Introduction

## Transverse-momentum, threshold and joint resummation for slepton-pair production at hadron colliders

Benjamin Fuks (LPSC Grenoble)

in collaboration with Giuseppe Bozzi and Michael Klasen

SUSY 07 Karlsruhe (Germany), July 27, 2007

- Introduction and motivations
- Resummation formalisms
  - Main features of the resummation
  - The resummed component
  - Matching procedure
- **Applications** 
  - q<sub>T</sub>-distribution
  - Invariant-mass distribution
  - Total cross sections
- Summary and outlook

Introduction

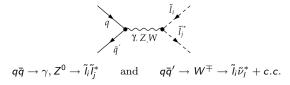
- Introduction and motivations
- - Main features of the resummation
  - The resummed component
  - Matching procedure
- - Invariant-mass distribution
  - Total cross sections

# Slepton-pair production at hadron colliders

Drell-Yan like process

Introduction

•00



- Sleptons are often light  $\Rightarrow$  decays into LSP + SM lepton  $\Rightarrow$  clean signal.
- Cross sections given by

$$\sigma = \sum_{a,b} \int_{\tau}^{1} \mathrm{d}x_{a} \int_{\tau/x_{a}}^{1} \mathrm{d}x_{b} f_{a/h_{1}}(x_{a},\mu_{F}) f_{b/h_{2}}(x_{b},\mu_{F}) \hat{\sigma}_{ab}(\mathbf{z},\mathbf{M};\alpha_{s}(\mu_{R}),\frac{\mathbf{M}}{\mu_{F}},\frac{\mathbf{M}}{\mu_{R}})$$

where  $\hat{\sigma}_{ab}$  is computed perturbatively

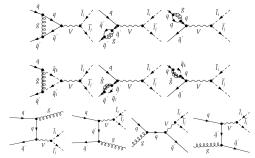
$$\hat{\sigma}_{ab}(z,M;\alpha_s(\mu_R),\tfrac{M}{\mu_F},\tfrac{M}{\mu_R}) = \sum_{n=0}^{\infty} \left(\tfrac{\alpha_s(\mu_R)}{\pi}\right)^n \hat{\sigma}_{ab}^{(n)}(z,M;\tfrac{M}{\mu_F},\tfrac{M}{\mu_R}) \ .$$



# Next-to-leading order calculations

Feynman diagrams:

Introduction



- Squark mixing included in the SUSY-loops.
- Partonic invariant-mass and transverse-momentum distributions at  $\mathcal{O}(\alpha_s)$ :

$$\begin{array}{rcl} \frac{\mathrm{d} \hat{\sigma}_{ab}}{\mathrm{d} M^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \, \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M,z,q_T) + \mathcal{O}(\alpha_s^2), \\ \frac{\mathrm{d}^2 \hat{\sigma}_{ab}}{\mathrm{d} M^2 \, \mathrm{d} q_T^2} & = & \hat{\sigma}_{ab}^{(0)}(M) \, \delta(q_T^2) \, \delta(1-z) + \frac{\alpha_s}{\pi} \, \hat{\sigma}_{ab}^{(1)}(M) \, \delta(q_T^2) + \frac{\alpha_s$$

where  $z = M^2/s$ .

#### Soft and collinear radiations:

Introduction

- \*  $\frac{\alpha_s^n}{a_-^2} \ln^m \frac{M^2}{a_-^2}$  or  $\alpha_s^n \left( \frac{\ln^m (1-z)}{1-z} \right)_+$  terms in the distributions  $(m \le 2 n 1)$ .
- \* Large at small  $q_T$  or  $z \leq 1$ .
- \* Fixed-order theory unreliable in these kinematical regions.
- \* Resummation to all orders needed
  - $\Rightarrow q_T$ -resummation.
  - ⇒ Threshold resummation.
  - ⇒ loint resummation

#### Advantages of resummation:

- Reliable perturbative results.
- \* Correct quantification of these radiations (even far from critical regions).
- \* Accurate invariant-mass and  $q_T$  spectra.

 $q_T$ -distribution  $\Rightarrow$  stransverse mass  $\Rightarrow$  spin and mass determination.

[Lester, Summers (1999); Barr (2006)]

M-distribution and total cross section  $\Rightarrow$  accurate mass determination.

[Bozzi, BF, Klasen (2007)]



- Resummation formalisms
  - Main features of the resummation
  - The resummed component
  - Matching procedure
- - Invariant-mass distribution
  - Total cross sections

#### iain reacures of the resummation

Reorganization of the cross section

$$d\sigma = d\sigma^{(res)} + d\sigma^{(fin)}$$
.

- $d\sigma^{(res)}$ 
  - \* Contains all the logarithmic terms.
  - \* Resummed to all orders in  $\alpha_s$ .
  - \* Exponentiation (Sudakov form factor).
- $d\sigma^{(fin)}$ 
  - \* Remaining contributions.

