

A New Constraint System Applied to Modifying Gravity

hep-th/0609150
astro-ph/0702002
+ ongoing work

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Fact 1

There are not many covariant field theoretic

constraint mechanisms.

Familiar example: Auxiliary fields (Lagrange multipliers)

WZ model

$$N e^{iS_{\text{eff}}} = \int D F D F^\dagger e^{iS_1 + i \int d^4x \{ |F|^2 + \partial_\phi W F + F^\dagger (\partial_\phi W)^\dagger \}}$$

$$= N \int D F e^{iS_1} \delta(F + (\partial_\phi W)^\dagger) e^{i \int d^4x \partial_\phi W F}$$

$$= N e^{iS_1} e^{i \int d^4x |\partial_\phi W|^2}$$

Same as using EOM: $F = (\partial_\phi W)^\dagger$

Note: $\pi^0 = \frac{\partial \mathcal{L}}{\partial \dot{F}} = 0$

No kinetic term.

Fact 2

Gauge theories usually have a constraint sector.

e.g. QED: Coulomb's law

Einstein-Hilbert gravity: Friedmann eq.

Scalar metric perturbations

Hence

new covariant constraint mechanism

||

possibly novel constraint sector of gauge theories

This talk

- 1) Present a novel constraint mechanism: cuscuton
- 2) Classical properties in flat space
- 3) Novel gravitational theory + phenomenology
- 4) Some preliminary results on quantization

what is this novel constraint mechanism?

Cuscuton

An action of the form

$$S[\varphi] = \int d^4x \sqrt{|g|} \left[\frac{1}{2} F((\partial\varphi)^2, \varphi) - V(\varphi) \right]$$

where in a coordinate in which $d\tilde{t}$ is normal to a constant φ patch *locally*, the kinetic term becomes a total derivative.

By an appropriate field redefinition, for a single real scalar field, we have a unique form

$$S = \int d^4x \sqrt{-g} \left[m^2 \sqrt{|g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi|} - V(\varphi) \right]$$

for $(\partial\varphi)^2 > 0$ (i.e. absolute value is arbitrary). [signature (1, -1, -1, -1)]

Intuition:

$$\boxed{\int d^4x \sqrt{|(\partial\varphi)^2|}} \xrightarrow[\text{change}]{\text{coord.}} \int d^4x \sqrt{\dot{\varphi}^2} = \int d^4x \dot{\varphi}$$

total derivative!

General Properties

EOM

$$\frac{\mu^2}{\sqrt{-g}} \partial_\alpha \left[\sqrt{-g} \frac{\partial^\alpha \varphi}{\sqrt{(\partial\varphi)^2}} \right] + V'(\varphi) = 0$$

1) Classical EOM is not well defined for $(\partial\varphi)^2 = 0$
(quantum corrections can smooth this out)

a) there are solutions that never access $(\partial\varphi)^2 = 0$.

b) $\int D\varphi e^{iS[\varphi]}$ can be defined as usual since $S[\varphi]$ is not singular even when $(\partial\varphi)^2 = 0$.

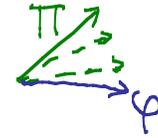
2) Canonical conjugate momentum is non-trivial

$$\Pi = \mu^2 \frac{\partial^0 \varphi}{\sqrt{(\partial\varphi)^2}}$$

in contrast w/ Lagrange multipliers.

claim: still a pure constraint system

3) By construction, the symplectic structure degenerates in the homog. limit $D\varphi \wedge D\pi \rightarrow 0$



4) The speed of sound is ∞ .

For this reason, this type of theories were rejected by Aharonov, Komar, & Susskind (1969)

As we will see, harmless constraint system.

5) Highest order derivative operator acting on the field is two (same order as KG)

6) Why the funny name cuscuton?

φ = auxiliary field = "life" of φ depends on other fields

cuscuton = Latin name of parasitic plant *cuscuta*

Classically nonperturbative proof for lack of dynamics

Choose locally a coordinate in which $d\tilde{t}$ is normal to a local patch of constant φ .

$$\therefore S = \int d\varphi \Sigma(\varphi) - \frac{1}{\mu^2} \int d\tilde{t} \Sigma(\varphi(\tilde{t})) V(\varphi(\tilde{t}))$$

