Supersymmetry breaking by constant boundary superpotentials in warped space

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Based on collaboration with Nobuhito Maru and Norisuke Sakai
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SUSY and Extra dimensions

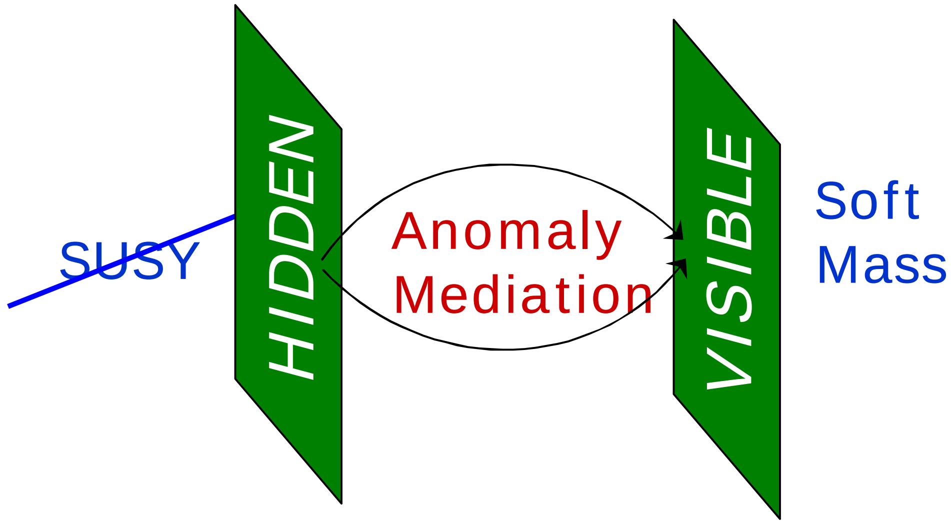
Solving the gauge hierarchy problem

Arkani-Hamed Dimopoulos Dvali 98

Randall Sundrum 98

Luty Sundrum 99

Mediation of SUSY breaking



Scalar masses depend only on the gauge quantum # of the scalars

Unlike 4D SUGRA, flavor symmetry is not necessary

Other contributions except for anomaly mediaiton are from non local and can be small

No SUSY FCNC problem

~~SUSY~~ in higher dimensinal space

Scherk-Schwarz mechanism 79

$$\begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix} (y + 2\pi R) = e^{2\pi i q \sigma_2} \begin{pmatrix} \phi \\ \phi^{c\dagger} \end{pmatrix} (y)$$

Distinct twists for bosons and fermions $\longrightarrow$ mass splitting

Scherk-Schwarz ~~SUSY~~ → **In flat space** ← ~~SUSY~~ by constant superpotentials

Marti Pomarol 01 Bagger Feruglio Zwirner 02

The same mass spectrum

In warped space ? Bagger Belyaev 03 Hall Nomura Okui Oliver 04 Abe Sakamura 06

- Whether SUSY is broken by the Scherk-Schwarz twist or not may depend on the way of gauging
  - Gauging in supergravity

gauge covariant derivative for gravitino

SUSY theory

$\Big\}$ bulk cosmological constant

- Even for ~~SUSY~~ by constant superpotentials a number of issues remain to be examined

~~SUSY~~ by constant superpotentials in warped space

- Soft mass?
- No FCNC problem?
- Kaluza-Klein mass spectrum?
- Radius stabilization?
- Radion mass?

Scherk-Schwarz ~~SUSY~~

Effective potential

Bagger Belyaev 03
Bagger Redi 04

The radius is not stabilized in the Randall Sundrum model if only the gravity multiplet is used

- Hypermultiplet background
- Hypermultiplet bulk mass parameter

Plan

Model

Background solution

Potential

Anomaly-mediated soft mass

Radion mass

Gravitino mass

Hyperscalar Kaluza-Klein mass spectrum

Summary

Future work

Model

- **metric** $ds^2 = e^{-2R\sigma}\eta_{\mu\nu}dx^\mu dx^\nu + R^2 dy^2, \quad \sigma(y) \equiv k|y|$ radius R

  $\eta_{\mu\nu} = \mathrm{diag.}(-1,+1,+1,+1),$ AdS5 curvature k

  5th coordinate $y (0 \le y \le \pi)$ of the orbifold S_1/Z_2

- **Lagrangian** (in terms of superfields)

