

Brane Phrenology:

Dark matter and collider constraints on brane oscillations

Tonnis ter Veldhuis
(Macalester College)

In collaboration with:

Thomas Clark, Sherwin Love, Chi Xiong (Purdue University)

Muneto Nitta (Keio University)

Susy'07, Karlsruhe, Germany, July 28.

Introduction

- We consider the effective theory that describes the Standard Model if it is localized on a flexible brane which is embedded in a higher dimensional space.
- This effective theory generically contains additional scalar fields (global case) or massive vector fields (local case) associated with the spontaneous breaking of translational symmetries.
- These additional degrees of freedom can be stable and they can have weak scale interactions and masses. Hence they are cold dark matter candidates.
- We consider the bounds on the parameter space of the effective theory based on collider data (LEP), the observed dark matter relic abundance (WMAP), and results of direct dark matter detection experiments (CDMSII and Xenon-10).

Brane dynamics

The action describing the motion of a thin brane embedded in a higher dimensional space can be constructed in terms of the induced brane metric. For simplicity, we consider the case with co-dimension equal to one.

The ISO(4,1) invariant interval is

$$ds^2 = dx^M \eta_{MN} dx^N \quad \text{with } M, N = 0..4$$

Consider a p=3 brane oriented perpendicular to the x^4 direction. The induced metric on the brane is

$$\begin{aligned} & \underline{x^4 = \varphi(x^\mu)} \\ & \quad \downarrow \\ ds^2 &= dx^\mu \eta_{\mu\nu} dx^\nu - dx^\mu \partial_\mu \varphi \partial_\nu \varphi dx^\nu = dx^\mu g_{\mu\nu} dx^\nu \quad \text{with } \mu, \nu = 0..3 \\ g_{\mu\nu} &= \eta_{\mu\nu} - \partial_\mu \varphi \partial_\nu \varphi \end{aligned}$$

The ISO(4,1) invariant action describing the motion of the brane into the extra dimension is

$$S = - \underset{\substack{\uparrow \\ \text{brane tension}}}{\sigma} \int dx^4 \sqrt{-\det(g)} = - \sigma \int dx^4 \sqrt{1 - \partial^\mu \varphi \partial_\mu \varphi}$$

Nambu Goto action in static gauge

Coupling to Matter

(Standard Model on a brane)

The vierbein and the connection follow from the induced metric g . The ISO(4,1) invariant extension of the Standard Model is obtained as

$$S = -\sigma \int dx^A \sqrt{-g} + \int dx^A \sqrt{-g} L_{SM}(\partial_\mu \rightarrow D_\mu)$$

Expanding up to quadratic power in terms of the scalar field yields the Lagrangian density

$$L = -\sigma + \frac{1}{2} \sigma \partial^\mu \varphi \partial_\mu \varphi + L_{SM} + \frac{1}{2} \partial^\mu \varphi \partial^\nu \varphi T_{\mu\nu}^{SM}$$

Rescaling the scalar field so that it obtains its usual engineering dimension and canonical kinetic term gives

$$L = -\sigma + \frac{1}{2} \partial^\mu S \partial_\mu S + L_{SM} + \frac{1}{2\sigma} \partial^\mu S \partial^\nu S T_{\mu\nu}^{SM} \quad S = \frac{\varphi}{\sqrt{\sigma}}$$

Massless scalar field

Energy momentum tensor

The massless scalar field is the Goldstone boson associated with the spontaneous breaking of the translational symmetry.

When the extrinsic curvature is considered, additional ISO(4,1) invariant terms linear in the Goldstone field can also be constructed. Such terms break the discrete symmetry that is manifest here.!

Gauging translations

If all translational symmetries are gauged, then regular gravity is also included in the model. Here we focus on gauging the translational symmetry perpendicular to the brane. The locally invariant interval is obtained by the introduction of a vector gauge field and a covariant derivative.

$$ds^2 = dx^\mu \eta_{\mu\nu} dx^\nu - dx^\mu D_\mu \varphi D_\nu \varphi dx^\nu$$

with

$$D_\mu \varphi = \partial_\mu \varphi + g X_\mu$$

and we define the gauge coupling as $g = \frac{M_X}{\sqrt{\sigma}}$

In unitary gauge, $\partial_\mu \varphi = 0$ and therefore $D_\mu \varphi = \frac{M_X}{\sqrt{\sigma}} X_\mu$.

To obtain the locally invariant action, replace $\partial_\mu \varphi \longrightarrow \frac{M_X}{\sqrt{\sigma}} X_\mu$ in the globally invariant action and add a kinetic term for the vector field. The leading part of the invariant action thus obtained is

$$L = -\sigma + \frac{1}{2} M_X^2 X^\mu X_\mu + L_{SM} + \frac{M_X^2}{2\sigma} X^\mu X^\nu T_{\mu\nu}^{SM} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

↑ Mass through Higgs mechanism
 ↑ $\frac{M_X^2}{2F_X^4}$

The model

$$L_{\text{SM}X} = \frac{1}{2} \frac{M_X^2}{F_X^4} X_i^{1\ 0} X_i^{0\ 1} T_{1\ 0}^{\text{SM}} + \frac{1}{2} M_X^2 X_i^{1\ 0} X_{i+1}^{0\ 1} - \frac{1}{4} F_i^{1\ 0} F_{i+1}^{0\ 1}$$

Parameters:

Mass of the vectors M_X

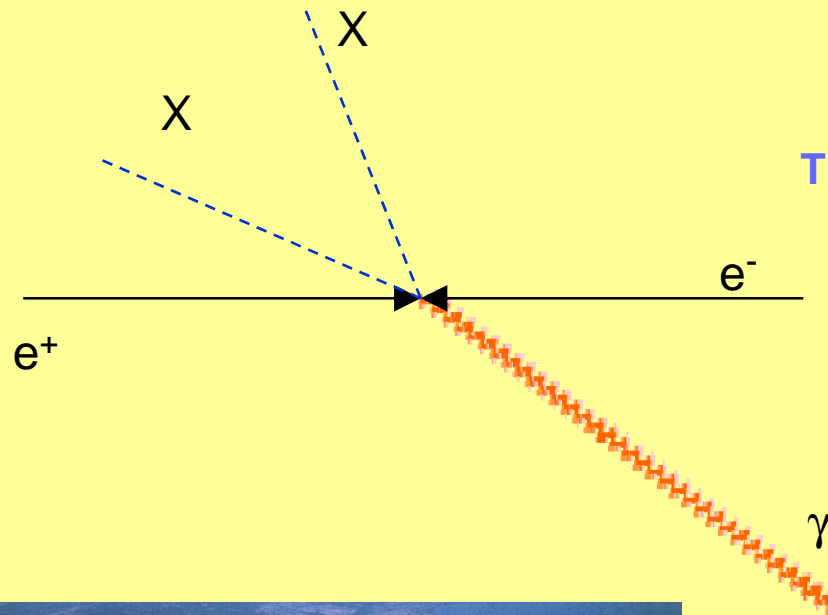
Interaction scale F_X

Number of flavors $i = 1 \dots N$

The X vector is stable due to an SO(N) (or, discrete, for N=1) symmetry. It is therefore a dark matter candidate.

