Very Light Cosmological Scalar Fields from a Tiny Cosmological Constant

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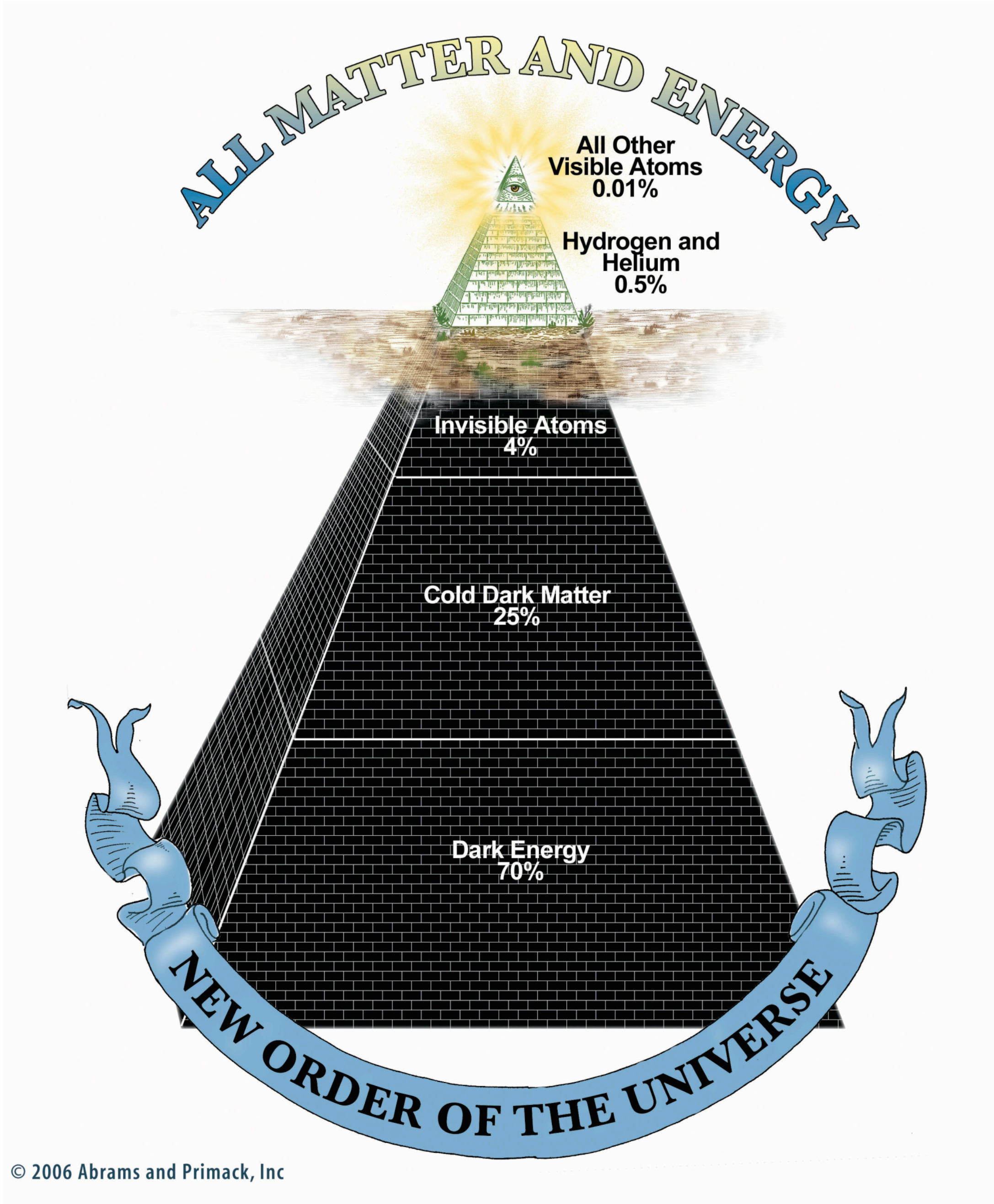
New picture of the Universe



From astro-ph/0609541 (J. R. Primack)

Scalar fields in Cosmology

- The phenomenology of scalar fields in the standard model can be rather exotic, especially the Higgs sector is a gateway to hidden sectors.
- What about cosmology?
- Scalar fields are used to explain different phenomena (dark energy, inflation…)
- Let me have a different perspective and raise the following question: given what we know of cosmology (ΛCDM), what does it take for a scalar field to fluctuate today and hence to impact our universe today?
- The expansion of a scalar field in a expanding universe is given by

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi + ... = 0$$

- Deriving this equation is trivial: assume Robertson-Walker metric and use Einstein's equations.
- Finding a solution is easy:

$$\phi(t) = \mathrm{Re}(c_1 \exp{(w_1 t)} + c_2 \exp{(w_2 t)})$$

with

$$w_{1/2} = -3/2H \pm \sqrt{9/4H^2 - m^2}$$

- Thus oscillations at time H are possible iff $m > 3/2H$.