Marti Pomarol 01

$$\mathcal{L}_5 = \int d^4\theta \frac{1}{2}\varphi^\dagger\varphi(T+T^\dagger)e^{-(T+T^\dagger)\sigma}(\Phi^\dagger\Phi + \Phi^c\Phi^{c\dagger} - 6M_5^3)$$
$$+ \int d^2\theta \left[\varphi^3 e^{-3T\sigma}\left\{\Phi^c\left[\partial_y - \left(\frac{3}{2}-c\right)T\sigma'\right]\Phi + W_b\right\} + \text{h.c.}\right]$$

Φ, Φ^c *hypermultiplet* with *bulk mass parameter* c $\quad \sigma' \equiv \frac{d\sigma}{dy}$

The Z_2 parity is assigned to be even (odd) for $\Phi(\Phi^c)$.

$W_b \equiv 2M_5^3(w_0\delta(y) + w_\pi\delta(y-\pi))$ constant superpotentials

w_0, w_π dimensionless constants

$\varphi = 1 + \theta^2 F_\varphi$ compensator chiral supermultiplet (of supergravity)

$T = R + \theta^2 F_T$ radion chiral supermultiplet

Equations of motion for auxiliary fields

$$F = -\frac{e^{-R\sigma}}{R}\left[-\partial_y\phi^{c\dagger} + \left(\frac{3}{2}+c\right)R\sigma'\phi^{c\dagger} + \frac{\phi}{2M_5^3}W_b \right.$$
$$\left. + \frac{1}{6M_5^3}\phi^\dagger\phi\partial_y\phi^{c\dagger} + \frac{1}{3M_5^3}\phi^{c\dagger}\phi\partial_y\phi^\dagger - \frac{1}{6M_5^3}\phi^\dagger\phi\phi^{c\dagger}\left(\frac{9}{2}-c\right)R\sigma'\right]$$

$$F^c = -\frac{e^{-R\sigma}}{R}\left[\partial_y\phi^\dagger - \left(\frac{3}{2}-c\right)R\sigma'\phi^\dagger + \frac{\phi^c}{2M_5^3}W_b \right.$$
$$\left. + \frac{1}{6M_5^3}\phi^c\phi^\dagger\partial_y\phi^{c\dagger} + \frac{1}{3M_5^3}\phi^{c\dagger}\phi^c\partial_y\phi^\dagger - \frac{1}{6M_5^3}\phi^c\phi^\dagger\phi^{c\dagger}\left(\frac{9}{2}-c\right)R\sigma'\right]$$

$$F_\varphi = -\frac{e^{-R\sigma}}{R}\left[-\frac{1}{6M_5^3}\phi^\dagger\partial_y\phi^{c\dagger} - \frac{1}{3M_5^3}\phi^{c\dagger}\partial_y\phi^\dagger + \frac{1}{6M_5^3}\phi^\dagger\phi^{c\dagger}\left(\frac{9}{2}-c\right)R\sigma' - \frac{1}{2M_5^3}W_b \right.$$
$$\left. -\frac{3(1-2R\sigma)}{r}\phi^{c\dagger}\partial_y\phi^\dagger - \frac{3(1-2R\sigma)}{r}W_b + \frac{1-2R\sigma}{r}\phi^{c\dagger}\phi^\dagger\left(\frac{3}{2}-c\right)R\sigma'\right]$$

$$F_T = -\frac{e^{-R\sigma}}{r}\left[6\phi^{c\dagger}\partial_y\phi^\dagger - 2\phi^{c\dagger}\phi^\dagger\left(\frac{3}{2}-c\right)R\sigma' + 6W_b\right]$$

$$r \equiv \phi^\dagger\phi + \phi^{c\dagger}\phi^c - 6M_5^3$$

where the partial integration has been performed in the equation for *F*

Background solution

- $w_0 = 0$

$w_\pi = 0$
except for calculation of KK spectrum

SUSY solution $F = F^c = F_\varphi = F_T = 0$

$$\phi(y) = N_2 \exp\left[\left(\frac{3}{2} - c\right) R\sigma\right]$$
$$\phi^c(y) = 0$$