Objective: study how the model parameters are constrained by the LEP data, the WMAP dark matter relic abundance result, and the CDMSII and Xenon-10 direct dark matter detection experiments.

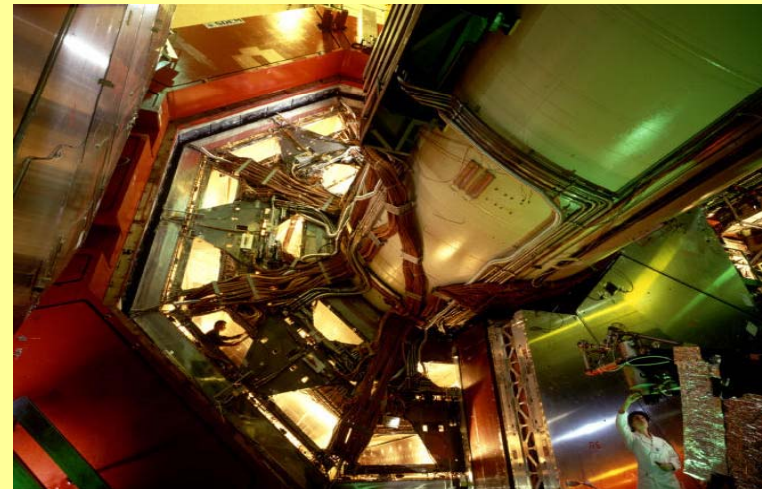
Lep II constraint: single photon signal



The X vectors escape the detector un-noticed.



The Cern accelerator complex. (CERN)

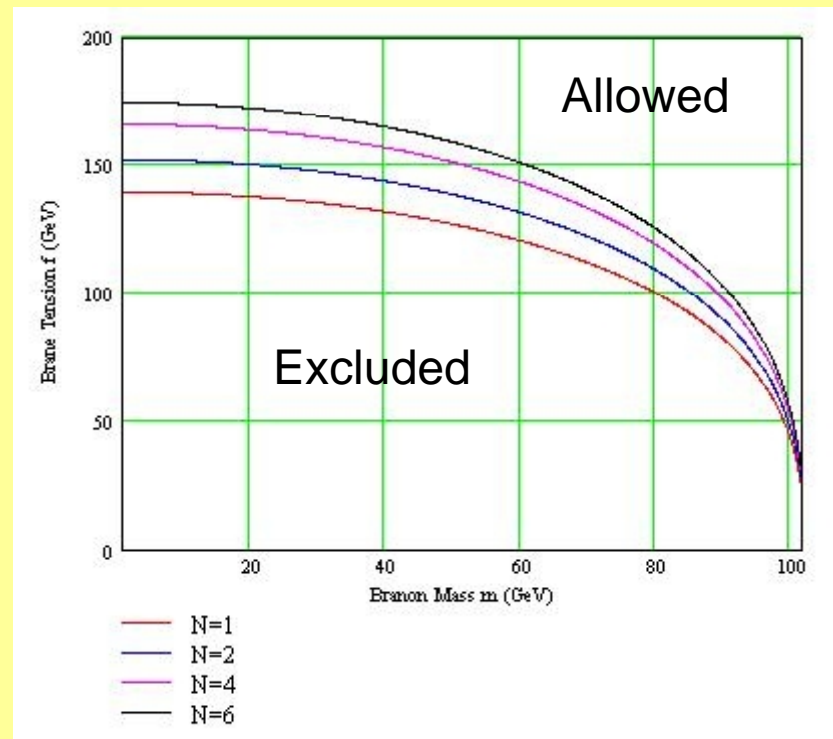


The L3 detector at LEP. (Peter Ginter, CERN)

Branon Scattering Summary: LEP II average center of mass energy = 206 GeV

Massive Scalar Branon

LEP II Excluded/Allowed Parameter Regions



Creminelli and Strumia: Nucl. Phys. **B596**, 125 (2001);

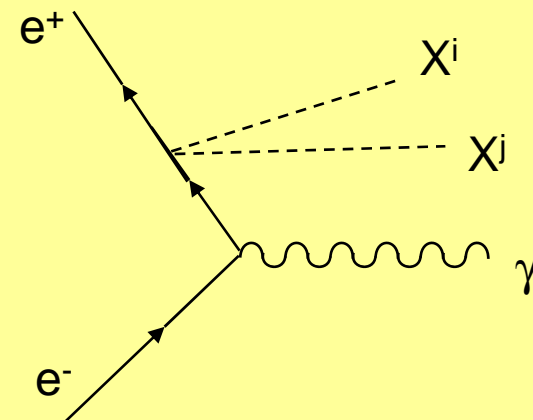
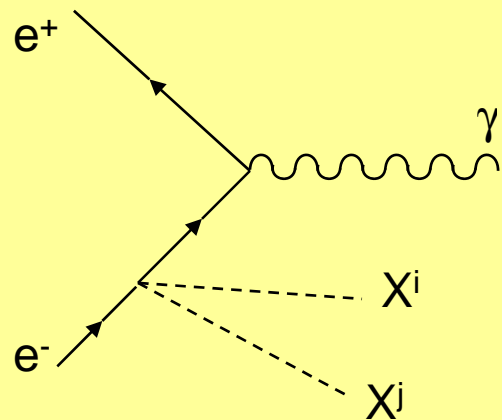
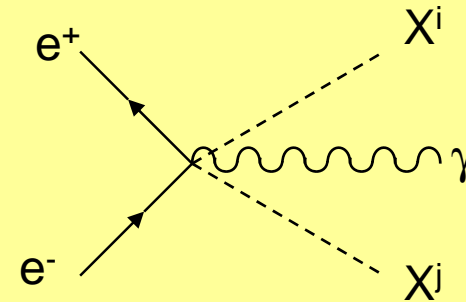
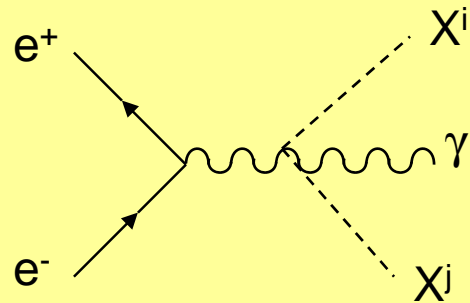
Alvarez, Cembranos, Dobado and Maroto: Phys. Rev. D **67**, 075010 (2003);

