# Transverse-momentum, threshold and joint resummation for slepton-pair production at hadron colliders 

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## Outline

1) Introduction and motivations
(2) Resummation formalisms

- Main features of the resummation
- The resummed component
- Matching procedure
(3) Applications
- $q_{T}$-distribution
- Invariant-mass distribution
- Total cross sections

4 Summary and outlook

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(1) Introduction and motivations


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## Slepton-pair production at hadron colliders

- Drell-Yan like process


$$
q \bar{q} \rightarrow \gamma, Z^{0} \rightarrow \tilde{I}_{i} \tilde{I}_{j}^{*} \quad \text { and } \quad q \bar{q}^{\prime} \rightarrow W^{\mp} \rightarrow \tilde{I}_{i} \tilde{\nu}_{l}^{*}+c . c .
$$

- Sleptons are often light $\Rightarrow$ decays into LSP + SM lepton $\Rightarrow$ clean signal.
- Cross sections given by

$$
\sigma=\sum_{a, b} \int_{\tau}^{1} \mathrm{~d} x_{a} \int_{\tau / x_{a}}^{1} \mathrm{~d} x_{b} f_{a / h_{1}}\left(x_{a}, \mu_{F}\right) f_{b / h_{2}}\left(x_{b}, \mu_{F}\right) \hat{\sigma}_{a b}\left(z, M ; \alpha_{s}\left(\mu_{R}\right), \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}\right)
$$

where $\hat{\sigma}_{a b}$ is computed perturbatively

$$
\hat{\sigma}_{a b}\left(z, M ; \alpha_{s}\left(\mu_{R}\right), \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}\right)=\sum_{n=0}^{\infty}\left(\frac{\alpha_{s}\left(\mu_{R}\right)}{\pi}\right)^{n} \hat{\sigma}_{a b}^{(n)}\left(z, M ; \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}\right) .
$$

## Next-to-leading order calculations

- Feynman diagrams:

- Squark mixing included in the SUSY-loops.
- Partonic invariant-mass and transverse-momentum distributions at $\mathcal{O}\left(\alpha_{s}\right)$ :

$$
\begin{aligned}
\frac{\mathrm{d} \hat{\sigma}_{a b}}{\mathrm{~d} M^{2}} & =\hat{\sigma}_{a b}^{(0)}(M) \delta(1-z)+\frac{\alpha_{s}}{\pi} \hat{\sigma}_{a b}^{(1)}(M, z)+\mathcal{O}\left(\alpha_{s}^{2}\right) \\
\frac{\mathrm{d}^{2} \hat{\sigma}_{a b}}{\mathrm{~d} M^{2} \mathrm{~d} q_{T}^{2}} & =\hat{\sigma}_{a b}^{(0)}(M) \delta\left(q_{T}^{2}\right) \delta(1-z)+\frac{\alpha_{s}}{\pi} \hat{\sigma}_{a b}^{(1)}\left(M, z, q_{T}\right)+\mathcal{O}\left(\alpha_{s}^{2}\right)
\end{aligned}
$$

where $z=M^{2} / s$.

## $q_{T}$ and invariant-mass distributions

- Soft and collinear radiations:
* $\frac{\alpha_{s}^{n}}{q_{T}^{2}} \ln ^{m} \frac{M^{2}}{q_{T}^{2}}$ or $\alpha_{s}^{n}\left(\frac{\ln ^{m}(1-z)}{1-z}\right)_{+}$terms in the distributions $(m \leq 2 n-1)$.
* Large at small $q_{T}$ or $z \lesssim 1$.
* Fixed-order theory unreliable in these kinematical regions.
* Resummation to all orders needed.
$\Rightarrow q_{T}$-resummation.
$\Rightarrow$ Threshold resummation.
$\Rightarrow$ Joint resummation.
- Advantages of resummation:
* Reliable perturbative results.
* Correct quantification of these radiations (even far from critical regions).
* Accurate invariant-mass and $q_{T}$ spectra.
$q_{T}$-distribution $\Rightarrow$ stransverse mass $\Rightarrow$ spin and mass determination.
[Lester, Summers (1999); Barr (2006)]
$M$-distribution and total cross section $\Rightarrow$ accurate mass determination.
[Bozzi, BF, Klasen (2007)]


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## Main features of the resummation

Reorganization of the cross section

$$
\mathrm{d} \sigma=\mathrm{d} \sigma^{(\mathrm{res})}+\mathrm{d} \sigma^{(\mathrm{fin})}
$$

- $\mathrm{d} \sigma^{\text {(res) }}$
* Contains all the logarithmic terms.
* Resummed to all orders in $\alpha_{s}$.
* Exponentiation (Sudakov form factor).
- $\mathrm{d} \sigma^{(\mathrm{fin})}$
* Remaining contributions.