# The resummed component: conjugate spaces

- Conjugate spaces: Mellin, impact-parameter  $\Rightarrow$  kinematics naturally factorizes.
- Factorization of the hadronic cross sections:

$$\frac{\mathrm{d}\sigma^{(\mathrm{res})}}{\mathrm{d}M^{2}}(\tau, M) = \sum_{a,b} \int_{\tau}^{1} \mathrm{d}x_{a} \int_{\tau/\chi_{a}}^{1} \mathrm{d}x_{b} f_{a/h_{1}}(x_{a}, \mu_{F}) f_{b/h_{2}}(x_{b}, \mu_{F}) \hat{\sigma}_{ab}^{(\mathrm{res})}(z; \alpha_{s}(\mu_{R}), \frac{M}{\mu_{F}}, \frac{M}{\mu_{R}}) 
\downarrow 
\frac{\mathrm{d}\sigma^{(\mathrm{res})}}{\mathrm{d}M^{2}}(N, M) = \sum_{a,b} f_{a/h_{1}}(N+1, \mu_{F}) f_{b/h_{2}}(N+1, \mu_{F}) \hat{\sigma}_{ab}^{(\mathrm{res})}(N; \alpha_{s}, \frac{M}{\mu_{R}}, \frac{M}{\mu_{F}}),$$

and

Introduction

$$\frac{\mathrm{d}^{2}\sigma^{(\mathrm{res})}}{\mathrm{d}M^{2}\mathrm{d}q_{T}^{2}}(\tau,M,q_{T}) = \sum_{a,b} \int_{\tau}^{1} \mathrm{d}x_{a} \int_{\tau/X_{a}}^{1} \mathrm{d}x_{b} f_{a/h_{1}}(x_{a},\mu_{F}) f_{b/h_{2}}(x_{b},\mu_{F}) \hat{\sigma}_{ab}^{(\mathrm{res})}(z,q_{T};\alpha_{s}(\mu_{R}),\frac{M}{\mu_{F}},\frac{M}{\mu_{R}})$$

$$\frac{\mathrm{d}^{2}\sigma^{(\mathrm{res})}}{\mathrm{d}M^{2}\mathrm{d}q_{T}^{2}}(N,M,q_{T}) = \sum_{a,b} f_{a/h_{1}}(N+1,\mu_{F}) f_{b/h_{2}}(N+1,\mu_{F}) \int_{z}^{b} \mathrm{d}b J_{0}(b\,q_{T}) \mathcal{W}_{ab}^{F}(N,b;\alpha_{s},\frac{M}{\mu_{R}},\frac{M}{\mu_{F}}).$$

ullet The logarithms are included in the functions  $\hat{\sigma}^{(\mathrm{res})}$  and  $\mathcal{W}^{\mathit{F}}$ :

$$\left(\frac{\ln(1-z)}{1-z}\right)_{+} \to \ln^{2} \overline{N} \quad \text{with} \quad \overline{N} = N \, \exp[\gamma_{E}] \qquad \qquad \frac{1}{q_{T}^{2}} \, \ln \frac{M^{2}}{q_{T}^{2}} \to \ln \overline{b}^{2} \quad \text{with} \quad \overline{b} = \frac{b \, M}{2} \, \exp[\gamma_{E}]$$

• The process-dependence is factorized outside the exponent:

$$\mathcal{W}_{ab}^{F}(N,b) = \mathcal{H}_{ab}^{F}(N) \exp \left\{ \mathcal{G}(N,b) \right\},$$

$$\hat{\sigma}_{ab}^{(\text{res})}(N) = \sigma^{(LO)} \tilde{C}_{ab}(N;\alpha_s) \exp \left\{ \mathcal{G}(N,L) \right\}.$$

- $\mathcal{H}^F$  and  $\tilde{C}$ -functions:
  - \* Can be computed perturbatively and are process-dependent.
  - \* Contain real and virtual collinear radiation, and hard contributions.
- The Sudakov form factor contains the soft-collinear radiation:
  - Can be computed perturbatively and is process-independent.
- Used formalisms:
  - \* Universal q<sub>T</sub>-resummation. [Catani, de Florian, Grazzini (2001); Bozzi, Catani, de Florian, Grazzini (2006)]
  - \* Threshold resummation including collinear radiation. [Sterman (1987); Catani, Trentadue (1989, 1991); Krämer, Laenen, Spira (1998); Catani, de Florian, Grazzini (2001)]
  - \* Universal joint resummation. [Laenen, Sterman, Vogelsang (2001); Kulesza, Sterman,
     Vogelsang (2002, 2004); Bozzi, BF, Klasen (in prep.)]