L3 Collaboration, P. Achard et al.: Phys. Lett. B **597** (2004) 145;

S. Mele, Search for Branons at LEP, Int. Europhys. Conf. on High Energy Phys., PoS(HEP2005)153.

Re-visit missing energy for Massive Vector Brane Oscillation Fields

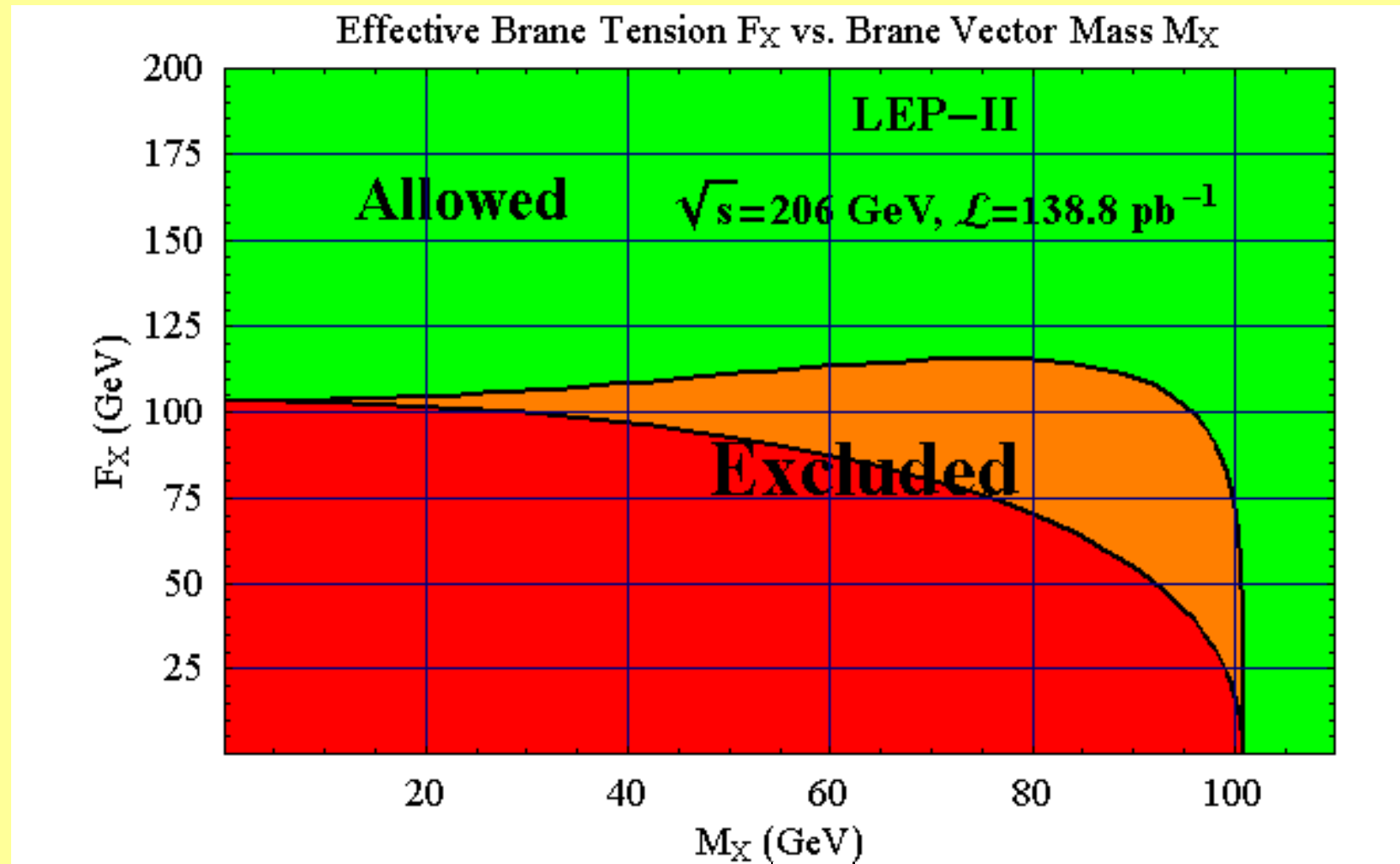
The Feynman Diagrams for Brane Particle Production:



The doubly differential cross-section for spin averaged e^+ and e^- collisions producing a photon and 2 X particles with summed over polarizations and the X species, $i=1,2,\dots,N = \#$ of extra dimensions

$$\begin{aligned} \frac{d^2\sigma_\gamma}{dx d(\cos\theta)} &= \frac{\alpha}{4\pi} \frac{1}{15,360\pi} \left[\frac{N}{F_X^8} \right] \frac{\sqrt{s} \sqrt{s(1-x) - 4M_X^2}}{\sqrt{(1-x)}} \times \\ &\times \left[\left[s(1-x) - 4M_X^2 \right]^2 + 20M_X^2 \left(s(1-x) + 2M_X^2 \right) \right] \times \\ &\times \left[x(3-3x+2x^2) - x^3 \sin^2\theta + \frac{2(1-x) \left[1 + (1-x^2) \right]}{x \sin^2\theta} \right] \end{aligned}$$

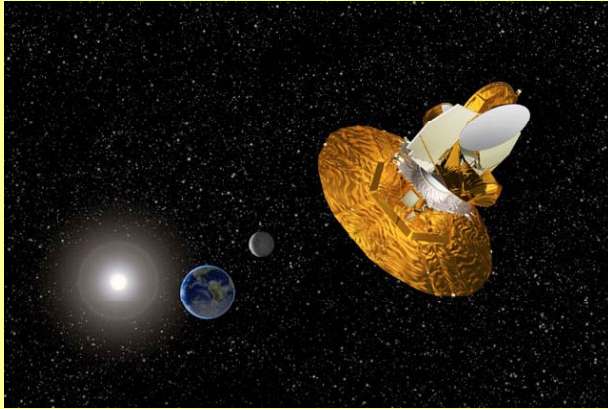
$E_{CM} = \sqrt{s}$ and the outgoing γ carries the fraction x of the beam energy $E_\gamma = x\sqrt{s}/2$ and θ is the photon's polar angle from the beam axis. α is the electromagnetic fine structure constant $\alpha = 1/128$.