N_2 a complex parameter

Perturbation

- $w_0 \ll 1$

$$\phi(y) = \phi_s(y) + \chi(y)$$
$$\phi^c(y) = \hat{\epsilon}(y)\chi^c(y)$$
$$\hat{\epsilon}(y) \equiv \begin{cases} +1, & 0 < y < \pi \\ -1, & -\pi < y < 0 \end{cases}$$

We work out solutions of the eom for $\chi(y)$ and $\chi^c(y)$ as deviations from the SUSY solution

Potential

Substitution of the background solutions and y -integration

$$
\begin{aligned}
V \;=\; & \frac{3M_5^3 k w_0^2}{2}\left\{\frac{-2(1-2c)}{(1-2c)(e^{2Rk\pi}-1)\hat{N}+2(e^{(2c-1)Rk\pi}-1)}\hat{N}^{4-2c-\frac{1}{3-2c}}\right. \\
& \left. +\frac{\hat{N}}{1-\hat{N}}\left(-4c^2+12c-6+\frac{3-2c}{3(1-\hat{N})}\right)\right\} \qquad \hat{N}\equiv\frac{|N_2|^2}{6M_5^3}
\end{aligned}
$$

Stationary conditions for the radius R and the modulus N_2

$$\frac{\partial V}{\partial R}=0 \quad \text{and} \quad \frac{\partial V}{\partial \hat{N}}=0$$

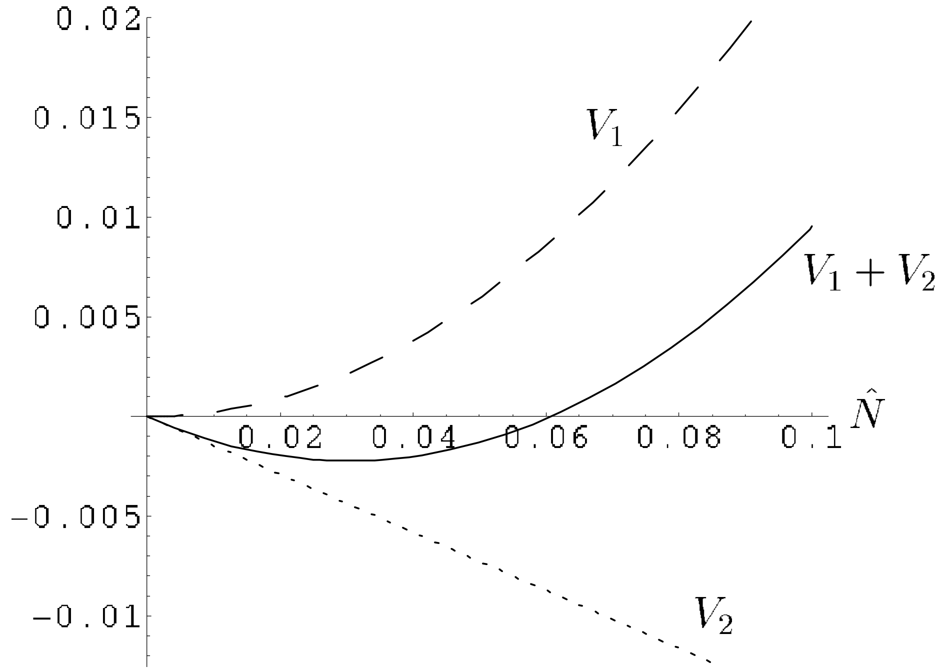
We find that there is a unique **nontrivial minimum** provided $c < c_{cr}$ with

$$c_{cr} \equiv \frac{17-\sqrt{109}}{12} \approx 0.546$$

- At c_{cr} the minimum occurs at infinite radius

$$\hat{N}(c_{cr}) = 0, \quad R(c_{cr}) = \infty$$

- To examine the stabilization more closely, We will parametrize $c = c_{cr} - \triangle c$ with a small $\triangle c$.