## The resummed component: conjugate spaces

- Conjugate spaces: Mellin, impact-parameter $\Rightarrow$ kinematics naturally factorizes.
- Factorization of the hadronic cross sections:

$$
\begin{aligned}
& \frac{\mathrm{d} \sigma^{(\mathrm{res})}}{\mathrm{d} M^{2}}(\tau, M)=\sum_{a, b} \int_{\tau}^{1} \mathrm{~d} x_{a} \int_{\tau / x_{a}}^{1} \mathrm{~d} x_{b} f_{a / h_{1}}\left(x_{a}, \mu_{F}\right) f_{b / h_{2}}\left(x_{b}, \mu_{F}\right) \hat{\sigma}_{a b}^{(\mathrm{res})}\left(z ; \alpha_{s}\left(\mu_{R}\right), \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}\right) \\
& \quad \Downarrow \\
& \frac{\mathrm{d} \sigma^{(\mathrm{res})}}{\mathrm{d} M^{2}}(N, M)=\sum_{a, b} f_{a / h_{1}}\left(N+1, \mu_{F}\right) f_{b / h_{2}}\left(N+1, \mu_{F}\right) \hat{\sigma}_{a b}^{(\mathrm{res})}\left(N ; \alpha_{s}, \frac{M}{\mu_{R}}, \frac{M}{\mu_{F}}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\mathrm{d}^{2} \sigma(\mathrm{res})}{\mathrm{d} M^{2} \mathrm{~d} q_{T}^{2}}\left(\tau, M, q_{T}\right)=\sum_{a, b} \int_{\tau}^{1} \mathrm{~d} x_{a} \int_{\tau / x_{a}}^{1} \mathrm{~d} x_{b} f_{a / h_{1}}\left(x_{a}, \mu_{F}\right) f_{b / h_{2}}\left(x_{b}, \mu_{F}\right) \hat{\sigma}_{a b}^{(\mathrm{res})}\left(z, q_{T} ; \alpha_{s}\left(\mu_{R}\right), \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}\right) \\
& \frac{\mathrm{d}^{2} \sigma(\mathrm{res})}{\mathrm{d} M^{2} \mathrm{~d} q_{T}^{2}}\left(N, M, q_{T}\right)=\sum_{a, b} f_{a / h_{1}}\left(N+1, \mu_{F}\right) f_{b / h_{2}}\left(N+1, \mu_{F}\right) \int \frac{b}{2} \mathrm{~d} b J_{0}\left(b q_{T}\right) \mathcal{W}_{a b}^{F}\left(N, b ; \alpha_{s}, \frac{M}{\mu_{R}}, \frac{M}{\mu_{F}}\right) .
\end{aligned}
$$

- The logarithms are included in the functions $\hat{\sigma}^{(\text {res })}$ and $\mathcal{W}^{F}$ :

$$
\left(\frac{\ln (1-z)}{1-z}\right)_{+} \rightarrow \ln ^{2} \bar{N} \text { with } \bar{N}=N \exp \left[\gamma_{E}\right] \quad \frac{1}{q_{T}^{2}} \ln \frac{M^{2}}{q_{T}^{2}} \rightarrow \ln \bar{b}^{2} \quad \text { with } \bar{b}=\frac{b M}{2} \exp \left[\gamma_{E}\right]
$$

## The resummed component: the partonic cross section

- The process-dependence is factorized outside the exponent:

$$
\begin{aligned}
\mathcal{W}_{a b}^{F}(N, b) & =\mathcal{H}_{a b}^{F}(N) \exp \{\mathcal{G}(N, b)\} \\
\hat{\sigma}_{a b}^{(\text {res })}(N) & =\sigma^{(L O)} \tilde{C}_{a b}\left(N ; \alpha_{s}\right) \exp \{\mathcal{G}(N, L)\}
\end{aligned}
$$

- $\mathcal{H}^{F}$ - and $\tilde{C}_{\text {-functions: }}$
* Can be computed perturbatively and are process-dependent.
* Contain real and virtual collinear radiation, and hard contributions.
- The Sudakov form factor contains the soft-collinear radiation:
* Can be computed perturbatively and is process-independent.
- Used formalisms:
* Universal $q_{T}$-resummation. [Catani, de Florian, Grazzini (2001); Bozzi, Catani, de Florian, Grazzini (2006)]
* Threshold resummation including collinear radiation. [Sterman (1987); Catani, Trentadue (1989, 1991); Krämer, Laenen, Spira (1998); Catani, de Florian, Grazzini (2001)]
* Universal joint resummation. [Laenen, Sterman, Vogelsang (2001); Kulesza, Sterman, Vogelsang (2002, 2004); Bozzi, BF, Klasen (in prep.)]