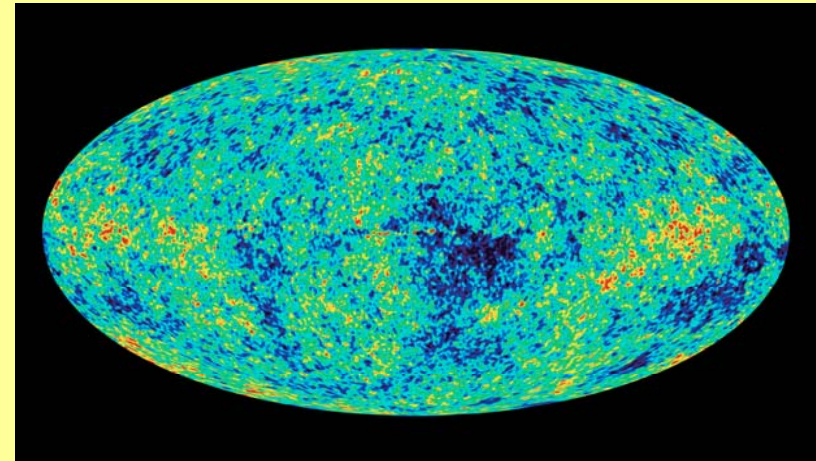
Excluded red shaded region from longitudinal component of vector (branon); Orange shaded region from transverse components of vector.

Experimental limit: $\sigma(e+e^- \rightarrow \gamma/E) < .45 \text{ pb}$

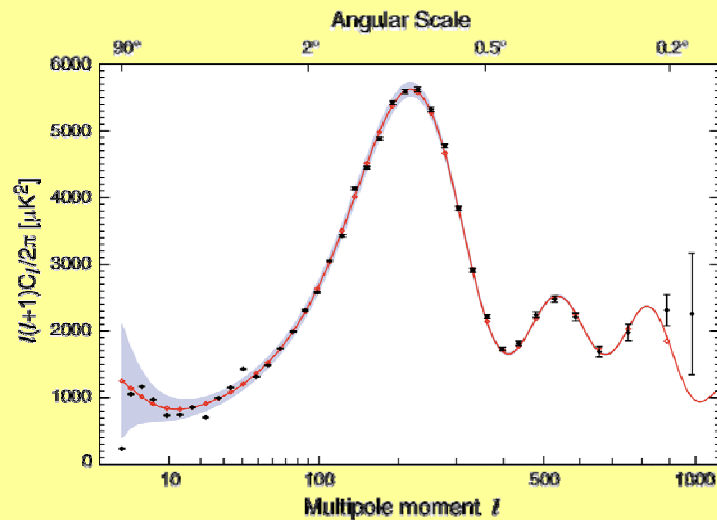
Relic dark matter abundance constraints



Artists impression of the WMAP satellite at the L2 point. (NASA)



WMAP (Wilkinson Microwave Anisotropy Probe) image of the CMB (Cosmic microwave background radiation) anisotropy.



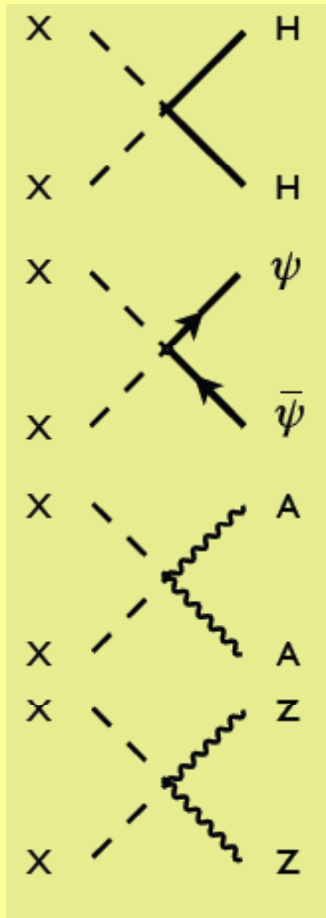
(WMAP science team.)

Relic abundance calculation

- It is assumed that at some point during the evolution of the Universe the population of X-vector particles is in thermal equilibrium with the populations of Standard Model particles.
- The density of the X-vector particles as a function of time/temperature follows from simple thermodynamics while the X-vector is in thermal equilibrium.
- However, at some point in time, the expansion rate of the Universe starts to exceed the annihilation rate of the X-vector particles. At that time, the X-vector falls out of thermal equilibrium, and from then on the X-vector density only changes because of the expansion of the Universe, and annihilation effectively ceases.
- Typically, freeze out occurs at temperatures that are about $1/20$ times the mass of the X-vector. The X-vectors are therefore non-relativistic at freeze out, and consequently they form cold dark matter.
- In the numerical calculation, the freeze-out temperature is determined from the non-relativistic annihilation cross-sections of a pair of X-vector particles into all relevant Standard Model particles, and the relic X-vector abundance is obtained.