The radius is stabilized at

At the leading order of Δc and $\widehat{N}$

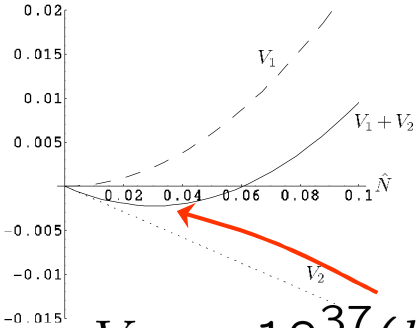
$$V \approx \frac{3M_5^3 k w_0^2}{2}(V_1 + V_2),$$

$$V_1 \equiv \frac{2(2c_{\rm cr} - 1)}{3 - 2c_{\rm cr}} \widehat{N}^{\frac{4c_{\rm cr}^2 - 12c_{\rm cr} + 10}{3 - 2c_{\rm cr}}},$$

$$V_2 \equiv -\widehat{N}\left(-8c_{\rm cr} + \frac{34}{3}\right)\Delta c$$

$$R \approx \frac{1}{10k}\left(\ln\frac{1}{\Delta c} - 3.4\right)$$

$Rk > 1$ for $\Delta c < 10^{-6}$



$$V \approx -10^{37}(kw_0)^2(\Delta c)^{1.2}$$

Canceling the cosmolgoical constant with
F–term or D-term localized at $y=0$
so as not to affect radius stabilization

F-term

Soft mass induced by F

Brane-to-brane mediation
by gravity Gregoire Rattazzi Scrucca Strumia Trincherini 04

Brane-to-brane mediation
by hypermultiplet
Maru Okada 03

They are small

D-term

Soft mass induced by
Fayet-Iliopoulos term

If matter in the visible sector
is neutral under this U(1),
additional contributions to
soft mass are not generated

The contributions of these secors to gravitino mass are also small

Anomaly-mediated soft mass

Luty 02

$$\tilde{m}_{\text{AMSB}} = \frac{g^2}{16\pi^2}(F_\varphi - F_T\sigma)|_{y=\pi}$$

$$\sim 10^{-4} \times g^2 k w_0$$

$$\tilde{m}_{\text{AMSB}} = \frac{g^2}{16\pi^2}\left\langle \frac{F_\omega}{\omega} \right\rangle$$

$$\omega = \varphi e^{-T\sigma}$$

For $g^2 k w_0 \sim 10^6\text{GeV}$

$$\tilde{m}_{\text{AMSB}} \sim 100\text{GeV}$$ dominant No FCNC

Radion and moduli masses

Quantum fluctuation fields $R + \tilde{R}, \quad N_2 + \tilde{N}_2, \quad \tilde{N}_2 = \tilde{N}_{2R} + i\tilde{N}_{2I}$

The lighter mode is almost exclusively made of the radion

$$m^2_{\text{light}} \approx k^2 w_0^2 0.38(3.4 + \ln \Delta c)^2 (\Delta c)^{1.7}$$

$$m^2_{\text{heavy}} \approx k^2 w_0^2 0.47(\Delta c)^{0.70}$$

For $w_0 \sim (10^7\text{GeV}/k)$ and $\Delta c \sim 10^{-6}$

$$m_{\text{light}} \sim 1\text{TeV}, \qquad m_{\text{heavy}} \sim 100\text{TeV}$$

radion complex moduli

Gravitino mass

Lagrangian (bulk + brane)

Gherghetta Pomarol 00

$$\mathcal{L}_{\text{bulk}} = iM_5\sqrt{-g}\left[\bar{\Psi}_M\gamma^{MNP}D_N\Psi_P - \frac{3}{2}\sigma'\bar{\Psi}_M\gamma^{MN}(\sigma_3)\Psi_N\right]$$

$$\mathcal{L}_{\text{boundary}} = -i\frac{W_b}{2M_5^2}\left[\psi_\mu^1\sigma^{\mu\nu}\psi_\nu^1 - \bar{\psi}_\mu^1\sigma^{\mu\nu}\bar{\psi}_\nu^1\right]$$

$$\frac{m_n}{k} \ll 1,\quad \frac{m_n}{k}e^{Rk\pi} \ll 1 \qquad m_{\text{lightest}} \approx 2w_0 k$$

$$\sim 10^7\,\text{GeV for } w_0 \sim (10^7\,\text{GeV}/k)$$

$$\frac{m_n}{k} \gg 1,\quad \frac{m_n}{k}e^{Rk\pi} \gg 1 \qquad m_n \approx \left(n - \frac{6w_0}{2\pi}\right)\pi k e^{-Rk\pi}$$