↑ spacelike 3-surface volume of constant φ

$$= \int d\tilde{t} \left(\frac{d\varphi}{d\tilde{t}} \right) \Sigma(\varphi) - \frac{1}{\mu^2} \int d\tilde{t} \Sigma(\varphi(\tilde{t})) V(\varphi(\tilde{t}))$$

contrast w/
Ordinary scalar

$$\int d\tilde{t} \Sigma(\varphi(\tilde{t})) \left(\frac{d}{d\tilde{t}} \varphi(\tilde{t}) \right)^2$$

No second
order in time
derivative

Manifestly non-dynamical
except at locations
where the gauge fixing
is singular

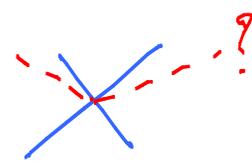
Perturbative Proof

EOM:
(Minkowski)

$$\partial_\nu \left[\frac{\partial^\nu \varphi}{\sqrt{|\partial_\alpha \varphi \partial^\alpha \varphi|}} \right] + \frac{V'(\varphi)}{m^2} = 0$$

$$\left(\eta^{\mu\nu} - \frac{\partial^\mu \varphi \partial^\nu \varphi}{|\partial_\alpha \varphi \partial^\alpha \varphi|} \right) \partial_\mu \partial_\nu \varphi$$

Linearize: $\varphi = \varphi_0 + \delta\varphi$

perturbative metric: $\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{\partial_\mu \varphi_0 \partial_\nu \varphi_0}{|\partial_\alpha \varphi_0 \partial^\alpha \varphi_0|}$ 

$$= \frac{1}{\dot{\varphi}_0^2 - |\vec{\nabla} \varphi_0|^2} \begin{pmatrix} -|\vec{\nabla} \varphi_0|^2 & -\partial_0 \varphi_0 \partial_i \varphi_0 \\ -\partial_0 \varphi_0 \partial_i \varphi_0 & -[\dot{\varphi}_0^2 - |\vec{\nabla} \varphi_0|^2] \delta_{ij} - \partial_i \varphi_0 \partial_j \varphi_0 \end{pmatrix}$$

$$\det \tilde{g}_{\mu\nu} = 0$$

Theory not hyperbolic

Lower dimensional theory

Exact Solutions in Minkowski space

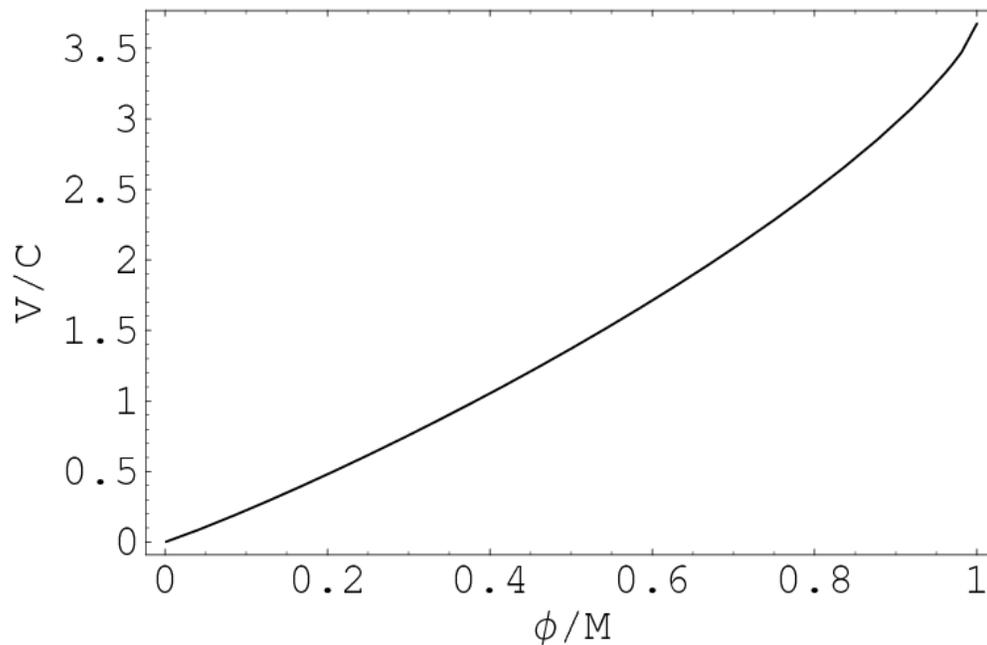
These are solutions to non-linear PDE

$SO(3,1)$

$$\varphi(x) = M e^{-\lambda(t^2 - \vec{x}^2)^2}$$

$$V(\varphi) = 3\mu^2 M \lambda^{1/4} \Gamma\left(\frac{3}{4}, -\ln \frac{\varphi}{M}\right)$$

$$C \equiv 3\mu^2 M \lambda^{1/4}$$



"Plane waves"

$$V(\varphi) = \text{const}$$

$$\varphi(x) = F(k \cdot x)$$

$k_\mu = \text{constant}$
(no on shell cond)

↑
any smooth function

Why do these solutions fail to correspond to dynamical degrees of freedom?