Relic abundance

Annihilation cross-sections: (non-relativistic limit shown)

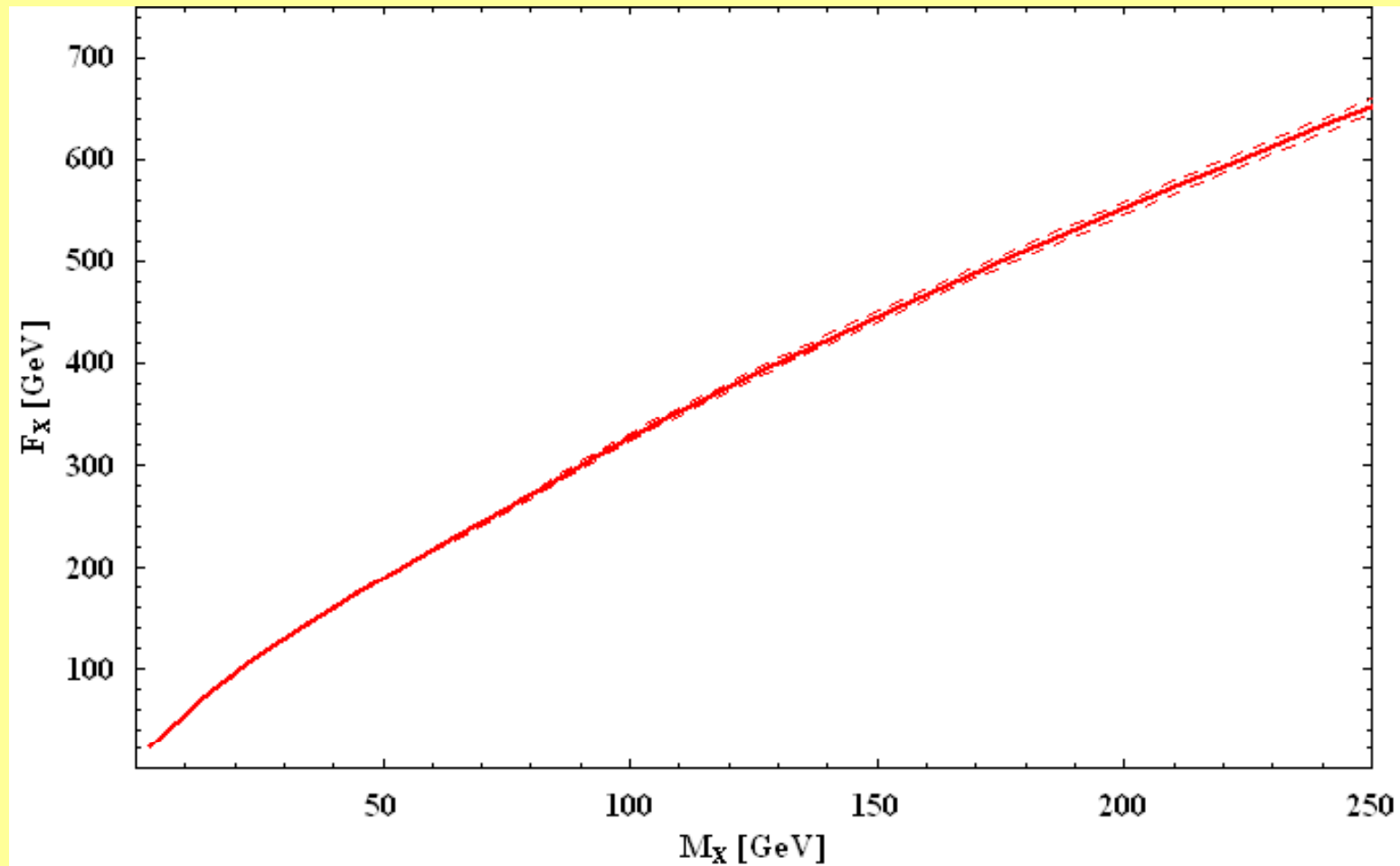


$$\sigma_{HH}^{3/4} = \frac{1}{144^{1/4}} \frac{M_X^2}{F_X^8} (2M_X^4 + M_H^4) \rho \frac{\overline{M_X^2} \cdot \overline{M_H^2}}{s \cdot 4M_X^2}$$

$$\sigma_{A\tilde{A}}^{3/4} = \frac{1}{72^{1/4}} \frac{M_X^2}{F_X^8} (M_X^4 - M_{\tilde{A}}^4) \rho \frac{\overline{M_X^2} \cdot \overline{M_H^2}}{s \cdot 4M_X^2}$$

$$\sigma_{AA}^{3/4} = \frac{1}{18^{1/4}} \frac{M_X^6}{F_X^8} \rho \frac{M_X}{s \cdot 4M_X^2}$$

$$\sigma_{ZZ}^{3/4} = \frac{1}{144^{1/4}} \frac{M_X^2}{F_X^8} (10M_X^4 + 8M_X^2 M_Z^2 + 3M_Z^4) \rho \frac{\overline{M_X^2} \cdot \overline{M_Z^2}}{s \cdot 4M_X^2}$$



Red line: $\Omega_c h^2 = 0.105^{+0.009}_{-0.009}$ WMAP Collaboration, astro-ph/0603449

Below the red line the X vector only gives a partial contribution to the observed dark matter density.

Direct dark matter detection



The Soudan underground laboratory is located 780 meters below the surface in a former iron mine.
(University of Minnesota)

**CRESST, DAMA, EDELWEISS,
GENIUS, HDMS, PICASSO, WARP,
XENON...**



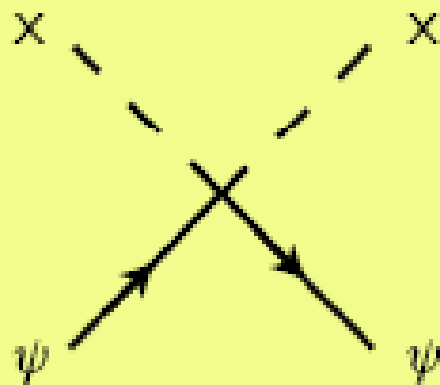
CDMS silicon dark matter detector.
(CDMS Collaboration)



CDMS cryostat. (CDMS collaboration)

Direct detection

Elastic X vector - target cross-section:



Proportional to square of target mass

$$\sigma = \frac{2}{\pi} \frac{M_X^2 M_\psi^2}{(M_X + M_\psi)^2} \frac{M_X^2 M_A^2}{F_X^8}$$