The gravitino and hyperscalar are affected by the constant superpotential classically

Hyperscalar mass

$$\frac{m_n}{k} \gg 1,\quad \frac{m_n}{k}e^{Rk\pi} \gg 1 \qquad m_n \approx \frac{k}{e^{Rk\pi}-1}\left(n\pi \pm \frac{6w_0}{2\sqrt{3}}\right)$$

In general, scalars in the visible sector can receive mass corrections by mediation of bulk fields or all the KK modes

Antoniadis Quiros 97

The mass induced in our model is small

Hyperscalar Kaluza-Klein mass spectrum

The equations for ϕ and ϕ^c couple only through W_b

$w_0 \sim w_\pi \sim \mathcal{O}(1)$ $\quad \frac{m_n}{k} \ll 1, \quad \frac{m_n}{k} e^{Rk\pi} \gg 1$

Dominantly ϕ mode $\quad \phi^c = 0$ if $w_0 = w_\pi = 0$ $\quad \alpha \equiv |c + \frac{1}{2}|$

- $|c| \geq \frac{1}{2}$ $$m_n \approx k e^{-Rk\pi} \left[\left(n + \frac{2\alpha + 1}{4} \right) \pi \pm \frac{|w_\pi|}{2\sqrt{3}} \right]$$ $+$ for $\frac{1}{2} \leq c \leq 1$; $-$ for $c \leq -\frac{1}{2}$ or $c > 1$
- $|c| < \frac{1}{2}$ $$m_n \approx k e^{-Rk\pi} \left[\left(n + \frac{2\alpha + 1}{4} \right) \pi + \frac{w_\pi^2 + 12}{24 \tan c\pi} \left(1 - \sqrt{1 + w_\pi^2 \frac{\tan^2 c\pi}{3}} \right) \right]$$

Dominantly ϕ^c mode $\quad \phi = 0$ if $w_0 = w_\pi = 0$ $\quad \beta \equiv |c - \frac{1}{2}|$

- $|c| \geq \frac{1}{2}$ $$m_n \approx k e^{-Rk\pi} \left[\left(n - \frac{1}{2} + \frac{2\beta + 1}{4} \right) \pi \mp \frac{|w_\pi|}{2\sqrt{3}} \right]$$ $-$ for $\frac{1}{2} \leq c \leq 1$; $+$ for $c \leq -\frac{1}{2}$ or $c > 1$
- $|c| < \frac{1}{2}$ $$m_n \approx k e^{-Rk\pi} \left[\left(n - \frac{1}{2} + \frac{2\beta + 1}{4} \right) \pi - w_\pi^2 \frac{\tan c\pi}{12} \right]$$

In order to take appropriate flat limit *k* ->0, *c* should be large

Our result for large |*c*| shows a linear dependence on w_π, which is quite simitar to the flat Scherk-Schwarz ~~SUSY~~ case

Summary

In numerically evaluating various masses, we have chosen

$$w_0 \sim (10^7\text{GeV}/k),\ M_5 \sim (M_4^2 k)^{1/3}$$

$$c = c_{\text{cr}} - \Delta c$$

$$c_{\text{cr}} \approx 0.546,\ \Delta c \sim 10^{-6}$$

Radius stabilized k^{-1}

__Masses__

Radion 1TeV

Moduli 100TeV

Such a small radion mass appears as a common feature of warped space model and its value is in experimentally allowed region (PDG).

Soft 100GeV

It is generated by anomaly mediation.
There is no FCNC problem.

Gravitino 10^7GeV

Such a large gravitino mass is similar to that of the anomaly mediation scenario given before.

Hyperscalar k

It is much heavier than other fields. The hyperscalar primarily acts as a part of the background configuration.

w -dependent behavior of hyperscalar Kaluza-Klein mass depends on bulk mass parameter *c*

Future work

- Solving negative slepton mass-squared problem

- The radius stabilization has been studied also in the AdS_4.

  For arbitrary values of c ?

- Radiative corrections for potential etc also remain to be examined

- Find different behavior of hypermultiplet mass spectrum for different values of c in gauged supergravity