## The finite component: matching procedure

- Fixed-order theory
* Reliable far from the critical kinematical regions $\left(z \ll 1, q_{T} \gg 0\right)$.
* Spoiled in the critical regions $\left(z \sim 1, q_{T} \sim 0\right)$.
- Resummation
* Needed in the critical regions.
* Not justified far from the critical regions.
- Both contributions important in the intermediate kinematical regions.
- Information from both fixed-order and resummation needed.
- Need to avoid double-counting.
- Consistent matching procedure required:

$$
\mathrm{d} \sigma^{(\mathrm{fin})}=\mathrm{d} \sigma^{(\mathrm{f} . \mathrm{o} .)}-\mathrm{d} \sigma^{(\exp )} .
$$

## Summary: complete resummation formulae

- Invariant-mass spectrum

$$
\begin{aligned}
\frac{\mathrm{d} \sigma}{\mathrm{~d} M^{2}}(\tau, M) & =\frac{\mathrm{d} \sigma^{(\mathrm{F} . \mathrm{O} .)}}{\mathrm{d} M^{2}}(\tau, M) \\
& +\oint_{C_{N}} \frac{\mathrm{~d} N}{2 \pi i} \tau^{-N}\left[\frac{\mathrm{~d} \sigma^{(\mathrm{res})}}{\mathrm{d} M^{2}}(N, M)-\frac{\mathrm{d} \sigma^{(\exp )}}{\mathrm{d} M^{2}}(N, M)\right]
\end{aligned}
$$

- Transverse-momentum spectrum

$$
\begin{aligned}
\frac{\mathrm{d}^{2} \sigma}{\mathrm{~d} M^{2} \mathrm{~d} q_{T}^{2}}\left(\tau, M, q_{T}\right) & =\frac{\mathrm{d}^{2} \sigma(\mathrm{~F} \cdot \mathrm{O} \cdot)}{\mathrm{d} M^{2} \mathrm{~d} q_{T}^{2}}\left(\tau, M, q_{T}\right) \\
& +\oint_{C_{N}} \frac{\mathrm{~d} N}{2 \pi i} \tau^{-N} \int \frac{b \mathrm{~d} b}{2} J_{0}\left(q_{T} b\right)\left[\frac{\mathrm{d}^{2} \sigma^{(\mathrm{res})}}{\mathrm{d} M^{2} \mathrm{~d} q_{T}^{2}}(N, b)-\frac{\mathrm{d}^{2} \sigma^{2}(\exp )}{\mathrm{d} M^{2} \mathrm{~d} q_{T}^{2}}(N, b)\right] .
\end{aligned}
$$

* Far from the critical regions, $\mathrm{d} \sigma^{(\mathrm{res})} \approx \mathrm{d} \sigma^{(\exp )} \Rightarrow$ Perturbative theory.
* In the critical regions, $\mathrm{d} \sigma^{(\mathrm{F} . \mathrm{O} .)} \approx \mathrm{d} \sigma^{(\exp )} \Rightarrow$ Pure resummation.
* In the intermediate regions $\Rightarrow$ Consistent matching.


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## $q_{T}$-distribution at the LHC



[Bozzi, BF, Klasen (2006; in prep.)]

* SPS1a and BFHK-B SUSY scenarios (slepton masses $\approx 100-200 \mathrm{GeV}$ ).
* Finite results at small $q_{T}$; enhancement at intermediate $q_{T}$; finite total $\sigma$.
* Improvement of scale dependences: (NLL+F.O. $\lesssim 5 \%$; F.O. 10\%).
* Effects of the threshold-enhanced contributions in the intermediate- $q_{T}$ region.


## Invariant-mass distribution at the LHC



[Bozzi, BF, Klasen (2007; in prep.)]

* SPS1a and BFHK-B SUSY scenarios (slepton masses $\approx 100-200 \mathrm{GeV}$ ).
* Normalization to LO cross section.
* Small $M: \mathrm{d} \sigma^{(\text {res })} \approx \mathrm{d} \sigma^{(\exp )} ;$ Large $M: \mathrm{d} \sigma^{(\mathrm{F} . \mathrm{O} .)} \approx \mathrm{d} \sigma^{(\exp )}$.
* Reduced SUSY-loop effects.
* Joint-exponent reproduces $q_{T}$-exponent.
$\Rightarrow$ some differences with threshold-resummation (however under control).


## Threshold-resummed total cross sections at the Tevatron


[Bozzi, BF, Klasen (2007)]

* SPS7 slope.
* NLO and threshold-resummation effects important.
* Resummation more important for heavier sleptons.
* Shift in $m_{\tilde{e}_{L}}$ if deduced from total $\sigma$ measurement.


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## Conclusion and outlook

- Full NLO SUSY-QCD calculations, including squark mixing.
- Threshold, $q_{T}$ and joint resummations.
- Comparison with the Monte Carlo approach.
- Study of other SUSY particle production processes.