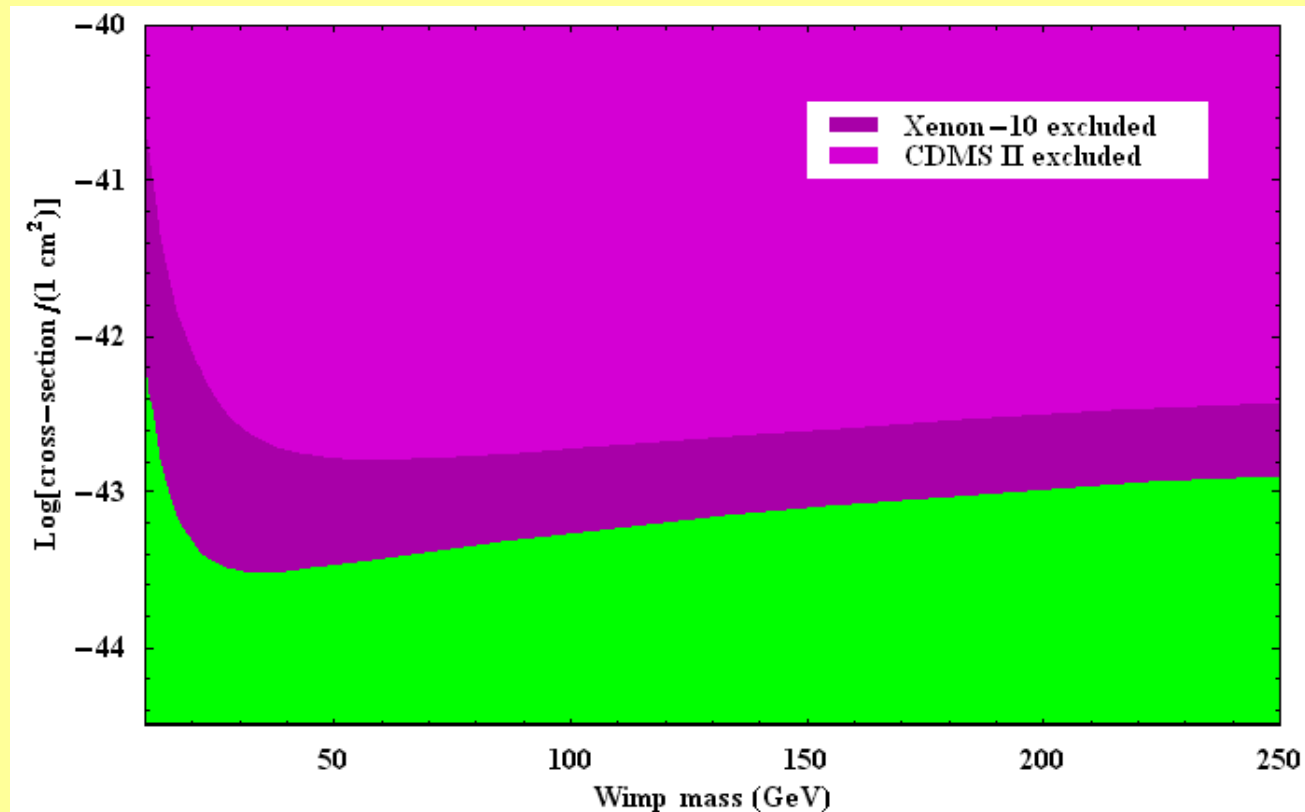
square of reduced mass

Relation between scattering cross-sections X - nucleus
and X - nucleon:

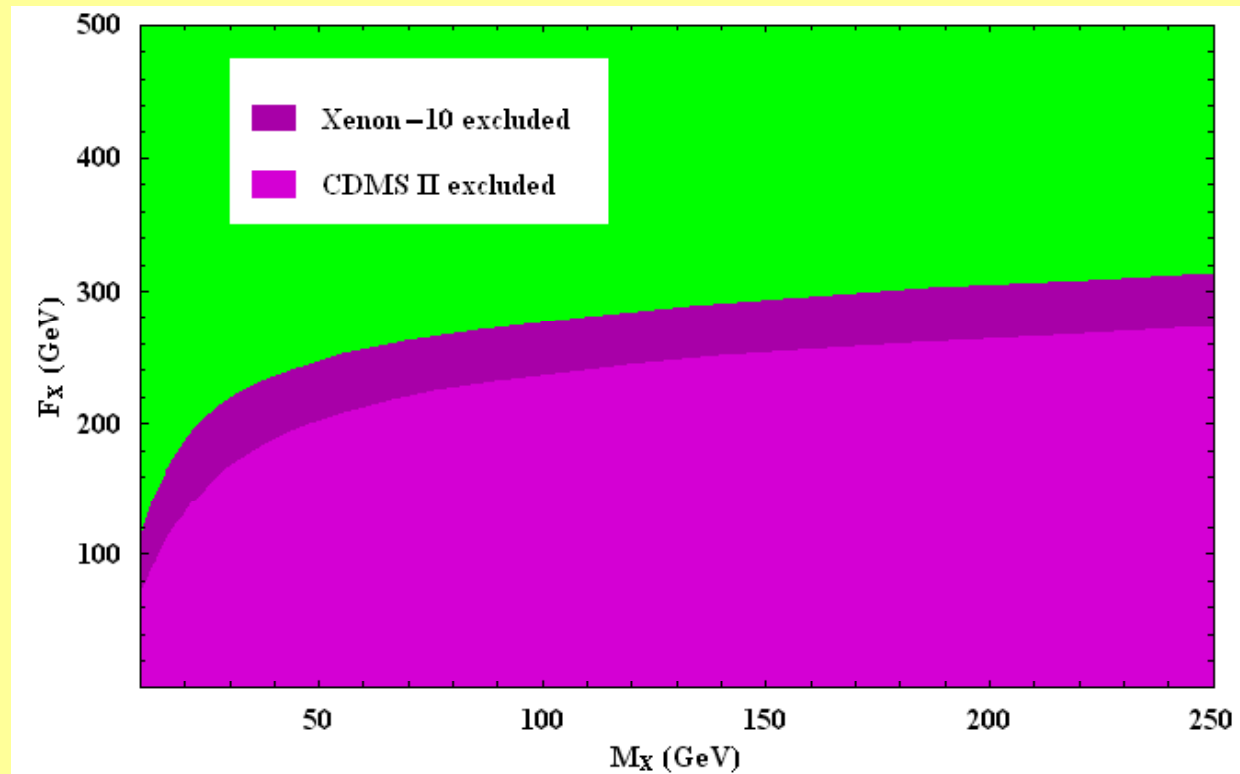
$$\sigma_{Nucleus} = N_n^2 \frac{\mu(M_X, M_{Nucleus})^2}{\mu(M_X, M_n)^2} \sigma_n$$

