution f with these "experimental" upper bounds:

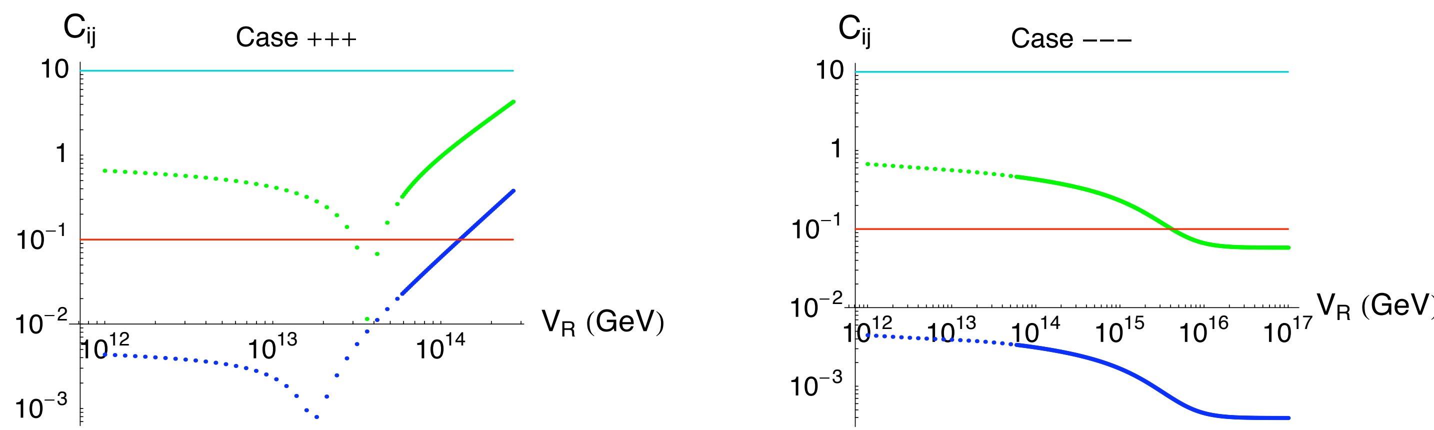


Figure 9: Coefficients C_{12} and C_{23} as a function of v_R for the solutions $(+,+,+)$ and $(-,-,-)$ in the case of a hierarchical light neutrino mass spectrum with $m_1 = 10^{-3}$ eV, $\beta = \alpha$, and no CP violation beyond the CKM phase. The green [light grey] curve corresponds to $|C_{23}|$, and the blue [black] curve to $|C_{12}|$. The horizontal lines indicate the "experimental" constraints $|C_{23}| < 10$ and $|C_{12}| < 0.1$ (see text).

The predictions lie significantly below the experimental bounds, except in the large v_R region where, depending on the supersymmetric parameters, $\mu \to e\,\gamma$ can exceed its present upper limit

Due to the small CKM angles [$V_L = V_{CKM}$], the type II contribution always dominates, except in the large v_R region of solutions (–,–,–) [type I limit] and (+,–,–)

Conclusions

- The possibilities to account for the observed neutrino data is much richer in the left-right symmetric seesaw mechanism than in the case of type I or type II dominance, with interesting implications for leptogenesis and LFV

- In particular, the mixed solutions where both seesaw mechanisms give a significant contribution to neutrino masses provide new opportunities for successful leptogenesis in SO(10) GUTs

(the final asymmetry strongly depends on the values of the Yukawa couplings, so a correct description of charged fermion masses is an important ingredient in the analysis)

Back-up slides

Note: diagonalization of a complex symmetric matrix by a complex orthogonal matrix

1) the eigenvalues of Z are the roots of $\mathrm{Det}\,(Z - z\mathbf{1}) = 0$

2) the eigenvectors associated with z_i are the solutions of $Z.\vec{v} = z_i\vec{v}$

It is always possible to find solutions of the latter equation, but in case of multiple solutions, it is not always possible to find an orthonormal basis of the eigenspace. The problem arises when one non-trivial solution has a zero norm in the SO(3, C) sense, i.e. $\vec{v}.\vec{v} = 0$; then Z cannot be diagonalized.

If all eigenvalues of Z are distinct, the eigenvectors automatically satisfy $\vec{v}.\vec{v} \neq 0$, hence Z is diagonalizable (it can be written as $O_Z \mathrm{Diag}\,(z_1, z_2, z_3) O_Z^T$)

Flavour effects in leptogenesis

Barbieri, Creminelli, Strumia, Tetradis
Endoh et al. - Pilaftsis et al. - Nardi et al. - Abada et al.
Blanchet, Di Bari, Raffelt - Pascoli, Petcov, Riotto - ...

"one-flavour approximation": leptogenesis described in terms of a single direction in flavour space, the lepton $\mathcal{L}_1 \propto \sum_\alpha Y_{1\alpha} L_\alpha$ to which N_1 couples
$\Rightarrow$ valid as long as the charged lepton Yukawas λ_α are out of equilibrium

At $T \lesssim 10^{12}\,\mathrm{GeV}$, λ_τ is in equilibrium and destroys the coherence of $\mathcal{L}_1$
$\Rightarrow$ 2 relevant flavours: L_τ and a combination of L_e and L_μ

At $T \lesssim 10^{9}\,\mathrm{GeV}$, λ_τ and λ_μ are in equilibrium $\Rightarrow$ must distinguish between L_e, L_μ and L_τ

Relevant parameters for the discussion of flavour effects:

$$\epsilon^\alpha_{N_1} \equiv \frac{\Gamma(N_1 \to L_\alpha H) - \Gamma(N_1 \to \bar{L}_\alpha H^\star)}{\Gamma(N_1 \to L_\alpha H) + \Gamma(N_1 \to \bar{L}_\alpha H^\star)} \qquad \tilde{m}^\alpha_1 \equiv \frac{|Y_{1\alpha}|^2 v^2}{M_1}$$

qualitatively $Y_B \approx \sum_\alpha \epsilon^\alpha_{N_1} \eta(\tilde{m}^\alpha_1)$ $\Rightarrow$ can deviate from the one-flavour approximation if e.g. $\epsilon^\tau_{N_1} \gg \epsilon^e_{N_1}, \epsilon^\mu_{N_1}$ and $\tilde{m}^\tau_1 \ll \tilde{m}^e_1, \tilde{m}^\mu_1$