If dynamical, cauchy data can be imposed.

cauchy $\sim \varphi|_\Sigma$ and $\partial_\mu \varphi|_\Sigma$ chosen

For these solutions, $\varphi|_\Sigma$ specification leaves

less than one real functional degree of

freedom for $\partial_\mu \varphi|_\Sigma$ (e.g. in $D=1+1$, couple of numbers instead of 1 function)

Coupling to a Dynamical Scalar Field

$$\mathcal{L}(\varphi, \psi) = \frac{1}{2} \partial_\alpha \varphi \partial^\alpha \varphi + \mu^2 \sqrt{|\partial_\alpha \varphi \partial^\alpha \varphi|} - V(\varphi, \psi)$$

$$\varphi = \varphi_0(t) + \delta\varphi(t, \vec{x}) \quad \psi = \psi_0(t) + \delta\psi(t, \vec{x})$$

Solve $\delta\psi$ equation of motion +
insert into $\delta\varphi$ equation of motion:

Resulting dispersion for $\delta\varphi$:

$$\omega^2 = \frac{k^4 + (\partial_\varphi^2 V + \partial_\psi^2 V) k^2 + \partial_\varphi^2 V \partial_\psi^2 V - (\partial_\varphi \partial_\psi V)^2}{k^2 + \partial_\varphi^2 V}$$

$$\text{Stability: } \partial_\varphi^2 V + \partial_\psi^2 V > 0 \quad \partial_\varphi^2 V \partial_\psi^2 V - (\partial_\varphi \partial_\psi V)^2 > 0$$

$$\text{causality: } v_g = \frac{d\omega}{dk} = 1 - \frac{\partial_\psi^2 V}{2k^2} + \mathcal{O}\left(\frac{V_{,\varphi\varphi}}{k^4}\right) < 1$$

No
Problems
w/
causality

Coupling to Gravity

$$S = \int d^4x \sqrt{|g|} \left[\mu^2 \sqrt{|g^{mn}} \partial_m \varphi \partial_n \varphi - \underbrace{V(\varphi)}_{\text{contains all the model dependence}} \right] - \frac{M_p^2}{2} \int d^4x \sqrt{|g|} R$$

+ $\int d^4x \sqrt{|g|} \mathcal{L}_M$ → For this talk, CDM only

Standard Einstein gravity

$g_{\mu\nu}$: 10 real functions

- 4 gauge

- 4 constraint EOM (analog: Coulomb's law)

2 functional d.o.f.

Cuscuton = scalar constraint

⇒ modifies constraint sector of Einstein equations e.g. Newton's Law

1 Homogeneous system: $ds^2 = dt^2 - a^2(t) |d\vec{x}|^2$

intuition: How could a total derivative in the homogeneous limit lead to a constraint?

$$\int dt a^3 \dot{\varphi} = - \int dt 3a^2 \dot{a} \varphi$$

⇒ Modified Friedmann eq.

$$H^2 = \frac{1}{3M_p^2} \left\{ \rho_m + V \left[V'^{-1}(3\mu^2 H) \right] \right\}$$

only degree of freedom

H^2 is no longer linearly dependent on ρ_m .

This not the same as solving for an ordinary scalar field and putting that into ρ_{tot} .

Reason: There is no boundary condition for $\varphi(t)$.