# The finite component: matching procedure

- Fixed-order theory
  - \* Reliable far from the critical kinematical regions ( $z \ll 1$ ,  $q_T \gg 0$ ).
  - \* Spoiled in the critical regions ( $z \sim 1$ ,  $q_T \sim 0$ ).
- Resummation

Introduction

- \* Needed in the critical regions.
- \* Not justified far from the critical regions.
- Both contributions important in the intermediate kinematical regions.
- Information from both fixed-order and resummation needed.
- Need to avoid double-counting.
- Consistent matching procedure required:

$$d\sigma^{(fin)} = d\sigma^{(f.o.)} - d\sigma^{(exp)}$$



Invariant-mass spectrum

$$\begin{array}{lll} \frac{\mathrm{d}\sigma}{\mathrm{d}M^{2}}(\tau,M) & = & \frac{\mathrm{d}\sigma^{(\mathrm{F.O.})}}{\mathrm{d}M^{2}}(\tau,M) \\ & + & \oint_{C_{N}} \frac{\mathrm{d}N}{2\pi i} \, \tau^{-N} \Big[ \frac{\mathrm{d}\sigma^{(\mathrm{res})}}{\mathrm{d}M^{2}}(N,M) - \frac{\mathrm{d}\sigma^{(\mathrm{exp})}}{\mathrm{d}M^{2}}(N,M) \Big]. \end{array}$$

Transverse-momentum spectrum

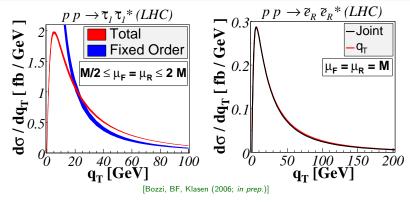
$$\begin{array}{lcl} \frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(\tau,M,q_T) & = & \frac{\mathrm{d}^2\sigma^{(\mathrm{F.O.})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(\tau,M,q_T) \\ & + & \oint_{\mathcal{C}_N} \frac{\mathrm{d}N}{2\pi i}\,\tau^{-N}\int \frac{b\mathrm{d}b}{2}J_0(q_T\,b) \left[ \frac{\mathrm{d}^2\sigma^{(\mathrm{res})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b) - \frac{\mathrm{d}^2\sigma^{(\mathrm{exp})}}{\mathrm{d}M^2\,\mathrm{d}q_T^2}(N,b) \right]. \end{array}$$

- \* Far from the critical regions,  $d\sigma^{(res)} \approx d\sigma^{(exp)} \Rightarrow \text{Perturbative theory.}$
- \* In the critical regions,  $d\sigma^{(F.O.)} \approx d\sigma^{(\exp)} \Rightarrow \text{Pure resummation}$ .
- \* In the intermediate regions ⇒ Consistent matching.



- - Main features of the resummation.
  - The resummed component
  - Matching procedure
- **Applications** 
  - q<sub>T</sub>-distribution
  - Invariant-mass distribution
  - Total cross sections

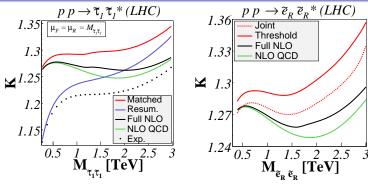
# $q_T$ -distribution at the LHC



- \* SPS1a and BFHK-B SUSY scenarios (slepton masses  $\approx$  100-200 GeV).
- \* Finite results at small  $q_T$ ; enhancement at intermediate  $q_T$ ; finite total  $\sigma$ .
- \* Improvement of scale dependences: (NLL+F.O. ≤ 5%; F.O. 10%).
- \* Effects of the threshold-enhanced contributions in the intermediate- $q_T$  region.



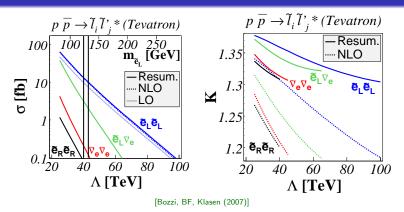
#### Invariant-mass distribution at the LHC



[Bozzi, BF, Klasen (2007; in prep.)]

- \* SPS1a and BFHK-B SUSY scenarios (slepton masses  $\approx$  100-200 GeV).
- \* Normalization to LO cross section.
- \* Small  $M: d\sigma^{(res)} \approx d\sigma^{(exp)}$ ; Large  $M: d\sigma^{(F.O.)} \approx d\sigma^{(exp)}$ .
- Reduced SUSY-loop effects.
- Joint-exponent reproduces q<sub>T</sub>-exponent.
   ⇒ some differences with threshold-resummation (however under control).





- \* SPS7 slope.
- \* NLO and threshold-resummation effects important.
- \* Resummation more important for heavier sleptons.
- \* Shift in  $m_{\tilde{e}_t}$  if deduced from total  $\sigma$  measurement.



Outlook

- - Main features of the resummation
  - The resummed component
  - Matching procedure
- - Invariant-mass distribution
  - Total cross sections
- Summary and outlook

#### Conclusion and outlook

- Full NLO SUSY-QCD calculations, including squark mixing.
- Threshold, q<sub>T</sub> and joint resummations.
- Comparison with the Monte Carlo approach.
- Study of other SUSY particle production processes.