- Note that if the mass is much bigger than H, the field has reached a minimum a long time ago and will not impact our present universe.

- However today $H \sim 10^{-33}\mathrm{eV}$

- How do we get such a small scalar mass?

- A regular mass term $m^2\phi\phi$ will not do the work!

- Let us study the operator

$$\alpha \int d^4x \sqrt{-g} R\phi\phi$$

- The action we are considering is given by:

$$\int d^4x\sqrt{-g}\left(-\frac{1}{16\pi G}(R-2\Lambda)+\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\alpha R\phi^2\right)$$

- The corresponding field equations are:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=-8\pi G S_{\mu\nu}$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi+\alpha R\phi=0$$

with

$$S_{\mu\nu}=\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi-\alpha\phi^2R_{\mu\nu}+\frac{1}{2}g_{\mu\nu}\alpha\phi^2R-\alpha(g_{\mu\nu}g^{\alpha\beta}\phi^2_{;\alpha\beta}-\phi^2_{;\mu\nu})\right)$$

where αR plays the role of a mass term.

- It is useful to rewrite the field equation as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{G_{eff}}{G}\Lambda g_{\mu\nu} =$$

$$-8\pi G_{eff}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi - \alpha(g_{\mu\nu}g^{\alpha\beta}\phi^2_{;\alpha\beta} - \phi^2_{;\mu\nu})\right)$$

- With

$$G_{eff} = \frac{G}{1 - 8\pi G\alpha\phi^2}$$

- Newton's constant is space-time dependent, this could easily lead to a time dependence of the couplings of the standard model.

- Using the contracted Einstein equation, we get

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 4\alpha\Lambda\phi - 8\pi G\alpha\phi(\partial_\mu\phi\partial^\mu\phi - \alpha\phi^2 R + 3\alpha\nabla_\mu\nabla^\mu\phi^2) = 0$$

- The scalar field is now massive!
- Using $\Lambda = 8\pi G\rho_{vac}$ and $\rho_{vac} \sim (2.4 \times 10^{-3}\ \mathrm{eV})^4$ we find:

$$m = 4.7 \times 10^{-33}\ \mathrm{eV}$$

- where we assumed $\alpha = 1$ we thus find $m \sim 3/2H$
- This scalar is thus relevant in today's universe!
- For the time change of the Newton constant we obtain:

$$\frac{G_{eff}(t_0) - G_{eff}(0)}{G_{eff}(t_0)} = -8\pi G\alpha\frac{\Delta\phi^2}{1 - 8\pi G\alpha\phi^2}$$

- Our action can be mapped to a Jordan-Brans-Dicke action:

$$\int d^4x\sqrt{-g}\frac{1}{16\pi}\left(\Phi R+\omega\frac{g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi}{\Phi}\right)$$

- with a space-time dependent parameter

$$\omega=(1-8\pi\phi^2G\alpha)/(32\pi\phi^2G\alpha^2)$$

- If we assume that the scalar field oscillates slowly we can use the bound on the parameter of the JBD-theory ($\omega > 500$) and obtain:

$$\phi/\Lambda_{Planck}<4\times10^{-3}$$

- and thus the time change since the Big Bang of the Newton constant is bounded

$$|\Delta G/G|<4\times10^{-4}$$

- A consequence can be a time variation of physical “constants”.
- If the controversial observation of Webb et al:

$$\Delta\alpha/\alpha \;=\; (-0.57 \pm 0.10) \times 10^{-5}$$

$$z \;\approx\; 0.5 \ldots 3.5$$

turned out to be correct, a natural way to describe it is a very light scalar field.

- We could then interpret the time variation as a renormalization effects (the details depend on the unification scheme).

$$\frac{1}{\alpha_i}\frac{\dot{\alpha}_i}{\alpha_i} = \left[\frac{1}{\alpha_u}\frac{\dot{\alpha}_u}{\alpha_u} - \frac{b_i}{2\pi}\frac{\dot{\Lambda}_G}{\Lambda_G}\right]$$

- This effect is expected in Kaluza-Klein models as shown by Marciano in 1984.