Linear Perturbations

$$ds^2 = (1 + 2\Phi) dt^2 - a^2(t) (1 - 2\Phi) |d\vec{x}|^2$$

$$\varphi \rightarrow \varphi(t) + \delta\varphi(t, \vec{x}) \quad \rho_m \rightarrow \rho_m(t) + \delta\rho_m(t, \vec{x})$$

All important gravitational potential evolution:

$$(1 + C_2) \ddot{\Phi} + (4H + C_1 + C_2 H + C_3) \dot{\Phi} + (3H^2 + \dot{H} - \frac{3}{2} \Omega_m H^2 + C_1 H + C_2 \dot{H} + C_3 H) \Phi = 0$$

$$C_1 \equiv \frac{3(\ddot{H} + 3H\dot{H})}{(\frac{k}{a})^2 - 3\dot{H}}$$

$$C_2 \equiv \frac{3(2\dot{H} + 3H^2\Omega_m)}{2((\frac{k}{a})^2 - 3\dot{H})}$$

$$C_3 \equiv \frac{3[2H(\frac{k}{a})^2 + 3\ddot{H}](2\dot{H} + 3H^2\Omega_m)}{2((\frac{k}{a})^2 - 3\dot{H})^2}$$

Consequences

- 1) There is a dynamical scale generated: $(\frac{k}{a})^2 = \dot{H}$
- 2) Cuscuton effects = $\{C_i \neq 0\} \Rightarrow \Phi$ is generically not a constant (if $C_i = 0$ and $\Omega_m = 1$, $\Phi = \text{const}$)

Explicit Examples

a) $V(\varphi) = \frac{1}{2} m^2 \varphi^2$

Background solution

"Same" as usual Friedmann (renormalize M_p only

$$M_p^2 \rightarrow M_p^2 - \frac{3\mu^4}{2m^2} \quad \text{for homog mode)}$$

Cuscuton effect on perturbation

$$\Theta_{\ell,k}^{\text{ISW}} = 2 \int_0^{\eta_0} d\eta \frac{\partial \Phi_k}{\partial \eta} e^{-\tau(\eta)} j_\ell[k(\eta_0 - \eta)]$$

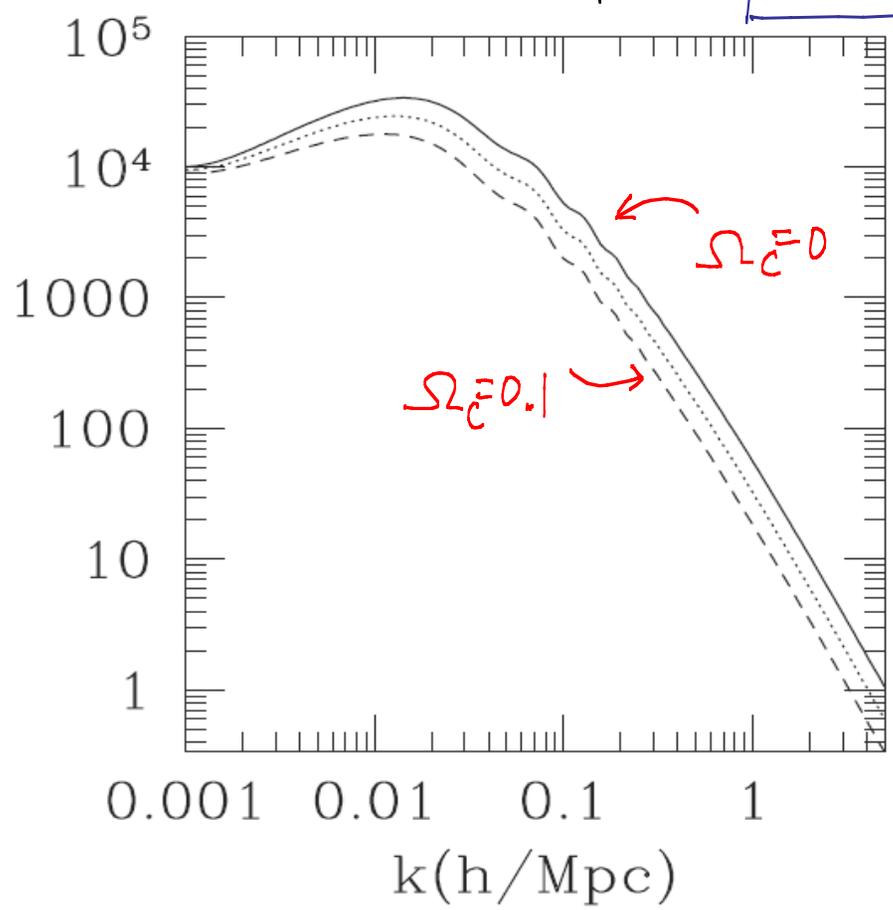
↑ previous slide $\frac{\partial \Phi_k}{\partial \eta} \neq 0$ is generic

depends on $\Omega_c \equiv \frac{3\mu^4}{2M_p^2 m^2}$ ← $\mu^2 \sqrt{|(2\varphi)^2|}$