Nucleon mass M_n

Number of nucleons in Nucleus N_n

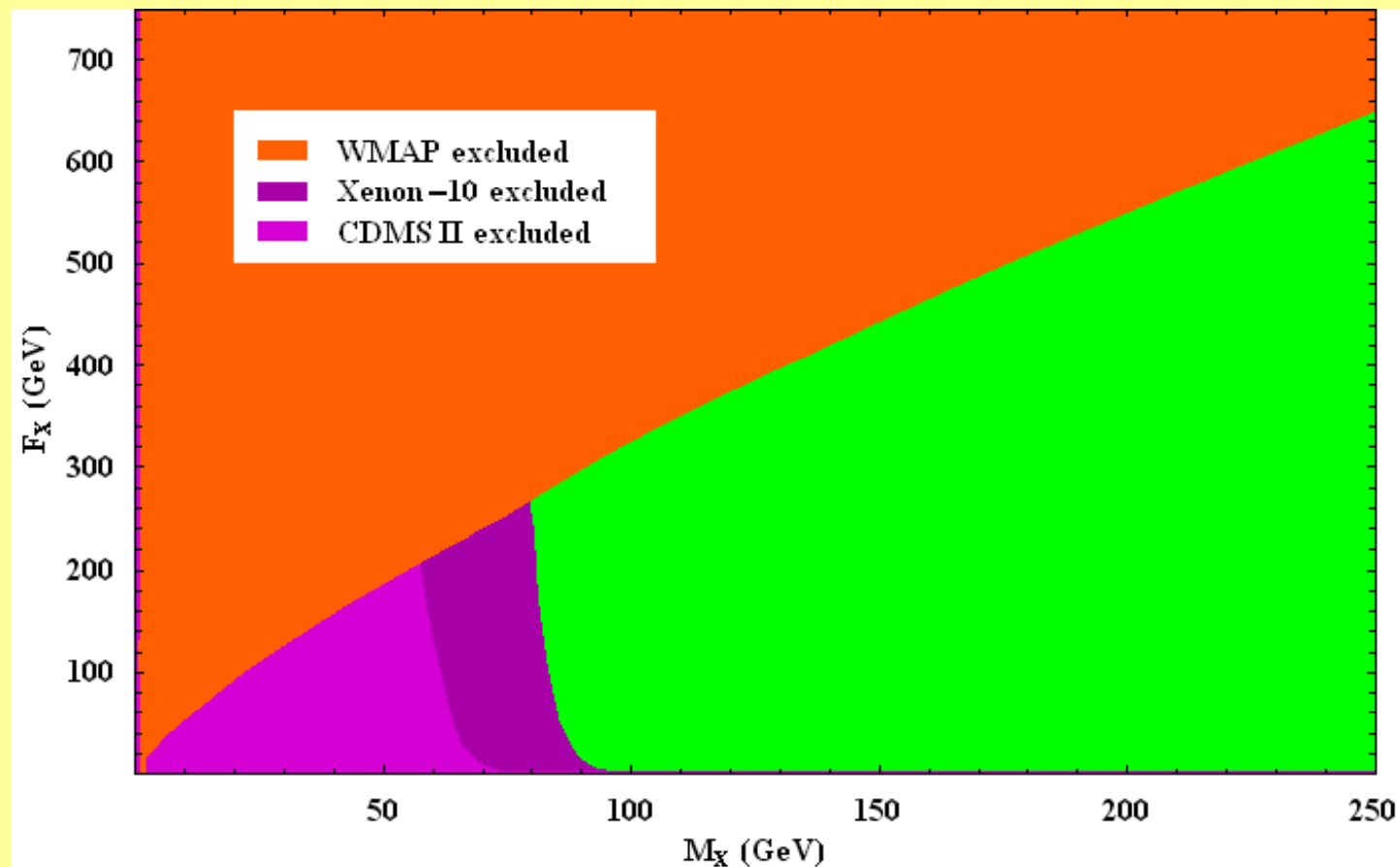


- The experimental results for experiments with different nuclei are always reduced by the experimental groups to a bound on the elastic (in this case spin independent) Wimp-nucleon cross-section as a function of the Wimp mass.
- The green area is still allowed, and the purple areas are excluded.
- Note that this graph assumes a specific model for the Milky Way dark matter halo, and that all the dark matter in the Halo is of the same kind.



- This graph is a direct translation of the experimental data to bounds on the parameter space of the model.
- Under the assumption that the X-vector provides all the dark matter in the Milky Way Halo, the purple areas in the parameter space are excluded, while the green area is still allowed.
- Note however that it follows from the X-vector relic abundance calculation that the X-vector only forms a fraction of the total dark matter density in a large area of the parameter space.
- Therefore, this graph can be improved by taking this fraction into account.

Combined relic abundance and direct detection plot



In principle, an additional unknown dark matter component can have a vanishing elastic scattering cross-section with the nucleon, which results in a more conservative bound on the parameter space of the model.

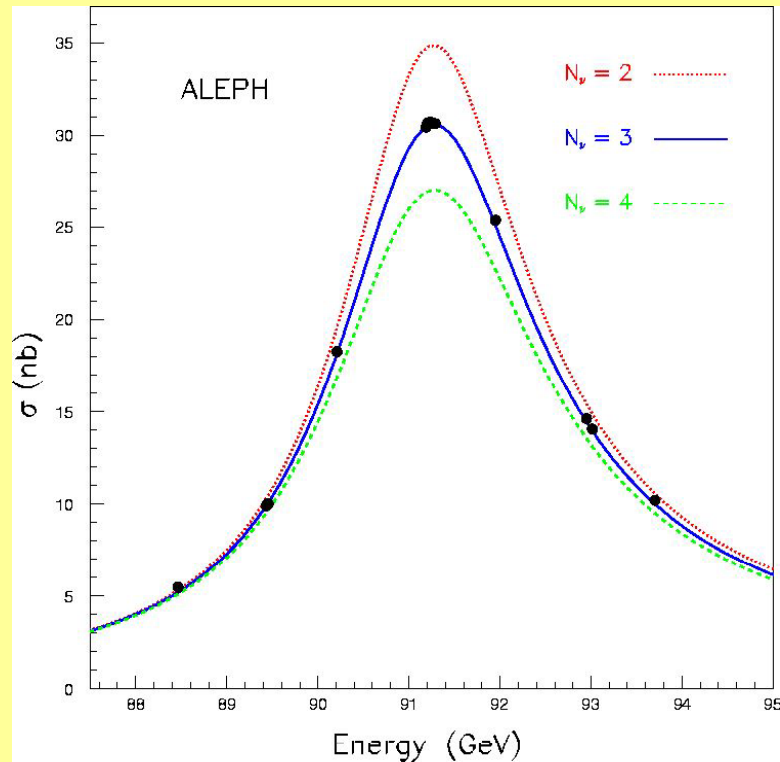
Coupling to Extrinsic Curvature terms

- The extrinsic curvature measures curvature of the embedded brane relative to the enveloping higher dimensional geometry.
- In unitary gauge: $K_i^{10} = i \frac{M_X}{F_X^2} @ X_i^0 + \dots$
- Invariant couplings to the Standard Model fields can be constructed by contracting the extrinsic curvature tensor to other tensors.
- One possibility is to couple to the Standard Model hypercharge field-strength:

$$L_{\text{extr}} = \frac{M_X^2}{F_X^4} (K_1 B_{10} + K_2 \tilde{B}_{10}) @ X_i^{1/2} @ X_i^0$$