- Let us now look at the theory in the Einstein frame:

$$g_{\mu\nu} = \cosh^2\left(\frac{\hat{\phi}\sqrt{\alpha}}{M_r}\right)\hat{g}_{\mu\nu} \qquad \phi = \sqrt{\frac{1}{\alpha}}M_r \tanh\left(\frac{\hat{\phi}\sqrt{\alpha}}{M_r}\right)$$

where $M_r = \sqrt{1/(8\pi G)}$

- One gets:

$$\int d^4x\sqrt{-\hat{g}}\left(\frac{1}{16\pi G}\left(\hat{R} - 2\Lambda\cosh^4\left(\frac{\hat{\phi}\sqrt{\alpha}}{M_r}\right)\right) + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}\right)$$

- Note that physics is not identical in both frames.

- Let us now expand the cosh term:

$$\int d^4x\sqrt{-\hat{g}}\left(\frac{1}{16\pi G}\left(\hat{R}-2\Lambda\right)-2\alpha\Lambda\hat{\phi}^2-\frac{5}{24\pi G}\alpha^2\Lambda\hat{\phi}^4-\mathcal{O}\left(\frac{\hat{\phi}}{M_r}\right)^6+\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}\right)$$

- Could we in principle have a Higgs effect? Let us assume for a second that the scalar field is gauged.
- If $\Lambda > 0$ (de Sitter) and $\alpha > 0$: no Higgs effect
- If $\Lambda < 0$ (anti-de Sitter) and $\alpha > 0$ or $\alpha < 0$: no Higgs effect
- If $\Lambda > 0$ (de Sitter) and $\alpha < 0$: Higgs effect possible
- Note that we could have introduced a self-interaction term: $\lambda\phi^4$
- In that case Higgs mechanism is possible both in anti-de Sitter and de Sitter cases.

- So far we had to rely on fine-tuning to obtain a small scalar mass. However local conformal symmetry can be imposed in the scalar sector:

$$\int d^4x \frac{1}{2}\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6}R\phi^2\right)$$

- Self-interaction term is also possible $\sqrt{-g}\lambda_\phi\phi^4$ however it does not introduce interesting effects.
- We are assuming that conformal invariance is broken in the gravity sector, this will induce a conformal symmetry breaking in the scalar sector. This is rather exotic physics. Let us thus have a model independent approach and assume only that we are living in an expanding universe.
- The expansion of our scalar field in a Robertson-Walker universe is given by:

$$\ddot{\phi} + 3H\dot{\phi} + (1-q)H^2\phi = 0$$

- with the deceleration parameter given by:

$$q(z) = \frac{3}{2}\frac{\sum_i \Omega_i^0(1+\omega_i)(1+z)^{3(1+\omega_i)}}{\sum_i \Omega_i^0(1+z)^{3(1+\omega_i)}} - 1$$

- We thus obtain:

$$m(z) = \sqrt{(1 - q(z))}H(z)$$

- using the input $\Omega_m^0 = 0.3$ and $\Omega_\Lambda = 0.7$ i.e. $q(0) = -0.55$

- we find:

$$m(0) = 1.9 \times 10^{-33} \text{ eV}$$

- In other words: because the cosmological constant is of the same order of magnitude as today’s Hubble time, a scalar field coupled in a conformal manner to gravity would have a mass term of the order of the Hubble time and thus will be active in today’s universe.

How does the scalar field couple to SM?

- Coupling to Higgs field is dangerous!

$$h^\dagger h \phi^2$$

- It thus has to couple only gravitationally to the SM which is fine since it's a gauge singlet.

- Local conformal invariance is required to maintain a light scalar field. What about conformal invariance in other sectors? Much progress on conformal invariance in the Higgs sector: Coleman-Weinberg does not work in the minimal SM but does work if a singlet is added (e.g. Meissner and Nicolai).

- Much progress has been done as well in conformal gravity (e.g. Mannheim):

$$-\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$$

- This action leads to a fourth order PDG. However ghost is not an issue if you look at the quantum theory from the PT symmetric point of view (Bender and Mannheim): spectrum is real.

- Do we have any hint of how scales are introduced in the SM? Maybe: typical thought experiments lead to two bounds:
  - QM:

$$\Delta x \equiv max\left[\Delta x(0), \Delta x(t)\right] \geq \sqrt{\frac{t}{2M}}$$

  - Gravitational bound:

$$t > R > M$$

- Could nature be described by a theory which is scale invariant at tree level?

Conclusions

- We have considered a scalar field coupled in a non minimal way to the Ricci scalar.
- This mechanism naturally leads to a very light cosmological scalar field which is active today and could lead for example to a time variation of the Newton constant.
- The reason is that the cosmological constant is of the same order of magnitude as today's Hubble time: we live at an interesting time.
- There has been some interesting progress in conformal gravity (see recent papers by Mannheim, Bender and Mannheim): developments in PT-symmetric quantum mechanics open the door to a viable alternative to Einstein's gravity. If this mechanism is correct, one typically ends up with scalar fields couple in a non-minimal way to gravity.
- Thank you for your attention!