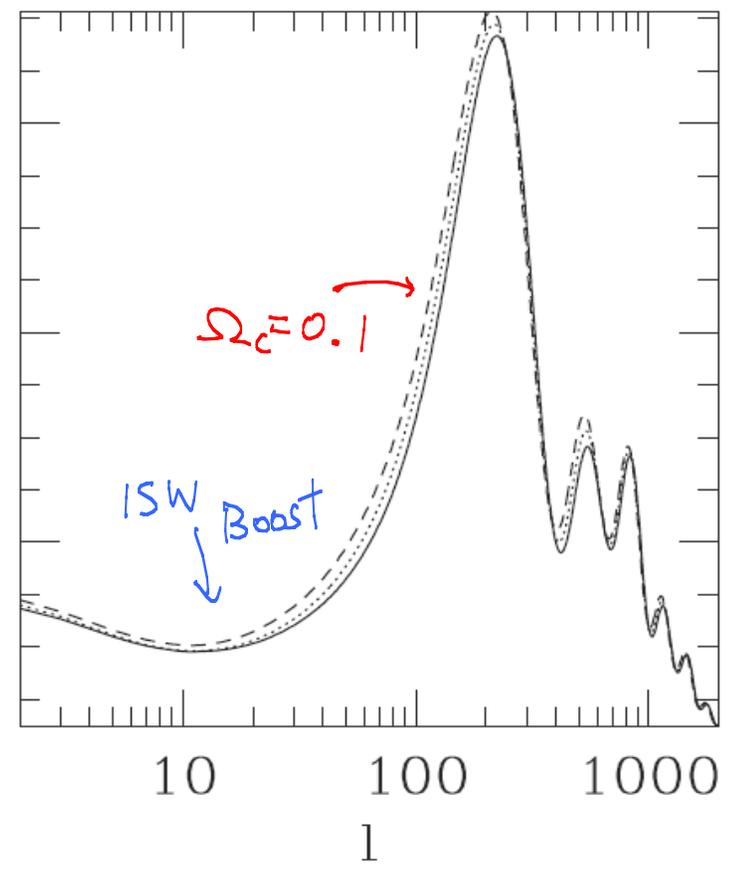
$$\Omega_c \equiv \frac{3 \mu^4}{2 M_p^2 m^2} = \text{const}$$

Matter power

CMB



$l(l+1)C_l(\mu K^2)/(2\pi)$



WMAP alone

$\Omega_c < 0.036$ (95%)

+ SNIa + b acoustic osc. +
SDSS gal survey

$\Omega_c < 0.027$

+ SDSS Ly- α

$\Omega_c < 0.016$

Model 2

$$V(\varphi) = V_0 \exp \left[- \left(\frac{\mu^2 r_c}{M_p^2} \right) \varphi \right]$$

Background solution:

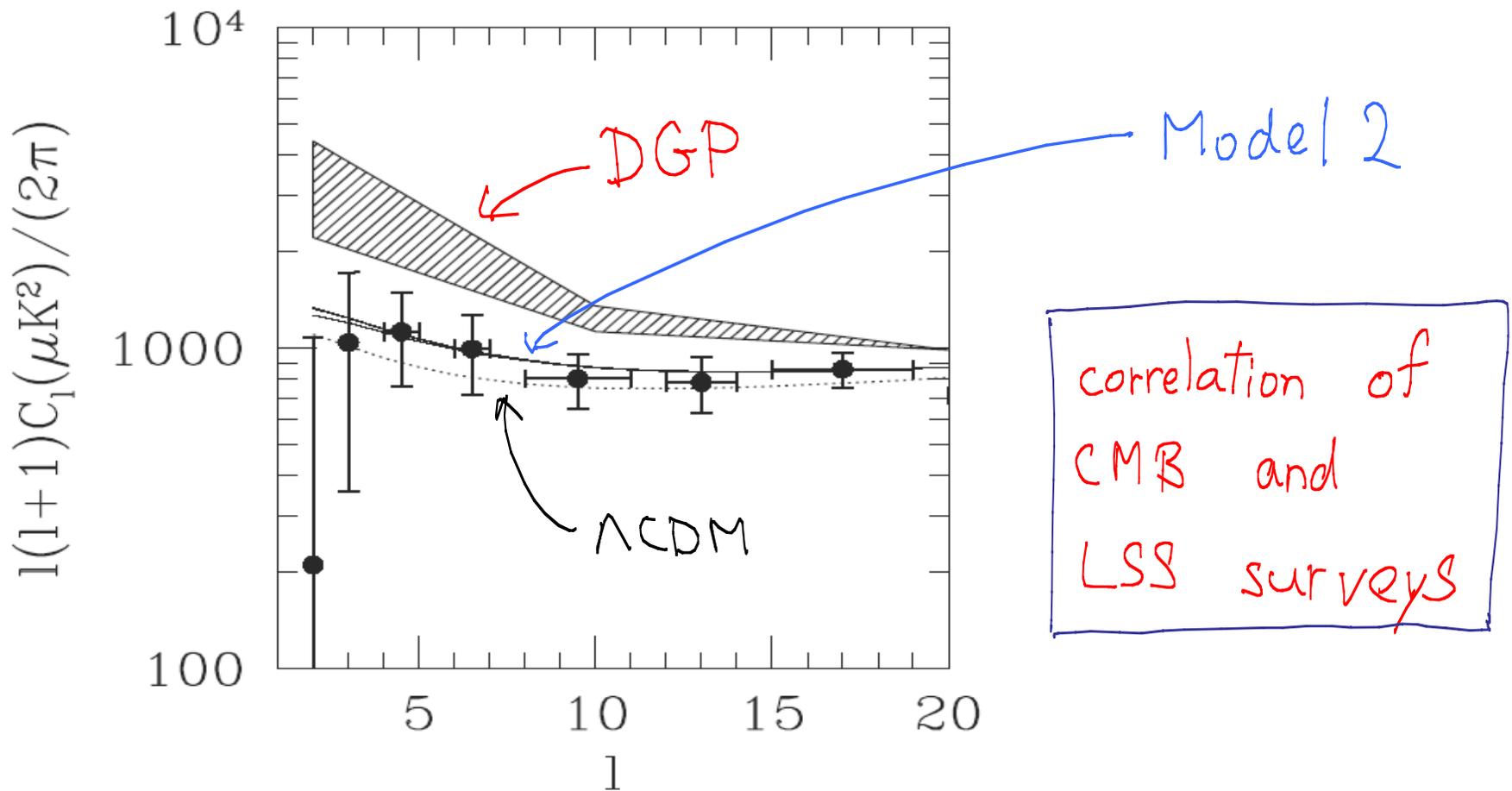
$$H = \frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{\rho_m}{3M_p^2}}$$

identical to DGP model
adapted for acc. cosmo.
hep-th/0010186

\therefore It is not possible to distinguish the two models based on the traditional geometrical tests:

- a) supernovae Ia
- b) distance to last scattering surface
- c) scale of baryonic acoustic oscillations

Largest difference between DGP and Model 2 :
large ($\sim 4\times \Lambda\text{CDM}$) ISW induced by the
anisotropic stress in DGP. (Anisotropic
stress in DGP approximated in astro-ph/0606286.)



Quantization (preliminary)

(work in progress w/ A. Hashimoto & I. Ellwood)

Integrate in two more auxiliary fields $\{V^m, \lambda\}$

cuscuton sector

$$Z = \int DV^m D\Phi D\lambda \exp \left[\frac{i}{\hbar} \int d^d x \left\{ \frac{\lambda}{2} V_m V^m + V^m \partial_m \Phi + \frac{\lambda}{2} \mu^4 + U(\Phi, \chi) \right\} \right]$$

↑
dynamical
field

signature $(-, +, +, \dots)$

- 1) Singularity of classical eq. may be smoothed out by quantum effects
- 2) UV embedding issues are non-trivial
- 3) Supersymmetrizing is straight forward.

Conclusion

1) cuscuton: a new constraint system

of the form $\int d\varphi \int_{\Sigma(\varphi)} \dots = \int d^4x \sqrt{|g|} V(\varphi, \chi)$

intuitive form: $\int d^4x \sqrt{g} (m^2 \sqrt{|\partial\varphi|^2} - V)$

↑ spacelike (pointing to $\Sigma(\varphi)$)
metric (pointing to $\sqrt{|g|}$)
↑ dynamical sector (pointing to χ)

2) When coupled to gravity, constraint sector of gravitational EOM is modified.

(No new d.o.f. unlike scalar-tensor theories of gravity)

3) Cosmologically enters at a dynamical scale of $\frac{dH}{dt}$.

4) ISW + power spectrum observables can probe this.

5) Quantization & susy version seems hopeful.