- The coefficients K_1 and K_2 are additional dimensionless constants of the effective action.

Z-boson width constraint

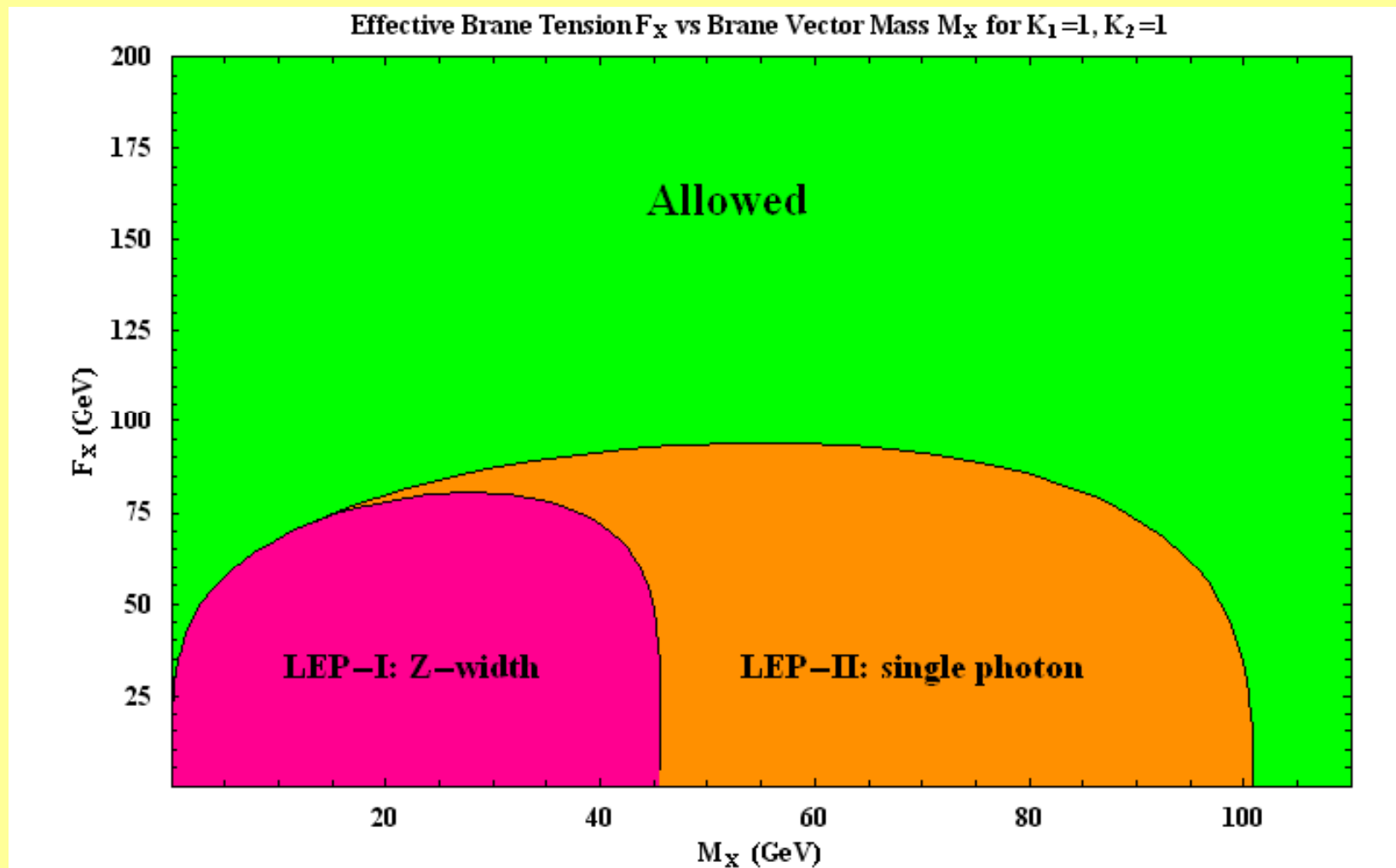


Aleph Z-boson lineshape measurement at LEP.

For sufficiently light X vectors, the Z boson can now (invisibly) decay into a pair of these particles. This leads to constraints on the coupling constants K_1 and K_2 from the LEP Z -boson line-shape measurement.

Experimental bound:

$$i Z \rightarrow X X < 2 \text{ MeV}$$



Extrinsic curvature coupling (to $B_{\mu\nu}$) only!

Transverse vector modes required. (These interactions vanish in the global limit.)

Conclusions

- We embedded 4-dimensional probe brane into D dimensional space-time which breaks extra dimensional translation invariance. Dynamics of associated Nambu-Goldstone mode describes oscillations of brane into extra dimension.
- Gauging the broken translations leads to massive Proca vector fields X_i which are Standard Model singlets.
- Coupled X_i to the Standard Model using both intrinsic and extrinsic curvatures.
- Isotropic codimensions: Massive vector is stable.
- Anisotropic codimensions: Massive vector is narrow resonance. (not discussed in this talk; very different phenomenology)
- Examined constraints on the brane tension and vector mass arising from electron-positron collider data (LEP I, II), and dark matter relic abundance (WMAP) and direct detection experiments (CDMS II, Xenon-10) .
- Our approach: investigate experimental and observational consequences of the existence of extra dimensions through universal features that are only related to symmetry breaking patterns and that therefore are not dependent on the details of short distance physics.
- These features may give hints about extra dimensions without the necessity to produce the heavy degrees of freedom that propagate into the bulk.