Backup

A minimal length from QM and GR

Claim: GR and QM imply that no operational procedure exists which can measure a distance less than the Planck length.

Assumptions:

- Hoop Conjecture (GR): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.
- Quantum Mechanics: uncertainty relation.

Minimal Ball of uncertainty:

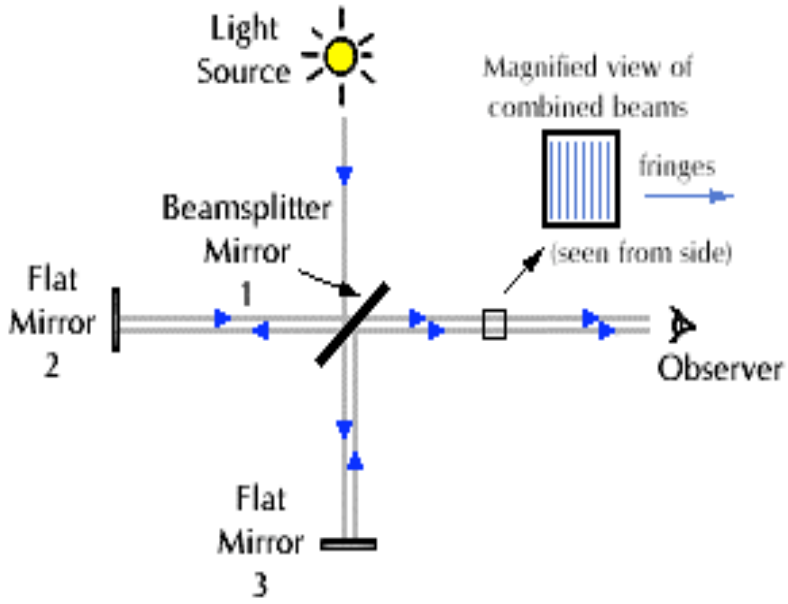
Consider a particle of Energy E which is not already a Black hole. Its size r must satisfy:

$$r \gtrsim \max\,[1/E\,,\,E]$$

where 1/E is the Compton wavelength and E comes from the Hoop Conjecture. We find:

$$r \sim l_P$$

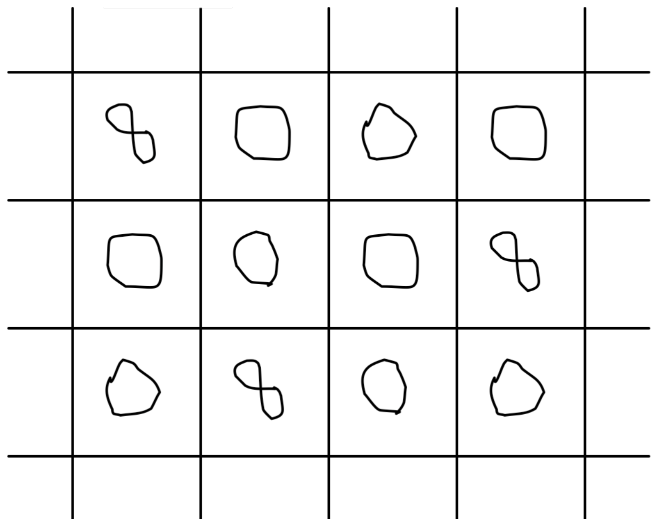
Could an interferometer do better?



Michelson Interferometer

Our concrete model:

We assume that the position operator has discrete eigenvalues separated by a distance l_P or smaller.



- At least one of the uncertainties $\Delta x(0)$ or $\Delta x(t)$ must be larger than:

$$\sqrt{t/2M}$$

- A measurement of the discreteness of $x(0)$ requires two position measurements, so it is limited by the greater of

$\Delta x(0)$ or $\Delta x(t)$:

$$\Delta x \equiv max\left[\Delta x(0), \Delta x(t)\right] \geq \sqrt{\frac{t}{2M}}$$

- This is the bound we obtain from Quantum Mechanics.

- To avoid gravitational collapse, the size R of our measuring device must also grow such that R > M.
- However, by causality R cannot exceed t.
- GR and causality imply: $t > R > M$
- Combined with the QM bound, they require Δx > 1 in Planck units or

$$\Delta x > l_P$$

- This derivation was not specific to an interferometer - the result is device independent: no device subject to quantum mechanics, gravity and causality can exclude the quantization of position on distances less than the Planck length.